DEVELOPMENT OF OPTIMIZATION TECHNIQUES FOR FUEL MANAGEMENT IN HEAVY WATER MODERATED REACTORS

By

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I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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I, hereby declare that the minor corrections suggested by the examiners have been incorporated.

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NOMENCLATURE

Alpha (α)	Weighing Factor	Describes the weight of previously evaluated population in updating the distribution function
B	Burn-up	Burn-up of individual channel
CP	Channel Power	Channel power of individual channel
DF	Distribution Function	A function used to generate a population of loading patterns
D	Discharge burn-up	Design / Target discharge burn-up of individual type of fuel
RF	Refueling factor	Describe the priority of a channel for refueling
Resh F	Reshuffling factor	Describe the priority of a channel for reshuffling
	Refueling Schemes	
	On- Power Refueling	Refueling operation is carried out during reactor operation
	Off Power / Batch Refueling	Refueling operation is carried out after shuting down the reactor
	Direct Refueling	The burnt fuel is replaced by fresh fuel
	Refueling with reshuffling	The fresh bundle is placed at a site of partially burnt fuel and the partially burnt fuel is placed to a more burnt fuel location and the fully burnt fuel is discharged

LIST OF ABBREVIATIONS

ACO	Ant Colony Optimization
AHWR	Advanced Heavy Water Reactor
ANN	Artificial Neural Network
BWR	Boiling Water Reactor
CARSH	Code for AHWR Refueling Strategy using Heuristics
DF	Distribution Function
DU	Depleted Uranium
DUPIC	Direct Use of PWR fuel in Candu reactors
EA	Evolutionary Algorithm
EDA	Estimation of Distribution Algorithm
FEMINA	Flux Expansion Method in Nodal Analysis
FPD	Full Power Days
GA	Genetic Algorithm
LEU	Low Enriched Uranium
LP	Loading Pattern
LPO	Loading Pattern Optimization
LWR	Light Water Reactor
MCP	Maximum Channel Power
MMP	Maximum Mesh Power
MOX	Mixed Oxide Fuel
NU	Natural Uranium
OF	Objective Function
OpenMP	Open Multi Processing

PHWR	Pressurized Heavy Water Reactor
PSO	Particle Swarm Optimization
Pu	Plutonium
PWR	Pressurized Water Reactor
RF	Refueling Factor
Resh F	Reshuffling Factor
SA	Simulated Annealing
SDS	Shut Down System
Th	Thorium

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SYNOPSIS

Nuclear energy is a superior alternative to conventional (petroleum and coal) energy resources as it is clean, economic and capable of large scale power production. With the reactor operational knowledge of more than sixty years [1] and better understanding of nuclear technology, the design and safety of nuclear reactors have considerably improved. Over this period of time, several different nuclear reactor concepts have emerged because of technology progress. There are persistent research efforts for safe and economic operation, lower waste generation and better fuel utilization in nuclear reactors. The main aim for the fuel management in a nuclear reactor is to realize higher fuel utilization without compromising the safe operation. Optimum utilization of fuel in all stages of reactor operation is a complex problem for all reactors. The focus of this thesis is to review the optimization problems related to fuel management in heavy water moderated reactors with unique fuel cycle features like Advanced Heavy Water Reactor (AHWR) [2,3]. Different optimization techniques in general were studied and specific techniques were developed for AHWR which have ascertained enhanced utilization of fuel at any stage of reactor operation.

For commercial power production, different types of thermal reactors are operational in different regions of the world. In USA, Europe, China and Japan, majority of the thermal reactors presently being operated for commercial power production are light water reactors (LWRs). Pressurized Heavy Water Reactors (PHWRs) [4] plays a greater role for nuclear power production in case of Canada and India. The neutron economy in PHWRs is better than LWRs as it uses heavy water moderator, small length Natural Uranium NU bundles, on-power refueling and a very small amount of excess reactivity is managed. Whereas, in case of LWRs, slightly enriched Uranium (SEU) is used along with light water moderator and refueling is always off-power (batch refueling). The use of SEU results in large initial excess reactivity which is compensated by dissolving a neutron poison (Boron) in the moderator.

In India, about 18 pressurized heavy water reactors (PHWRs) are in operation. The very well defined three stage program of India is outlined below:

Stage-I: Development and expansion of NU based PHWRs

Stage-II: Development of Fast breeder Reactors (FBRs) using re-processed Pu from discharged NU of stage I reactors

Stage-III: Development of advanced Th-U-233 based sustainable reactors

As a step towards developing technologies for Th-U233 based next generation reactors, Advanced Heavy water reactor (AHWR) is being designed. Purpose of AHWR is to design a unique reactor with all the advanced safety features, where Thorium based reactor technologies will be used and developed by utilizing past experience and expertise of PHWRs & LWRs. The AHWR has a very unique fuel cycle where full length cluster having high target discharge burn-up fuel (40-60 GWd/Te) is being used with on-power refueling. Due to this unique fuel cycle feature of AHWR, the fuel management in equilibrium core will be quite different from any conventional reactor presently in operation or construction. The distinctive fuel cycle of AHWR necessitates the development of special refueling scheme for efficient fuel utilization in equilibrium core and transition phase. A good portion of this thesis has been devoted to development of special refueling scheme, which can demonstrate a safe and economic operation of AHWR during transition phase and equilibrium phase.

The initial core of AHWR is also quite different from LWRs and PHWRs. The AHWR initial core will require minimum two types of clusters for initial core loading for flat flux distribution. The applicability of different modern optimization algorithms has been analyzed for AHWR initial core loading pattern optimization (LPO) problem. In second portion of this thesis, population based algorithms (Genetic algorithm (GA) and Estimation of distribution algorithm (EDA)) have been developed to address AHWR initial core loading pattern optimization problem.

A brief description of these two problems (in-core fuel management for AHWR and initial core LPO of AHWR) and development of optimization techniques to address these problems is given below.

Loading pattern optimization (LPO) during each refueling stage is the main challenge in LWRs like pressurized water reactors (PWRs) and boiling water reactors (BWRs). Whereas, in case of heavy water reactors like pressurized heavy water reactors (PHWRs), the development of refueling scheme is relatively simple mainly due to on-line refueling, use of small length bundle and natural uranium as fuel. The use of small length bundle and flexibility of multi-bundle shift scheme helps in controlling the ripples in power due to refueling. Even though the PHWRs possess best neutron economy, the use of natural uranium fuel in PHWRs limits the achievable burn-up to ~7000 MWd/Te and as a result large amount of discharged fuel has to be managed. The Advanced Heavy Water Reactor (AHWR) uses both the features of PWRs (high discharge burn-up fuel) and PHWRs (on-power refueling) and has several inherent passive safety features. The AHWR uses (Th, U, Pu) MOX or (Th, LEU) MOX fuel with boiling light water as coolant and heavy water as moderator. The AHWR is designed to have good neutron economy and owing to higher discharge burn-up, lower waste is expected. However, during on-power refueling with full length channel and high enrichment fuel, the challenge is to control the refueling ripples and maintaining the power distribution and operational parameters under their design limits. The special refueling strategy has an objective to control the flux distribution and hence power peaking due to on-power refueling. In the special refueling scheme, it is proposed that each refueling operation should be followed by reshuffling operations so that refueling ripples are contained within their design limit. The special refueling scheme requires selection of two channels (one for refueling and one for reshuffling) at each refueling. For selecting two appropriate channels from a typical core consisting of ~444 fuel channels, we have to study ${}^{444}P_2$ number of possible combinations. Simulation of all these combinations and finding out the best one seems very cumbersome and time consuming. The best combination found out this way will be used for one refueling only. For every refueling one has to simulate same number of combinations. Selection of channels for refueling and reshuffling is a complex problem. AHWR has a very long transition period (~10-15 years), where different types of fuel are to be managed. In our work, we have introduced a concept of refueling factor and reshuffling factor. These factors will give the priority for a particular cluster for refueling and reshuffling and are assigned to each fuel cluster of the core. Based on these factors, a set of refueling inputs is generated and simulated. If the refueling input gives satisfactory result, we will burn the core for few days till the excess reactivity is exhausted and refueling factors and reshuffling factors are modified to proceed for the next refueling. If the refueling input does not give satisfactory results, then next set of channels with lower priority is considered. Two different computer codes have been developed, where the selection of fuel channels for refueling/reshuffling has been automated. These programs are developed such that the fuel cluster which has achieved its target discharge burn-up is selected for discharge on priority keeping the power peaking in control. Several 3D diffusion calculations are required for simulating all these refueling schemes. Parallel processing on shared memory interface has been used to reduce the time for fuel cycle study for AHWR. The new optimization technique developed as part of this thesis could successfully demonstrate the in-core fuel management in AHWR and an improved fuel utilization.

In order to maximize the power output, it is required to achieve a flat flux distribution in the reactor core. The flux flattening in equilibrium stage of any reactor is achieved by differential burn-up zones. In initial core of a reactor, flux flattening is achieved with different types of fuels which have differential reactivity (different fissile content). In AHWR, the equilibrium cycle cluster has been designed to have a high discharge burnup (~40-60 GWd/Te) and hence has a higher excess reactivity. The direct use of equilibrium core cluster in initial core will lead to requirement of large quantity of poison in moderator to suppress this excess reactivity, which will adversely affect different reactivity feedbacks and worth of reactivity devices and shut down system (SDS). In LWRs, the initial core fuel clusters have lower enrichment than equilibrium core fuel clusters. In PHWRs, for flux flattening in initial core, Th or depleted Uranium (DU) or deeply depleted U (DDU) has been used along with NU bundles [5-6]. For AHWR, two initial core clusters with lower enrichment (2% fissile) is being considered for initial core clusters. The loading of initial core clusters in the core locations is a complex combinatorial optimization problem. The problem is to find out the location and number of channels of different types of fuels to be loaded for best fuel utilization in initial core and maintaining safe and continuous operation. In second part of this thesis, initial core LPO problem of AHWR has been solved. This type of problem can be solved by defining one objective function and then maximize or minimize it. Two types of optimization methods are available in literature to find the optimum solution to this problem named as conventional methods and modern methods. The conventional methods include gradient based techniques like Gauss Newton Method [5], Steepest decent method [7] etc. These techniques require a good understanding of problem and how the parameters are related to objective function. The application of these methods is very difficult and a very small area of search space is explored. Modern methods based on evolutionary algorithms are frequently being used for fuel loading pattern optimization problems. Genetic algorithm (GA) [8-10] simulated annealing (SA) [11] and Ant Colony Algorithm (ACO) [12] are few examples of population based evolutionary algorithms which have been successfully applied for core reloading optimization problems of Pressurized water reactors (PWRs). In these modern optimization algorithms, a two step procedure is followed. In first step, a pool of randomly generated solutions is evaluated and in second step, new set of solutions is generated by considering feedback from the current evaluations. In case of EDA, the weighing factor ‘α’ considered for updating the probability distribution function, initial distribution function and population of one pool of candidate solution are the feedback parameters. A very small value of ‘α’ (<0.1) has been considered good for better search space exploration. In our study, we have observed that the proper value of various parameters in different feedback parameters is very important. Considerable improvement in optimized solution is observed when, a better choice of these internal parameters is considered. In this thesis, we have observed that a small value of ‘α’ is not adequate for AHWR initial core LPO problem. A comprehensive study has been done which has given a new direction for more improvement in population based algorithms for LPO problems.

As a part of this thesis, a computer code based on estimation of distribution (EDA) [13] has been developed to optimize the initial core of AHWR. The distributed memory parallel computer system AGGRA at BARC was used for parallelization. Suitable values for various internal parameters (‘α’ and population size) to be considered for AHWR initial core loading pattern optimization problem have been estimated. For the sake of comparison and completeness, the initial core optimization of AHWR by using Genetic algorithm (GA) has also been addressed. The thesis comprises of seven chapters.

Chapter 1 presents brief introduction to the nuclear reactors. It is followed by a summary of scope and review of previous work of the studies carried out in the present thesis. The design features and fuel cycle of AHWR is described. An outline of the thesis is given at the end of this chapter.

In Chapter 2, a basic description of reactor core calculations in steady state has been presented. A brief description of the computational tools used for simulation of steady state of nuclear reactor is also given. The transport theory code ITRAN [14] has been used to perform lattice level calculations. The calculations were performed by using 69-group library based on ENDF/B-VI.8 nuclear data obtained from IAEA. The whole core simulation were done in to two energy groups using a 3D diffusion theory based code FEMINA [15] based on nodal expansion method..

In Chapter 3, a special refueling scheme for AHWR core is described, which is an optimal combination of refueling strategies used in LWRs and PHWRs. Each refueling operation is combination of refueling and one reshuffling operation to control the local power peaking. Generally the experience and knowledge base in a form of ''IF-THEN'' rule-sets [16] are used along with other specific search control strategies. However, the direct implementation of heuristic information in the form of factors has been tried. These factors are further translated for describing the preference of a channel for refueling or reshuffling. A computer code CARS has been developed based on the above principles. The computer code CARS provides all the micro details like fresh fuel requirement, storage space for discharged fuel, behavior of maximum channel power (MCP), maximum mesh (bundle) power (MMP), channels selected for refueling/reshuffling, discharge burn-up achieved and boron requirement, etc. for the entire life of reactor.

Chapter 4 describes the problems observed during core follow-up studies for AHWR done with CARS:

(a) Power peaking problem during the pre-equilibrium phase (transition phase).

(b) Due to the quarter core mirror symmetry, certain channels near to axis symmetry were getting over due for refueling (described later). As a result some other channels were getting discharged pre-maturely and leading to overall loss in discharge burnup.

(c) Very long computation time (7-10 days)

The following improvements in CARS were done to address these problems.

(a) Adoption of double reshufflings along with refueling

(b) Use of $\pi/2$ rotational symmetry instead of mirror symmetry

(c) Use of parallel processing to accelerate the speed

A refueling strategy based on a suitable combination of three refueling schemes viz. refueling without reshuffling, refueling with single reshuffling and refueling with double reshuffling scheme is adopted. The refueling strategy was evolved after including the zonal power of each zone as an important parameter of the strategy. As a result the flux distribution at each refueling was maintained near to the equilibrium core flux distribution. The use of CARS has reduced the manual efforts for the selection of channels for refueling to a great extent. The CARS has made the study of various types of equilibrium core clusters for AHWR in a very short time. It has also helped in the design and optimization of burnable poison content in the equilibrium core cluster.

In Chapter 5, the basic loading pattern optimization (LPO) problem is defined. Different optimization techniques (Modern methods and Conventional methods) are discussed for solving LPO problem. The size and search space for AHWR initial core LPO is described. The AHWR core consists of 444 fuel lattice locations. By exploiting symmetry of the core, the problem size for AHWR initial core optimization reduces to 2^{62} (10^{18}), which is also a very large size problem. Simulation of all these loading patterns to choose the best loading pattern is not practical in a finite time scale. This type of combinatorial optimization problem can be solved by defining an objective function and then maximizing it. The objective function of the problem is based on K-effective, SDS-1 worth, boron in moderator and radial peaking. The objective function is defined by using penalty method. In appendix referred in this section, the most adequate value of various parameters in objective function has been estimated. We have applied Estimation of distribution algorithm (EDA) (modern method) to optimize initial core loading pattern (LP) of AHWR. In EDA, new solutions are generated by sampling the probability distribution model estimated from the selected best candidate solutions. The weighing factor 'α' decides the fraction of current best solution for updating the probability distribution function after each generation. A comprehensive study on parameters like population size, weighing factor ‘α’ and initial probability distribution function has been done. It is observed that choosing a very small value of ‘α’ may limit the search of optimized solutions in the near vicinity of initial probability distribution function and better loading patterns which are away from initial distribution function may not be considered with due weightage. It is also observed that increasing the population size improves the optimized loading pattern, however, the algorithm still fails if the initial distribution function is not close to the expected optimized solution. We have tried to find out the suitable values for ‘α’ and population size to be considered for AHWR initial core loading pattern optimization problem.

In Chapter 6, the initial core optimization of AHWR has been tackled by applying Genetic algorithm (GA). The objective function in GA is defined in the same way as in chapter 5, such that during the optimization process, all the variable of objective function (MCP, MMP and SDS-1 worth) meet the pre-determined designed limit. The dependence on population size and initial distribution function on the optimized loading pattern is studied and most adequate values for AHWR initial core LPO have been estimated. It was observed that the algorithm has failed with population size of 24. However, when the initial distribution function is near to optimized solution, the results are better. By increasing the population size to 240 or 1200, optimized loading pattern similar to EDA is achieved. A discussion on the computational cost and simulation time is also given.

Finally, Chapter 7 gives a summary of the research work carried out in this thesis and scope for future extension of this work. In this chapter, the various aspects of optimum fuel utilization during initial phase, transition phase and equilibrium phase have been emphasized. This study allows us to demonstrate fuel cycle of AHWR, where, on-power refueling and high target discharge burnup clusters are used. The new technique developed for AHWR for in-core fuel management has opened a scope for further study to develop refueling strategy for use of slightly enriched Uranium (SEU) or DUPIC fuel (Direct Use of PWR fuel In CANDU reactors) and Thorium based fuels like (Th, Pu) MOX or (Th, U) MOX, in PHWR for lower waste generation and achieving higher discharge burnup. In the present thesis work, a better understanding of the fundamental principles that govern fuel management during initial, transition and equilibrium stage of any reactor has been achieved. The outcome of the research work is also useful to provide guidance for improvement in current optimization techniques to tackle LPO problems.

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CHAPTER 1

Introduction

1.1 Nuclear Power

The faster consumption of conventional (petroleum and coal) energy resources and their environmental effects, makes nuclear energy a superior alternative as it is cost effective, clean source and capable of large scale power generation. As of today, ~10% (WNA, 2016) of total electricity generated in the world is being produced from nuclear power reactors. The design and safety of nuclear reactors has improved with the better understanding of nuclear technology and reactor operational experience of more than sixty years (NEI, 2016). With this knowledge and technology advancement, several different nuclear reactor concepts have emerged. The nuclear reactors can be broadly classified in two categories as thermal reactors and fast reactors (Stacey (2001); Duderstadt and Hamilton (1976)). In thermal reactors, slow neutrons (average neutron energy < 1eV) are used to do the fission reaction. The high value of fission cross section at lower neutron energy is helpful to maintain a fission chain reaction even with very small fissile inventory. As the neutrons produced in a fission reaction are fast (Energy in MeV), a moderator (low mass material like light water, heavy water or graphite etc.) is used to slow down the neutrons. The fast reactors have average neutron energy in keV range and thus any low mass material in the core region of reactor is avoided. The large number of neutrons liberated per fission reaction with fast neutrons is useful to convert the fertile atoms to fissile nuclei (breading), however, to maintain the fission chain reaction larger fissile inventory will be required because of lower fission cross section in fast energy region of neutrons.

Due to simpler design and lesser complexities, thermal reactors are more explored and are installed in all parts of the world. Most of the thermal reactors presently being operated in the world for commercial power production are light water reactors (LWRs). The LWRs require slightly enriched Uranium (SEU) as fuel and off-power refueling (batch refueling). There is another class of thermal reactors called Pressurized Heavy Water Reactors (PHWRs). The PHWRs require Natural Uranium (NU) as fuel and heavy water as moderator along with on-power refueling. In India, about 18 pressurized heavy water reactors (PHWRs) (Bhardwaj (2006)) are in operation. India has a very well defined three stage program, where in first stage; NU will be used in PHWRs. In second stage, The Plutonium (Pu) reprocessed from the discharged fuel of PHWRs will be used in fast breeder reactors (FBRs) and in third stage, advanced (Th, U-233) based sustainable reactors will be developed. As a step towards developing technologies for Th-U233 based next generation reactors, Advanced Heavy water reactor (AHWR) (Sinha and Kakodar (2006)) is being designed. In AHWR, operational experience and expertise of PHWRs is being utilized and advanced safety features are incorporated along with development of Thorium based technologies

1.2 Fuel management in thermal reactors

The fuel management for any type of reactor is a topic of research and there are incessant efforts to improve the fuel utilization for better economic operation and lower waste generation. The main objective in multi-stage decision process of fuel loading, safe and continuous operation and refueling of a nuclear power reactor is to minimize the unit cost of electricity produced. Optimization problems in LWRs, such as optimum fuel assembly pattern with burnable poison rods and optimum core loading pattern with different fuel types or different burn-up zones etc. have been focused in many studies and as these problems deal with multiple variables which interact with each other, they are quite complex problems. For example, enrichment optimization of fuel for Gd-bearing assembly takes into consideration several variables such as loading of Gd, radial and axial peaking, cycle length, number of feed assembly, fuel cycle cost, etc. The primary aim for the fuel management in a nuclear reactor is to achieve higher fuel utilization without compromising the safety during operation. Optimum utilization of fuel in all stages of reactor operation is a complex problem for all reactors. The focus of this thesis is to review the optimization problems related to in-core fuel management, specifically, initial core loading pattern optimization and problems related to refueling in heavy water moderated reactors. The basic fuel cycle of any thermal reactor has three stages named as initial core, transition core and equilibrium core. The reactor is designed with the aim to live its ~90% life in equilibrium core only. In the equilibrium core the global parameters are fixed and only certain variation in local parameters is permitted. The fuel in equilibrium core has different burn-up zones for flat flux distribution. However, in case of initial core only fresh fuel is available. Therefore, to achieve flat flux distribution, differentially reactive (different fissile content or poison Gd etc content) clusters are used.

The three loading pattern optimization (LPO) problems in the reactor fuel cycle are, 1) Initial core LPO problem 2) Optimum utilization of fuel during transition phase and management of different kinds of fuels 3) LPO for equilibrium core at each refueling.

The initial core LPO problem is to find the number of different types of fuel clusters and their location in the core such that k-effective is maximized and worth of reactivity devices is in design limit. Now if the core has 400 locations and we have 3 different fuels then there are 3^{400} different ways to load the core. The symmetry of core can reduce the problem size but still it is observed that the search space is too large to carry out an exhaustive search, therefore, different optimization techniques are developed to find the optimum solution.

With the initial reactivity suppressed by addition of poison in the moderator, the reactor will operate for sometime till refueling is required. During first few refuelings, initial core clusters are required to be discharged with priority (lower design discharge burn-up) and the equilibrium core clusters are loaded. In this case, a cluster with higher burn-up may be ignored to be discharge if it has not reached its design discharge burn-up. In this case we have a restriction of considering the fuel type and discharge burn-up.

Once the core has been loaded with equilibrium core clusters, we have to optimize the core loading pattern at each refueling for target discharge burn-up and full power operation. Now let us consider, if the core (400 channels) is required to be refueled with 80 fuel clusters (20% of core). Therefore, about ${}^{400}P_{80}$ number of possible candidate solutions will be available.

It is observed that both PHWRs and LWRs have fuel management optimization described above however the size of search space may be different. The fuel management and LPO problems in case of PHWRs and LWRs has been studied many times. The AHWR has unique features like high discharge burn-up fuel and on-power refueling and low power density, which requires a different refueling scheme than considered in conventional reactors. Therefore, in this thesis the fuel management during initial, transition and equilibrium phase of AHWR fuel cycle has been considered and optimization techniques have been developed for efficient utilization of fuel during various stages of reactor.

1.3 Design features of AHWR

As recalled from section 1.1, AHWR (Sinha and Kakodkar, 2006; Kakodkar, 1998) plays a very significant role in the road to development towards third stage of Indian nuclear power program. AHWR is a Th based reactor, where better features of PWRs (like use of high discharge burn up fuel, lower waste generation) and PHWRs (like on power refueling low excess reactivity, better neutron economy) are amalgamated together. The AHWR is a unique reactor which is designed for commercial utilization of Thorium and integrated technological demonstration of the Thorium cycle. The AHWR is a 920 MWth (fission power 980 MWth), vertical pressure tube type thorium-based reactor cooled by boiling light water and moderated by heavy water, where inherent safety features like negative coolant void reactivity and heat removal through natural circulation are employed. The AHWR design incorporates our experiences in design, operation and safety analyses/aspects of PHWRs and BWRs. The aims of the design are to achieve relatively higher fraction of power from Th-^{233}U, self-sustenance in ^{233}U, a high discharge burnup with minimum makeup fuel and negative void reactivity coefficient. Plutonium is used as makeup fuel to achieve high discharge burnup and self-sustaining characteristics of Th-^{233}U fuel cycle. Since plutonium is being used as the makeup fissile feed, it is required to minimize the inventory of plutonium and also its consumption, which is another important objective. The plutonium being considered is from discharged fuel from PHWRs. The fuel cycle of AHWR is based on (Th-^{233}U-Pu) MOX fuel in closed cycle mode with target discharge burnup of ~ 40 GWd/Te. The fuel cluster for AHWR is known as D5 cluster (A Chakraborty et al. (2015)) as shown in Fig 1.1. It consists of a circular array of 54 fuel pins with central support rod and support tube.

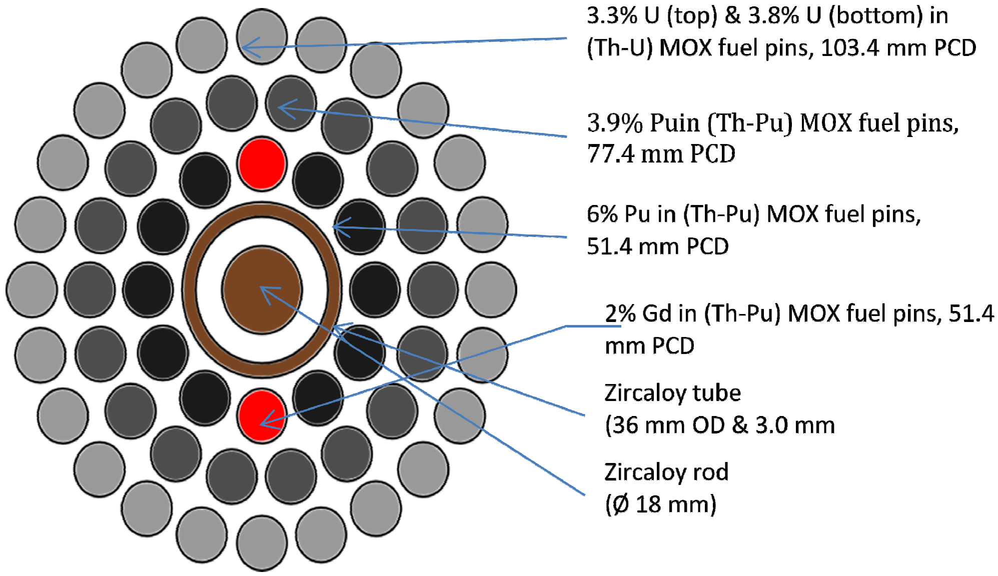


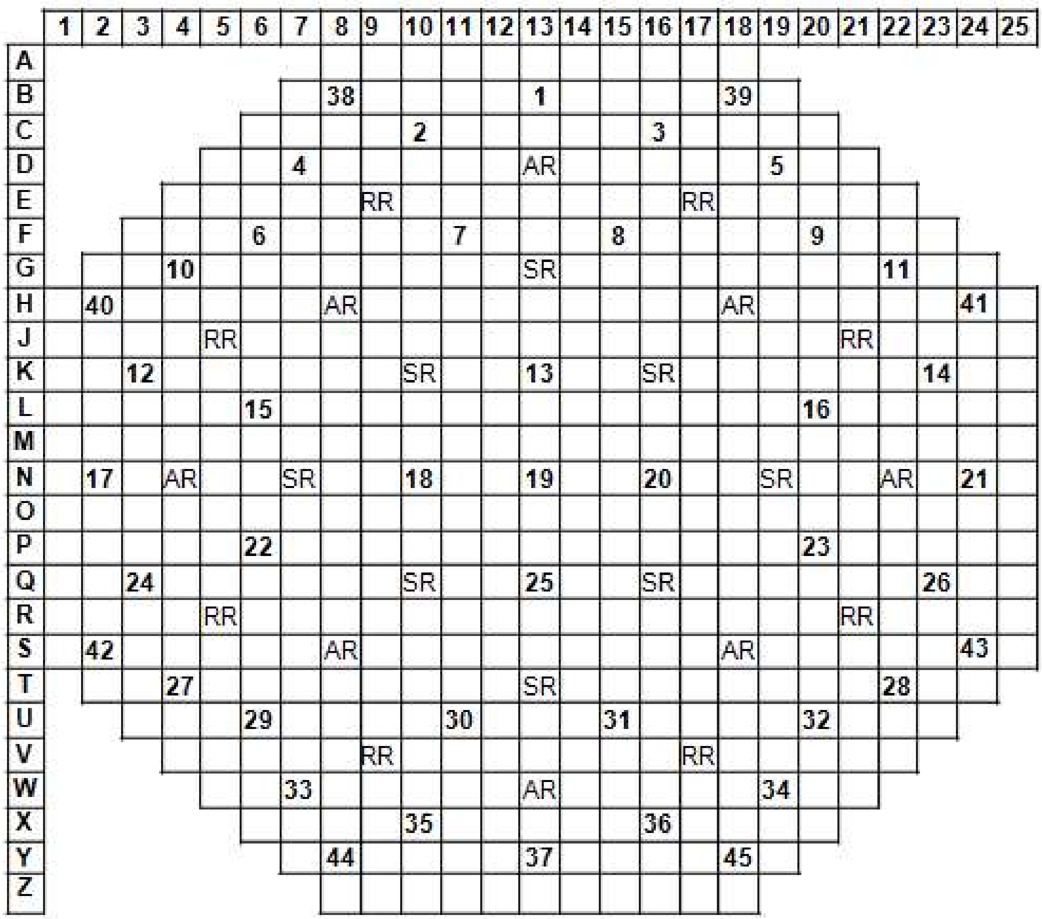
Figure 1.1 Cross section of the AHWR (Eq. Core) fuel cluster

The fuel cluster or assembly has a central solid structural support rod of OD 18 mm and made of Zircalloy-2. The central support rod is termed as displacer rod. The displacer rod is surrounded by a central structural tube of OD 36 mm and ID 30 mm made of Zircaloy-2. The structural tube is termed as displacer tube and it is perforated to inject coolant directly on fuel pins during LOCA. The displacer tube surrounded by three concentric arrays of fuel pins containing 12, 18 and 24 fuel pins in the inner, intermediate and outer rings respectively. For equilibrium core cluster, the 12 pins of the innermost ring contain 6% Pu in (Th, Pu) MOX and the 18 pins of the middle contain 3.9% Pu in (Th, Pu) MOX. The 24 pins of the outer ring contain average 3.55% of Uranium in (Th, U) MOX (the lower half of the active fuel contain 3.8 % U and the upper half contain 3.3 % U). The U-233 content in the Uranium has been assumed to be about 78%. To make the cluster suitable for on-power refueling the excess reactivity of the cluster has to be suppressed by adding 2% Gadolinium (burnable poison) in two fuel pins of the inner ring. The cross section of the fuel cluster is shown in Figure 1.1. The basic fuel cycle is based on the fact that the AHWR core should be self-sustaining in U-233. The U-233 required is to be bred in situ, therefore it is proposed to have an initial core with Depleted Uranium – Plutonium MOX as fuel to conserve Plutonium resources and U-233 is being produced in-situ in the intermediate core using (Th, Pu) MOX. There will be gradual transition from the initial core to intermediate core and from intermediate core to equilibrium core. Therefore the initial core needs to be refueled with intermediate core fuel clusters containing (Th, Pu) MOX as fuel. After sufficient inventory build-up of U-233, the intermediate core shall be refueled with equilibrium core clusters to attain self-sustaining in U-233 in equilibrium state of AHWR.

The reactor core of AHWR consists of 513 lattice locations in a square lattice pitch of 225 mm. Of these, fuel assemblies occupy 452 locations and 61 locations are reserved for the reactivity control devices and shut down system-1. Among the 61 locations for the reactivity devices, 37 locations are used for housing the 37 Shutoff Rods (SORs) of Shut Down System#1 (SDS#1). The remaining 24 are used for housing the control rods (CRs) for short-term reactivity compensation and power maneuvering during normal operation. The boron carbide (B4C) packed in SS tubes placed between SS shells is used as control element of the control rods and shut off rods.

An alternate fuel cycle for AHWR consisting of Low Enriched Uranium (LEU) in Thorium matrix is also considered in once through mode. The LEU has been assumed to be composed of 19.75% U-235 and 80.25% U-238. The AHWR with (Th, LEU) MOX fuel is known as AHWR-LEU (DAE, 2016; Thakur et al., 2011) and is being designed with average fissile content of

~4.2% in fuel such that the target discharge burnup of ~ 60GWd/Te is achieved in once through mode. The core layout of AHWR-LEU is shown in Fig 1.2.



N	Shut off Rod (1-45)
AR	Absorber Rod
RR	Regulating Rod
SR	Shim Rod

Figure 1.2 Layout of the AHWR / AHWR-LEU core

1.4 Optimization techniques related to fuel management in nuclear reactors

Real world optimization problems often require the maximization or minimization of certain function or parameter. It is observed that many times the different variables involved in the problem have contradicting nature. For example, in case of optimization problem related to chip design, the designer has the primary objective to compactly fit millions of circuits in small area to reduce the signal processing delay. But a too closely packed circuit will lead to immense heat generation and may affect the working of the chip. Therefore, the designer has to optimize the chip satisfying the primary aims like compactness, less signal processing delays, lower noise and lower heat generation etc. Similar kind of optimization problems are formulated in all fields like manufacturing industry, traffic handling, airline staff management, lecture scheduling in universities and inventory management etc. In general, any optimization problem is defined as;

$$\textit{Minimize} \text{ or } \textit{Maximize } f(x_1,\ldots,x_n) \qquad \text{(objective function)}$$

$$\textit{subject to} \quad g_i(x_1,\ldots,x_n) \geq 0 \qquad \text{(functional constraints)}$$

$$x_1,\ldots,x_n \in S \qquad \text{(set constraints)}$$

$x_1,\ldots,x_n$ are called decision variables. In other words, the target is to find maximum or minimum value of objective function $f(x_1,\ldots, x_n)$ such that $x_1,\ldots,x_n$ satisfy the constraints.

One of the most common optimization problems is known as traveling salesman problem (TSP) which is a special case of quadratic assignment problem (QAP). In QAP, the problem is to assign a set of n facilities to set of n locations. For each pair of locations, a distance is specified and for each pair of facilities a weight or flow is specified. The problem is to assign all facilities to different locations such that cost is minimized. In other words, the aim is to minimize the sum of the distances multiplied by the corresponding flows. In case of TSP, there are n cities. The salesman starts his tour from City 1, visits each of the cities exactly once and returns to City 1. For each pair of cities i,j there is a cost c_{ij} associated with traveling from City i to City j. The goal is to find a minimum-cost tour. The reason for discussing these optimization problems here is their obvious similarities to the nuclear reactor fuel management problems. A reactor fuel loading pattern optimization for initial core or refueling is also an assignment problem, where, the different fuel types are allotted to different locations in the core with the aim of maximizing fuel utilization or minimizing cost of generated electricity and uninterrupted reactor operation. As an example, consider the initial core optimization problem of PHWR 220 (Mishra et al., 2009). There are 306 channels in PHWR 220. The left–right symmetry of the core can be exploited to reduce the size of optimization problem. Therefore, 153 channels can be considered. Each channel consists of 12 fuel bundles. Therefore, there are $153 \cdot 12 = 1836$ fixed bundle locations. The initial core optimization problem is to arrange the given number of Th bundles in these locations in an optimum manner. For example, for 30 Th bundles, the problem size is ${}^{1836}C_{30}$ (approximately 10^{65}). It is necessary to make 3D diffusion calculations in different states of reactor to estimate the merit of any solution candidate. Further, the number of Thorium bundles may not be fixed and the size of problem will be significantly more (2^{1836}) in that case.

1.5 Review of optimization techniques used for nuclear reactor fuel management

Loading pattern optimization (LPO) is required for initial core as well as at each refueling in case of LWRs. Significant research (Ahn and Levine (1985), Linear Programming (Sauar (1971)), dynamic programming (Wall and Fench 1965)), knowledge based methods (Galprin and Kimhy (1991), Lin et al. (1998), Genetic algorithm (GA) (Goldberg, 1989; Parks, 1996; Chapot et al., 1999), simulated annealing (SA) (Stevens et al., 1995) and Ant Colony Algorithm (ACO)

(Machado and Schirru, 2002)) has been carried out on subject of optimum utilization of fuel in LWRs and different optimization techniques have been explored.

In-core fuel management in reactors with on-power refueling like PWHRs is much easier as the search space at each refueling is quite small and maximum burn-up channel in the preferred zone is chosen for refueling. Multiple short bundles in PHWRs provides great flexibility to adopt suitable bundle shift scheme to control the local peaking due to refueling in PHWRs and makes the choice easier. However, for use of high burn up fuel in PHWRs (Choi, 2008, Gupta et al, 2008), the complexities will increase many fold and special refueling schemes are required.

LPO of initial core of PHWR is also a complex combinatorial optimization problem. Research (Balakrihnan and Kakodkar (1994), Mishra et al, (2009)) has been carried out to improve fuel utilization in initial core of PHWR also.

The loading pattern optimization techniques for initial core LPO or refueling can be broadly divided in to two classes' namely deterministic class and stochastic class.

1.5.1 Deterministic Class

As the name suggests, in case of deterministic class, the relation between control variables and target parameters will be described explicitly in form of mathematical equations. The understanding of problem and the relation of optimization target with internal parameters is required.

As an example, in Balakrishnan and Kakodkar (1994), to optimize initial of PHWR with Natural U and Thorium (Th) bundles, the objective function has been considered as:

$$OF = (W_bB)^2 + (W_cC)^2 + (W_tT)^2 + (W_1S_1)^2 + (W_2S_2)^2 \tag{1.1}$$

Where, B is the maximum bundle power; C is the maximum channel power; T is the maximum coolant channel outlet temperature; S_1 and S_2 are the decreases in worth of SDS-1 and SDS-2 from their nominal values respectively; and Ws are a set of weights for different decision objectives. Five decision variables have been considered which are represented as:

$$X = [x_1, x_2, x_3, x_4, x_5] \tag{1.2}$$

x_1 is number of Th bundles, x_2 & x_3 is average distance of Th bundles from SDS-1 and SDS 2 respectively. x_4 and x_5 is average distance of Th bundles from core center and core periphery, respectively.

A five dimensional vector which is a function of decision variable (x_1, x_2, x_3, x_4, x_5) is defined.

$$Y = \begin{vmatrix} W_bB(x_1, x_2, x_{3,}x_4, x_5) \\ W_cC(x_1, x_2, x_{3,}x_4, x_5) \\ W_tT(x_1, x_2, x_{3,}x_4, x_5) \\ W_1S_1(x_1, x_2, x_{3,}x_4, x_5) \\ W_2S_2(x_1, x_2, x_{3,}x_4, x_5) \end{vmatrix} \tag{1.3}$$

the objective function can be written as:

$$OF = [Y^T] \cdot [Y] \tag{1.4}$$

The jacobian matrix of the objective function is

$$J = \begin{pmatrix} W_b \dfrac{\partial B}{\partial x_1} & \cdots & W_b \dfrac{\partial B}{\partial x_5} \\ \vdots & \ddots & \vdots \\ W_2 \dfrac{\partial S_2}{\partial x_1} & \cdots & W_b \dfrac{\partial S_2}{\partial x_5} \end{pmatrix} \tag{1.5}$$

The minimization has been performed using Gauss method to minimize a function which is defined as sum of squares of variables,:

$$[\, J^T J] \cdot [\, \Delta X] = -\, [J^T] \cdot [\, Y\,] \tag{1.6}$$

By solving equation (1.6), ΔX is evaluated, and all the five decision variables are modified to generate the next candidate solution. The process is repeated till the desired results are achieved. The implementation of this method is very difficult and a lot of intuition and previous experience is required. Firstly, it is not known, what should be the first guess LP for starting the process of optimization? It is observed that if the first guess is far away from optimized solution, the optimized solution may not be very good and algorithm may trap in local minima. Secondly, a very limited area of search space is explored as every iteration is derived by results of previous solutions. Thirdly, at each iteration new decision variables are used to map a new core configuration. It is very difficult manual process.

