Transport of Heavy Quarks In Quark-Gluon Plasma

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree/diploma at this or any other Institution/University.

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List of Publications arising from this thesis

Journals :

1. *S. Mazumder*, T. Bhattacharya, J. Alam and S. K. Das, Momentum dependence of drag coefficients and heavy flavor suppression in quark gluon plasma, Phys. Rev. C 84, 044901(2011).

2. *S. Mazumder* and J. Alam, Effects of equation of state on nuclear suppression and the initial entropy density of quark gluon plasma, Phys. Rev. C 85, 044918(2012).

3. *S. Mazumder*, T. Bhattacharya and S. K. Das, A random walk with heavy flavours, Advances in High Energy Physics, 136587(2013).

4. *S. Mazumder*, T. Bhattacharya and J. Alam, Gluon bremsstrahlung by heavy quarks - its effects on transport coefficients and equilibrium distribution,
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1. Proceedings of the DAE Symposium on Nucl. Phys. 56 (2011) 902-Vishakhapatnam, India, December 26-30, 2011.

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3. S. Mazumder, T. Bhattacharyya and J. Alam, Effect of gluon radiation off Heavy Quark on Transport coefficients inside Quark Gluon Plasma, International Conference on Matter at Extreme Conditions, 17th January, 2014.

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Dedicated to my parents who always give me inspiration

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This part is the most difficult and challenging of all parts in the entire thesis, at least to me. But, let us face it with bravery and truthfulness! It appears to me as an injustice to the persons I want to pay my gratitude to regarding the thesis, because there is no apt word which can describe their role in my life. I always fear that if I forget to name any one of the persons who has held me up at the time of odds and has inspired me throughout this whole path of writing the thesis, my conscience will be shattered. So, I decided not to mention any specific names in this acknowledgement. I am really very much grateful to all my teachers who motivated me to pursue physics and also to all my friends who devoted their precious moments in hours of discussions. Above all, I want to thank all the persons who surround me and encourage to live a beautiful life which would not have been possible without their presence.

SYNOPSIS

Quark Gluon Plasma: A novel state of QCD matter

Quantum Chromo Dynamics (QCD) is the non-abelian gauge theory with the gauge group SU(3), coupled to quarks in the fundamental representation, which describes one of the basic interactions of nature, the strong interaction. Due to its non-abelian nature, QCD, in many aspects, is different from Quantum Electro Dynamics (QED), the theory of abelian gauge group U(1) describing the electromagnetic interactions of nature. Unlike QED, where the mediatory particle or the gauge particle, photon, does not interact among themselves, in QCD gluons (gauge boson of QCD) self-interact because they carry colour charge themselves.

QCD is the governing theory of the interactions among the coloured degrees of freedom, quarks and gluons having the following Lagrangian:

$$\mathcal{L} = \bar{\psi}_i(i\not{D} - m_i)\psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \tag{1}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \tag{2}$$

Here, i is the flavour, a is the colour index over the generators of the non-abelian gauge group G, fermion multiplet ψ belongs to an irreducible representation r of G, m is the mass of the fermion and $\not{D} = \gamma^\mu(\partial_\mu - ig_s A^a_\mu t^a_r)$, where t^a_r are the Gellmann matrices. The last term in the field strength tensor, $F^a_{\mu\nu}$, is the typical of QCD which exists due to the self-interaction of gluons. Here, f^{abc} are the structure constants of the gauge group, G and g_s is the colour charge related to the coupling of QCD, α_s by the relation, $\alpha_s = (g_s^2/4\pi)$. The coupling of QCD stands apart due to its unique dependence on the relevant energy scale. The behaviour of this coupling at different regime of energy is very different. At lower energies or larger distances, the value of α_s becomes larger, so that it is impossible to separate individual quarks from a hadron. This is the reason behind confinement of quarks into colour-neutral hadrons.

But, at high enough temperature and baryon density, it is possible to realise a state in which quarks are no longer bound to their parent hadrons, rather they interact, out of the influence of their parent hadron, within a bigger volume, i.e. the nuclear volume. This phenomenon is called deconfinement where the pertinent degrees of freedom are coloured quarks and gluons. This can be attributed to another fantastic feature of the QCD coupling, the asymptotic freedom, which predicts that α_s decreases with the increase in energy or decrease in distance. Therefore, the interaction between the quarks and gluons becomes weaker when two of them approach each other.

There can be two physical situations in which the quarks can be ‘deconfined ’from their parent hadrons to form a medium of deconfined matter.

i) Naoki Itoh in 1970 [1] and Collins and Perry [2] in 1975 proposed that at high enough baryon density (i.e. if a nucleus is compressed to a critical value of pressure), hadrons within the nucleus will overlap with each other. As a result of this, quarks within the overlapping hadrons behave as asymptotically free particles when they are no longer bound to their parent hadrons.

ii) Same sort of situation can also arise when the nucleus is ‘heated up ’(i.e. provided with enough kinetic energy) to create more and more hadrons up to a critical value of temperature. These hadrons, again, start to overlap resulting into the formation of such a system, called Quark Gluon Plasma (QGP), in which each quark loses its identity of belonging to its parent hadron, but interact as independent degrees of freedom inside the nuclear volume.

This novel state of deconfined quark matter might have existed just a few microseconds after the Big Bang in the early universe and may still exist in the core of a neutron star. It is also believed, after a stunning testimonial at a CERN press release in 2000, that when two heavy ions are collided at relativistic/ultra-relativistic energies in the laboratories (as has been performed in Relativistic Heavy Ion Collider, RHIC at BNL and Large Hadron Collider, LHC at CERN) a system of Quark Gluon Plasma (QGP) is formed.

Open Heavy Quark as a probe of QGP

The physics of Quark Gluon Plasma is a contemporary field of research, both theoretically and experimentally, and it will continue to intrigue scientific minds all over the world in the upcoming years. This is because of the fact that there is ample opportunity to test Quantum Chromo Dynamics (QCD) when applied to a medium at a particular temperature and baryon density. There are various ways in which QCD in vacuum modified by the non-zero temperature and baryon density. Therefore, Quark Gluon Plasma is a good tool to explore different aspects of QCD in a medium. As, QGP formed due to the heavy ion collisions, lasts only for a short scale, both spatially and temporally, it is not possible to measure its properties directly, rather one needs different probes to study different properties of the medium. There are various probes like electromagnetic probe (such as photons, dileptons which do not interact via QCD with the medium particles and carry information of the initial state of the medium), Quarkonia (bound state of heavy quark and its corresponding anti quark) suppression, quenching of Jet (collimated beam of high energetic particles) etc., which give the signature of the formation of a deconfined system, QGP.

In this dissertation, we endeavour to explore Quark Gluon Plasma with yet another class of probes, open Heavy Quark (HQ) originated at a very early stage of heavy ion collision by hard processes. HQ has the following advantages as an 'external'probe: i) They are produced before the formation of QGP due to the early hard scatterings. It can experience the whole evolution of the medium starting from the beginning, ii) Being heavy (mass>> temperature of the bath), there will be negligible thermal production, iii) Total number of HQ is conserved as the probabilities of creation and annihilation are small inside the medium, iv) Thermal equilibration time of HQ is larger than that of light quarks and gluons. Therefore, they can be treated as Brownian particles moving inside a thermal fluid composed of light quarks, light anti-quarks and gluons.

The heavy quark, as a Brownian particle, evolves inside the medium according to the Boltzmann Transport Equation (BTE):

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial f}{\partial \vec{p}} = \left[\frac{\partial f}{\partial t}\right]_{\text{coll}} \tag{3}$$

where $f(\vec{x}, \vec{p}, t)$ is the single-particle distribution function of an ensemble of HQ immersed inside a fluid of light partons. In the absence of any external force, $\vec{F}$, in a uniform plasma, BTE becomes:

$$\frac{\partial f}{\partial t} = \left[\frac{\partial f}{\partial t}\right]_{\text{coll}}. \tag{4}$$

The right hand side of the above Eq. 2.26 is called the collision integral which entails that heavy quark is suffering collisions with the medium particles. In general, it comes under an integral. After linearising this integro-differential equation with the approximation of 'soft 'scattering, one can arrive at a partial differential equation, called the Fokker-Planck Equation (FPE) [7]:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}[A_i(\vec{p})f + \frac{\partial}{\partial p_j}(B_{ij}(\vec{p})f)] \tag{5}$$

One of the main motivations of this dissertation is to solve for this differential equation. In order to do that the following informations are required:

i) The inputs, A_i and B_{ij}, which are basically related to the drag and diffusion coefficients of the heavy quark travelling inside Quark Gluon Plasma. The interaction of the HQ with the medium particles is encoded within these transport coefficients.

ii) The initial distribution function of Heavy Quark, calculated from the initial hard processes at the very early stage of Heavy Ion Collision (HIC). It has been supplied from the MNR code [9] for charm and bottom.

iii) That the HQ is propagating inside an expanding (in space-time) plasma is reflected in the evolution equation along with the proper initial conditions for the background QGP.

Having calculated the transport coefficients of the Heavy Quark and discussing about the space-time expansion of QGP, the method of the solution of the Fokker Planck Equation has been elaborated.

