STUDIES ON OPTIMAL DESIGN OF PID CONTROLLER FOR SYSTEMS WITH TIME DELAY

By

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Dedicated to my family.

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CONTENTS

Synopsis

The Proportional-Integral-Derivative (PID) controller is one of the favorite and widely used controllers in the field of control engineering covering the simple temperature controller to the complicated reactor and accelerator control systems. It is popular due to its simple design and structure, satisfactory control effort and acceptable robustness. Design of the controller is simple because it can be made using a single operational amplifier, resistances and capacitances. The combination of resistances and capacitances gives three tuning parameters proportional (K_p), integral (K_i) and derivative (K_d) in the analog domain. In digital domain the PID controller can be easily implemented with the use of fast processor such as Field programmable gate array (FPGA), Digital signal processor (DSP) etc. The satisfactory performance represents the desired closed loop time response such as rise time, settling time, overshoot and steady-state error whereas robustness represents the stability of the closed loop system under the perturbation in parameters.

The design of PID controller for a given system involves tuning of three parameters K_p, K_i and K_d. The tuning of PID controller as first proposed by Ziegler-Nichols and is still widely used in many process industries. The Ziegler-Nichols PID tuning method is based on an empirical formula that covers most of the plants in the process industries. In the process industries, there are different systems such as stable first order system with time delay, unstable first order system with time delay, stable and unstable second order system with time delay, integrating process etc. In order to achieve better output response, the controller of different type of systems needs to be tuned differently. Ziegler-Nichols tuning method being based on empirical formula sometimes fails to provide proper tuning of controller parameters. The requirement of high performance from the controller and due to availability of fast computers in the present days, various tuning method based on optimization have been developed. A few popular methods among them are PID tuning using nonlinear optimization, particle swarm algorithm, genetic algorithm based optimization etc.

Among the various methods used for tuning of PID controller, the dominant pole placement is a very common technique in the state space design where a pair of conjugate poles is chosen to meet the required closed loop time response. This method is widely used for systems with no time delay. However, for the single input single output systems with time delay, the dominant pole placement leads to a characteristic quasi-polynomial with infinite number of roots in the characteristics equation of the closed loop transfer function. The presence of time delay reduces the phase margin of the control system and therefore, is a major source of instability and performance deterioration. An arbitrary placement of dominant poles leads to the sluggish response and poor robustness. In the controller design, attempts have been made for the pole placement by reducing infinite number of poles with poles near the origin in the complex s-plane. For the systems with time delay, the tuning of PID controller using dominant pole placement is tricky but provide better results and therefore needs special attention to achieve the desired closed loop performance measures.

In the present thesis, three analytical methods will be presented for the tuning of PID controller using dominant pole placement for plants with time delay. The first one is based on a graphical tuning method. In the formulation no approximation has been used to the exponential term that arises in the characteristics equation due to the time delay. This method provides the scope of choosing PID parameters in the parameters plane with desired closed loop performance measures. The second PID tuning method, applicable for the standard second order systems with time delay, is based on utilizing dominant pole placement and Linear Quadratic Regulator (LQR) approach. This method provides the optimum setting of PID parameters with user defined damping ratio and settling time with very less control efforts. The third methodology focuses on developing of two degree of freedom (2-DOF) PID controller tuning method utilizing LQR and dominant pole placement approach especially for integrating plus time delay plants. This method gives lots of flexibility in choosing PID parameters considering the trade-off between performances and robustness. The main goal and motivation of the present thesis is to develop an analytical formulation for PID controller that gives desired closed loop response with specified robustness with less control effort for time delay systems. The thesis will consist of five chapters and main features are described below.

In **Chapter 1** the introduction about the PID controller and its important parameters and motivation for the present research work together with detailed literature survey on the subject will be discussed.

The **Chapter 2** of the thesis will present a graphical tuning method of PI/PID controller for time delay systems using dominant pole placement with specified gain margin (GM) and phase margin (PM). In this approach no approximation has been taken to the exponential term in the characteristic equation which arises due to the delay term. GM and PM are related with the robustness and performance measures of a closed loop system. A choice of large GM and PM provides comparatively better stability but leads to the sluggish time response. On the other hand, a choice of small GM and PM gives faster time response but provides relatively less stability. For an appropriate design there is a trade off between speed of response and stability for time delay systems. A graphical stability criterion applicable to the time-delay systems has been used to identify the stabilizing regions in the proportional-integral plane. This helps to choose all feasible sets of PI/PID controller parameters in the vicinity of the dominant poles with pre-specified GM and PM. In order to achieve these stability margins to maintain the robustness, a Gain Phase Margin Tester (GPMT) has been inserted in the forward part of the closed-loop control system. The range of PI parameters has been obtained for first order plus time delay (FOPTD) system by sweeping the closed loop natural frequency (ω_{cl}) for a fixed closed loop damping ratio (ζ_{cl}). In order to obtain the PID parameters for second order plus time delay (SOPTD) system, first the derivative gain is fixed and then the range of PI parameters are obtained using dominant poles that satisfy the specified GM and PM.

In order to illustrate the utility and confirm the validity of the proposed technique, three different examples have been considered and results of simulation performed in MATLAB are presented. It is observed from the simulation that sets of K_p and K_i which are on the constant ζ_{cl} curve in the K_p-K_i plane, on the part of the curve that is straight with positive slope, are more appropriate and produce closed loop time response performance measures very close to the specified values. On the other hand, the sets of K_p and K_i which are on the part of the curve with negative slope, give poor closed loop time response measure. In the case of FOPTD system, simulation results indicate that the choice of controller parameters corresponding to comparatively lower value of ω_{cl} is more suitable for the robust controller design under the perturbation of system parameters. For SOPTD system, the sets of K_p and K_i depend on the pre-defined choice of K_d in addition to certain other parameters.

The main focus in **Chapter 3** of the thesis will be on the development of optimal tuning of PID controller for the standard SOPTD systems. It is well known that the SOPTD model of the plant more closely resembles the higher order plant than does the FOPTD model. Standard SOPTD models are very rich in dynamics and include under damped, critically damped and over damped systems. An analytical optimum PID controller tuning method has been developed for the SOPTD plants using LQR technique and dominant pole placement approach. It is well known that by tuning the controller using LQR an infinite gain margin and at least 60° phase margin for single input and single output processes without time delay is guaranteed. However for the time delay systems this cannot be guaranteed and needs further evaluation.

The tuning of PID controller for SOPTD system using LQR is based on rewriting the state equations in two parts; one for $t < L$ and other for $t \geq L$, where L is the time delay. The initial value of PID controller settings for $t < L$ are generally large and time varying. They need large controller action and most of the times cause saturation of the actuator. It is also difficult to implement them practically, particularly in analog domain. It is obvious that a choice of constant parameters of PID controller throughout eases the practical implementation, needs low control effort and maintains the state optimality for all values of $t \geq L$. Thus, constant parameters of PID controller obtained for $t \geq L$ have been used in the simulation. The effectiveness of the proposed methodology has been demonstrated via simulation of stable open loop oscillatory, over damped, critical damped and unstable open loop systems. Results show improved closed loop time response (around 50% reduction in overshoot and around 30% reduction in settling time) with less control effort over the recently developed LQR based PI/PID controller tuning methods. A comparison of simulation results with other time domain performance indices such as Integral of Absolute Error (IAE), Integral of Square Error (ISE), Integral of Time Absolute Error (ITAE), Integral of Time Squared Error (ITSE), is also obtained. It is observed that the present method gives an overall better closed loop time response with comparatively less control effort. The effect of location of non-dominant pole has also been studied. It affects the rise time however reduces the overshoot considerably. This behavior is completely opposite to the cases of delay free processes.

In **Chapter 4** of the thesis, the development of a method for finding the optimum tuning of PID controller for the integrating processes with time delay will be discussed. Integrating processes contain at least one pole at the origin. Open loop integrating processes are difficult to manipulate because a small load disturbance can easily destroy the balance between the input and output resulting in increasing or decreasing output without limit. The formulations of Chapter 3 cannot be used for integrating plants as they cannot be represented in the form of standard second order transfer function with time delay. The method developed here is an extension of the formulations discussed in Chapter 3 for integrating processes with time delay. The tuning of PID controller for integrating processes with time delay needs special attention in terms of output performance, robustness, noise sensitivity, analytical tunability and applicability over a wide range of processes. It is difficult to develop a One Degree of Freedom (1-DOF) PID controller for integrating processes as one can either achieve a good load disturbance rejection or a good set point response. An effective solution to the above problem is the use of a 2-DOF control system. A 2-DOF PID control system separately tunes the servo response using a set point filter, without affecting the regulatory response tuned by main PID controller in the loop. The set point filter of 2-DOF control system is also used to avoid actuator saturation problem during the initial start up.

In this chapter PID controller parameters for integrating process with time delay are obtained analytically using LQR and dominant pole placement approach to meet the closed loop design criteria. The method is also based on rewriting the state equations in two parts; one for $t < L$ and other for $t \geq L$. PID parameters obtained for $t \geq L$ are used for regulatory control and used throughout. The initial servo control is handled by using a set point filter which is uniquely designed in terms of the PID controller parameters obtained for $t \geq L$ and a single filter time constant λ. The transfer function of set point filter is designed to make the closed loop response of the whole system equal to the output response of a first order system with same time delay. The initial control effort of the controller depends on parameter λ. The value of λ can be suitably tuned to optimize the servo response so that the actuator saturation problem can be avoided during transient.

In order to demonstrate the effectiveness of the proposed tuning methodology, four categories of integrating systems have been considered. These are: First Order Integrator Plus Time Delay (FOIPTD), Double Integrator Plus Time Delay (DIPTD), Pure Integrator Plus Time Delay (PIPTD) and Unstable First Order Integrator Plus Time Delay (UFOIPTD). The whole range of positive PID parameters has been obtained for all categories of plants in terms of regulatory response measured in terms of IAE criteria and robustness measured in terms of maximum sensitivity M_s. In addition, the entire range of PID settings obtained using the present approach is also evaluated in terms of smoothness of the controller measured by Total Variation (TV). Simulation results indicate that both good robustness and load regulation cannot be achieved simultaneously. Better robustness occurs at lower value of ω_{cl} and at higher value of ζ_{cl}, whereas better load regulation can be achieved at higher ω_{cl} and lower ζ_{cl}. Thus a proper design needs a tradeoff between robustness and load regulation. It is observed that tuning of set point filter constant λ plays a crucial role in reducing the overshoot and initial control effort. On average the proposed tuning method of 2-DOF PID controller gives a reasonably good closed loop performance measures for most of the integrating plants. The main advantage of the proposed method is the flexibility of choosing the PID parameters depending on the requirement of a given system.

Finally in **Chapter 5** some general conclusions on the present research work together with the perspective of future work will be outlined.

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List of Abbreviations

PID……………………………………………………….....Proportional-Integral-Derivative

PI…………………………………………………………………….Proportional-Integral

PD…………………………………………………………….....Proportional-Derivative

LQR……………………………………………………….....Linear-Quadratic-Regulator

2-DOF…………………………………………………………….Two degree of freedom

SISO……………………………………………………….....Single Input Single Output

GPMT………………………………………………………….Gain Phase Margin Tester

FOPTD……………………………………………………First Order Plus Time Delay

SOPTD………………………………………………….Second Order Plus Time Delay

GM/ A_m ………………………………………………………………………Gain Margin

PM/ φ_m ……………………………………………………………………Phase Margin

Z-N…………………………………………………………………......Ziegler Nichols

ISE………………………………………………………….Integral of Squared Error

IAE…………………………………………………………....Integral of Absolute Error

ITSE…………………………………………………….Integral of Time Squared Error

ITAE…………………………………………………....Integral of Time Absolute Error

ARE…………………………………………………….....Algebraic Riccati Equation

IMC…………………………………………………………….Internal Model Control

M_s……………………………………………………………....Maximum Sensitivity

FOIPTD……………………………………......First Order Integrator Plus Time Delay

DIPTD……………………………………………...Double Integrator Plus Time Delay

PIPTD…………………………………………….....Pure Integrator Plus Time Delay

UFOIPTD…………………………Unstable First Order Integrator Plus Time Delay

CHAPTER 1

Introduction

1.1 Background and motivation

Proportional-Integral-Derivative (PID) controllers have been in use for control of industrial processes for over seven decades and now are considered as the second most important control decision and communication instrument of the 20th century, after the microprocessor [1-4]. A PID controller is easier to understand for control engineers due to the intuitive simplicity of the algorithm and simple meaning of its three tuning parameters viz. the proportional term K_p, integral term K_i and the derivative term K_d. These parameters of a PID controller are required to be tuned individually to match the process dynamics in order to provide a good closed loop time response and robust performance. Improper setting of the controller parameters results in sluggish, oscillatory closed loop time response and poor robustness [5-11] accompanied by poor disturbance and noise rejection characteristics. Historically, the first tuning rule for setting up controller parameters was defined in 1934 for the design of a proportional-derivative (PD) controller for a process exactly modeled as an Integrator Plus Delay (IPD) system [12]. Subsequently, tuning rules were defined for PI and PID controllers, assuming that the process can be exactly modeled by a First Order Lag Plus Delay (FOLPD) model [13] or a pure delay model [13,14]. Until about 20 years ago, in process control applications, more than 95% of the controllers were of PI or PID type [15-22].

1.1.1 Methods for tuning, design and synthesis of PID controllers

A considerable number of tuning rules for PID controllers have been developed in the last few decades [4, 23-26]. It has been reported that around 154 tuning rules were developed for PI and more than 258 for PID controller [26]. Numerous PID tuning algorithms have been studied. PID tuning methods can be broadly classified as

(i) Method based on a process reaction curve [27-28]: This is an experimental open-loop tuning method and is only applicable to open-loop stable plants. In this method the plant parameters are obtained graphically from the open loop step response [1-2, 27] due to a step input variation. The PID rules for selecting the controller gains using this method aim to provide quarter decay of first overshoot after one oscillation. Examples of these are the Ziegler –Nichols [27].

(ii) Methods based on ultimate cycle or ultimate frequency [2, 5, 29-31]: The ultimate frequency can be obtained by increasing proportional gain (putting integral and derivative gain to zero) up to a limit, so that plant shows constant frequency sustained oscillations in a closed loop. The procedure to obtain ultimate frequency many times leads the system towards instability condition and damages the plant. These methods are popularly known as the Z-N closed-loop methods. Astrom and Hagglund [31] proposed an alternate method based on relay auto tuning that induce a self sustaining oscillation in the loop. This method provides good set point response but poor stability margin.

(iii) Methods based on minimizing an appropriate performance criterion [32-35]: In this approach the performance criteria are chosen based on the specific requirement. Minimization of a chosen performance criterion such as Integral of Squared Error (ISE), Integral of Time Squared Error (ITSE), Integral of Absolute Error (IAE), Integral of Time Absolute Error (ITAE) and Linear Quadratic Regulator (LQR) are some of the popular methods which derive the controller gains either analytically or numerically. In the frequency domain, these methods can be designed to meet specific gain and phase margin criteria as well as sensitivity and complementary sensitivity requirements [34-35].

(iv) Methods based on dominant pole placement [36-38]: The specified closed loop response is achieved through a choice of dominant closed-loop poles. The dominant pole placement technique is a very common technique where a pair of conjugate poles is chosen to meet the required closed loop time response [38]. A pair of complex-conjugate poles of the closed loop system is chosen using a desired closed loop damping ratio (ζ_{cl}) and a natural frequency (ω_{cl}). However, the technique cannot always guarantee the dominance of chosen poles and thus sometimes results in a poor control performance. The Cohen-Coon PID method is a dominant pole design method based on First Order Plus Time Delay (FOPTD) model of the plant. Like Z-N [27] method this method also attempts to locate three dominant poles, a pair of complex poles and one real pole, such that the amplitude decay ratio for load disturbance response is one quarter and the integrated error is minimized.

(v) Robust control methods [39-43]: In these methods, the controller is design considering wide range of uncertainty in the process model. PID controller design based on H_∞ optimization and Internal Model Control (IMC) methods belong to this genre. The principal idea in robust control is to specify suitable weighting functions to reflect the desired performance and stability, and then perform controller optimization over all possible perturbations of the process to achieve robustness. PID controller based on H_∞ optimization is generally based on H_∞ loop shaping [41-43]. PID controller based on IMC is a two step process and provides a suitable tradeoff between performance and robustness. In step 1 a stable controller is obtained that is optimal with respect to some performance criteria; the second step augments the controller from Step 1 with a filter to insure that the IMC controller is proper [39-40].

These methods have been applied for a variety of processes ranging from delay-free to processes with time delay with varying performance in terms of time-response and control effort.

1.1.2 Time Delay

The time delay or dead time is an integral part of many industrial processes. The time delay in a process plant may be traced to various reasons such as (a) time needed for transport of information (b) buildup of delay due to the presence of low-order systems connected in series (c) processing time of sensors and controllers etc. Thus, a common approach is to model a process plant as a transfer function with time delay [2, 44-50]. The common templates are:

(i) First Order Plus Time Delay (FOPTD) system defined as

$$G(s) = \frac{Ke^{-Ls}}{1+Ts} \tag{1.1}$$

(ii) Integrator Plus Time Delay (IPTD) system defined as

$$G(s) = \frac{Ke^{-Ls}}{Ts} \tag{1.2}$$

(iii) First Order Integrator Plus Delay (FOIPTD) system defined as

$$G(s) = \frac{Ke^{-Ls}}{s(1+Ts)} \tag{1.3}$$

(iv) Second Order Plus Time Delay (SOPTD) system defined as

$$G(s) = \frac{Ke^{-Ls}}{s^2+as+b} \tag{1.4}$$

$$\text{or } G(s) = \frac{Ke^{-Ls}}{(1+T_1 s)(1+T_2 s)} \tag{1.5}$$

The delay can be random or constant and an estimate of it is not free from parametric uncertainties. The presence of time delay in the transfer function of a plant deteriorates its phase margin and many times leads to instability of the closed-loop system. [51] presents a detailed treatment of a tuning methodologies for PID controllers used for controlling systems with time delay. In [51] a methodology based on a generalization of the Hermite-Biehler theorem to derive the set of all stabilizing PI or PID controllers for delay-free systems is first presented. A suitable subset of controller gains may then be chosen to meet certain closed-loop performance specifications from this set.

An approximate method for handling systems with time delay for a PID controller design is the Pade's approximation where the delay term e^{-Ls} is often approximated as

$$e^{-Ls} = \frac{P(L,s)}{Q(L,s)} \tag{1.6}$$

where $P(L,s) = \sum_{r}^{k} \frac{(2r-k)!}{k!(2r-k)!}(-Ls)^k$ (1.7)

$$Q(L,s) = \sum_{r=0}^{k} \frac{(2r-k)!}{k!(2r-k)!}(Ls)^k \text{ , r = 0, 1,} \ldots k \tag{1.8}$$

Though it might seem intuitive to extend the approach for delay-free systems presented in [51] to systems with time delay, however an approximate system with Pade's approximation might not always lead to a stable closed-loop system by extending the methodology for delay-free systems to cover systems with time delay. The alternate approach is to use Pontryagin's quasi polynomials to find the set of all stabilizing PID controllers, which has been developed for systems with delay in the measurement channel i.e. in the feedback path only as presented in [51]. A PID controller design approach based on the LQR methodology as presented in [52] gives the PID parameters for a FOPTD system with desired closed loop performance measures with certain constraints. The method works well for FOPTD systems but fails to provide optimal response in case of SOPTD plants; this is because one of the controller parameters is determined by the largest system poles in the left half of the complex plane. In another approach e.g. as presented in Ref [53-54] the set of stabilizing PID controller gains are obtained utilizing Kharitonov theorem for the SOPTD systems with uncertain delays.

Of the methods discussed above, the PID controller design method based on dominant pole placement achieves a given control performance specifications by developing a simple analytical formulation using dominant pole placement technique for tuning a PID controller for a plant with time-delay and obtaining a desired closed loop response with a specified robustness and less control effort. To handle the time delay the quasi polynomial representing characteristics equation is first broken into real and imaginary parts and then these are equated with the real and imaginary parts of the dominant poles in order to satisfy desired time domain specifications. To bring the desired closed loop performance with minimum control effort, the LQR approach is used together with dominant pole placement for the case of SOPTD systems. The method is extended to cover the various categories of IPTD systems that bring the widest range of PID controller parameters in a trade-off between sensitivity and performance.

In the present thesis, three analytical methods are presented for tuning a PID controller for controlling plants with time delay using dominant pole placement. The first is based on a graphical tuning method where no approximation in formulation has been used to deal with the exponential term in the system's transfer function. This method helps in choosing the PID parameters in the parameters plane with desired closed loop performance measures. The second PID tuning method presented in this thesis blends the dominant pole placement and Linear Quadratic Regulator (LQR) approaches. In Ref. [52] author's have developed this approach for a FOPTD system and also extended for SOPDT systems by cancelling the largest process pole by suitably choosing derivative gain of the PID controller. Hence for SOPTD systems, the method proposed by [52] is not optimal and fails in case of complex system poles. The methodology presented in this thesis extended the approach of [52] to a standard SOPTD system and provides optimum setting of PID parameters with user defined damping ratio and settling time. PID controller developed using the second method presented in this thesis is further extended as a third method to cover wide range of integrating systems. In this method the tradeoff between sensitivities and performance is rigorously addressed for a wide range of industrial integrating systems. This method provides a two degree of freedom control design. First, the load regulation PID controller is designed utilizing dominant pole placement and LQR approach; second using these PID parameters and single tuning parameters lambda λ a set point filter is designed for reference tracking irrespective of disturbance. The λ corresponds to the time constant of the assumed FOPTD model. Finally a range of PID parameters is provided that is optimum with respect to load regulation, sensitivity and controller energy. The presented PID tuning algorithm has been verified by numerical simulation covering wide ranges of industrial processes.

1.2 Problem statement and primitives

Constructing a PID controller is simple because it can be made using a single operational amplifier, resistances and capacitances. The combination of resistances and capacitances gives three tuning parameters proportional (K_p), integral (K_i) and derivative (K_d) in the analog domain. In digital domain the PID can be easily implemented with the use of fast processor such as FPGA, DSP etc. The satisfactory performance represents the desired closed loop time response such as rise time, settling time and overshoot, whereas robustness represents the stability of the closed loop system under the perturbation in the parameters. For single-input single-output (SISO) feedback system the closed loop as shown in Fig. 1-1 is taken throughout this thesis both for the design of PID controller an for analysis. Set point filter F(s), also known as servo controller, is generally used for the input tracking. Proper tuning of servo controller reduces the initial overshoot irrespective of the main controller C(s). C(s) is known as the main controller or controller of the regulatory response. The load disturbance D(s) in Fig. 1-1 represents the disturbances that drive the process away from its desired behaviour. Noise source N(s) is generally used in the simulation in order to realize the actual response of the closed loop system before it can be applied to the real world. In order to reduce the effect of noise in the control signal generally noise filters are used in the measurement device [55-59].

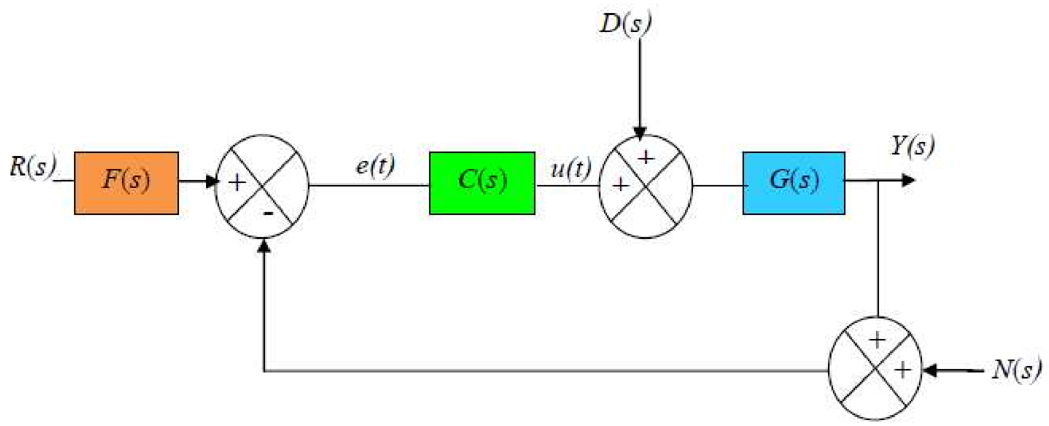


Figure 1-1: Typical closed loop feedback system with set point R(s), load disturbance D(s), noise N(s), act on the set point filter F(s), PID controller C(s) and plant G(s).

1.2.1 PID controller

Though very old design, PID controller is still one of the favorite and most widely used controllers in the industries. It is simple to understand due to simple meaning of its three tuning parameters defined mathematically as follows.

Proportional gain (K_p): The proportional term produces an output which is proportional to the current error value. The proportional gain K_p of the PID controller is defined as

$$u(t) = K_p e(t) \tag{1.9}$$

where, $e(t)$ is the error value and $u(t)$ is the controller output. A Proportional controller alone cannot guarantee zero static control errors except for the case of integrating systems with unit step response. The magnitude of the static error depends on K_p. The speed and noise sensitivity of the closed-loop system increase with an increase in K_p however, at the same time the robustness decreases.

Integral gain (K_i): The contribution from the integral term in controller output is proportional to both the magnitude of the error and the duration of the error. The integral part of a PID controller is the sum of the instantaneous error over time and gives the accumulated offset that should have been corrected previously. The accumulated error is then multiplied by the integral gain (K_i) and added to the controller output i.e.

$$u(t) = K_i \int_0^t e(\tau)d\tau \tag{1.10}$$

The integral term accelerates the movement of the process towards set point and eliminates the residual steady-state error that occurs with a pure proportional controller. Since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the set point value.

Derivative gain (K_d): The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain K_d.

The magnitude of the contribution of the derivative term to the overall control action is given as:

$$u(t) = K_d \frac{de(t)}{dt} \tag{1.11}$$

Derivative action predicts system behavior and thus helps in improving settling time and stability of the system. A major disadvantage of the derivative part is that the differentiation of the control error makes it very noise sensitive. Noise filters are generally used either in the derivative part or in the measurement part in order to minimize the effect of measurement noise [20 (pp. 99), 60 (pp. 171-172)].

1.2.2 Linear Quadratic Regulator

The LQR reduces the amount of work done by the controller and helps control systems engineer to optimize the controller. In the case of PID controller $u(t)$ is related with the work done. Chapters 3 and 4 of the thesis are focused on obtaining desired closed loop response with minimum control effort $u(t)$. LQR controller works on state space equation. State space equation can be continuous in time or discrete. The Present thesis uses continuous state space. To get minimum $u(t)$ one needs to minimize the quadratic cost function given by

$$J = \int_0^\infty \left(\mathbf{X}^T(t)\,\mathbf{Q}\,\mathbf{X}(t) + \mathbf{u}^T(t)\,\mathbf{R}\,\mathbf{u}(t) \right) dt \quad , \tag{1.12}$$

where **Q** is the semi positive definite state weighting matrix and **R** is the positive definite control weighting matrix. The LQR solution gives the optimal control vector $\mathbf{u}(t)$ as [61]

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{X}(t)\,, \tag{1.13}$$

where **X** is the state vector, **B** is the control matrix and **P** is the symmetric positive definite Riccati coefficient matrix which can be obtained by solving continuous Algebraic Riccati Equation (ARE)

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} = 0. \tag{1.14}$$

A is the system matrix. PID controller parameters are arranged in state weighting matrix, while system parameters are contained inside state transition and control matrix. Chapter 3 and 4 will highlight how user defined ζ_{cl} and ω_{cl} will be used in the ARE in order to find the optimum setting of PID controller that will give desired closed loop with minimum control effort for second order and integrating systems with time delay.

1.2.3 Simulation software

The optimal PID tuning method that is developed in this thesis is simulated using MATLAB and SIMULINK software trademark of Mathworks corporation [62]. Control system analysis is performed using the readymade command set available in MATLAB e.g. *bode(), margin(), fsolve()* etc. The tuning procedure of PID controller is written as a subroutine in *m-file* of the MATLAB text editor. The closed loop simulations are performed using SIMULINK toolbox of the MATLAB.

1.3 Contribution of the thesis

As mention before, the present thesis discusses three analytical methods to design PID controller covering most of the transfer functions of plants of real world process industries. These include FOPTD systems, various dynamics of SOPTD systems such as over damped, critically damped, highly oscillatory and unstable processes as well as integrating systems.

First method is based on graphical tuning method of PI/PID controller for the industrial processes represented by first order and second order plus time delay systems. The stability equation method and gain phase margin tester have been used to portray constant gain margin and phase margin boundaries. The novelty of this method is that the range of parameters of PID controller can be determined with user specified closed loop specification i.e. percentage overshoot, settling time and required gain and phase margin without taking any approximation to the exponential term that arises due to time delay part in the characteristic equation. The method uses dominant pole placement approach for achieving desired closed loop time responses.

In the second method a linear quadratic regulator is used together with dominant pole placement approach to obtain optimal setting of PID parameters to get desired closed loop response with minimum control effort for standard second order plus time delay systems. It is observed that placing the third non-dominant pole far away from the dominant poles, results in more robust controller in the case of mismatch between the process delay time and the delay time at which the controller is designed. However, the penalty one has to pay is the increase in the rise time. The present method gives an overall better closed loop time response with comparatively less control effort. The third method is fully focused on the design of 2-DOF PID controller using LQR and pole placement approach for various categories of integrating systems such as pure integrator with time delay, double integrator with time delay etc. This method is an extension of the second method and gives flexibility in choosing PID parameters in a tradeoff between the performance and robustness. The entire range of PID settings obtained using the present approach is found reasonably well when smoothness of the controller is evaluated. On average, the proposed 2-DOF PID controller tuning method gives a reasonably good closed loop performance measures for most of the integrating plants.

1.4 Outline of the thesis

The organization of thesis is as follows: At first the introduction and motivation for the research is discussed in the present Chapter 1.

The Chapter 2 of the thesis presents a graphical tuning method of PI/PID controller for time delay system using dominant pole placement with specified GM and PM. A graphical stability criterion applicable to the time-delay systems has been used to identify the stabilizing regions in the proportional-integral plane. This helps to choose all feasible sets of PI/PID controller parameters in the vicinity of the dominant poles with pre-specified GM and PM. In order to illustrate the utility and confirm the validity of the proposed technique, three different examples have been considered and results of simulation performed in MATLAB are presented. This contribution is published in Journal of Process Control [63].