For example, if we start with and initial guess loading pattern with 30 Th bundles, and let us say it is observed after solving (1.6), Δx_1 is +4. Then, we have to map a new core configuration where number of Th bundles is 34 and their average distance from SDS-1 and SDS-2 is $x_2 + \Delta x_2$, $x_3 + \Delta x_3$ respectively and average distance from core center and core periphery is $x_4 + \Delta x_4$, $x_5 + \Delta x_5$ respectively. Designing a new core configuration based on these decision variables require more intuition and is very time consuming. These types of difficulties have been observed in various types of gradient based methods used for LPO problem (Ahn and Levine (1985), Linear Programming (Sauar (1971)). In general, the calculus based methods require very few 3D diffusion calculations, depend on the existence of derivatives and are too much problem dependent.

1.5.2 Stochastic Class

The best way to solve a combinatorial optimization problem is to check all the feasible solutions in the search space. However, checking all the feasible solutions is not always possible, especially when the search space is large. Due to the difficulties observed in gradient based techniques and considerable enhancement in computer technology, more aggressive methods were developed in which random numbers have been used and a vast area of search space is explored. The use of random variable does not mean that the search is directionless. The direction of search is governed by the fittest candidate of previous generation. In these methods, relation between decision variables and optimization objective is not used. The objective function is defined in similar way as in deterministic class; however, no explicit equations are used to update the decision variables. Many Meta-heuristic algorithms have been devised and modified to solve these problems. The Meta-heuristic approaches are not guaranteed to find the optimal solution since they evaluate only a subset of the feasible solutions, but they try to explore different areas in the search space in a smart way to get a near-optimal solution in less cost and time. Methods based on simulated annealing (SA) (Park (1987) and genetic algorithm (GA) (Goldberg (1989), Poon and Parks (1993) have been introduced and a fair bit of success is achieved in finding a better optimized solution. Further heuristic information has been used in many cases (Jiang et al. (2006), Hamaida et al. (1999)) to improve the performance of algorithm. There are other examples like Artificial neural network (ANNs, Sadighi et. al (2002), Ant

Colony optimization (ACO, Machado and Scirru (2002), Tabu Search (Lin et al.(1998))) where nuclear LPO problems have been solved using modern techniques. These modern population based evolutionary algorithms are very efficient and simple to implement.

The GAs have been used more frequently for LPO problems and considerable number of research papers are available ((Poon and Parks (1993), Parks (1996), Chapot et al. (1999), Pereira and Lapa (2003 and Ziver et al. (2004), Jiang et al. (2006)). This is the reason that many times researchers consider GA as a benchmark to their applied / developed new method. As we have applied GA and Estimation of distribution algorithm (EDA) in chapter 6 and 5 respectively and detail description of how to apply this method to nuclear fuel LPO problems has been given there. However, the general description of how this population based method work is described through an example of function maximization using GA (Goldberg (1989));

Let us consider, the objective is to maximize the function $F(x) = x^2$ where $x \in [0, 31]$ using GA. In the first step of optimization, the parameter x is coded in binary form as a finite length string. x in binary base can take values from 00000 to 11111 for $x \in [0, 31]$ in decimal base. A pool of candidate solution by random number generator is generated and the fitness value of function is evaluated for this candidate solution pool as given in table 1.1. The basic implementation of GA consists of three steps namely;

1) Reproduction

2) Crossover

3) Mutation.

As can be observed from Table 1.1, the number of counts to be considered based on fitness value of current strings, is 1 for string 1 & 4 , 2 for string 2 and 0 for string 3. It means when random number is used to generate the next population for reproduction, string 3 will not appear.

Table 1.1 Description of GA (step 1)

String No	Initial population	X value	F(x)	$F_i/\Sigma F$	F_i/avg. F	Actual count
1	0 1 1 0 1	13	169	0.14	0.58	1
2	1 1 0 0 0	24	576	0.49	1.97	*2*
3	0 1 0 0 0	8	64	0.06	0.22	*0*
4	1 0 0 1 1	19	361	0.31	1.23	1
Sum			1170	1.00	4.00	4.00
Avg			293	0.25	1.00	1.00
Max			576	0.49	1.97	2.00

Therefore, while selecting the probable candidates for mating and reproduction, string 2 will appear twice while both strings 1 and 4 will appear only once and string 3 will not appear at all as shown in Table 1.2.

Table 1.2 Description of GA (step 2)

String No	Initial population	Mate *	Crossover site*	New Population	X value	F(x) value
1	0 1 1 0 | 1	2	4	01100	12	144
2	1 1 0 0 | 0	1	4	11001	25	*625*
3	1 | 1 0 0 0	4	2	11011	27	*729*
4	1 | 0 0 1 1	3	2	10000	16	256
Sum						1754
Avg						439
Max						729

*Randomly selected

As a next step, crossover site is chosen randomly. Many different cross over and mutation operators have been developed to deal with the data structure representation. A two point cross over operator is used here. After the crossover, new population is generated as shown in Table 1.2. Finally the fitness value of F(x) has been evaluated for new population. It is observed that the maximum and average value of F(x) has improved from 576 & 293 to 729 & 239 respectively. This process is repeated till the maximum value of f(x) saturates.

In this section, we have reviewed different optimization algorithms which include conventional calculus based methods and modern population based algorithms. We have also tried to understand the basic implementation, advantages and disadvantages of both types of algorithms. In present thesis, the applicability of various optimization techniques discussed above will be used for fuel management of AHWR at various stages. In the next section, the outline of the thesis is given.

1.6 Outline of thesis

In this chapter, we have first introduced the importance of nuclear power in relation to world and for India's perspective. Then the India's three stage program is mentioned and significance of AHWR has been highlighted. The various optimization problems in context to nuclear reactor fuel management have been described in the conventional LWRs and PHWRs. The unique design features of AHWR have been described. Further, to solve the optimization problems, different optimization techniques already studied in literature have been investigated.

As the primary focus of this thesis is on the development of optimization techniques for fuel management in heavy water moderated reactors, we have considered the complete fuel cycle of AHWR-LEU as a case study in this thesis.

In Chapter 2, a basic description of reactor core calculations in steady state has been presented. A brief description of the computational tools used for simulation of steady state of nuclear reactor is also given.

In Chapter 3, a special refueling scheme for AHWR core is described, which is an optimal combination of refueling strategies used in LWRs and PHWRs. Each refueling operation is combination of refueling and one reshuffling operation to control the local power peaking. Generally the experience and knowledge base in a form of ‘‘IF-THEN’’ rule-sets are used along with other specific search control strategies. However, the direct implementation of heuristic information in the form of factors has been tried. These factors are further translated for describing the preference of a channel for refueling or reshuffling. A computer code CARS has been developed based on the above principles. The computer code CARS provides all the micro details like fresh fuel requirement, storage space for discharged fuel, behavior of maximum channel power (MCP), maximum mesh (bundle) power (MMP), channels selected for refueling/reshuffling, discharge burn-up achieved and boron requirement, etc. for the entire life of reactor.

Chapter 4 describes the problems observed during core follow-up studies for AHWR done with CARS:

(a) Power peaking problem during the pre-equilibrium phase (transition phase).

(b) Due to the quarter core mirror symmetry, certain channels near to axis symmetry were getting over due for refueling (described later). As a result some other channels were getting discharged pre-maturely and leading to overall loss in discharge burnup.

(c) Very long computation time (7-10 days)

The following improvements in CARS were done to address these problems.

(a) Adoption of double reshufflings along with refueling

(b) Use of π/2 rotational symmetry instead of mirror symmetry

(c) Use of parallel processing to accelerate the speed

A refueling strategy based on a suitable combination of three refueling schemes viz. refueling without reshuffling, refueling with single reshuffling and refueling with double reshuffling scheme is adopted. The refueling strategy was evolved after including the zonal power of each zone as an important parameter of the strategy. As a result the flux distribution at each refueling was maintained near to the equilibrium core flux distribution.

In Chapter 5, the basic loading pattern optimization (LPO) problem is defined. Different optimization techniques (Modern methods and Conventional methods) are discussed for solving LPO problem. The size and search space for AHWR initial core LPO is described. The AHWR core consists of 444 fuel lattice locations. By exploiting symmetry of the core, the problem size for AHWR initial core optimization reduces to 2^{62} (10^{18}), which is also a very large size problem. Simulation of all these loading patterns to choose the best loading pattern is not practical in a finite time scale. This type of combinatorial optimization problem can be solved by defining an objective function and then maximizing it. The objective function of the problem is based on K-effective, SDS-1 worth, boron in moderator and radial peaking. The objective function is defined by using penalty method. In appendix referred in this section, the most adequate value of various parameters in objective function has been estimated. We have applied Estimation of distribution algorithm (EDA) (modern method) to optimize initial core loading pattern (LP) of AHWR. In EDA, new solutions are generated by sampling the probability distribution model estimated from the selected best candidate solutions. The weighing factor 'α' decides the fraction of current best solution for updating the probability distribution function after each generation. A comprehensive study on parameters like population size, weighing factor 'α' and initial probability distribution function has been done. It is observed that choosing a very small value of 'α' may limit the search of optimized solutions in the near vicinity of initial probability distribution function and better loading patterns which are away from initial distribution function may not be considered with due weightage. It is also observed that increasing the population size improves the optimized loading pattern, however, the algorithm still fails if the initial distribution function is not close to the expected optimized solution. We have tried to find out the suitable values for 'α' and population size to be considered for AHWR initial core loading pattern optimization problem.

In Chapter 6, the initial core optimization of AHWR has been tackled by applying Genetic algorithm (GA). The objective function in GA is defined in the same way as in chapter 5, such that during the optimization process, all the variable of objective function (MCP, MMP and SDS-1 worth) meet the pre-determined designed limit. The dependence on population size and initial distribution function on the optimized loading pattern is studied and most adequate values for AHWR initial core LPO have been estimated. It was observed that the algorithm has failed with population size of 24. However, when the initial distribution function is near to optimized solution, the results are better. By increasing the population size to 240 or 1200, optimized loading pattern similar to EDA is achieved. A discussion on the computational cost and simulation time is also given.

Finally, Chapter 7 gives a summary of the research work carried out in this thesis and scope for future extension of this work. In this chapter, the various aspects of optimum fuel utilization during initial phase, transition phase and equilibrium phase have been emphasized. This study allows us to demonstrate fuel cycle of AHWR, where, on-power refueling and high target discharge burnup clusters are used. The new technique developed for AHWR for in-core fuel management has opened a scope for further study to develop refueling strategy for use of slightly enriched Uranium (SEU) or DUPIC fuel (Direct Use of PWR fuel In CANDU reactors) and Thorium based fuels like (Th, Pu) MOX or (Th, U) MOX, in PHWR for lower waste generation and achieving higher discharge burnup. In the present thesis work, a better understanding of the fundamental principles that govern fuel management during initial, transition and equilibrium stage of any reactor has been achieved. The outcome of the research work is also useful to provide guidance for improvement in current optimization techniques to tackle LPO problems.

CHAPTER 2

Simulation Tools

2.1 Introduction

In last chapter we have discussed that the main focus of this thesis is related to fuel management in heavy water moderated reactors and in particular AHWR fuel cycle has been considered. Before discussing the development of optimization techniques for efficient fuel utilization at all stages of reactor operation, it is necessary to understand the simulation of a nuclear reactor in any configuration. Therefore, in this chapter, we have briefly discussed the basic simulation tools used for nuclear reactor simulation. A three dimensional model is required which can simulate the reactor with all its heterogeneities and different materials like fuel, control assembles and structural materials etc. Solution of reactor core problems with minimum approximation can be reached by solving neutron transport equation and using Monte-Carlo methods in some special cases. However, there is continues stress to reduce the computational time and computational cost to perform these calculations. Therefore, a fairly good approximation and efficient way of doing neutronics simulations in a very short time is to divide the core in to similar looking regions (lattice) like fuel channels. In these regions, heterogeneities are homogenized by solving exact transport equation and average few energy group diffusion theory parameters are generated. These few group parameters describe the interaction probability of a particular neutron-nuclear reaction and are commonly known as macroscopic cross sections (∑). These parameters are used for the given reactor geometry to solve the diffusion equation to estimate various reactor operational parameters like k-effective and core flux distribution etc.

In section 2.2, Neutron transport equation has been discussed and capabilities of lattice code ITRAN used to simulate AHWR lattice is also described. In section 2.3, three dimensional diffusion theory code FEMINA used to simulate AHWR core is described. Section 2.4 outlines the summery of this chapter.

2.2 Neutron transport equation

The equation for angular neutron density n(r, E, Ω, t) at any arbitrary volume V in a system can be derived by considering the various processes by which neutrons are gained or lost. The various mechanisms by which the neutrons can appear or disappear or leave the volume V are defined below (Duderstadt and Hamilton (1976); Lewis and Miller (1984):-

Gain terms

I. Neutron sources like fission or external neutron source
II. Neutrons streaming in to V from other regions
III. Neutron in V with initial parameters E' and Ω´ can be scattered from a nucleus and have final energy E and Ω

Loss terms

IV. Leakage out term from the region V
V. Collision term (absorption or scattering)

The rate of change of number of neutrons in volume V = I + II + III – IV – V (2.1)

If we define each gain and loss term in equation (2.1) then we find

$$\int_V d^3r \left[\frac{\partial n}{\partial t} + v\,\Omega \cdot \nabla n + v\textstyle\sum_t(r,E)\, n - \int_0^\infty dE' \int_{4\pi} d\Omega' v' \sum_s(E' \rightarrow E, \Omega' \rightarrow \Omega)\, n(r,E',\Omega',t) - S\,(r,E,\Omega,t)\right] dE\, d\Omega = 0 \qquad (2.2)$$

For this integral to be zero for any volume V, the integrand is =0

Hence the neutron balance equation is

$$\frac{1}{v}\frac{\partial \varphi}{\partial t} + \Omega \cdot \nabla\varphi + \textstyle\sum_t(r,E)\varphi(r,E,\Omega,t) = \int_{4\pi} d\Omega' \int_0^\infty dE' \sum_s(E' \rightarrow E, \Omega' \rightarrow \Omega)\, \varphi\,(r,E',\Omega',t) + S\,(r,E,\Omega,t) \qquad (2.3)$$

Where, φ (r, E, Ω, t) is neutron flux at r with energy E and moving in direction Ω at time t.

$\sum_t(r,E)$ = Total macroscopic cross section for energy E at r

$\sum_s\left(E' \rightarrow E, \Omega' \rightarrow \Omega\right)$ = Macroscopic scattering cross section

S(r, E, Ω, t) is source term. It includes fission source and external source.

Therefore, S(r, E, Ω, t) = S_f(r, E, Ω, t) + S_e(r, E, Ω, t)

Where S_f(r, E, Ω, t) is fission source and S_e(r, E, Ω, t) is external source term

The fission source term can be defined as

$$S_f(r,E,\Omega,t) = \frac{\chi(E)}{4\pi} \int_{4\pi} d\Omega' \int_0^\infty dE'\, \nu(E') \textstyle\sum_f(r,E')\, \varphi\,(r,E',\Omega',t) \qquad (2.4)$$

Here, $\chi(E)$ is the neutron energy distribution given by fission spectrum.

$\nu(E)$ is the total number of fission neutrons produced in one fission

$\sum_f(r, E')$ = Macroscopic fission cross section

As, our main concern is to solve steady state equation only, we can write the K- eigenvalue problem without any external source as:

$$\Omega \cdot \nabla\varphi\,(r, E, \Omega) + \sum_t(r, E)\varphi(r, E, \Omega) - \int_{4\pi} d\Omega' \int_0^\infty dE' \sum_s\left(E' \rightarrow E, \Omega' \rightarrow \Omega\right) \varphi\left(r, E', \Omega'\right) = \frac{1}{K}\, S_f\,(r, E, \Omega) \tag{2.5}$$

Where S_f is given by equation (2.4)

The homogenized few group cross sections for AHWR lattice have been generated by solving neutron transport equation using lattice code ITRAN. In all our calculations, the ENDF-B/ VI.8 has been used as the basic evaluated nuclear data cross section library.

ITRAN can solve the neutron transport equation and perform the calculations based on the first flight collision probability (CP) method (Krishnani, 1981) or a method based on combination of interface current (IC) formalism and CP method (Krishnani, 1982) for various geometries like Slab, Spherical, Cylindrical, cluster geometries, 2-D rectangular geometry of LWR (BWR and PWR). The code ITRAN has been used starting from the 69 group WIMSD nuclear data library (ENDF/B-VI.8) to generate 2 group lattice cross-sections for various fuel elements, control materials, guide tubes and structural materials for AHWR core. These condensed cross sections are used in the second step of reactor simulation using diffusion theory code FEMINA (Kumar and Srivenkatesan, 1984).

2.3 Neutron Diffusion equation

As discussed in section 2.1, a good approximate and quick solution of real reactor physics problem can be achieved by dividing the core into lattices where heterogeneities are homogenized by solving exact transport theory. The few energy group parameters are then used to solve the diffusion equation. The diffusion equation is simplified form of transport equation with the following assumptions:

I. First assumption is that the angular flux is weakly dependent on angle (linearly anisotropic)

II. Secondly, the rate of time variation of the current density is much slower than the collision frequency.

III. Fission sources of neutrons are isotropic

The final form of time dependent diffusion equation after considering above assumptions to transport equation is

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} - \nabla \cdot D(r,E)\nabla\Phi + \sum_t(r,E)\Phi(r,E,t) = \int_0^\infty dE' \sum_s(E' \rightarrow E)\ \Phi\ (r,E-,t) + S\ (r,E,t) \quad \text{.........(2.6)}$$

Here, $\Phi(r,E,t) = \int_0^\infty d\Omega\ \ \varphi\ (r,E,\Omega,t)$ (2.7)

D(r, E) is diffusion coefficient. Fick's law has been used to relate neutron current density J (r,E) to Diffusion co-efficient as shown below

$$J(r,E) = -D(r,E)\frac{\partial\Phi(r,E)}{\partial r} \tag{2.8}$$

The final form of steady state multi-group diffusion equation in form of K-eigen value can be written as

$$-\nabla \cdot D_g(r)\nabla\Phi_g(r) + \Sigma_g^r(r)\Phi_g(r) - \sum_{g'\neq g}\left(\Sigma_{g'\rightarrow g}\Phi_{g'}(r)\right) = \frac{1}{K}\chi_g \sum_{g'} \nu\Sigma_{g'}^f(r)\,\Phi_{g'}(r)$$

Here g=1,2,3.. (2.9)

The practical problems can be solved by using neutron diffusion equation by various numerical techniques like finite difference method (Menon et al. (1981) and nodal expansion method etc. The continuous diffusion equation is converted in to a set of linear equations by descretization of space in to meshes. Finite difference method is very straightforward and simple method to solve diffusion equation. But it has a requirement that the mesh size should be of the order of diffusion length and hence a very large number of unknowns will appear for a typical power reactor. Therefore, too much computer time is consumed as for accurate results, dense packing of meshes is required. As the main aim is to estimate assembly level power distribution and by considering mesh size smaller than the assembly size, unnecessary computational time will be involved. This has resulted in development of coarse mesh techniques like nodal methods (Kumar and Srivenkatesan (1984)).

Nodal methods give an algorithm which is quite similar to finite difference method and are used for a long time because of simple implementation. In nodal methods, the diffusion equation is integrated over the large homogeneous regions called nodes to obtain a nodal balance relation with average surface currents and fluxes as unknowns. In the conventional nodal (Delp et al. (1964); Goldstein et al. (1967)) methods, the currents are eliminated from the equation using special coupling coefficients, which are defined as ratio of average surface current and fluxes in a node. The special coupling coefficients are estimated a priori by applying simplifying assumptions or by using auxiliary fine mesh calculations. This makes the methods less accurate, still they provide acceptable results for static problems. But then conventional methods are not suited for kinetics calculations, as the coupling coefficients tend to change from time to time and thus will have to be calculated repeatedly.

To circumvent the problem of re-estimation of coupling coefficients many higher order nodal schemes such as nodal expansion method (NEM) (Finnemann (1975); Finnemann et al. (1977)), polynomial method (Sime and Henry (1976); Shober and Henry (1976)), analytical nodal method (Shober et al. (1977)), nodal green's function method (Lawrence and Dorning (1978)) have been developed. In these methods, currents are treated explicitly to obtain coupling between adjacent nodes. This is done by first integrating the 3D diffusion equation over transverse directions. The set of one dimensional equations thus obtained is then solved in different manner in various methods. These methods are proved to be very computationally efficient compared to FD methods because the calculation of currents is simple and inexpensive. The power distribution obtained from these methods are also accurate for nodes as large as size of fuel assembly. The computer code FEMINA has used nodal expansion method for calculation of flux distribution.

With reactor operation, the fuel composition as well as flux distribution, both changes concurrently. Therefore, for fuel depletion (burn-up) calculations, it is assumed that for a short interval of time (known as burn-up step) the variation in flux is insignificant and thus the fuel can be depleted at the same rate. For next burn-up step, new fluxes are calculated by considering the modified fuel composition and the fuel is depleted for next burn-up step.

In general the flow chart of reactor core simulation is shown in Fig 2.1

2.4 Summary

In this chapter, general simulation of reactor core has been described. A brief description and capability of various tools used for reactor core simulation of AHWR has been given.

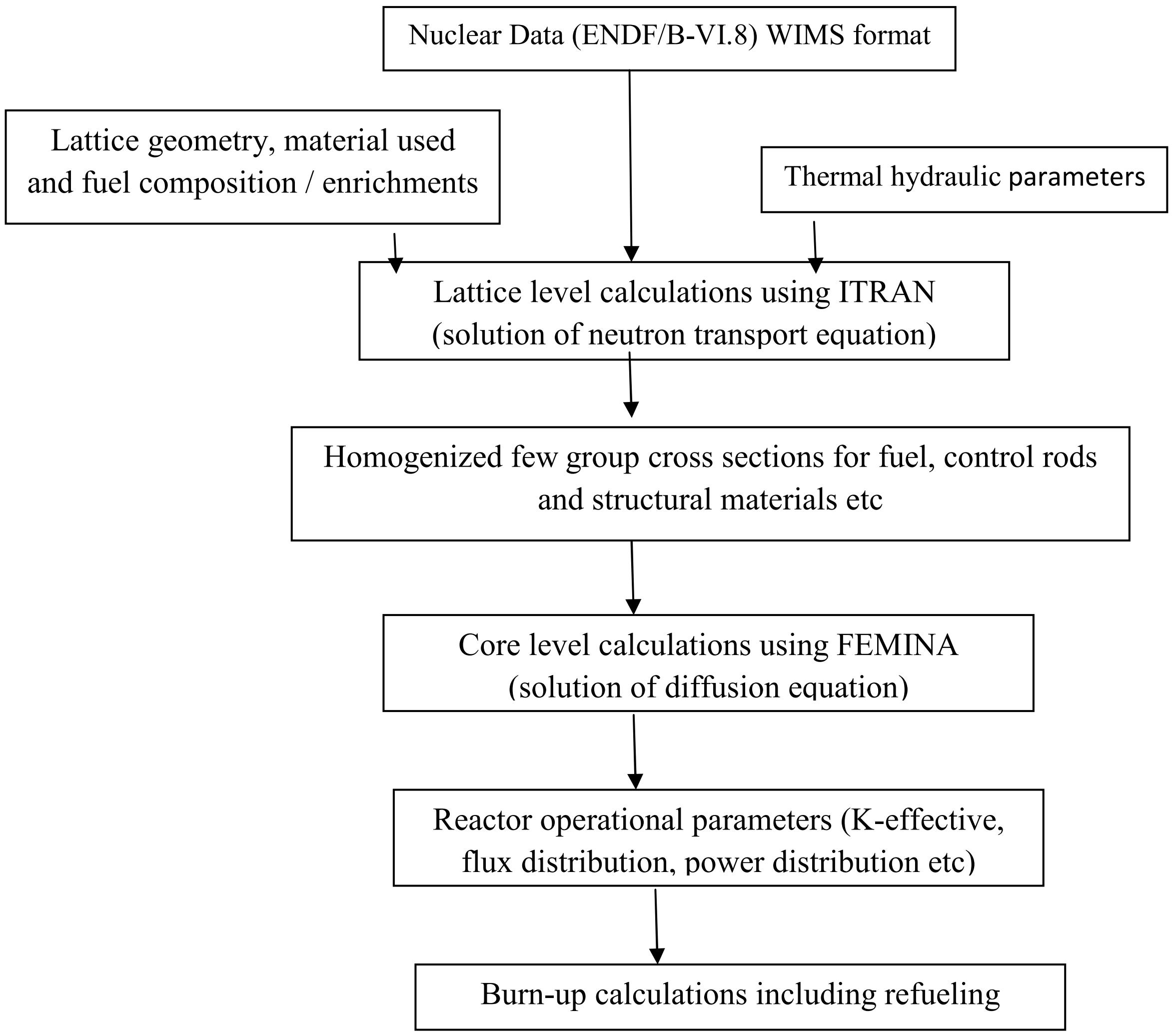


Fig 2.1 Flow chart of reactor core simulations

The lattice calculations for the AHWR equilibrium core cluster were performed by using Neutron Transport Theory computer code ITRAN (Krishnani, 1982). The calculations were performed by using 69-group library based on ENDF/B-VI.8 nuclear data obtained from IAEA (IAEA, 2016). Two group cross sections with burn up were generated using ITRAN. These two group cross sections are further used in core calculations by diffusion theory code FEMINA

(Kumar and Srivenkatesan, 1984). The steady state calculations for follow up and refueling are also performed by FEMINA. In the optimization techniques developed for refueling studies of AHWR-LEU or for initial core optimization of AHWR-LEU, executive file of FEMINA has been used multiple times with different inputs. The operational parameters estimated by FEMINA for all the input cases have been used to decide the direction of search for optimization solution.

CHAPTER 3

Special Refueling Scheme for AHWR

3.1 Introduction

The primary aim for fuel management in any reactor is to have best fuel utilization and keeping all the safety parameters in their design limit along with maintaining 100% full power of reactor. As discussed in Chapter 1, with respect to fuel management, thermal reactors can be broadly divided into two categories defined as reactors with on power refueling and reactors with off power (batch refueling). In both types of reactors, the fuel management is a complex combinatorial optimization problem with several constraints and has many solutions. The search space is very large and optimal solution requires an exhaustive study which may not be feasible in finite scale of time in many cases.

Fuel reload problems for reactors adopting batch refueling like PWRs are being addressed in a better way using evolutionary algorithms such as simulated annealing (SA) (Stevens et. al., 1995), genetic algorithm (GA) (Goldberg, 1989; Parks, 1996; Chapot et al., 1999), estimation of distribution algorithm (EDA) (Jiang et. al. 2006), artificial neural network (ANN) (Sadighi et al., 2002) and ant colony algorithm (ACO)(Machado and Schirru, 2002) etc. Evolutionary algorithms belong to stochastic optimization techniques which are inspired by natural evolution processes. Galperin et. al. (Galperin and Nissan, 1988; Galperin et.al, 1989) proposed a solution to this nuclear reload problem using heuristic search method and knowledge based method. Due to the vast search space involved in this problem, the use of the artificial intelligence will give more satisfactory results. These methods usually require high end computational machines and parallel processing for efficiently and effectively scanning a large space of the solutions. On the other hand, in heuristic search methods, very few possible solutions are simulated and the best among them is referred as final solution.

In-core fuel management in reactors with on-power refueling like PWHRs is much easier as the search space at each refueling is quite small and maximum burn-up channel in the preferred zone is chosen for refueling. Multiple short bundles in PHWRs provides great flexibility to adopt suitable bundle shift scheme to control the local peaking due to refueling in PHWRs and makes the choice easier. Choi (Choi, 2000) developed an automated fuel management program for CANDU reactors. The use of this program for developing refueling strategy and generating the core parameters for about 500 FPDs of operation in CANDU-6 reactor takes a day. Manually doing this job will take a much larger time. In his later work (Choi, 2008) he studied the use of DUPIC (**D**irect **U**se of **P**WR spent fuel in **C**ANDU) fuel during transition phase and in equilibrium phase. The task of optimum fuel utilization was more complicated and the refueling scheme was changed from 8-bundle shift to 2-bundle shift scheme to control the local peaking due to replacement of burnt fuel with higher fissile content fuel. The studies have shown that the use of DUPIC fuel will give better fuel utilization. The average enrichment in DUPIC fuel is ~1.5%. If higher enrichment of ~3-4% is used, the complexity involved in fuel cycle study will increase significantly and the refueling rate will decrease. Gupta et.al (Gupta et.al, 2008) has shown that the use of 3- 4% ^{235}U in Th matrix in PHWR cluster will give a better fuel utilization.

But the on-power refueling challenges involved in the high burn-up PHWR design needs to be addressed properly. If we have to study the performance and feasibility for achieving high discharge burn-up for reactors having on-power refueling (like PHWRs and AHWRs), the micro details of the fuel cycle studies are mandatory.

The AHWR (Sinha and Kakodkar, 2006; Kakodkar, 1998) is being designed as 920 MWth (300 MWe), vertical, pressure tube type reactor with boiling light water as coolant and heavy water as moderator. The fuel cycle of AHWR is based on (Th-^{233}U-Pu) MOX fuel in closed cycle mode with target discharge burnup of ~ 36.5 GWd/Te. However, an alternate fuel cycle consisting of Low Enriched Uranium (LEU) in Thorium matrix is also considered in once through mode. The LEU has been assumed to be composed of 19.75% ^{235}U and 80.25% ^{238}U. The AHWR with (Th, LEU) MOX fuel is known as AHWR-LEU (DAE, 2016; Thakur et al., 2011) and is being designed with average fissile content of ~4.2% in fuel such that the target discharge burnup of ~ 60GWd/Te is achieved in once through mode. On-power refueling, negative coolant void reactivity and heat removal through natural circulation are salient features of AHWR /AHWR-LEU. Table 3.1 gives a description of important physical parameters of AHWR-LEU. The AHWR-LEU is being designed for 100 years life and the transition period from initial phase to equilibrium phase is fairly long time (~7-10 Full Power Years). To ascertain that the core operational parameters like maximum channel power (MCP) and maximum mesh power (MMP) are within their design limits and to find out the requirement of fresh fuel and storage space for discharged fuel inventory, the full refueling study from initial phase to equilibrium phase is required. The preliminary study has shown that the natural circulation limits the heat removal capacity of coolant for a channel to only 2.85 MW(th) for a maximum cluster peaking of 1.15. The design parameters MCP and MMP have a limit of 2.85 MW(th) and 200kW(th) respectively.

Table 3.1Description of the AHWR LEU reactor core

No. of channels	444
Lattice pitch, mm	225
No of RR / AR/ SR / SORs	8 / 8 / 8 / 45
Fuel	(Th-LEU)MOX
No. of rings in a cluster / total no. of pins	3 / 54
No of fuel pins in each ring	12 / 18 / 24
Multi-purpose displacer tube (annular)	
Material / OD / Thickness , mm	Zr-2 / 36 / 3
Solid rod inside displacer, Material/OD	Zr-2 /18 mm
Pressure tube	
ID / OD, mm	120 / 128
Material / Density, g/cc	Zr-2.5%Nb / 6.55
Calandria tube	
ID / OD, mm	163.8 / 168
Material / Density, g/cc	Zircaloy-2 / 6.55
Avg. fuel temperature, *K*	723
Avg. fuel density, g/cc	9.3
Coolant material	Light water
Temperature, *K*	558
Average density, g/cc	0.45
Moderator material	Heavy water
Temperature, K	340.5
Average density, g/cc	1.089

Generally, initial core consists of at least two types of fuel clusters which are different from equilibrium core clusters. Hence during transition phase (period of operation between initial core and equilibrium core), various types of fuel clusters have to be managed properly such that the safety margins are maintained and operational parameters are within the design limits.

The on-power refueling of AHWR is more challenging vis-à-vis PHWRs in two respects: (1) The PHWRs employ small fuel clusters whereas AHWR employ full length fuel cluster thereby reducing the flexibility of refueling small number of clusters at a time in PHWRs, (2) the fuel in AHWR has high discharge burnup which necessitates high fissile content causing severe ripple effect. This may also require fuel shuffling as well (see Sec. 3.2 below). *The main focus in this chapter is to study the feasibility of on-power refueling for reactors like AHWR or AHWR-LEU designed for high discharge burn-up.* In this chapter we have accentuated that the conventional methods for optimum fuel utilization do not give satisfactory results for reactors like AHWR-LEU. Therefore specialized tools have been developed for optimal utilization of fuel in equilibrium phase and pre-equilibrium phase and specialized refueling strategy for AHWR-LEU has been formulated. Section 3.2 describes the requirement of special refueling scheme for AHWR. It also describes the results of a refueling study of AHWR with direct refueling. The section 3.3 gives the description of the factors based on heuristic approach. The section 3.4 gives the implementation of this approach in the form of a computer code. Section 3.5 describes the use of this code to study a cluster for AHWR-LEU core. Section 3.6 discusses the conclusions. The results have been published in international scientific journal *Thakur et. al., 2013* and in international conference *Thakur et. al., 2011*.

3.2 Requirement of special refueling scheme for AHWR

The unique features (on power refueling and high discharge burn-up fuel) of AHWR imply that its fuel management is a new and complex problem. As a first step, fuel management problem was addressed in a similar way as was done for PHWRs and is described as direct refueling. In this direct refueling scheme, one of the lower power producing channel which has highest discharge burn-up was chosen for refueling. During the transition phase, both initial core clusters and equilibrium core clusters are present in the core. Therefore, the channels were chosen considering their fuel type also. It means the initial core clusters were prioritized for refueling first. For each refueling, a set of four quarterly symmetric channels were considered which were having higher burn-up than others. No criteria were put on the maximum channel power (MCP) and maximum mesh power (MMP) after refueling to observe the level of peaking during core follow-up. The core refueling studies were carried out from ~ 300 FPDs (on-set of refueling for initial core) to ~ 5500 FPDs.