Transport coefficients of HQ

The first step towards solving FPE is to evaluate drag and diffusion coefficients from the interaction of HQ with the light partons constituting the medium. It can be understood from the following discussions that the transport coefficients are really the average (over the relevant phase space) of the momentum transfer between the HQ and the bath particles or the square of the momentum transfer depending upon whether it is drag or diffusion. As we are dealing with the relativistic HQ, it is imperative to take into account the dependence of drag/diffusion coefficients on the momentum of HQ and the temperature of the bath. Heavy Quark, while propagating inside the medium, can lose energy via two basic processes: i) elastic collisions with the medium particles and ii) gluon radiation off HQ due to the inelastic collision.

Collisional transport coefficients

The elastic scattering processes which have been considered here are: $Qq \to Qq$, $Q\bar{q} \to Q\bar{q}$ and $Qg \to Qg$. Whereas Q stands for the Heavy Quark, q, $\bar{q}$ and g are light quark, their corresponding anti-quark and gluon, respectively, constituting the medium. If X_{coll} is denoted to be the notation for elastic drag/diffusion coefficients, it can be expressed symbolically as:

$$X_{\mathrm{coll}} = \int \text{phase space} \times \text{interaction} \times \text{transport part.} \tag{6}$$

The transport part is either the momentum transfer of HQ (in case of drag) or the square of the momentum transfer (in case of diffusion). As all the discussions are dealing with the relativistic Heavy Quark, there will be one drag coefficient and two diffusion coefficients, transverse and longitudinal. These three transport coefficients can be calculated from the above Eq. 2.40. The interaction part is actually the invariant amplitude squared evaluated from the relevant Feynman diagrams. As a first attempt to shield the infra-red divergence (appearing in the t-channel Feynman diagram) due to soft gluon exchange, Debye mass ($m_D \sim gT$) has been used in an ad-hoc manner. Later, the gluon propagator, resummed with the help of the Hard Thermal Loop (HTL) technique, has been employed for the computation of the elastic matrix elements for a more self-consistent shielding in the weak coupling regime [4]. The magnitude of the transport coefficients are enhanced, for the entire range of the momentum of heavy quark and the temperature of the bath, when HTL is used. This is due to the inclusion of full spectral function (inclusion of processes like Landau damping etc.) instead of using only the static approximation, i.e. the Debye mass. Fig. 1 is an example of drag of HQ when varied with temperature, T. Same kind of plots will follow for diffusion and also for the case when varied with respect to the momentum of HQ.

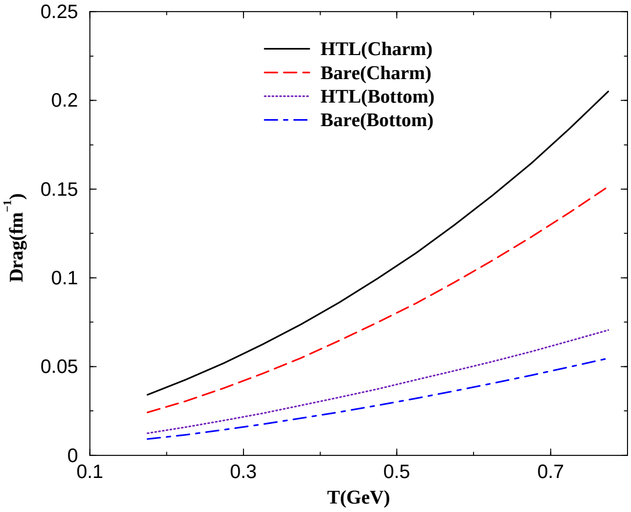

Figure 1: Drag vs Temperature of the bath:comparison between HTL with bare (shielded with Debye mass) propagator for a 5 GeV charm and bottom [4]

Radiative transport coefficients

Transport coefficients have been calculated also when the heavy probe suffers gluon bremsstrahlung while interacting with the QCD medium. The present calculation has been done within the ambit of pQCD and kinetic theory and it is shown that the radiative transport coefficients can be expressed in terms of the elastic (collisional) transport coefficients as long as the emitted gluons are soft, i.e. their energies are much less than that of the HQ. Therefore, in this factorisation limit, the radiative transport coefficient can be generically written as:

$$X_{\rm rad} = X_{\rm coll} \times \int \text{gluon emission spectrum} \times \text{phase space factor of emitted gluon.} \qquad (7)$$

In the above formula, the gluon emission spectrum has been taken for the whole range of rapidity of the radiated gluon [5]. The approximations used in the derivation of the spectrum for the generic process, $Q(k_1) + q/\bar{q}/g(k_2) \rightarrow Q(k_3) + q/\bar{q}/g(k_4) + g(k_5)$:

i) Gluons are soft: $E_5 \ll E_1$. Here, $k_5 = (E_5, \vec{k_\perp}, k_{5z})$ and $k_1 = (E_1, \vec{k_1})$.

ii) No recoil of the HQ due to elastic scattering, i.e. eikonal trajectory1: $q_\perp \ll E_1$, $q = k_1 - k_3$.

iii) No recoil of HQ due to soft gluon emission, i.e. eikonal trajectory2: $k_\perp \ll E_1$.

The approximation which has been relaxed here is the collinear approximation, $E_5 \gg k_\perp$. Instead here, we use $E_5 \geq K_\perp$. It is evident from the figures 3.7, 3.11 and 3.9 that the radiative transport coefficients are larger in magnitude than the collisional ones especially at higher momentum of heavy quark. The same statement is also true for higher temperature and for the case of bottom quark.As bottom is heavier than charm, the momentum dependence of the transport coefficients is less prominent for bottom.

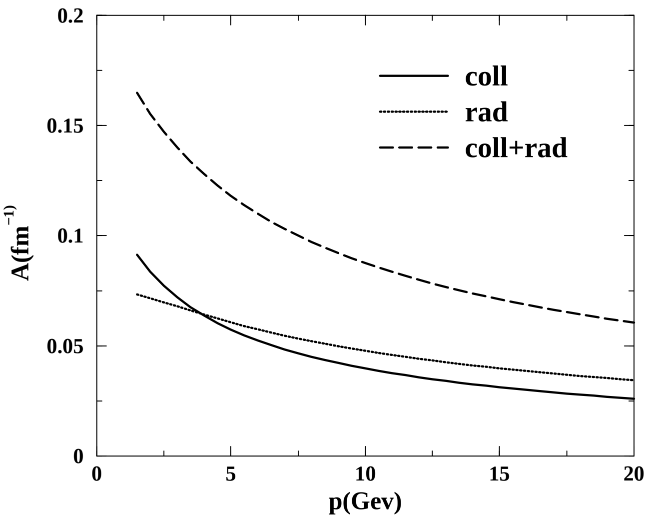

Figure 2: Drag of charm vs momentum at bath temperature, T=525 MeV [7]

Fig. 3.7 shows a dependence of the drag coefficient of charm quark on its momentum. As the momentum of heavy quark increases, the drag decreases because drag is approximately the measure of the inverse of the relaxation time. Higher the momentum of the particle, harder for the medium to relax the particle to equilibrium. Likewise drag, diffusion also has dependence

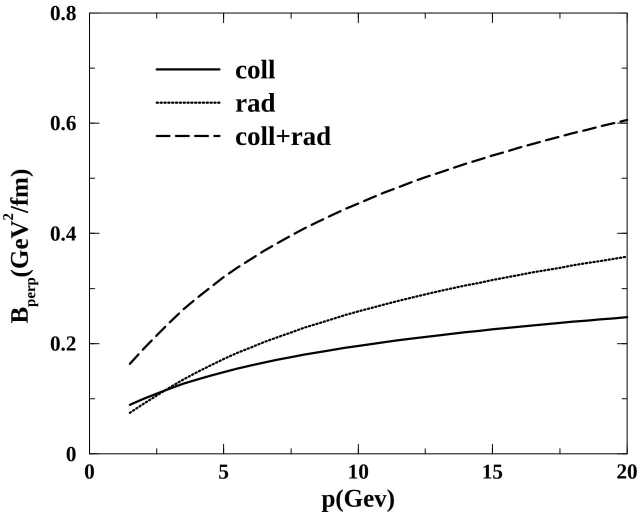

Figure 3: Transverse diffusion of charm vs momentum at bath temperature, T=525 MeV [7]

on the momentum of charm as indicated in Fig. 3.11 and 3.9. So, it is seen that the momentum dependence of the transport coefficients should not be neglected for a relativistic heavy quark because they will have effect on the solution of the Fokker Planck Equation.

It is important to mention that we have performed a calculation of the dead-cone factor in the gluon spectrum of heavy quark when HQ recoils after scattering elastically with the medium. Therefore, this recent effort[Ref. [6]] has relaxed eikonal trajectory1 approximation and has introduced a non- eikonality parameter into the picture.