Chapter 3 of the thesis focuses on the development of an analytical method for optimal tuning of PID controller for the standard SOPTD systems using LQR and dominant pole placement. The effectiveness of the proposed methodology has been demonstrated via simulation of stable open loop oscillatory, over damped, critical damped and unstable open loop systems. Results show improved closed loop time response with less control effort over the recently developed LQR based PI/PID controller tuning methods. Results of this work have been published in ISA Transaction [64].

In Chapter 4 of the thesis, the optimal tuning of a 2-DOF PID controller for the integrating processes has been discussed. The PID controller parameters are obtained analytically using LQR and dominant pole placement approach to meet the closed loop design criteria. The initial transient response is handled by using a set point filter which is uniquely designed in terms of the PID controller parameters and a single filter time constant. The effectiveness of the proposed tuning methodology has been demonstrated by performing simulations on four categories of integrating systems. Results of this work have been published in ISA Transaction [65].

Finally in Chapter 5 some general conclusions on the present research work together with the perspective of future work have been discussed.

CHAPTER 2

PI/PID controller for time delay systems fulfilling desired closed loop response with guaranteed gain and phase margin

2.1 Introduction

As discussed in Chapter 1, the presence of the time-delay in a control loop reduces the phase margin of the control system and therefore, is a major source of instability and performance deterioration [65-70] and needs special attention. Designing a PI or a PID controller that will fulfill user defined closed loop performance measures such as settling time and percentage overshoot with appropriate robustness is always a subject of interest. Robustness of the closed loop system is another important design issue for a controller and this is generally measured either by maximum sensitivity function in Nyquist plane or via gain and phase margin specifications.

The dominant pole placement, [38, 71-72] is a very common technique in the state space design where a pair of conjugate poles is chosen in terms of user defined ζ_{cl} and ω_{cl} to meet the required closed loop time response. However, for the single input single output systems (SISO) with time delay, the pole placement leads to a characteristic quasi-polynomial with infinite number of roots for the closed-loop. In Ref. [72], controller design based on quasi-direct pole placement has been attempted for the time delay system by shaping the frequency response of the closed loop system. For a system with time-delay, a popular approach is Pade approximation and techniques exist to define a set of controller gains which ensure stability [51, 73] known as the approximate sets. While Pade approximation works well for small delays, approximate sets converge to true sets only with higher order approximations, as lower order approximations do not guarantee reliable results [51]. So, larger delay makes the controller design difficult by analytical means. It is also not possible to predict a priori the order of approximation for a time delay system for the true and approximate sets for the controller design. In the controller design, attempts have been made for the pole placement by reducing infinite numbers of poles with poles near the origin in the complex *s*-plane [49, 66-67].As the time delay systems are infinite dimensional, the design of the controller is more complex than the delay free case.

In recent years, considerable attention has been paid for finding the set of stabilizing PID parameters for time delay systems [53-54]. In [54] the admissible range of stabilizing proportional gain (K_p) is first derived by a version of Hermite–Biehler Theorem. Then, the stabilizing region in integral-derivative plane (K_i- K_d), for a fixed proportional-gain, is drawn and identified directly in terms of a graphical stability criterion applicable to time-delay systems. Ref [74] discussed the sets of obtaining PID values using, Pade approximation and using Hermite-Biehler theorem. Ref [75] discussed the use of Smith predictor for handling the systems with time delay. Use of Smith predictor is generally applicable to the nominal model of the plant and is constructed digitally along with the PID controller. In Ref [76], a method is discussed to obtain the ranges of stabilizing PID parameters within the GM and PM boundaries drawn in the parameters plane using the stability equation and Gain-Phase Margin Tester (GPMT) [54]. The well known D-Decomposition method is applied in [77-79] to draw the stability boundary graphically in the parameters space. A computationally efficient method is described in

Ref. [73] which helps to characterize all the stabilizing parameters of the PID controller for any given plant. Several methods for the selection of controller parameters based on GM and PM specifications for the robustness considerations have been developed in [80, 81].

The design of a discrete controller using LQR approach by placing the poles within the unit circle in a predefined location has been discussed in Ref. [82], but only for delay free systems. An analytical method to tune the PI/PID parameters in an optimal way using LQR techniques with user specified ω_{cl} and ζ_{cl} is presented in [52] for the first order plus time delay (FOPTD) model. This approach also works for some specific second order plus time delay (SOPTD) models, where the system poles have real roots. Although the resultant LQR system for a SISO process provides at least a PM of 60^0 and GM of infinity, however, in general this property cannot be carried over to the case of a system with time delay [52]. In Ref [83] authors have worked on the design of a PID controller using nonlinear optimization technique and maximized the bandwidth with constraints on both GM, PM and sensitivity so that criteria on robustness and closed loop performance are both satisfied simultaneously.

This Chapter presents a graphical method to tune PI/PID controller for time delay systems using dominant pole placement with specified GM and PM without taking any approximation to the exponential term in the characteristic equation. A graphical stability criterion applicable to the time-delay systems has been used to identify the stabilizing regions in the proportional-integral plane. This helps to choose all feasible sets of PI/PID controller parameters in the vicinity of the dominant poles with pre-specified GM and PM. In order to assess these stability margins and robustness, a GPMT has been inserted in the forward part of the closed-loop control system. The range of parameters of PI controller for

FOPTD system is obtained by sweeping the closed loop natural frequency for a fixed closed loop damping ratio. To obtain the PID parameters for SOPTD system, first a fixed value for the derivative gain is chosen and then the range of PI parameters is calculated using dominant poles that satisfy the specified GM and PM.

2.2 K_p-K_i sets for FOPTD systems

In this section, first the region of K_p-K_i in the controller parameters plane that stabilizes the given first order system with time delay is obtained. After that, region inside the stabilization region is ascertained that gives guaranteed gain and phase margin. Finally, a method is discussed to obtain the sets of K_p-K_i that gives desired closed loop time response measures with pre specified GM and PM. In order to draw the stability boundary in the parameter space the D-Decomposition method [77, 78] has been utilized. This method essentially maps the boundary of instability from the root plane to the parameters plane. In other words, it establishes a direct correlation between the variable parameters of the closed-loop characteristic equation and the stability region of the controller. By mapping the boundaries of the asymptotic stability domain onto the parameter space, boundaries can be constructed in the parameter space and then the stable regions of the controller can be confirmed [78, 79].

2.2.1 Stabilizing region of K_p-K_i

Consider a linear feedback control system as shown in Fig. 2-1. Plant $G(s)$ is an FOPTD system characterized by three parameters model in the frequency domain as

$$G(s) = \frac{K}{s+a} e^{-sL}, \tag{2.1}$$

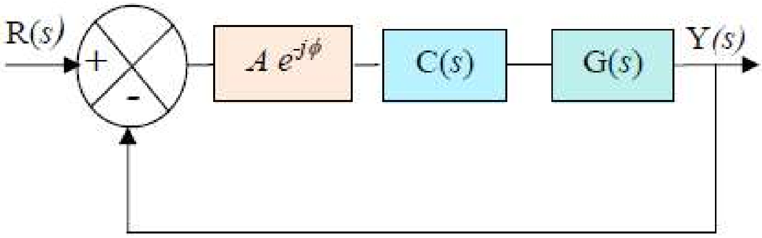


Figure 2-1: Block diagram of a general feedback control system with GPMT.

Where K/a, $1/a$ and L are the static gain, time constant and time delay respectively. An FOPTD system is generally tuned with PI controller; it can also be tuned with a PID controller. The PI controller is preferred over PID controller because the noise sensitivity introduced by derivative part can give undesirable control signal variations which leads to the expensive wear and tear of actuator. The general form of a PI controller in terms of proportional term K_p and integral term K_i , is

$$C(s) = K_p + \frac{K_i}{s}. \tag{2.2}$$

Now, our objective is to find the stabilizing sets of PI controller parameters in K_p-K_i plane using the graphical stability criteria. Following Fig. 2-1, the closed loop characteristic quasi-polynomial without GPMT can be expressed as

$$\delta(s) = s^2 + as + \left(KK_p s + KK_i\right)e^{-sL} \tag{2.3}$$

Multiplying both sides of Eq. (2.3) by e^{sL} and using $s = j\omega$,

$$\delta^{\#}(j\omega) = \left(-\omega^2 + j\omega a\right)e^{j\omega L} + j\omega KK_p + KK_i \tag{2.4}$$

Let us assume that

$$\delta^{\#}(j\omega) = \delta_r^{\#}(\omega) + j\delta_i^{\#}(\omega) \tag{2.5}$$

where, $\delta_r^{\#}(\omega)$ and $\delta_i^{\#}(\omega)$ are real and imaginary parts of the characteristics quasi-polynomial respectively. Comparing real and imaginary parts in Eqs. (2.4) and (2.5), one get

$$\delta_r^{\#}(\omega) = KK_i - a\omega\sin(\omega L) - \omega^2\cos(\omega L) \tag{2.6}$$

$$\delta_i^{\#}(\omega) = KK_p\omega + a\omega\cos(\omega L) - \omega^2\sin(\omega L) \tag{2.7}$$

Let us consider that there is a point (K_p^0, K_i^0, ω) in $K_p - K_i$ plane such that real and imaginary parts of the characteristics quasi-polynomial are equal to zero. Under such condition, if the Jacobian J defined by

$$J = \begin{bmatrix} \dfrac{\partial\delta_r^{\#}(\omega)}{\partial K_p} & \dfrac{\partial\delta_r^{\#}(\omega)}{\partial K_i} \\ \dfrac{\partial\delta_i^{\#}(\omega)}{\partial K_p} & \dfrac{\partial\delta_i^{\#}(\omega)}{\partial K_i} \end{bmatrix}_{(K_p^0, K_i^0, \omega)} \tag{2.8}$$

is nonsingular, then according to implicit function theorem [54], Eqs. (2.6) and (2.7) have a unique local solution curve ($K_p(\omega)$, $K_i(\omega)$) and the sets of K_p and K_i which make Eqs. (2.6) and (2.7) equal to zero provide a marginal stability curve with increasing ω. The controller points on the marginal stability curve will neither give asymptotically stable nor unstable time response. In fact marginal stability curve divides the parameter plane between stable and unstable zones. Furthermore, following proposition holds [54, 84 (pp. 250)]:

In the parameter space of K_p and K_i the critical roots which lead to instability will lie towards the left side of the curve ($K_p(\omega)$, $K_i(\omega)$) for det $J < 0$ *and to the right side of the curve for* det $J > 0$ *when one follows this curve in the direction of increasing ω.*

The marginal stable boundary curve in $K_p - K_i$ plane can be obtained by solving Eqs. (2.6) and (2.7) after putting $\delta_i^{\#}(\omega)$=0 and $\delta_r^{\#}(\omega)$=0. For positive value of ω, one have,

$$K_p = \frac{1}{K\omega}\left(\omega^2 \sin(\omega L) - a\omega\cos(\omega L)\right)$$

$$K_i = \frac{1}{K}\left(\omega^2 \cos(\omega L) + a\omega\sin(\omega L)\right) \tag{2.9}$$

Using Eq. (2.8), one can easily evaluate $\det J = -K^2\omega$ which is < 0, $\forall\omega$> 0. This implies that the stabilizing sets of K_p and K_i will lie on the right side of the marginal stable boundary curve in $K_p - K_i$ plane with increasing ω.

2.2.2 Region of K_p - K_i with guaranteed GM and PM

In order to find the suitable parameters in the controller parameters plane that provide the guaranteed GM and PM one has to introduce GPMT in the forward part of the feedback system. GPMT is a computational aid often used in the simulation to define the GM and PM boundaries. The term $Ae^{-j\phi}$ in the forward path of the control system in Fig. 2-1 is known as GPMT. Here A represents the gain margin and ϕ represents the phase margin of the closed loop system. By putting A=1 for finding PM because PM is the amount of phase required to reach 180 degree when gain is 0dB (i.e. A=1). Similarly by putting ϕ= 0 (i.e. phase is kept 180degree) for finding GM. As the value of ω is changes the ranges of K_p-K_i gives the boundary of controller parameters that give desired GM and PM. The detail description is mention in Ref [85].

The closed loop characteristics quasi polynomial in the presence of GPMT can be expressed as

$$\delta(s) = s^2 + as + \left(KK_p s + KK_i\right)e^{-sL}Ae^{-j\phi} \tag{2.10}$$

Following the procedure outlined in subsection 2.2.1, expressions for K_p and K_i for positive ω in the presence of GPMT, can be obtained as

$$K_p = \frac{1}{AK\omega}\left(\omega^2 \sin(\omega L + \phi) - a\omega\cos(\omega L + \phi)\right)$$

$$K_i = \frac{1}{AK}\left(\omega^2 \cos(\omega L + \phi) + a\omega\sin(\omega L + \phi)\right). \tag{2.11}$$

$\det J = -A^2K^2\omega$,which is < 0, $\forall\omega > 0$, implies that the sets of K_p and K_i that will provide GM or PM either greater than or equal will lie on the right side of the GM or PM marginal boundary curve in K_p-K_i plane with increasing ω. It is clear from Eq. (2.11) that a higher value of GM reduces the available region for K_p and K_i in the parameters plane.

2.2.3 Sets of K_p-K_i with guaranteed GM , PM and desired closed loop time response

In this subsection a method is discussed that provides the sets of K_p and K_i for desired closed loop time response using closed loop damping ratio ζ_{cl} and natural frequency ω_{cl} within the specified GM and PM. The ranges of K_p and K_i parameters can be obtained for different dominant poles by sweeping ω_{cl} for a fixed ζ_{cl} .

Let the desired dominant poles are $s = -\zeta_{cl}\omega_{cl} \pm j\omega_{cl}\sqrt{1-\varsigma_{cl}^2} = \sigma \pm j\omega$. If such poles exist for a particular FOPTD system, then they must be roots of Eq. (2.3). Multiplying both sides of Eq. (2.3) with e^{sL} , putting $s = \sigma + j\omega$ and rearranging the terms one get,

$$\delta^{\#}(\sigma + j\omega) = \left((\sigma + j\omega)^2 + a(\sigma + j\omega)\right)e^{(\sigma + j\omega)L} + (\sigma + j\omega)KK_p + KK_i \tag{2.12}$$

Using $\delta^{\#}(\sigma + j\omega) = \delta_r^{\#}(\sigma,\omega) + j\delta_i^{\#}(\sigma,\omega)$ and comparing real and imaginary parts of Eq. (2.12),

$$\delta_r^{\#}(\sigma,\omega) = (x_1\cos(\omega L) - x_2\sin(\omega L))e^{\sigma L} + KK_p\sigma + KK_i \quad (2.13)$$

$$\delta_i^{\#}(\sigma,\omega) = (x_1\sin(\omega L) + x_2\cos(\omega L))e^{\sigma L} + KK_p\omega \quad (2.14)$$

where, $x_1 = \sigma^2 - \omega^2 + a\sigma$; $x_2 = 2\sigma\omega + a\omega$.

Putting $\delta_r^{\#}(\sigma,\omega) = 0$, $\delta_i^{\#}(\sigma,\omega) = 0$ in Eqs. (2.13) and (2.14), one can easily evaluate K_p and K_i as,

$$K_p = \frac{-1}{K\omega}\left(x_1 e^{\sigma L}\sin(\omega L) + x_2 e^{\sigma L}\cos(\omega L)\right) \quad (2.15)$$

$$K_i = \frac{1}{K}\left(x_2 e^{\sigma L}\sin(\omega L) - x_1 e^{\sigma L}\cos(\omega L) - KK_p\sigma\right). \quad (2.16)$$

By choosing different ζ_{cl} and ω_{cl} and hence σ and ω using relations $\sigma = -\zeta_{cl}\omega_{cl}$ and $\omega = \omega_{cl}\sqrt{1-\varsigma_{cl}^2}$,the possible sets of K_p and K_i can be obtained. The values of K_p and K_i within the boundaries of GM and PM, give the required set of controller parameters with guaranteed GM and PM. As these controller parameters are obtained with chosen ω_{CL} and ζ_{CL}, it is expected that the controller will provide desired response time.

2.3 PID parameters for SOPTD systems

Dynamics of a real plant can be more closely approximated using SOPTD model than FOPTD model. The SOPTD processes are very rich in dynamics as they include under damped, critically damped and over damped systems. For SOPTD model the PID controller is found to be more effective as compare to the PI controller. This is due to the availability of an extra tuning parameters i.e. derivative part which helps to control any overshoot in the response. In the following subsections a procedure to obtain PID parameters for SOPTD system to get desired closed loop time response with specified GM and PM is discussed.

2.3.1 Stabilizing region of PID parameters

The transfer function of the standard second order system with delay can be written as:

$$G(s) = \frac{K}{s^2 + as + b} e^{-sL} \quad . \tag{2.17}$$

The transfer function $C(s)$ of PID controller is

$$C(s) = K_p + \frac{K_i}{s} + K_d s \,. \tag{2.18}$$

The closed loop characteristic quasi-polynomial in this case is given by

$$\delta(s) = s^3 + as^2 + bs + \left(KK_d s^2 + KK_p s + KK_i\right) e^{-sL} \tag{2.19}$$

Following the procedure outlined in subsection 2.2.1, real and imaginary parts of Eq. (2.19) are given by

$$\delta_r^{\#}(\omega) = KK_i - a\omega^2 \cos(\omega L) - (b\omega - \omega^3)\sin(\omega L) - KK_d \omega^2 \tag{2.20}$$

$$\delta_i^{\#}(\omega) = KK_p \omega - a\omega^2 \sin(\omega L) + (b\omega - \omega^3)\cos(\omega L) \tag{2.21}$$

Notice that the imaginary part $\delta_i^{\#}(\omega)$ contains only the proportional term, whereas the real part $\delta_r^{\#}(\omega)$ is a function of both integral and derivative terms. In order to find the full stabilizing sets of PID controller parameters from Eqs. (2.20) and (2.21) consisting of three unknowns K_p, K_i and K_d, it is necessary to fix the value of either K_i or K_d. Considering a predefined value for K_d, the marginal stable boundary curve in $K_p - K_i$ plane can be obtained by setting $\delta_r^{\#}(\omega) = 0$, $\delta_i^{\#}(\omega) = 0$ in Eqs. (2.20) and (2.21). The expressions for K_p and K_i for positive ω can be obtained as

$$K_p = \frac{1}{K\omega}\left(\omega^3 \cos(\omega L) + a\omega^2 \sin(\omega L) - b\omega \cos(\omega L)\right) \tag{2.22}$$

$$K_i = \frac{1}{K}\left(KK_d\omega^2 + b\omega \sin(\omega L) + a\omega^2 \cos(\omega L) - \omega^3 \sin(\omega L)\right) \tag{2.23}$$

The stabilizing sets of K_p and K_i in this case will also lie on the right side of the marginal stable boundary curve in $K_p - K_i$ plane with increasing ω as $\det J = -K^2\omega$, which is < 0, $\forall\omega > 0$. There are various methods such as Ziegler-Nichols (Z-N) stability criteria, Integral of Square Error (ISE), Integral of Time Square Error (ITSE), Integral of Absolute Error (IAE) etc. by which one can choose the derivative gain K_d [2,27,70,33].

2.3.2 Region of PID parameters with guaranteed GM and PM

Following the procedure outlined in section 2.2.2, the expressions for K_p and K_i, with a given K_d, after introducing GPMT, can be obtained as

$$K_p = \frac{1}{AK\omega}\left(\omega^3 \cos(\omega L + \phi) + a\omega^2 \sin(\omega L + \phi) - b\omega \cos(\omega L + \phi)\right) \tag{2.24}$$

$$K_i = \frac{1}{AK}\left(AKK_d\omega^2 + b\omega \sin(\omega L + \phi) + a\omega^2 \cos(\omega L + \phi) - \omega^3 \sin(\omega L + \phi)\right) \tag{2.25}$$

As $\det \mathbf{J} = -A^2K^2\omega$,the stabilizing sets of K_p and K_i will lie on the right side of the marginal stable boundary curve in (K_p, K_i) plane with increasing ω .

2.3.3 PID parameters with guaranteed GM, PM and desired closed loop time response

The mathematical steps in this case are similar as outlined in subsection 2.3 for PI controller. With little bit of algebra the real and imaginary parts of the characteristics quasi-polynomial of the closed loop system can be obtained as

$$\delta_r^{\#}(\sigma,\omega) = X_1 e^{\sigma L}\cos(\omega L) - X_2 e^{\sigma L}\sin(\omega L) + KK_p\sigma + KK_i + KK_d(\sigma^2 - \omega^2) \quad (2.26)$$

$$\delta_i^{\#}(\sigma,\omega) = X_2 e^{\sigma L}\cos(\omega L) + X_1 e^{\sigma L}\sin(\omega L) + KK_p\omega + 2KK_d\omega\sigma \quad (2.27)$$

where,

$$X_1 = \sigma^3 - 3\sigma\omega^2 + a\sigma^2 - a\omega^2 + b\sigma$$

$$X_2 = 3\omega\sigma^2 - \omega^3 + 2a\sigma\omega + b\omega$$

Predefining the value of K_d and setting $\delta_r^{\#}(\sigma,\omega) = 0$, $\delta_i^{\#}(\sigma,\omega) = 0$ in Eqs. (2.26) and (2.27), it is easy to obtain expressions for K_p and K_i as

$$K_p = -\frac{1}{K\omega}\left(X_1 e^{\sigma L}\sin(\omega L) + X_2 e^{\sigma L}\cos(\omega L) + 2\sigma\omega KK_d\right) \quad (2.28)$$

$$K_i = \frac{1}{K}\left(X_2 e^{\sigma L}\sin(\omega L) - X_1 e^{\sigma L}\cos(\omega L) - \sigma KK_p - KK_d(\sigma^2 - \omega^2)\right) \quad (2.29)$$

By sweeping ζ_{cl} and ω_{cl} in Eqs.(2.28) and (2.29) using relations $\sigma = -\zeta_{cl}\omega_{cl}$ and $\omega = \omega_{cl}\sqrt{1-\varsigma_{cl}^2}$, one can obtain the range of K_p and K_i in the parameter plane which are within the intersection region of the GM and PM boundaries. These values of K_p and K_i will satisfy the requirement of the closed loop time response with guaranteed GM and PM, although, the real results should be checked with the simulation results.

2.4 Illustrative examples with Simulation

In order to demonstrate the application of present method three different types of process are chosen and performed simulations using MATLAB and SIMULINK. We will first consider a FOPTD process and obtain the appropriate range of K_p and K_i that satisfy both the GM and PM specifications as well as the user defined closed loop time response specifications i.e. percentage overshoot (controlled by ζ_{cl}) and settling time (controlled by ζ_{cl} and ω_{cl}) [70]. It is to be noted here that a choice of large GM and PM provides comparatively better stability but leads to sluggish time response. On the other hand, a choice of small GM and PM gives fast time response but provides relatively less stability. For an appropriate design there is a trade-off between the time response performance measures and stability. Designer is, therefore required to choose suitable GM and PM specifications before applying the proposed method.

2.4.1 Example 1: FOPTD process

Let the FOPTD process [86] is given by

$$P_1 = \frac{K}{s+a} e^{-Ls} \tag{2.30}$$

where $K = 1$, $a = 1$, and $L = 0.3s$. The objective here is to find the sets of K_p and K_i of the PI controller to get the closed loop time response specifications with guaranteed GM and PM. First find the stability region in the parameter plane of K_p and K_i using Eq. (2.9) and then draw the GM and PM boundaries using Eq. (2.11). Finally, the set of K_p and K_i within the specified GM and PM is obtained by sweeping ω_{cl} for different ζ_{cl} .

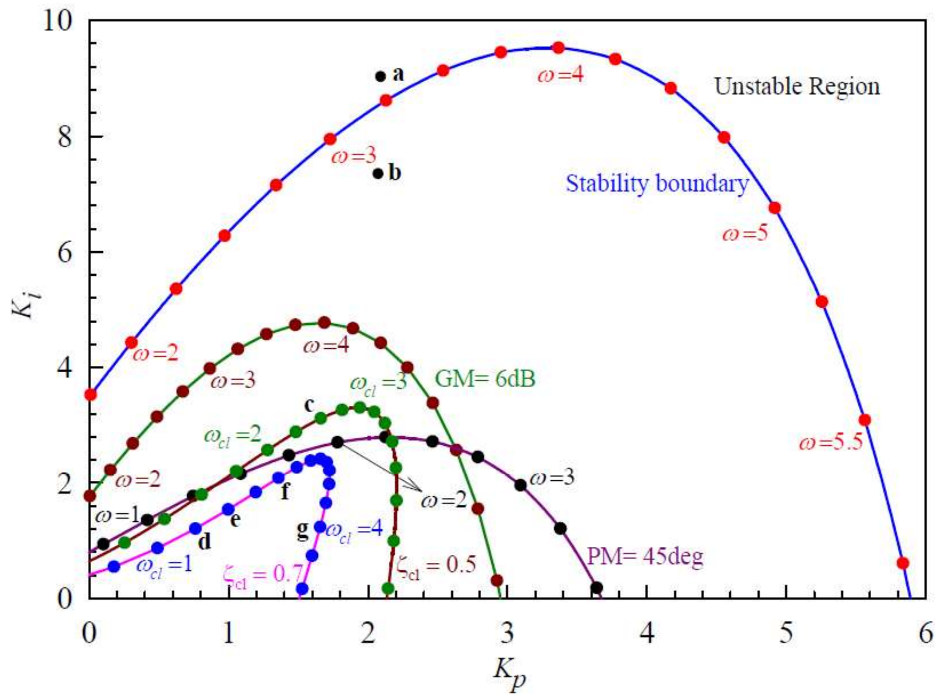


Figure 2-2: Stabilizing region, GM and PM boundaries and the range of K_p and K_i obtained via dominant pole placement.

Figure 2-2 shows the stabilizing region and the range of K_p and K_i. Boundaries of constant GM of 6 dB and constant PM of 45 deg are indicated by curves GM and PM respectively. Any point (K_p , K_i) within the intersection region of these curves, provides guaranteed GM and PM. For illustration purpose, two curves one with ζ_{cl} = 0.5 and other with ζ_{cl} = 0.7 are shown. The range of K_p and K_i is obtained by sweeping ω_{cl}. In order to check the behavior of time response seven points **a, b, c, d, e, f** and **g** are selected covering different regions in the parameters plane. For example, point **a** (K_p=2, K_i =9) is outside the stability region while point **b** **(**K_p=2, K_i=7.5) is inside. The time response with controller parameters corresponding to point **b** is found to be stable whereas that corresponding to point **a** shows an unstable and oscillatory behavior as shown in Fig. 2-3.

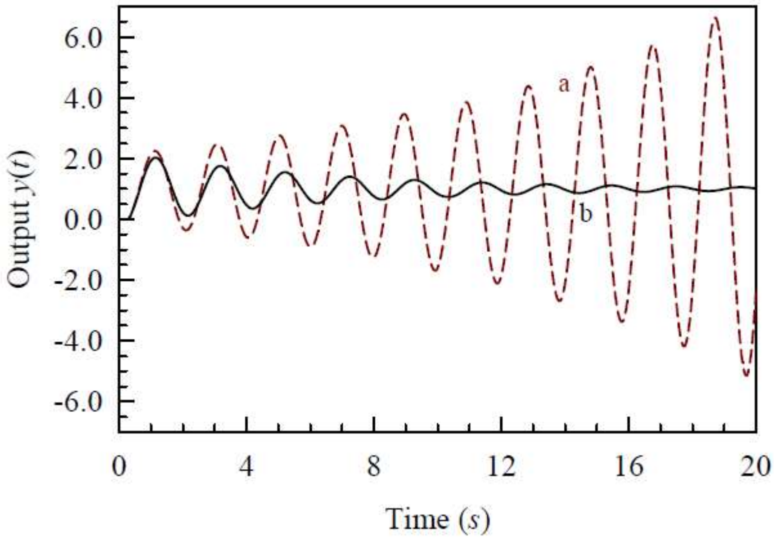


Figure 2-3:Closed loop time response of the process P_1 corresponding to points ***a*** and ***b***.

The time responses of process P_1 for controller settings corresponding to points **c, d, e, f** and **g** are shown in Fig. 2-4. In all cases a load disturbance of 20% is introduced at $t = 10s$. PI parameters and closed loop performance measures such as percentage overshoot (% *OS*), settling time (T_s) and rise time (T_r) obtained from the simulation are presented in Table 2-1. It can be readily seen that the performance measures obtained from the simulations are very close to the values chosen for the design of controllers. For example, simulation with controller parameters corresponding to point **c** which is chosen outside the PM specification, gives lower value (PM=38.32 deg, specified PM=45 deg) as expected. Simulation results indicate that controller parameters corresponding to smaller ω_{cl} not only satisfy the required closed loop response but also need small control efforts on the same ζ_{cl} curve. From the simulation results it is observed that points which are on the positive slope of the curve $\zeta_{cl} = 0.7$ i.e. points **d**, **e** and **f** show the performance measures very close to the specified values. Controller parameters corresponding to the points on the curve with negative slope do not meet the desired specifications. For example, in the case of point **g,** the settling time T_s = 5.3s, obtained from the simulation is much larger than the designed specification of 1.42s this is because it has large PM (76.75). Simulation results indicate that suitable values of K_p and K_i belong to the region on the linear part of the curve with positive slope. Note that the GM and PM obtained from the simulation (see Table 2-1) are within the specified values.

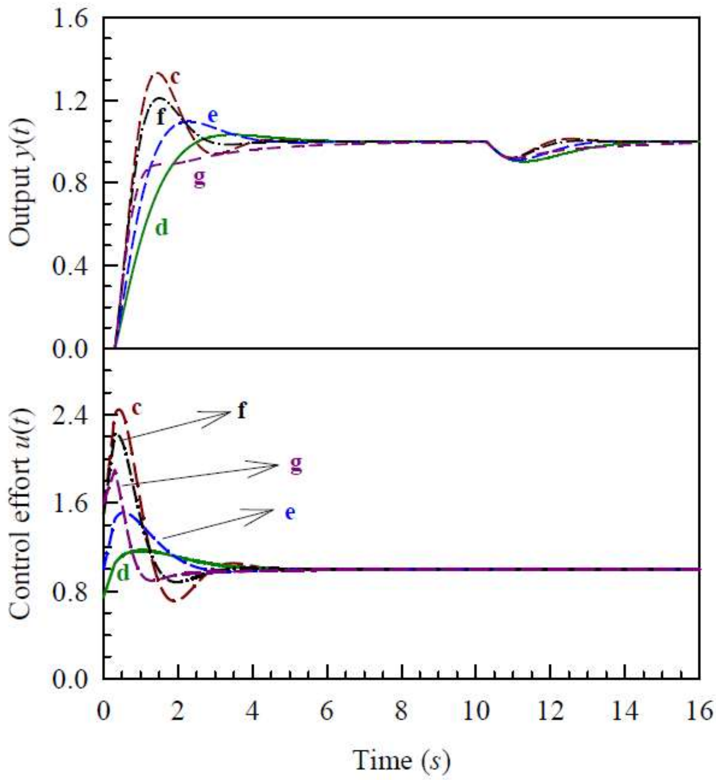


Figure 2-4: Time responses and control efforts of process P_1 with controller parameters corresponding to points**c, d, e, f** and **g**. Load disturbance of 20% is introduced at t = 10s.