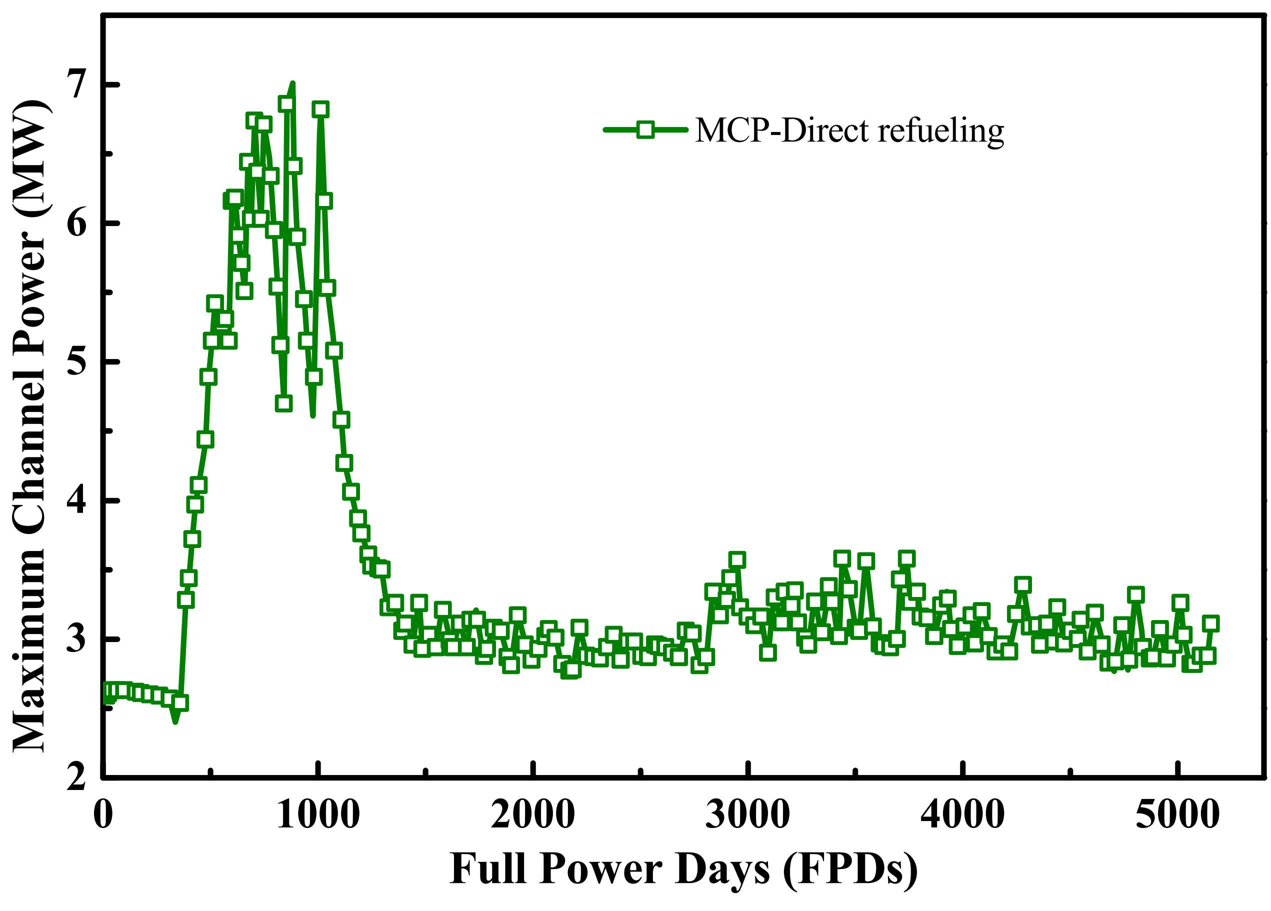


Fig 3.1 Maximum channel power vs full power days for direct refueling

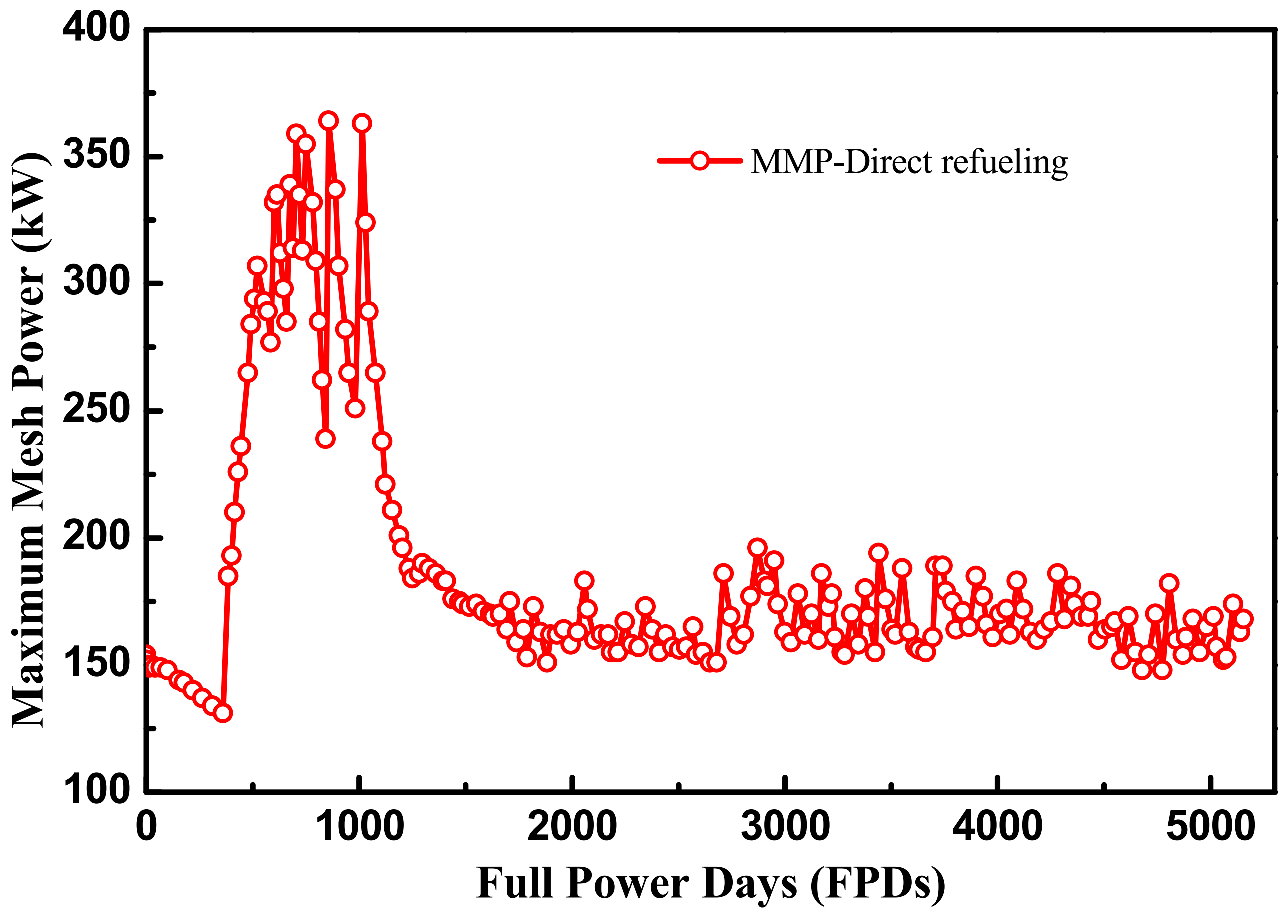
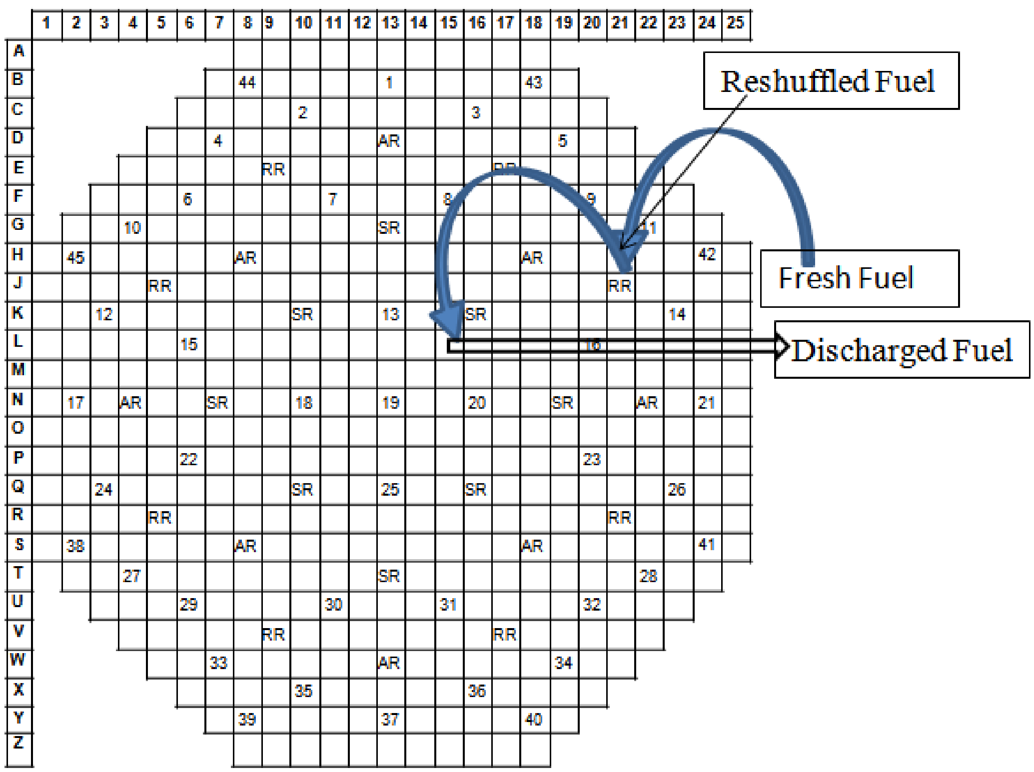


Fig 3.2 Maximum mesh power vs full power days for direct refueling

Figs. 3.1 and 3.2 show the behavior of MCP and MMP respectively for direct refueling for AHWR. It is observed that during the transition phase (300 – 1800 FPDs), the MCP is in range of 3.0 MW to 7.0 MW. This is way beyond the design limit of 2.85 MW. After ~1800 FPDs, the MCP reduces but is still in range of 3.0 MW – 3.8 MW. Although a reduction in peak power after transition period is observed but still the MCP is not able meet its design limit of 2.85 MW. The trend for MMP is also similar. During the transition phase, MMP is in range of 200 – 400 kW. However, The MMP is below 200 kW after the transition phase. Therefore, the MMP can be maintained in its design limit after the transition phase is over. The main reason for such high peaking during transition period is the difference in reactivity of discharged cluster and the refueled fresh equilibrium core cluster. The burnt initial core clusters are being replaced with highly reactive fresh equilibrium core clusters having ~ 4.2 % fissile. To reduce the peaking during transition phase and equilibrium phase of the core, it is proposed to reduce this reactivity difference by adding a reshuffling operation also, which will result in lower peaking after each refueling. Therefore, special refueling scheme is proposed in this thesis, which is an optimal combination of refueling strategies used in LWRs and PHWRs. In the proposed special refueling scheme, each refueling operation is a combination of refueling and one or two reshuffling operations (see Fig.3.3) to control the local power peaking.



N	Shut off Rod (1-45)
AR	Absorber Rod
RR	Regulating Rod
SR	Shim Rod

Fig 3.3 Schematic representation of refueling with single reshuffling scheme in AHWR core

Generally the experience and knowledge base in a form of "IF-THEN" rule-sets (Galperin et. al., 1989) are used along with other specific search control strategies. However, we have tried the direct implementation of heuristic information in the form of factors. These factors are further translated for describing the preference of a channel for refueling or reshuffling and a computer code is developed.

This computer code can automatically perform the refueling study for a given period of operation and provide all the micro details like fresh fuel requirement, storage space for discharged fuel, behavior of maximum channel power (MCP), maximum mesh (bundle) power (MMP), channels selected for refueling / reshuffling, discharge burn-up achieved and boron requirement *etc*. for the entire life of reactor. In this chapter, we have considered that each refueling is followed by one reshuffling only. To maintain the quarter core symmetry, always a mini-batch of four channels is selected for refueling and reshuffling operations.

3.3 Proposed factors for refueling and reshuffling based on heuristic approach

The main objectives for developing refueling strategy are: -

i. Maximise the fuel utilization by maximizing its discharge burnup.
ii. The reactor should operate at rated power (full power)
iii. The operation parameters like MCP and MMP should be maintained within their design limits during operation
iv. If more than one type of fuel (different fissile content) exists in the core, fuel reaching its design discharge burnup should be discharged on priority.

v. Core power distribution should be maintained close to the equilibrium core power distribution to maintain worth of reactivity devices close to design values

vi. The number of fuel handling operations should be minimum

As discussed in section 3.2, for facilitating high discharge burnup fuel with on-power refueling, the special refueling scheme requires selection of two channels (one for refueling and one for reshuffling) at each refueling. For selecting two appropriate channels from a typical core consisting of~400 fuel channels, we have to study ${}^{400}P_2$ number of possible combinations. Simulation of all these combinations and finding out the best one seems very cumbersome and time consuming. The best combination found out this way will be used for one refueling only. For every refueling one has to simulate same number of combinations. The challenge is to find appropriate channels at each refueling. The brief description of a few proposed important factors used to describe priority of a channel to be considered for refueling or reshuffling is given below. These factors are based on simple heuristic logics.

3.3.1 Refueling factor

Refueling factor (RF) is the direct measure of the maturity level of a channel for refueling. The refueling factor is defined in such a way that it is proportional to its instantaneous burnup of that channel. Higher the burnup, higher is the maturity level for its refueling and as a result such a channel is given higher priority for refueling. For different fuel types used in the core having different design discharge burnup, the refueling factor for a given type of fuel is also inversely proportional to its design discharge burnup.

$$RF \propto \frac{B}{D} \tag{3.1}$$

'B' is the burn-up of channel and '*D'* is the design discharge burn-up of fuel type present in channel. Based on the burnup criteria, if a channel is matured for refueling but there is local power peaking in its neighborhood due to refueling of the channel or due to movement of reactivity devices, then the refueling of such a channel may further aggravate the local peaking. Hence, the power of the channel to be refueled (relative to its equilibrium power) as well as average power of its neighboring channels has also been considered for evaluation of refueling factor for a channel. This factor also decides the priority between the two channels having the same fuel type and same burnup but their channel powers are different or they are present in different flux locations in the core. The refueling factor is inversely proportional to the channel power of a channel due for refueling as well as to the channel power of the first and second nearest neighbors.

$$RF \propto \frac{1}{CP \cdot C1 \cdot C2}$$

Where '*CP'* is channel power of channel, '*C1*' is average power of 1st neighbors of channel and '*C2*' is average power produced by 2nd neighbors of channel

Or
$$RF = K \frac{B}{D \cdot CP \cdot C1 \cdot C2} \tag{3.2}$$

Here, '*K*' is proportionality constant. The value of *K* can be taken as 1. All the channels are arranged in descending order of their refueling factors. The preference order for channels to be discharged has been decided.

3.3.2 Reshuffling factor

As discussed in section 3.2, to control the power peaking during refueling, refueling with one or more reshufflings may be opted. In the reshuffling process, a partially burnt fuel located in low flux region is moved to the high flux region, where it replaces highly burnt fuel (the fuel due for refueling). The highly burnt fuel is discharged to spent fuel bay. The vacancy created in the low flux region (due to movement of partially irradiated fuel) is filled with fresh fuel (Fig. 3.3). This process is called refueling with single reshuffling. Similarly, if one more step is added in between then it is called refueling with double reshuffling. The fuel which is moved is assigned a parameter termed as reshuffling factor (*Resh F*). The reshuffling factor is assigned to only partially burnt channels and the factor is weighted differently for different reshuffling scheme used. The reshuffling factor is modified to account its channel power as well as the channel power of its neighboring channels. The channels which are having very low burnup or which are near to discharge cannot serve the purpose for reshuffling. Therefore, the channels which have burnup nearly half of the design discharge burnup could be appropriate candidates for single reshuffling provided it does not result into local peaking in its neighborhood. Keeping this in mind, the reshuffling factor is assigned for channels, which are having their burnup in the range of one fifth of core average burnup to 1.5 times core average burnup. In this burnup range, the channels having lower burnup and located in the lower neutron flux region will be preferred for reshuffling. We have defined *Resh F* as:-

$$\text{Resh F} = 0 \qquad \text{if } \mathrm{B} < \frac{\mathrm{Cab}}{5} \text{ or } \mathrm{B} > (1.5 \cdot \mathrm{Cab})$$

$$= K \frac{D}{B \cdot CP \cdot C1 \cdot C2} \quad \text{if } \frac{\mathrm{Cab}}{5} < \mathrm{B} < (1.5 \cdot \mathrm{Cab})$$

Where ‘Cab’ is core average burnup.

Here also, 'K' is proportionality constant and its value is taken as 1. It is to be noted that in the above burnup range, the channels which are having lower burnup and present in low flux regions will be more suitable candidates for reshuffling.

3.4 Development of Code for Automated Refueling Strategy using Heuristics (CARSH)

The development of CARSH started after careful study of the various refueling strategies followed in LWRs & PHWRs. The refueling strategies followed in PHWRs as well as in light water reactors were translated into logics such that these can be easily programmed. Based on the experiences gained during the study of various fuel cycles of AHWR or AHWR-LEU, these logics were suitably tuned to the requirements of fuel cycle used. The basic input to the computer program is core power distribution, core burnup distribution, types of various fuels present or likely to be present in the core, design limits on mesh power as well as channel power and the characteristics of the various types of fuel used in the core. The code generates the list of fuel channels for refueling in the order of priority. A set of most suitable channels is picked up from the list and refueling of these channels (with suitable burnup step) is simulated by the 3D diffusion code used to estimate the new power and burnup distribution. The new power and burnup distribution is used in the next step for selection of another set of channels for refueling and so on the process continues. Presently, its main objective is to automatically develop the refueling strategy of AHWR / AHWR-LEU meeting all the design objectives and the development of CARSH has been done considering AHWR core.

The lattice calculations for the AHWR equilibrium core cluster were performed by using Neutron Transport Theory computer code ITRAN (Krishnani, 1982) with the ENDF/B-VI.8

based IAEA 69 multigroup WIMSD nuclear data library. The calculations were performed by using 69- group library based on ENDF/B-VI.8 nuclear data obtained from IAEA (IAEA, 2016). Two group cross sections with burn up were generated using ITRAN. These two group cross sections are further used in core calculations by diffusion theory code FEMINA (Kumar and Srivenkatesan, 1984). The steady state calculations for follow up and refueling are also performed by FEMINA.

The brief description of structure of computer code CARSH is given below.

3.4.1 Outline of CARSH

The computer program CARSH reads the burnup distribution, power distribution and the fuel type distribution in the core from the output of FEMINA (from where refueling operation has to start). Other inputs like fuel types to be loaded in the core, design discharge burnup of different fuel types, different zones of the core, operating limits on MCP and MMP etc. are also made available. Based on the various refueling rules, the refueling factor is evaluated for all the channels. The quarter core symmetry of AHWR core is used to reduce the complexities of the problem. Hence the refueling scheme used is mini batch refueling scheme where one mini-batch consists of four quarterly symmetric channels.

The channel with highest refueling factor is chosen for refueling from one quadrant of core. As a next step the refueling scheme is chosen. If the channel selected for refueling resides in low flux region then direct refueling is chosen otherwise refueling with single reshuffling is adopted. If the refueling scheme adopted is refueling with single reshuffling then a channel from same quadrant of core is selected for reshuffling based on the reshuffling factors. Based on the channel selected from one quadrant of the core, the symmetric channels from the other three quarters are picked to make a mini batch of 4 channels. The selected mini batch of the channels is simulated for refueling using FEMINA and the MCP and MMP are observed in the output of FEMINA. If the MCP and MMP are beyond the design limits then next preferred set of channel for refueling and / or reshuffling (next highest refueling and /or reshuffling factor) is selected and core is again simulated. The program this way tries a lot of combinations till it finds a suitable set of channels for refueling and / or reshuffling.

The computer program CARSH thus has two basic modules:

- *Refueling without reshuffling*
- *Refueling with single reshuffling*

In both modules, executive file of 3D diffusion theory code FEMINA is called to simulate the core with refueling inputs. The decision regarding the adaption of simple refueling or refueling with reshuffling is taken based on the channel power and average channel power of nearest neighbors of the channel selected for refueling as well as from the comparison of channel power distribution with time averaged equilibrium core power distribution.

A brief flow chart of CARSH is described in Fig3.4. If any preferred input is not available even after code execution of few hours, then the design limit is increased (or re-rating of reactor is required). The code has a restart option. Hence, whenever required it can be stopped and can be started later on. Moreover, the burnup, power distribution, refueling inputs and other outputs are stored after each refueling.

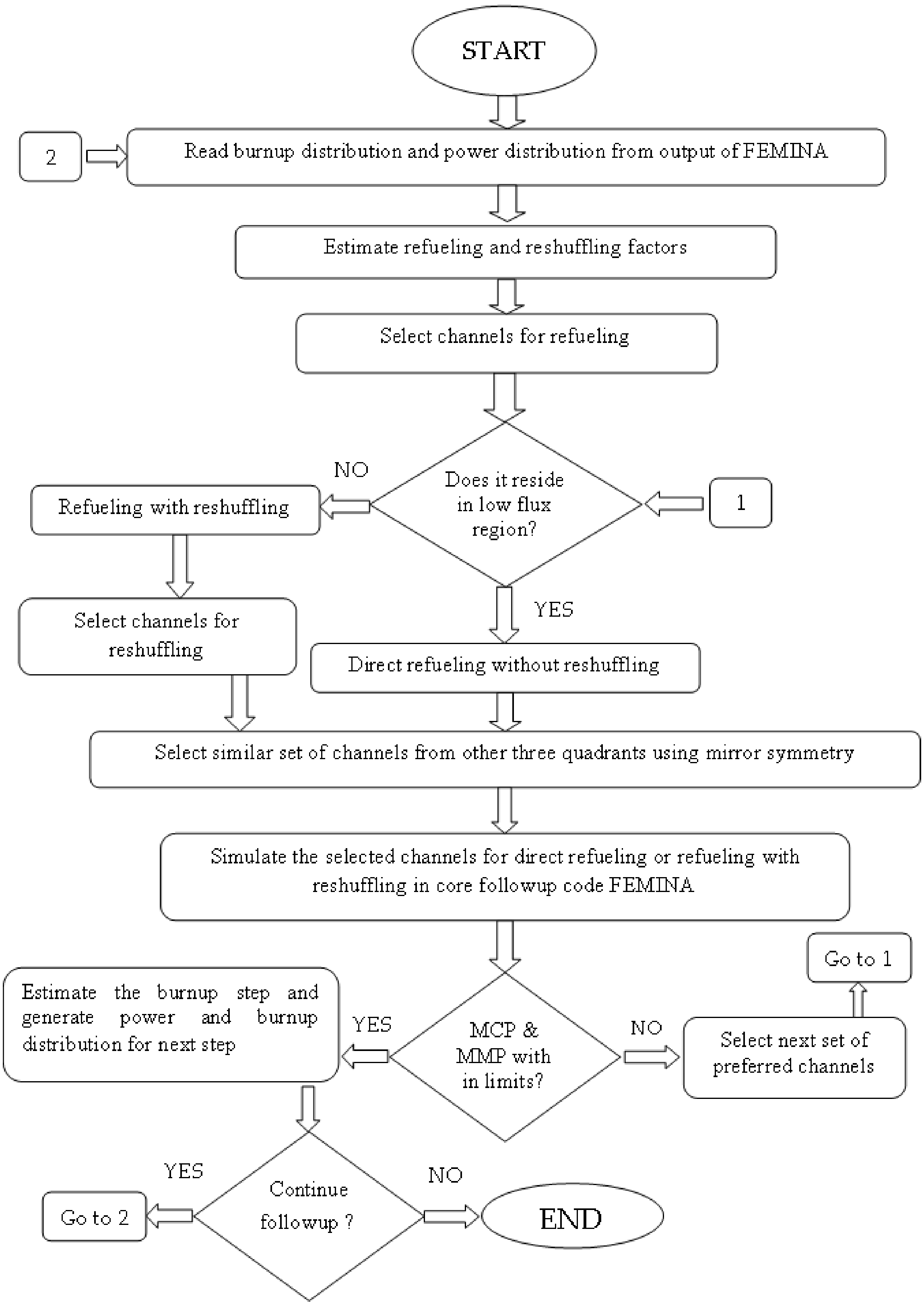


Fig 3.4 Flow chart of CARSH

3.5 Results of simulations carried out by the computer code CARSH

3.5.1 Refueling of AHWR-LEU using CARSH

The computer code CARSH has been extensively used for refueling of AHWR-LEU core. This exercise was taken up as a test for the capabilities of CARSH and during the process many improvements have been carried out. The AHWR-LEU equilibrium core cluster described in Table 3.2 has been used to refuel the core from onset of refueling of initial core to ~5100 full power days (FPDs) using CARSH. The equilibrium core cluster is having radial and axial gradation in enrichment.

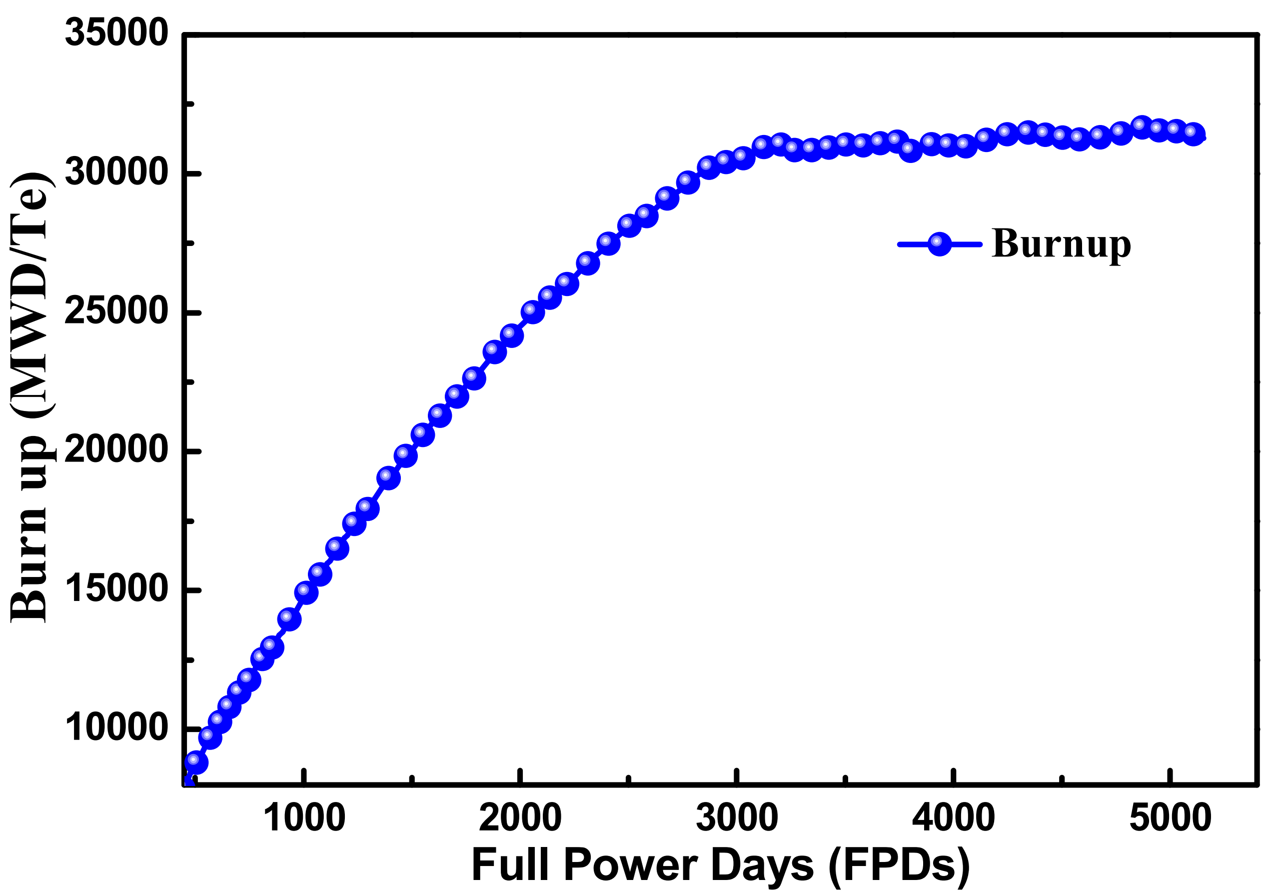


Fig 3.5a Core average burnup vs full power days

Table-3.2 Description of AHWR-LEU cluster studied

Gd content	Fuel Type (Fissile content)	Axial gradation	LEU content (%) in various rings of cluster				U-235 content (%)	K_∞ (0 MWd/TeXe sat.)	Design discharge burnup
			Inner	Middle	Outer	Cluster Average			
5% Gdin 2 pins of inner ring	^{235}U-Th (4.29%)	Upper Half	30	24	14	20.88	4.12	1.173	61.0 GWd/Te
		Lower Half	30	24	18	22.66	4.47	1.215	

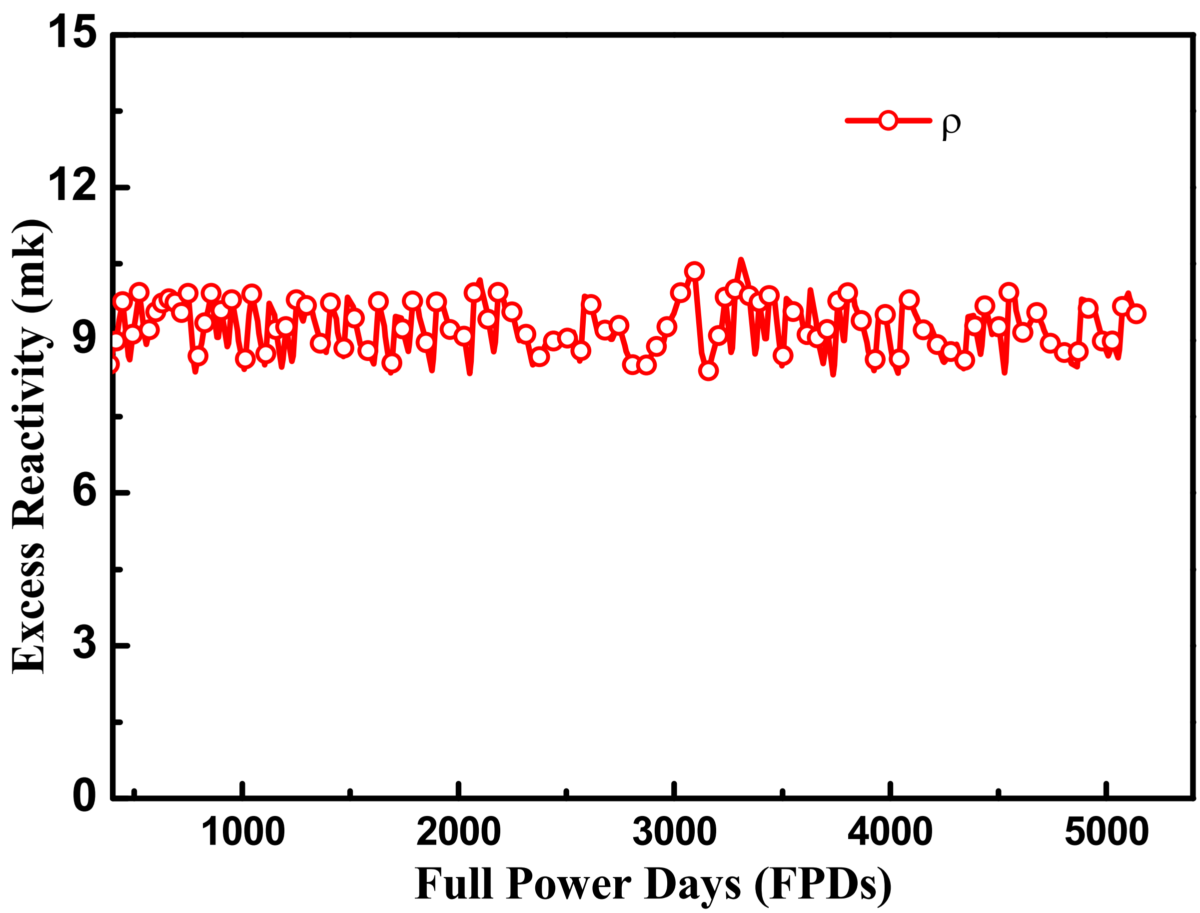


Fig 3.5b Core excess reactivity vs full power days

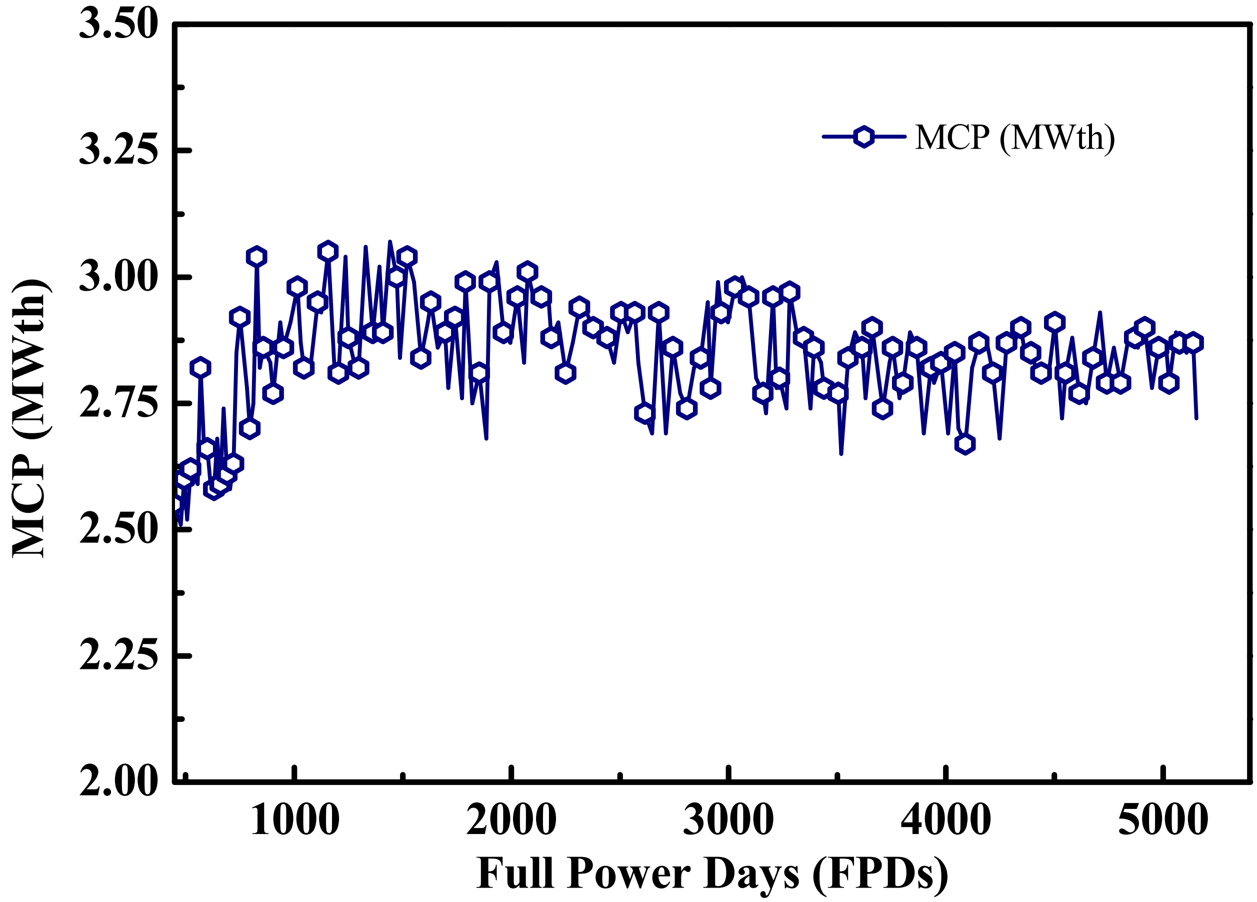


Fig 3.5c Maximum channel power vs full power days

It should be noted that the initial core of AHWR LEU is being designed with two different types of clusters (central 80 channels and remaining 364 contain differential reactive fuel) for flux flattening. The description of the initial core clusters and the initial loading pattern is not discussed in this chapter. However, the design discharge burnup for *Type-1* and *Type-2* cluster is 3.5 GWd/Te and 28 GWd/Te respectively. Gadolinium (Gd) has been mixed in the 2 pins of innermost ring of the cluster to suppress the initial excess reactivity. The target discharge burnup of this cluster is estimated to be about 60GWd/Te using time averaged calculations. The burnup penalty of Gd is ~ 3 GWd/Te.

The core refueling study using CARSH has shown that this cluster with Gd could help to control the local power peaking to some extent. The study has shown that the average in-core burnup achieves a constant value of ~31 GWd/Te at ~ 3300 FPDs (Fig. 3.5a). The in-core excess reactivity has been maintained near 10mk throughout the core followup (Fig. 3.5b).

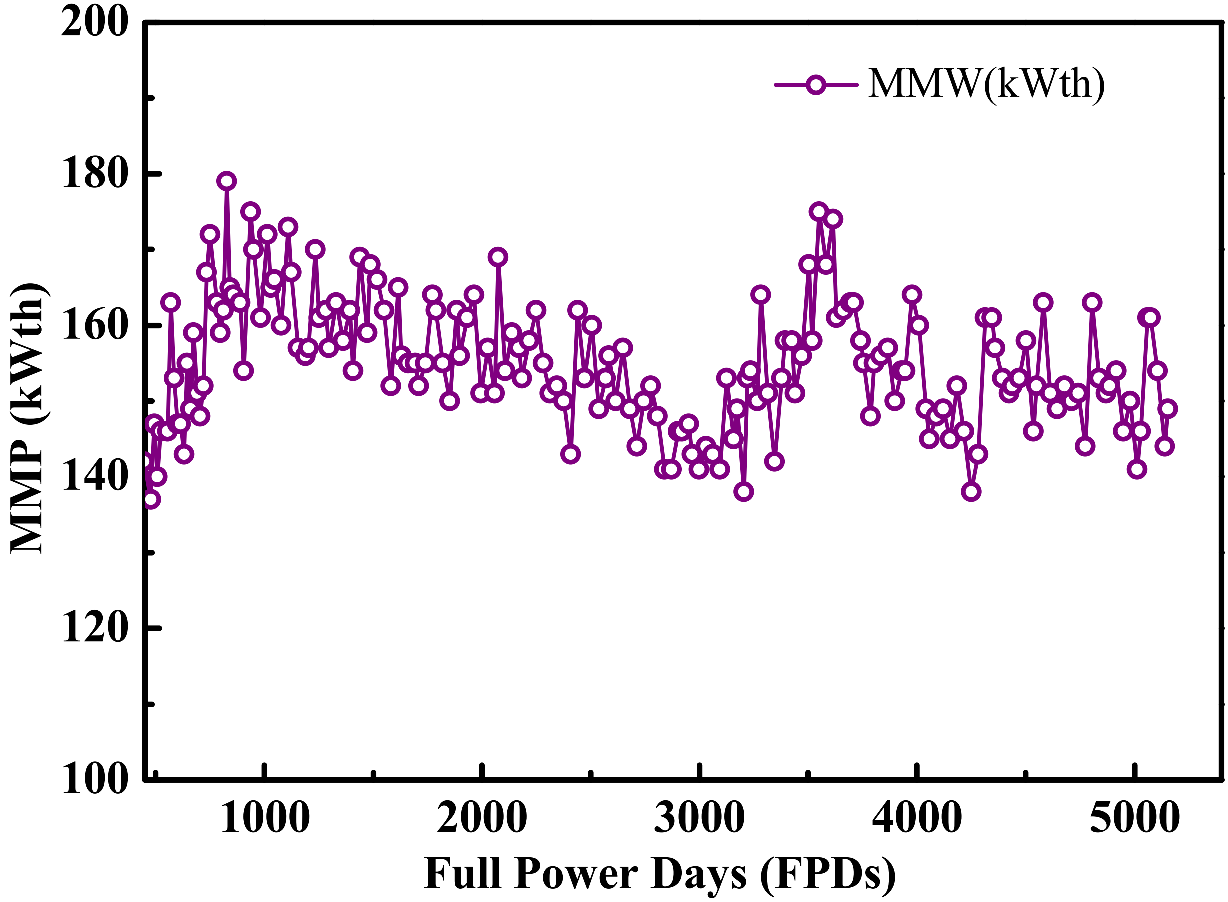


Fig 3.5d Maximum mesh power vs full power days

The MCP was observed to be ~3.1 MW(shown in Fig.3.5c) against the design limit of 2.85 MW. Hence, re-rating of the reactor may be required. For calculation purposes, the channel has been divided in to 24 axial meshes and the maximum mesh power (MMP) has an upper limit of 200kW.

The MMP was maintained well below 200kW during the refueling study. It was also observed that after all the initial core clusters have been discharged (at 3300 FPDs), the MCP can be maintained close to 2.90 MW. The initial core clusters have been discharged on priority (Fig. 3.5e). The cumulative discharge burnup of all equilibrium core cluster discharged is ~ 56 GWd/Te. Further, the discharge burnup of last 100 clusters discharged is ~ 59 GWd/Te. During the above mentioned study of refueling strategy for AHWR-LEU core, a mini batch (4 channels with quarter core mirror symmetry) refueling scheme was used to control the power peaking.