Effect of gluon radiation on the shear viscosity to entropy density ratio of QGP

Now that the procedure for determination of the radiative transport coefficients is known, its effect can be explored in estimating different observables like the shear viscosity of QGP. The value of the shear viscosity, η to entropy density, s ratio, η/s plays a pivotal role in determining the nature of the QGP medium. It has been shown within certain framework that η/s can be estimated by evaluating the transport parameter, $\hat{q}$, which is the square of the average transverse momentum exchange between the fast parton (probe) and the medium per unit

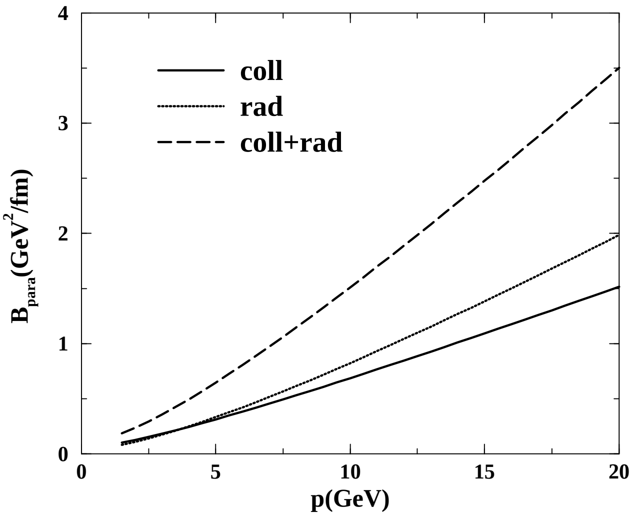

Figure 4: Longitudinal diffusion of charm vs momentum at bath temperature, T=525 MeV [7]

length. In this work, $\hat{q}$ has been related to the transverse diffusion coefficient, $B_\perp$ of the charm quark and the value of $\hat{q}$ turns out to be $\sim 1GeV^2/fm$. It can be shown that η/s of QGP comes closer to the experimental band when gluon radiation by charm quark has been taken into account on top of the elastic collisions [7][Fig. 6.4].

Gluon radiation and the equilibrium distribution function of HQ

The fate of the equilibrium distribution function of a charm quark has also been investigated (we assume charm quark to be equilibrated in the first place) when charm quark undergoes elastic as well as inelastic, i.e. radiative interactions with the medium particles. This equilibrium distribution function of charm never follows that of the thermal Boltzman distributions of the particles of the background medium, rather it is described by Tsallis class of distribution, which might be thought of as a superposition of many Boltzmann distributions. We have used the generalised Einstein relation obtained in the Ref. [11] to show that raditaion has negligible effect on the shape of the equilibrium distribution function of charm quark [7], i.e. whether a heavy quark undergoes elastic collisions only or it also radiates gluons has nothing to do with the ultimate shape of the equilibrium distribution of the charm quark.

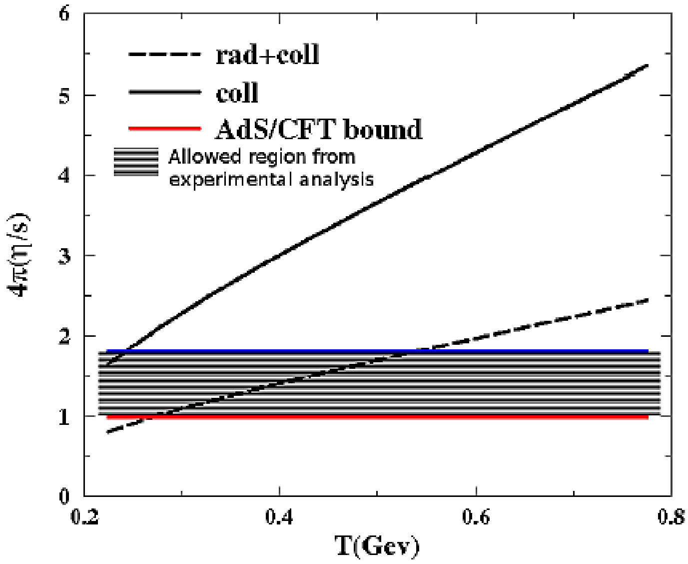

Figure 5: For a charm quark with momentum, $< p_T >= 5$ Gev propagating in QGP of temperature, T [7]

Initial Condition and Expansion of QGP

If one tries to investigate the effect of gluon radiation on another kind of observable like the nuclear modification factor of heavy flavour, R_{AA}, the first and foremost requirement is to solve Fokker Planck Equation. Before solving, it has to be kept in mind that not only the HQ, but also the background medium is evoling with time. So, one must have to take into account this space-time expansion. The initial temperature, T_i and the initial thermalisation time, τ_i for the background QGP expected to be formed in RHIC and LHC can be constrained to the experimentally obtained (final) total mutiplicity in the following way:

$$T_i^3 \tau_i \approx \text{Constant} \times \frac{dN}{dy} \tag{8}$$

Here, we have made use of the boost invariant model of relativistic hydrodynamics proposed by Bjorken [7], [12] for the space-time evolution of the expanding QGP. Therefore, in the previous section when the theoretical result of the nuclear modification factor was compared with that of the experiments, one would require to take particular values for the initial temperature and the initial thermalisation time satisfying the above relation 8. In this way, we got the initial temperature, T_i to be 300 MeV and 550 MeV at RHIC($\sqrt{s} = 200$ GeV) and at LHC ($\sqrt{s} = 2.76$ TeV) respectively.

Solution of FPE and the nuclear modification factor

Now that the drag and diffusion coefficients of the Heavy Quark for elastic scattering as well as for the gluon bremsstrahlung have been calculated and the space-time evolution of the background QGP has been discussed, one can go for solving the Fokker Planck Equation. Being a second order partial differential equation, it needs an initial condition, i.e. the initial distribution function of the heavy quark, $f_{in}(p_T, \tau_i)$ to be supplied when it first sees the medium of QGP around it. These distribution functions for charm and bottom have been taken from the MNR code [9] for proton-proton collision in the absence of the medium. The ratio between the solution of FPE at the critical temperature, $T_c \sim 175$ MeV and the initial distribution, $f_{in}(p_T, T_i)$ is the theoretical estimate of the nuclear modification factor, R_{AA} of open charm and bottom in QGP. To compare our result with experimental data from RHIC and LHC, Peterson fragmentation function has been used to hadronise the heavy quark. Finally, one can obtain the distribution functions for the single electrons originated from the decays of D and B mesons. Therefore, theoretical expression for the nuclear suppression factor becomes:

$$R_{AA}^{D(B)\to e} = \frac{f^{D(B)\to e}(p_T, T_c)}{f^{D(B)\to e}(p_T, T_i)} \tag{9}$$

The momentum dependence of the transport coefficients has been found to be crucial in reproducing the experimental data [10]. It is also observed that the gluon radiation from HQ plays a dominant role in heavy quark propagation, especially in high momentum and temperature domain. Finally, we have been able to reproduce the experimental data for R_{AA} including radiation from HQ[Fig. 7].

Therefore, in conclusion we can comment that i) energy loss of bottom is less than that of charm quark, ii) in order to reproduce experimental data, we should take into account the momentum

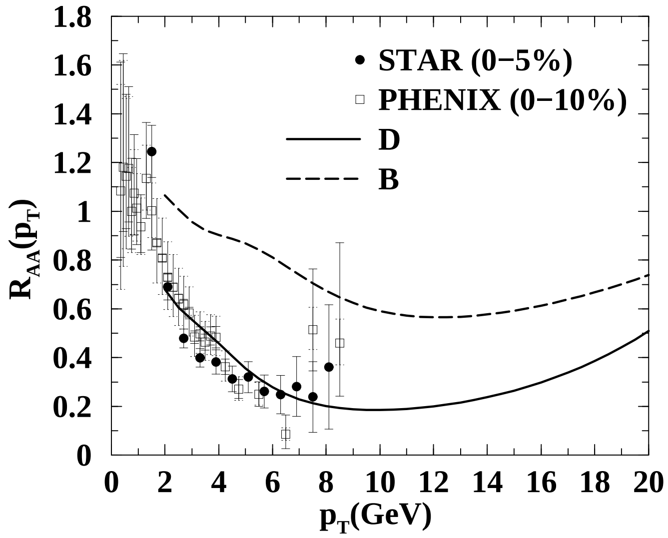

Figure 6: R_{AA} as a function of p_T for D and B mesons at RHIC at centre of mass energy, 200 GeV/nucleon [10]

dependent transport coefficients of heavy quark as well as the contribution of radiation of gluons from the heavy quark.

Effect of Equation of State on the initial parameters of QGP

When Eq. 8 has been used for the QGP expansion, ideal hydrodynamics, where the presuure and energy density relationship is $P = \frac{1}{3}\epsilon$, i.e. the square of the velocity of sound, c_S^2 is 1/3, has been considered. But, lattice QCD (lQCD) result shows some dependence of c_S^2 with temperature, T. Keeping this in mind our motivation has been to use temperature dependent c_S^2 instead of using 1/3 and to try to prdict the a range of the initial entropy density for the background medium. The equation used:

$$s_i \tau_i \propto \frac{dN}{dy}, \quad i.e. \tag{10}$$

$$T_i^{1/c_S^2} \tau_i \propto \frac{dN}{dy} \tag{11}$$

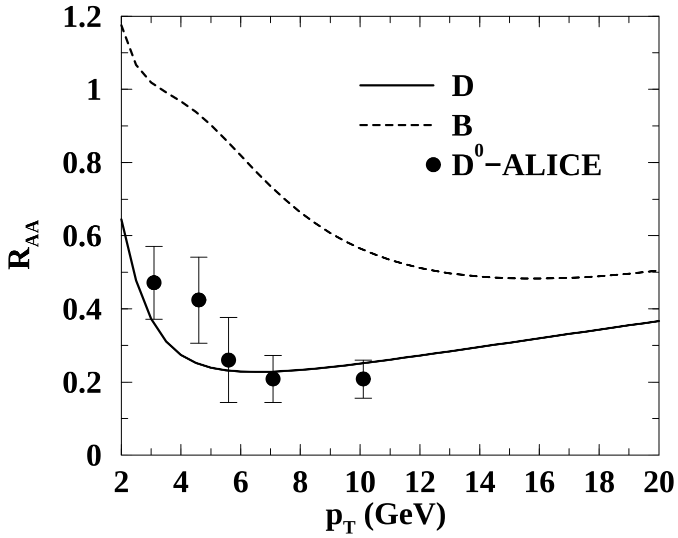

Figure 7: R_{AA} as a function of p_T for D and B mesons at LHC for centre of mass energy, 2.76 TeV/nucleon [10]

For this purpose, the experimental data on the charged particle mutiplicity, $\frac{dN_{ch}}{d\eta}$ (which is connected to $\frac{dN}{dy}$ by a constant numerical factor) and the nuclear suppression factor, R_{AA} of single electron spectra originated from the semileptonic decays of D and B mesons have been employed. It has been mentioned earlier that the plot c_S^2 vs T from lattice has been used to minimise the model dependence. We observe that the value of the initial entropy density, s_i of the QGP varies from 20 to $59/fm^3$ depending on the value of c_S^2 dictated by lattice [13].