Table 2-1: Time response performance measures of process P_1.

Points	Specified GM=6dBPM=45°				Simulated Results				
	K_p	K_i	ζ_{cl}	ω_{cl}(rad/s)	GM (dB)	PM (deg)	$\%OS$	$T_r(s)$	$T_s(s)$
c	1.5	3.0	0.5	2.5	9.13	38.32	36	0.5	3.7
d	0.75	1.0	0.7	1.25	16.4	66.62	3.0	1.6	4.2
e	1.0	1.5	0.7	1.5	13.62	57.41	10	1.1	4.1
f	1.5	2.4	0.7	2.0	9.92	47	20	0.6	2.6
g	1.6	1.0	0.7	4.0	10.74	76.75	0.0	1.7	5.3

In order to see the effect of Pade approximation, the first order Pade approximation is implemented on time delay part. The PI value is obtained by equating real and imaginary parts of characteristics equations with real and imaginary parts of dominant poles corresponding to ω_{cl} = 1.5 and ζ_{cl} = 0.7 (equivalent to controller parameters corresponds to point ***e***). The PI value obtained are K_p = -2.6853 and K_i = 1.2763, while in present approach where no approximation is taken in time delay parts positive values of PI were obtained i.e. K_p = 1 and K_i = 1.5.

2.4.2 Robustness analysis

Simulations have been also performed by varying parameters K, a and L of the process P_1 by ± 50% from the original values and the behavior of time response using the same controller parameters corresponding to points **c, d, e, f** and **g** (Fig. 2-2) is studied. Results are summarized in Table 2-2. Fig. 2-5(a) shows the time responses when values of K, a and L are increased by 50% i.e. K=1.5, a=1.5, and L=0.45s. It is to be noted that points ***f*** and ***g*** do not meet the desired GM and PM specifications. Point **c,** which was within the GM boundary earlier, is now outside the GM specification as a result of the perturbation in system parameters. Fig. 2-5 (b) shows the time responses when values of *K*, *a* and *L* are decreased by 50% (*K*=0.5*, a*=0.5*,* and *L*=0.15*s*).

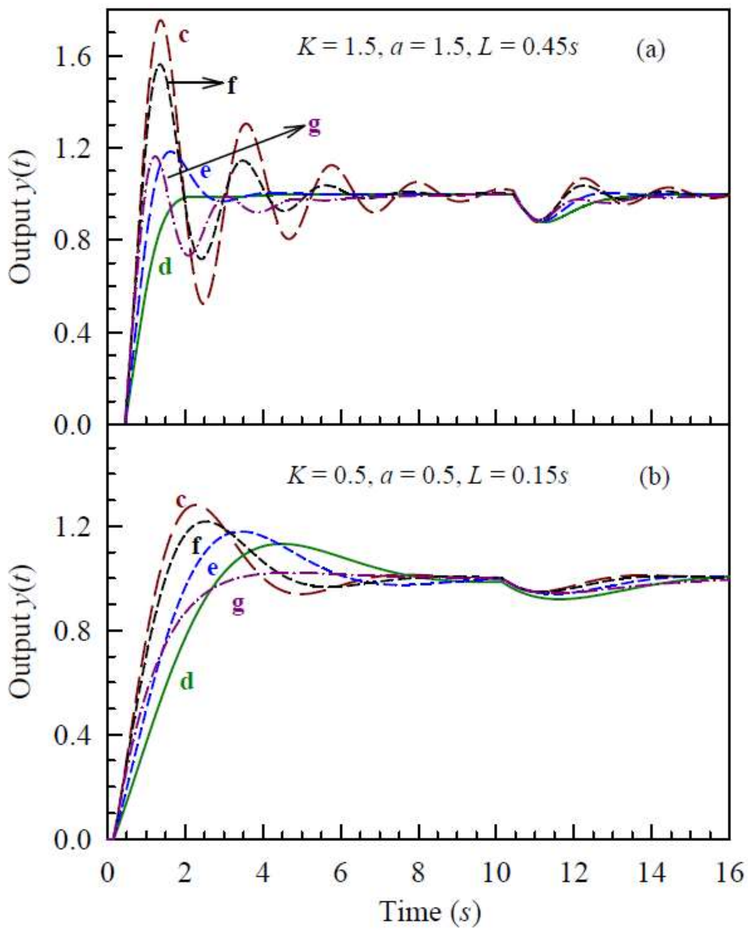


Figure 2-5:Time responses of process P_1 for the controller parameters of points **c, d, e, f** and **g** obtained with (a) 50% increase in the value of parameters (*K*=1.5*, a*=1.5*,* and *L*=0.45*s*.) (b) 50% decrease in the value of parameters (*K*=0.5*, a*=0.5*,* and *L*=0.15*s*).

It is easy to observe from Fig. 2-5 that for the same set of controller parameters, an increase in the value of system parameters leads to more oscillations in the time response as compared to the case when the system parameters are decreased from the original values.

Table 2-2: Time response performance measures of process P_1 when the process parameters are changed by ± 50% from the original values. (A) K=1.5, a=1.5, and L=0.45s. (B) K=0.5, a=0.5, and L=0.15s.

Points	Specified GM=6dBPM=45°				Simulated Results					
	K_p	K_i	ζ_{cl}	ω_{cl}(rad/s)		GM (dB)	PM (deg)	$\%OS$	$T_r(s)$	$T_s(s)$
c	1.5	3	0.5	2.5	A	2.56	18.48	75	0.5	10.5
					B	21.8	42.9	29	0.8	7
d	0.75	1	0.7	1.25	A	10.19	66.34	0	1.2	2
					B	28.34	57.6	12	2.4	8.2
e	1	1.5	0.7	1.5	A	7.335	51.32	20	0.6	2.8
					B	25.75	52.56	19	1.8	8.4
f	1.5	2.4	0.7	2	A	3.58	29.09	55	0.5	8
					B	25.75	52.56	19	1.8	8.4
g	1.6	1	0.7	4	A	3.58	29.08	18	0.6	8
					B	22.26	76.8	0	2	4.2

A comparison of results presented in Table 2-2, indicates that the controllers corresponding to parameters defined by points **d** and **e** (at comparatively lower value of ω_{cl}) are more robust under the perturbation of system parameters. The simulation is also performed keeping the value of K constant and varying the parameters aL. Parameters aL is known as laggardness [87-88] of the system. Results of simulation are shown in Fig.

2-6.The closed loop performance measures are given in Table 2-3. In the first case the value of *aL* is varied by ± 50% by changing only *L* and keeping *a* fixed. In the second case the value of *aL* is varied by changing only *a.* The time response is found to be less sensitive in the case of controller parameters ***d*** and ***e*** for the change in *aL* (either due to *a* or *L*) when the value of *L* is increased (*K*, *a* fixed) and when *a* is decreased (*K*, *L* fixed) when compared to controller parameters corresponding to other points.

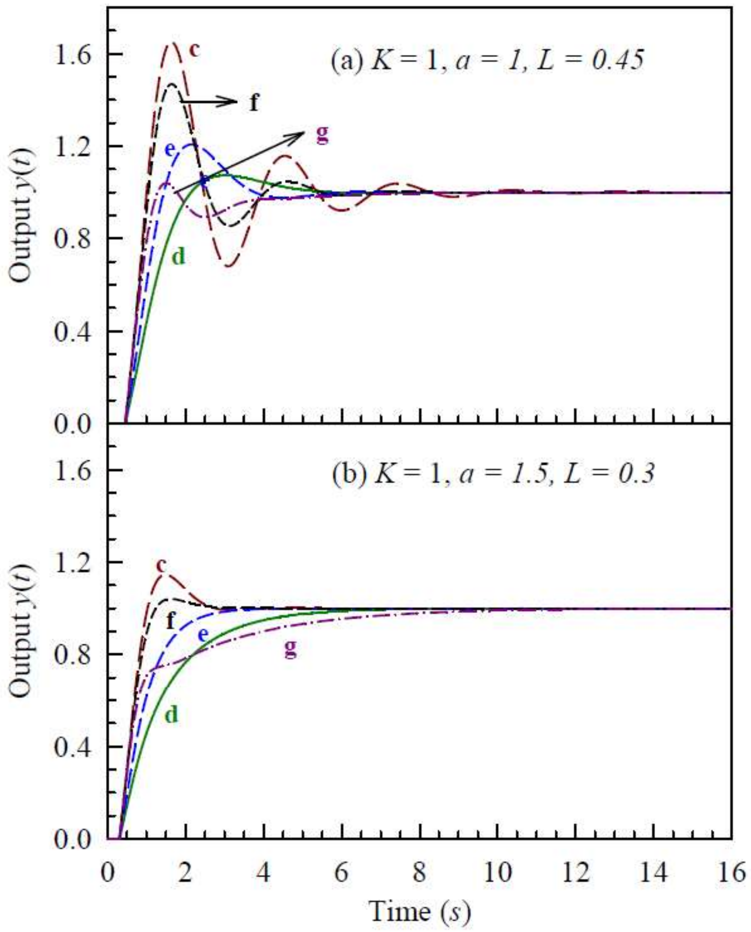


Figure 2-6: Time responses of process P_1 for the controller parameters of points **c**, **d**, **e**, **f** and **g** obtained at constant '*aL = 0.45* (increased by 50% with the original value)' and changing *a* and *L*(a) (*K*=1*, a*=1*,* and *L*=0.45 *s*.) (b) (*K*=1*, a*=1.5*,* and *L*=0.3*s*).

Simulations are also performed with controller parameters corresponding to point **d** by varying the values of both *a* and *L* by ± 20% and keeping *aL* constant (*aL* = 0.3). It is found that by changing *a* or *L* within 20% does not affect time response drastically and all the performance measures remain within specified value. A decrease in the value of *a* produces overshoot whereas an increase in *a* produces slow response.

Table 2-3: Time response performance measures of process P_1 at constant '*aL* = 0.45(increased by 50% with the original value)' and changing *a* and *L* (A) *K* = 1, *a* = 1, and *L* = 0.45*s*. (B) *K* = 1, *a* = 1, and *L* = 0.15*s*.

Points	Specified GM=6dBPM=45°				Simulated Results					
	K_p	K_i	ζ_{cl}	ω_{cl}(rad/s)		*GM* (dB)	*PM* (deg)	*%OS*	T_r(s)	T_s(s)
c	1.5	3	0.5	2.5	A	10.28	21.8	65	0.4	10
					B	23.17	52.01	10	0.4	2
d	0.75	1	0.7	1.25	A	28.93	58.9	5	1.4	5.3
					B	39.38	80.92	0	3.6	6
e	1	1.5	0.7	1.5	A	22.13	46.88	20	0.9	4.5
					B	33.11	72.81	0	2	3
f	1.5	2.4	0.7	2	A	13.34	31.9	45	0.4	6.4
					B	24.66	61.522	5	4.2	2.2
g	1.6	1	0.7	4	A	17.21	64.45	3	0.6	5
					B	26.05	95.62	0	6.2	8

2.4.3 Example 2: SOPTD process

Consider a non-minimum phase process defined by the transfer function [52]

$$G(s) = \frac{1-s}{(1+s)^2(2+s)} \; . \tag{2.31}$$

The corresponding over damped SOPTD model of the above process is

$$P_2 = \frac{1}{s^2+3s+2} e^{-1.64s} \; . \tag{2.32}$$

Following the formulations developed in section 2.3, the stabilizing sets of PID parameters are obtained to draw the stability boundary and constant GM and PM boundaries.

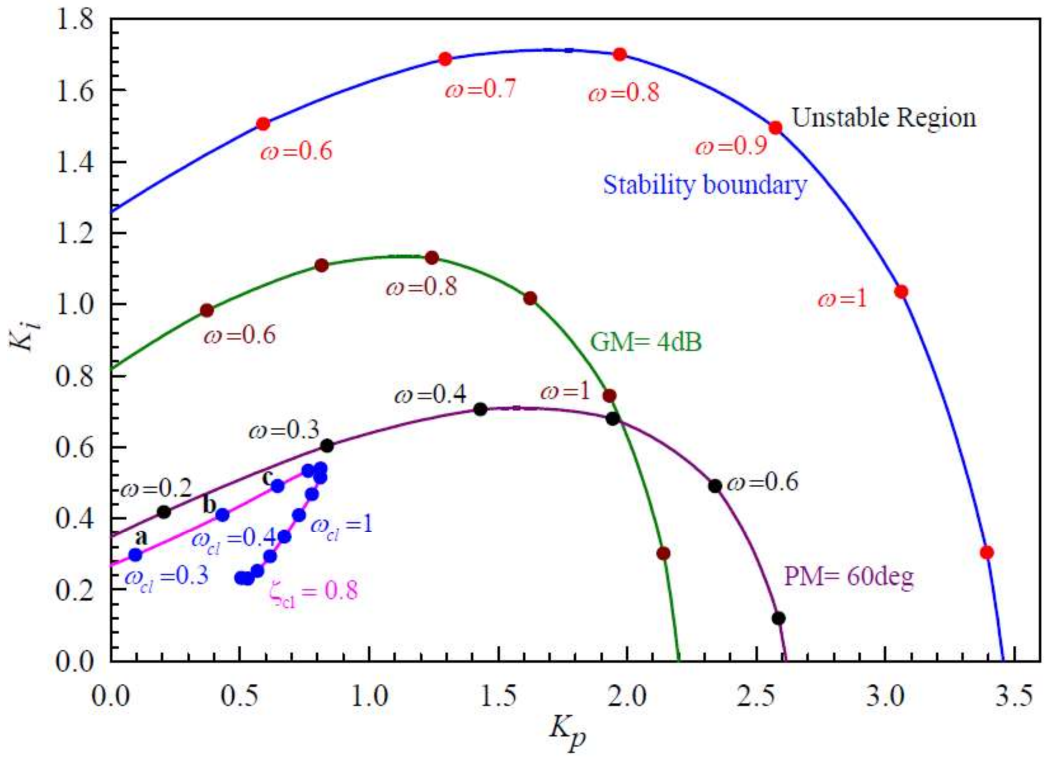


Figure 2-7: Stabilizing region, GM and PM boundaries and the range of K_p and K_i for process P2 using dominant pole placement with K_d = 0.2452.

Figure 2-7 shows the stabilizing regions and the range of controller parameters in (K_p, K_i) plane obtained via dominant pole placement with specified GM of 4 dB and PM of 60 deg. The sets of K_p and K_i are obtained using Eqs. (2.28) and (2.29). The value of K_d used in the calculation is obtained from Z-N stability criteria. These PID parameters are used for simulation of the actual process P_2 given by Eq. (2.31). Any another appropriate value of K_d can be chosen provided solution gives positive K_p and K_i with an improved time response. Figure 2-8(a) shows the time responses of process P_2 where K_d is chosen utilizing the different methods to obtain K_p and K_i at ζ_{cl}= 0.8andω_{cl}= 0.3rad/s. It can be readily seen that time responses are almost similar in all the cases except for Z-N criteria. For comparison, Fig. 2-8 (b) shows time responses with controller parameters obtained using Z-N, ISE, IAE, ITSE and ITAE methods. Table 2-4 summarizes various performance measures. A close inspection indicates that the controllers designed by other methods do not fulfill the desired PM and also produce comparatively large percentage overshoot. However, the rise and settling times are better as compared to the present method. In the present method there is a flexibility to tune the controller for desired performance measures within the set robustness by appropriately choosing ζ_{cl} and ω_{cl} (provided solution exits). This feature is not available with other methods discussed above.

Figure 2-9 presents time response and control effort of process P_2 for controllers indicated by points **a**, **b** and **c** in the (K_p, K_i) plane (see Fig. 2-7) for ζ_{cl} = 0.8 and at three values 0.3, 0.4 and 0.5 of ω_{cl}. Table 2-5 lists the K_p and K_i obtained using the present method with K_d = 0.2452 (Z-N method). Notice that controller parameters corresponding to point **c** are within the specified PM boundary; but simulation gives a lower value ~56.8 deg. This may be due to the fact that the simulation is performed with actual process given by Eq. (2.31) whereas GM and PM boundaries are drawn using the model of non-minimum phase process given by Eq. (2.32). The performance measures with other points are very close to the values specified by ζ_{cl} and ω_{cl}.

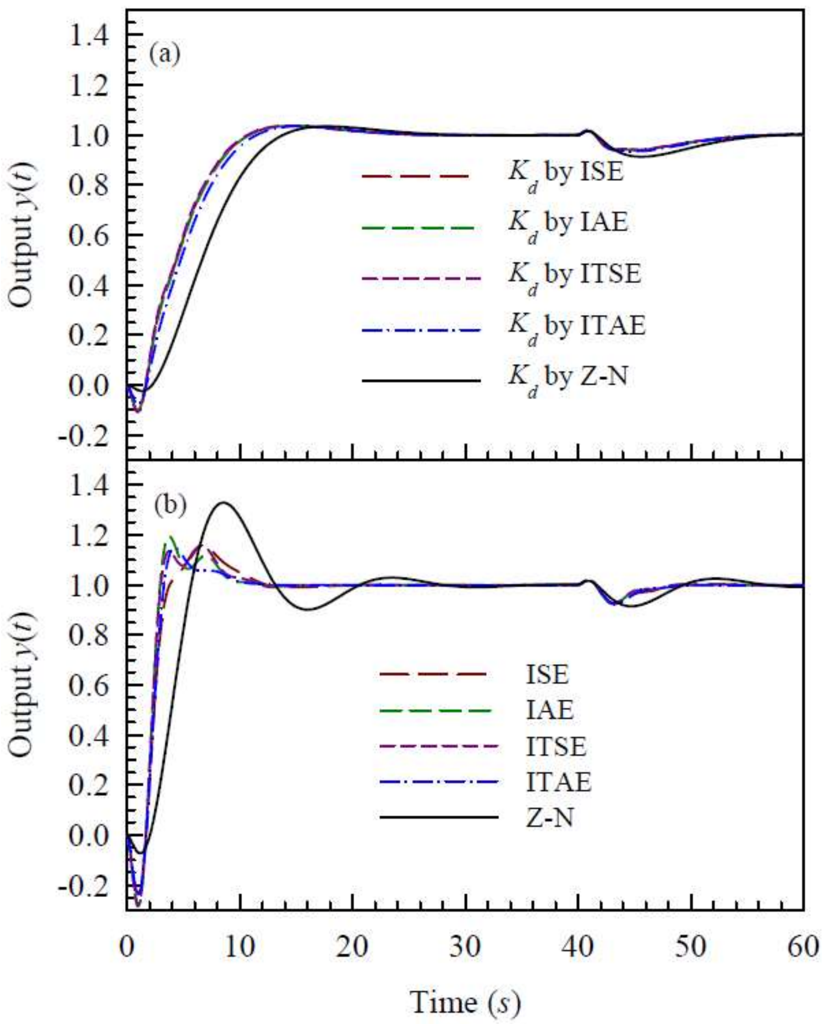


Figure 2-8:Closed loop time responses of process P2 : (a) using present method at different value of K_d, (b) using ISE, IAE, ITSE, ITAE and Z-N methods. In both cases a load disturbance of 20 % is applied at t = 40 s.

Table 2-4: Comparison of performance measures of process P_2 obtained with present method and ISE, IAE, ITSE, ITAE and Z-N methods.

Methods	Specified GM=4dB and PM=60°					Simulated Results				
	K_p	K_i	K_d	ζ_{cl}	ω_{cl}(rad/s)	GM (dB)	PM (deg.)	% OS	T_r(s)	T_s(s)
Present	0.8497	0.4378	1.815	0.8	0.3	4.86	79.2	4	8	24
ISE	1.834	0.9521		---	---	4.99	59.32	15	1	11
Present	0.8305	0.4342	1.775	0.8	0.3	5.06	78.8	4	8	24
IAE	2.217	1.035		---	---	4.44	55.56	20	1	11
Present	0.8924	0.4458	1.904	0.8	0.3	4.42	79.9	4	8	24
ITSE	2.133	1.048		---	---	4.26	56.9	15	1	11
Present	0.6548	0.4013	1.409	0.8	0.3	7.20	75.9	4	8	24
ITAE	2.01	0.909		---	---	5.78	57.28	15	1	11
Present	0.0962	0.2965	0.245	0.8	0.3	15.5	64	4	9	24
Z-N	0.499	0.699		---	---	7.91	40.32	32	3	29

Table 2-5: Time response performance measures of process P_2 with $K_d = 0.2452$.

Points	Specified GM=4dB and PM=60°				Simulated Results				
	K_p	K_i	ζ_{cl}	ω_{cl}(rad/s)	GM (dB)	PM (deg)	% OS	T_r(s)	T_s(s)
a	0.0962	0.2965	0.8	0.3	15.5	64	4	9	24
b	0.4335	0.4097	0.8	0.4	15.95	62.6	5	5	18
c	0.6468	0.5333	0.8	0.5	11.92	56.8	10	4	14

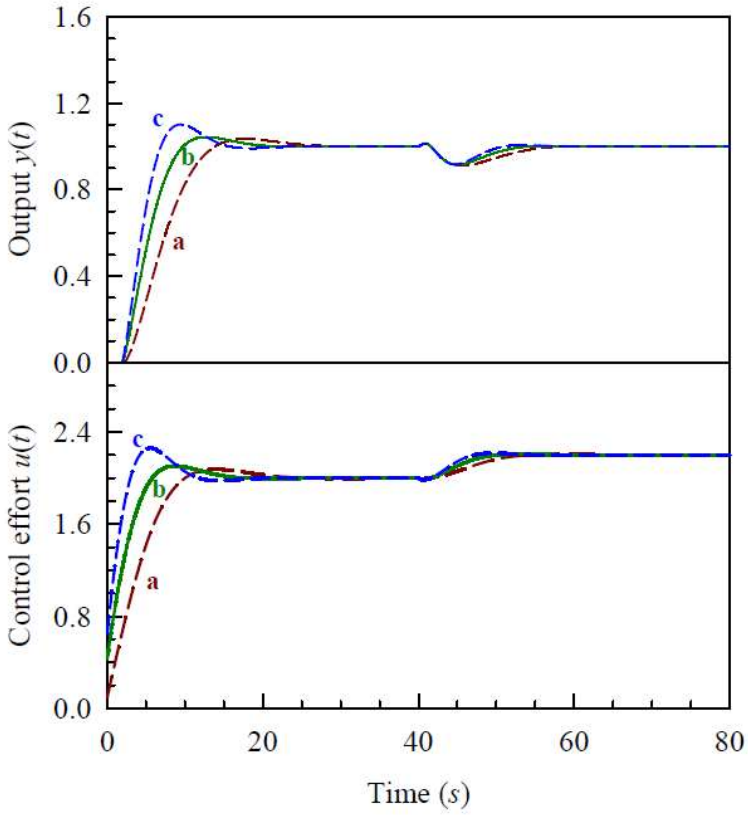


Figure 2-9: Time response and control effort of process P_2 corresponding to controller parameters indicated by points **a**, **b** and **c** with a load disturbance of 20% at t = 40s.

2.4.4 Example 3: SOPTD process of highly oscillatory behavior

Consider a highly oscillatory process [38] defined by

$$P_3 = \frac{1}{s^2 + s + 5} e^{-0.1s} \tag{2.33}$$

Let us design a PID controller with following requirements: the overshoot should not be more than 10%, the settling time should be below 15*s,* and the closed loop system should have at least 4 dB GM and 40 deg PM. The former two requirements demand $\zeta_{cl} \geq 0.6$ and $\omega_{cl} \geq 0.47$ rad/s. The different stabilizing regions and the range of parameters K_p and K_i using K_d= 8.8654(Z-N method) are shown in Fig. 2-10.

Table 2-6: Performance measures of process P_3 with K_d = 8.8654.

Points	Specified GM= 4dB PM= 40°				Simulated Results				
	K_p	K_i	ζ_{cl}	ω_{cl}(rad/s)	GM (dB)	PM (deg)	% OS	$T_r(s)$	$T_s(s)$
a	1.087	2.426	0.6	0.5	5.13	42.3	10	4.2	12.5
b	2.271	3.446	0.6	0.6	5.086	41.5	10	3.2	10.5
c	3.442	4.628	0.6	0.7	5.045	40.8	11	2.5	9
d	4.6	5.962	0.6	0.8	5.003	40.05	12	2	8

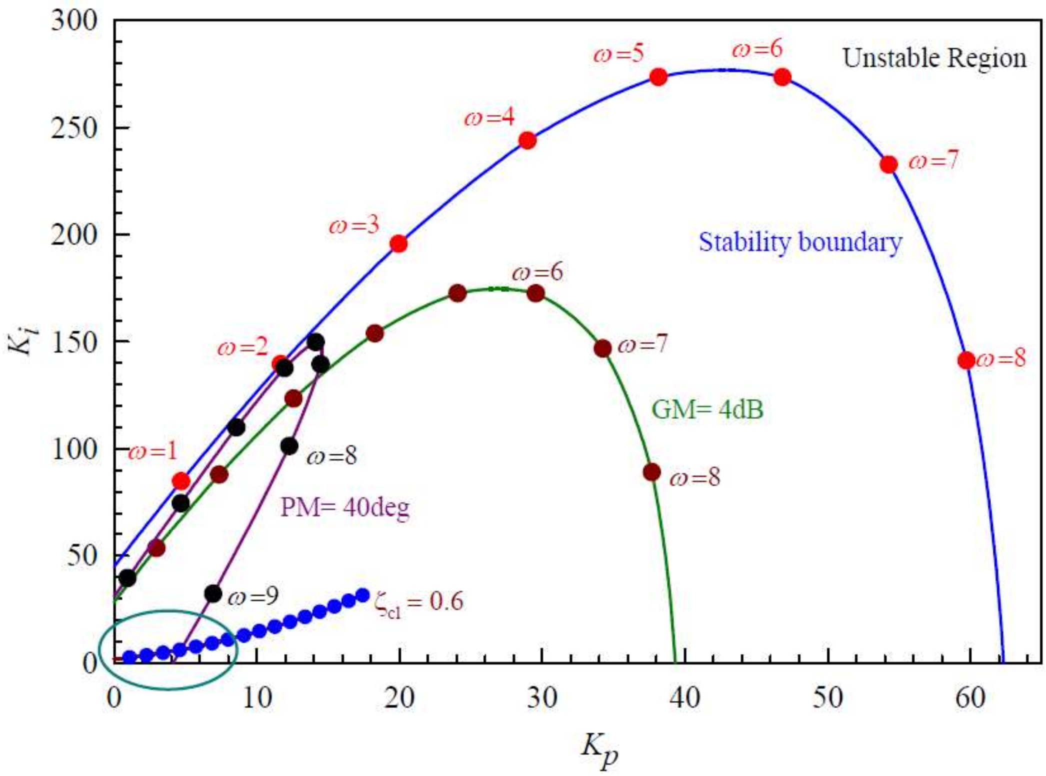


Figure 2-10: Stabilizing region, GM and PM boundaries and the range of K_p and K_i obtained for process P_3 via dominant pole placement with K_d = 8.8654.

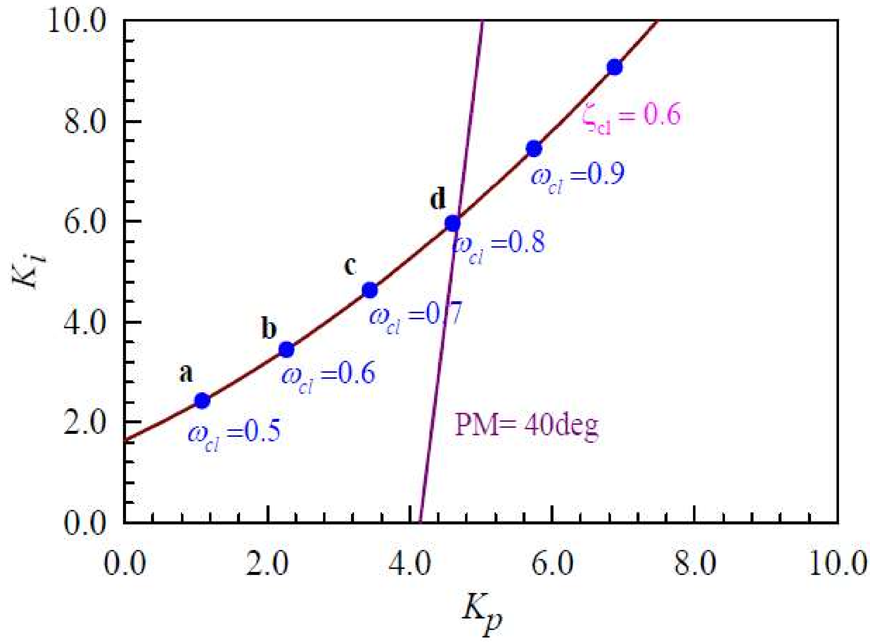


Figure 2-11: Zoomed view of the encircled region in the K_p-K_i plane.

Figure 2-11 shows the enlarged view of the encircled portion of Fig. 2-10. The marked points **a, b, c** and **d** on the curve ζ_{cl} = 0.6 are within the specified GM and PM boundaries. From the time responses of process P_3 shown in Fig. 2-12, it is easy to observe that controller parameters corresponding to these points reasonably satisfy the desired design criteria (see Table 2-6). Although the value of ζ_{cl} is same in all the cases, controller designed with higher value of ω_{cl}**,** gives comparatively more percentage overshoot. This may be due to the less phase margin available with higher value of ω_{cl} in the parameters plane.