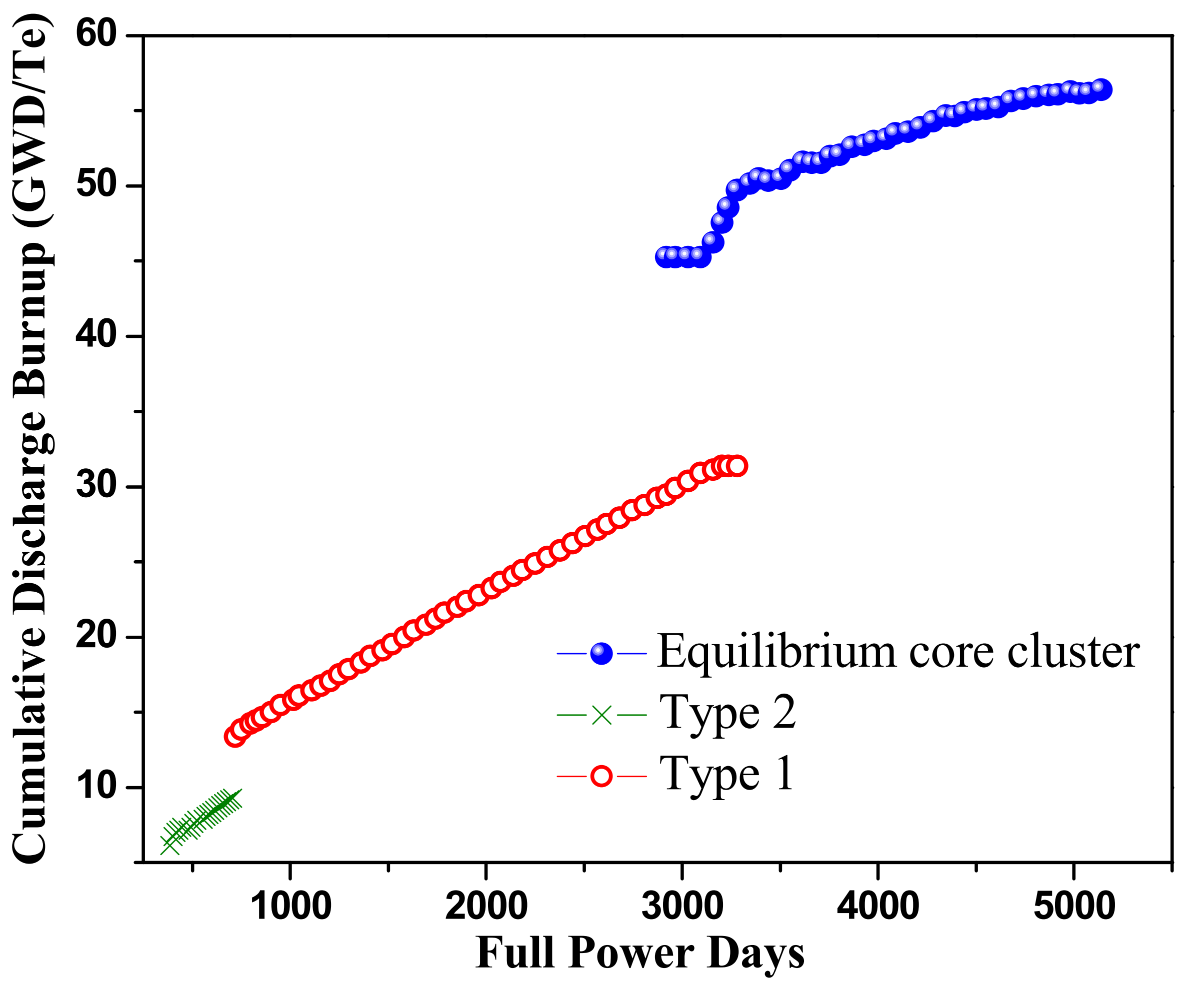


Fig 3.5e Cumulative discharge burnup vs full power days

3.6 Conclusions

In this chapter, the problems related to fuel management were discussed for reactors with on-power refueling and off- power refueling. The unique features related to fuel cycle of AHWR-LEU were also discussed. The in-core fuel management in AHWR-LEU requires on-power refueling strategy with high fissile content fuel cluster. Incorporating these two diverse features (on-power refueling and high fissile content fuel) belonging to two different types of reactors namely PHWRs and LWRs which have contradicting requirements, into one reactor (AHWR-LEU) is a new and complex problem. It was observed that the conventional way of doing refueling in AHWR will not serve the purpose as it is not possible to control the peaking using direct refueling. Therefore, a special refueling strategy which is optimal combination of PHWR and LWRs refueling strategy was devised. In this special refueling strategy, it is proposed that if each refueling operation is followed by reshuffling operation, significant reduction in power peaking can be achieved. It is also proposed that by defining a few factors like refueling and reshuffling factor, the priority of channels can be defined for selection for refueling or reshuffling by giving weightage (assigning refueling and reshuffling factor) to each channel based on its channel power, fuel type, location and burn up.

To choose a set of refueling and reshuffling inputs from a large number of available combinations, the channels based on their refueling factor and reshuffling factor can be selected for one refueling simulation. A computer code CARSH was developed by using the remedy worked out in this chapter. CARSH can automatically generate the various refueling inputs (selection of channels for refueling and reshuffling) required for special refueling strategy and carry out the refueling simulations for AHWR-LEU core.

It is observed that by using this special refueling strategy, peaking during the transition phase and equilibrium phase has been substantially reduced. The MCP during transition phase is observed below 3.1 MW as compared to 7.0 MW observed during direct refueling. And after the transition period is over the MCP can be maintained below 2.9 MW which is very close to design limit of 2.85 MW. The MMP can be maintained below its design limit of 200 kW throughout the reactor operation period. It is also observed that the target discharge burn-up is also achievable if this special refueling scheme is implemented. However, continuous operation of reactor is not possible at 100 % FP and there is still scope for improvement by adding more reshuffling operations.

CHAPTER 4

Refueling With Double Reshuffling

4.1 Introduction

The special refueling scheme devised for AHWR (as described in chapter 3) could successfully demonstrate its equilibrium fuel cycle to a limited extent. It was observed that a considerable scope of enhancement in computer code CARS is there to develop a better refueling strategy which can further improve the core operational parameters like MCP and MMP during transition period of core. To identify the areas in which improvement is feasible, the micro-details of the core parameters extracted in our last study using computer code CARS were carefully scrutinized and following observations were made.

a) The power peaking is more dominant during the pre-equilibrium phase (transition phase). And it becomes difficult to control power peaking below design limit of 2.85 MWth by adopting single reshuffling along with refueling. Further, the in-core burnup distribution profile at 3300 FPDs was observed to be different from equilibrium core burnup distribution estimated from time average calculations. This leads to increase in time to reach the target discharge burnup of cluster.

b) To maintain a symmetric power distribution and lower radial peaking, a mini batch of 4 channels which are quarterly symmetric is chosen at each refueling. We have used quarter core mirror symmetry and it was observed that due to the quarter core mirror symmetry, certain channels near to axis of symmetry (boundary of quarter core) were not getting selected for refueling as it causes power peaking in that region. Therefore these channels are getting overdue for refueling (as shown in Fig 4.1). As a result some other channels were getting discharged pre-maturely and leading to overall loss in discharge burnup.

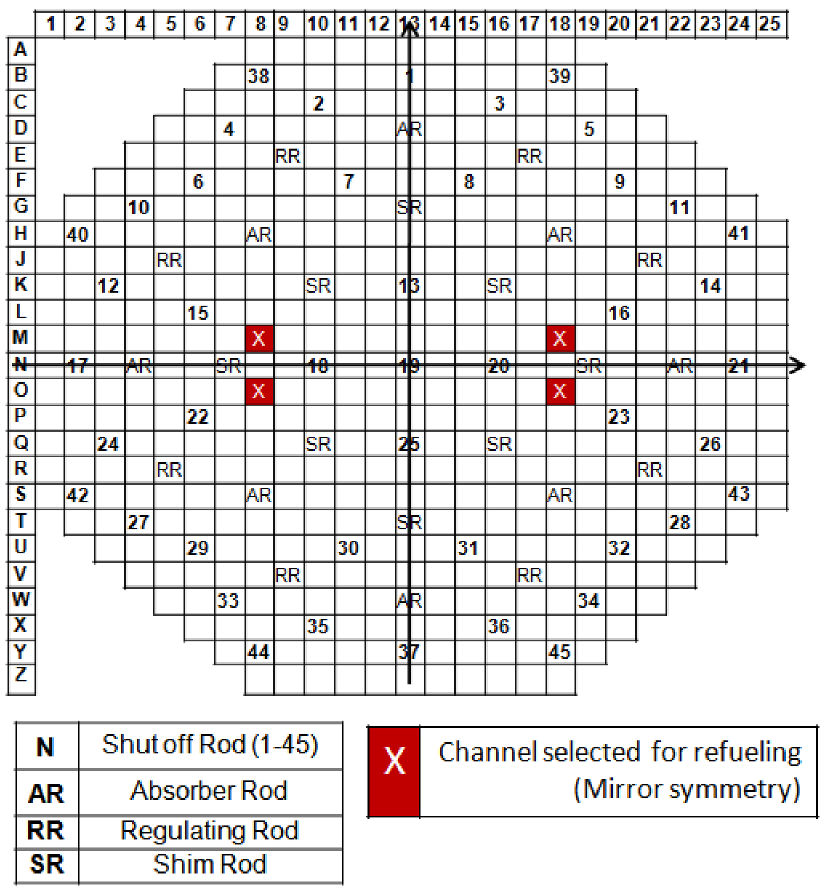


Fig 4.1 Mirror symmetry of AHWR core

c) It takes a long time (7-10 days) to develop the refueling strategy on a personal computer. The code has been running everyday for 8-10 hrs.

In this chapter, we have tried to address the problems faced with the refueling strategy developed using single reshuffling scheme by adding few enhancements in computer code CARS. Development of a new refueling scheme has been described where each refueling operation is followed by two reshuffling operations. A new subroutine was developed which can do automatic refueling for AHWR using this scheme. An efficient refueling strategy was developed which is a mixture of all the refueling schemes proposed namely direct refueling, refueling with single reshuffling and refueling with double reshuffling scheme. To minimize the peaking problem near the axis boundary of core, we have tried to exploit rotational symmetry of core. To reduce the time period of simulations, parallel processing has been used. Section 4.2 describes the various modifications suggested for improvement in results during refueling studies of AHWR-LEU and further development of computer code CARS. Section 4.3 details the results obtained after developing refueling strategy using the modified version of computer code CARS incorporating various new features. Section 4.4 gives the conclusions and scope for future research. The results have been published in International scientific journal *Thakur et. al., 2013* and in international conference *Thakur et. al., 2011*.

4.2 Modifications in CARS for a better refueling strategy

As discussed in last section, necessary modifications are required in computer code CARS to maintain 100 % FP operation of reactor during transition period. To improve the peaking problem during transition phase of core, it is advised that more number of reshuffling operations should be done with each refueling. Therefore, we have added one more subroutine in code

CARS which selects the set of channels such that each refueling is followed by two reshuffling operations. This refueling scheme is named as refueling with double reshuffling scheme. To rectify the problem arising due to selection of quarterly mirror symmetric channels, rotational symmetry is considered as another modification. Further to reduce the time of simulations, parallel processing has been considered. The details of all the modifications are given below.

4.2.1 Refueling with double reshuffling scheme

During the transition phase from initial core to equilibrium core, single reshuffling along with refueling was not capable to control the power peaking to desirable extent.

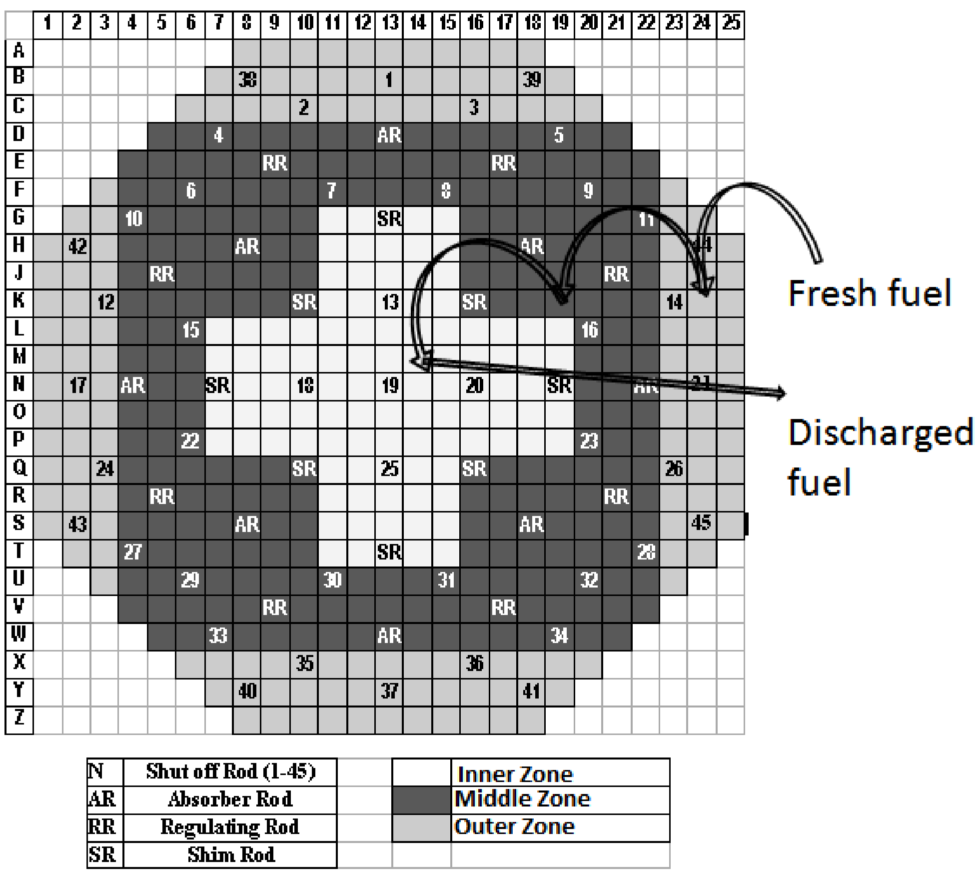


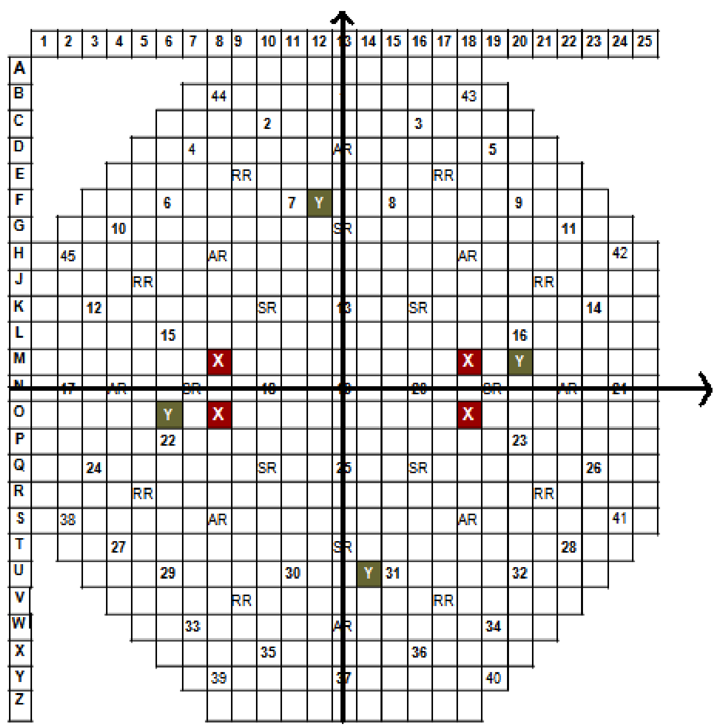
Fig 4.2 Schematic representation of refueling with double reshuffling scheme

In order to control the power peaking in transition phase (for ~ 33000 FPDs), double reshuffling along with refueling scheme was adopted. For incorporating double reshuffling scheme, one has to select three quarterly symmetric channels from one quadrant which comprise of ${}^{111}P_3$ (~ 1330800) combinations. However, in our remedy, we have avoided simulations of all these combinations and tried to minimize the simulation to the best candidates for refueling and reshuffling channels. In this scheme the core is assumed to be divided into three zones (namely inner, middle and outer zones). In the present case, the three zones have been chosen to be same as burnup zones, decided after equilibrium core burnup optimization for time averaged core calculations. This will ensure that the burnup profile and power distribution will remain comparable to equilibrium core burnup profile and power distribution. The outer zone channel with lower burnup is replaced with fresh fuel cluster and the irradiated cluster from the outer zone is shifted to middle zone to replace the cluster with relatively higher burnup. Finally the irradiated cluster from the middle zone is shifted to the inner (central) zone to replace the fuel clusters which are due for discharge to spent fuel bay (Fig. 4.2). The burnup ranges for shifting the fuel from outer zone to middle zone and middle zone to inner zone are decided based on fuel type used and burnup distribution available in the core.

To incorporate this scheme into CARS, the core has been divided into three zones (Fig. 4.2) and the refueling factors (as described in chapter 3) have been assigned to all the channels in the core. Thus the three channels from three zones (outer, middle and inner) of the same quadrant of the core with maximum refueling factors are chosen for refueling with double reshuffling. Similarly by adopting quarter core symmetry the channels from other three quadrants are picked up for reshuffling and refueling. These combinations are simulated by FEMINA and the operational parameters like MCP, MMP and core power distribution is estimated. Further, if the operational parameters are beyond their designed limit then the zone of maximum powered channel is observed. The channel with next higher refueling factor from this zone is considered for modified refueling input. The process is repeated till we find suitable operational parameters after refueling.

4.2.2 Use of π/2 rotational symmetry instead of mirror symmetry

It was also observed that the quarter core mirror symmetry was not suitable for refueling of certain channels near the quarter core boundary (near the axis of symmetry) and it causes power peaking.



N	Shut off Rod (1-45)
AR	Absorber Rod
RR	Regulating Rod
SR	Shim Rod

X	Channel selected for refueling (mirror symmetry)
Y	Channel selected for refueling (90^0 rotational symmetry)

Fig 4.3 Mirror symmetry and rotational symmetry of AHWR core

More explicitly, the refueling of the channels marked as '*X*' in Fig 4.3 was commonly being rejected due to power peaking, as the symmetric channel in the nearby quadrant is quite close. The adoption of reshuffling along with refueling scheme was capable to control the peaking to limited extent only. As a result these channels were getting over due for refueling.

The AHWR core also has a $\pi/2$ rotational symmetry. Hence, adoption of $\pi/2$ rotational symmetry provides sufficient distance between the two channels in the neighboring quadrants. As an example the channels marked as '*Y*' in Fig 4.3 are sufficiently away from the $\pi/2$ rotational symmetric channels of the other quadrants. Adoption of $\boldsymbol{\pi}/2$ rotational symmetry was proved to be very useful in reducing power peaking. It was observed that by adopting refueling with double reshuffling scheme along with $\pi/2$ rotational symmetry, the power peaking during the pre-equilibrium phase of refueling was controlled substantially. However, following problems were observed by adopting double reshuffling scheme;

- The number of refueling machine operations increased.
- After sometime during refueling it was observed that there is flux depression in the inner region of the core.

The above mentioned problems were resolved by adopting a refueling scheme based on the combination of three refueling schemes *viz. refueling without reshuffling (direct refueling), refueling with single reshuffling and refueling with double reshuffling* scheme. For this purpose the zonal power of each zone is monitored and compared with equilibrium core power distribution at each step. The flux distribution at each refueling has been maintained near to the equilibrium core flux distribution by adopting the desired refueling scheme. The improved flow chart of CARS is shown in Fig 4.4.

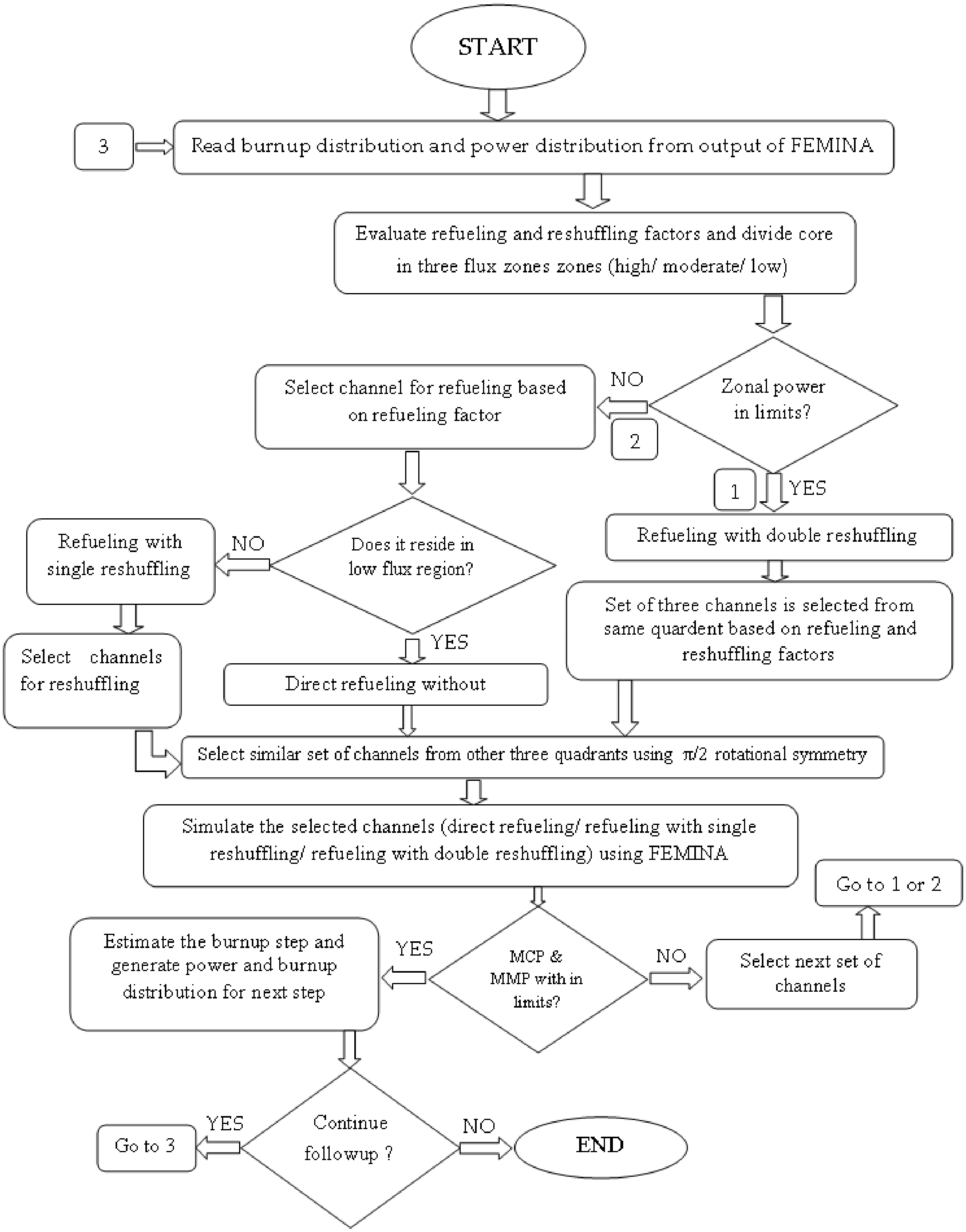


Fig 4.4 *Flow chart of CARSH (modified for incorporating double reshuffling scheme along with single reshuffling scheme)*

4.2.3 Use of parallel processing to accelerate the speed

In this advance era of computing and with the increasing computational power, it is possible to enhance the efficiency and speed of a code using parallel processing even on shared memory architecture like personal computers. In case of CARS, the parallel processing has been used to increase its speed. A parallel processing interface like MPI (Massage passing interface) or OpenMP (Open Multi Processing) can be used for task sharing, increasing the efficiency and reducing the time for simulations.

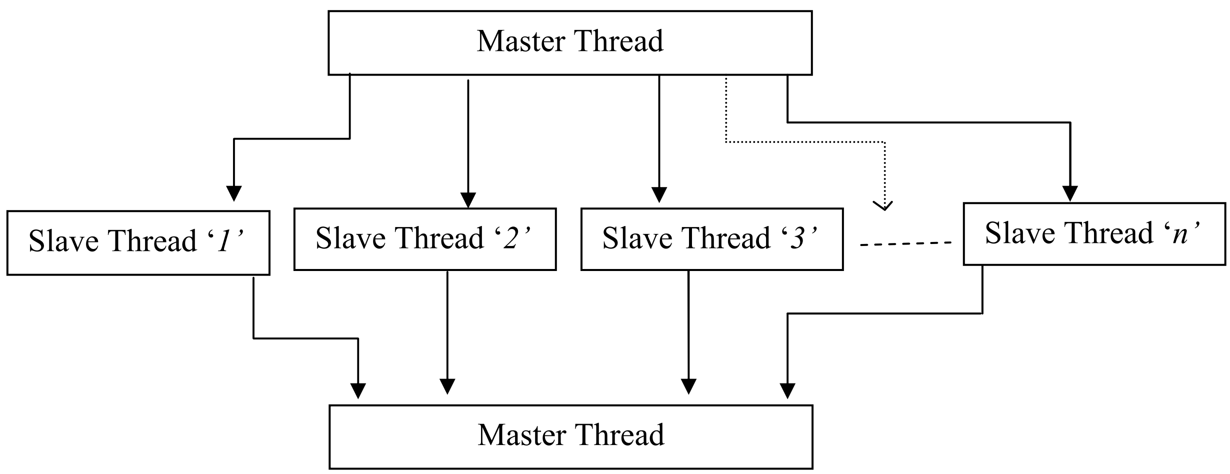


Fig 4.5 Schematic diagram of task sharing in parallel processing

It is easier to convert a sequential program to parallel on a shared memory model using OpenMP. Therefore, we have used OpenMP (Barney, https://computing.llnl.gov/tutorials/openMP/) which is an application processing interface for parallel processing on shared memory architecture. MPI could have been a better alternative if our requirement is higher than 8 cores and across the server parallelization is necessity. OpenMP is an implementation of multithreading, a method of parallelizing whereby a master thread generates a specified number of slave threads and a task is divided among them (Fig.4.5). The threads then run concurrently, with the runtime environment allocating threads to different processors. The idea is to generate pre-decided preferable refueling inputs and then run multiple FEMINA executables with these different inputs in parallel mode. We have used OpenMP to parallelize CARS on INTEL i7 960 (3.20GHz) computer. Right now eight slave threads are being generated which simulates different refueling inputs concurrently. The most compatible input is chosen by comparing the outputs. If no input is found compatible, then next set of eight inputs will be considered. We could reduce the time for simulation of 180 refueling (5700 FPDs) from ~8 days to ~ 1 day, thereby getting 8 times improvement in speed utilizing the 8 core computer.

4.3 Results of Simulations carried out by modified computer code CARSH

4.3.1 Refueling studies for AHWR-LEU

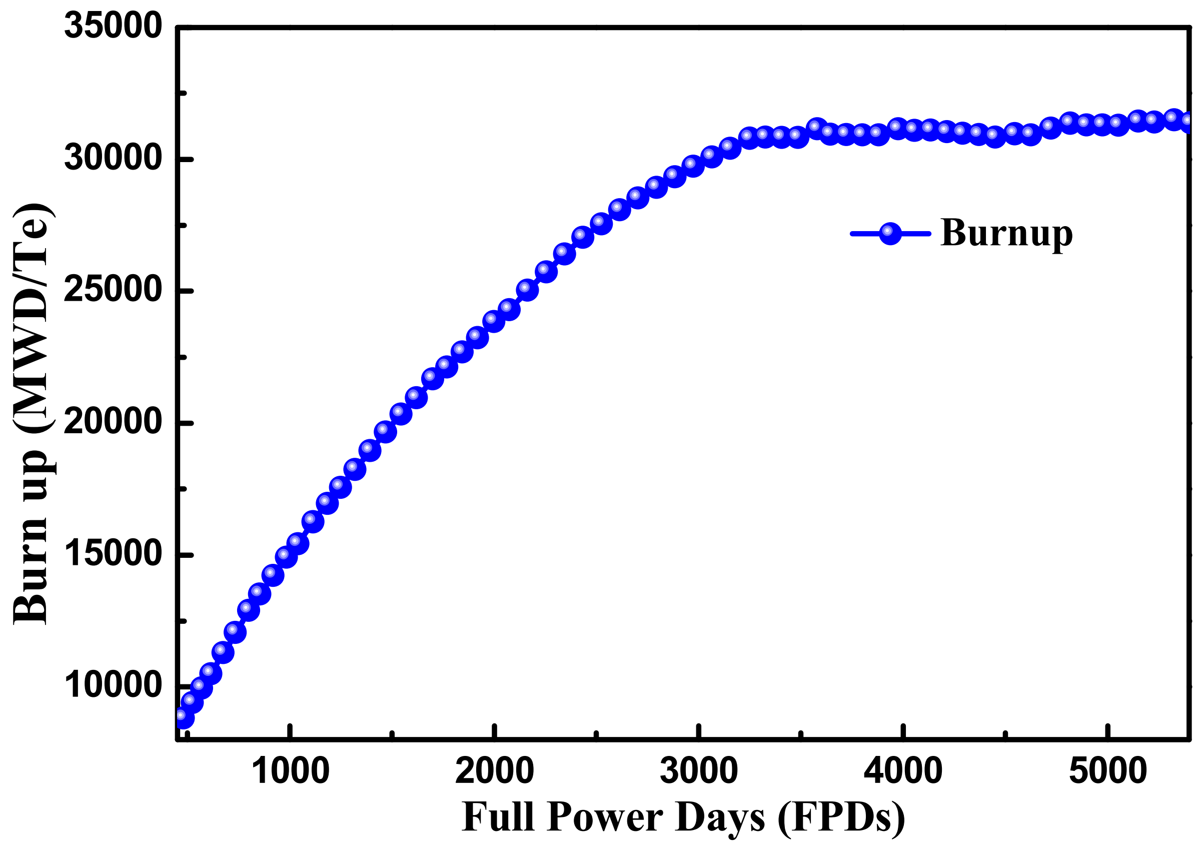


Fig 4.6 Core average burnup vs full power days

Based on the earlier studies on the equilibrium core cluster and making all the improvements from past experience in to CARS, the same equilibrium core cluster (Table 3.2, chapter 3) was studied to facilitate the on-power refueling of AHWR-LEU.

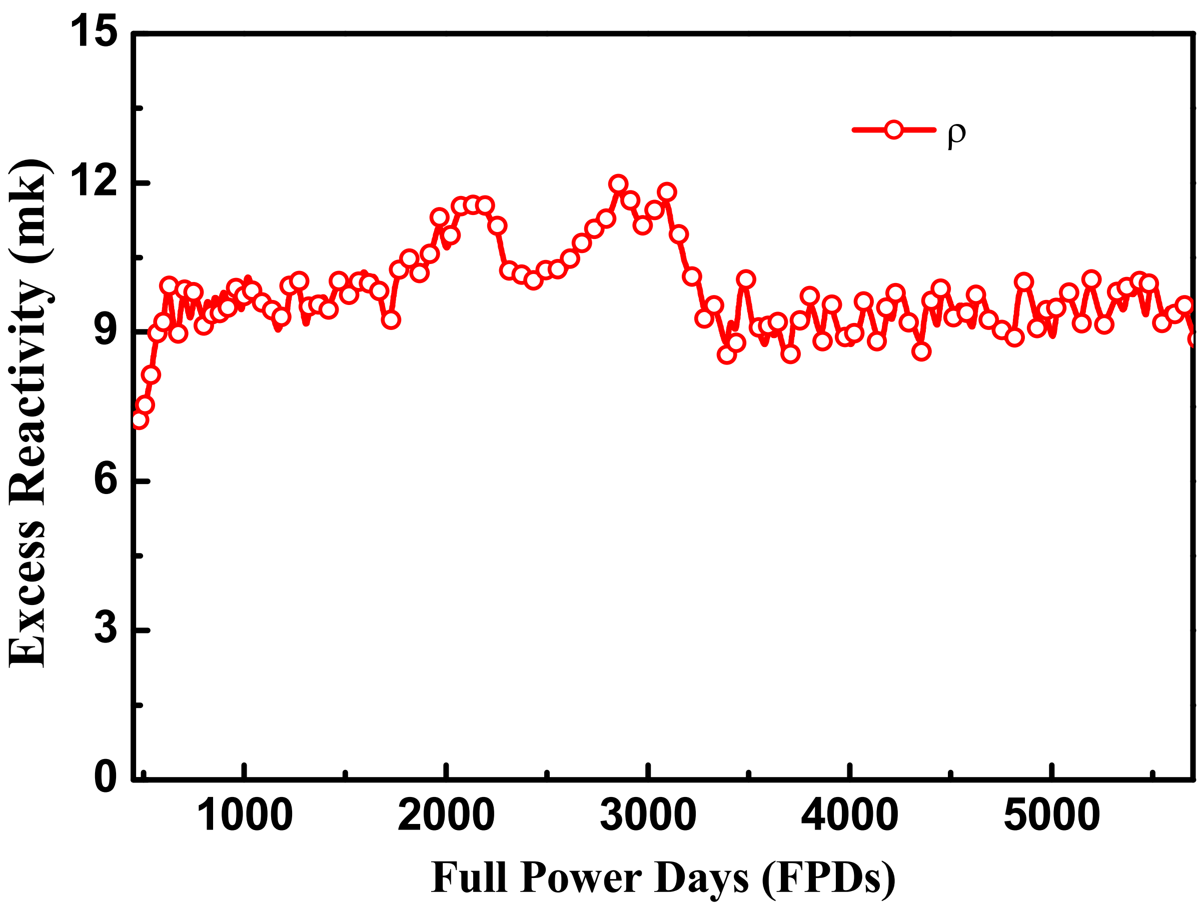


Fig 4.7. Core excess reactivity vs full power days

The improved computer code CARSH was used for the refueling of initial core of AHWR-LEU from the onset of refueling (350 FPD) to the equilibrium core (5700 FPD). The refueling scheme used is a combination of refueling with single reshuffling as well as refueling with double reshuffling. During the core follow up, the observed variation of in-core average burnup with full power days (FPDs) is shown in Fig 4.6. It is observed from Fig. 4.6 that the core reaches the equilibrium state in ~ 3300 FPDs and the average in-core burnup achieves a constant value of~31GWd/Te. The core excess reactivity has been maintained nearly 10 mk throughout the follow up as shown in Fig 4.7. The variation of MCP with FPDs is shown in Fig 4.8. It is observed that the MCP can be maintained below 2.85 MWth throughout the core followup studies. The refueling strategy devised by the modified computer code shows that 100% FP reactor operation is possible in all stages of reactor core. The variation of MMP with FPDs is shown in Fig 4.9. The MMP has been maintained within the design limit of 200 kW. Figure 4.10 shows time of the discharge of various types of clusters from the core and it is evident that the computer code CARSH has discharged the various types of fuel clusters in the desired order. The initial core clusters have been completely discharged from the core at about 3300 FPDs.

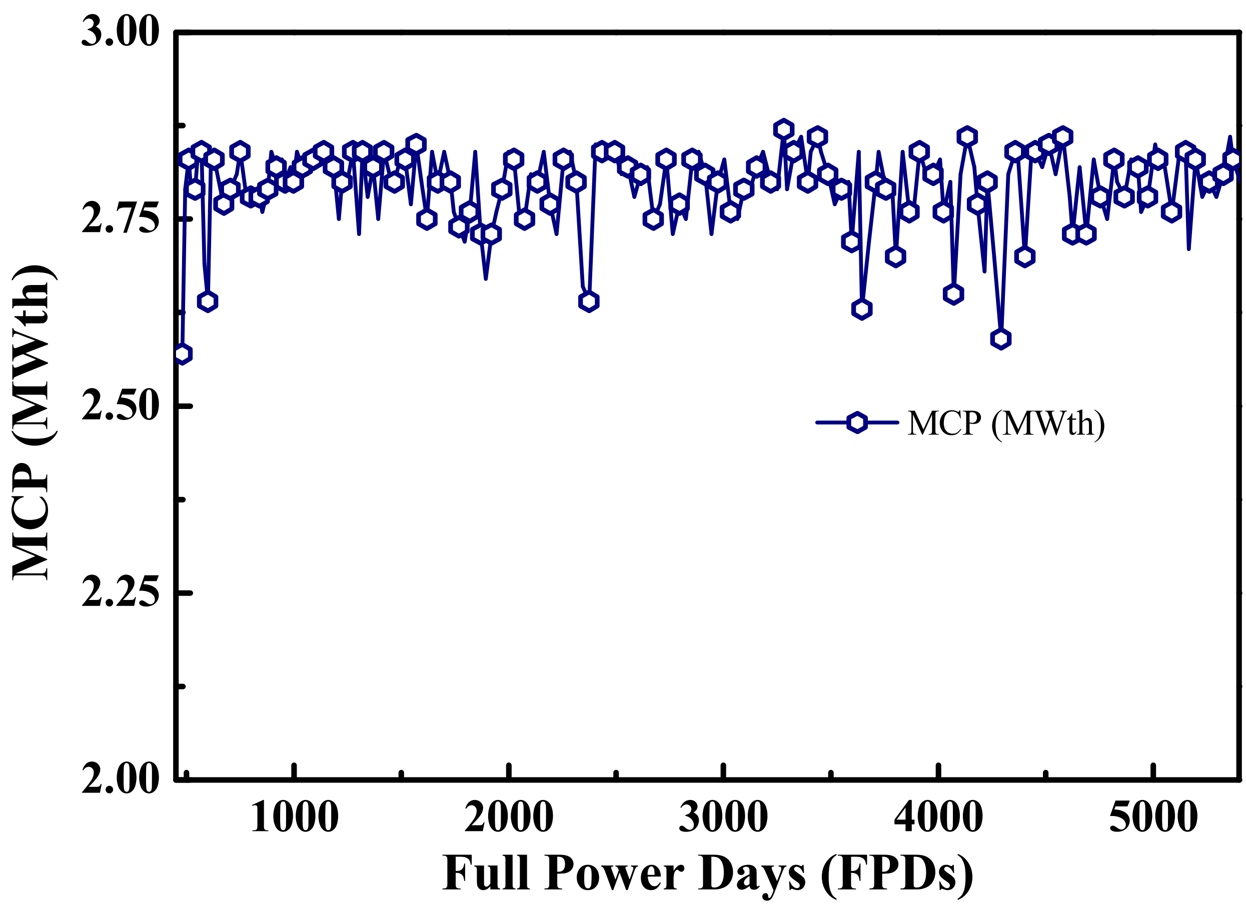


Fig 4.8. MCP vs full power days

Hence from the 3300 FPDs to 5700 FPDs the refueling operation has been carried out by replacing the irradiated equilibrium core cluster with the fresh equilibrium core cluster. A total number of 800 channels (200 mini-batches of 4 channels) have been refueled from onset of refueling to 5700 FPDs. The discharge burnup of the equilibrium core clusters have been given in Fig 4.11. It shows that the discharge burnup of the equilibrium core clusters progressively improves to ~ 60.0 GWd/Te. It has been observed that the refueling in the initial 1000 FPDs of the pre-equilibrium phase is dominated by refueling with double reshuffling scheme. After 1000 FPDs onwards the refueling with single reshuffling scheme becomes dominant. The details of refueling schemes adopted during the followup are given in Table-4.1.