Summary

In conclusion, the entire work presented here can be summarised into the following important points:

i) A detailed account of the open heavy quark propagation inside a thermalised medium of light partons is presented in this dissertation.

ii) Both the cases, when the Heavy Quark interacts only elastically as well as when it suffers gluon bremsstrahlung also, have been treated and drag/diffusion coefficients have been computed for both cases.

iii) For the case of a relativistic heavy quark, transverse and longitudinal diffusion coefficients become equally important along with the drag coefficient.

iv) Gluon bremsstrahlung off HQ plays dominant role, especially at the higher momentum of HQ and at higher temperature of the bath, as opposed to the elastic processes in the evaluation of the transport coefficients. These transport coefficients, in turn, are responsible for explaining experimental data on the nuclear modification factor, R_{AA} of HQ.

v) It has been observed that the dependence of the transport coefficients on the momentum of the relativistic heavy probe plays a crucial role in estimating R_{AA} of heavy flavours at RHIC and LHC energies.

vi) It has also been explored that the gluon radiation has quite significant effect in calculating the shear viscosity to entropy density ratio of QGP, whereas it has negligible contribution in determining the shape of the equilibrium distribution function of charm quark.

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Chapter 1

Introduction

1.1 Towards Quark Gluon Plasma:

One of the most exciting and fascinating things happens in physics when a theoretical prediction of significant extent inspires tremendously rigorous and challenging experiments which finally lead to satisfactory results. Such was the excitement when it has been declared on 10th February, 2000 by the then CERN director General that the experiments performed at CERN were giving a clear picture of "a new state of matter" containing coloured quarks and gluons which are not confined to the respective parent hadrons any more.

The previous paragraph has lot of new terms introduced. As we will go deeper into this dissertation we will try to develop a general idea about what is called "a new state of matter" or what does confinement or deconfinement mean. Before going into the details our first and foremost job is to understand what are known as "quarks" and "gluons" and what are the basic laws of physics governing those.

The quest of understanding nature depends, in many ways, upon knowing the basic building blocks of matter. If one looks back in the history of how we have come to know about the elementary or indivisible particles of nature, we might understand the amount of hard work it took to arrive at today's scenario. Starting from ancient times, continuous effort has been on to probe deep inside the matter in search of its basic constituents. For many decades, atoms were known to be the smallest indivisible blocks of matter. But, in 1911, Ernest Rutherford performed a scattering experiment which made it clear that an atom is basically composed of negatively charged electrons and a concentrated small sized heavy positively charged nucleus. It was established later that whereas the electrons are elementary particles themselves, nucleus is made of positively charged protons and electrically neutral neutrons. Both protons and neutrons are hadrons which have been thought of as composite particles made up of more sub-structures. In 1968, Deep Inelastic Scattering was performed at Stanford linear Accelerator (SLAC), where a high energy beam of electrons was scattered off a proton target. This is just like taking a photograph of the inside of a proton in pursuit of knowing its basic constituents. The sub structure of proton can be "seen" simply by bombarding electrons where the momentum transfer is very large. The Deep Inelastic Scatterings like $e+p \to e+X$, where X is anything constituting the inner structure of proton, showed that the cross sections manifest scale invariance at the higher energies. This means that the form factors start to lose their dependence on the mass scales relevant in the problem.

The DIS experiment results were successfully explained by the parton(the name came from 'part of nucleon) model developed by Feynman. This is a very naive model where the constituents of the proton are described by free point-like particles. Despite being a simple model it could throw light on many qualitative features of DIS including Bjorken scaling. But, the problem with parton model lies elsewhere: proton, being a bound state of a the then unknown force, should be described dominantly by non-perturbative effects, whereas in the parton model, at extremely high energies the partons could be described by free point-like particles.

With the advent of Quantum field theory of strong interactions, i.e. Quantum Chromo Dynamics (QCD), the explanation of the DIS experiments was remarkable. It has been shown that the renormalised strong coupling varies with relevant scale of energy. At asymptotically high energies, quarks may be treated as free point particles due to the fact that coupling becomes vary small in that regime of energy. Therefore, inside a hadron the partons will behave almost like free particles. That is why parton model has been so much successful in explaining the DIS experiments. This phenomenon is called *Asymptotic freedom*[1]. This feature is attributed to the fact that QCD is non-abelian in nature.

The opposite feature exhibited by QCD is the *confinement*, through which it may be justified why quarks are colour neutral, i.e. they are confined within the small volume of hadrons. This phenomenon happens because at smaller and smaller energies, the QCD coupling becomes so strong that quarks cannot fly apart from hadrons.

It is evident that whereas the leptons like electrons are governed by the field theory of electromagnetic interactions, Quantum Electro Dynamics (QED), the governing law for describing the interactions between quarks and gluons is QCD. Through this QCD interactions, they remain confined in colour neutral hadrons.

In this dissertation, our one of the principal aims is to be able to creat such a state where quarks can be made deconfined, aparently, within a volume greater than the hadronic volume. We will try to understand and answer the following question: Is it possible to have such a scenario when quarks might be deconfined from their parent hadrons? Though at very high energies, asymptotic freedom will let us treat the partons more or less as free particles, but as we try to isolate a quark from a hadron, the coupling will become so strong that it would be impossible to liberate the quark from its parent hadron (confinement feature of QCD). There must have to be other processes through which one is able to create this kind of state. Starting from Deep Inelastic Scattering the emergence of the concept of such a state is not at all a straightforward story. Though, here, we are not interested in the methodical development, it will be really constructive to know why two heavy ions are being collided in the laboratories at increasingly higher energies all over the world.

Therefore, asymptotic freedom is the very reason responsible for the creation of the deconfined coloured quark matter. To this end, let us discuss the running coupling of QCD which can be related to a function called 'β-function'through an equation called the 'Renormalisation Group Equation'. The solution of that equation will give the dependence of the QCD coupling with the energy scale. Before doing that we should answer a couple of questions like:(i) what is a renormalisation group equation in the first place? and (ii) what is known as the β-function of a gauge theory? The answer to the first question will require a basic knowledge about the gauge theory that it might encounter certain divergences resulting from the integrations like:$\int p^n dp$, where n is a positive integer and p is any momentum ranging from 0 to ∞. So, one needs a cut off in the momentum scale or a subtraction point, μ to make sure that the final results are not diverging due to the divergent integration. The differential equation showing the dependence of the running coupling of QCD on this cut off or the subtraction point is called the 'Renormalisation Group Equation'. It also reflects the fact that the final results of a theory or the basic physics should not depend on the subtraction point. The coefficient of this equation (i.e the right hand side of this equation) is what is called the 'β-function'. The first order differential looks like:

$$\mu \frac{\partial g}{\partial \mu} = \beta(g), \tag{1.1}$$

where, g is the coupling of QCD. The β-function of QCD can be seen to have the following form (the method of calculation is not given here, any text book on QCD can be consulted for this):

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(11 - \frac{2}{3}N_f\right), \tag{1.2}$$

where, N_f is the number of flavour taken in this context. The fact that the β-function is negative in case QCD is the reason why QCD is an asymptotically free theory. Knowing this QCD β-function one can solve the differential Eq. 1.1 for the coupling whic varies with respect to the energy scale.

After this brief account of the non-abelian nature(asymptotic freedom) of QCD, let us come back to the present context of this thesis: the creation of Quark Gluon Plasma(QGP) in heavy ion collisions. Though we are continually using the word "deconfined" state, it is very important to keep it in mind that the quarks and gluons are not deconfined within any possible volume. However, in this context, it is the nuclear volume instead of the hadronic volume within which quarks are bound by strong interaction. The meaning of deconfinement can easily be understood by the following simple picture of how a matter of freely moving quarks and gluons are formed from the hadronic state[2]: The hadrons are considered as the quark bags within which they are confined by the interaction between themselves. As the mediator of the strong interaction is the gluon, therefore, hadrons are really a composite system of bound quarks and gluons. A nucleus can thus be thought of as a dense system of quark bags, i.e. neutrons and protons. Now, we start to imagine that what will happen when the nucleus is compressed, i.e.when we increase the baryon density or we heat it up keeping the volume fixed such that the pair creation of pions (there might be generation of other hadrons too) comes into the picture. At these extreme scenarios quark bags will start to overlap with one another and a quark matter will be formed where all the quarks and gluons will no longer be bound to their parent hadron rather they will be deconfined within a bigger volume, i.e. the nuclear volume. The situation also arises in condensed matter physics in case of metal-insulator transition called Mott Transition. In case of an insulator where the distances between the atoms are large, electron density is small leading to a weak Coulomb screening, therefore the electrons are bound. If the atoms are moved closer together the electron density increases and the electron feels a strong Coulomb screening. Therefore, the energy levels move up and after a certain point there are no available bound states for the valence electrons and the insulator becomes a metal. There is a critical screening length after which the last electron state is bound no longer.