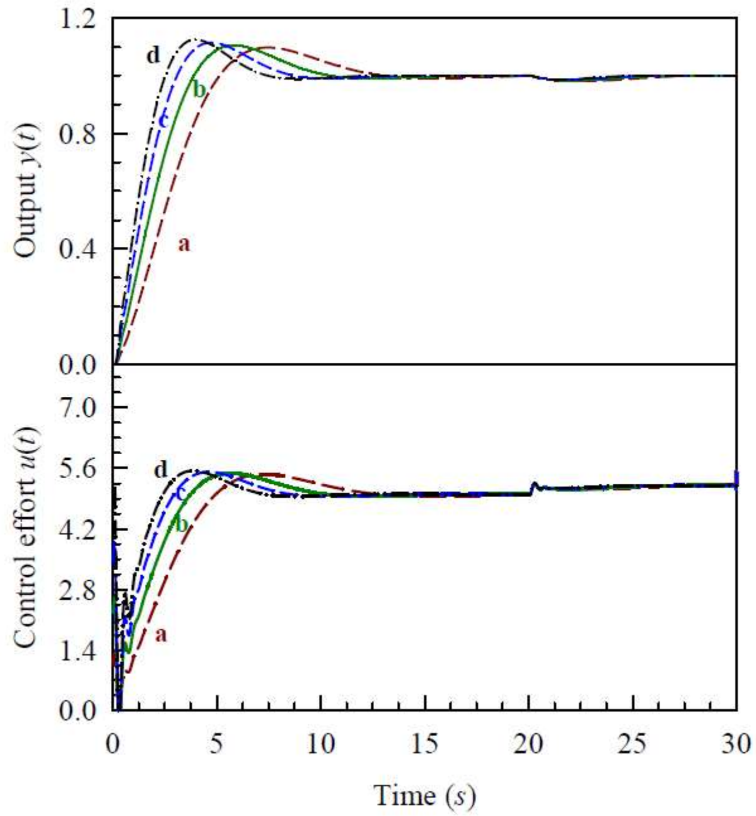


Figure 2-12: Time response and control effort of process of process P_3 for the controller parameters corresponding to points **a**, **b**, **c** and **d** with 20% load disturbance at $t = 20\ s$.

2.5 Summary

A new methodology is developed in this chapter to obtain the set of PI/PID parameters graphically for the specified closed loop time response measures using dominant pole placement with guaranteed GM and PM.

A graphical stability criterion has been utilized to get the stabilizing regions in the proportional-integral plane to choose sets of PI/PID controller parameters that would result in a closed loop system with of the dominant poles with pre-specified GM and PM. The range of parameters of PI controller for FOPTD system has been obtained by sweeping the

ω_{cl} for a fixed ζ_{cl}. To obtain the PID parameters for SOPTD system, first a predefined value of K_d is used and then the range of PI parameters is obtained. Simulations performed with one FOPTD system and two SOPTD systems demonstrated the effectiveness and confirmed the validity of the proposed methodology.

In the case of FOPTD system it is observed from the simulation that sets of K_p and K_i which are on the positive slope and linear part of fixed ζ_{cl} curve with increasing ω_{cl} in the parameter plane, are more appropriate and produce closed loop time response performance measures very close to the specified values. On the other hand, the sets of K_p and K_i which are on the negative slope of the above mentioned curve, give poor closed loop time response measure. At this point we would like to point out that the observed simulation results needs to be studied analytically. This opens the direction of future research in the design of PID controllers.

A non minimum phase process and a highly oscillatory process have been considered for numerical simulation in the case of SOPTD system. The value of K_d is chosen by Z-N rules. Simulation results indicate a high value of all the PID parameters in the case of highly oscillatory process as compared to that of the non minimum phase process. The choice of controller parameters corresponding to comparatively lower value of ω_{cl} is found more suitable for the robust controller design under the perturbation of system parameters. The main advantage of the present method is the flexibility to tune the controller for desired performance measures within set robustness by appropriately choosing the closed loop damping ratio and natural frequency.

CHAPTER 3

Optimal PID controller design using LQR and pole placement technique for standard SOPTD systems

3.1 Introduction

The design techniques based LQR are well known in modern control theory and have been widely used in many applications [61, 71, 52]. In a recent article Saha et al. [71] have obtained the PID controller parameters for second order systems via LQR using the dominant pole placement technique. However, their approach is applicable only for systems having no time delay. Most of the real industrial plants have time delay in their transfer function. Since the presence of time delay in a control loop is a source of instability and performance degradation [10, 60], it is therefore, necessary to design the PID controller optimally to achieve good stability. Many researchers have worked on the tuning of controller for the systems having time-delay [89-91] with pole placement and mentioned the challenges due to the presence of exponential term in the characteristics equation which leads to the infinite roots. They have used different approaches to design the controller with some limitations. He et al. [52] have proposed an analytical method to tune the PI/PID controller parameters in an optimal way using LQR techniques with user specified closed loop damping ratio and natural frequency for the FOPTD model. His method is based on the decomposition of state equation in two parts one for $t < L$ and another for $t \geq L$ in such a way that the state equation for $t \geq L$ becomes independent of L and then applied the usual LQR approach for obtaining the PI parameters for FOPTD. They have compared simulation results of their method with the gain-phase margin method [92] and presented much improved results.

Most of the real plants can be more closely approximated using SOPTD model as compared to FOPTD model. The SOPTD processes are very rich in dynamics as they include under damped, critically damped and over damped systems. Very few tuning rules are available for such processes. He et al. [52] have also extended their approach for SOPTD systems by equating the larger process pole with the derivative term of the PID controller and then applied the PI tuning approach using LQR to obtain other two parameters. This approach works satisfactory for SOPTD model if the system poles are real, but does not provide the optimum parameters of the PID controller as one of the parameters is prefixed. This technique cannot be applied for SOPTD systems with complex poles (such as highly oscillatory processes) of the system as they are always in pairs and cannot be eliminated with single complex zero of the controller.

In the present work we have combined the concept of LQR based PI/PID controller tuning method together with the dominant pole placement approach to derive the PID controller parameters analytically for SOPTD systems. It is shown that the present technique gives a good closed loop time response for various processes as compared with the existing PI/PID controller tuning methods using LQR. In order to illustrate the utility of the present technique, simulations performed in MATLAB [62] have been presented for different types of SOPTD models. These include critically damped and over-damped processes as well as processes having complex poles. The effect of non-dominant pole on the control signal and on the stability of the closed loop system has also discussed.

3.2 LQR based PID parameters for standard SOPTD systems

In this section the conventional pole placement method and technique of LQR have been ingeniously used for SOPTD systems where the time delay part is handled in the controller output equation instead of characteristic equation. This approach eases the use of pole placement for time delay systems without involving the exponential term in the characteristics equation as discussed in Chapter 2.

3.2.1 Converting delay based LQR problem to delay free

Consider a linear plant with time delay represented in state space as

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t - L) \tag{3.1}$$

where **A, B, X** and L are the system matrix, input matrix, state vector and the time delay term respectively. For $t < L$, no control signal will be effective and thus one has Eq. (3.1) as control free equation. Control signal will be effective only for $t \geq L$. So by decomposing Eq. (3.1) into two components, one for $t < L$ and other for $t \geq L$, we have

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t), \qquad 0 \leq t < L \quad , \tag{3.2}$$

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}^m(t), \qquad t \geq L \quad , \tag{3.3}$$

where, $\mathbf{u}^m(t) = \mathbf{u}(t - L)$. Since Eqs. (3.2) and (3.3) are now delay free, one can easily apply the standard LQR approach [61] used for delay free processes to find the optimum control vector $\mathbf{u}^m(t)$ subjected to the minimization of the cost function defined by

$$J = \int_0^\infty \left(\mathbf{X}^T(t)\, \mathbf{Q}\, \mathbf{X}(t) + \mathbf{u}^{m^T}(t)\, \mathbf{R}\, \mathbf{u}^m(t) \right) dt \quad , \tag{3.4}$$

where **Q** is the semi positive definite state weighting matrix and **R** is the positive definite control weighting matrix. The LQR solution gives the optimal control vector $\mathbf{u}^m(t)$ as

$$\mathbf{u}^m(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{X}(t), \tag{3.5}$$

where **P** is the symmetric positive definite Riccati coefficient matrix. It can be obtained by solving continuous time algebraic Riccati equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} = \mathbf{0}. \tag{3.6}$$

Putting the value of $t = t + L$, in Eq. (3.5) we can write

$$\mathbf{u}(t) = \mathbf{u}^m(t+L) = -\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}\mathbf{X}(t+L). \tag{3.7}$$

Here one can see that $\mathbf{u}(t)$ gives the control signal in the whole time horizon of $t \geq 0$, however $\mathbf{X}(t+L)$ is not directly known at time t. With the use of Eqs. (3.2), (3.3) and (3.5), $\mathbf{X}(t+L)$ can be expressed in terms of transmission of $\mathbf{X}(t)$ as

$$\mathbf{X}(t+L) = e^{(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P})t} e^{\mathbf{A}(L-t)}\mathbf{X}(t) \text{ for } 0 \leq t < L$$

and

$$\mathbf{X}(t+L) = e^{(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P})L}\mathbf{X}(t) \text{ for } t \geq L.$$

The optimal control vector $\mathbf{u}(t)$ for the present case, thus can be written as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{(\mathbf{A}_c)t}e^{\mathbf{A}(L-t)}\mathbf{X}(t), \qquad 0 \leq t < L, \tag{3.8}$$

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{(\mathbf{A}_c)L}\mathbf{X}(t), \qquad t \geq L \quad, \tag{3.9}$$

where , $\mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}$. (3.10)

The beauty of the above mathematical formulation lies in the fact that the optimal control vector $\mathbf{u}(t)$ handles the delay part as given by Eqs. (3.8) and (3.9). As the system matrix A_c

given by Eq. (3.10) does not contain any time delay for $t \geq L$, one can easily apply the approach of direct pole placement to get `the desired closed loop time performance measures.

3.2.2 Optimal control using pole placement and LQR

In order to obtain the optimal feedback gain $\mathbf{u}(t)$ we need to calculate $e^{(\mathbf{A_c})t}$ and $e^{\mathbf{A}(L-t)}$. By substituting $\mathbf{u}^m(t)$ from Eq. (3.5) into Eq. (3.3) we have for $t \geq L$,

$$\dot{\mathbf{X}}(t) = \mathbf{A_c}\mathbf{X}(t)\,. \tag{3.11}$$

The matrix $\mathbf{A_c}$ can be determined by setting the characteristic equation of the closed loop system $\Delta(s) = |s\mathbf{I} - \mathbf{A_c}|$ equal to the desired closed loop equation. For example, in the case of FOPTD process, where the matrix $\mathbf{A_c}$ is a 2×2 matrix, we have

$$\Delta(s) = |s\mathbf{I} - \mathbf{A_c}| = (s + p_1)(s + p_2) = (s^2 + 2\varsigma_{cl}\omega_{cl}s + \omega_{cl}{}^2), \tag{3.12}$$

where, $p_1 = \zeta_{cl}\omega_{cl} + i\omega_{cl}\sqrt{1-\zeta_{cl}^2}$, $p_2 = \zeta_{cl}\omega_{cl} - i\omega_{cl}\sqrt{1-\zeta_{cl}^2}$,

with ζ_{cl} and ω_{cl} as the desired closed loop damping ratio and natural frequency.

For the SOPTD process the dimension of matrix $\mathbf{A_c}$ will be 3×3. Utilizing the help of dominant pole placement technique, matrix $\mathbf{A_c}$ can be evaluated in terms of known parameters ζ_{cl} and ω_{cl} from the equation

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}_c| &= (s + p_1)(s + p_2)(s + p_3) \\ &= (s + m\zeta_{cl}\omega_{cl})(s^2 + 2s\zeta_{cl}\omega_{cl} + \omega_{cl}^2). \end{aligned} \tag{3.13}$$

The location of non-dominant pole $p_3 = m\zeta_{cl}\omega_{cl}$ is placed m times away from the real part of the dominant poles of the closed loop system. Let's call this m as the relative dominance and as per the literature its value should be chosen around 3 or more [50].

3.2.3 Determination of matrices A, B, X, Q, R and P for SOPTD systems and its relation with optimal PID parameters

In the case of second order process, matrices **Q**, **R** and **P** are generally taken as

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, \quad \mathbf{R} = [r], \quad \mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}. \tag{3.14}$$

In the optimal control it is a standard practice to design regulator by varying **Q** and keeping **R** fixed [52, 71]. A schematic of closed loop system with PID controller for SOPTD process is shown in Fig. 3-1. The state variables for the present case are

$$\mathbf{X}(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T, \tag{3.15}$$

where, $x_1(t) = \int e(t)\,dt$, $x_2(t) = e(t)$, $x_3(t) = \dfrac{de(t)}{dt}$, (3.16)

with error $e(t) = r(t) - y(t)$. Here $r(t)$ and $y(t)$ are the reference and output signals respectively. The control signal can be expressed in terms of the state variable as

$$u(t) = K_p x_2(t) + K_i x_1(t) + K_d x_3(t). \tag{3.17}$$

The transfer function of the PID controller can be express in s domain as

$$C(s) = K_p + \frac{K_i}{s} + K_d s. \tag{3.18}$$

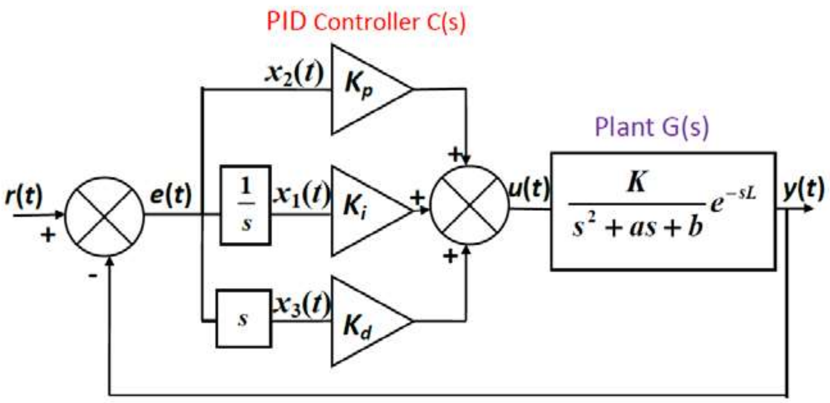


Figure 3-1: Schematic of closed loop system with PID controller.

In the case of unity output feedback system such as shown in Fig. 3-1, if we put the reference signal $r(t)=0$, we have $e(t)=-y(t)$. with this condition, the second order transfer function with time delay can be written as

$$G(s)=\frac{y(s)}{u(s)}=\frac{K\,e^{-sL}}{s^2+as+b}=\frac{-e(s)}{u(s)}, \tag{3.19}$$

Here, $a=2\zeta_{ol}\omega_{ol}$ and $b=\omega_{ol}^2$, where ζ_{ol} and ω_{ol} are the damping ratio and natural frequency of the open loop plant respectively. Using Eq. (3.16) one can express Eq. (3.19) in terms of state variables as $\dot{x}_3(t)=-ax_3(t)-bx_2(t)-Ku(t-L)$.

In terms of state-space formulation the derivative of the state variables can be written as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -b & -a\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ -K\end{bmatrix}u(t-L)\,. \tag{3.20}$$

Comparing Eq. (3.20) with Eq. (3.1), it is straightforward to obtain matrices **A** and **B** as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -b & -a \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix}. \tag{3.21}$$

Using Eqs. (3.10), (3.14) and (3.21)

$$\left| s\mathbf{I} - \mathbf{A}_c \right| = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ \eta p_{13} & b + \eta p_{23} & s + a + \eta p_{33} \end{vmatrix}. \tag{3.22}$$

where $\eta = r^{-1}K^2$. Now from Eqs. (3.22) and (3.13)

$$\begin{aligned} & s^3 + (a + \eta p_{33})s^2 + (b + \eta p_{23})s + \eta p_{13} = \\ & s^3 + ((2+m)\zeta_{cl}\omega_{cl})s^2 + (\omega_{cl}{}^2 + 2m\zeta_{cl}{}^2\omega_{cl}{}^2)s + m\zeta_{cl}\omega_{cl}{}^3 \ . \end{aligned} \tag{3.23}$$

By comparing the coefficients of powers of s from both sides of Eq. (3.23), the elements p_{13}, p_{23} and p_{33} can be obtained as

$$p_{13} = \frac{m\zeta_{cl}\omega_{cl}^3}{\eta},$$

$$p_{23} = \frac{\omega_{cl}^2 + 2m\zeta_{cl}^2\omega_{cl}^2 - b}{\eta},$$

$$p_{33} = \frac{(2+m)\zeta_{cl}\omega_{cl} - a}{\eta}. \tag{3.24}$$

The remaining three elements of the matrix **P** and three elements of the matrix **Q** can be obtained by solving Riccati equation Eq. (3.6), which gives six equations for six variables in terms of known parameters. With some algebraic manipulations, one can obtain

$$p_{11} = \frac{m\zeta_{cl}\omega_{cl}^5(1 + 2m\zeta_{cl}^2)}{\eta},$$

$$p_{12} = \frac{(2+m)m\zeta_{cl}^2\omega_{cl}^4}{\eta},$$

$$p_{22} = \frac{2\omega_{cl}^3(\zeta_{cl} + 2m\zeta_{cl}^3 + m^2\zeta_{cl}^3) - ab}{\eta},$$

$$q_1 = \frac{m^2\zeta_{cl}^2\omega_{cl}^6}{\eta},$$

$$q_2 = \frac{\omega_{cl}^4(1 + 4m^2\zeta_{cl}^4 - 2m^2\zeta_{cl}^2) - b^2}{\eta},$$

$$q_3 = \frac{\omega_{cl}^2(4\zeta_{cl}^2 + m^2\zeta_{cl}^2 - 2) + 2b - a^2}{\eta} \tag{3.25}$$

3.2.4 Evaluation of matrix $e^{\mathbf{A}(L-t)}$

The value of $e^{\mathbf{A}(L-t)}$ can be evaluated as follows.

$$e^{\mathbf{A}(L-t)} = \text{Laplace inverse}\left(\left[(s\mathbf{I} - \mathbf{A})^{-1}\right]_{t=L-t}\right)$$

$$= \ell^{-1}\left(\begin{bmatrix} \frac{1}{s} & \frac{(s+a)}{s(s+p_{01})(s+p_{02})} & \frac{1}{s(s+p_{01})(s+p_{02})} \\ 0 & \frac{s+a}{(s+p_{01})(s+p_{02})} & \frac{1}{(s+p_{01})(s+p_{02})} \\ 0 & \frac{-b}{(s+p_{01})(s+p_{02})} & \frac{s}{(s+p_{01})(s+p_{02})} \end{bmatrix}\right)$$

$$= \begin{bmatrix} f'_{11}(t) & f'_{12}(t) & f'_{13}(t) \\ f'_{21}(t) & f'_{22}(t) & f'_{23}(t) \\ f'_{31}(t) & f'_{32}(t) & f'_{33}(t) \end{bmatrix}_{t=L-t}. \tag{3.26}$$

Here p_{01} and p_{02} are the poles of the open loop system (see Eq. (3.19)) given by

$$p_{01} = \frac{a - \sqrt{a^2 - 4b}}{2}, \qquad p_{02} = \frac{a + \sqrt{a^2 - 4b}}{2}, \tag{3.27}$$

Using partial fraction approach $f'_{11}, f'_{12}, f'_{13}$, $f'_{21}, f'_{22}, f'_{23}, f'_{31}, f'_{32}$ and f'_{33} can be evaluated as

$$f'_{11}(L-t) = 1\,,$$

$$f'_{12}(L-t) = \frac{p_{02}e^{-p_{01}(L-t)}}{p_{01}(p_{01}-p_{02})} - \frac{p_{01}e^{-p_{02}(L-t)}}{p_{02}(p_{01}-p_{02})} + \frac{a}{b}\,,$$

$$f'_{13}(L-t) = \frac{e^{-p_{01}(L-t)}}{p_{01}(p_{01}-p_{02})} - \frac{e^{-p_{02}(L-t)}}{p_{02}(p_{01}-p_{02})} + \frac{1}{b}\,,$$

$$f'_{21}(L-t) = 0\,,$$

$$f'_{22}(L-t) = \frac{-p_{02}e^{-p_{01}(L-t)}}{(p_{01}-p_{02})} + \frac{p_{01}e^{-p_{02}(L-t)}}{(p_{01}-p_{02})}\,,$$

$$f'_{23}(L-t) = -\frac{e^{-p_{01}(L-t)}}{(p_{01}-p_{02})} + \frac{e^{-p_{02}(L-t)}}{(p_{01}-p_{02})}\,,$$

$$f'_{31}(L-t) = 0\,,$$

$$f'_{32}(L-t) = \frac{be^{-p_{01}(L-t)}}{(p_{01}-p_{02})} - \frac{be^{-p_{02}(L-t)}}{(p_{01}-p_{02})}\,,$$

$$f'_{33}(L-t) = \frac{p_{01}e^{-p_{01}(L-t)}}{(p_{01}-p_{02})} - \frac{p_{02}e^{-p_{02}(L-t)}}{(p_{01}-p_{02})}\,. \tag{3.28}$$

3.2.5 Evaluation of matrix $e^{(A_c)t}$

Using Eqs. (3.10), (3.14) and (3.21) have

$$\mathbf{A_c} = (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\gamma & -\beta & -\alpha \end{bmatrix}.$$

where, $\gamma = \eta\, p_{13}$, $\alpha = a + \eta\, p_{33}$ and $\beta = b + \eta\, p_{23}$.

Now,

$$e^{(\mathbf{A}_c)t} = \ell^{-1}\left[(s\mathbf{I} - \mathbf{A}_c)^{-1}\right]$$

$$= \ell^{-1}\left(\frac{1}{|s\mathbf{I}-\mathbf{A_c}|} \begin{bmatrix} s^2+\alpha s+\beta & s+\alpha & 1 \\ -\gamma & s^2+\alpha s & s \\ -s\gamma & -s\beta-\gamma & s^2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} f_{11}(t) & f_{12}(t) & f_{13}(t) \\ f_{21}(t) & f_{22}(t) & f_{23}(t) \\ f_{31}(t) & f_{32}(t) & f_{33}(t) \end{bmatrix}. \tag{3.29}$$

Using Eq. (3.13) in Eq. (3.29) and with some algebraic manipulations, it is straightforward to get f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32} and f_{33} as

$$f_{11}(t) = \sum_{i=1}^{3} \frac{p_i^2 - \alpha p_i + \beta}{D_i} e^{-p_i t} ,$$

$$f_{12}(t) = \sum_{i=1}^{3} \frac{-p_i + \alpha}{D_i} e^{-p_i t} ,$$

$$f_{13}(t) = \sum_{i=1}^{3} \frac{1}{D_i} e^{-p_i t} ,$$

$$f_{21}(t) = \sum_{i=1}^{3} \frac{-\gamma}{D_i} e^{-p_i t} ,$$

$$f_{22}(t) = \sum_{i=1}^{3} \frac{p_i^{\,2} - \alpha p_i}{D_i} e^{-p_i t} ,$$

$$f_{23}(t) = \sum_{i=1}^{3} \frac{-p_i}{D_i} e^{-p_i t} ,$$

$$f_{31}(t) = \sum_{i=1}^{3} \frac{\gamma p_i}{D_i} e^{-p_i t} ,$$

$$f_{32}(t) = \sum_{i=1}^{3} \frac{\beta p_i - \gamma}{D_i} e^{-p_i t} ,$$

$$f_{33}(t) = \sum_{i=1}^{3} \frac{p_i^{\,2}}{D_i} e^{-p_i t} , \tag{3.30}$$

where, $D_1 = -(p_1 - p_2)(p_3 - p_1)$, $D_2 = -(p_1 - p_2)(p_2 - p_3)$ and $D_3 = -(p_3 - p_1)(p_2 - p_3)$.

3.2.6 Evaluation of PID parameters for $0 \le t < L$,

Using Eqs. (3.28) and (3.30) in Eq. (3.8), the optimal value of control $\mathbf{u}(t)$ for $0 \le t < L$, can be expressed as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{\mathbf{A}_c t}e^{\mathbf{A}(L-t)}\mathbf{X}(t) \ ,$$

$$= r^{-1}K\begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}^T \begin{bmatrix} f_{11}(t) & f_{12}(t) & f_{13}(t) \\ f_{21}(t) & f_{22}(t) & f_{23}(t) \\ f_{31}(t) & f_{32}(t) & f_{33}(t) \end{bmatrix} \begin{bmatrix} f'_{11}(L-t) & f'_{12}(L-t) & f'_{13}(L-t) \\ f'_{21}(L-t) & f'_{22}(L-t) & f'_{23}(L-t) \\ f'_{31}(L-t) & f'_{32}(L-t) & f'_{33}(L-t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} . \tag{3.31}$$

Comparing the coefficients of $x_1(t)$, $x_2(t)$ and $x_3(t)$ in Eqs. (3.17) and (3.31), one can easily obtain the PID parameters for $0 \le t < L$,as

$$K_i(t) = r^{-1}K(p_{13}\sum_{i=1}^{3} f_{1i}(t)f'_{i1}(L-t) + p_{23}\sum_{i=1}^{3} f_{2i}(t)f'_{i1}(L-t) + p_{33}\sum_{i=1}^{3} f_{3i}(t)f'_{i1}(L-t)) \ ,$$

$$K_p(t) = r^{-1}K(p_{13}\sum_{i=1}^{3} f_{1i}(t)f'_{i2}(L-t) + p_{23}\sum_{i=1}^{3} f_{2i}(t)f'_{i2}(L-t) + p_{33}\sum_{i=1}^{3} f_{3i}(t)f'_{i2}(L-t)) \ ,$$

$$K_d(t) = r^{-1}K(p_{13}\sum_{i=1}^{3} f_{1i}(t)f'_{i3}(L-t) + p_{23}\sum_{i=1}^{3} f_{2i}(t)f'_{i3}(L-t) + p_{33}\sum_{i=1}^{3} f_{3i}(t)f'_{i3}(L-t)) \ . \tag{3.32}$$

3.2.7 Evaluation of PID parameters for $t \ge L$

Using Eqs. (3.9) and (3.30) the optimal control $\mathbf{u}(t)$ for $t \ge L$ can be evaluated as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{\mathbf{A}_c L}\mathbf{X}(t) \ ,$$

$$= r^{-1}K\begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}^T \begin{bmatrix} f_{11}(L) & f_{12}(L) & f_{13}(L) \\ f_{21}(L) & f_{22}(L) & f_{23}(L) \\ f_{31}(L) & f_{32}(L) & f_{33}(L) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}. \tag{3.33}$$

Comparison of coefficients of $x_1(t)$, $x_2(t)$ and $x_3(t)$ in Eqs. (3.33) and (3.17) gives the PID parameters for $t \geq L$ as

$$K_i = r^{-1}K\left(p_{13}f_{11}(L) + p_{23}f_{21}(L) + p_{33}f_{31}(L)\right),$$

$$K_p = r^{-1}K\left(p_{13}f_{12}(L) + p_{23}f_{22}(L) + p_{33}f_{32}(L)\right),$$

$$K_d = r^{-1}K\left(p_{13}f_{13}(L) + p_{23}f_{23}(L) + p_{33}f_{33}(L)\right). \tag{3.34}$$

Note that for L=0, the matrix elements f_{ij} = 1 when $i = j$ and f_{ij} = 0 for $i \neq j$ and Eq. (3.34) leads to the optimal PID parameters for systems having no time delay.

3.3 Simulation results and discussion

In order to demonstrate the application of the PID tuning methodology proposed in this Chapter, simulation results for different processes performed using MATLAB will be presented. The examples considered are under damped, critically damped and over damped SOPTD processes. Plants with unstable open loop response and highly oscillatory behavior are also discussed using present approach considering the challenge of present day's requirement in the control industries.

3.3.1 Example 1: Non Minimum Phase Process with delay

Let consider an over damped SOPTD model of a non minimum phase process. The closed loop time response is compared with the previously developed LQR-based PI/PID tuning method [52], where the derivative term of the PID controller for SOPTD process is set equal to one of the process pole and thus is not obtained in an optimum way. For fair comparison, similar values of ζ_{cl} = 0.8 and ω_{cl} = 0.793 rad/s are taken in the simulation. The non-dominant pole is placed 6 times away from the desired dominant real poles i.e. m = 6.

The transfer function of the non minimum phase process considered here is

$$\mathrm{P1} = \frac{1-s}{(1+s)^2(2+s)}\ , \tag{3.35}$$

and its corresponding over damped SOPTD model is

$$\mathrm{P1} = \frac{1}{s^2+3s+2}e^{-1.64s}\ . \tag{3.36}$$

Matrices **A** and **B** can be obtained from Eqs. (3.36) and (3.21) as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}. \tag{3.37}$$

Using Eqs. (3.24) and (3.25), matrices **P** and **Q** with **R** = [1] can be evaluated as

$$\mathbf{Q} = \begin{bmatrix} 5.7158 & 0 & 0 \\ 0 & 1.4889 & 0 \\ 0 & 0 & 9.8290 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 13.0394 & 12.1288 & 2.3908 \\ 12.1288 & 19.2785 & 3.4540 \\ 2.3908 & 3.4540 & 2.0732 \end{bmatrix}. \tag{3.38}$$

The eigen values of matrices **P** and **Q** are

$$eig[\mathbf{P}] = \begin{bmatrix} 1.4050 \\ 3.6579 \\ 29.3282 \end{bmatrix} \text{ and } eig[\mathbf{Q}] = \begin{bmatrix} 5.7158 \\ 1.4889 \\ 9.8290 \end{bmatrix}. \tag{3.39}$$

The positive eigen values of matrices **P** and **Q** indicate that the positive definite condition of LQR is satisfied. Finally, the PID parameters for $0 \le t < 1.64s$ can be obtain using Eq.