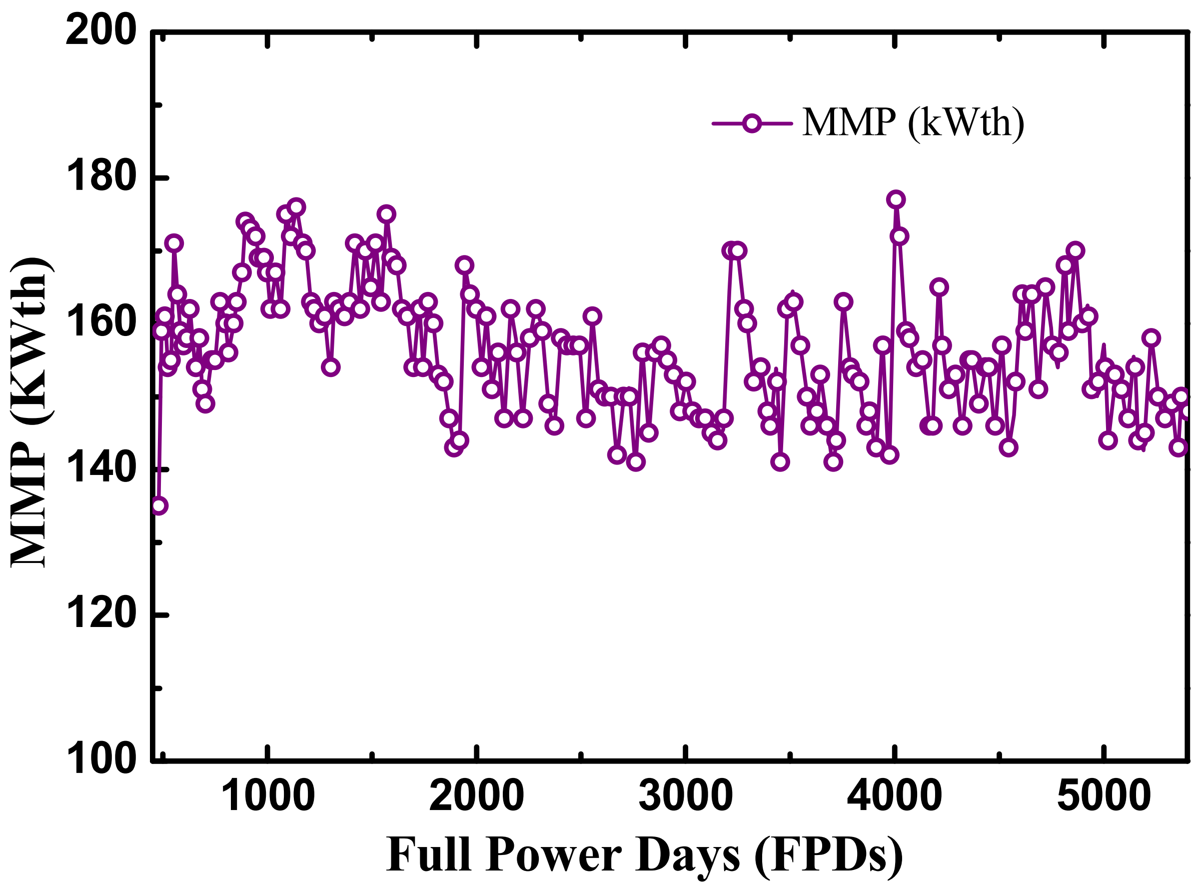


Fig 4.9 MMP vs full power days

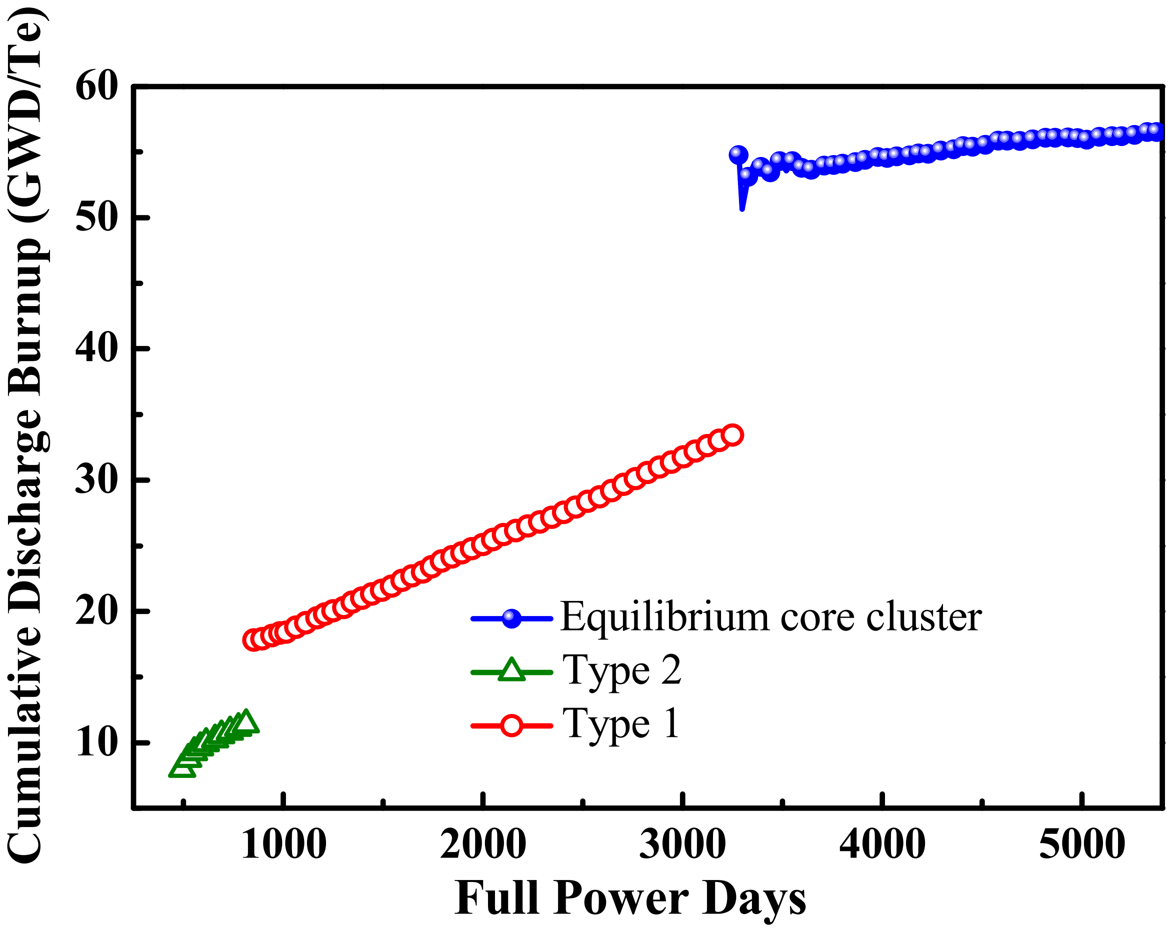


Fig 4.10 Cumulative discharge burnup vs full power days

The average refueling machine operations (including refueling and reshuffling) during 350-1380 FPDs are 3.8 channels / week. From 1380-2380 FPDs the average refueling machine operations reduces to 2.3 channels / week. After 2380 FPs the refueling machine operations further decreases to 1.9 channels/ week.

The refueling strategy has been evolved after a trial of different refueling schemes (direct refueling / single reshuffling / double reshufflings). The very first trail was direct refueling (no reshuffling) scheme. The priority for refueling of channel was decided based on its maturity level (burn-up). However, by using this scheme the core key parameters were beyond the design limits

(Fig 4.12 and 4.13). To improve core key parameters, single reshuffling scheme was incorporated. However, the results were still not very satisfactory during transition phase. Hence, double reshuffling scheme has also been used to maintain all the core key parameters within design limits.

The comparison of few core key parameters generated from these refueling strategies is given in Table-4.2. The Table-4.2 shows that by using direct refueling scheme (no reshuffling), it is impossible to maintain MCP and MMP within the design limits. Also, the discharge burn-up achieved is ~55 GWd/Te.

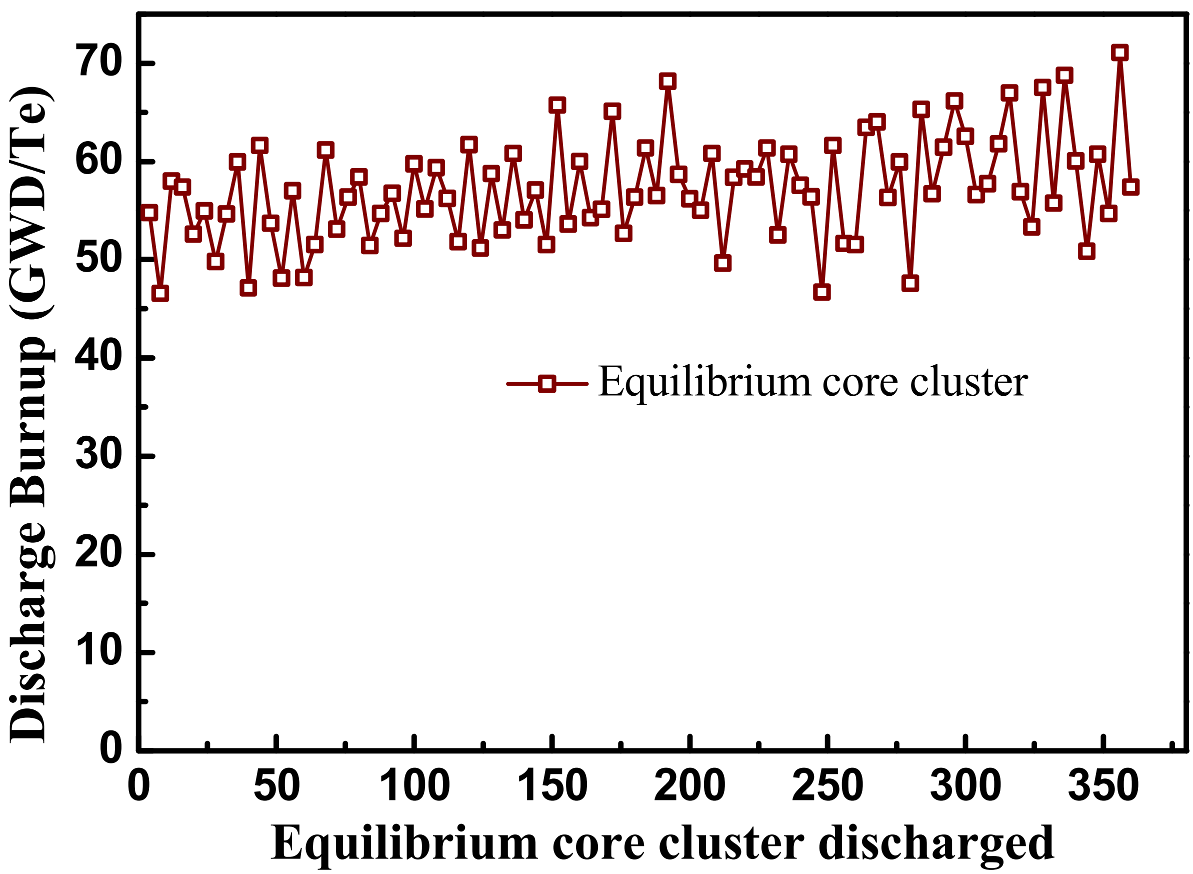


Fig 4.11 Discharge burnup vs number of equilibrium core clusters discharged

This is ~5GWd/Te lower than the target discharge burn-up of ~60 GWd/Te. The loss in discharge burn-up may be due to refueling ripples and un-even flux distribution due to the followed refueling strategy. The adoption of single reshuffling scheme has helped to control the power peaking to a significant extent. Moreover, the discharge burn-up also improves to ~58 GWd/Te.

Table-4.1 Details of different refuelling schemes adopted during follow up studies

Burnup interval (in FPDs)	*Total refuelings (mini batch)*	*Double reshuffling refueling scheme*	*Single reshuffling scheme*	*No reshuffling (Direct refueling scheme)*
0 - 350	0	-	-	-
350 - 1380	50	41	9	0
1380 - 2380	36	8	28	0
2380 - 5700	114	0	113	1

Table-4.2 also shows that all the core key parameters can be maintained within design limits by incorporating double reshuffling scheme. Figures 4.12 and 4.13 compare the MCP and MMP respectively for the different refueling schemes as discussed above. It is observed that the variation in MCP and MMP for direct refueling is ranging from 2.5 MW to 6.8 MW and 140kW to 360 kW respectively.

Table-4.2 Comparison of core key parameters for different refuelling schemes

Refueling Scheme adopted	*MCP (MW)*	*MMP (kW)*	*Discharge burnup achieved (GWd/Te)*	*In-core excess reactivity maintained (mk)*
Direct Refuelling	2.5<MCP<6.9	140<MMP<380	55	~10
Single Reshuffling	2.5<MCP<3.15	140<MMP<185	58	~10
Double Reshuffling	2.5<MCP<2.85	140<MMP<180	59	~10

These variations are very large. The variations in MCP and MMP have been controlled to a large extent by adopting single reshuffling scheme, where the variation in MCP and MMP have been ranging from 2.5 MW to 3.1 MW and 140kW to 180kW respectively. These results were further refined by adopting double reshuffling scheme.

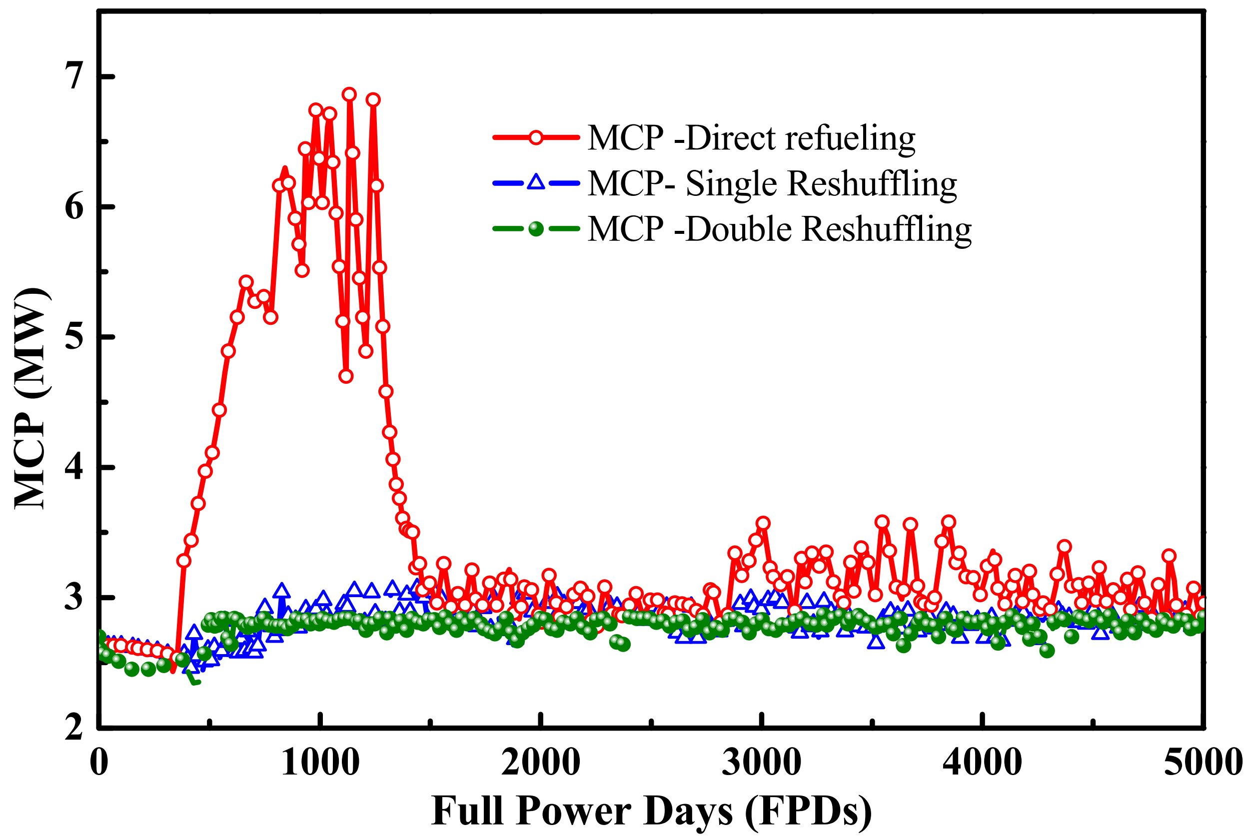


Fig 4.12 MCP variation for different refueling schemes

The variation in MCP and MMP were ranging between 2.5MW to 2.85MW and 140kW to 175kW respectively. We have found that the program CARSH adopts a very suitable refueling strategy to maintain the full power operation of reactor without compromising the discharge fuel burn-up.

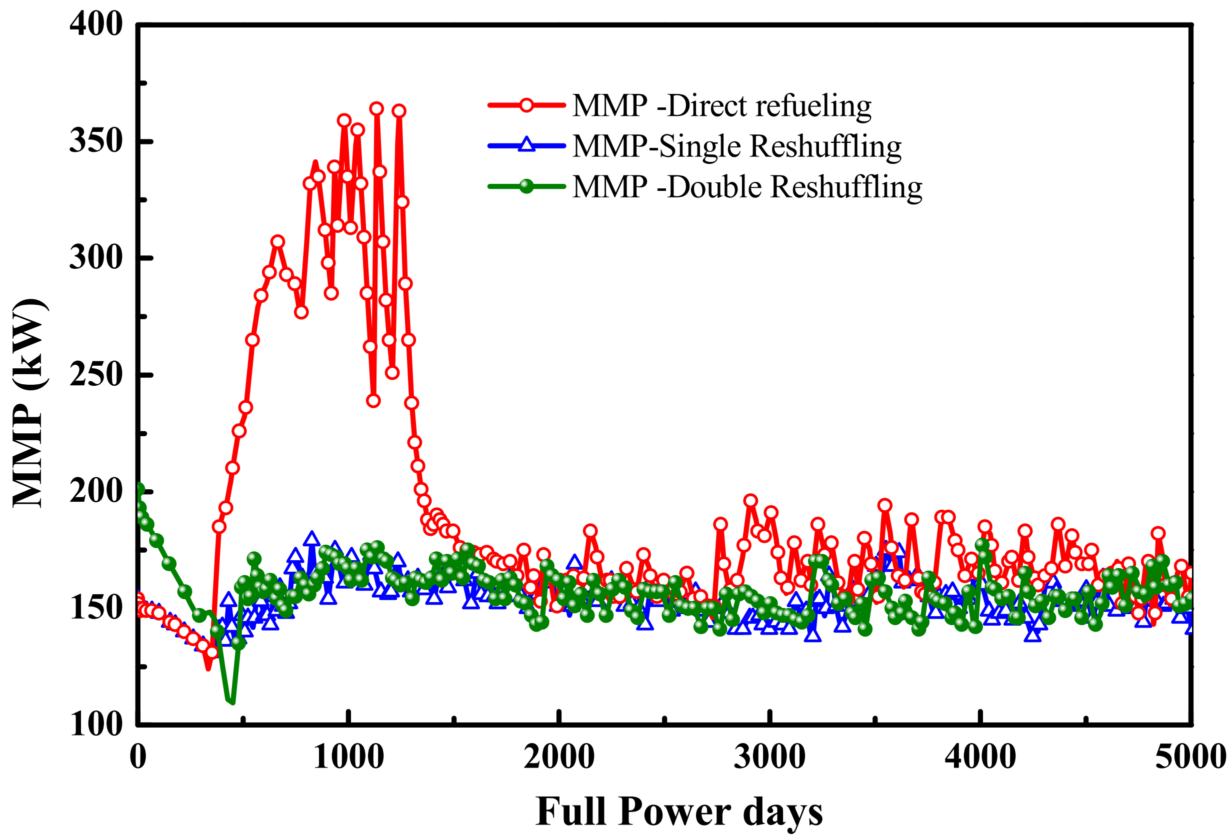


Fig 4.13 MMP variation for different refueling schemes

4.4 Conclusions

In this chapter, a modified refueling scheme where each refueling is followed by two reshuffling operations is proposed to reduce the power peaking. A special refueling strategy, which is a mixture of direct refueling, refueling with single reshuffling and refueling with double reshuffling is projected for facilitating the use of high discharge burnup fuel with on-power refueling. The demonstration of this refueling scheme is given through the development and modification of a computer code CARSH based on heuristic approach. The direct implementation of heuristic rules has been done to define maturity level of a channel for refueling or reshuffling in the form of refueling factor and reshuffling factor respectively. It is observed that more reshuffling operations are required during the beginning of transition phase (up to 1400 FPDs) of core to control the peaking. However, the refueling machine operations decrease from 3.8 channels/week to 1.9 channels/week as we move from transition phase towards the equilibrium phase of core. In the equilibrium core, the refueling with single reshuffling plays the dominant role in proposed refueling strategy. To reduce the time period of simulations, parallel processing has been used. It is observed that the time for simulations has been reduced by a factor of eight.

The modified computer code CARSH has been used for automatic selection of channels for on-power refueling study and for developing refueling strategy of AHWR-LEU. It is observed that the core operational parameters like MMP, MCP are remained below their design limit at all stages of reactor operation. The target discharge burn-up of ~59 GWd/Te is also achievable by using this refueling strategy. The use of CARSH has reduced the manual efforts for the selection of channels for refueling to a great extent. The CARSH has made the study of various types of equilibrium core clusters for AHWR-LEU in a very short time. It has also helped in the design and optimization of burnable poison content in the equilibrium core cluster. The CARSH has given the knowledge about the micro details of the on-power refueling of the AHWR-LEU core. The study has brought many important features of the refueling strategy for AHWR LEU core. The present study has been able to develop confidence that the on-power refueling in AHWR-LEU can be reality.

For future work, refueling strategy can be planned on the similar lines for Indian PHWRs for facilitating the use of high fissile content U based fuel or Th based fuel for lower waste generation and higher burnup. As PHWR uses small length bundles and is a forced circulation system, better margins are expected as compared to AHWR for controlling the power peaking.

The studies described (in chapter 3 and 4) have shown that the efficient fuel utilization could be possible in pre-equilibrium (transition phase) and equilibrium phase of reactor operation in reactors like AHWR by following specific refueling scheme. However, the fuel utilization during initial phase of core is also important and needs to be addressed properly for a complete fuel cycle study for AHWR.

CHAPTER 5

Loading Pattern Optimization Using Estimation of Distribution Algorithm (EDA)

5.1 Introduction

The unique features of AHWR (on power refueling and high discharge burn-up fuel) necessitates the requirement of a special refueling scheme for an efficient fuel utilization in transition and equilibrium phase. In the last two chapters (chapter 3 and 4) in-core fuel management in AHWR-LEU has been demonstrated through the development of specialized refueling scheme. However, fuel management during initial phase is also a challenging problem and needs to be addressed. The average U-235 content in equilibrium core cluster of AHWR-LEU is about 4.29 %. Loading of equilibrium core cluster in initial core of AHWR-LEU will impose the requirement of large quantity of poison (Boron) in the moderator to suppress initial excess reactivity. The large quantity of Boron dissolved in moderator will adversely affect the worth of control and shut down system in addition to its adverse effect on various reactivity feedbacks. Therefore, the initial core clusters for AHWR are being designed to have lesser excess reactivity. For flux flattening in initial core of AHWR-LEU, two fuel clusters with differential reactivity have been considered. The cluster with more reactivity is named as Type-1 and with lesser reactivity is called Type-2. The AHWR-LEU core consists of 444 fuel lattice locations. By exploiting symmetry of the core, the problem size for AHWR-LEU initial core optimization is 2^{62} ($\sim 10^{18}$). Simulation of all these loading patterns is not practical in a finite time scale to choose the best loading pattern.

In this chapter, the complex combinatorial optimization problem of optimization of initial core loading pattern (LP) has been solved using one modern population based algorithm named as estimation of distribution algorithm (EDA). This optimization problem has similarities with PHWR initial core LPO (Balakrishnan and Kakodkar (1994), Mishra (2009)) and with various loading pattern optimization problems in LWR fuel cycle. As discussed in chapter 1 & chapter 3 (section 3.1), population based algorithms are more effective and are frequently being used for fuel loading pattern optimization (LPO) problem. Genetic algorithm (GA) (Goldberg, 1989; Parks, 1996; Chapot et al., 1999), Simulated Annealing (SA) (Stevens et al., 1995) and Ant Colony Algorithm (ACO) (Machado and Schirru, 2002) are few examples of population based evolutionary algorithms which have been successfully applied for core reloading optimization problems of Light water reactors (LWRs). Estimation of Distribution Algorithm (EDA) (Jiang et. al. 2006) has been applied successfully to CONSORT research reactor where five different types of fuels are to be loaded in 24 locations with the objective of maximization of k-effective. The typical size in this problem is $\sim 10^{12}$. Jiang et al. has considered a population size of 50 in each generation along with a very small value of weighing factor *'α'* (0.001). Jiang et. al. carried out more than 2000 generations and it is only possible because the objective function considered is maximization of k-effective only. And the prediction of k-effective is done by Artificial Neural Network (ANN) (Jiang et. al. 2006). By using ANN, a large number of loading patterns can be analysed for k-effective values in very short time. Mishra et. al., 2009 has successfully applied EDA to initial core loading optimization of pressurized heavy water reactors (PHWRs). The problem size is ~10^{65}. They have tried to find the optimized loading pattern with fixed number of Th or depleted U fuel bundles. They have defined objective function based on penalty method (Michalewicz, 1999) and have also used EDA for optimization of initial core of PHWR. Due to the complexity of objective function, full 3D diffusion calculations are required which necessitate the use of parallel processing. Mishra et. al, 2009 have used earlier experience (Balakrishnan and Kakodkar, 1994) for fixing the number of Thorium bundles. It is also observed (Mishra et. al., 2009) that the optimized loading pattern with lower population is better than the optimized loading pattern with higher population size in some cases. In their analysis, they have considered same value of weighing factor 'α' (0.05) for all the population sizes.

In the optimization analysis presented in this chapter, we have applied EDA to optimize initial core of AHWR-LEU and it is observed that the optimization results are very sensitive to weighing factor 'α' and population size of each generation. It is also observed that while choosing a very small value of weighing factor 'α' with large population size, algorithm may lead to unnecessary computations and may not always lead to a good optimized loading pattern.

The main focus in this chapter is to optimize the initial core of AHWR-LEU using Estimation of Distribution Algorithm (EDA). During this study, it was observed that EDA itself is very sensitive to the various internal parameters used for updating the probability distribution function after each generation. An extensive study was done to determine adequate parameters used in EDA for better optimized loading pattern. We have studied the effect of variation of weighing factor 'α', population size in each generation and initial distribution function on the final optimized loading pattern.

In this chapter, we have tried to establish that considering a very small value of weighing factor 'α' for any population size during each generation results in search around local area only. Therefore the good solutions which are far away from initial distribution function may not get explored. Therefore, with increase in population size, value of weighing factor 'α' should also be increased for better optimization. The plan of this chapter is as follows

Section 5.2 gives the details of the initial core optimization problem for AHWR-LEU. Section 5.3 gives a brief description of earlier attempts for initial core LPO of AHWR. Section 5.4 describes the objective function considered. Section 5.5 gives the description of EDA and the various parameters used in this algorithm. Section 5.6 gives the numerical results for variation of weighing factor 'α', population size and initial probability distribution function. Conclusions are discussed in Section 5.7.

5.2 AHWR-LEU initial core loading pattern optimization problem

As described in chapter 1, the AHWR-LEU core consists of 513 lattice locations. Out of 513, there are 444 fuel lattice locations and 69 locations are occupied by various control and shut down devices comprising of 45 SORs, 8 RRs, 8 ARs, and 8 SRs. There are two types of fuel clusters being considered for initial core of AHWR-LEU for flux flattening. Table-3.1 (chapter 3) presents the core configuration of the AHWR-LEU core and Table-5.1 gives the description of initial core clusters considered for present study. Neutron Transport Theory computer code ITRAN (Krishnani, 1981; Krishnani, 1982a (pp. 255–260); Krishnani, 1982b (pp. 287–296)) has been used to perform the lattice calculations for the AHWR-LEU initial core clusters. The calculations were performed by using 69-group library based on ENDF/B-VI.8 nuclear data obtained from IAEA (IAEA, 2016). Two energy group cross sections for both the clusters were generated using ITRAN. Diffusion theory code FEMINA (Kumar and Srivenkatesan, 1984) based on nodal expansion method has been used to carry out the core level calculations with two group crosssections generated using ITRAN.

Table-5.1 Description of AHWR-LEU cluster studied for initial core optimisation

<table>
<tr><th rowspan="2">Cluster type</th><th rowspan="2">Fuel Type</th><th rowspan="2">Gd content</th><th colspan="4">LEU content (%) in various rings of cluster</th><th rowspan="2">U-235 content (%)</th><th rowspan="2">K_∞ (0 MWd/TeXe sat.)</th></tr>
<tr><th>Inner</th><th>Middle</th><th>Outer</th><th>Cluster Average</th></tr>
<tr><td>Type-1</td><td rowspan="2">(LEU-Th) MOX</td><td>No Gd</td><td>13</td><td>13</td><td>13</td><td>13</td><td>2.56</td><td>1.084</td></tr>
<tr><td>Type-2</td><td>7% Gd in 2 pins of inner ring</td><td>13</td><td>13</td><td>13</td><td>13</td><td>2.56</td><td>0.9813</td></tr>
</table>

There are 2^{444} ($\sim 10^{133}$) different possible ways with which the two types of fuels can be loaded in the core. The quarter core symmetry reduces the number of combinations to 2^{111}($\sim 10^{33}$). The problem size can be further reduced to 2^{62} ($\sim 10^{18}$) by exploiting 1/8th mirror symmetry of few channels as shown in Fig 5.1. The main objectives for initial core optimization have been defined as:

1. K-effective is maximized.
2. Worth of Shut down system should be always greater than design requirement (63mk).
3. Maximum channel power (MCP) should be below the design limit of 2.6MWth.
4. Maximum mesh power (MMP) should be below the design limit of 200kWth.

It is a complex combinatorial optimization problem.

5.3 First attempt to optimize initial core LP of AHWR-LEU

The AHWR initial core LPO was first dealt with using a two zone distribution approach (Thakur et. al. 2008). In this approach, the core is divided into two zones namely inner zone and outer zone. For flat flux distribution, inner zone is loaded with low reactive clusters and outer zone is loaded with high reactive clusters. The optimal number of channels for each type of fuels was estimated by optimizing the size of inner and outer zone such that the MCP and MMP achieve its design limit. However, in the optimized LP, it was observed that the inner zone has power dip as it consists of mainly lower reactive fuel clusters. Further, no constraint was applied for SDS worth therefore the required shut down margin was also not available for the optimized LP.

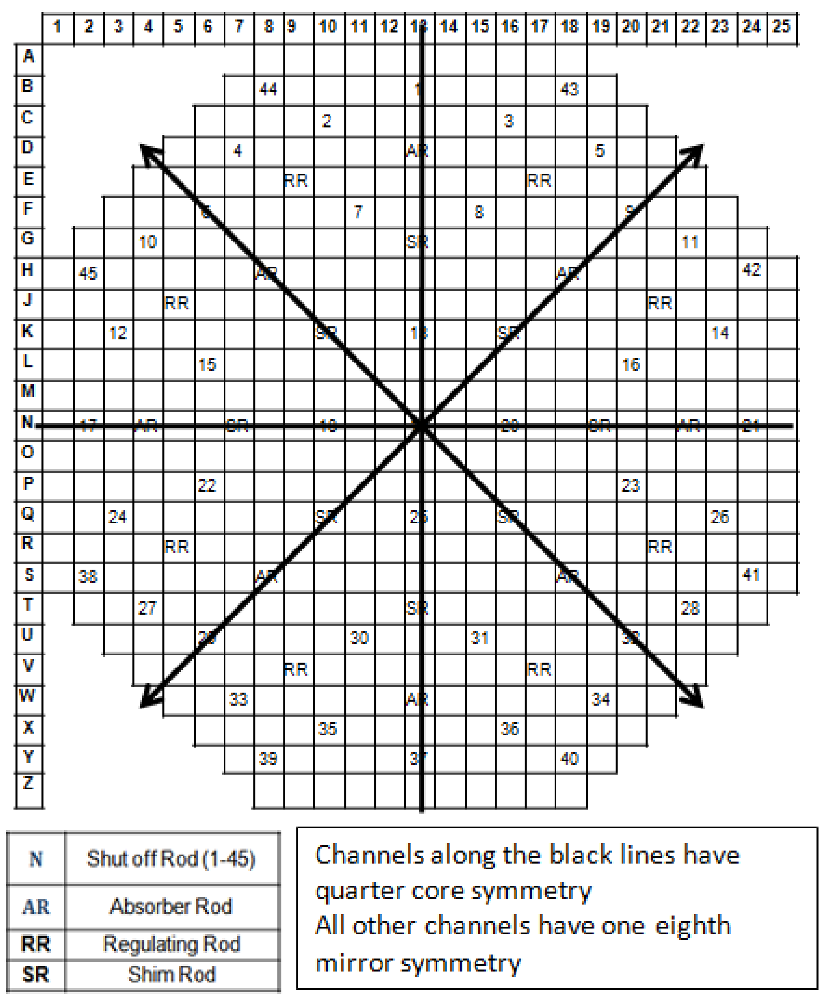


Fig 5.1 Symmetry of AHWR-LEU core

As a next step, the implementation of conventional methods like Gauss Newton method (Balakrishnan and Kakodkar, 1994) to solve AHWR initial core LPO problem was tried. It was observed that use of Gauss Newton method is very difficult in two respects

(i) This method requires a very good initial guess LP to start with and near to which the optimized solution is searched.

(ii) The variable chosen for this method (like average distance of some type of clusters to core boundary or core center) needs to be mapped back to position-wise core configuration to form the updated loading pattern at each step of optimization process.

This method was applied for AHWR LPO problem in a similar way as described in chapter 1 (Balakrishnan and Kakodkar, 1994) and the optimized loading pattern generated from two zone studies as described in starting of this section was considered as a initial guess solution. However, it was very difficult to update the variables by mapping back to position-wise core configuration and we could not succeed in achieving a better optimized loading pattern. Therefore, it was decided to use more efficient population based modern methods (evolutionary algorithms) for solving initial core LPO problem.

5.4 Objective function defined for evolutionary algorithms

As described in chapter 1, Evolutionary Algorithms (EA) uses a black box approach where the knowledge of how the objectives are related to the control variables is not used, but an evaluation of the candidate solutions is done at each iteration / generation and different learning strategies are used to structure information in order to find near-optimal solutions.

For applying EA to initial core LPO of AHWR, an objective function is defined and is maximized. For AHWR initial core, penalty method has been used to define the objective function in a similar way to Mishra et al., 2009.

The objective function (OF) for this problem is defined as

$$OF = (A_1 \cdot k\text{-}eff) - A_2(MCP - 2.6) - A_3(MMP - 200) - A_4(63.0 - worth\ of\ SDS\text{-}1[43 rods]) \quad (5.1)$$

where A_1, A_2, A_3 and A_4 are constants. The description about choice of these constants has been given in appendix-I. In this optimization problem, the value of A_1 is taken as 1. For optimization studies *SDS-1* (43 rods, two maximum worth rods failure criteria) worth requirements of *63.0 mk* have been considered.

Further, if MCP of a LP is < 2.6, then A_2 =0, else A_2=0.384.

Similarly, if MMP< 200, then A_3=0, else A_3=0.05.

And worth of SDS-1> 63.0, then A_4=0, else A_4=0.333.

Hence, we are considering only the penalty due to design parameters not meeting their design limit, but we do not prefer any LP which is having very high margins in peaking or SDS-1 worth. The criteria are to maximize the K-effective without compromising full power operation and safety of reactor.

The AHWR initial core is expected to require small amount of neutron poison (~25 ppm of Boron) in moderator to compensate initial excess reactivity. Therefore, all the simulations have considered with 25 ppm of B in moderator.

5.5 Estimation of Distribution Algorithm (EDA)

We have used estimation of distribution algorithm (EDA) to optimize initial core LP of AHWR. EDA is a population based evolutionary algorithm where the optimized solution is achieved by sampling the probability distribution model which is generated based on current best solutions. We start with an initial probability distribution function and generate probable pool of candidate solutions based on this probability distribution function. After evaluation of the objective function value for all the candidates in the present sample, the probability distribution function is modified by giving some weight to current best solutions. Univariate marginal distribution algorithm (UMDA) (Muhlenbein and Paab, 1996;Jiang et al., 2006) has been used to estimate the probability distribution in present study. Jiang et. al., 2006 has described three variants of EDA named as EDA-S, EDA-G and EDA-H. All the three variants have similar description as given below:-

Step-1: Generate a population (N) of different loading patterns based on initial distribution function. This population consists of solution candidates for fitness estimation. In each generation, same population size (N) is maintained and is evaluated. However, the candidates in the pool are modified from the feedback given by previous generation.

Step-2: Simulate all the candidates of this population by solving diffusion equation using FEMINA and objective function for all the candidates are evaluated.

Step-3: Select best $M < N$ candidates based on objective function values

In present study, M is considered as 25% of N.

Step-4: The distribution function (DF) for generating loading pattern is modified

For EDA-S

$$\mathrm{DF}\ (t+1) = \mathrm{DF}\ (t) \cdot (1\text{-}\alpha) + \alpha \cdot \mathrm{X} \tag{5.2}$$

where, 'α' (weighing factor) is a constant and its value is between 0 and 1. *DF(t)* and *X* are having same structure. *'t'* represents the generation.

$$\text{Where} \quad \mathrm{X} = \frac{1}{\mathrm{M}} \cdot \sum_{i=1}^{\mathrm{M}} \mathrm{X_i}(t) \tag{5.3}$$

$X_i(t)$ is 0 or 1 based on the loading of a particular fuel in fuel lattice location.

This is the most basic form of EDA. However the algorithm can be made more efficient by using heuristic information to update the DF. The EDA-H use of heuristic information also

For EDA-H

$$\mathrm{DF}\ (t+1) = \mathrm{DF}\ (t) \cdot (1\text{-}\alpha) + \alpha \cdot \mathrm{X} \tag{5.4}$$

$$\mathrm{DF}(t+1)=\mathrm{DF}(t+1) \cdot \mathrm{H}^{\beta} \tag{5.5}$$

Where, 'H' contains the heuristic information and 'β' is a scalar adjusting the weight between population based learning and heuristic information.

For EDA-G

Similarly, another way of enhancing the performance is to use elitism strategy. That is at each iteration, current best solution is kept and due weight is given to it for updating DF

$$\mathrm{DF}\ (t+1) = \mathrm{DF}\ (t) \cdot (1\text{-}\alpha) + \alpha \cdot \mathrm{X} + \eta \cdot \mathrm{X_b} \tag{5.6}$$

Step-5: Again generate the population of different loading patterns based on new distribution function

Step-6: Go to step-2 and repeat the cycle till optimization is achieved

We have used most basic form of EDA named as EDA-S given by equation 5.1. In our present study, two initial distribution functions have been considered. As discussed earlier, the Type-2 fuel is lesser reactive than Type-1 fuel. It is evident that the optimized core will have more number of Type-1 fuel clusters than Type-2 fuel clusters. Hence, both the initial distribution functions should generate loading pattern with more number of Type-1 fuel cluster and less number of Type-2 fuel clusters. Following is the details of initial distribution function considered:-

I. The distribution function is such that the loading pattern generated by it should have ~60 % of Type-1 fuel cluster and ~40% of Type-2 fuel clusters
II. The distribution function is such that the loading pattern generated by it should have ~80 % of Type-1 fuel cluster and ~20% of Type-2 fuel clusters

The loading patterns are generated using random numbers by following the criteria that for a channel if random number is less than 0.6 or 0.8 then Type-1 will be loaded else Type-2 will be loaded. Both the cases have been analysed for three different population sizes (24, 240 and 1200) and three different values of 'α' (0.05, 0.1 and 0.5). The distributed memory parallel computer system AGGRA at BARC was used for parallelization and computation of N (24, 240 or 1200) independent objective functions by running FEMINA.

5.6 Results of EDA on AHWR-LEU initial core loading pattern optimization

The study of varying initial distribution function has been divided in two parts as described above.

Case-I: Initial distribution function is 0.6 for Type-1 and 0.4 for Type-2

In this case the loading patterns are generated by generating random number for each fuel channel and setting the criteria that if the random number generated is less than 0.6, Type-1 cluster will be loaded else Type-2 cluster will be loaded. In this way, the Type-1 clusters will be ~60% and Type-2 clusters will be ~40%. Now three different values of population sizes (24, 240 and 1200) have been considered.