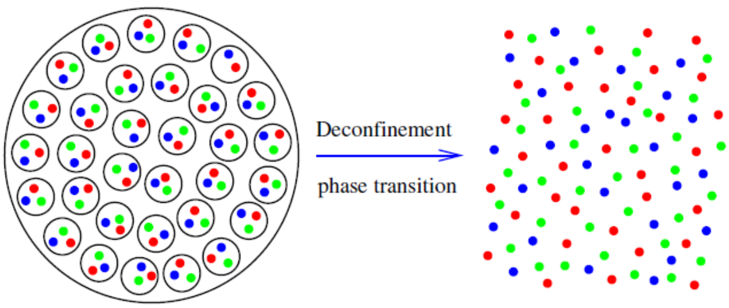

Figure 1.1: Transition from hadronic to quark matter [2]

The above example is just to motivate that this type of transitions are also available in other physical situations. Just like the Mott transition, there are certain critical values for the baryon density and the temperature beyond which the hadrons will no longer exist and a matter is formed which is "coloured". It has been predicted theoretically that in order that such melting of quark bags will occur the critical baryon density (which is taken care through the baryochemical potential, μ_B, to be non-zero) needed is ten times the normal nuclear matter density, $\rho_0 = 0.125\ GeV/fm^3$ and the temperature required is of the order of $160 - 180\ MeV$. This concept of the creation of the deconfined matter due to asymptotic freedom was first proposed by Naoki Itoh in 1970 and then by Collins and Perry in 1975 [3].

Such a state of deconfined quark matter might have existed a few microseconds after the Big Bang in the early universe. Also, the core of the neutron star might contain this state. If we wish to analyse these systems, one of the best ways is to recreate such a system in the laboratories. It is expected that when two heavy ions are collided at relativistic energies, a medium composed of deconfined quarks and gluons will be formed. Recently, such a system, beleived to be created at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC), intrigues scientific mind all over the world with its mutifarious interesting aspects.

We can have a rough and handwaiving idea about the pressure and energy density needed to create the deconfined quark matter at $\mu_B = 0$ [4]

The simplest example of confined or hadronic matter can be a purely massless pionic gas. The pressure of such non-interacting gas, where we have taken all three pion charge states, is given by:

$$P_\pi = \frac{\pi^2}{90} g_{\rm pion} T^4 \approx= \frac{1}{3} T^4 \tag{1.3}$$

g_{pion}=pion degrees of freedom=3. The most simple example of the deconfined matter is the ideal QGP for which the pressure can be written as:

$$P_{\rm qgp} = \frac{\pi^2}{90} g_{\rm qgp} T^4 - B \tag{1.4}$$

where, $g_{\rm qgp}$ is the number of degrees of freedom in QGP=$[2\times 8 + \frac{7}{8}(2\times 2\times 2\times 3)]$, gluon having two polarisations and eight colours, massless quarks and anti-quarks having two spins, two flavours and three colours. B is the bag pressure which is actually taking care of the pressure exerted by the vacuum on the coloured medium. Since the state with minimum free energy or highest pressure is favourable in nature, therefore, the transition is from a low temperature pion gas to a high temperature QGP. The critical temperature for this first order transition will be determined by the condition $P_\pi = P_{\rm qgp}$, and which is $T_c = (0.3 \times B)^{1/4} = 150$ MeV if $B^{1/4} = 200$ MeV extracted from the quarkonia spectroscopy. The relation between pressure and energy density, ϵ for massless ideal constituents is $\epsilon = 3P$. Therefore, the corresponding energy densities are $\epsilon_\pi \approx T^4$ and $\epsilon_{\rm qgp} \approx 12T^4 + B$. The above calculation was really a back of the envelope one and are very much idealistic, a real QGP is never massless, non-interacting and the relation between energy density and pressure is not so trivial (a real QGP is really non-conformal). But, we can have a order of magnitude estimate of the critical energy density which is 0.5 to 1.0 GeV/fm^3.

1.2 Probing the Quark Matter:

Once the deconfinement sets in, it is of prime importance to understand its different properties fully, i.e. to study its behaviour at different temperature and baryon density. The medium produced in the heavy ion collisions expands very fast. Its lifetime ($5 - 10$fm$/c$) and the spatial volume both are far too short to measure any of the properties directly. Therefore, we need to search for a probe. A probe is a particle produced in Heavy Ion Collisions and interacts with the medium in such a way that it carries relevant informations of the medium through which it has propagated and has come out retaining its identity but being somewhat modified by the medium. By looking at the modifications of the probe and analysing them we can retrace back various characteristics of the medium.

We will have a brief discussion about the possible probes [5] of the quark matter produced in heavy ion collisions particularly emphasising on the type of probe we will be dealing with in the rest of the chapters. The choice of probe actually depends upon the sensitivity of that probe with certain properties of the medium. Therefore, we can see the response of the system towards the probe and can gain insight in different characteristics of the system.

1.2.1 Electromagnetic probe

The medium formed due to the heavy ion collision at relativistic energies are considered, by definition, to be hotter than its surroundings (vacuum). Therefore, in the hot QGP, due to the quark anti-quark annihilation real and virtual photons are created. These virtual photons in turn give rise to the electron-positron or muon-antimuon pairs. The intearctions are in the leading order $q\bar{q} \to \gamma$ or $q\bar{q} \to \gamma^* \to l\bar{l}$. Also there are other various sources of photons and dileptons. The main processes through which electromagnetic probes (photons and dileptons) can be generated are the following:

1. Directly from the hard collisions of the partons at the time of nuclear collision $q\bar{q} \to g\gamma, q(\bar{q})g \to q(\bar{q})\gamma$ etc.

2. Directly emitted from the thermalised QGP and the hadronic sector,

3. decay photons originated from the hadronic decays like, $\pi^0 \to \gamma\gamma, \eta^0 \to \gamma\gamma$ etc. Dileptons can be generated from Dalitz decay. Electromagnetic signals probe the structure of the electromagnetic current-current correlation function (response function):

$$W_{\mu\nu}(q^2) = \int d^4x\ d^4y\ e^{iq(x-y)} < j_\mu(x) j_\nu(y) > \tag{1.5}$$

where, $j_\mu(x)$ is the electromagnetic current. As they do not interact with the strongly intearcting medium, they have a large mean free path compared to the size of the system. Therefore, they will emit unaffected and will carry the information from whole of the evolution of the medium. They are one of the most clean probes of QGP as they help us see the most earliest and hottest part of the evolution [6, 7]. But, the problem with them is that they can be emitted from all the evolution stages and there will be a lot of backgroud emission from hadronic decay processes. If we can differentiate the early stage radiation from backgroud it will be a very good signal of QGP [8, 9].

1.2.2 Quarkonia suppression

The basic mechanism of the deconfinement of the matter produced in the heavy ion collision is the Debye scrrening. When an external static charge is put into a QED plasma in equilibruim, its electric field is screened by the charges in the plasma. The Debye mass or the transverse Debye screening length is given by the pole of the photon propagator. In case of QCD, which is actually a non-abelian gauge theory, as far as the non-perturbative treatment is valid, the Debye screening has the same sort of interpretation like that in case of QED. In the leading order, it looks like:

$$m_D = (N/3 + N_f/6)^{1/2} \, gT \tag{1.6}$$

where N_f is the number of flavours in the SU(N) gauge theory [10]. As the definition of the Debye screening is not complete in perturbative domain because at next to leading order the problem becomes highly non-perturbative, there are other methods of estimating Debye screening such as Lattice gauge theory, some effective field theories etc. Here, what we need is the very basic idea that when the screening radius, r_D becomes less than the binding radius of the hadrons,r_H, deconfinement sets in. We will discuss, very briefly, what happens to a heavy quarkonia when it is put inside such a deconfined medium. Quarkonia are basically bound states of heavy quark anti-quark pairs ($c\bar{c}$, $b\bar{b}$). They are much smaller than the light hadrons and are much more tightly bound with binding enegries up to 0.5 to 1.0 GeV. Due to the early parton-parton hard scattering, a $c\bar{c}$ pair will be produced before QGP is formed. The resonance intearction of the $c\bar{c}$ system will then lead to J/ψ production. After being produced the $c\bar{c}$ pair finds itself in a deconfined medium. If the temperature of the surrounding medium is enough high and if the Debye screening radius is smaller than the size of J/ψ, the resonance interaction will not be operative and the produced $c\bar{c}$ will not result into a j/ψ, rather they will propagate separately inside the medium. To understand the above concept in more details, let us look at the typical radius of J/ψ. From a non-relativistic treatment at temperature, $T = 0$, we can get an idea about the radius of J/ψ. In vacuum, the non-relativistic $c\bar{c}$ potential is

$$V(r) = \sigma r - \alpha_{eff}/r \tag{1.7}$$

where σ is the string tension and α_{eff} is the Coulomb interaction coupling. Now, if we calculate, for an isolated system of $c\bar{c}$ at $T = 0$ and with a mass of charm to be 1.5 Gev, the magnitude of the J/ψ radius will be $r_{J/\psi} = 0.2fm$. If we vary different parameters like mass, string tension and α_{eff}, we will have an estimate: $0.2 \leq r_{J/\psi} \leq 0.5fm$. Now, we will have to see what happens when this J/ψ falls into a thermal medium with temperature, $T \geq T_c$. At $T \geq T_c$, there will be no string tension (as $\sigma(T_c) = 0$) which leads to the following form of the Coulomb Screened potential:

$$V(r) = -\frac{\alpha_{eff}}{r} exp(-r/r_D) \tag{1.8}$$

where r_D is the Debye screening radius. But, this potential, as it stands, can still provide bound state description. Again, with this potential, by minimising the binding energy, we can show that the smallest value of the screening radius which permits a Coulombic bound state is;

$$r_D^{min} = [0.84m\alpha_{eff}(T)]^{-1}. \tag{1.9}$$

This is just a rough estimate and according to this estimate when the Debye screening radius goes below the above mentioned value it becomes smaller than $r_{J/\psi}$. It has been observed from different studies that the existence of J/ψ will be excluded when $T/T_c = 1.2$ or even less. Except J/ψ's, there are other quarkonia states having different sizes and binding energies. Therefore, at first, the larger and more loosely bound excited states are dissolved and finally the smallest and the most tightly bound states. This is called the sequencial suppression of quarkonia. This is indeed a good probe of QGP, because it is really sensitive to the changes in the colour response function of the medium, i.e. whether a 'coloured' system is formed or not can be inferred from the study of sequencial melting of quarkonia [11, 12].