(3.32). The time varying PID parameters are plotted in Fig. 3-2. The PID parameters for $t \geq 1.64\,s$ can be calculated using Eq. (3.34) as

$$[K_p \quad K_i \quad K_d] = [0.6984 \quad 0.4602 \quad 0.1543]. \tag{3.40}$$

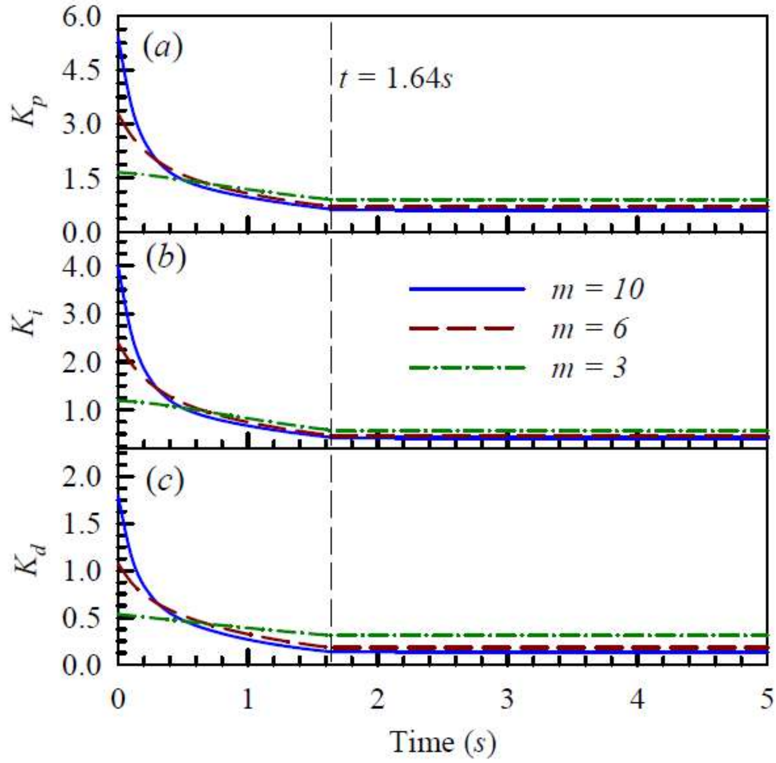


Figure 3-2: PID parameters K_p, K_i and K_d as a function of time.

Figure 3-2 shows the variation of PID parameters used in the simulation at $m = 6$. For comparison, PID parameters evaluated at $m = 3$ and $m = 10$ are also shown. It is clear that values of all the PID parameters are very high at the beginning (t = 0s), followed by a decrease with t up to $t = 1.64s$ and then remain constant thereafter. Note that design of PID controller at higher value of m leads to lower values for all the PID parameters for $t \geq 1.64\,s$ whereas the situation is completely reverse in the case of $t < 1.64\,s$.

The time response of the time varying PID controller parameters with step input for process P1 with 20% disturbance at $t = 40s$ is shown in Fig. 3-3 by solid black line. It can be noted that controller with time varying parameters needs higher control effort than any other constant parameters. The observed behavior of the closed loop time response during the initial period is due to the high values of initial controller parameters, which are responsible for the decrease in the system rise time and hence enhancement in the overshoot. Note that PID controller parameters between $0 \le t < L$ are time varying and large initially. This leads to a comparatively larger control efforts and may cause the actuator saturation in some cases. It is also difficult to implement them practically, particularly in analog domain. It is obvious that a choice of constant PID controller parameters throughout eases the practical implementation, needs low control effort and maintains the state optimality for all values of $t \ge L$. The plots of time response using only constant PID parameters throughout (i.e. for $t \ge 0$) obtained for $t \ge 1.64s$ using Eq. (3.34) for various values of relative dominance m are also shown in Fig. 3-3 for comparison. In the simulation all other parameters have been kept constant. It can be seen from Fig. 3-3 that as the value of m is decreased from 6 to 3; the PID controller based on constant parameters tries to cope with the actual time varying PID controller and produces overshoot with improved rise time.

An increase in the value of m reduces the overshoot but at the same time increases the rise time. At higher values of m, say around m = 50, this effect saturates and further increase in m has no significant effect on the time response. Thus the choice of m depends upon a particular requirement whether one needs fast rise time or less overshoot. In our experience a good choice for m is between 3 to10.

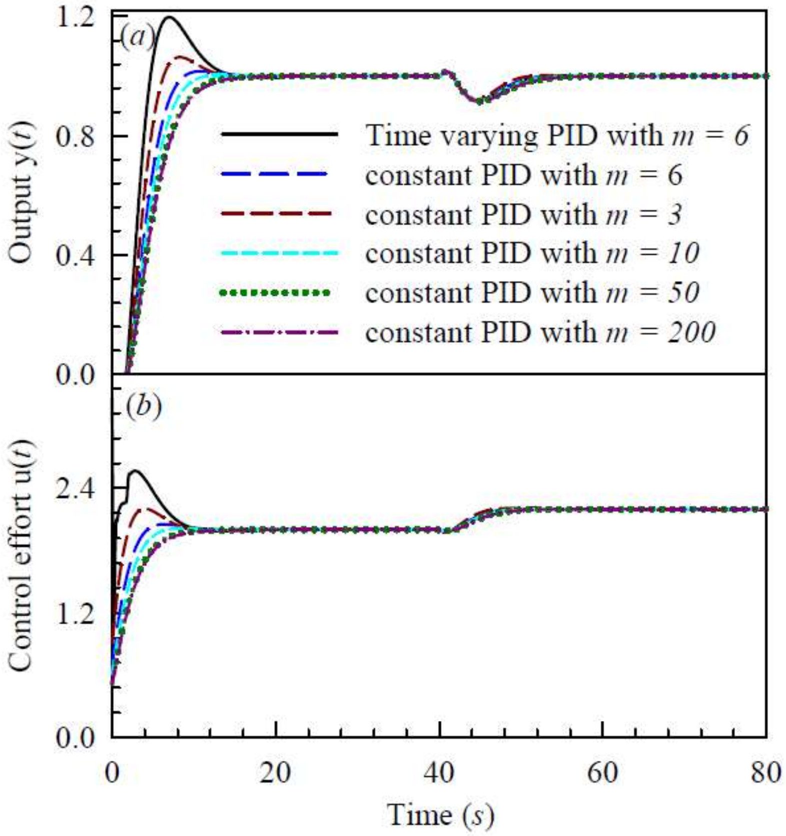


Figure 3-3: Comparison of (a) time responses and (b) control efforts of time varying PID with constant PID parameters for process P1 at different *m*.

It is interesting to point out here that increase in the rise time with *m* in the delayed processes is just opposite to the LQR based PID tuning with no delay [71], where an increase in *m* decreases the rise time of the closed loop time response. For a given process, our simulation results indicate that an increase in the value of *m* results in the lower values of PID controller parameters as shown in Fig. 3-2 and thus a reduction in the control effort. This fact can also be explained using Eqs. (3.9), (3.24) and (3.25) where an increase in *m* increases the value of matrix elements of **P.** This finally causes reduction in the control effort **u**(t) due to the presence of the term $\mathbf{P}e^{(\mathbf{A}_c)L}$ which decreases with increase in the value of elements of matrix **P**. Note that in the case of delay free process $e^{(\mathbf{A}_c)L} = 1$ and **u**(t) is proportional to matrix **P.**

Observing the simulation results shown in Fig. 3-3, it appears that the time varying part of PID parameters though, improves the rise time of the closed loop time response, but at the same time it produces substantial overshoot as compared to the cases where only constant PID parameters are used. As it will be easy to implement practically, therefore, in subsequent examples consider only constant value of PID parameters evaluated for $t \geq L$ using Eq. (3.34).

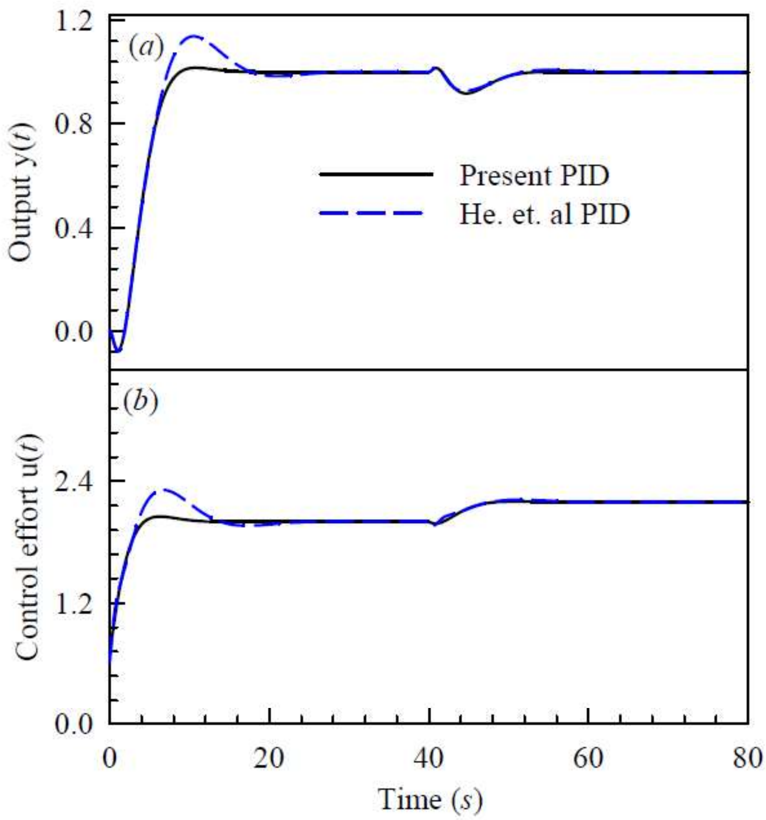


Figure 3-4: Time response and controller response for process P1 with 20% disturbance at $t = 40s$.

To shows the effectiveness of the present method, we now compare our results with those of the previously developed LQR based PI/PID tuning method at same values of closed loop damping ratio and natural frequency. The optimal PID controller for process P1 with $\zeta_{cl} = 0.8$, $\omega_{cl} = 0.793$ rad/s and $m = 6$ obtained for $t \geq 1.64s$ is

$$C_1[\text{present}] = 0.6984 + \frac{0.4602}{s} + 0.1543\, s\,,$$

and the PID controller used in Ref. [52] is given by

$$C_1[51] = 0.6138 + \frac{0.5561}{s} + 1.0\, s\,.$$

Figure 3-4(a) compares the step responses with 20% disturbance at $t = 40s$. It is easy to observe that the present method gives very less overshoot, only 4% as compared to the 14% of the earlier method. Note that the value of derivative gain in controller $C_1[51.]$ is larger than the controller $C_1[\text{present}]$. From the simulation it is clear that a choice of larger derivative gain does not necessarily reduce the overshoot. The main reason for the reduction in overshoot is the optimal tuning of derivative parameter K_d in the present case is taken as one of the real pole of the open loop system in earlier case and thus, is not optimum one. Due to the optimal design of all the three parameters in the present method, there is almost 46% reduction in the settling time together with a substantial reduction in the overshoot. Figure 3-4(b) compares the control energy required to achieve good closed loop time response. Since the cost function is optimized properly in the present method, the required control energy is also less. PID controller parameters and closed loop performance measures such as percentage overshoot (%OS), settling time (T_s) and rise time (T_r) are presented in Table 3-1 for comparison.

3.3.2 Example 2: Higher order process with delay

Consider a higher order process [31] given by

$$P2 = \frac{1}{(1+s)^8}. \tag{3.41}$$

The corresponding over damped SOPTD model of this process is

$$P2 = \frac{0.3360}{s^2 + 1.3878s + 0.3360} e^{-4.3s}. \tag{3.42}$$

The controller parameters for this model calculated using the method presented by [52] where one of the pole is taken equal to the K_d are given in Table 3-1.

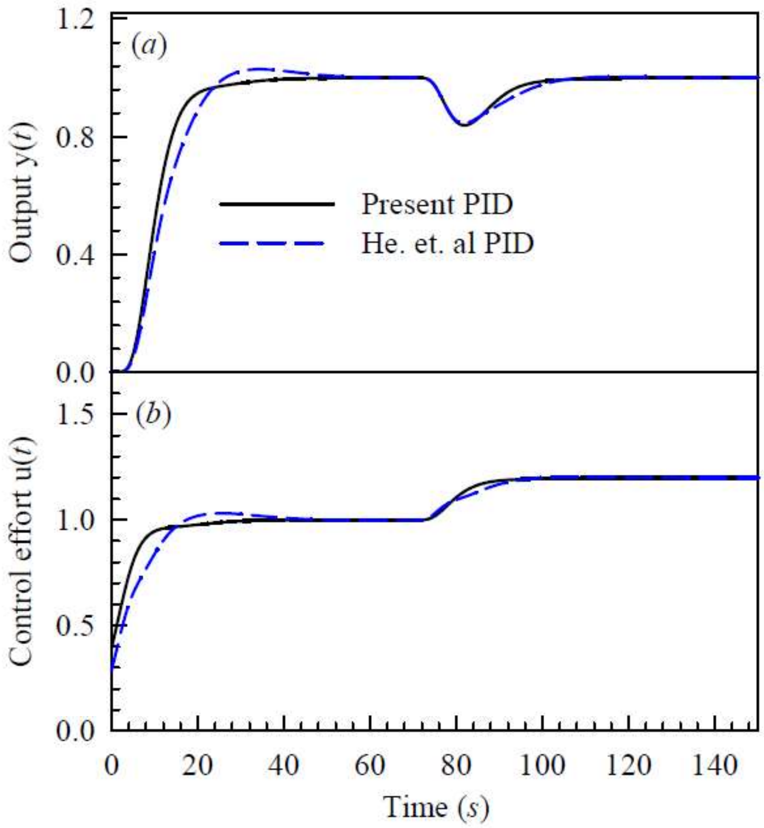


Figure 3-5: Time response and controller response for higher order process P2 with 20% disturbance at $t = 70s$.

The optimal PID controller for $t \geq 4.3s$ for the above process using present method with $m = 4$ is

$$C_2 = 0.3919 + \frac{0.0912}{s} + 0.2834s\,. \tag{3.43}$$

In both the cases same value for $\zeta_{cl} = 0.9$, $\omega_{cl} = 0.3$ rad/s is used. The eigen values of matrices **P** and **Q** in this case are also positive and therefore satisfying the condition of LQR. Figure 3-5(a) compares the step response with 20% disturbance at $t = 70s$. Due to optimal design of all the three parameters, overshoot is almost negligible with an improvement in the rise time and disturbance rejection. The control effort required for desired time response, plotted in Figure 3-5(b), is also slightly less in the present optimization method.

In order to test the present method with large time delay, we have varied the time delay of process P2 from 4.3*s* to 44.3*s* in steps of 10*s* and performed simulations. In all the cases fixed value of $\omega_{cl} L = 1.3$ is used. Simulation results indicate a satisfactory closed loop time response. As usual, the response time to become slow with increase in the time delay *L*.

3.3.3 Example 3: Critically damped process with delay

Consider a critically damped SOPTD process [93] given by

$$\text{P3} = \frac{e^{-0.2s}}{(1+s)^2}\,. \tag{3.44}$$

The optimal PID controller designed for $\zeta_{cl} = 0.98$, $\omega_{cl} = 2$ rad/s and $m = 4$ is

$$C_3 = 3.7238 + \frac{1.9858}{s} + 1.6867s\,. \tag{3.45}$$

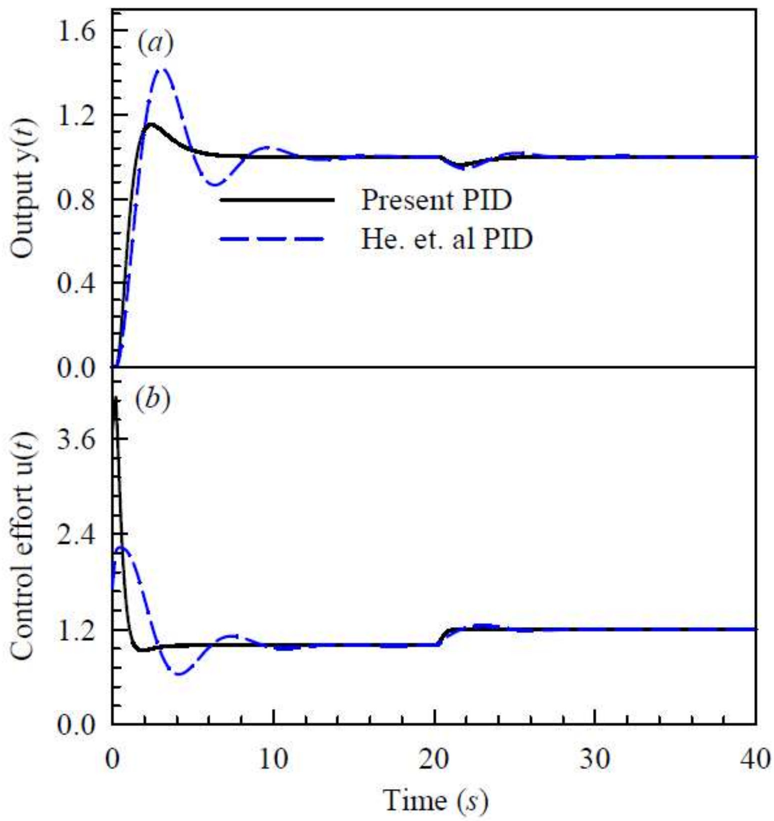


Figure 3-6: Time response and controller response for process P3 with 20% disturbance at $t = 20s$.

Figure 3-6(a) shows the comparison of step responses of the critically damped SOPTD process with 20% disturbance at $t = 20s$. Clearly, the present method gives an improved performance. Both the overshoot and settling time are improved by considerable amount (see Table 3-1) with slight improvement in the rise time and disturbance rejection time. Although the present tuning method takes slightly more control signal initially (Fig. 3-6(b)), but one can easily verify that the total control cost is almost identical in both the cases using *trapz*() function in MATLAB.

Table 3-1: Comparison of closed loop performance measures

Processes	K_p	K_i	K_d	ζ_{cl}	$\omega_{cl}L$	m	%OS	$T_r(s)$	$T_s(s)$
P1 (He et al.)	0.6138	0.5561	1	0.8	1.3	--	14	4.5	15
P1(Present)	0.6984	0.4602	0.1543	0.8	1.3	6	4	4.5	8
P2 (He et al.)	0.2873	0.0851	1.0753	0.9	1.3	--	4	16	23
P2(Present)	0.3919	0.0912	0.2834	0.9	1.3	4	0	12	20
P3 (He et al.)	1.7342	2.1759	1	0.98	0.4	--	35	1.2	8
P3(Present)	3.7238	1.9858	1.6867	0.98	0.4	4	15	1.1	5

3.3.4 Example 4: Unstable SOPTD process

Now we consider an unstable plant [94] given by

$$\text{P4} = \frac{1.5}{(0.5s+1)(s-1)} e^{-0.3s} . \tag{3.46}$$

With some algebraic manipulation one can easily write Eq. (3.46) in standard second order TF as given in Fig. 3-1 and get the value of a = 1, b = -2 and K = 3. The optimal LQR based PID controller obtained with $\zeta_{cl} = 0.9$, $\omega_{cl} = 0.8$ rad/s and $m = 4$ is

$$C4[\text{present}] = 1.2153 + \frac{0.1688}{s} + 0.5682s . \tag{3.47}$$

The controller designed by adopting the method of He et.al. [52] taking $K_d = 2$, the larger real system pole with same ζ_{cl} and ω_{cl} is given by

$$C_4[\text{He et al.}] = 0.5619 + \frac{0.0824}{s} + 2.0\,s\,. \tag{3.48}$$

The time response plotted in Fig. 3-7 clearly shows the advantage of the proposed method for control of unstable plant dynamics. Expect for slightly higher percentage overshoot all other closed loop performance measures are quite reasonable ($T_r = 1.8s$, $\%OS = 150$ and $T_s = 5s$). Simulation results indicate that the range of $\omega_{cl}L$ is limited. In the case of stable system the appropriate range is $\omega_{cl}L \in (1.0, 1.5)$ and for the case of unstable systems it is $\omega_{cl}L \in (0.1, 0.4)$.

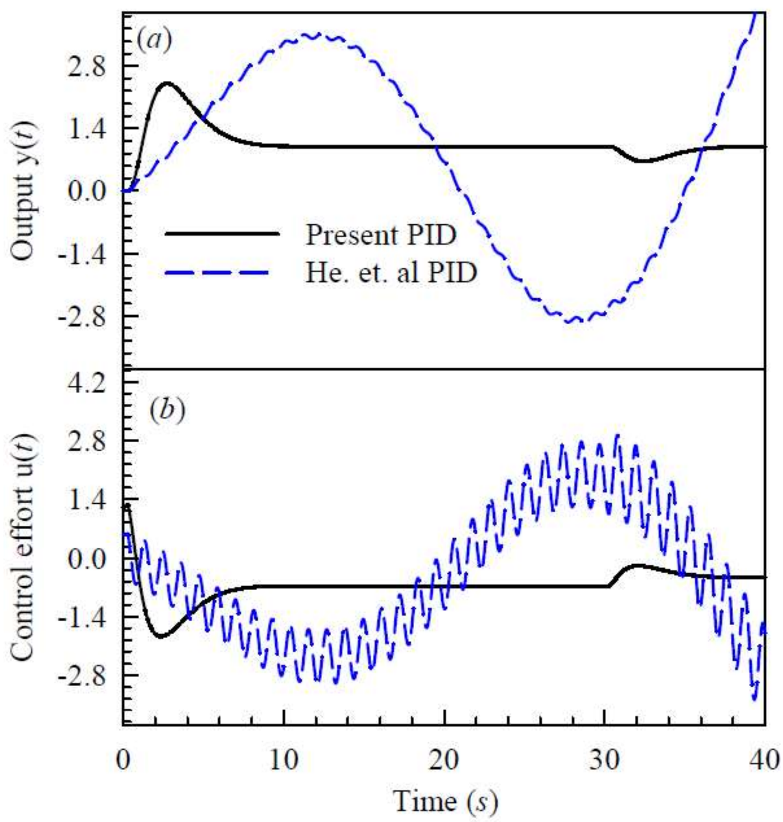


Figure 3-7: Time response and controller response for process P4 with 20% disturbance at $t = 30s$.

3.3.5 Example 5: Highly Oscillatory SOPTD Process

Here we consider a SOPTD process with highly oscillatory open loop response [38] with transfer function given by

$$\mathrm{P5} = \frac{1}{s^2 + s + 5} e^{-0.1s} \ . \tag{3.49}$$

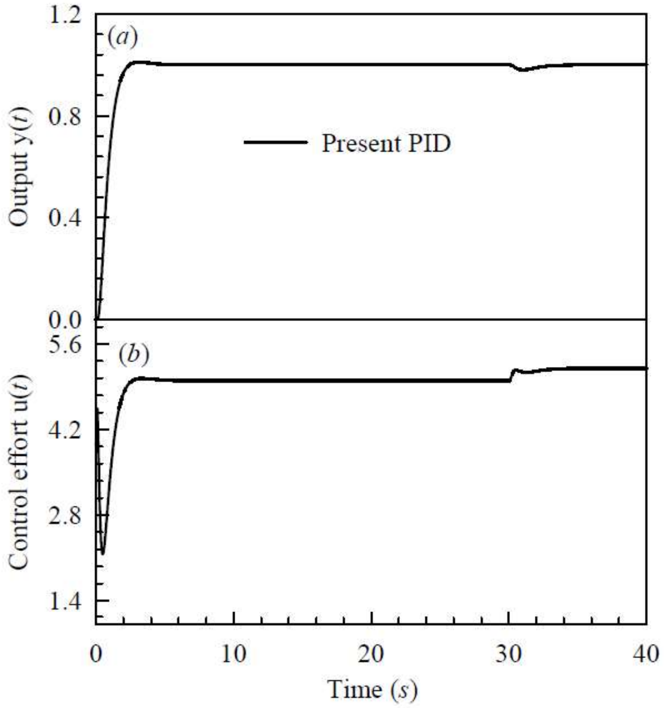


Figure 3-8: Time response and controller response for process P5 with 20% disturbance at $t = 30$ s

Our aim is to design a controller with very small percentage overshoot and settling time. Since the roots of the process are complex, the method used by He et al. [52] for SOPTD process cannot be applied here. The controller with $\zeta_{cl} = 0.9$, $\omega_{cl} = 1.5$ rad/s and $m = 4$ is $C_5 = 3.9434 + \frac{5.8325}{s} + 3.6339s$. Simulation result presented in Fig. 3-8 shows a remarkable time response ($T_r = 1.5s$, $\%OS = 2$ and $T_s = 3.3s$) for process P5.

3.3.6 Comparison with other time domain tuning methods

In order to check the relative merits and demerits of the present method, simulations have been performed for processes P1, P2, P3 and P5 using other time domain tuning methods [31-33].

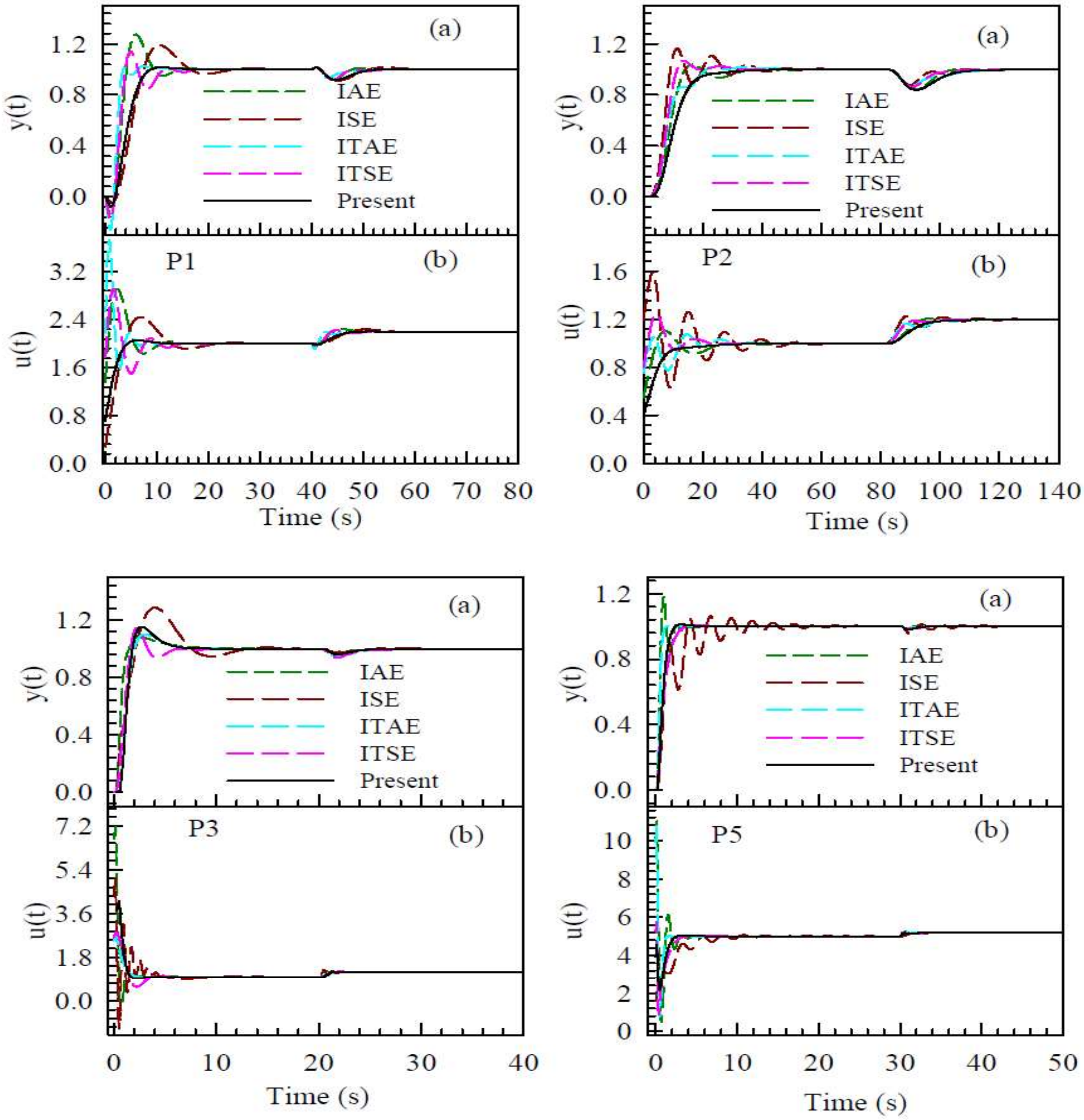


Figure 3-9: Time response and controller response for processes P1, P2, P3 and P5 obtained using methods based on different time domain performance measures.

These tuning methods are Integral of Square Error (ISE), Integral of Time Square Error (ITSE), Integral of Absolute Error (IAE), Integral of Time Absolute Error (ITAE). We have used the *fmincon*() function of the MATLAB optimization toolbox for finding the sets of optimized PID controller parameters subjected to a given time domain performance index based cost function. In all the cases the optimization started with same initial value of PID parameters equal to 0.3. i.e. $K_p = 0.3$, $K_i = 0.3$, $K_d = 0.3$. Results are compared in Fig. 3-9. It is easy to observe that present method gives overall satisfactory closed loop time response. In other cases the response time is fast but with substantial overshoot and oscillations. We have also calculated the control energy using the square of the MATLAB function *norm*(*u*(t),2). Except for process P3, where ITAE and ITSE require slightly less control energy, controllers designed with present method need comparatively less control energy for all other cases.