Case-I(a): **Population size of 24 in each generation**

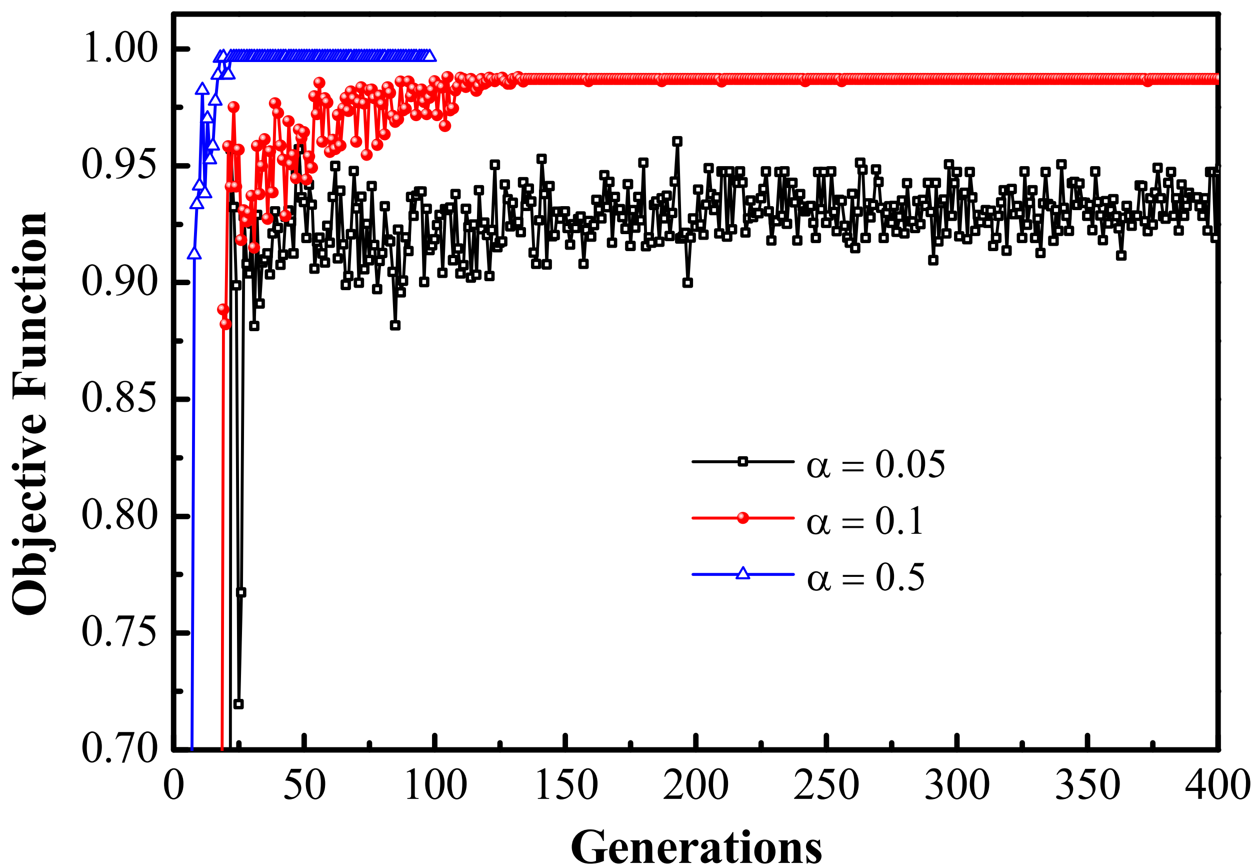


Fig.5.2 (Case-Ia) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 24 in each generation for initial distribution of 0.6 for Type-1 and 0.4 for Type-2.

The three values of α considered are 0.05, 0.1 and 0.5. The first case of population size of 24 and α=0.05 belongs to very small population size and very small 'α'. The best value of objective function in 400 generations is 0.9870. After increasing the value of α *to* 0.1, the best value of objective function in 400 generations improved to 0.98790. By further increasing the value of $\boldsymbol{\alpha}$ to 0.5, the best value of objective function in 150 generations improves marginally to 0.9965.

Case-I(b):- **Population size of 240 in each generation**

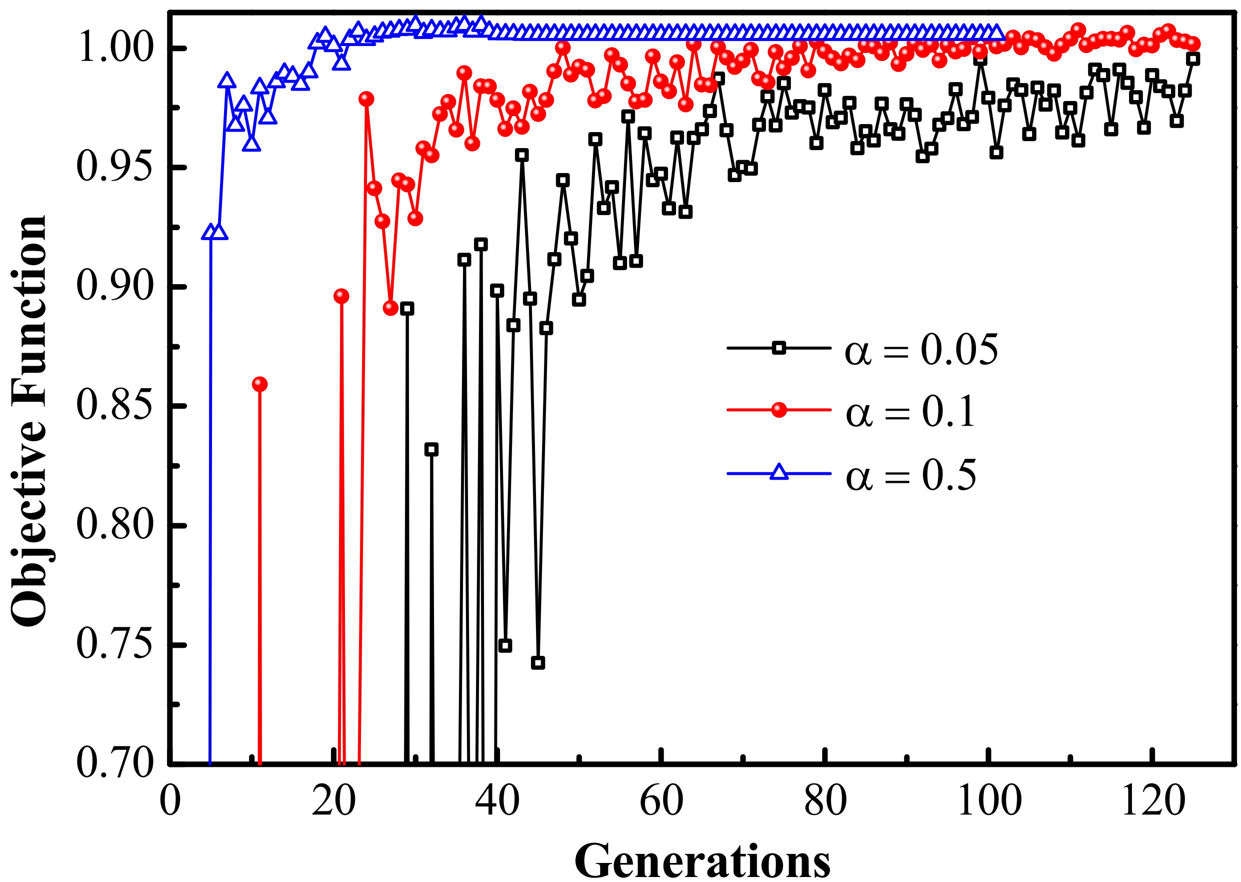


Fig.5.3 (Case-Ib) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 240 in each generation for initial distribution of 0.6 for Type-1 and 0.4 for Type-2

The variation of best value of objective function with generation number for different values of α, is shown in Fig 5.2. It is observed that there is very small or negligible improvement in optimized value of objective function by increasing α from 0.05 to 0.5.

In the second step, we tried the simulations considering higher population size of 240 in each generation. The three values of $\boldsymbol{\alpha}$ considered are 0.05, 0.1 and 0.5. The first case of population size of 240 and α=0.05 belongs to moderate population size and very small 'α'. The best value of objective function in ~130 generations is 0.9954. After increasing the value of α to 0.1, the best value of objective function in 130 generations improved to 1.0074. By further increasing the value of α to 0.5, the best value of objective function in 150 generations improves marginally to 1.009530. The variation of best value of objective function with generation number for different values of α is shown in Fig 5.3. On comparison with case-I(a), it is observed that there is significant improvement in optimized value of objective function. It is also observed that for very small value of α (0.05), the optimized value of objective function is poor (0.9954) for both cases (case 1a & 1b).

Case-I(c):- **Population size of 1200 in each generation**

By further increasing the population size to 1200 in each generation and considering three values of α as 0.05, 0.1 and 0.5, following observations were made. The first case of population size of 1200 and α=0.05 belongs to large population size and very small 'α'. The best value of objective function in ~130 generations is 0.9955.

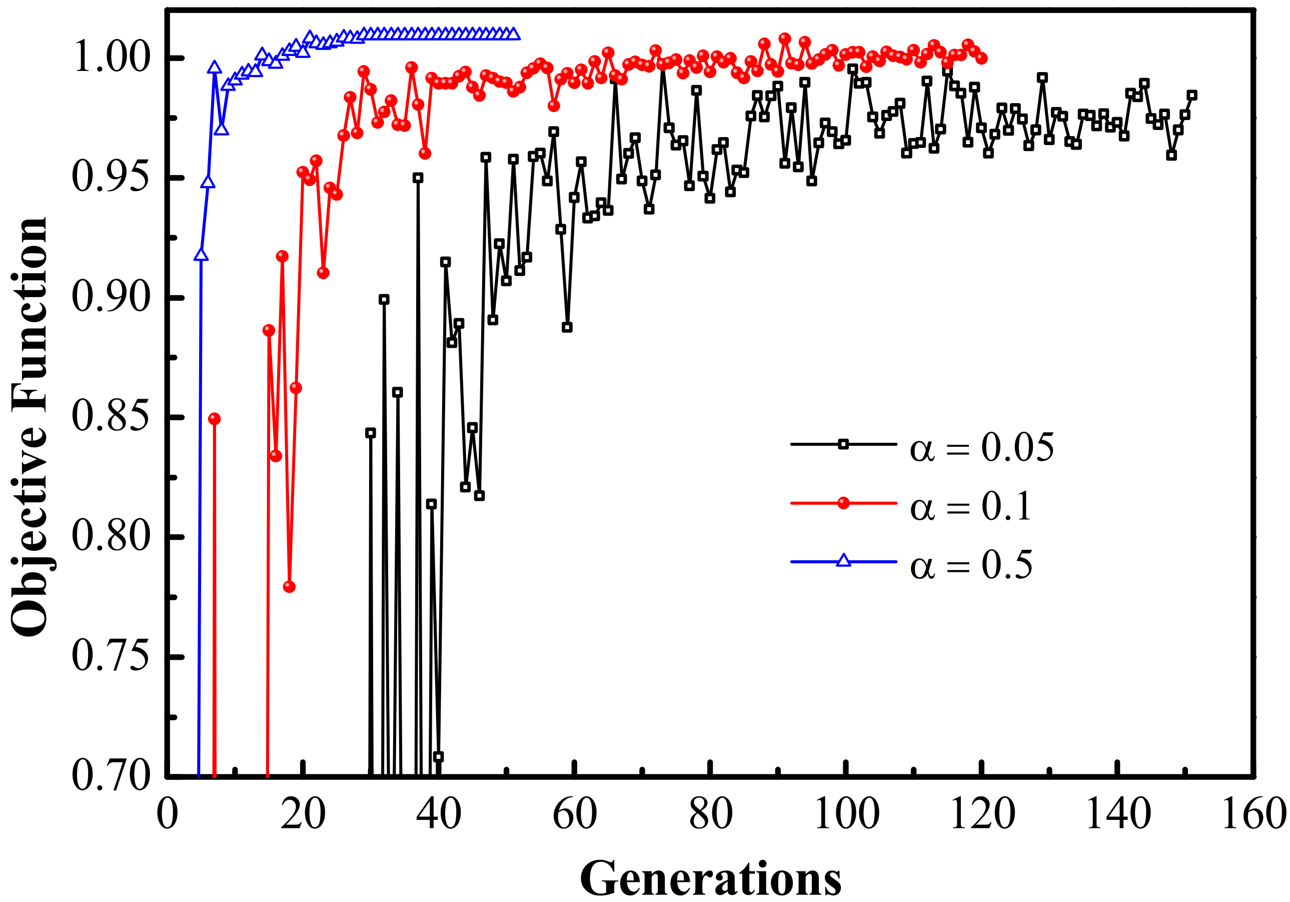


Fig.5.4 (Case-Ic) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 1200 considered in each generation for initial distribution of 0.6 for Type-1 and 0.4 for Type-2.

This value is comparable to case-I(a) and I(b) for α =0.05. It means the algorithm still fails for α=0.05. After increasing the value of α to 0.1, the best value of objective function in 130 generations improved to 1.0079. By further increasing the value of α to 0.5, the best value of objective function in 150 generations improves marginally to 1.009530. The variation of best value of objective function with generation number for different values of α is shown in Fig 5.4.

It is evident from the above study that the population size of 24 is too small and the best optimized value of objective function in 400 generations is 0.9976 for α=0.05. After increasing the population size from 24 to 240 or 1200 for same value of α=0.05, there is very slight or negligible improvement in optimized value of objective function to 0.9936 and 0.9956 respectively. However, after increasing the value of α to 0.1, value of objective function improves to 1.0074 in 130 generations for population size of 240 and 1.0079 in 125 generations for population size of 1200. By further increasing the value of α to 0.5, value of objective function improves to 1.009530 for both the population sizes of 240 and 1200.

Table-5.2 Properties of loading pattern optimised using EDA for variation of population size and α for initial distribution function of 0.6 for Type-1 and 0.4 for Type-2

α	**Population size (Generations)**	***Initial distribution function of 0.6 for Type-1 and 0.4 for Type-2***						
		Max. Objective function	**K-eff**	**MCP (MW)**	**MMP (kW)**	**Worth of SDS-1 (mk)**	**Type-1**	**Type-2**
0.05	**24 (400)**	0.98704	0.98794	2.68	159	63.3	260	184
	240 (125)	0.99536	0.99536	2.60	153	63.6	296	148
	1200 (155)	0.99551	0.99551	2.59	153	63.3	300	144
0.1	**24 (400)**	0.98790	0.98790	2.60	153	63.3	260	184
	240 (125)	1.00740	1.00740	2.60	155	63.8	344	100
	1200 (125)	1.00790	1.00790	2.59	155	63.9	348	96
0.5	**24 (100)**	0.99650	0.99650	2.60	157	64.5	296	148
	240 (100)	1.00953	1.00953	2.58	154	63.1	356	88
	1200 (60)	1.00953	1.00953	2.58	154	63.1	356	88

Hence, considering very small value of α (0.05) is not correct for AHWR-LEU initial core loading optimization problem. By considering small value of α, the optimized solution in near vicinity of initial guess distribution function is observed. Table-5.2 shows the results of case-I.

Case-II: Initial distribution function is 0.8 for Type-1 and 0.2 for Type-2

From our earlier experience (Thakur, et. al., 2010), we have observed that the optimized initial core of AHWR is having ~20% Type-2 clusters and 80% Type-1 clusters. Hence a case is considered where the initial distribution is 80% and 20% for Type-1 and Type-2 respectively. In this case the loading patterns are generated by generating random number for each fuel channel and setting the criteria that if the random number generated is less than 0.8, Type-1 cluster will be loaded else Type-2 cluster will be loaded. In this way Type-1 clusters will be ~80% and Type-2 clusters will be ~20%. Now three different values of population sizes have been considered similar to case-I.

Case-II(a):- **Population size of 24 in each generation**

The three values of α considered are 0.05, 0.1 and 0.5. The first case of population size of 24 and α=0.05 belongs to very small population size and very small 'α'. The best value of objective function in 400 generations is 1.0040. This is better value than 0.98704 observed with same α and population size but with different initial distribution function (Case-Ia). After increasing the value of α to 0.1, the best value of objective function in 400 generations improved to 1.0044. This value is again better value than 0.99536 observed with same α and population size but with different initial distribution function (Case-Ia). It shows that EDA gives significantly better optimization for small value of α and small population size (24), if the initial distribution function is closer to the optimized solution.

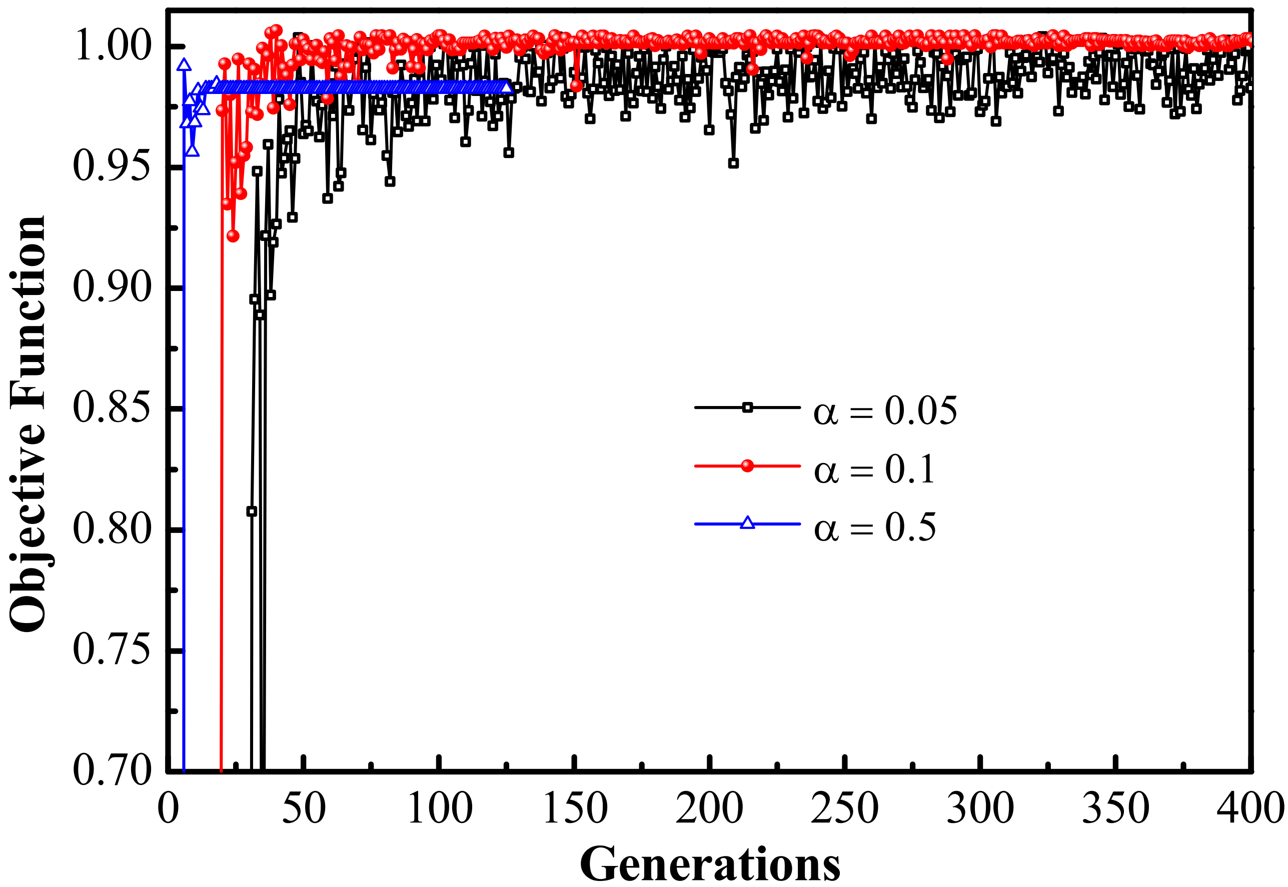


Fig.5.5 (Case-IIa) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 24 in each generation for initial distribution of 0.8 for Type-1 and 0.2 for Type-2

By further increasing the value of *α* to 0.5, the best value of objective function in 130 generations reduces drastically to 0.9825. This is worse value than 0.9965 observed with same *α* and population size but with different initial distribution function (Case-Ia). It is showing that the algorithm fails for population size of 24 and *α*=0.5**.** This is because the population size is very small. The variation of best value of objective function with generation number for different values of *α* is shown in Fig 5.5.

Case-II(b):- **Population size of 240 in each generation**

The three values of α considered are 0.05, 0.1 and 0.5. The first case of population size of 240 and α=0.05 belongs to moderate population size and very small 'α'. The best value of objective function in ~250 generations is 1.0072. This again is a better optimised value than case-I(b) where optimized value is only 0.9954.

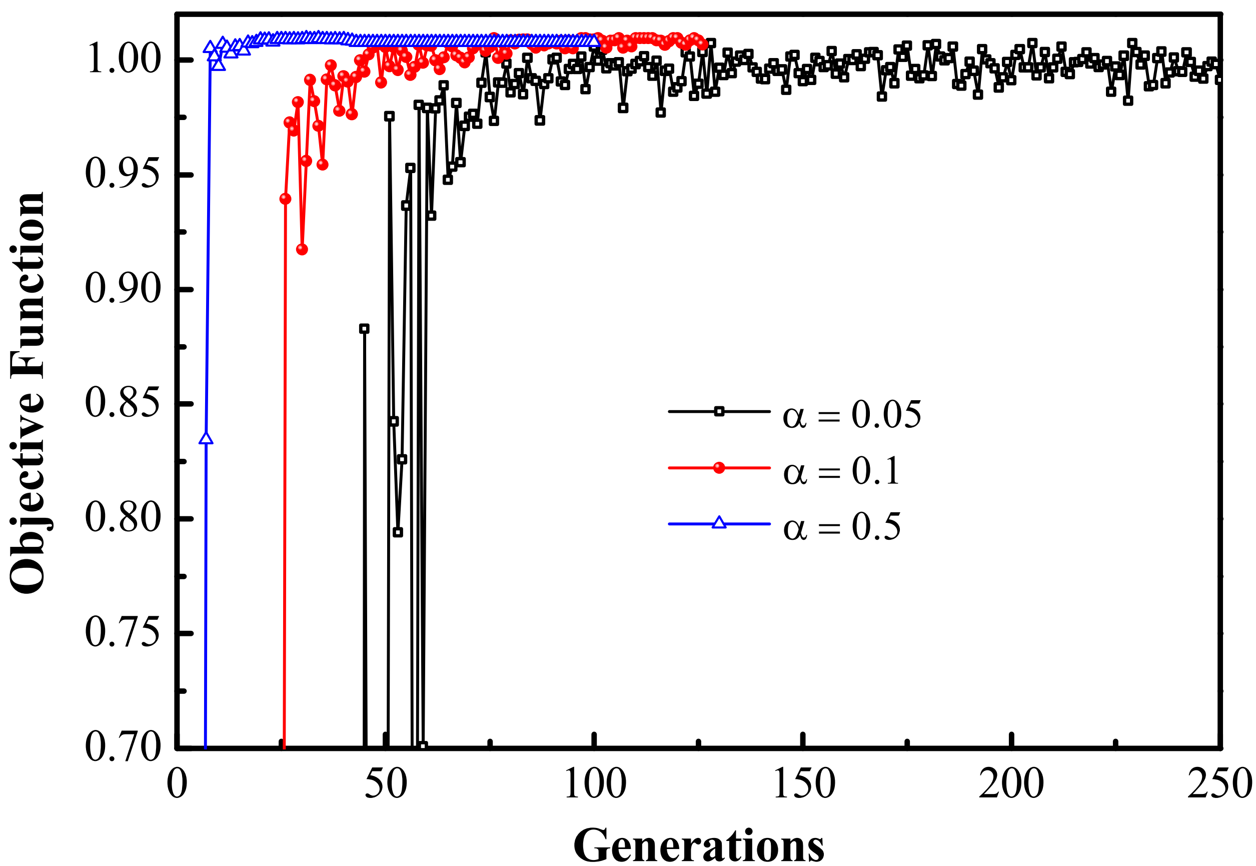


Fig.5.6 (Case-IIb) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 240 in each generation for initial distribution of 0.8 for Type-1 and 0.2 for Type-2.

After increasing the value of α to 0.1, the best value of objective function in 130 generations improved to 1.00953. By further increasing the value of α to 0.5, the best value of objective function in 130 generations remains same as 1.00953. The variation of best value of objective function with generation number for different values of α is shown in Fig 5.6.

Case-II(c):- Population size of 1200 in each generation

The three values of $\boldsymbol{\alpha}$ considered are 0.05, 0.1 and 0.5. The first case of population size of 1200 and α=0.05 belongs to large population size and very small 'α'.

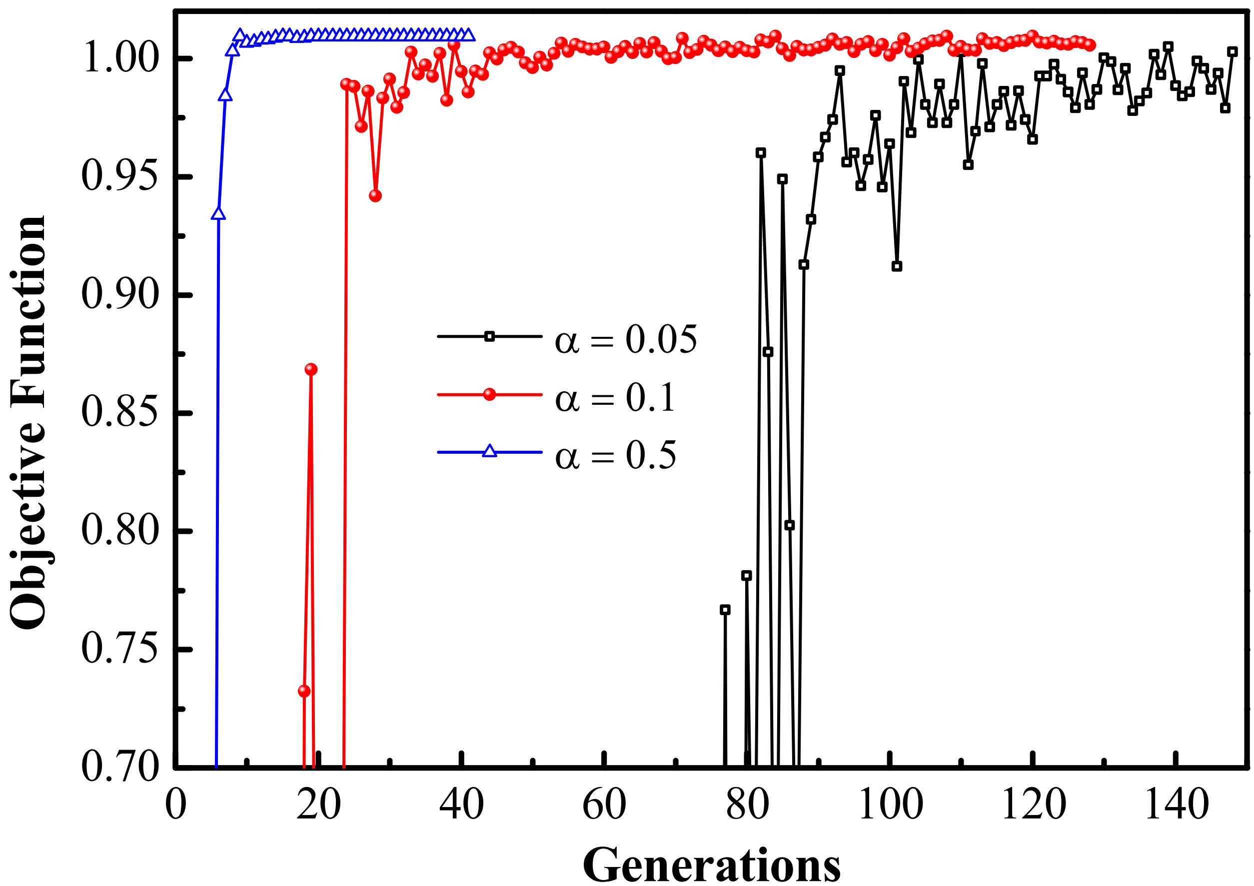


Fig.5.7 (Case-IIc) Variation of objective function with generations for three values of α (0.05, 0.1 and 0.5) for population size of 1200 in each generation for initial distribution of 0.8 for Type-1 and 0.2 for Type-2

The best value of objective function in ~130 generations is 1.0050. After increasing the value of α to 0.1, the best value of objective function in 130 generations improved to 1.00953. By further increasing the value of α to 0.5, the best value of objective function in 130 generations remains same as 1.009530. The variation of best value of objective function with generation number for different values of α, is shown in Fig 5.7. Table-5.3 describes the results of case-II.

Table-5.3 Properties of loading pattern optimised using EDA for variation of population size and α for initial distribution function of 0.8 for Type-1 and 0.2 for Type-2

α	Population size (Generations)	Initial distribution function of 0.8 for Type-1 and 0.2 for Type-2						
		Max. Objective function	K-eff	MCP (MW)	MMP (KW)	SDS-1 Worth (mk)	Type-1	Type-2
0.05	24 (400)	1.0040	1.0040	2.60	154	65.1	328	116
	240 (250)	1.0072	1.0072	2.59	154	63.1	344	100
	1200 (145)	1.0050	1.0050	2.60	156	63.4	336	108
0.1	24 (400)	1.0044	1.0044	2.60	155	64.5	332	112
	240 (125)	1.00953	1.00953	2.58	154	63.1	356	88
	1200 (125)	1.00953	1.00953	2.58	154	63.1	356	88
0.5	24 (100)	0.98259	1.0056	2.66	159	64.0	340	104
	240 (100)	1.00948	1.00948	2.59	155	63.2	356	88
	1200 (50)	1.00953	1.00953	2.58	154	63.1	356	88

From comparison of Table-5.2 and Table-5.3, it is clear that for all the cases studied, the optimized value of objective function is better for case-II than respective case-I. Now, case-I and case-II differ in respect of initial distribution function only. It is observed that Case-I the algorithm completely failed for population size of 24. However, we observed much better optimization in case-II for same population size of 24. But in both the cases the best optimized value is not observed in this population size (24). Hence, it can be concluded that the population size of 24 is too small and is not correct for use in EDA on initial core LPO analysis of AHWR-LEU.

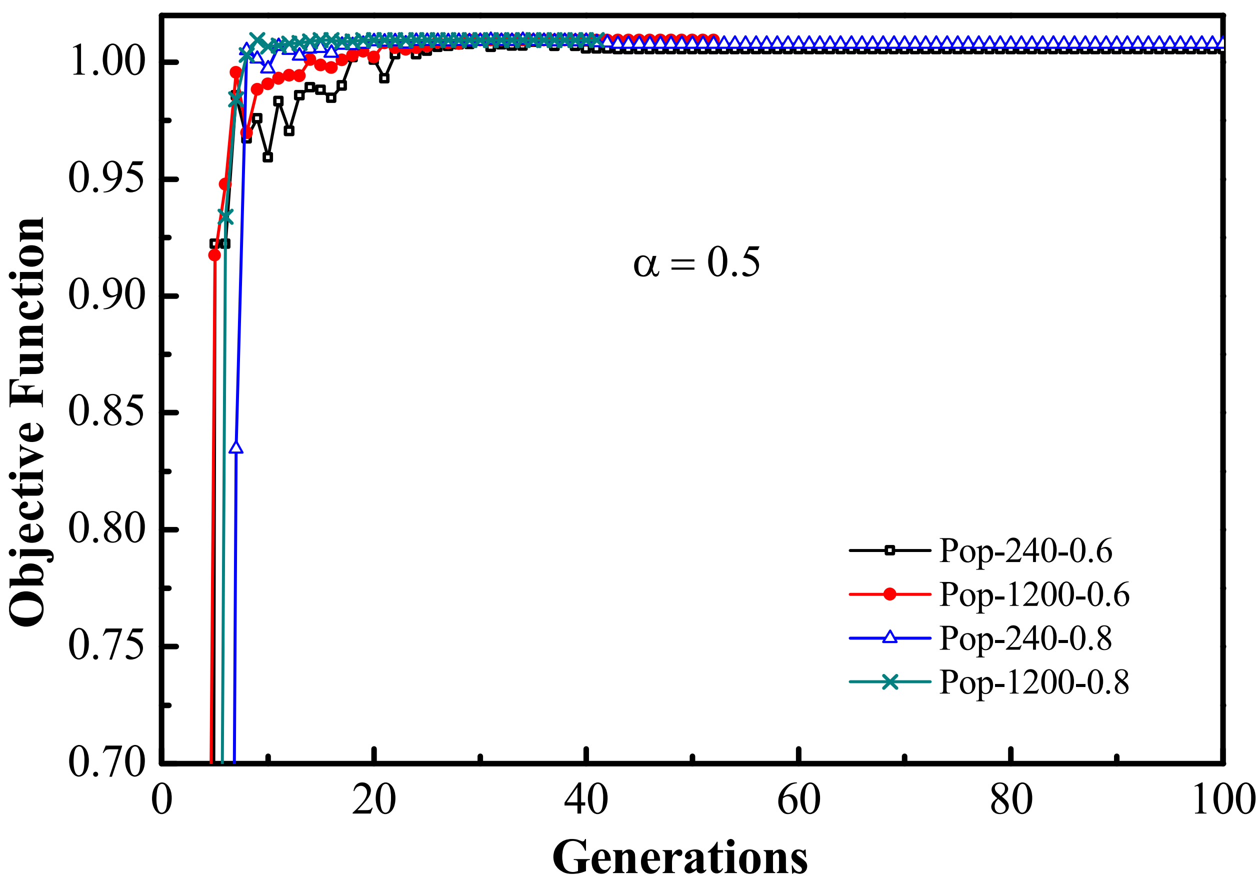


Fig.5.8 Variation of objective function with generations for two population sizes viz. 240 and 1200 for α=0.5 in EDA and initial distribution of 0.6 for Type-1 and 0.4 for Type-2 as well as for initial distribution of 0.8 for Type-1 and 0.2 for Type-2.

Similarly, the small value of α=0.05 has not resulted in best value of objective function for any case. Therefore, it can also be concluded that the value of α should be greater than *0.05*. For population size of 240, best value of objective function is observed for α=0.1 and 0.5, however, the results are dependent on initial distribution function. For the case where α=0.5 and population size is 1200, same value (1.00953) of objective function is observed and there is no dependence on initial distribution function.

The loading pattern corresponding to this value has 88 Type-2 clusters and 356 Type-1 clusters. The same optimized value of 1.009530 is observed for α=0.1 and population size of 1200 and initial distribution function of 0.8 and 0.2 for Type-1and Type-2 clusters. It shows that if our initial distribution function is near to optimized loading pattern, α=0.1 is also adequate and minimum population size is 240. However, in no case, α=0.05 has given an adequate optimization. Fig 5.8 shows the variation of objective function with generations for two population sizes viz. 240 and 1200 for α=0.5 initial distribution of 0.6 for Type-1 and 0.4 for Type-2 as well as for initial distribution of 0.8 for Type-1 and 0.2 for Type-2. It is observed that almost similar optimization is achieved in all the cases. However, the convergence is faster for the case where initial distribution of 0.8 for Type-1 and 0.2 for Type-2 is considered. The behavior of average value of DF will give the information about how many Type-1 and Type-2 clusters will be there in next generation. In Fig 5.9, 5.10 and 5.11 the average value of DF has been plotted for all the cases studied. In Fig 5.9a, 5.10a and 5.11a, the starting point is 0.6. This is because initially we have uniformly filled 0.6 in all the 62 elements of DF. Now with each generation, individual values of 62 element of DF will be updated. This will result in change in the average value of DF. Fig 5.9b, 5.10b and 5.11b represents the cases with initial DF of 0.8 for Type-1 and 0.2 for Type-2 therefore the starting value is 0.8. From Fig 5.9a and 5.9b, it is clear that the DF is not showing any improvement after ~75 generations for α=0.05 and 0.1. It shows that the algorithm has failed. For α=0.5, it has achieved a constant value after ~30 iterations and there is no further improvement in DF. It represents that few elements of DF have approached the value 1 and others have reached to 0. Since the finalized solution does not show good parameters of an optimization, it implies that the algorithm has struck in to local minima. From Fig 5.10a and 5.10b, it is observed that for α=0.05 the algorithm has failed similar to the case with population size of 24. For α=0.1 and α=0.5, the average value of DF has progressed towards 0.8 value. For α=0.5, DF has achieved a constant value. The optimized LP achieved shows that the MCP, MMP and SDS worth requirements are meeting their corresponding design limits. Therefore the optimized LP is a good solution to problem. Similarly for α=0.1, the best optimized LP is also a good solution. It is observed that the optimized solutions achieved by population size of 240 and α=0.1 or 0.5 are better. From fig 5.11a and 5.11b, it is observed that for α=0.05 the average value of DF always remains near initial DF. This shows that only area near to initial DF is explored. For case with α=0.1, the improvement in DF is very-2 slow. However, for case with α=0.5, the average value of DF approaches 0.8 even when initial DF is 0.6. A very good LP is achieved by having population size of 1200 and moderate value of α=0.5 and is shown in Fig 5.12.