1.2.3 Jet Quenching

As the quark and gluon Jets are coloured objects, they can very well probe the coloured structure of QCD matter. When a Jet propagates inside QGP, it can lose energy in two different ways: i) one by colliding with the medium particles elastically and ii) other by radiating gluons after elastic collision in its course of propagation. At the time of collision between two heavy ions, hard scatterings give rise to fast partons which after some time due to strong interaction develop a shower of particles around itself. The fast leading parton along with other daughter partons constitute a collimated beam called a jet. When this Jet is created in a heavy ion collision, it finds a thermal and dense medium around it. In the process of its propagation through the medium, a jet interacts with the medium particles and loses its energy/momentum untill it gets out of the medium and hadronises.

Let us find out what information we can extract from the energy loss of a jet. Let us suppose that a back to back (at the leading order, due to the momentum conservation) jet is created at the edge of the medium 1.2, then one jet which is near the edge can leave the medium without interacting much with the medium particles while the other jet will have to interact with the medium particles and consequently its energy will be attenuated.

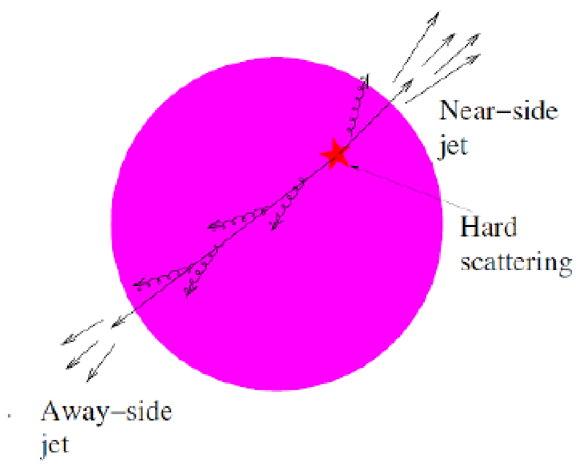

Figure 1.2: Energy loss of a away side jet[13]

The energy loss of a fast charged particle is a well-known problem in QED. Just like the case in QED, when a highly energetic parton travels through a QCD medium it will lose energy or in other words, its transverse momentum will be broadened. This energy loss is expected to be greater in heavy ion collison (A+A) than in case of p+p or p+A collisions. This is called the "jet quenching".

It can be shown that the loss of energy per unit length by the fast partons due to the elastic collision with the medium particles is proportional to the square of the strong coupling[14]:

$$-\frac{dE}{dx} \propto \alpha_s^2 T^2 (1 + N_f/6) ln(\frac{E}{\alpha_s T}) \tag{1.10}$$

where T is the temperature of the thermal medium and E is the energy of the fast parton. Because of the dependence on T^2, it has been pointed out by Bjorken that $-dE/dx$ can be shown to be proportional to $\sqrt{\epsilon}$ [15], where ϵ is the energy density of the QGP medium. There is a rather self-consistent way of calculating the collisional energy loss using the resummed Hard Thermal Loop propagator in order to take into account the proper screening effect. These calculations have also been done in many literatures.

In the QCD medium, gluon bremsstrahlung is also a major mechanism of losing energy. After its production from hard processes a fast parton emits gluon radiation and both the leading parent parton and the radiated gluon would have to traverse the medium of length L, suppose. It has been shown that the energy loss of the leading parton due to the soft gluon radiation is proportional to the strong coupling (α_s) and the energy (ω) of the emitted gluon[16]:

$$\Delta E \simeq \alpha_s \omega \tag{1.11}$$

where

$$\omega = \frac{1}{2} \hat{q} L^2 \tag{1.12}$$

Here, in the above expression $\hat{q}$ is called the transport coefficient connected to the jet which is propagating. The physical interpretation of this transport parameter is the average of the square of the transverse momentum transfer between the medium and the fast parton per unit length of the medium:

$$\hat{q} = \frac{< q_\perp^2 >}{L} \tag{1.13}$$

This relation is very much important in the context of probing QGP medium because it has a dependence on the energy density of the medium produced and its value in QGP is more than that in hadronic medium. Therefore, it is obvious that the jet will be more suppressed inside the coloured matter.

1.2.4 Heavy Quark Jet

Thus far, we have been discussing about the energy loss of jets where the leading fast parton is either a light qquark or a gluon. Also, a heavy quark produced due to the early hard collision before the medium formation can very well form a collimated parton shower resulting into a jet. These heavy quark jets while propagating through the quark matter will also lose energy via elastic as well as inelastic interactions with the medium particles. It has been observed in the experimental data of the nuclear modification factor, R_{AA} that heavy quarks lose quite an amout of energy in the medium. Therefore, they can also be considered as a very good probe for detecting QGP. Besides, they have certain nice extra characteristics which make them excellent probes:

i) We know that the QCD interaction conserves the flavour, i.e. it is flavour independent. Gluons only couple to the colour charge of the quark. Therefore, as such there should be no basic differences between light and heavy quark jets. Nonetheless, the presence of quite a heavy mass ($m \gg T$) might change the scenario as far as the kinematics, i.e. the phase space substantially. It can limit the phase space in case radiation of a gluon off a heavy quark.

ii) As the mass of heavy quark is significantly larger than the typically attained temperature of the medium produced in heavy ion collisions and the other non-perturbative scales, $m \gg T, \Lambda_{QCD}$, the production of heavy quarks is constrained to the early stage of the collision. It is very unlikely that the heavy quark numbers will be changed due to their production inside QGP medium.

iii) Thermalisation time of heavy quark ought to be larger than that of the light quarks/gluons by a factor m/T. Rough estimations show that whereas light quarks/gluons thermalises within 0.3 to 1 fm/c time, the heavy quark thermalisation time, τ_Q are almost 5-20 times larger and may be comparable to or even larger than the QGP lifetime, $\tau_{QGP} \simeq 5fm/c$ in a central Au+Au collisions.

iv) Being much heavier than the medium particles, the heavy quark can be described by the theory of Brownian motion. Non-relativistically, the typical thermal momentum of a heavy quark is $p_t^2h \approx 3mT \gg T$ which is much larger than the typical momentum transfer scale ($\sim T$) of heavy quark from the medium. consequently, the whole treatment of the heavy quark can be performed on the assumption of small momentum transfer with the help of Fokker Planck equation quite effectively.

We will elaborate on the heavy quark energy loss regarding the elastic collision as well as the radiative loss by the way of gluon bremsstrahlung in the later chapters, subsequently.

1.3 Organisation of the thesis:

The thesis will mainly stress upon the motion and the transport coefficient of heavy quark when used to probe the system of QGP.

In Chapter-II, the motion of heavy quarks will be discussed in details after it enters the thermalised quark matter. Therefore, HQ is a so-called "external" probe to the medium. We will see that the evolution of heavy flavours is described by the theory of Brownian motion. Different transport coefficients of heavy quarks have been evaluated in the framework of the Fokker-Planck Equation (FPE) while they are traversing the deconfined medium. The formalism of FPE will be illustrated in quite details and different underlying assumptions to use this kind of formalism will be discussed accordingly. In this chapter, we will only deal with the elastic binary collisions suffered by the probe with the light medium particles. The fact that the medium is a thermalised one will be taken care of by shielding the infra-red divergence of the dynamics by using Debye mass and then by using the resummed gluon propagator in the framework of leading order Hard Thermal Loop (HTL) theory. The drag and diffusion coefficients of the probe are plotted as functions of the momentum of the probe and temperature of the medium and different implications are pointed out.

Chapter-III will mainly deal with the gluon radiation emitted by the Heavy Quarks by virtue of their interaction with the medium partons. First, there will be a generic discussion regarding the spectrum of emitted gluon from the heavy probe with a small overview of the different approximations and assumptions used in this work. The transport coefficients like drag, transverse and longitudinal diffusion coefficients are calculated in case of radiative energy loss and they are plotted showing a comparison with their values when HQ is suffering elastic collisions only.

Once the transport coefficients of HQ are known for both elastic as well as inelastic collisions with the medium, Fokker Planck Equation is now solved in Chapter-IV. In order to solve this equation, we also took into account the background for which the space-time evolution is governed by the hydrodynamic conservation equations. Bjorken boost invariant hydrodynamic model (which is a one dimensional hydro) has been considered here. In this way we observe the effect of the medium on the distribution function of the Heavy Quark when it emerges from the coloured medium. The medium effect thus encoded in the distribution function of the probe is studied considering nuclear modification factor, R_{AA} as the relevant observable. Theoretically extracted R_{AA} has been contrasted with the experimental data as observed in the heavy ion collision at RHIC/LHC and it is seen that they quite agree with each other.