3.3.7 Robustness Analysis

Most of the real plants operate in a wide range of operating conditions and it is required that the controller must be able to stabilize the system with slight change in the operating conditions. In such situation, the robustness of the closed loop system is an important feature. The purpose of this study is to check the robustness property of the optimal LQR-PID controller when there is a mismatch between the delay time of the process and the delay time for which the PID controller is designed. Consider an under damped SOPTD process given by

$$\text{P6} = \frac{9}{s^2 + 1.2s + 9} e^{-2s} \; . \tag{3.50}$$

It is easy to observe that the open loop system poles are complex. We have designed the PID controller with closed loop parametric demand of $\zeta_{cl} = 0.98$, $\omega_{cl} = 2$ rad/s. The optimal LQR based PID controller obtained with $m = 3$ is

$$C_6 = 0.0979 + \frac{0.1913}{s} + 0.0111s\,. \tag{3.51}$$

Figure 3-10(a) shows the closed loop step response of the under damped SOPTD process P4 with 20% disturbance at $t = 30s$ and the corresponding control effort is plotted in Fig. 3-10(b). It can be readily seen that the stabilization of load disturbance by present controller is quite satisfactory. We have also studied the robustness of the present controller by varying the mismatched delay time L_m from 0.5 s to 4 s covering both sides of the actual delay $L = 2s$ for which the controller is designed. Results of simulation are presented in Fig. 3-10 for comparison. The time response with designed parameters is shown by black solid line. It can be readily seen that an increase in the value of time L_m from the designed value of $L = 2s$, causes an overshoot in the time response and finally leads to the oscillation if the value of L_m becomes larger. In contrast, the mismatched value of L_m less than L is responsible for the increase in the rise time as well as in the settling time and thus making the response sluggish. In the present LQR based PID method, one have an extra tuning factor that is the value of relative dominance m which one can utilized to improve the robustness of the controller in the case of a mismatch in the delay time. To explain the effect of m on robustness of the controller we have designed another optimal LQR-PID controller keeping all the parameter same except the value of m. The optimal PID controller for $m = 10$ is

$$C_{m10} = 0.0658 + \frac{0.1586}{s} + 0.0029s\,. \tag{3.52}$$

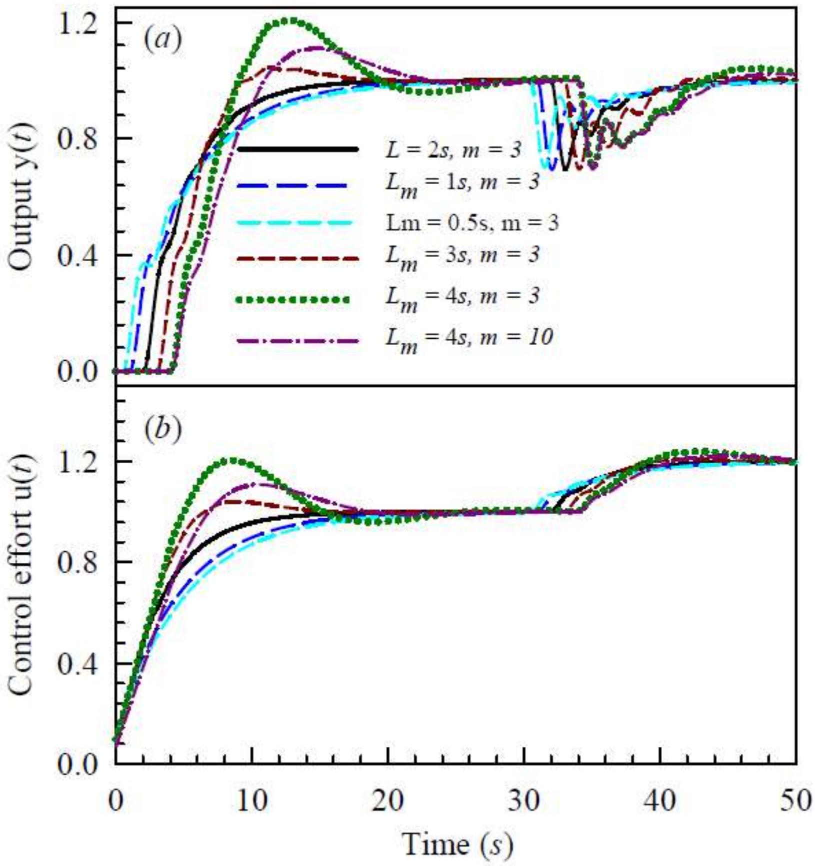


Figure 3-10: The plots of (a) time response and (b) controller response for under damped SOPTD process P6 with 20% disturbance at $t = 30s$ and under mismatched delay time L_m.

A comparison of time response curves for cases $L_m = 4s$, $m = 3$ and $L_m = 4s$, $m = 10$ clearly indicates that a controller designed at higher m shows less overshoot and thus will be more robust in the case of mismatch between the process delay time and the delay time at which the controller is designed. However, the penalty one has to pay is the increase in the rise time.

3.4 Summary

An improved design methodology of PID controller for standard SOPTD system has been developed in this chapter combining the optimal approach of LQR and the dominant pole placement technique. The proposed tuning method allows more flexible pole placement, which results in better time response. The PID parameters have been calculated analytically using user defined closed loop damping ratio and natural frequency. It is demonstrated by simulation that present tuning methodology gives improved closed loop time response with less control effort as compared to the earlier developed LQR based PI/PID tuning method. Simulation results indicate that present method works well for most of the SOPTD models such as under-damped, critically-damped, over-damped, unstable and highly oscillatory processes. It is observed from the simulation that most appropriate range of $\omega_{cl}L$ for stable SOPTD processes is $\omega_{cl}L \in (1.0,1.5)$ and for the unstable SOPTD process is $\omega_{cl}L \in (0.1,0.4)$. A comparison of simulations results with other time domain performance indices indicates that the present methods gives an overall better closed loop time response with comparatively less control effort.

It is observed that the location of non-dominant pole (value of m) affects the closed loop time response provided all others parameters are kept constant. An increase in the value of m, increases the rise time with a substantial control on the overshoot. This observed behavior of the closed loop time response with m in the case of processes with time delay is completely opposite to the cases of delay free processes. A slightly higher value of m adds an extra robustness to the closed loop time response in the case of mismatch between the process delay time and the delay time at which the controller is designed. The proposed analytical tuning method to obtain optimum PID parameters for SOPTD process will be helpful for the online applications. We would like to point out here that the present approach cannot be applied to integrating processes because they cannot be represented in the form of standard second order transfer function.

CHAPTER 4

Design of a two degree of freedom LQR based optimal PID controller for integrating processes with time delay

4.1 Introduction

Open loop integrating processes, which contain at least one pole at the origin, are difficult to manipulate. A small load disturbance can easily destroy the balance between the input and output which can cause increasing or decreasing output without limit. Building controller for such systems needs special attention and is always a challenging task. There are several real world industrial processes whose transfer functions exhibit pure integrator plus time delay. Some typical examples of integrating systems are: distillation column level control in chemical processes [95], a jacketed continuous stirred tank reactor carrying out an exothermic reaction [96], vertical take-off of airplanes [97], high pressure steam flowing to a steam turbine generator in a power plant [98] etc.

The tuning of PID controller for integrating processes needs special attention in terms of output performance, robustness, noise sensitivity, analytical tunability and applicability over a wide range of processes. It is difficult to develop a PID controller for integrating systems which can tune both regulatory response (load disturbance regulation) and servo response (set point tracking) simultaneously using a single PID controller, known as One Degree of Freedom (1-DOF) PID controller. With a 1-DOF PID controller, one can either achieve a good load disturbance or a good set point response [99]. In control engineering most of the controllers are designed to keep good regulation of load disturbance as the primary concern [100-102]. In the cases, where the set point changes frequently, the 1-DOF controller may lead to very high undesirable overshoot. An effective solution to this problem, where one can obtain both good regulatory response and servo response, is the use of a 2-DOF control system [103-104]. A 2-DOF PID control system separately tunes the servo response using a set point filter, without affecting the regulatory response tuned by main PID controller in the loop. The set point filter of 2-DOF control system is also used to avoid actuator saturation problem during the initial start up. Disturbance observer based design of controller is also very popular in 2-DOF control systems [105-108]. It allows independent tuning of disturbance rejection characteristics, which is particularly helpful in situations in which gains need to be tuned on-line.

There are several methods documented in the literature for the tuning of integrating systems with time delay using PID controllers. Some popular methods among them are: Integral Model Control (IMC) method [109-110], coefficient equating method [111], optimization method [112-113], direct synthesis method [114-115] etc. Designers have also proposed more than one controller in the main control loop for controlling integrating processes [116]. Survey of literature indicates that there is still scope to improve the performance and robustness of the PID controller for integrating systems. In the latest IMC based PID controller [109] authors have presented empirical formula obtained by curve fitting and tuning IMC filter constant to get PID parameters at fixed value of $M_s = 2$ for a limited range of time delay. For other values of M_s one needs to retune and repeat the entire lengthy task of curve fitting. Jin and Liu [110] have also proposed IMC based PID in terms of performance and robustness tradeoff. Performance is evaluated in terms of minimum IAE criteria while robustness is measured by M_s.

In this chapter, a 2-DOF LQR based PID controller together with uniquely designed set point filter for integrating systems is presented. The PID parameters are obtained analytically using LQR and dominant pole placement approach to meet the design criteria based on the closed loop natural frequency and closed loop damping ratio. The method is based on rewriting the state equations in two parts; one for $t < L$ and other for $t \geq L$ [64,117], where L is the time delay. As mentioned in Chapter 3, the initial value of PID settings for $t < L$ are generally large and time varying. In such cases a large controller action is needed and most of the time it causes saturation of the actuator. To handle this difficulty, a set point filter is used which is uniquely designed in terms of the PID parameters obtained for $t \geq L$ and a single filter time constant λ. The transfer function of the set point filter is designed to make the closed loop response of the whole system equal to the output response of a first order system with same time delay. The initial response of the controller depends on λ, which can be suitably tuned to control the transient response so that the actuator saturation problem can be avoided. In order to find the optimum settings with respect to the load disturbance and robustness, the whole range of positive PID parameters has been obtained for all categories of plants in terms of regulatory response, measured in terms of IAE criteria, and robustness, measured in terms of maximum sensitivity M_s.

4.2 Classification of integrating systems with time delay

In order to demonstrate the effectiveness of the proposed tuning methodology, four categories of integrating systems have been considered. The transfer function of these integrating processes is listed below

4.2.1 First Order Integrator Plus Time Delay (FOIPTD)

The transfer function of the plant in general form can be written as

$$G(s) = \frac{K\,e^{-sL}}{s(s+a)}. \tag{4.1}$$

examples of frequently encountered FOIPTD processes in real time applications are liquid storage tank [118], continuous stirred tank reactor [96], paper drum dryer cans [119] etc. unstable first order integrator plus time delay (UFOIPTD) [120] processes also fall under this category with '*a*' lies on the right side of complex *s*-plane.

4.2.2 Double Integrator Plus Time Delay (DIPTD)

The transfer function of the plants under this category can be represented as

$$G(s) = \frac{K\,e^{-sL}}{s^2}. \tag{4.2}$$

Oxygen control in feed batch fermentation reactors [121] , DC motors with high-speed disk drives [116] , vertical take-off of airplanes [97] etc. are examples of DIPTD type integrating systems. The DIPTD process is one degree higher in the unstable category when compared with the FOIPTD process. The presence of two poles at the origin, gives the parabolic response to a small disturbance and thus the amplitude saturation reaches more quickly as compared to that of FOIPTD process.

4.2.3 Pure Integrator Plus Time Delay (PIPTD)

Transfer function of the model has the following form

$$G(s) = \frac{K\,e^{-sL}}{s}. \tag{4.3}$$

High-pressure steam flowing to a steam turbine generator in a power plant [98], totally heat integrated distillation columns [122] etc. are few examples of PIPTD type integrating processes.

4.2.4 General transfer function of integrating processes with delay

As the main aim of the present chapter is to find out the optimal PID controller parameter analytically for the above mentioned models, it will be convenient to represent the transfer function of all the models in a single unified form as

$$G(s) = \frac{K\,e^{-sL}}{s\big(s(1-\delta)+a\big)}. \tag{4.4}$$

With $\delta = 0$ and $a = a$ one have FOIPTD process, $\delta = 0$ and $a = 0$ correspond to DIPTD process and $\delta = 1$ and $a = 1$ result in PIPTD process.

4.3 2-DOF LQR based PID tuning for the integrating systems

In most of the industrial control applications the primary concern is to achieve a good load regulation against disturbance. However, in the case of integrating processes the operating conditions change frequently and thus a good set point tracking is also necessary. To fulfill the both requirements an effective solution is to use a 2-DOF PID control system.

4.3.1 2- DOF control system

Consider the conventional 2-DOF control system block diagram as shown in Fig. 4-1(a) which is also known as feed forward type control systems. Here $d(t)$ is the load disturbance. The closed loop transfer function of 2-DOF control system from set-point variable $r(t)$ to the output $y(t)$ and that from the disturbance $d(t)$ to $y(t)$ are given by

$$\frac{Y(s)}{R(s)} = \frac{C_f(s)G(s)}{1+C(s)G(s)} + \frac{C(s)G(s)}{1+C(s)G(s)} \tag{4.5}$$

$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1+C(s)G(s)} \tag{4.6}$$

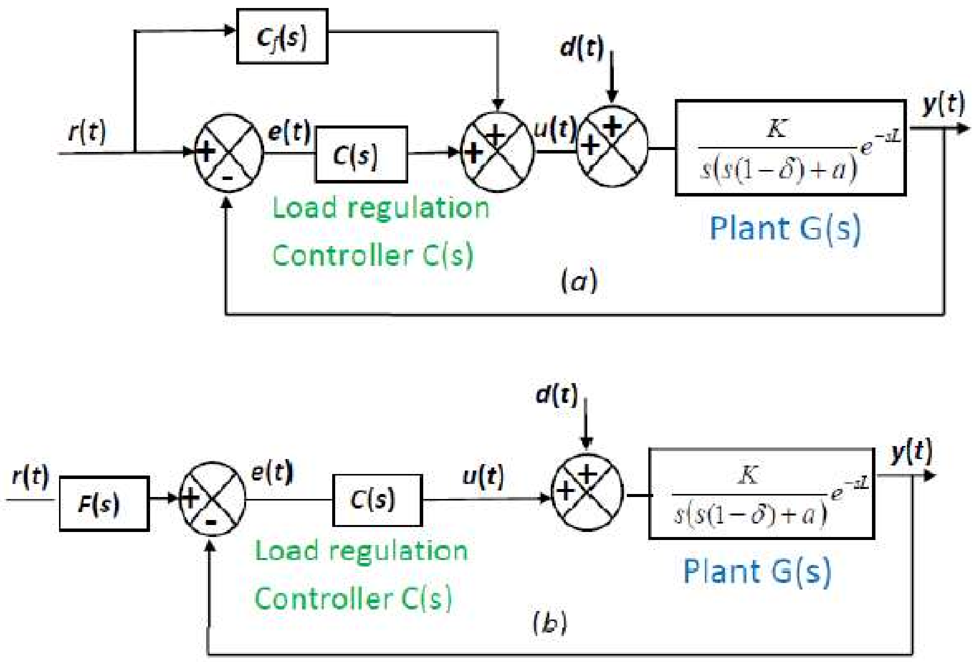


Figure 4-1: (*a*) Feed forward type 2-DOF control system (*b*) Set point filter based 2-DOF control system

From the feed forward type 2-DOF control system it is very clear from Eqs. (4.5) and (4.6) that controller $C_f(s)$ plays only in the tuning of set point response while the load disturbance is control by the main controller $C(s)$ only. Fig. 4-1(b) is identical to Fig. 4-1(a), only the configuration is different. Eq. (4.5) can also be written as $\frac{Y(s)}{R(s)} = \frac{F(s)C(s)G(s)}{1+C(s)G(s)}$ where $F(s) = 1 + \frac{C_f(s)}{C(s)}$. The reason of showing two different configurations is to simplify the problem statements and to show how the 2-DOF control system can be used to regulate load regulation and set point change separately. In the present chapter the configuration of Fig. 4-1(b) is followed to represent the 2-DOF control systems.

4.3.2 LQR based PID controller parameters for integrating systems with time delay

The procedure to obtain LQR based PID parameters for standard SOPTD a system has been already discussed in Chapter 3. Here we will outline only relevant points for continuity. Linear plant with time delay is given by

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t-L) \tag{4.7}$$

Eq. (4.7) can be decomposed into two components, one for $t < L$ and other for $t \geq L$, as

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t), \qquad 0 \leq t < L \quad , \tag{4.8}$$

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{Bu}^m(t), \quad t \geq L \quad , \tag{4.9}$$

where **A, B, X** and L are the state transition matrix, control matrix, state matrix and the time delay term respectively as defined earlier. Here $\mathbf{u}^m(t) = \mathbf{u}(t-L)$. As Eqs. (4.8) and

(4.9) are now delay free, the application of standard LQR approach [60] subjected to the minimization of the cost function gives the optimum control vector $\mathbf{u}^m(t)$

$$\mathbf{u}^m(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{X}(t)\,, \tag{4.10}$$

where **P** is the symmetric positive definite Riccati coefficient matrix. It can be obtained by solving continuous algebraic Riccati equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} = \mathbf{0}. \tag{4.11}$$

where **Q** is the semi positive definite state weighting matrix and **R** is the positive definite control weighting matrix. With some algebraic manipulations, the optimal control vector $\mathbf{u}(t)$ for the present case, thus can be written as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{(\mathrm{A}_c)t}e^{\mathrm{A}(L-t)}\mathbf{X}(t)\,, \qquad 0 \le t < L\,,$$

(4.12)

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}e^{(\mathrm{A}_c)L}\mathbf{X}(t)\,, \qquad t \ge L\,, \tag{4.13}$$

$$\text{where}\,,\ \mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\,. \tag{4.14}$$

4.3.3 Determination of matrices A, B, Q and P for integrating systems and its relation with optimal PID parameters

In the case of unity output feedback system (Fig. 4-1(*b*)), by putting $r(t) = 0$, we have $e(t) = -y(t)$. With this condition, we have,

$$G(s) = \frac{y(s)}{u(s)} = \frac{K\,e^{-sL}}{s\left(s(1-\delta)+a\right)} = \frac{-e(s)}{u(s)}\,,$$

which can be used to obtain the derivative of state variable $x_3(t)$ as

$$\dot{x}_3(t) = -\frac{a}{1-\delta}x_3(t) - \frac{K}{1-\delta}u(t-L) \quad . \tag{4.15}$$

As defined earlier here, $\dot{x}_1(t) = x_2(t)$ and $\dot{x}_2(t) = x_3(t)$, where $x_1(t) = \int e(t)\,dt$. $x_2(t) = e(t)$, $x_3(t) = \dfrac{de(t)}{dt}$. The state variables **X**(*t*) can be represented in matrix form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\dfrac{a}{1-\delta} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\dfrac{K}{1-\delta} \end{bmatrix} u(t-L) \tag{4.16}$$

Comparing Eqs. (4.16) and (4.7) matrices **A** and **B** can be obtained for integrating systems as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\dfrac{a}{1-\delta} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{K}{1-\delta} \end{bmatrix}.$$

In this case also we use **Q** as the same diagonal matrix, **R** = [1], and **P** as symmetric semi definite matrix as defined in Eq. (3.14) in Chapter 3.

Putting $\mathbf{u^m}(t)$ from Eq. (4.10) into Eq. (4.9), it is straightforward to obtain the characteristic equation of the system for $t \geq L$ as

$$\Delta(s) = \left| s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \right| = 0 \quad ,$$

or,

$$s^3 + s^2\left(\frac{a}{1-\delta} + \frac{K^2}{(1-\delta)^2}p_{33}\right) + s\left(\frac{K^2}{(1-\delta)^2}p_{23}\right) + \frac{K^2}{(1-\delta)^2}p_{13} = 0 \tag{4.17}$$

At this point let us assume that the closed loop of the system under consideration can be represented as a user defined standard second order transfer function with the closed loop characteristics equation given by

$$(s+m\zeta_{cl}\omega_{cl})(s^2+2s\zeta_{cl}\omega_{cl}+\omega_{cl}^2)=0 \qquad (4.18)$$

The third non dominant pole $m\zeta_{cl}\omega_{cl}$, is chosen *m* times away from the real part $\zeta_{cl}\omega_{cl}$ of the dominant pole. Equating the coefficients of similar power of *s* in Eqs. (4.17) and (4.18), the three elements of matrix **P** can be obtained as

$$p_{13}=\frac{m\zeta_{cl}\omega_{cl}^3(1-\delta)^2}{K^2} \quad ,$$

$$p_{23}=\frac{\left(\omega_{cl}^2+2m\zeta_{cl}^2\omega_{cl}^2\right)(1-\delta)^2}{K^2},$$

$$p_{33}=\frac{(2+m)(1-\delta)^2\zeta_{cl}\omega_{cl}-a(1-\delta)}{K^2} \quad . \qquad (4.19)$$

The remaining elements of matrix **P** and elements of matrix **Q** can be obtained by solving the algebraic Riccati equation given by Eq. (4.11) as

$$p_{11}=\frac{K^2}{(1-\delta)^2}p_{13}p_{23} \quad ,$$

$$p_{12}=\frac{a}{(1-\delta)}p_{13}+\frac{K^2}{(1-\delta)^2}p_{13}p_{33} \quad ,$$

$$p_{22}=\frac{a}{(1-\delta)}p_{23}+\frac{K^2}{(1-\delta)^2}p_{23}p_{33}-p_{13} \quad ,$$

$$q_{11}=\frac{K^2}{(1-\delta)^2}p_{13}^2 \quad ,$$

$$q_{22} = \frac{K^2}{(1-\delta)^2} p_{23}^2 - 2p_{12} \quad ,$$

$$q_{33} = \frac{K^2}{(1-\delta)^2} p_{33}^2 - 2p_{23} + 2\frac{a}{(1-\delta)} p_{33} \quad . \tag{4.20}$$

4.3.4 PID controller parameters for $t \geq L$

To get the PID controller parameters for $t \geq L$ using Eq. (4.13), it is needed to evaluate the elements of matrix $e^{(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})L}$. These matrix elements can be obtained using the inverse Laplace transform as:

$$e^{(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})t}\Big|_{t=L} = \ell^{-1}\left[(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})^{-1}\right]_{t=L}$$

$$= \begin{bmatrix} f_{11}(t) & f_{12}(t) & f_{13}(t) \\ f_{21}(t) & f_{22}(t) & f_{23}(t) \\ f_{31}(t) & f_{32}(t) & f_{33}(t) \end{bmatrix} \quad , \tag{4.21}$$

where,

$$f_{11}(t) = \sum_{i=1}^{3} \frac{1}{\alpha_i}\left(p_i^2 - \left(\frac{a}{(1-\delta)} + \frac{K^2}{(1-\delta)^2} p_{33} \right) p_i + \frac{K^2}{(1-\delta)^2} p_{23} \right) e^{p_i L} \quad ,$$

$$f_{12}(t) = \sum_{i=1}^{3} \frac{1}{\alpha_i}\left(-p_i + \frac{a}{(1-\delta)} \right) e^{p_i L} \quad ,$$

$$f_{13}(t) = \sum_{i=1}^{3} \frac{1}{\alpha_i} e^{p_i L} \quad ,$$

$$f_{21}(t) = \sum_{i=1}^{3} \frac{1}{\alpha_i}\left(\frac{-K^2}{(1-\delta)^2} \right) e^{p_i L} \quad ,$$

$$f_{22}(t) = \sum_{i=1}^{3} \frac{1}{\alpha_i}\left(p_i^2 - \left(\frac{a}{(1-\delta)} + \frac{K^2}{(1-\delta)^2} p_{33} \right) p_i \right) e^{p_i L} \quad ,$$

$$f_{23}(t)=\sum_{i=1}^{3}\frac{-p_i}{\alpha_i}e^{p_i L},$$

$$f_{31}(t)=\sum_{i=1}^{3}\frac{1}{\alpha_i}\left(\frac{K^2}{(1-\delta)^2}p_{13}p_i\right)e^{p_i L},$$

$$f_{32}(t)=\sum_{i=1}^{3}\frac{1}{\alpha_i}\left(\frac{K^2}{(1-\delta)^2}p_{23}p_i-\frac{K^2}{(1-\delta)^2}p_{13}\right)e^{p_i L},$$

$$f_{33}(t)=\sum_{i=1}^{3}\frac{p_i^{\,2}}{\alpha_i}e^{p_i L}\ .$$

Here $p_1=-\zeta_{cl}\omega_{cl}+i\omega_{cl}\sqrt{1-\zeta_{cl}^2}$, $p_2=-\zeta_{cl}\omega_{cl}-i\omega_{cl}\sqrt{1-\zeta_{cl}^2}$ and $p_3=-m\zeta_{cl}\omega_{cl}$ are the desired roots of Eq. (4.18) and

$$\alpha_1=-(p_1-p_2)(p_3-p_1),$$

$$\alpha_2=-(p_1-p_2)(p_2-p_3),$$

$$\alpha_3=-(p_3-p_1)(p_2-p_3)\ .$$

Using the values of **P**, **Q**, **R** and $e^{(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})L}$ in Eq. (4.13) the PID controller parameters for $t \geq L$ can be written as

$$K_i=\frac{K}{(1-\delta)}\big(p_{13}f_{11}(L)+p_{23}f_{21}(L)+p_{33}f_{31}(L)\big),$$

$$K_p=\frac{K}{(1-\delta)}\big(p_{13}f_{12}(L)+p_{23}f_{22}(L)+p_{33}f_{32}(L)\big),$$

$$K_d=\frac{K}{(1-\delta)}\big(p_{13}f_{13}(L)+p_{23}f_{23}(L)+p_{33}f_{33}(L)\big)\ . \tag{4.22}$$

Equations (4.22) give the PID parameters for $t \geq L$. In the simulation these setting were used for the entire time domain. It should be noted that in this chapter we have not obtained the PID parameters for $0 \le t < L$ as mentioned earlier. The reason is that PID parameters for this region are generally large in value and time varying and thus need more control effort which is difficult to implement in analog domain. Problems associated with the startup i.e. for the time horizon of $0 \le t < L$ will be minimized using the set point filter $F(s)$ which make the overall closed loop system 2-DOF control system as discussed in the following subsection.

4.3.5 Set point filter *F*(*s*)

As discussed earlier (page 23), the set point filter is used to improve the servo performance of the closed loop system independently from the employed main load disturbance controller. In this chapter the set point filter $F(s)$ is designed uniquely which contains the PID parameters obtained using Eq. (4.22) and a single tuning parameter λ in such a way that the overall servo performance represents the response of a first order system with time delay L.

From Fig. 4-1(*b*) the relation between output variable $y(t)$ and input variable $r(t)$ can be represented as

$$\frac{Y(s)}{R(s)} = \frac{F(s)C(s)G(s)}{1+C(s)G(s)}, \tag{4.23}$$

If one wants the desired output to be equal to the first order system with time delay, then Eq. (4.23) must be equal to

$$\frac{Y(s)}{R(s)} = \frac{1}{1+\lambda s} e^{-Ls} \quad . \tag{4.24}$$

From Eqs. (4.23) and (4.24) $F(s)$ can be obtained as

$$F(s) = \frac{1 + C(s)G(s)}{(1+\lambda s)C(s)G(s)} e^{-Ls} \quad . \tag{4.25}$$

Putting the value of $G(s)$ from Eq. (4.4), $C(s) = K_p + \frac{K_i}{s} + K_d s$ and keeping only the first order term in the expansion of e^{-Ls} i.e. $e^{-Ls} = 1 - Ls$. $F(s)$ can be obtained as

$$F(s) = \frac{s^3\left(\frac{1-\delta}{K} - K_d L\right) + s^2\left(\frac{a}{K} - K_p L + K_d\right) + s\left(K_p - K_i\right) + K_i}{s^3\left(\lambda K_d\right) + s^2\left(\lambda K_p + K_d\right) + s\left(\lambda K_i + K_p\right) + K_i} \quad . \tag{4.26}$$

4.4 Results and discussion

In order to highlight the effectiveness of the proposed 2-DOF LQR based PID control system for the integrating processes, we now present results of simulation using MATLAB for all the integrating models discussed in section 2 and compare the results with some latest reported tuning methods of Kumar & Sree [109] and Jin and Liu [110] for integrating systems. In addition, we also discuss the response of an unstable integrating system. Eqs. (4.22) give the values of PID parameters in terms of closed loop design parameters ζ_{cl} , ω_{cl} , m and process parameters. A choice of an arbitrary ζ_{cl} or ω_{cl} sometimes may lead to the negative value of PID parameters. In the simulation, only those ζ_{cl} and ω_{cl} are considered for which the PID parameters are positive.

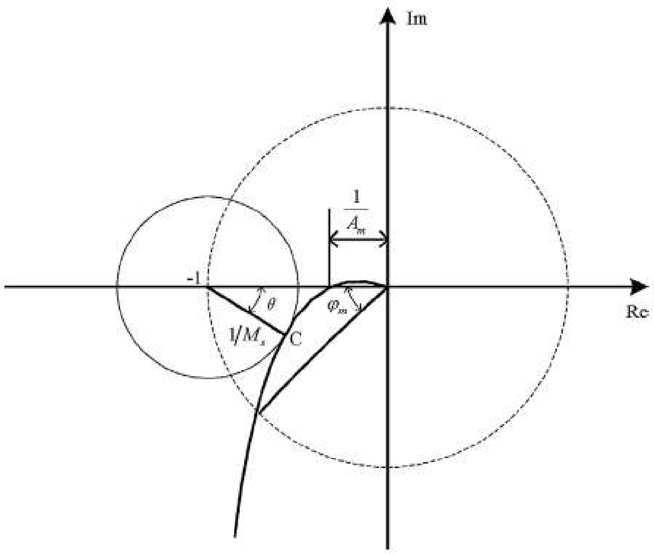


Figure 4-2: Graphical interpretation of maximum sensitivity function

The performance of the proposed controller is measured in terms of minimum IAE criteria and maximum sensitivity M_s. The gain margin A_m and phase margin φ_m obtained from the simulation are used to calculate the maximum sensitivity M_s using the relations $A_m \geq M_s/(M_s-1)$ and $\varphi_m \geq 2\sin^{-1}(1/(2M_s))$ as shown in Fig. 4-2 [123]. Higher value of M_s is quoted in the paper obtained from gain and phase margin relations to ensure that the available maximum sensitivity cannot be more than this value. Note that for a stable system, generally a value of M_s less than 2 is preferred for controllers to be robust. For a given process model, ω_{cl} is swept with different ζ_{cl} and m to obtain the range of feasible PID parameters. At the same time the controller smoothness is evaluated which is measured in terms of total variation (TV) defined as $\mathrm{TV} = \sum_{0}^{\infty} |u_i - u_{i-1}|$ i.e. sum of the differences between the present output and the previous output of the controller. The Total

Energy (TE) of the controller is also estimated using the square of the L-2 norm using MATLAB function *norm*(*u*(*t*),2).

In order to evaluate the performance of each tuning method on same footage considering different performance indexes (M_s, IAE_{ld}, IAE_{sp}, TV_{ld}, TV_{sp}), a global performance index (GPI) has been computed [124]. To compute GPI first normaliz each performance index against its highest value ranging over the set of tuning methods. Then, the normalized indexes are summed up and the result is divided by the number of measured performance indexes. In the present case the total number of performance index is five. Tuning method with lowest GPI is considered as the more balanced and optimum tuning method. The range of GPI is between 0 and 1.

4.4.1 Example 1: FOIPTD process

Consider an FOIPTD process model [110] with transfer function as

$$P1 = \frac{0.2}{s(4s+1)} e^{-s} \quad . \tag{4.27}$$

A comparison of the present FOIPTD model with the unified model of Eq. (4.4) gives δ = 0, a = 1/4 , L = 1 and K = 0.2/4. Fig. 4-3(a) shows the variation of robustness and load regulation level of process P1 covering all the positive values of PID parameters as a function of closed loop natural frequency ω_{cl} obtained at a fixed value of m = 2 and at three values of damping ratio ζ_{cl} equal to 0.7, 0.8 and 0.9.