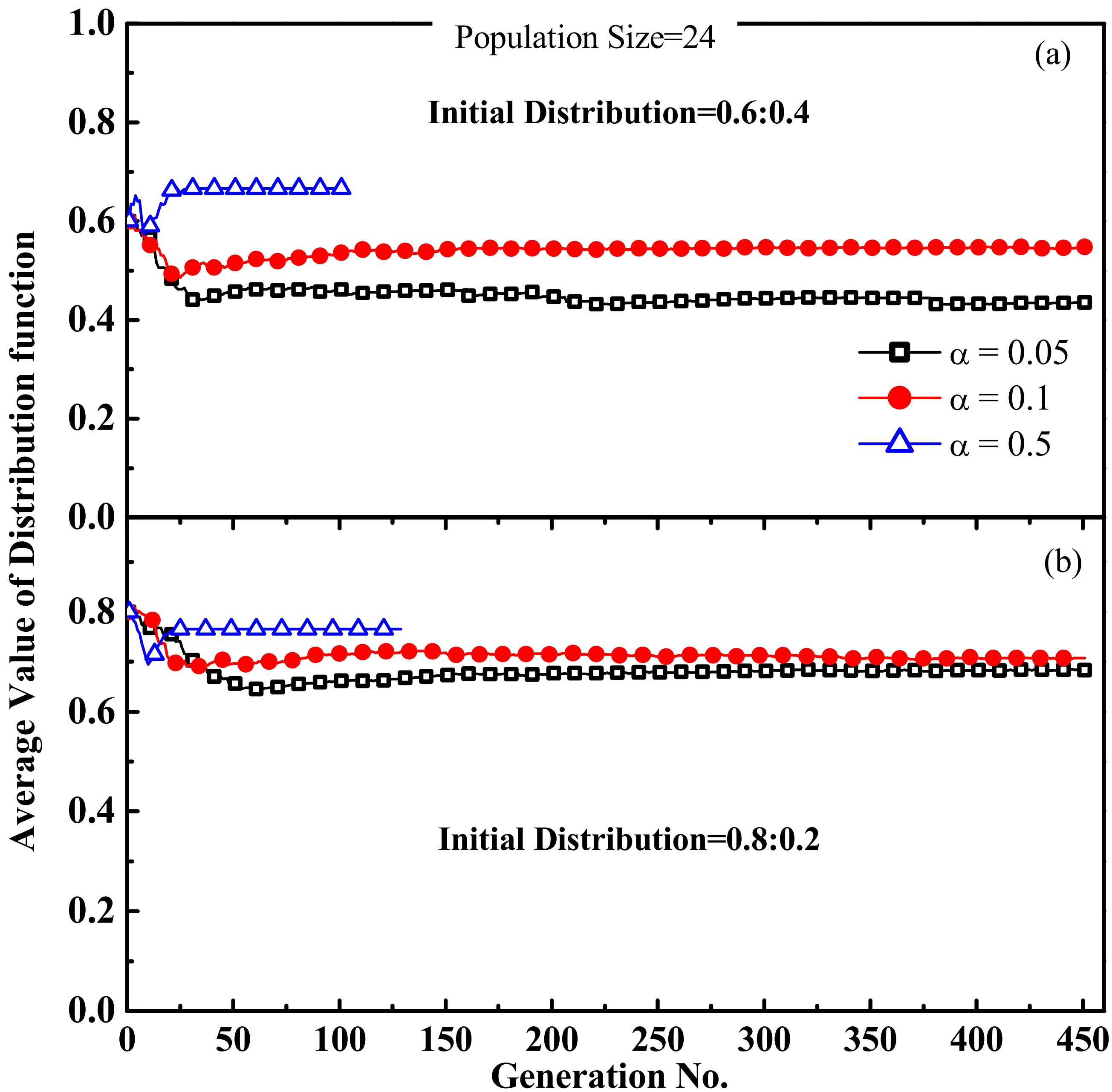


Fig.5.9 Variation of average value of distribution function with generations for population size 24 and three values of α (0.05, 0.1 and 0.5) with initial distributions of 0.6 & 0.4 (a) and 0.8 & 0.2 (b)

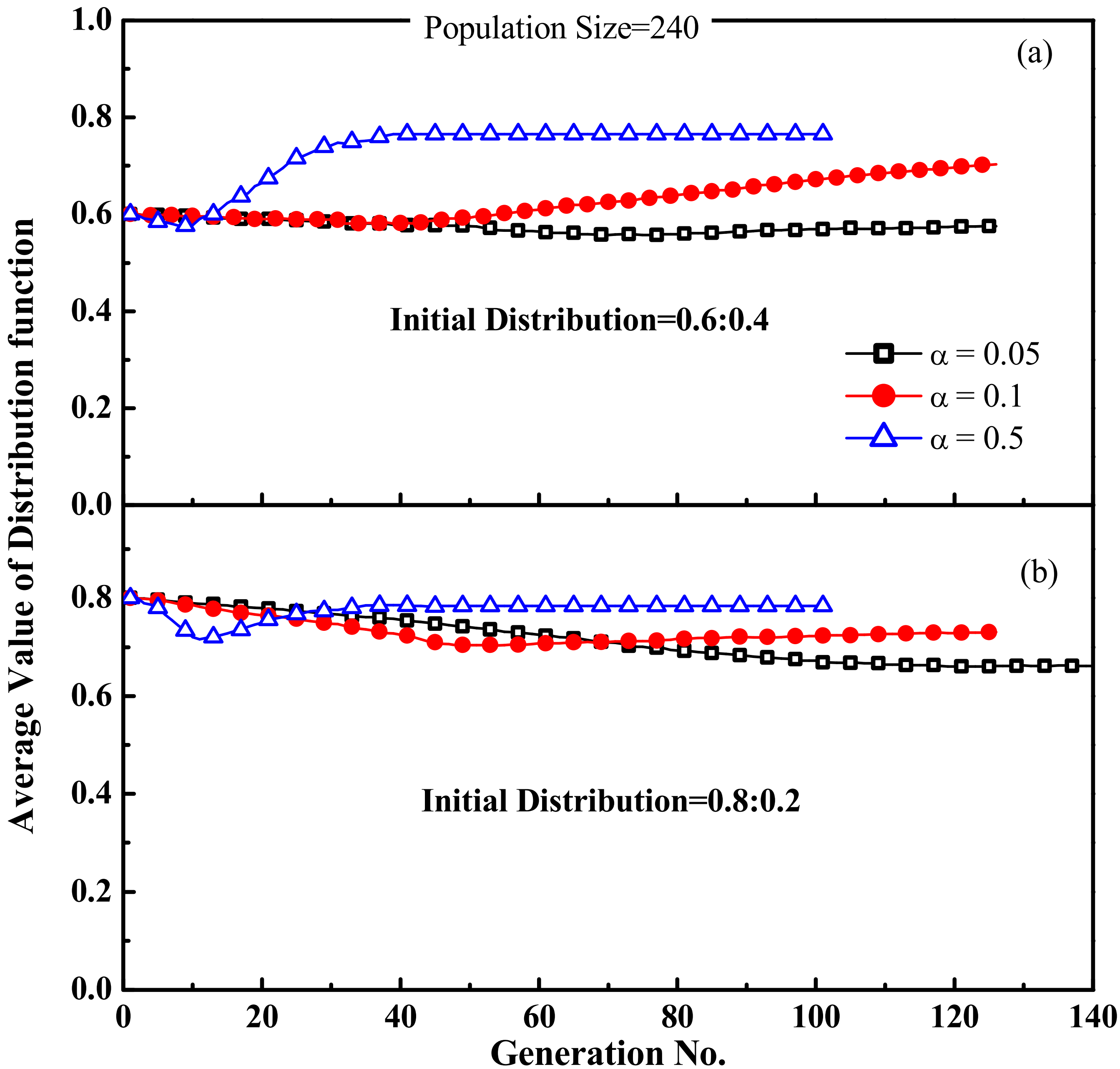


Fig.5.10 Variation of average value of distribution function with generations for population size 240 and three values of α (0.05, 0.1 and 0.5) with initial distributions of 0.6 & 0.4 (a) and 0.8 & 0.2 (b)

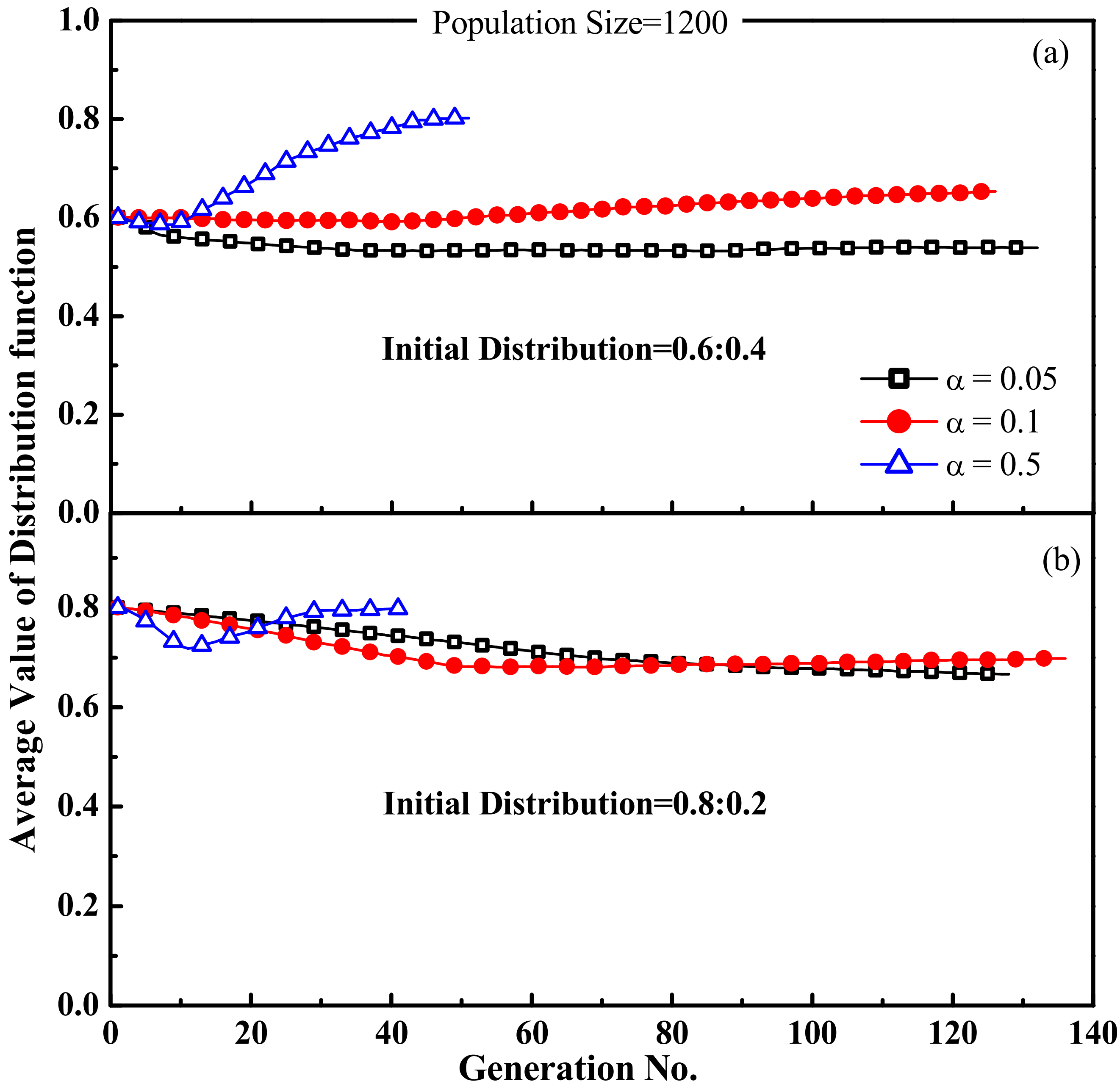


Fig.5.11 Variation of average value of distribution function with generations for population size 1200 and three values of α (0.05, 0.1 and 0.5) with initial distributions of 0.6 & 0.4 (a) and 0.8 & 0.2 (b)

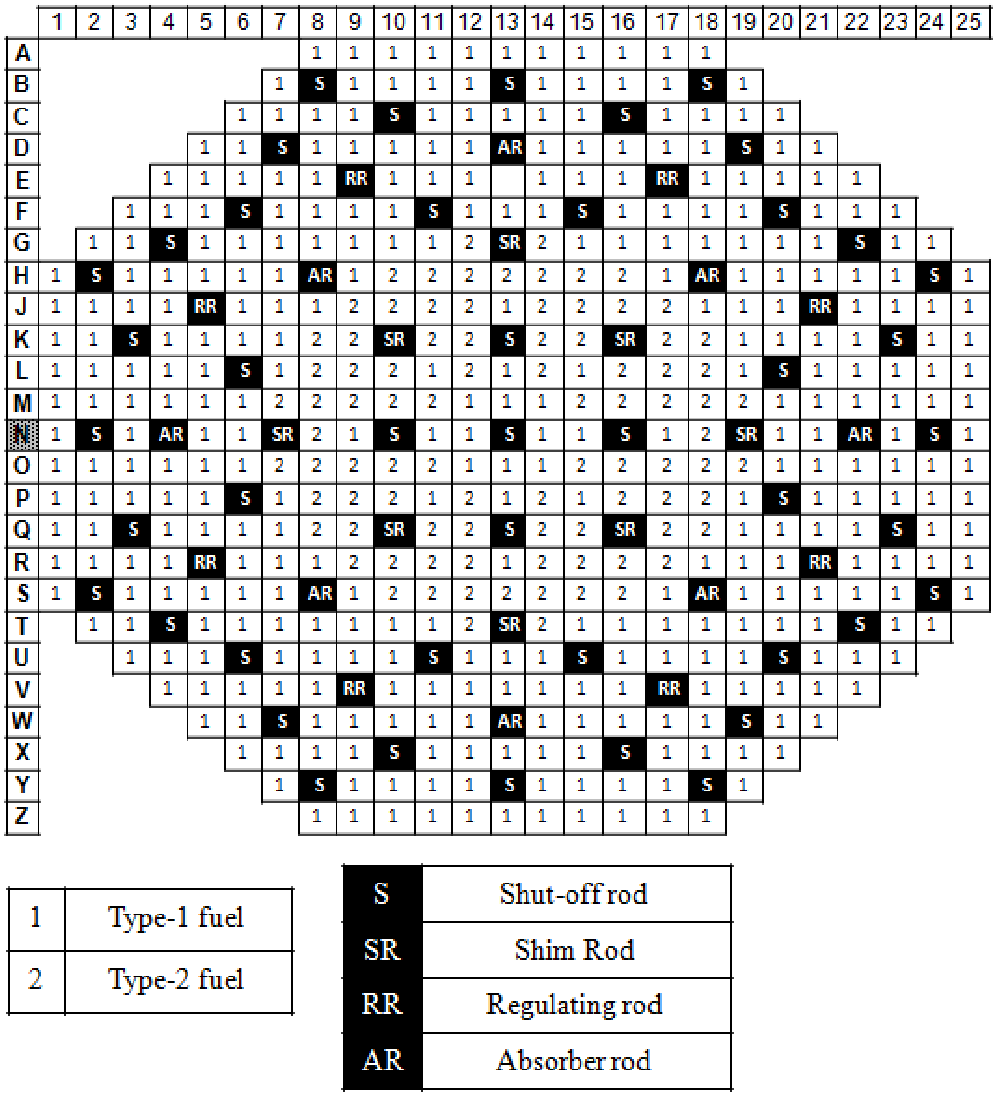


Fig 5.12 Core loading pattern of optimized core (1 and 2 represents Type-1 and Type-2 clusters respectively) (S, SR, RR and AR shows the locations of SORs, Shim rods, Regulating rods and Absorber rods respectively).

5.7 Conclusions

In this work, an effort has been made to optimize the initial core of AHWR-LEU. As a first step, use of conventional methods was explored for finding the optimal solution. The optimized solution achieved by considering these methods was not satisfactory and therefore use of evolutionary algorithms was considered and the initial core LPO problem was dealt with using estimation of distribution algorithm (EDA).

While applying EDA it was observed, unlike in other optimization studies, (Mishra et. al., 2009, Jiang et. al., 2006) very small value (≤0.05) of weighing factor *'α'* is not producing desired optimization. By considering a very small value of *'α'*, the optimization is slowed and more solutions are explored to reduce the probability of falling in local minima. In our study we have found that the real optimization is happening only in the evolving region of the process. When the value of objective function saturates, the new patterns are generated only with the probability distribution function and the probability distribution function itself does not changes much. Jiang et. al. (2006) has also considered elitism or heuristic information to improve the results. In Jiang et al. it is concluded that simple EDA (named as EDA-S) without elitism or heuristic information the maximum value of objective function is lower than EDA with elitism or heuristic information (called as EDA-G and EDA-H). In our case of initial core optimization of AHWR-LEU, by using the general form of EDA (EDA-S) with small value of weighing factor '*α'* and small population size (24), the optimization was not achieved at all. However, we observed that the results can be improved by choosing adequate alpha and population size. It is also observed that for a small population size (24), although more generations (400) have been simulated but true optimization was not achieved. In case of larger population size, fewer generations are simulated and better optimization solutions are achieved. It is to be noted that considering a higher population size will increase the computational cost but it improves the results. It is observed that choosing small population size and small '*α*' is less computationally costly (< 24×400= 10000 simulations) but results are inferior. By having large population size (240 or 1200) and keeping small '*α*'(0.05), computational cost is increased (~ 240×125 = 30000 or 1200×125 = 150000 simulations) but there is high probability that the optimized solution achieved will be near to initial distribution function (Fig 5.10a and 5.11a). The dependence on initial distribution function is reduced by increasing the value of 'α'. We have observed very good optimization for α=0.5 and population size of 1200. The results for α=0.5 show convergence in less than 50 generations. This shows that the computational cost for this case (1200×50 = 60000 simulations) is almost six times of population size of 24 (α=0.05 and 0.1) and is about twice the case with population of 240 (α=0.05 and 0.1). The time for doing so many simulations in case of higher population cases may be reduced by increasing parallelization. In present study we could parallelize up to 600 CPUs. A significant decrease in computational time was observed due to this increased parallelization. The cases with population size of 240 and 1200 were although more computationally costly than case with population size of 24 but required less simulation time (due to parallelization) as we simulated fewer generations in higher population cases.

In this chapter, we have studied the dependence of various parameters in EDA on the finalized solution. However, we have considered only one method EDA in this chapter. For verification of our results it will be better if the same problem is addressed with some other optimization technique also. Keeping this in mind, another optimization method GA was considered and was used to address same LPO problem for initial core of AHWR in the next chapter.

CHAPTER 6

Loading Pattern Optimization Using Genetic Algorithm (GA)

6.1 Introduction

In chapter 5, the initial core LPO problem for AHWR-LEU was introduced and applicability of conventional methods and modern methods was discussed. We have used EDA for initial core optimization of AHWR-LEU and it was observed that the algorithm is sensitive to value of internal parameters. A study was carried to choose the adequate value of these parameters. In the end, LP for initial core of AHWR-LEU was optimized. For verification of our results it will be better if the same problem is addressed with some other optimization technique. For sake of comparison and completeness, in this chapter, we have addressed the initial core LPO of AHWR-LEU by using Genetic algorithm (GA) (Goldberg, 1989). The same initial core clusters (Table 5.1) and problem size (2^{62}) was considered as it was done for EDA. Further, the objective function considered is also same as given in equation 5.2. A computer code was developed based on GA to optimize the initial core LP of AHWR-LEU.

The main focus in this chapter is to optimize the initial core of AHWR-LEU using Genetic Algorithm (GA). To compare our results with EDA, studies similar to chapter 5 were now carried out with GA. The dependence of internal parameters such as initial distribution function *and population size in each generation was studied on the optimized solution. An extensive study was done to determine adequate value of these parameters used in GA for better optimized loading pattern and the results were compared with earlier studies carried out studies using EDA.* In GA too, similar dependence on population size and initial distribution function in each generation is observed. However, by increasing the population size, the results in GA optimization improved drastically. And it is observed that our results are contrary to earlier published results (Mishra et. al., 2009) for other problems (PHWR initial core LPO problem), where EDA is found to be more efficient and produces better results than GA. We have observed that for AHWR initial core LPO problem, GA is more efficient than EDA. In the end few possible improvements are described which could enhance the performance of EDA. The plan of chapter is as follows;

Section 6.2 gives the description of GA and the various parameters used in this algorithm. Section 6.3 gives the numerical results for variation of population size and initial probability distribution function considered in GA for AHWR-LEU initial core LPO problem. The comparison of results produced by EDA & GA is discussed in Section 6.4. In section 6.5, possible improvements in EDA are discussed which could enhance its performance. Section 6.6 discusses the conclusions.

6.2 Genetic algorithm (GA)

The use of GAs has been very frequent for application to various nuclear reloading pattern optimization problems. They have been found to be very efficient and have always provided plausible solutions to complex combinatorial optimization problem. GA is also population based algorithm where optimized solution is evolved by selection of better candidate solutions and putting them in mating pool for recombination and generation of new candidates. In our work, we have used tournament selection (Ziver et al., 2004) method for generating the mating pool and we have used uniform crossover operator for recombination in mating pool to generate the new candidates. A brief description of GA used is as given below:-

Step-1 Generate a population (*N*) of different loading patterns based on random initial distribution function

Step-2 Simulate all the candidates of this population by solving diffusion equation using 3D diffusion theory code FEMINA and Objective function for all the candidates is evaluated

Step-3 two person tournament selection is done for creating the mating pool

Step-4 New candidates of population size *N* is generated by using uniform cross-over operator between different candidates of mating pool

Step-5 Go to step-2 and repeat the cycle till optimization is achieved

As discussed in chapter 5, the Type-2 fuel is lesser reactive than Type-1 fuel and therefore the optimized core will have more number of Type-1 fuel clusters than Type-2 fuel clusters. Hence, similar to chapter 5 studies, two initial distribution functions have been considered and their details are given below:-

III. The distribution function is such that the loading pattern generated by it should have ~60 % of Type-1 fuel cluster and ~40% of Type-2 fuel clusters

IV. The distribution function is such that the loading pattern generated by it should have ~80 % of Type-1 fuel cluster and ~20% of Type-2 fuel clusters

However, there is no weighing factor 'α' in GA instead a two point tournament selection is used to consider the feedback for generation of new candidate solutions in mating pool. In this way, following differences have been observed when we compare EDA and GA;

Selection of feedback candidates; In EDA the probability distribution model is modified by considering due weight of current **best solutions**. However in GA, tournament selection is used for creating the mating pool. Therefore, in case of GA the candidates which are at bottom of the solution pool also have a chance of getting selected. Therefore, more diverse search is possible in case of GA. To increase the diversity of search process in case of EDA, lower value of weighing factor 'α' is considered better. However, as we have observed during studies in chapter 5, small value of weighing factor 'α' is not adequate for our problem.

Generation of new pool of candidate solutions; In case of EDA, new candidates are generated by sampling the probability distribution function however in case of GA, mating of various candidates in the pool is done by using uniform crossover operator to generate new solutions. The process appears to be similar for both the algorithms in following manner;

In case of EDA, the updated DF will generate more candidate solutions which are near to the previous **best solutions** and in case of GA, the **better candidates** are mated multiple times which leads to generation of more solutions which are near to better candidates.

6.3 Results of GA on AHWR-LEU initial core loading pattern optimization

The GA was used to optimize the initial core loading pattern of AHWR-LEU and the study has been divided in to two parts as described below.

Case-I:- Initial distribution function is 0.6 for Type-1 and 0.4 for Type-2

Three cases were considered with population size of 24, 240 and 1200 in each generation as it was done in EDA. For each case for generating the very first population set, initial distribution function 0.6 for Type-1 and 0.4 for Type-2 was considered. It is observed that similar to EDA case the algorithm fails for case with population size of 24. However, the maximum value of OF is 0.9989, which is slightly better than for the same case with EDA (0.9965). For the cases with population size of 240 and 1200, the optimized value is 1.00953.

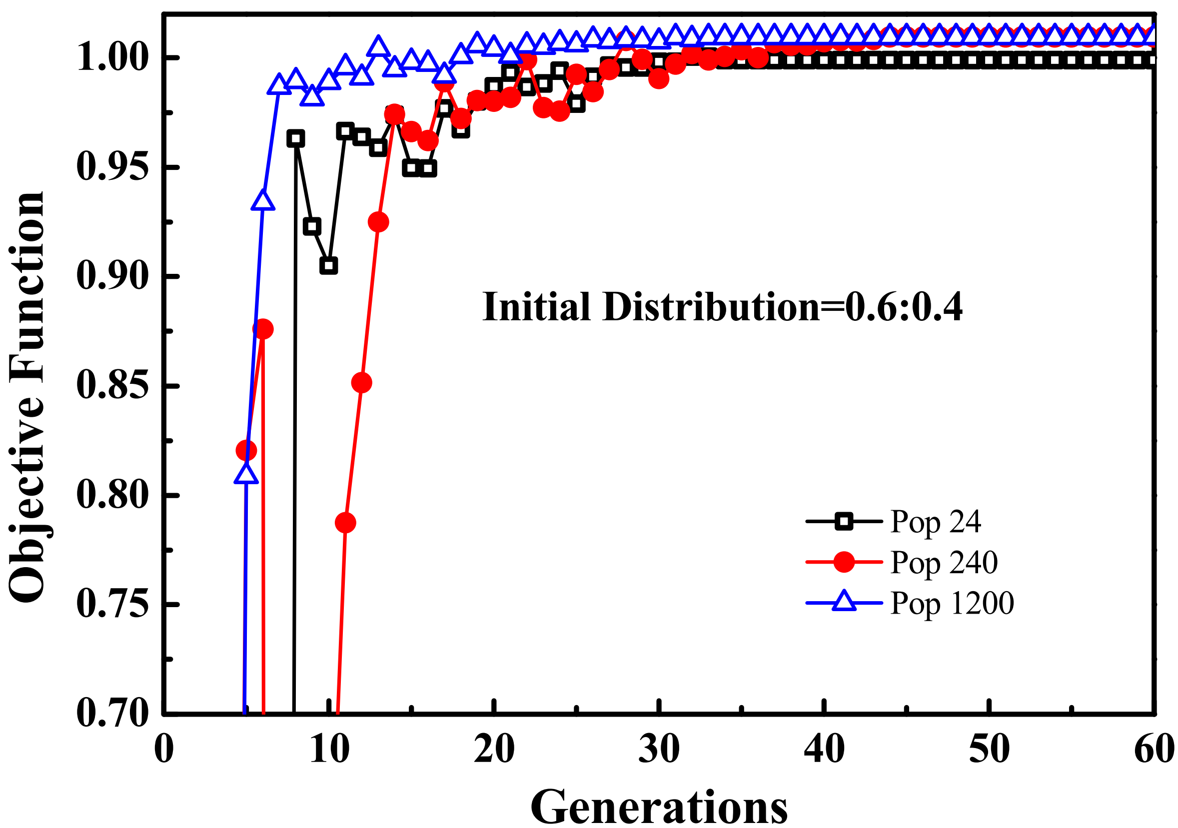


Fig.6.1 Variation of objective function with generations for three population sizes viz. 24, 240 and 1200 for GA and initial distribution of 0.6 for Type-1 and 0.4 for Type-2

The same loading pattern was achieved with EDA. Fig 6.1 shows the variation of best value of objective function value for three different population sizes of 24, 240 and 1200. Table-6.1 gives the details of optimized loading patterns achieved using GA.

Table-6.1 Properties of loading pattern optimized using GA for variation of population size for initial distribution function of 0.6 for Type-1 and 0.4 for Type-2

Population size	***Initial distribution function of 0.6 for Type-1 and 0.4 for Type-2***						
	Max. Objective function	**K-eff**	**MCP (MW)**	**MMP (KW)**	**Worth of SDS-1 (mk)**	**Type-1 Clusters**	**Type-2 Clusters**
24	0.99890	0.99890	2.58	149	64.6	308	136
240	1.00953	1.00953	2.58	154	63.1	356	88
1200	1.00953	1.00953	2.58	154	63.1	356	88

Case-II:- Initial distribution function is 0.8 for Type-1 and 0.2 for Type-2

Three cases were considered with population size of *24*, *240* and *1200* in each generation as it was done in EDA. For each case for generating the very first population set, initial distribution function *0.8 for Type-1 and 0.2 for Type-2*was considered.

Table-6.2 Properties of loading pattern optimized using GA for variation of population size for initial distribution function of 0.8 for Type-1 and 0.2 for Type-2

Population size	***Initial distribution function of 0.8 for Type-1 and 0.2 for Type-2***						
	Max. Objective function	**K-eff**	**MCP**	**MMP**	**Worth of SDS-1**	**Type-1 Clusters**	**Type-2 Clusters**
24	1.00560	1.00560	2.60	156	64.0	336	108
240	1.00953	1.00953	2.58	154	63.1	356	88
1200	1.00953	1.00953	2.58	154	63.1	356	88

It is observed that similar to EDA case the algorithm fails for case with population size of 24. However, the maximum value of OF is 1.0056, which is slightly better than for the same case with EDA (1.0042). For the cases with population size of 240 and 1200, the optimized value is 1.00953. The same loading pattern was achieved with EDA.

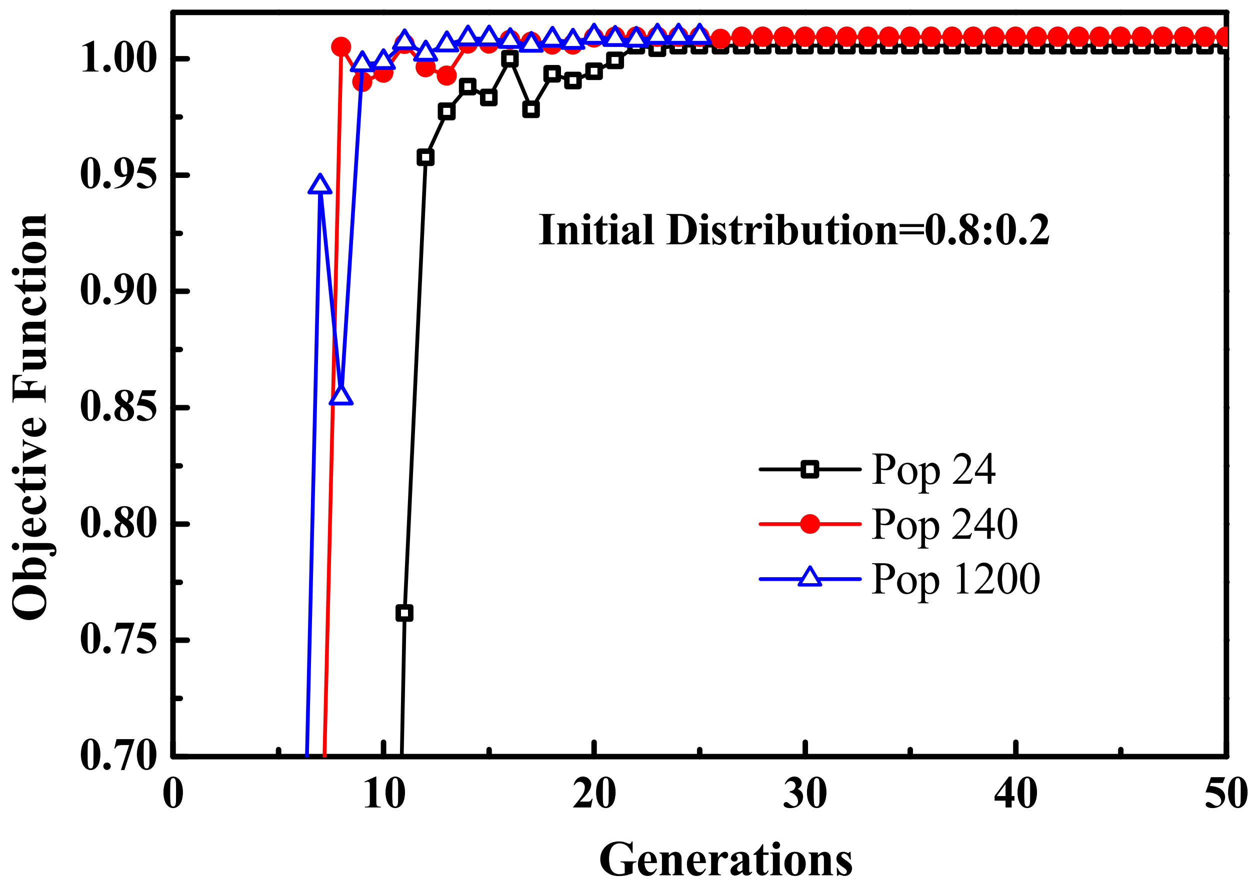


Fig.6.2 Variation of objective function with generations for three population sizes viz. 24, 240 and 1200 for GA and initial distribution of 0.8 for Type-1 and 0.2 for Type-2

Fig 6.2 shows the variation of objective function value for three different population sizes of 24, 240 and 1200. Table-6.2 gives the details of optimized loading patterns achieved using GA and it is observed that the best optimized loading pattern achieved with objective function value of 1.00953 is similar to the loading pattern shown in Fig 5.12.

6.4 Comparison of results of GA and EDA

From the results of chapter 5 & 6, two primary findings have been observed in contradiction to earlier reported studies. First, GA is more efficient than EDA for present initial core LPO problem of AHWR-LEU. It has been observed from table 6.2 and 6.3 that population size of 240 is sufficient for our problem whereas EDA requires a minimum population size of 1200 for achieving same optimization results. However, Mishra et al., 2009 has shown that EDA gives better results than GA for initial core LPO problem of PHWR. Secondly, very small value of weighing factor ‘α’ is generally considered good (Mishra et. al., 2009 and Jiang et al., 2006) for better exploration of search space. Our study has been started with considering a very small value of ‘α’ in EDA but we could not reach the desired optimization. Later, when we tried to do the parametric study by varying higher value of ‘α’, better optimization was observed. The difference in our results and earlier reported results may be attributed due to different size of problem. It should be noted that Mishra et al., 2009 has applied EDA in a little different way than as described in this chapter. We have not put any limit on number of Type-2 clusters but Mishra et al. has always tried to find the optimum solution in near vicinity of initial distribution function. Further, a similar parametric study of varying ‘α’ in EDA is done on PHWR initial core optimization problem; different optimized results may be achieved. It is observed for AHWR initial core optimization problem that considering very small value of ‘α’ may lead to unnecessary calculations. In the present case, even with α=0.5, algorithm takes more number of generation than for same case with GA with population size of 240 and a slightly inferior optimized LP has been observed. This clearly shows that GA has outperformed EDA. Therefore, a couple of improvements are suggested to enhance the performance of EDA (Thakur et. al., 2015).

6.5 Proposed improvements in EDA

The plus point of EDA is that by considering smaller value of 'α', more solutions in search space can be explored and convergence of the objective function is delayed. Keeping this in mind, and considering that Mishra et. al, 2009 and Jiang et. al, 2006 have emphasized a smaller value of 'α', we have tried to search for improving the algorithm and search for a better solution than earlier observed in Fig 5.12. Therefore two basic modifications have been proposed;

1) When we exploit the symmetry of core to reduce the problem size, there are ~62 locations. If we suppose that there are ~80% channels of Type-1 and 20% channels of Type-2 and define our initial distribution function (DF) in this way, then while generating loading patterns, we have to generate very small set (62 numbers only) of random numbers (RNs) multiple times. Such a small set is not truly random and it is observed that ~40% of loading patterns generated are having variation of more than ±20% from the average DF value. For better exploration of search space, a lower value α=0.1 is considered and we have put the restriction criteria for random numbers to generate the loading patterns within 20% of present DF. For doing this, first we generate a set of 62 random numbers and analyze that the LP generated by this set is in ±20% of the averaged DF. If it is NOT then, we ignore this set and generate a new set and the process is repeated, till we get the desired random number set. In the next step, the set of random numbers which satisfy the restriction criteria is used to generate the loading pattern for creating the candidate of pool. The process is repeated to generate all the candidates for pool.

Fig 6.3 shows the comparison for α =0.1 (population size 1200) with and without restriction criteria on random numbers of ±20% for case with initial distribution function of 0.8 for Type-1 fuel and 0.2 for Type-2 fuel. Fig 6.4 shows the comparison for α =0.1(population size 1200) with and without restriction criteria on random numbers of ±20% for case with initial distribution function of 0.6 for Type-1 fuel and 0.4 for Type-2 fuel.

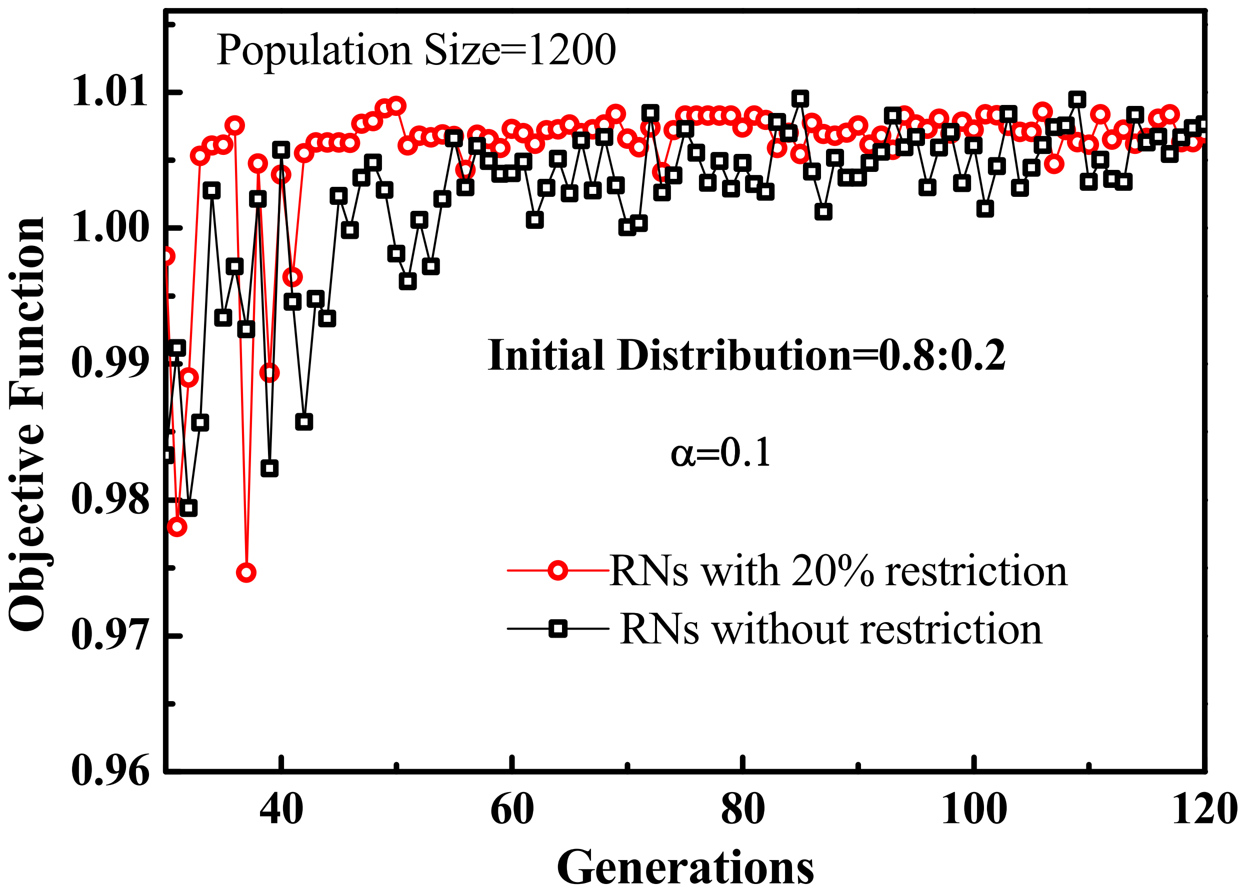


Fig 6.3 Restriction criteria for random numbers for initial distribution of 0.8 for Type-1 and 0.2 for Type-2

It was observed that the approach to higher value of objective function is faster when we use the restriction criteria on random numbers which results in slight reduction in computational cost. However, same value of objective function and similar loading patterns are achieved and no improvement in the best value of optimized LP (1.0095) is observed.