The initial thermalisation time, τ_i and the initial temperature, T_i of the medium of deconfined quarks and gluons cannot be calculated from any first principal theoretical considerations as yet. There is always a sort of model dependence in their values. In Chapter-V, we have presented a naive estimation of the initial entropy density, s_i/initial temperature, T_i of the medium by varying the square of the velocity of sound, c_S^2 (as per lattice results) with temperature, T and by comparing our theroretical result of R_{AA} (for each value of c_S^2 taken) with the experimental data obtained in Au+Au collisions at centre of mass energy, $\sqrt{s} = 200$ GeV/nucleon.

In Chapter-VI, the radiative energy loss off Heavy Quark has been recast as an important aspect in determining the shear viscosity of QGP. We observe here that the shear viscosity to entropy density ratio, η/s of QGP is closer to the experimental value when gluon radiation off charm quark has been taken into account. But, the story is very different if we want to see the effect of gluon bremsstrahlung on the equilibruim distribution function of the charm quark (if we consider charm to reach equilibruim finally). It will be shown that the soft gluon emission from charm has no effect on the shape of the charm quark distribution function.

In the final chapter, we will briefly discuss about different scpoes and outlooks in the direction of the heavy quark energy loss. All over the world, many groups are presently working on the derivation of the spectrum of the gluon radiated from the heavy probes. They are basically using different approximations and assumptions regarding the dynamics of the heavy quark propagation inside QGP. In the perspective of this dissertation, the radiated gluon spectrum really helps in calculating the transport coefficients of heavy quarks when it emits a gluon bremsstrahlung. It is not yet a settled issue that whether we should take into account both the radiative and collisional effect or whether and at what regime one effect overcasts the other. So, we will make some critical comments in this respect and try to address this unsettled issue with a more detailed description of the evaluation of the radiative transport coefficients with an up-to-date gluon emission spectrum.

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Chapter 2

Evolution of Heavy Quark in Quark Gluon Plasma

2.1 Motion of Heavy Quarks as Brownian particles: Formalism:

Let us start this chapter with a brief overview of the physics of Brownian motion. How heavy quarks satisfy the criteria of being a Brownian particle will be elaborated in details with a highlight on the underlying assumptions.

The problem of Brownian motion dates back to very early time (observed by Robert Brown in 1827 and explained by Einstein in 1905) in the field of non-equilibrium statistical mechanics. It is an age old classical problem which deals with the motion of a heavy particle immersed in a fluid made up of light particles. In this thesis, as we are interested to study the motion of a relatively (in comparison with the other scales of the problem) heavy mass particle, we will also remain within the classical regime as far as the equation of motion of the massive particle is concerned. In classical mechanics, if we need to understand the time evolution of the motion of a particle, we have to solve some sort of differential equation (like Newton's laws, Lagrange's equations etc.) and we can determine the exact values of the observables like position, velocity or momentum of the particle as functions of time for the given initial conditions. Therefore, those are called deterministic approach of solving the evolution equation for the relevant particle.

At this point we should pause and ponder over the fact that for Brownian motion, we are really tackling a statistical problem with an effect of many particles constituting the fluid on the heavy mass immersed in it and more over we have to keep it in mind that though the motion of heavy quarks has been treated as a classical one, but the microscopic interaction suffered by them with the fluid particles is governed by a quantum field theory of strong interactions,i.e. Quantum ChromoDynamics. The above two points are very critical and should be handled very carefully. First, we will consider the problem of Brownian motion as a probabilistic one and continue the journey upto the derivation of the equation of motion of HQ. After having completed this discussion we will incorporate QCD into our picture.

The problem of Brownian motion is a particular example of the general theory of random or stochastic processes. There are two ways to treat this problem: i) Langevin approach and ii) Fokker Planck approach. Although, the present work rests on the Fokker Planck formalism, yet it will be quite constructive to discuss the alternative approach, which is due to Langevin.

2.1.1 Langevin Formalism:

The evolution of heavy quarks traversing through the medium of Quark Gluon Plasma can be governed by Langevin Equation. This formalism has been employed by many authors in various literatures [1],[2],[3], [4],[5] etc.

Non-Relativistic Heavy Quark:

Let us first treat heavy quark as a non-relativistic Brownian particle. The equation of motion followed by the heavy quark will then be:

$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t) \tag{2.1}$$

$$\frac{dx_i}{dt} = \frac{p_i}{M} \tag{2.2}$$

Here, M is the mass of the heavy quark, η_D is the momentum drag coefficient and $\xi_i(t)$ takes care of the effect of the random collisions of the Brownian particle with other light fluid particles. Collisions are included via a Gaussian white noise term characterised by the correlation function of the fluctuating force.

$$< \xi_i(t)\xi_j(t') >= \kappa\delta_{ij}\delta(t-t') \tag{2.3}$$

3κ is the mean squared momentum transfer per unit time which arises from the cumulated effect of many uncorrelated momentum kicks. As heavy quark is considered to be a nonrelativistic one, there is only one diffusion coefficient, i.e. the medium appears to be isotropic to the heavy quark which is near to the thermal equilibrium, yet not in equilibrium.

The solution of the Eq. 2.2 is the following:

$$p_i(t) = \int_{-\infty}^{t} dt' e^{\eta_D(t-t')}\xi_i(t')x_i(t) = \int_0^t dt' \frac{p_i(t')}{M} \tag{2.4}$$

where $t \gg \eta_D^{-1}$ has been assumed. This formalism [6] has been developed for a nonrelativistic HQ, where $M \gg T$ with thermal momentum of HQ is $p \sim \sqrt{MT}$. Now, if one can determine momentum space diffusion coefficient, the frictional force or drag coefficient, η_D can be related to it from the non-relativistic fluctuation-dissipation theorem:

$$\eta_D = \frac{\kappa}{2MT} \tag{2.5}$$

Relevant time scales for non-relativistic Langevin Equation:

After we have discussed Langevin equation as the equation of motion of non-relativistic Brownian particle, we should also mention the range of validity within which this approach is relevant and useful. For a non-relativistic heavy quark($M \gg T$), one can calculate the interaction rate, Γ of HQ with the medium particles and it comes out to be $\Gamma \sim g^2T$ [7]:

$$\Gamma = g^2 T \int \frac{dq}{(2\pi)^3} \frac{\pi m_D^2}{(\vec{q}^2 + m_D^2)^2 q} \tag{2.6}$$

In the above equation, the only scale within the integral is the Debye mass, $m_D \sim gT$. Therefore, the typical momentum exchange in the interaction will be of the same order. Consequently, we can talk about two time scales:

i) The time interval between two collisions:

$$\Delta\tau = \frac{1}{\Gamma} \sim \frac{1}{g^2 T} \tag{2.7}$$

ii) Duration of a collison:

$$\tau_{coll} \sim \frac{1}{gT} \tag{2.8}$$

We could see from the above two estimations of the time scales that in the weak coupling regime, we can take $\tau_{coll} \ll \Delta\tau$, which may be the range of validity of the non-relativistic langevin treatment. Owing to its large mass which is very very large compared with the temperature of the bath, the total number of collisions needed to change the squared momentum by a factor of 1 is huge. Typically, the HQ relaxation time is greater than that of the light quark by the following amount [6]:

$$\tau_{heavy} \sim \frac{M}{T} \tau_{light}. \tag{2.9}$$

Therefore, while solving langevin equation by discretising the time derivative, one needs to keep the different time scales in mind. The time step, Δt should be sufficiently large to include many collisions, but should be much shorter than τ_{heavy}. The the time hierarchy would be:

$$\tau_{light} \ll \Delta t \ll \tau_{heavy}. \tag{2.10}$$

Relativistic Heavy Quark:

In the non-relativistic treatment of heavy quarks it has been considered that HQs are not too far from the equilibrium. But, it is observed that in heavy ion collisions, charm and bottom quarks are produced with sufficient transverse momentum. Therefore, it is quite necessary to study HQs relativistically, i.e. when they are fast moving. Still, it is worth mentioning that throughout the treatment the heavy quarks are always considered to be different than the light medium partons.