PID controller parameters are obtained using Eqs. (4.22). The load regulation is obtained by measuring IAE_{ld}, where subscript 'ld' denotes the load disturbance. From Fig. 4-3(a) it can be observed that at low ω_{cl}, controller parameters obtained at ζ_{cl} = 0.9 show better sensitivity (low M_s) and poor load regulation (high IAE_{ld}). Notice that controller parameters for a fixed ζ_{cl} provide better sensitivity at lower values of ω_{cl} whereas better load regulation occurs at comparatively higher values of ω_{cl}. Simulation performed with different models of FOIPTD system choosing different '*a*', '*K*' and '*L*' shows almost similar behavior with different M_s and IAE_{ld}.

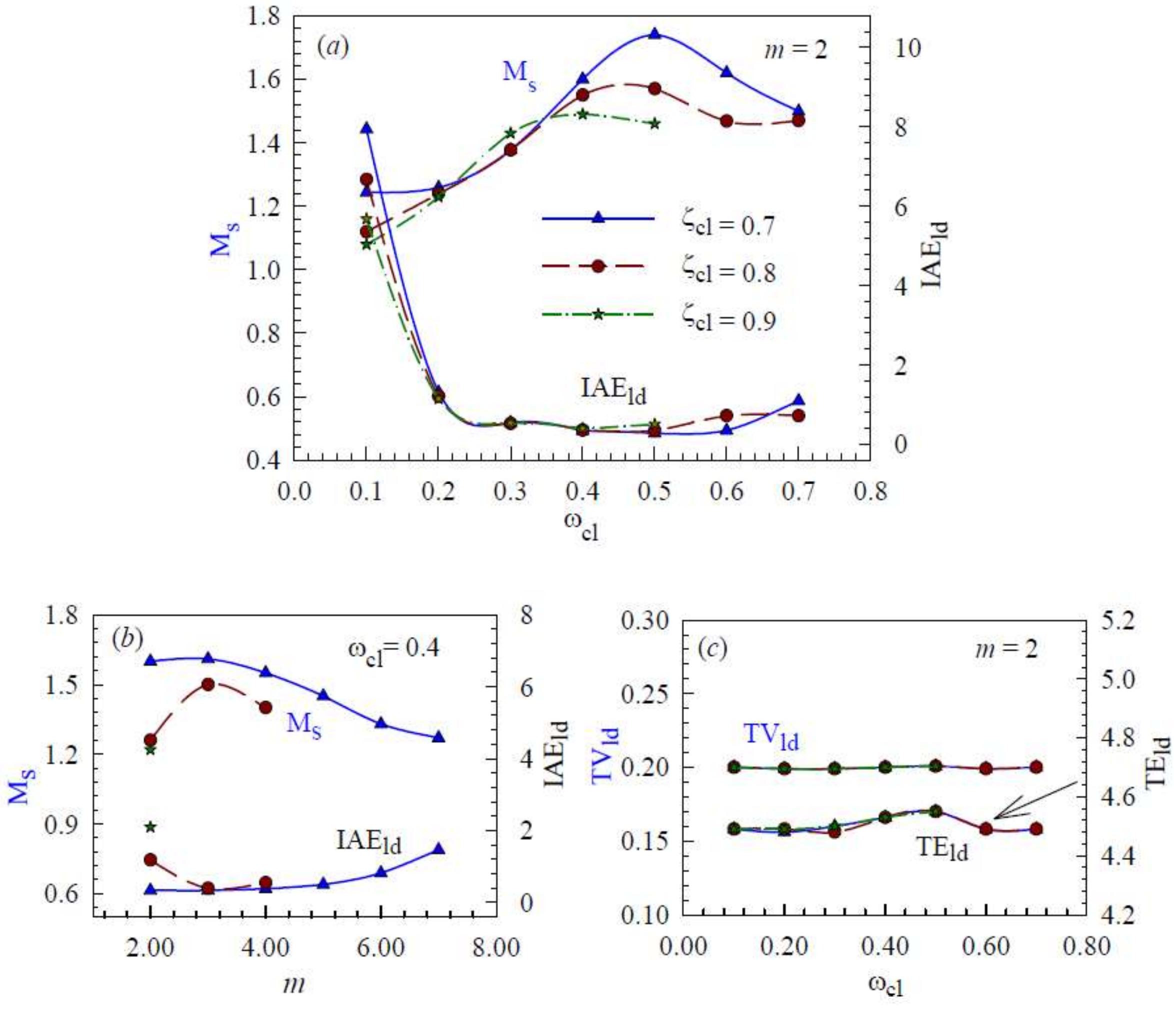


Figure 4-3: Variation of (*a*) robustness and load regulation level of process P1 covering all the positive values of controller parameters at m = 2, (*b*) M_s and IAE_{ld} with m for fixed ω_{cl} = 0.4, (*c*) TV_{ld} and TE_{ld} of PID controller with ω_{cl} at fixed value of m = 2.

Figure 4-3(*b*) shows the robustness and load regulation at different values of *m* at a fixed ω_{cl} = 0.4 where both sensitivity and load regulations (Fig. 4-3(*a*)) are better i.e. sensitivity M_s is within 1.8 and load regulation IAE_{ld} is within 0.6. It is to be noted that for the case of integrating systems M_s less than 2 and IAE_{ld} less than 1 are considered good. It can be easily seen that the choice of positive PID parameters are not available at higher *m* with increasing ζ_{cl} .

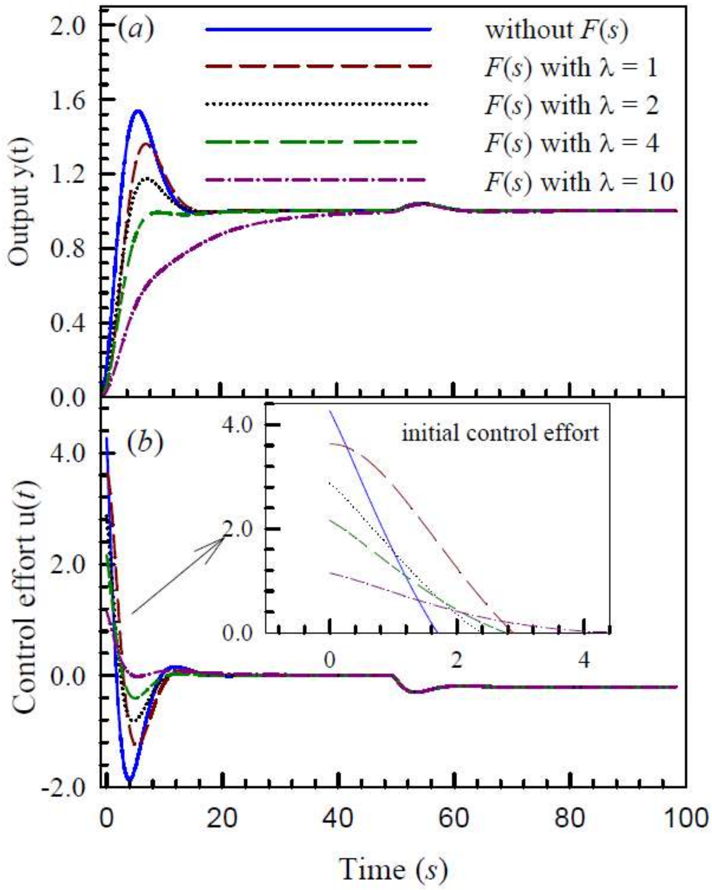


Figure 4-4: Closed loop time response and control effort of process P1 at different set point filter parameter λ with ζ_{cl} = 0.7, ω_{cl} = 0.5 and *m* = 2.

Figure 4-3(*c*) shows the variation of controller smoothness TV_{ld} and total controller energy TE_{ld}. Note that there is very little change in TV_{ld} and TE_{ld} in the entire range of controller parameters obtained using the proposed tuning method. The closed loop time response and control effort of process P1 with and without set point filter $F(s)$ are shown in Fig. 4-4. The PID controller parameters are obtained using ζ_{cl} = 0.7, ω_{cl} = 0.5 and m = 2. The corresponding Riccati coefficient and state weighting matrices are,

$$\mathbf{Q}_{P1} = \begin{bmatrix} 12.25 & 0 & 0 \\ 0 & 23.04 & 0 \\ 0 & 0 & 167 \end{bmatrix}, \quad \mathbf{P}_{P1} = \begin{bmatrix} 51.8 & 98 & 70 \\ 98 & 344.4 & 296 \\ 70 & 296 & 460 \end{bmatrix} \tag{4.28}$$

The eigen values of these matrices are positive, thus satisfying the positive definite condition of LQR. As shown in Fig. 4-4, the closed loop response curve without $F(s)$ shows very high overshoot and controller needs comparatively large initial control effort. Both overshoot and initial control effort are found to decrease as the value of λ increases. The zoomed view of initial control effort is also shown in Fig. 4-4(*b*). The regulation response to the load disturbance with different $F(s)$ is found to be almost same as expected in a 2-DOF control system.

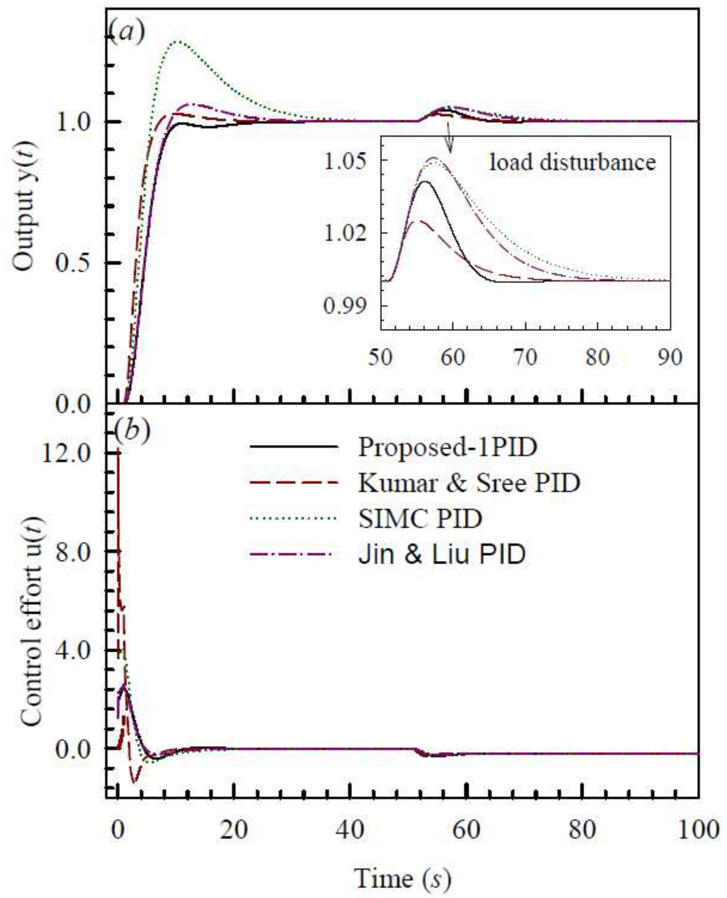


Figure 4-5: Comparison of (*a*) Time response (*b*) Control effort of the proposed 2-DOF control system for process P1 with some latest PID tuning methods for integrating processes.

Figure 4-5 compares the closed time response and control effort of the proposed method with latest IMC based 2-DOF tuning methods of Kumar & Sree [109], Jin & Liu [110] and well known Skogestad IMC (SIMC) [69]. The closed loop performance measures are listed in Table 4-1. For fair comparison, we have chosen the PID parameters for which M_s is 1.75 (the maximum value in our case). Observing the results shown in Table 4-1, it is clear that the present controller (proposed-1) gives reasonable value of the robustness and performance measures; better M_s and TV_{sp} as compared to Kumar & Sree and SIMC and almost comparable to Jin & Liu. Though IAE_{sp} is slightly poor than that of Kumar & Sree method but it is much better that the other two methods. As far as the load disturbance is concerned the proposed method gives better IAE_{ld} than Jin & Liu and SIMC and very close to Kumar & Sree as can be seen from Fig. 4-5(*a*) (zoom view). Note that the proposed method provides a better settling time (T_s) and almost negligible overshoot (%OS) i.e. T_s ~ 5*s* and %OS~ 0 than the other methods (Kumar & Sree: T_s ~ 9*s*, %OS~ 8; Jin & Liu: T_s ~ 20*s*, %OS~ 10). TV obtained in present case is less than that obtained with Kumar & Sree's method but at the cost of compromise in the performance as observed in IAE values.

Table 4-1: Performance measures of process P1 obtained with different tuning methods.

PID controller $C(s)$ and set point filter $F(s)$ for process P1			Performance and robustness measures					GPI
	$C(s)$ & $F(s)$	λ	M_s	IAE_{ld}	TV_{ld}	IAE_{sp}	TV_{sp}	
Proposed-1 (ζ_{cl} = 0.7, ω_{cl} = 0.5 m = 2)	$C(s)=4.979+\frac{0.7224}{s}+10.1078s$ $F(s)=\frac{9.892s^3+10.13s^2+4.257s+0.7224}{40.43s^3+30.02s^2+7.868s+0.7224}$	4	1.75	0.28	0.46	2.30	5.44	0.523
Kumar & Sree	$C(s)=7.415\left(1+\frac{1}{7.8s}+1.9487s\right)\frac{0.5s+1}{0.1863s+1}$ $F(s)=\frac{0.7s+1}{3.8s+1}$	-	1.98	0.23	0.54	1.96	30.8	0.714
SIMC	$C(s)=3.75\left(1+\frac{1}{12s}+2.667s\right)$	-	2.1	0.64	0.34	7.24	6.19	0.766
Jin & Liu	$C(s)=3.686\left(1+\frac{1}{10.392s}+\frac{2.473s}{0.2473s+1}\right)$ $F(s)=\frac{9.79s^2+6.266s+1}{25.7s^2+10.392s+1}$	-	1.99	0.56	0.35	3.88	5.34	0.635
Proposed-2 (ζ_{cl} = 0.8, ω_{cl} = 0.4, m = 2)	$C(s)=4.52+\frac{0.5668}{s}+9.7075s$ $F(s)=\frac{10.29s^3+10.19s^2+3.954s+0.5668}{38.83s^3+27.79s^2+6.787s+0.5668}$	4	1.57	0.34	0.42	2.7	5.03	0.518

The present method provides a more balanced controller where total variation and load regulation are quite reasonable with much better servo response. The most important feature of the proposed method is the flexibility of choosing the PID controller parameters. To show this fact we have chosen another controller Proposed-2 and performed simulation.

This choice is based on the close inspection of Fig. 4-3(*a*). It is clear that a choice of ζ_{cl} = 0.8 (keeping other terms fixed i.e. ω_{cl} = 0.5 and *m* = 2) gives better sensitivity M_s ~ 1.57 with reasonable IAE_{ld} ~0.34. The performance measures of this controller are also listed in Table 4-1 for comparison. It is interesting to note that the GPI of the controller's proposed-1 and proposed-2 designed by the present method are much lower than the GPI of the other methods.

4.4.2 Robustness against perturbation

Robustness of the present controller (Proposed-1) under the perturbation of system parameters is performed via simulation by varying parameters *K*, *a* and *L* and observing the behavior of overshoot and settling time. First, the time delay *L* is varied by ± 50% keeping *K* and *a* fixed. Simulation results shown in Fig. 4-6 indicate a 10% hike in the overshoot with small oscillations and 30% increase in the settling time when the time delay is increased by 50%. There is very little effect on the system performance when the time delay is decreased by 50%. It is observed that the present system remains stable for a process time delay within the region $0 < L < 2.07s$. The variation of parameter *a* by ± 50% (keeping *K* and *L* constant) shows a hike in overshoot by ~ 5% when *a* is increased and only 2% when *a* is decreased. There is almost negligible effect of the variation of *a* on the settling time. In the simulation, a significant effect can be observed on the system performance, 20% hike in overshoot and 80% increased in settling time when the parameter *K* (keeping *a* and *L* constant) is decreased by 50%. There is almost negligible effect on the system performance when *K* is increased by 50%.

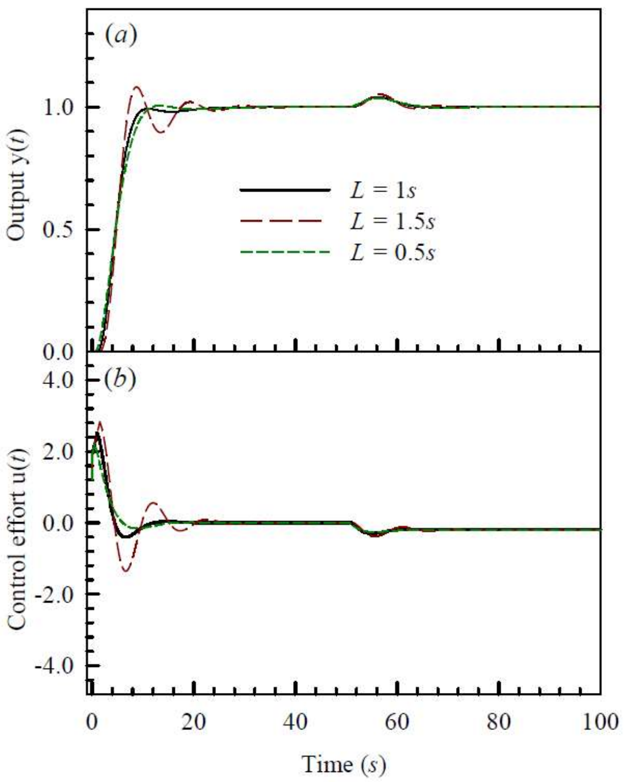


Figure 4-6: Comparison of the time response and control effort of the perturbed system P1 when the time delay $L = 1$ is varied by ± 50% i.e. $L = 1.5$ and $L = 0.5$.

4.4.3 Robustness against noise in measurement device

Integrating systems which are a part of unstable systems lead to infinite amplitude response with a small disturbance in open loop. These are more prone to noise disturbances as compared to that of FOPTD and SOPTD systems as discussed in Chapter 2 and 3. In this sub-section we have examine the effect of noise on robustness by introducing white noise in the measurement device of the present controller and other controllers. White noise, with a power spectrum of 0.00029 and a sample period of 0.01*s* in the measurement device is introduced in the simulation which corresponds to a variance (σ^2) of 0.029 i.e. 5% of the variance of control effort.

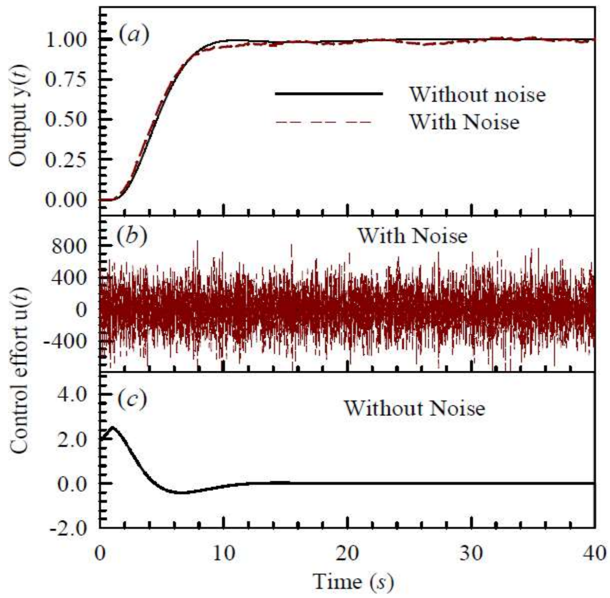


Figure 4-7: Time response and control effort of process P1 with added white noise with a power spectrum of 0.00029 and a sample period of 0.01*s*.

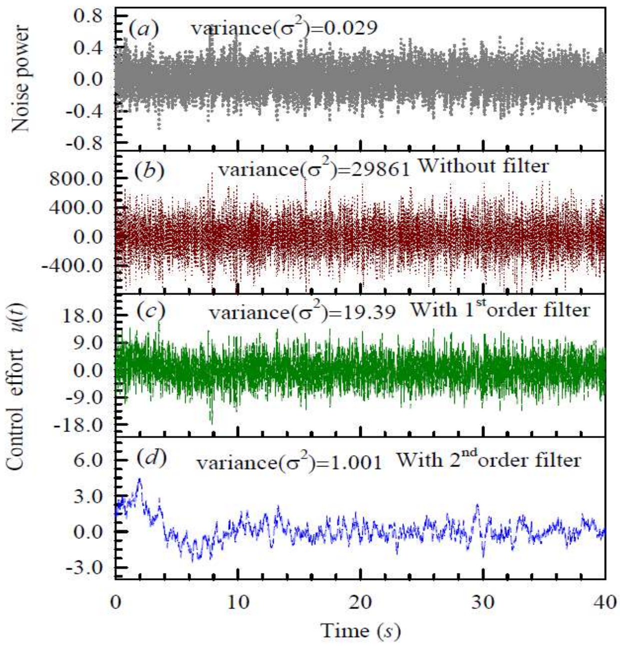


Figure 4-8: Reduction in the control effort with first order and second order noise filters in the proposed-1 controller for process P1.

Results shown in Fig. 4-7 indicate acceptable closed loop response but the increased value of control efforts (σ^2 = 29861) seems to be unacceptable. In order to reduce the effect of noise in the control signal generally noise filters are used in the measurement device [124-127]. We have also performed simulation using first order and second order noise filters using the methodology discussed in Ref [127]. Results are shown in Fig. 4-8. It is found that there is a considerable reduction in the control effort (σ^2 = 1.001) with the second order noise filter.

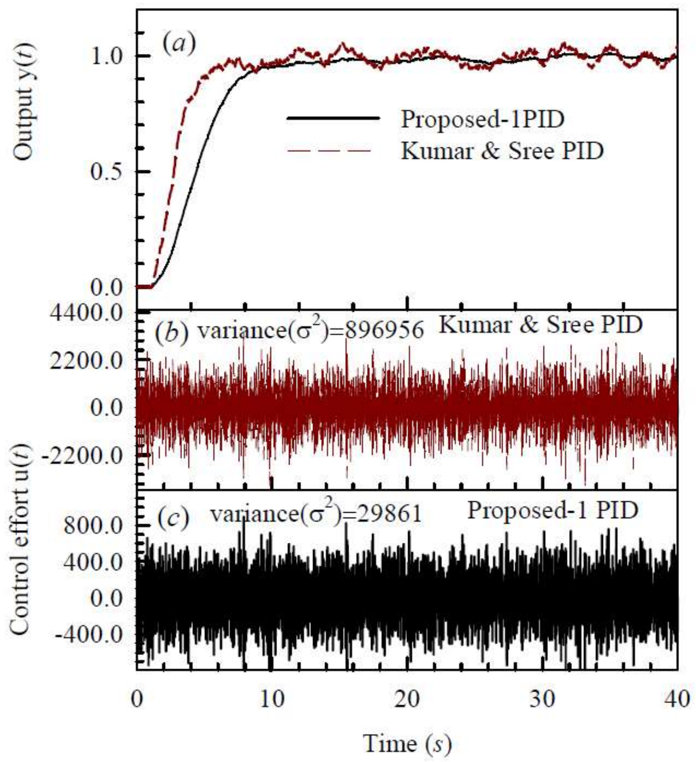


Figure 4-9: Comparison of time response and controller effort with proposed-1 PID and Kumar & Sree PID with same noise power.

Since the sensitivity of the present controller is better than those obtained with other methods (see Table 4-1), the effect of noise on the system output and manipulated variable should be expected to be less. This fact is also observed when simulation has been performed by other methods after introducing noise in the measurement device. Simulation performed by adding same noise power spectrum of 0.00029 and a sample period of 0.01*s* in the measurement device shows that control effort of present tuning method ($\sigma^2 = 29861$) is much less than that of Kumar & Sree method ($\sigma^2 = 896956$) as shown in Fig. 4-9.

4.4.4 Example 2: DIPTD process

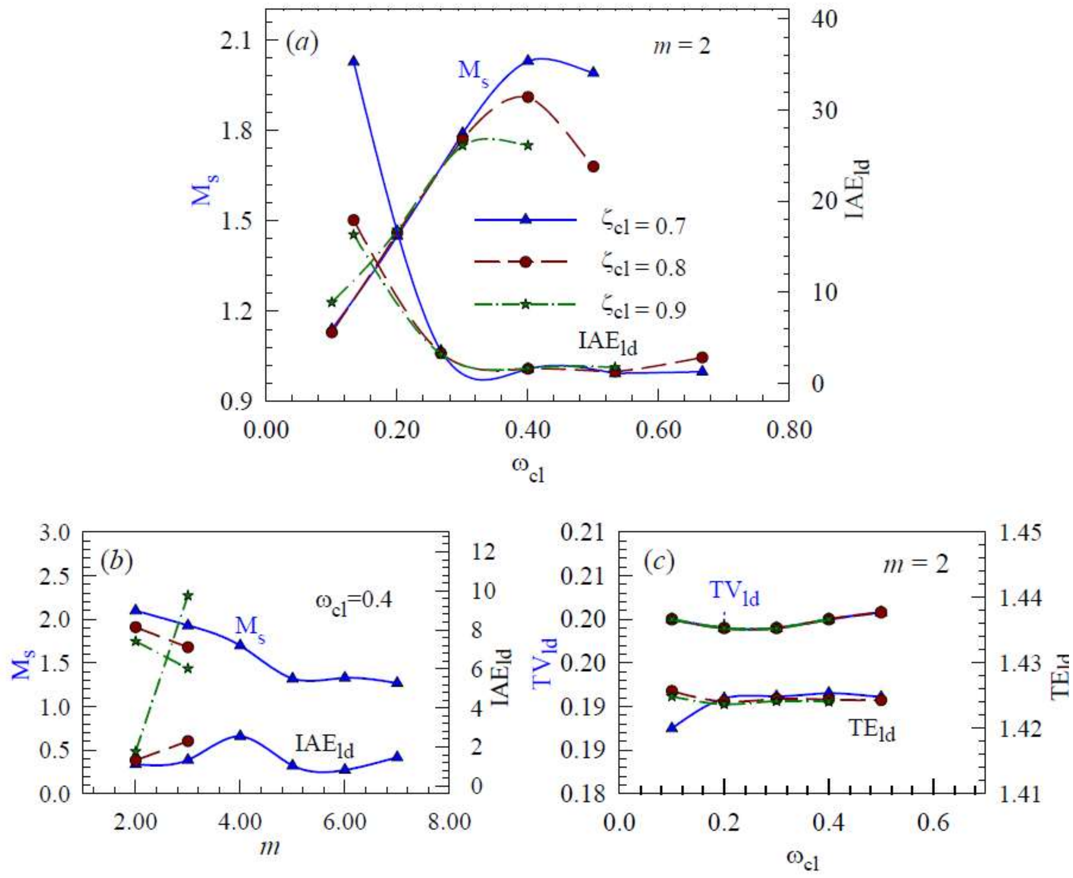


Figure 4-10: Variation of (*a*) robustness and load regulation of process P2 with ω_{cl} covering all the positive values of PID at $m = 2$, (*b*) M_s and IAE_{ld} with m for fixed $\omega_{cl} = 0.4$, (*c*) TV_{ld} and TE_{ld} of PID controller with ω_{cl} at fixed value of $m = 2$.

Consider a DIPTD process model [109] whose transfer function is

$$P2 = \frac{1}{s^2} e^{-s} \tag{4.29}$$

Here $\delta = 0$, $a = 0$ and $L = 1$. Fig. 4-10 shows the variation of M_s, IAE_{ld}, TV_{ld} and TE_{ld} covering the all positive PID parameters obtained using Eq. (4.14) by varying the tuning parameters ζ_{cl}, ω_{cl} and m. The variation of M_s and IAE_{ld} with ω_{cl} for the process P2 shows almost identical behavior as that of process P1 i.e. PID parameters that are obtained at fixed ζ_{cl} and m provide better robustness and poor load regulation at lower ω_{cl} whereas poor robustness and better load regulation at higher ω_{cl}.

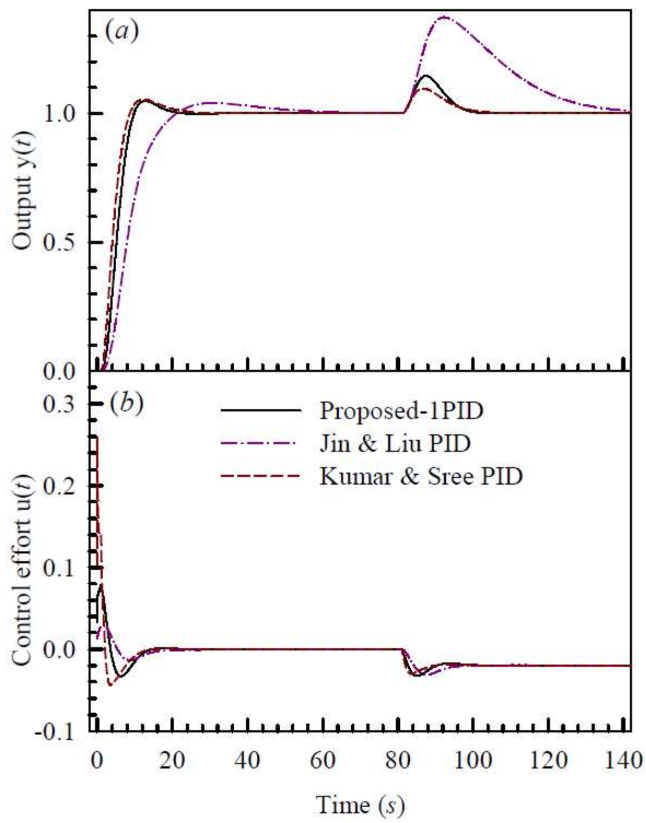


Figure 4-11: Comparison of time response and control effort of the proposed method for process P2 with some latest PID tuning methods of integrating processes.

Table 4-2: Performance measures of process P2 obtained with different tuning methods.