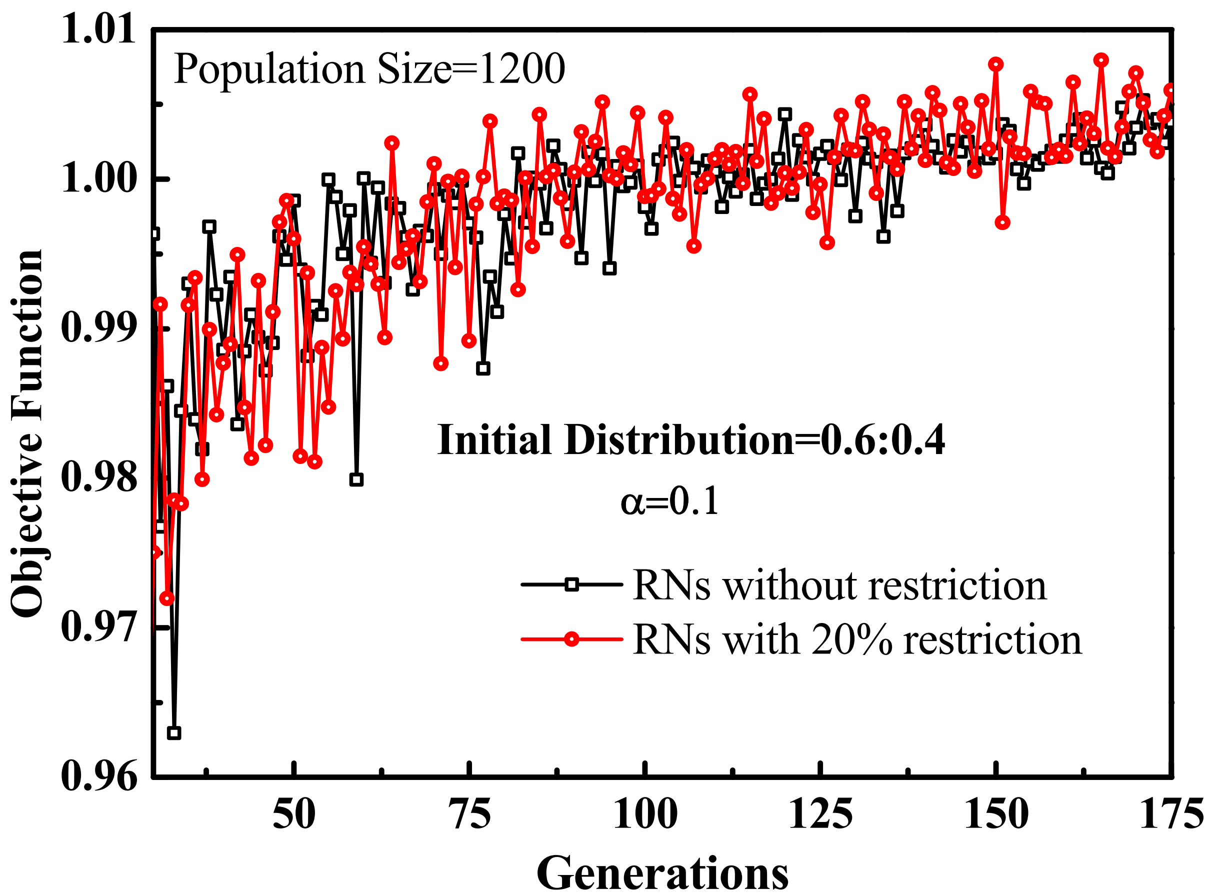


Fig 6.4 Restriction criteria for Random Numbers for initial distribution of 0.6 for Type-1 and 0.4 for Type-2

2) EDA with α=0.5 behaves in a similar way as GA. The convergence is achieved very fast in less than 60 generations. However, the search is influenced by the best solution in the previous pool. Therefore, to enhance the diversity, instead of choosing best candidates from the pool, tournament selection can be applied to choose the better candidates to modify the DF. In this way, solution which is at bottom of the pool can also be picked.

A few simulations were carried out with α=0.5, population size of 240 and initial distribution function of 0.6 for Type-1 and 0.4 for Type-2 was considered. It was observed that the optimized loading pattern is same to as given in Fig 5.12. Fig 6.5 gives comparison of best value of objective function in each generation for simple EDA and EDA with tournament selection. It is observed that EDA with tournament selection converges to higher value. However, the best value of objective function is 1.0095 and has not improved further.

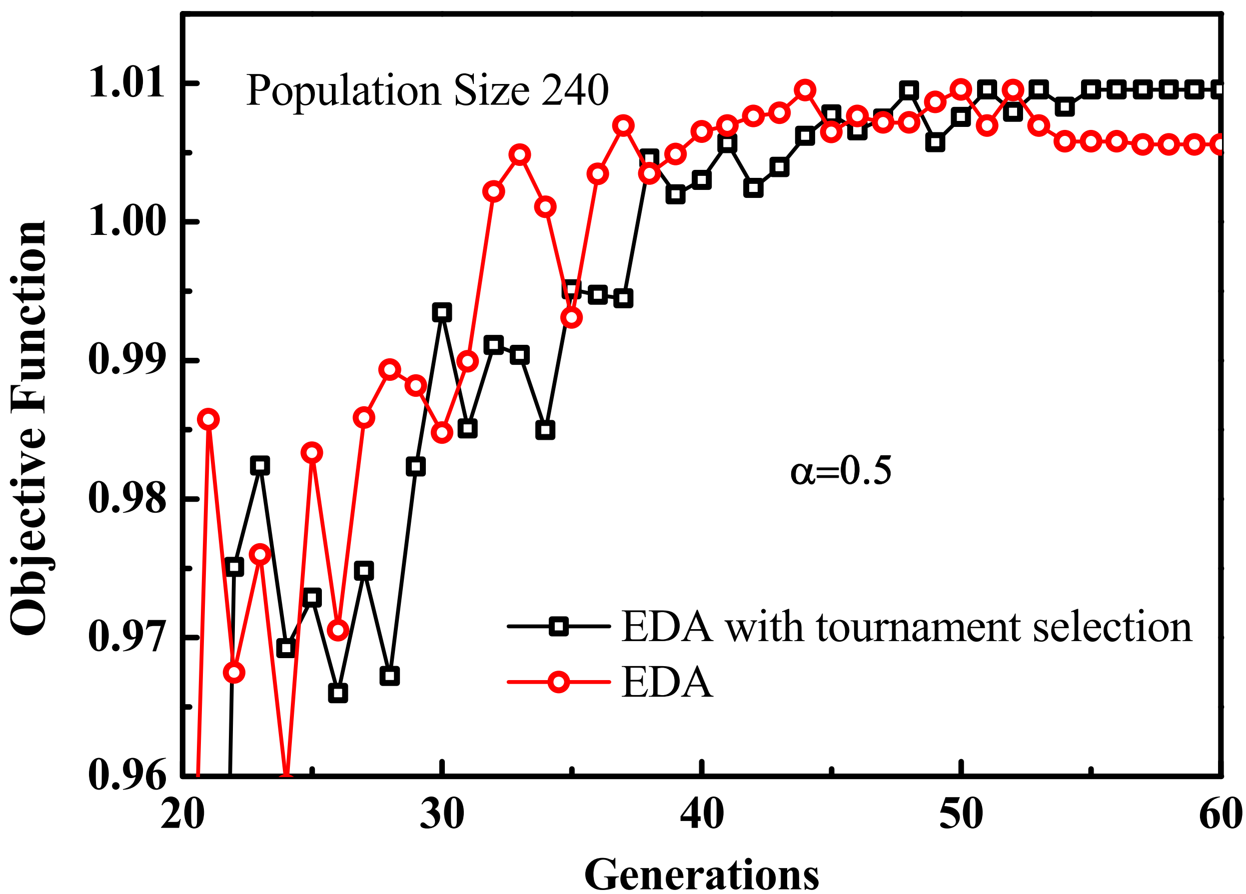


Fig 6.5 Tournament selection applied in EDA for initial distribution of 0.6 for Type-1 and 0.4 for Type-2 and population size of 240

6.6 Discussion and Conclusions

In this work, an effort has been made to optimize the initial core of AHWR-LEU using GA. In order to compare our results with EDA, we have addressed the same problem using GA. In GA too it was observed that the algorithm has failed with population size of 24. However, when the initial distribution function is near to optimized solution, the results are better. By increasing the population size to 240, the optimized loading pattern similar to EDA is achieved. By further increasing the population size to 1200 does not results in any improvement. It can be concluded that for any optimization study using GA, an adequate value of parameters used in the optimization algorithm should be obtained for the particular problem to enhance the performance of GA. It is remarkable that two initial probability distributions, two population sizes and two methods (GA and EDA) all give the same end result. It is also observed that contrary to previous published work, GA has a better performance than EDA for AHWR initial core LPO problem. For GA, a population size of 240 is sufficient as compared to EDA where population size of 1200 is adequate.

In this chapter, we have tried various modifications in EDA for improving the performance and search for a better optimized LP. Restriction criteria on random numbers and incorporation of tournament selection were done which has lead to slight improvement in the performance. A search for a better optimized loading pattern has lead us to a better understanding of both the algorithms (GA & EDA). In general it is difficult to say which algorithm is better, however, present research has elucidated that before applying any optimization algorithm, a study is required to choose the adequate value of internal parameters.

The increased computational parallelization has helped in better exploration of search space using these population based methods. However, the exhaustive study to find the optimized solution in a short time is still a distant dream. It is also known that there are many redundant candidate solutions which do not require exploration but the algorithm (black box) does not know about these redundant solutions and keeps on exploring and simulating these candidates also which leads to unnecessary increase in computation cost and time. For future study, it is planned to use some conventional method like linear programming or some gradient based method to limit the search space so that the redundant solutions can be ignored and an exhaustive study is possible.

CHAPTER 7

Conclusions and Scope of Future Research

7.1 Conclusions

In this thesis, we have investigated various fuel management problems at different stages of reactor operation and optimization techniques have been developed to solve these problems. As AHWR plays an important role in India's nuclear energy development program, fuel cycle of AHWR has been considered in detail for demonstration. The thesis has been divided into two parts.

In the first part, fuel management during pre-equilibrium and equilibrium phase of AHWR-LEU has been discussed. It was observed that due to the unique features of AHWR like use of high discharge burn-up fuel with on-power refueling and low power density, the conventional methods for fuel management are not applicable. Therefore, a specific technique in form of a special refueling scheme has been developed to demonstrate the proper utilization of fuel during pre-equilibrium and equilibrium phase. A special refueling strategy which is optimal combination of PHWR and LWRs refueling strategy was devised. In this special refueling strategy, it is proposed that if each refueling operation is followed by one or two reshuffling operations, significant reduction in power peaking can be achieved and full power operation is possible. However, it was observed that to choose one channel for refueling and one or two channels for reshuffling, a lot of manual effort is required. Therefore, it was also proposed that the priority of channels for refueling or reshuffling can be decided by defining few factors like refueling and reshuffling factor. The refueling and reshuffling factor are assigned to each channel by giving weightage based on its channel power, fuel type, location and burn up. In other words, refueling factor and reshuffling factors give the maturity of a channel for refueling and reshuffling respectively. By introducing the concept of these factors, automatization of simulation of refueling inputs is possible and manual efforts can be reduced. To choose a set of refueling and reshuffling inputs from a large number of available combinations, the channels based on their refueling factor and reshuffling factor can be selected for one refueling simulation. A computer code CARS was developed by using this remedy as described in chapters 3. CARS can automatically generate the various refueling inputs (selection of channels for refueling and reshuffling) required for special refueling strategy and carry out the refueling simulations for AHWR-LEU core. In the first step, the special refueling scheme was devised by considering each refueling operation is followed by one reshuffling operation. It is named as 'single reshuffling scheme'. The computer code CARS has been used for automatic selection of channels for single reshuffling scheme and for development of refueling strategy for AHWR-LEU. It was observed that the maximum channel power peaking is reduced significantly; however, it was still beyond the design limit of 2.85 MW.

Therefore, in chapter 4, another modification in CARS was done by adding a subroutine based on 'double reshuffling scheme'. In this double reshuffling scheme, each refueling operation is followed by two reshuffling operations. It is observed that the core operational parameters like

MMP, MCP remained below their design limit at all stages of reactor operation after using modified version of CARS which uses a suitable combination of single reshuffling and double reshuffling scheme thereby minimizing number of refueling machine operations. The target discharge burn-up of ~59 GWd/Te is also achievable by using this refueling strategy. The use of CARS has reduced the manual efforts for the selection of channels for refueling to a great extent. The CARS has made the study of various types of equilibrium core clusters for AHWR-LEU in a very short time. The present study has been able to develop confidence that the on-power refueling in AHWR-LEU can be reality.

In the second part of this thesis, initial core loading pattern optimization problem has been discussed for AHWR-LEU to have an efficient utilization of fuel and full power operation from beginning of the reactor operation. The general loading pattern optimization problems have been described and applicability of conventional methods and modern methods for solving these problems has been discussed. As a first step, use of conventional methods was explored for finding the optimal solution. The optimized solution achieved by considering these methods was not satisfactory as it requires more intuition, manual effort and it searches a very small area of the search space. Therefore, use of evolutionary algorithms was considered and the initial core LPO problem was dealt with using EDA and GA. While applying EDA it was observed, unlike in other optimization studies, (Mishra et. al., 2009, Jiang et. al., 2006) very small value (≤ 0.05) of weighing factor *'α'* is not producing desired optimization. Therefore, a parametric study was carried out to find out the adequate value of weighting factor *'α',* and population size considered in each generation. We have observed a very good optimization for $\alpha=0.5$ and population size of 1200. The results for $\alpha=0.5$ show convergence in less than 50 generations. This shows that the computational cost for this case ($1200 \cdot 50 = 60000$ simulations) is almost six times of population size of 24 (α=0.05 and 0.1) and is about twice the case with population of 240 (α=0.05 and 0.1). Parallelization on distributed memory systems can be used to significantly decrease the computational time. The cases with population size of 240 and 1200 were although more computationally costly than the case with population size of 24 but required less simulation time (due to parallelization) as we simulated fewer generations in higher population cases.

For verification of our results and in search of a better optimized solution, the same problem is addressed with another optimization method GA. In GA too, it was observed that the algorithm has failed with population size of 24. However, when the initial distribution function is near to optimized solution, the results are better. By increasing the population size to 240, the optimized loading pattern similar to EDA is achieved. By further increasing the population size to 1200 does not result in any improvement. It can be concluded that for any optimization study using GA or EDA, an adequate value of parameters used in the optimization algorithm should be obtained for the particular problem to enhance the performance of algorithm used. It is remarkable that two initial probability distributions, two population sizes and two methods (GA and EDA) all give the same end result. It is also observed that contrary to previous published work, GA has a better performance than EDA for AHWR initial core LPO problem. For GA, a population size of 240 is sufficient as compared to EDA where population size of 1200 is adequate.

Further, various modifications in EDA for improving the performance and search for a better optimized LP have been tried. Restriction criteria on random numbers and incorporation of tournament selection were done which has lead to slight improvement in the performance. A search for a better optimized loading pattern has lead us to a better understanding of both the algorithms (GA & EDA). In general it is difficult to say which algorithm is better, however, present research has elucidated that before applying any optimization algorithm, a study is required to choose the adequate value of internal parameters.

7.2 Future scope

For future work, refueling strategy can be planned on the similar lines for Indian PHWRs for facilitating the use of high fissile content U based fuel or Th based fuel for lower waste generation and higher burn-up. As PHWR uses small length bundles and is a forced circulation system, better margins are expected as compared to AHWR for controlling the power peaking.

For future study related to LPO problems, it is observed that the meta-heuristic algorithms like EDA or GA are very much dependent on the internal parameters. As we have seen in present thesis that the optimization results are very much sensitive to the value of population size and weightage factor ‘α’ considered in EDA. Similarly, optimization results for GA are sensitive to population size, cross-over operator and mutation. Therefore instead of doing a parametric study for finding out the adequate value of the individual parameter as we have done in this thesis, some mathematical model can be developed to achieve the adequate value of these internal parameters for a particular problem.

Another alternate approach to solve LPO problem is to find a way for carrying out the exhaustive search of optimal solution. As we have observed that, in case of population based algorithms, it is not possible to conclude which algorithm is better. Some algorithm may give better results for one problem and other may give better results for some other problem. However, the use of modern evolutionary algorithms along with increased computational parallelization has helped in better exploration of search space than conventional gradient based methods. But, the exhaustive study to find the optimized solution in a short time is still a distant dream even with parallelization as the search space is too large. It is also known that there are many redundant candidate solutions which do not require exploration but the evolutionary algorithms does not know about these redundant solutions and keeps on exploring and simulating these candidates also. This leads to unnecessary increase in computation cost and time. These redundant solutions form a good part of the search space and there is possibility that we can reduce the search space by ignoring these redundant candidates. Therefore, for future study, it is planned to use some conventional method like linear programming or some gradient based method to limit the search space so that the redundant solutions can be ignored and an exhaustive study is possible.

APPENDIX I

Objective Function used in EDA & GA for Initial Core LPO of AHWR

I.1 Introduction

Estimation of Distribution Algorithm (EDA) and Genetic Algorithm (GA) has been used to optimize initial core loading pattern (LP) of AHWR-LEU. The objective function (OF) defined for the optimization of initial core loading pattern for AHWR-LEU is of maximum importance and the variations in any parameter of objective function may change the direction of optimization process. As discussed in section 5.4, the penalty method has been used to define OF. The efficiency of the algorithm depends on how correctly the objective function defines the optimization problem. The algorithm will be slow or may not reach a true optimization if the penalty coefficients (A_1, A_2, A_3 and A_4) in OF are not suitably chosen. A comprehensive study about the effect of change in various parameters of objective function on optimized loading pattern was carried out for both GA and EDA. It was observed that the variation in any parameter of objective function leads to similar effect on optimized loading pattern for both the algorithms (GA and EDA).

The main objective in LPO problem is to maximize K-effective while keeping MCP and MMP within the specified limits and worth of SDS#1 greater than the specified value. Therefore, while estimating the OF value for a particular LP, the penalty on k-effective is forced whenever the specified limit on any operational parameter is breached. The objective function (OF) for AHWR-LEU initial core LPO problem is defined as given in equation (I.1)

$$OF = A_1 \cdot k\text{-}eff - A_2 \cdot (MCP - DL_{MCP}) - A_3 \cdot (MMP - DL_{MMP}) - A_4 \cdot (DL_{SDS} - \text{worth of SDS-1}) \qquad \text{(I.1)}$$

The A_1, A_2, A_3 and A_4 are constants.

Further, $DL_{MCP} = 2.6$, $DL_{MMP} = 200$ and $DL_{SDS} = 63.0$

It is to be noted that, IF MCP < DL_{MCP}, $A_2 = 0$

Similarly, If MMP < DL_{MMP}, $A_3 = 0$

And If SDS worth > DL_{SDS}, $A_4 = 0$

Hence, we are considering only the penalty due to design parameters not meeting their design limit, but we do not prefer any LP which is having very high margins in peaking or SDS-1 worth. The criteria are to maximize the K-effective without compromising full power operation and safety of reactor.

The sensitivity study of variation of parameters in OF has been carried out in two parts. In the first part, the effect of individual penalty parameter is studied while in second part the effect of change in specified limits of various design parameters (like DL_{MCP} & DL_{SDS}) used in OF has been studied. The study has been carried out using both the algorithms (GA and EDA) and similar results have been observed. A population size of 240 has been considered for all the cases simulated in this study.

I.2 Effect of variation in penalty coefficients A_1, A_2, A_3 and A_4

The effect of variation of A_1, A_2, A_3 and A_4 has been studied in three steps. In first step, we have studied the importance of individual coefficient (A_1, A_2, A_3 and A_4). This has been done by considering penalty due to one penalty coefficient at a time and making other coefficients equal to 0. Following four different cases have been studied

Case i. All the penalty coefficients are made 0. That is A_2, A_3 and A_4 are 0 & A_1 =1. It shows that the OF is only K-effective

Case ii. Penalty coefficients due to MCP and SDS worth are made 0. That is A_2, A_4 are 0 & A_3, A_1 are 1. It implies that the penalty due to MMP is considered.

Case iii. Penalty coefficients due to MMP and SDS worth are made 0. That is A_3, A_4 are 0 & A_2, A_1 are 1.It implies that the penalty due to MCP is considered.

Case iv. Penalty coefficients due to MCP and MMP are made 0. That is A_2, A_3 are 0 & A_4, A_1 are 1. It implies that the penalty due to SDS worth is considered.

These four cases describe the importance of each design parameters (k-effective, MCP, MMP and SDS worth). Table-I.1 describes the properties of optimized loading pattern obtained for these four cases. It is expected that operational parameters of the optimized loading pattern in the Case-i may not meet the design limit. As the OF does not care for any design parameter and progresses with the aim to maximize K-effective only. From Table-I.1, it is observed that the optimized loading pattern has K-effective of 1.0455 and all the channels are Type-1 (high reactive). The MCP, MMP and SDS worth of optimized LP are 5.77 MW, 334 kW and 41.9 mK respectively. It is observed that neither operational parameter nor SDS worth satisfies design requirements.

In the Case ii, the penalty due to maximum mesh power (MMP) is considered. It implies all the other parameters of optimized LP may not fulfill their design requirements. From Table-I.1, it is observed that the optimized loading pattern has K-effective of 1.0299 and there are 420 Type-1 clusters and 24 Type-2 clusters. The MMP of optimized LP is 200 KW which qualifies its design requirements. However, the MCP and SDS worth of optimized LP are 3.48 MW, and 40.5 mK respectively and design requirements of MCP and SDS worth are not satisfied.

In the Case iii, the penalty due to maximum channel power (MCP) is considered. It implies all the other parameters of optimized LP may not satisfy their design requirements. From Table-I.1, it is observed that the optimized loading pattern has K-effective of 1.0147 and there are 380 Type-1 clusters and 64 Type-2 clusters. The MCP and MMP of optimized LP are 2.6 MW and 150 KW which is below its design limit. However, the SDS worth of optimized LP is 52.4 mK and does not meet its design requirements.

Table-I.1 Properties of loading pattern optimized for Different objective function (considering penalty due to one parameter only)

Penalty parameter	Max. Objective function	K-eff	MCP (MW)	MMP (KW)	Worth of SDS-1 (mK)	Type-1 Clusters	Type-2 Clusters
None (case-i)	1.0455	1.0455	5.77	334	41.9	444	0
MMP (case-ii)	1.0299	1.0299	3.48	200	40.5	420	24
MCP (case-iii)	1.0147	1.0147	2.60	150	52.4	380	64
SDS worth (case-iv)	1.0097	1.0097	3.39	196	63.9	352	92

In the case iv, the penalty due to SDS worth is considered. It implies all the other parameters of optimized LP may not meet their design requirements. From Table-I.1, it is observed that the optimized loading pattern has K-effective of 1.0097 and there are 352 Type-1 clusters and 92 Type-2 clusters. The MCP of optimized LP is 3.39 MW and which does not meet its design requirements. However the SDS worth of 63.9 mk and MMP of 196 KW is observed which is below its design limit.

From the study of the four cases, it is observed that the maximum penalty is due to SDS worth limit followed by MCP and MMP. It is important to note that the MMP is a subset of MCP, therefore, the operational parameter MMP is taken care if penalty due to MCP is accounted duly. In the second step, the penalty coefficients A_2 and A_4 are varied in a range from 0.005 to 0.5. The value of A_1 and A_3 is considered as 1 and 0.05 respectively. Following nine cases have been considered:-

Case v. A_2=0.005 and A_4=0.005

Case vi. A_2=0.005 and A_4=0.05

Case vii. A_2=0.005 and A_4=0.5

Case viii. A_2=0.05 and A_4=0.005

Case ix. A_2=0.05 and A_4=0.05

Case x. A_2=0.05 and A_4=0.5

Case xi. A_2=0.5 and A_4=0.005

Case xii. A_2=0.5 and A_4=0.05

Case xiii. A_2=0.5 and A_4=0.5

Table-I.2 and Table-I.3 give details of maximum value of OF observed as well as value of other operational parameters (K-effective, MCP, MMP and SDS) in all these cases.

It is observed from Table-I.3 that by considering a very small value (= 0.005) of A_2 and A_4, the operational parameter MCP of optimized LP exceeds its design limit of 2.6. Now keeping a small value of A_2 and increasing A_4 to 0.5 (case vi, case vii. and x), the MCP of optimized LP further increases. However the worth of SDS comes near to design value. Similarly, when A_4 has a very small value of 0.005 and A_2 has a large value of 0.5 (case XI), the SDS worth of optimized LP is 62.9 which is slightly lower than its design requirement of 63mk.

Table-I.2 Maximum value of OF / K-effective for variation of penalty coefficients A2 and A4

A1 =1 & A3 =0.05	A4 = 0.005		A4 = 0.05		A4 = 0.5	
	OF	**K-effective**	**OF**	**K-effective**	**OF**	**K-effective**
A2 = 0.005	1.0089	1.0090	1.0090	1.0097	1.0089	1.0098
A2 = 0.05	1.0095	1.0095	1.0095	1.0095	1.0090	1.0097
A2 = 0.5	1.0079	1.0084	1.0095	1.0095	1.0095	1.0095

But MCP and MMP are observed to be having satisfactory value. By increasing A_4 to 0.5 (case xiii), the MCP of optimized LP and the worth of SDS comes near to design value. In the third step, minor variations in penalty coefficients are done to increase the efficiency.

Table-I.3 Parameters of optimized LP for variation of penalty coefficients A_2 and A_4

A1 =1 & A3 =0.05	A4 = 0.005			A4 = 0.05			A4 = 0.5		
	MCP	**MMP**	**SDS Worth**	**MCP**	**MMP**	**SDS Worth**	**MCP**	**MMP**	**SDS Worth**
A2 = 0.005	2.62	157	64.2	2.74	158	63.3	2.77	160	63.0
A2 = 0.05	2.58	154	63.1	2.58	154	63.1	2.74	158	63.3
A2 = 0.5	2.6	154	62.9	2.58	154	63.1	2.58	154	63.1

In last section, it is observed that better results are expected when penalty coefficients A_2 and A_4 are having similar values of 0.5. Further fine tuning of these coefficients has been done to achieve a faster convergence. From case xiv to case xviii, A_1, A_2, A_3 and A_4 are varied and the effect is observed on the optimized loading pattern and approach to reach optimized solution (minimum number of generations) is studied. Five different cases have been studied:-

Case xiv. A_1=1, A_2=0.5, A_3=0.7 and A_4=0.5

Case xv. A_1=1, A_2=0.7, A_3=0.7 and A_4=0.7

Case xvi. A_1=1, A_2=0.384, A_3=0.05 and A_4=0.5

Case xvii. A_1=1, A_2=0.5, A_3=0.05 and A_4=0.333

Case xviii. A_1=1, A_2=0.384, A_3=0.05 and A_4=0.333

Table-I.4 compares the maximum value of OF and other operational parameters for the cases xiv to xviii. In case xiv and xv, higher value of A_3 (0.7) has been considered. However, it is observed from Table-I.4 that slightly lower value of OF is observed than cases xvi – xviii. Therefore, better optimization is observed when low values (≤0.05) of A_3 is considered.

Table- I.4 Properties of loading pattern optimized using GA for variation in A_1, A_2, A_3 and A_4

Penalty parameter	Max. Objective function	K-eff	MCP (MW)	MMP (KW)	Worth of SDS-1 (mK)	Type-1 Clusters	Type-2 Clusters
Case-XIV	1.0094	1.0094	2.58	154	63.1	352	92
Case-XV	1.0091	1.0091	2.56	152	63.1	348	96
Case-XVI	1.0095	1.0095	2.58	154	63.1	356	88
Case-XVII	1.0095	1.0095	2.58	154	63.1	356	88
Case-XVIII	1.0095	1.0095	2.58	154	63.1	356	88

It is observed from Table-I.4 that all the cases except case xiv & xv have identical optimized loading pattern. It takes 34 generations in case xiv to reach the optimized loading pattern (1.0094). It was observed that Case xv requires 33 generations to reach maximum value of OF (1.0091). Case xvi and xvii requires 23 and 25 generations to reach maximum value of OF (1.0095). It is observed that case xviii requires minimum 21 generations to reach optimized LP. It is observed that the fastest approach to maximum value of objective function is observed in case xviii. Therefore, from all the above 18 cases it can be concluded that the best value of penalty coefficients A_1, A_2, A_3 and A_4 is considered in the case xviii with minimum number of generations.

I.3 Effect of variation in limits on MCP and SDS worth

In the second part of our study, we have tried to vary the design limit on MCP and SDS worth and effect on the optimized LP has been observed.

1.3.1 Sensitivity study due to variation of SDS worth limit in objective function

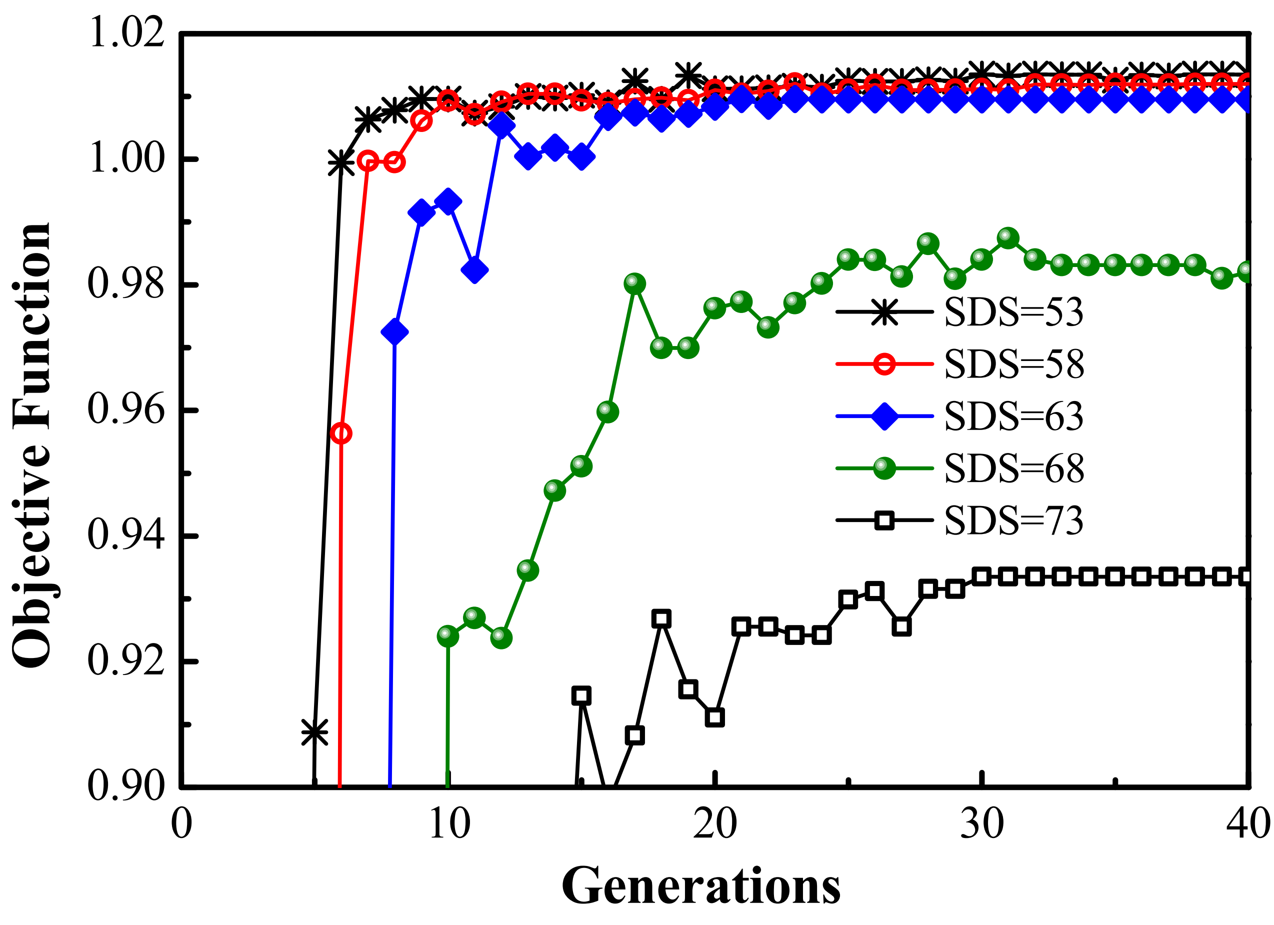


Fig-I.1 Effect of variation of DL_{SDS} in OF

In this part, the effect of variation of SDS worth limit (DL_{SDS}) was studied on the optimized loading pattern. Five different cases have been studied where the DL_{SDS} as described in equation-I.1 has been varied from 53 to 73 (53, 58, 63, 68 and 73). Fig I.1 describes behavior of OF in each generation for the five cases studied. Table-I.5 describes the properties of optimized loading pattern obtained for all the cases. It is observed from Table-I.5 that with increase in DL_{SDS} from 53 to 73, the k-effective of optimized LP decreases from 1.0134 to 0.9989 and number of type-2 cluster increases from 68 to 144. It shows that if the design requirement of SDS worth limit is stretched, the no. of type-2 clusters will increase and this will result in lower k-effective. For the last two cases in Table-I.5 where DL_{SDS} is 68 & 73 mk respectively, the optimized LPs have MCP greater than 2.6. This shows that for these cases, re-rating of reactor is required. For all other cases considered all the operational parameters meet their design limit.

Table-I.5 Properties of loading pattern optimized using GA for variation of SDS worth limit in objective function

SDS Worth Limit	Max. Objective function	K-eff	MCP (MW)	MMP (KW)	Worth of SDS-1 (mk)	Type-1 Clusters	Type-2 Clusters
53.0	1.0134	1.0134	2.55	148	53.4	376	68
58.0	1.0119	1.0119	2.60	153	59.0	368	76
63.0	1.0095	1.0095	2.58	154	63.1	356	88
68.0	0.9820	1.0013	2.65	155	68.4	316	128
73.0	0.9335	0.9989	2.75	162	73.0	300	144

1.3.2 Sensitivity study due to variation of MCP limit in objective function

In this study, the effect of variation of design limit of MCP was studied on the optimized loading pattern. Five different cases have been studied where the DL_{MCP} as described in equation-I.1 has been varied from 2.4 to 2.8 (2.4, 2.5, 2.6, 2.7 and 2.8). Fig I.2 describes behavior of OF in each generation for the five cases studied. Table-I.6 describes the properties of optimized loading pattern obtained for all the cases.

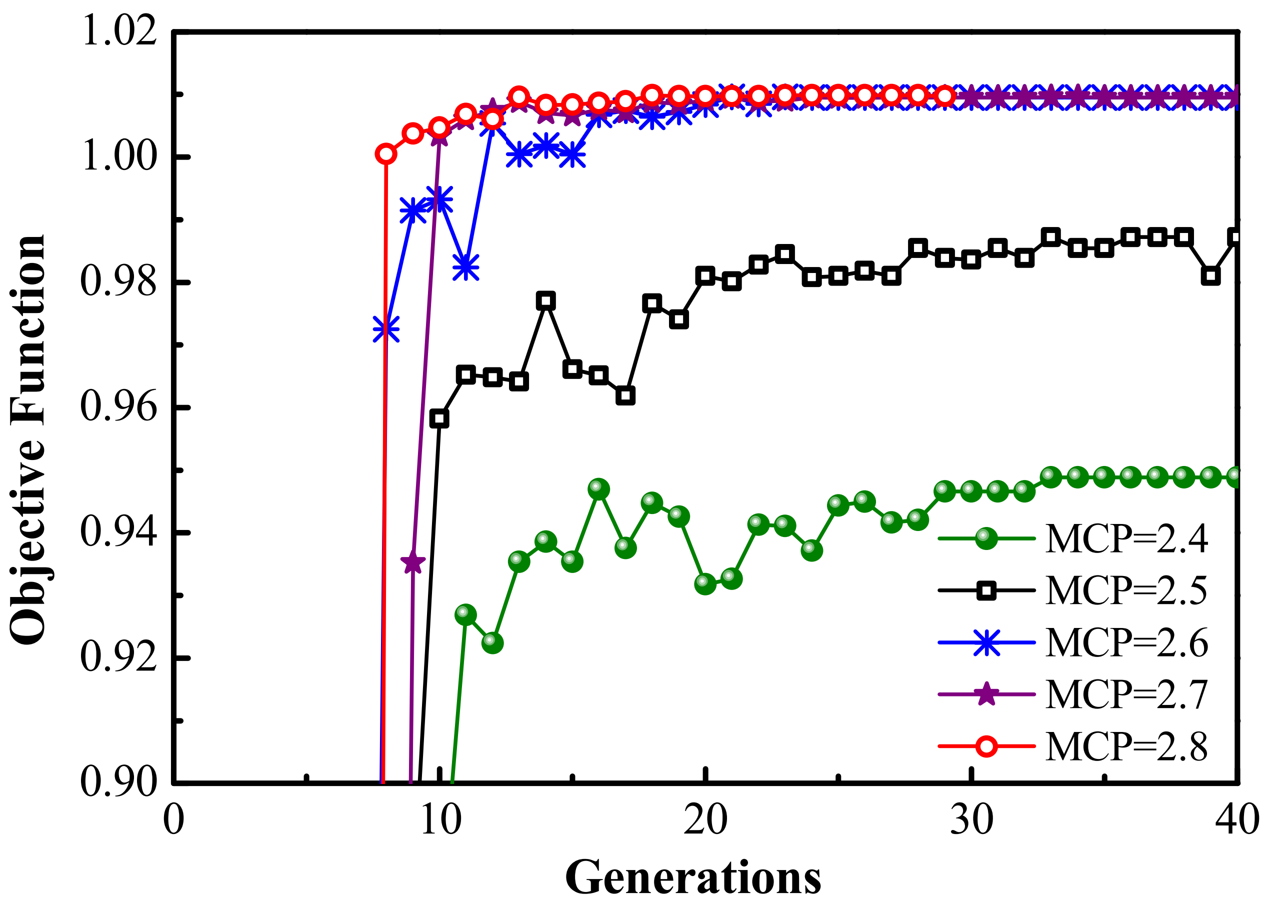


Fig-I.2 Effect of variation of DL_{MCP} in OF

It is observed from Table - I.6 that with increase in DL_{MCP} from 2.4 to 2.8, the k-effective of optimized LP increases from 1.0064 to 1.0098 and number of type-2 cluster decreases from 100 to 76. It shows that if the design requirement of MCP is relaxed, the no. of type-2 clusters will decrease and this will result in higher k-effective. For the first two cases in table-I.6 where

DL_{MCP} is 2.4 and 2.5 respectively, the optimized LP has MCP =2.55. This shows that it is very difficult to optimize an initial LP with Type-1 and Type-2 cluster with MCP lower than 2.54. For these cases, re-rating of reactor is required. For other cases considered, all the operational parameters meet their design limit.

Table-I.6 Properties of loading pattern optimized using GA for variation of MCP limit in objective function

MCP Limit	Max. Objective function	K-eff	MCP (MW)	MMP (KW)	Worth of SDS-1 (mk)	Type-1 Clusters	Type-2 Clusters
2.4	0.9488	1.0064	2.54	151	63.7	344	100
2.5	0.9872	1.0064	2.54	151	63.8	344	100
2.6	1.0095	1.0095	2.58	154	63.1	356	88
2.7	1.0096	1.0096	2.65	155	63.1	352	84
2.8	1.0098	1.0098	2.72	157	63.0	348	76

I.4 Conclusions

A study on change in various parameters in OF considered in GA and EDA was done and the effect on the optimized loading pattern was observed. It is observed that very small value of any one of the co-efficient makes the respective penalty almost insignificant, therefore the objective function is dependent on the remaining penalties. Similarly, high value of any one of the co-efficient makes the respective penalty most dominating compared to others. Therefore better results may be generated when the weightages to each penalty is almost comparable. In the present study, most appropriate value of penalty coefficients has been estimated to be as A_1=1, A_2=0.384 A_3 = 0.05 and A_4 =0.333. As the MMP is dependent on MCP or more precisely MMP is a sub-set of MCP, therefore, the value of A_3 does not play a major role and a very small value will be better. The effect due to variation of design limit in SDS worth and MCP limit was also studied. It is observed that that if the design requirement of SDS worth limit is stretched, the no. of Type-2 clusters will increase and this will result in lower k-effective. It is also observed that if the design requirement of MCP is relaxed, the no. of Type-2 clusters will decrease and this will result in higher k-effective. It is also observed that it is not possible to achieve MCP lower than 2.54, with Type-1 and Type-2 clusters.

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