PID controller $C(s)$ and set point filter $F(s)$ for process P2			Performance and robustness measures						
	$C(s)$ & $F(s)$	λ	M_s	IAE_{ld}	TV_{ld}	IAE_{sp}	TV_{sp}	GPI
Proposed-1 ($\zeta_{cl}=0.8$, $\omega_{cl}=0.4$ $m=2$)	$C(s)=0.1368+\frac{0.0152}{s}+0.5141s$ $F(s)=\frac{0.4859s^3+0.3773s^2+0.1216s+0.0152}{2.056s^3+1.061s^2+0.1975s+0.0152}$	4	1.92	1.31	0.56	3.1	0.21	0.58
Kumar & Sree	$C(s)=0.1883\left(1+\frac{1}{9.4s}+3.4489s\right)\frac{0.5s+1}{0.2199s+1}$ $F(s)=\frac{7.84s^2+5.6s+1}{32.42s^2+9.4s+1}$	-	2.01	1.08	0.59	2.98	0.65	0.73
Zin & Liu	$C(s)=0.046\left(1+\frac{1}{21.38s}+\frac{7.255s}{0.7255s+1}\right)$ $F(s)=\frac{46.23s^2+13.58s+1}{155.1s^2+21.38s+1}$	-	2.0	9.25	0.49	5.12	0.19	0.81
Proposed-2 ($\zeta_{cl}=0.7$, $\omega_{cl}=0.4$, $m=2$)	$C(s)=0.147+\frac{0.0185}{s}+0.5240s$ $F(s)=\frac{0.476s^3+0.3767s^2+0.1288s+0.0185}{2.096s^3+1.113s^2+0.2213s+0.0185}$	4	2.0	1.11	0.61	2.44	0.24	0.59

Notice that positive PID parameters are not possible at higher values of ω_{cl} and m. The range of positive PID reduces as the value of ζ_{cl} is increased. As shown in Fig. 4-10(c), there is very little change in the TV_{ld} and TE_{ld} over the entire range of ω_{cl} at m = 2. Figure 4-11(a) and (b) compare the time response and controller response of the proposed method with Kumar & Sree [109] and Jin & Liu [110] tuning methods. It can be readily seen that present method gives almost same transient response as compared with Kumar and Sree method (Proposed-1: T_s ~ 18*s*, %OS~ 8; Kumar & Sree: T_s ~ 18*s*, %OS~ 8; Jin & Liu: T_s ~ 50*s*, %OS~ 5) and better settling time than that of Jin & Liu.

The present method needs very less initial control effort, the load regulation is slightly poor but it is better than Jin & Liu. Table 4-2 shows the closed loop performance measures. The present controller is obtained at ζ_{cl} = 0.8, ω_{cl} = 0.4 and m = 2 (Proposed-1). In order to demonstrate the flexibility of the present method, the controller parameters obtained with ζ_{cl} = 0.7, ω_{cl} = 0.4 and m = 2 (Proposed-2) together with performance measures are also presented in Table 4-2 for comparison. It is very easy to see that all the performance measures of this controller are quite reasonable and some of them are better than those obtained with other methods. The evaluated GPI of the controllers proposed-1 and proposed-2 obtained by the present method, are much lower than the GPI of other methods.

4.4.5 Example 3: PIPTD process

Consider a PIPTD process model [109] with transfer function as

$$P3 = \frac{0.2}{s} e^{-7.4s} . \tag{4.30}$$

In this case, if we use δ = 1 in the unified process model given in Eq. (4.4), all the coefficients in Eqs. (4.19) and (4.20) will be either zero or infinite which is not desired. Here, we have use δ= 0.999 which is much closer to 1 and this replacement makes the estimated model to appear like

$$P3_{\text{estimated}} = \frac{0.2}{s(0.001s+1)} e^{-7.4s} . \tag{4.31}$$

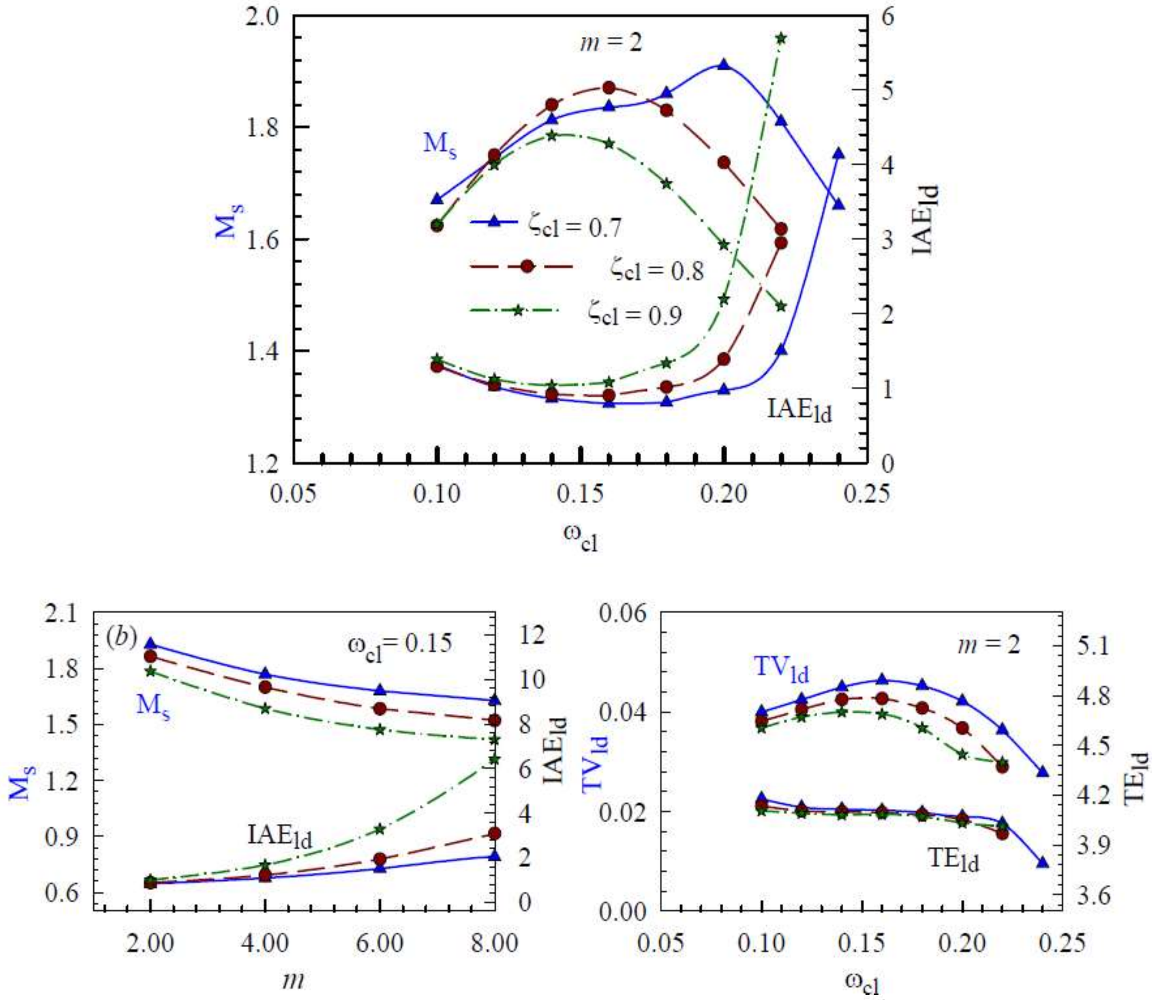


Figure 4-12: Variation of (*a*) robustness and load regulation of process P3 with ω_{cl} covering all the positive values of PID at m = 2, (*b*) M_s and IAE_{ld} with m for fixed ω_{cl} = 0.15, (*c*) TV_{ld} and TE_{ld} of PID controller with ω_{cl} at m = 2.

It can be easily checked that the frequency response of the original and estimated transfer function are almost similar within the frequency range of 0 to 10^7 rad/s. Figure 4-12 shows the variation of performance measures for a wide range of controller design parameters. In the simulation, the controller parameters are obtained using the estimated transfer function given by Eq. (4.31), whereas the performance measures are obtained using the actual transfer function given by Eq. (4.30). From Fig. 4-12(*a*) it is easy to notice that the range of ω_{cl} is limited for positive PID parameters in a small zone between 0.1 to 0.24. At higher value of ω_{cl} the sensitivity M_s improves whereas the load regulation IAE_{ld} deteriorates.

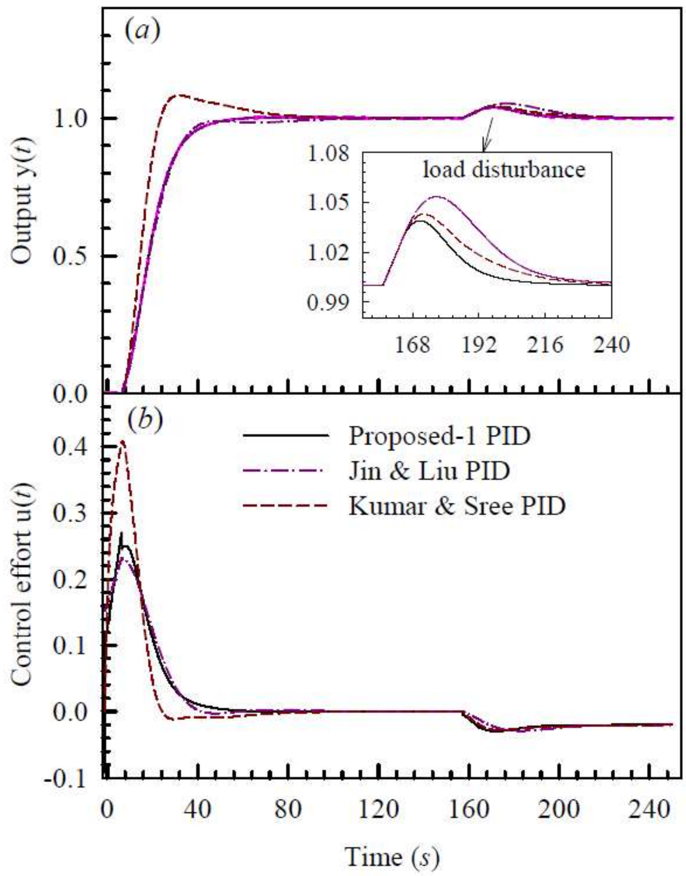


Figure 4-13: Comparison of the time response and control effort of the proposed method for process P3 with some latest PID tuning methods of integrating processes.

Notice from Fig. 4-12(*b*) that the higher value of *m* improves sensitivity at the expense of load regulation. At higher ω_{cl} both TV_{ld} and TE_{ld} show the improving behavior. Observing the simulation results, it is clear that the choice of PID controller parameters is available at higher value of ω_{cl} where all the performance measures show improving behavior except the load regulation.

Table 4-3: Performance measures of process P3 obtained with different tuning methods.

PID controller $C(s)$ and set point filter $F(s)$ for process P3			Performance and robustness measures					GPI
	$C(s)$ & $F(s)$	λ	M_s	IAE_{ld}	TV_{ld}	IAE_{sp}	TV_{sp}	
Proposed-1 ($\zeta_{cl} = 0.7$, $\omega_{cl} = 0.2$, $m = 2$)	$C(s) = 0.5827 + \frac{0.0214}{s} + 1.1782s$ $F(s) = \frac{-8.714s^3 + 1.866s^2 + 0.5613s + 0.0214}{23.56s^3 + 12.83s^2 + 1.011s + 0.0214}$	20	1.91	0.97	0.042	9.74	0.45	0.70
Kumar & Sree	$C(s) = 0.5169\left(1 + \frac{1}{34.1s} + 3.2985s\right)\frac{1}{1.4836s + 1}$ $F(s) = \frac{17.25s + 1}{30.4s + 1}$	-	2	1.4	0.035	17.5	0.72	0.88
Jin & Liu	$C(s) = 0.384\left(1 + \frac{1}{35.788s}\right)$ $F(s) = \frac{14.21s + 1}{35.788s + 1}$	-	2	2.13	0.039	15.3	0.38	0.85
Proposed-2 ($\zeta_{cl} = 0.7$, $\omega_{cl} = 0.15$, $m = 3$)	$C(s) = 0.5743 + \frac{0.0247}{s} + 1.0885s$ $F(s) = \frac{-8.05s^3 + 1.839s^2 + 0.5496s + 0.0247}{19.59s^3 + 11.43s^2 + 1.018s + 0.0247}$	18	1.90	0.847	0.045	10.16	0.51	0.72
Proposed-3 ($\zeta_{cl} = 0.8$, $\omega_{cl} = 0.15$, $m = 3$)	$C(s) = 0.5342 + \frac{0.0203}{s} + 0.9791s$ $F(s) = \frac{-7.24s^3 + 2.0s^2 + 0.5139s + 0.0203}{17.62s^3 + 10.59s^2 + 0.8993s + 0.0203}$	18	1.79	1.018	0.041	10.98	0.47	0.71

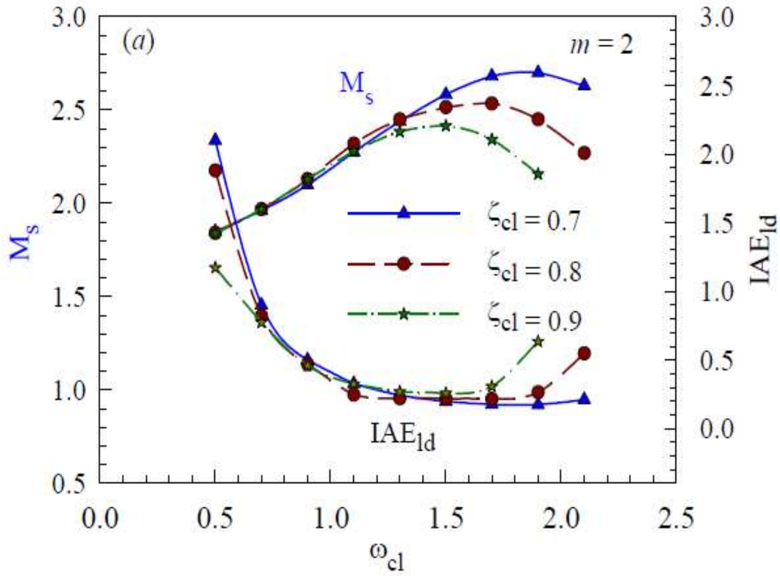
One can always make a tradeoff between load regulation and sensitivity according to the requirement of a given system. Figure 4-13(a) and (b) show the closed loop time response and controller response of the present PID controller obtained using ζ_{cl} = 0.7, ω_{cl} = 0.2 and m = 2 (Proposed-1) together with those obtained by Kumar & Sree and Jin & Liu methods. The value of λ is chosen 20 to reduce the overshoot which also affects the rise time. The proposed controller gives better load regulation and sensitivity as can be seen from Table 4-3. The closed loop performance measures are (Proposed-1: $T_s \sim 25s$, %OS~ 0; Kumar & Sree: $T_s \sim 70s$, %OS~ 10; Jin & Liu: $T_s \sim 25s$, %OS~ 0) same as that of Jin & Liu and much better than that of Kumar & Sree. The initial control effort is much less than that of Kumar & Sree [109]. Table 4-3 compares the closed loop performance measures of the controller designed by proposed method with that of Kumar & sree and Jin & Liu methods. To demonstrate the flexibility of the present method we have also listed parameters and performance measures of two more controllers indicated by Proposed-2 and Proposed-3. Here it can be seen that all the performance measures are quite reasonable and some of them are better than those of the controllers designed by other method. A comparison of the GPI of different controllers listed in Table 4-3 clearly indicates that the controllers Proposed-1, Proposed-2 and Proposed-3 are better and more balanced controllers.

4.4.6 Example 4: UFOIPTD process

Consider a challenging unstable integrating process [128] with transfer function as

$$P4 = \frac{e^{-0.2s}}{s(s-1)} \quad . \tag{4.32}$$

Here $\delta = 0$, a = -1 and L = 0.2. Figure 4-14 shows the performance measures of process P4 obtained by varying tuning parameters ω_{cl} , ζ_{cl} and m. As discussed earlier, in this case also the regulatory performance is better at the lower value of ζ_{cl} and at higher value of ω_{cl}, whereas the sensitivity shows the opposite behavior.



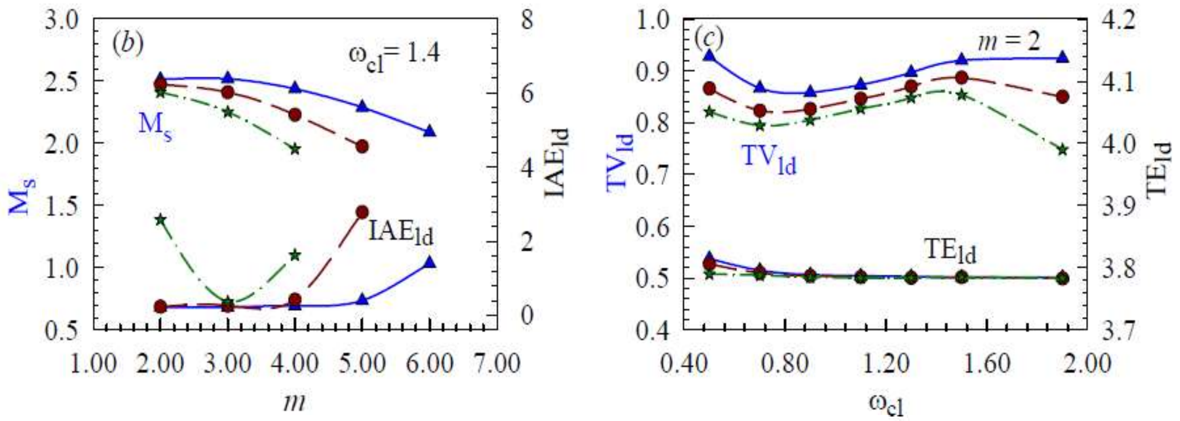


Figure 4-14: Variation of (*a*) robustness and load regulation of process P4 with ω_{cl} covering all the positive values of PID at m = 2, (*b*) M_s and IAE_{ld} with m for fixed ω_{cl} = 1.4, (*c*) TV_{ld} and TE_{ld} of PID controller with ω_{cl} at m = 2.

Table 4-4: Performance measures of process P4 obtained with different tuning methods.

PID controller $C(s)$ and set point filter $F(s)$ for process P4			Performance and robustness measures					GPI
	$C(s)$ & $F(s)$	λ	M_s	IAE_{ld}	TV_{ld}	IAE_{sp}	TV_{sp}	
Proposed-1 (ζ_{cl} = 0.7, ω_{cl} = 0.67 m = 2)	$C(s)=0.7275+\frac{0.2108}{s}+2.1425s$ $F(s)=\frac{0.572s^3+0.997s^2+0.5167s+0.2108}{1.285s^3+2.579s^2+0.854s+0.2108}$	1.0	1.94	0.98	0.81	1.68	0.85	0.81
Kumar & Sree	$C(s)=1.9421\left(1+\frac{1}{2.9s}+1.5459s\right)\frac{0.1s+1}{0.0488s+1}$ $F(s)=\frac{0.81s^2+1.8s+1}{4.4832s^2+2.9s+1}$	-	1.94	0.29	0.81	0.72	1.72	0.67
Chao et. al.	$C(s)=0.8594\left(1+\frac{1}{4.4s}+2.7s\right)$ $F(s)=\frac{1.93s^2+2.8s+1}{11.88s^2+4.4s+1}$	-	1.94	1.08	0.79	1.48	1.01	0.82
Proposed-2 (ζ_{cl} = 0.7, ω_{cl} = 1.5, m = 2)	$C(s)=2.0215+\frac{1.0515}{s}+2.761s$ $F(s)=\frac{0.4478s^3+1.357s^2+0.97s+1.0515}{1.657s^3+3.974s^2+2.652s+1.0515}$	0.6	2.5	0.20	0.92	0.67	1.37	0.68
Proposed-3 (ζ_{cl} = 0.8, ω_{cl} = 1.4, m = 3)	$C(s)=1.7748+\frac{0.780}{s}+2.6857s$ $F(s)=\frac{0.4629s^3+1.331s^2+0.9949s+0.78}{1.611s^3+3.751s^2+2.243s+0.78}$	0.6	2.4	0.25	0.87	0.74	1.33	0.67

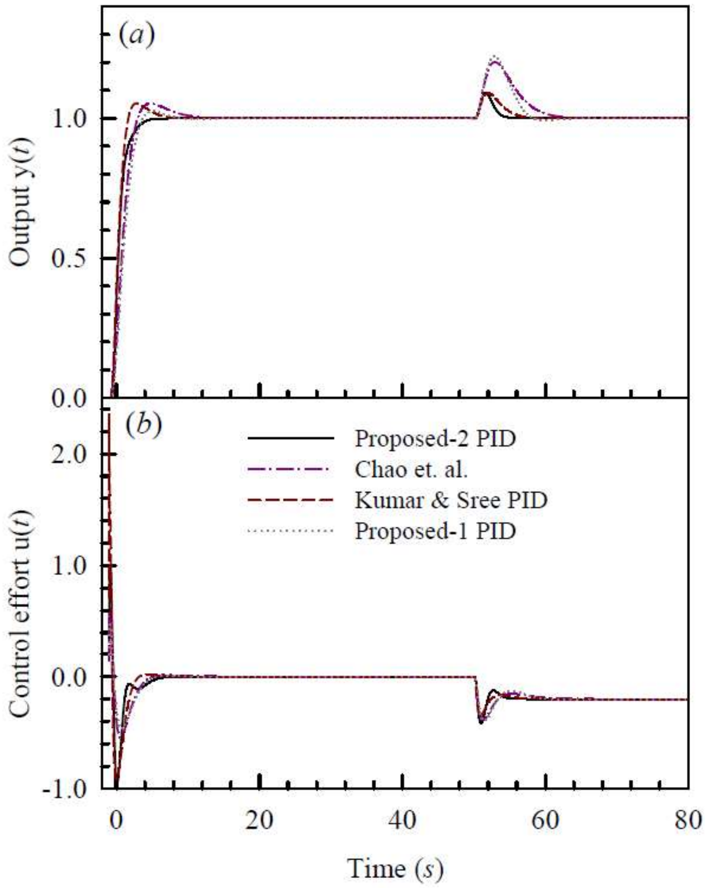


Figure 4-15: Comparison of process P4 with some latest PID tuning method for integrating processes (*a*) Time response (*b*) Control effort.

Figure 4-15(*a*) and 4-15(*b*) compare the time response and controller response of the process P4 with other reported tuning methods for integrating systems respectively. Table 4-4 lists the closed loop performance measures. It is readily observed from Fig. 4-14 that with same M_s (M_s = 1.94), the present controller (Proposed-1) gives a reasonable value of performance measures. The load regulation is better than Chao et. al. [128] only, the controller smoothness is almost identical and the initial control effort is much better than the both methods. In this case the estimated closed loop performance measures are much better than the other two methods (Proposed-1: T_s ~ 5*s*, %OS~ 3; Kumar & Sree: T_s ~ 7*s*,

%OS~ 5; Chao et. al.: $T_s \sim 9s$, %OS~ 5). The main feature of the proposed tuning method is the flexibility to choose PID controller parameters in a tradeoff between robustness and load regulation. A comparison of GPI of the controllers listed in Table 4-4 indicates that global performance indexes of controller's Proposed-2 and Proposed-3 are comparable to that of Kumar & Sree method. Observing Fig. 4-14(*a*) and 4-14(*b*), it is clear that if robustness is compromise, then much better load regulation can be achieved. To demonstrate this fact simulations are performed with controllers Proposed-2 and Proposed-3. Results are also listed in Table 4-4. Here one can see that all the performance measures are improved considerably.

4.5 Summary

The design of a 2-DOF LQR based PID controller for integrating systems considering the tradeoff between regulatory performance and sensitivity has been presented in this chapter. The PID tuning is based on LQR using dominant pole placement approach. The three design parameters, closed loop damping ratio ζ_{cl} , natural frequency ω_{cl} and relative dominance m are used to obtain the whole range of positive PID parameters for a given model. In the proposed method the transient response can be precisely tuned using a unique design of a set point filter based on single parameter λ and PID parameters obtained using the LQR method. The tuning rule of the proposed method is analytical for PID controller as well as for the set point filter and is applicable for a wide range of integrating processes.

The simulation has been performed covering a series of integrating plants such as FOIPTD, DIPTD, PIPTD and UFOIPTD. Simulation results indicate that both good robustness and load regulation cannot be achieved simultaneously. Better sensitivity occurs at lower value of ω_{cl} and at higher value of ζ_{cl}, whereas better load regulation can be achieved at higher ω_{cl} and lower ζ_{cl}. Thus a proper design needs a tradeoff between sensitivity and load regulation. It is observed that tuning of set point filter constant λ plays a crucial role in reducing the overshoot and initial control effort. Based on the simulation results of different types of integrating systems discussed in the present work, a guideline for choosing the range of suitable ζ_{cl} and ω_{cl} and λ can be summarized as follows. For FOIPTD system the range is $0.7 \le \varsigma_{cl} \le 0.8$ and $0.4 \le \omega_{cl} \le 0.6$, for DIPTD system $0.7 \le \varsigma_{cl} \le 0.8$ and $0.3 \le \omega_{cl} \le 0.45$, for PIPTD system $0.7 \le \varsigma_{cl} \le 0.9$ and $0.13 \le \omega_{cl} \le 0.18$, for UFOIPTD system $0.7 \le \varsigma_{cl} \le 0.9$ and $1.25 \le \omega_{cl} \le 1.75$. The value of λ can be taken around 3 to 5 for most of the integrating systems except for PIPTD system where it should be between18 to 22. On average, the proposed 2-DOF PID controller tuning method gives a reasonably good closed loop performance measures for most of the integrating plants. In order to get the positive PID parameters for a given system one needs to sweep the three tuning parameters ω_{cl} , ζ_{cl} and λ. Since the method is analytical, it takes very little time. Once the variation of the robustness and load regulation plot is obtained, it gives lots of flexibility to the designer for choosing the suitable PID controller parameters based on tradeoff between the robustness and performance. The controller designed by this method also requires minimum control effort as the method is based on LQR.

CHAPTER 5

Conclusion and future works

5.1 Conclusion

The present thesis has studied and developed tuning methods of PID controller parameters for variety of processes with time delay. This time delay can appear by various means. These delays can be random or constant. The delay arises from the measurement sensor during processing of the output data, from the reference signal due to remote operation of plants etc. Presence of time delay deteriorates the system performance and many times causes unwanted response. PID controller which is second most widely used controller in the process industry needs to be designed optimally to handle such time delay in the control systems.

Three analytical PID tuning methods have been developed in order to cover most categories of industrial plants which include first order systems with time delay, second order systems with time delay and integrating systems with time delay. Sub categories of second order system and integrating systems have also been studied. Systems studied in second order categories are systems with highly oscillatory behaviour, critically damped system, under damped systems and over damped systems. Systems studied in integrating categories are first order integrating systems, pure integrating systems and double integrating systems. In the all three tuning methods proposed in this thesis, the dominant pole placement approach is common.

The first tuning method discussed in Chapter 2 is a general one and can be applied to varieties of plants. This method is based on D decomposition method in which the range of PID parameters have been obtained graphically for pre-specified closed loop performance measures. The second tuning method, discussed in Chapter 3, is specific to the standard second order systems with time delay. This method uses LQR approach together with pole placement technique to tune PID controller for desired closed loop response with minimum control effort. The third tuning method for 2-DOF control system discussed in Chapter 4 applies to wide varieties of integrating systems. PID tuning rules in this case are also developed using LQR approach and pole placement technique to control both servo and regulatory responses simultaneously.

The salient points of the studies performed in this thesis can be summarized as follows:

The graphical tuning rule for PID controller developed in Chapter 2 is applicable to variety of first and second order systems with time delay. It is observed from the simulation that sets of proportional and integral gains which are on the linear part with positive slope on fixed damping ratio curve with increasing natural frequency in the parameter plane, are more appropriate and produce closed loop time response performance measures very close to the specified values. It is also observed that controller designed for small closed loop natural frequency requires very less control effort. On the other hand, the sets of proportional and integral gains which are on the negative slope of the above mentioned curve, give poor closed loop time response measure.

In the case of FOPTD system, the choice of controller parameters corresponding to comparatively lower value of natural frequency is more suitable for the robust controller design under the perturbation of system parameters.

The PID controller designed using LQR technique and dominant pole placement, discussed in Chapter 3, is shown to give a good closed loop time response for various second order processes with time delay as compared with the existing PID tuning methods using LQR.

The present tuning method is also found to work well to control the performance of an unstable second order plants. In this case the choice of closed loop natural frequencies is found to be much less for designing PID controller when compared with other stable second order systems.

A comparison of simulations results with other time domain performance indices such as ISE, ITSE and IAE indicates that the present method gives an overall better closed loop time response with comparatively less control effort.

Simulation results indicate that change in the location of non-dominant pole towards left hand side in complex s-plane increases the robustness of controller in the case of mismatch between the process delay time and the delay time at which the controller is designed. However this action increases the rise time which is completely opposite to the cases of systems with no time delay.

The third methodology discussed in Chapter 4 focuses on developing of 2-DOF PID controller tuning method utilizing LQR and dominant pole placement approach especially for integrating plants. The advantage of this method is the flexibility in choosing PID parameters considering the trade-off between performances and robustness.

Simulation results performed on different kinds of integrating plants indicate that better sensitivity occurs at the lower value of natural frequency and at higher value of damping ratio and vice versa.

The set point filter designed with PID parameters and system parameters consists of only one variable parameter λ to control the servo response and it plays a crucial role in reducing the overshoot and initial control effort. An increase in the value of λ causes slowing down in the transient response and reduces overshoot as well as initial control effort.

The present method gives lots of flexibility to the designer for choosing the suitable PID controller parameters based on tradeoff between the robustness and performance. Controller tuned using this also needs minimum control effort as the method is based on LQR.

Studies carried out on the introduction of noise in the measurement device indicate that the controller designed by present method gives better noise sensitivity than reported controllers. However, designed by other method better response first order or second order noise filter should be used after the measurement device.

5.2 Future works

Tuning of PID controller parameters is a research area with much opportunity for contribution. It needs special attention for different category of plants. In the present thesis, only few goals have been address in developing PID controller parameters tuning for the plants with time delay. The theoretical work presented in the thesis on the design of PID controller is developed for single input and single output system, disturbance observer etc. This can be extended in developing algorithm to control multiple input and multiple output systems which are more common in the present regime.

The present theoretical work on the design of PID controller, developed for analogue domain, can be further extended to develop same type of algorithm in digital domain. In developing digital PID controller proper care needs to be taken in choosing suitable sampling time as its results in quantization effect, frequency warping etc. The conversion of present s-domain PID controller setting to z-domain will be more advantageous for the industrial practitioner due to the availability of fast and less costly processors.

The graphical tuning procedures presented for obtaining PI controller parameters for fixed time delay systems, could be further extended for the case of network delay where the delay is random and changes frequently.

Finally the present analysis is fully focused on the integer order PID controller design. Introduction of fractional order PID would certainly open doors of new research in the design of optimum controller with respect to control efforts, robustness and desired closed loop time response.

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