The Langevin Equation for a relativistic heavy quark can be written as:

$$\begin{aligned} \frac{dp^i}{dt} &= -A(\vec{p})p^i + \xi^i(t), & (2.11)\\ < \xi^i(t)\xi^j(t') > &= B^{ij}(\vec{p})\delta(t-t'), & (2.12)\\ B^{ij}(\vec{p}) &= B_{\parallel}(p)\frac{p^i p^j}{p^2} + B_{\perp}(\delta^{ij} - \frac{p^i p^j}{p^2}). & (2.13)\end{aligned}$$

Here, the change of notation is in order:$\eta_D \rightarrow A(\vec{p})$ and $\kappa\delta_{ij} \rightarrow B_{ij}$. It should be mentioned in this note that for the relativistic heavy quark it is very important to consider both of the diffusion coefficients, longitudinal and transverse, and also to keep the full momentum dependence of the transport coefficients. In the above Eq. 2.13, A is the drag and $B_\perp$ and $B_\parallel$ are the transverse and longitudinal diffusion coefficients of the HQ. To this end, let us introduce a useful tensor which will enable us to separate out the momentum dependence of the noise term, i.e. the diffusion term, distinctly:

$$\begin{aligned} b^{ij} &= \sqrt{B_\parallel}\frac{p^i p^j}{p^2} + \sqrt{B_\perp}(\delta^{ij} - \frac{p^i p^j}{p^2}) & (2.14)\\ &= b_\parallel \frac{p^i p^j}{p^2} + b_\perp(\delta^{ij} - \frac{p^i p^j}{p^2}). & (2.15)\end{aligned}$$

With the help of the above equation, Langevin equation can be rewritten as:

$$\begin{aligned} \frac{dp^i}{dt} &= A^i(\vec{p}) + b^{ij}(\vec{p})\eta^i(t), & (2.16)\\ < \eta^i(t)\eta^j(t') > &= \delta^{ij}\delta(t-t'). & (2.17)\end{aligned}$$

The above equation can be solved explicitly only when it is discretised [2, 6] in small time steps Δt. The momenta at each time step are p^0, p^1,....., p^n. α is a parameter which specifies a whole family of discretisation and it ranges from 0 to 1. After solving the Langevin Equation small algebraic manipulation leads to the following results:

$$< \Delta p^i > \; = \; A^i(\vec{p^0})\Delta t + \alpha(\partial_k b^{ij}(\vec{p^0}))b^{kj}(\vec{p^0})\Delta t \;\; and \tag{2.18}$$

$$< \Delta p^i \Delta p^j > \; = \; b^{ik}(\vec{p^0})b^{jk}(\vec{p^0})\Delta t = B^{ij}(\vec{p^0})\Delta t. \tag{2.19}$$

Depending upon the value of α, there are many discretisation schemes, like Ito(for $\alpha = 0$), Stratonovich (for $\alpha = 1$). In the Ito scheme, it is possible to arrive at the equivalent Fokker Planck equation:

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial p^i}[A^i_{Ito}(\vec{p})P] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}[B^{ij}(\vec{p})P], \tag{2.20}$$

where, $A^i_{Ito} = -Ap^i$ and $P(\vec{p}, t|\vec{p^0}, t^0)$ is the conditional probability that a particle with momentum p^0 at time t^0 will have momentum $\vec{p}$ at time t.

Here, we will not illustrate the Langevin dynamics in further detail, because in our work, we have not followed this prescription, but the Fokker Planck formalism. The brief discourse on Langevin dynamics has been put forward just to make a comparative study of the two equivalent formulations for the evolution of heavy quark inside QGP. One can refer to different literatures like [Ref. [6, 7, 8]] for detailed account of Langevin simulation.

2.1.2 Fokker Planck Formalism:

We have already encountered one form of the Fokker Planck Equation (from Langevin Eqquation) in the previous section. It is high time to introduce the Fokker Planck formalism, formally, starting from another approach.

We have already mentioned that the problem of the heavy flavour propagation within a medium of light partons can be described by the theory of Brownian motion. if $f(\vec{x}, \vec{p}, t)$ is the single particle distribution function of an ensemble of heavy quarks immersed in the fluid consisting of light quarks and gluons, then the evolution of this distribution function will be governed by the Master Equation or the Boltzmann Transport Equation (BTE) [1]:

$$\left[\frac{\partial}{\partial t}+\frac{\vec{p}}{E}.\frac{\partial}{\partial \vec{x}}+\vec{F}.\frac{\partial}{\partial \vec{p}}\right] f(\vec{x},\vec{p},t)=\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}} \tag{2.21}$$

$\vec{F}$ is the force exerted on the HQ by the surrounding colour field. $\vec{p}$ and E denote the three momentum and the energy of the HQ respectively. The right hand side of Eq. 2.21, which is called the collision integral, $C[f]$, is attributed to the QCD interactions of HQ with light quarks, anti-quarks and gluons. One should, in principle, solve this integro-differential equation under the influence of $Q\bar{Q}$ potential and the background colour field which are to be included in the force term. But, here, we will set $\vec{F}=0$ and will treat QGP to be uniform. Therefore, the second and the third term of the left hand side of Eq. 2.21 vanishes under these approximations. Defining

$$f(\vec{p},t)=\frac{1}{V}\int d^3\vec{x} f(\vec{x},\vec{p},t) \tag{2.22}$$

which is the normalized probability distribution in the momentum space, we have

$$\frac{\partial f(\vec{p},t)}{\partial t}=\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}}. \tag{2.23}$$

Eq. 2.23 tells that the only physical reason for the evolution of the momentum distribution function is the collisions of heavy quark with the light particles inside the QCD matter.

Our main aim is to determine the collision integral of the transport Eq. 2.23. Once, we determine a particular form of $C[f]$, we can proceed towards solving the differential equation. There are various approximate methods to solve this integro-differential equation. In this note, it is constructive to mention that Eq. 2.23 can be reduced to the simple form in the relaxation time approximation:

$$\frac{\partial f(\vec{p},t)}{\partial t}=-\frac{f-f_0}{\tau} \tag{2.24}$$

which is a simplified and useful first approximation. Here, f_0 is the equilibrium distribution function and τ is the relaxation time that determines the rate at which the fluctuations in the system drive it to a state of equilibrium again. In this form, the equation is easier to solve. But, we have to arrive at a rigorous form of collision integral [1] corresponding to the present physical and more realistic situation. The approximation, that we apply here, is due to Landau, which allows only soft scatterings in the medium, i.e. the scatterings between the heavy quark and the medium particles where the magnitude of the exchanged three momentum is small. If we define $w(\vec{p},\vec{k})$ to be the rate of collisions which changes the momentum of the HQ from $\vec{p}$ to $\vec{p}-\vec{k}$, we have

$$\left[\frac{\partial f}{\partial t}\right]_{\text{collisions}} = \int d^3\vec{k}[w(\vec{p}+\vec{k},\vec{k})f(\vec{p}+\vec{k}) - w(\vec{p},\vec{k})f(\vec{p})]. \tag{2.25}$$

The second part of the integral corresponds to all those transitions that remove HQ from momentum $\vec{p}$ to $\vec{p}-\vec{k}$, and therefore, represents a net loss to the distribution function. Likewise, the first part of the integral represents a net gain to the distribution function of HQ. With these, Eq. 2.23 becomes:

$$\frac{\partial f(\vec{p},t)}{\partial t} = \int d^3\vec{k}[w(\vec{p}+\vec{k},\vec{k})f(\vec{p}+\vec{k}) - w(\vec{p},\vec{k})f(\vec{p})] \tag{2.26}$$

Eq. 2.26 is an equation in f and in the right hand side the w's contain the distribution functions of the medium particles, which are in equilibrium, in their expressions. To see the dependence of w on the distribution functions, Let us write down the expression for $w(\vec{p},\vec{k})$ when the heavy quark is interacting with the gluons of the QGP medium [1]:

$$w^g(\vec{p},\vec{k}) = \gamma_g \int \frac{d^3q}{(2\pi)^3}\hat{f}_g(\vec{q})v_{\vec{p},\vec{q}}\sigma^g_{\vec{p},\vec{q}\to\vec{p}-\vec{k},\vec{q}+\vec{k}} \tag{2.27}$$

where, $\hat{f}_g$ is the medium gluon distribution function, γ_g is the degeneracy of gluons in the medium and σ_g is the cross section for the heavy quark-gluon interaction. We can simplify the form of Eq. 2.26 by adopting the previously discussed Landau approximation. Mathematically, this approximation amounts to assuming $w(\vec{p},\vec{k})$ to fall of rapidly to zero with $|\vec{k}|$, i.e., transition probability function, $w(\vec{(p)},\vec{k})$ is sharply peaked around $|\vec{k}| = |\vec{p}|$. Therefore, if we expand the integrand in the right hand side of Eq. 2.26 in powers of $\vec{k}$, we have

$$w(\vec{p}+\vec{k},\vec{k})f(\vec{p}+\vec{k}) \approx w(\vec{p},\vec{k})f(\vec{p}) + \vec{k}\cdot\frac{\partial}{\partial\vec{p}}(wf) + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(wf) \tag{2.28}$$

Retaining terms up to the second order only, we obtain Landau Kinetic equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\vec{p})f]\right] \quad , \tag{2.29}$$

where the kernels are defined as the following:

$$A_i = \int d^3\vec{k}w(\vec{p},\vec{k})k_i \quad , \tag{2.30}$$

and

$$B_{ij} = \frac{1}{2}\int d^3\vec{k}w(\vec{p},\vec{k})k_ik_j. \tag{2.31}$$

When the distribution functions of the light quark, the corresponding anti quarks and gluons of the medium in the expression of $w(\vec{p},\vec{k})$ are that of the equilibrium ones, i.e. either Fermi-Dirac(in case of light quarks and anti-quarks) or Bose-Einstein(in case of gluons), Eq. 3.32 is named as the Fokker-Planck Equation. These coefficients A_i and B_{ij} are related to the drag and diffusion coefficients of the propagating heavy quark while it is suffering QCD interactions with the particles of the medium. These are the inputs to the Fokker Planck Equation and in the next section, I will discuss how they can be evaluated within the ambit of perturbative QCD.

2.2 Elastic collisions of heavy quark with the medium particles:

We will first consider the binary elastic collisions of the Heavy Quark (Q) with the light quark (q), their corresponding anti-quark ($\bar{q}$) and gluon (g) constituting the medium and then evaluate the transport coefficients of the heavy quark by computing the invariant matrix elements for the following processes:

$$Q(p) + q/\bar{q}/g(q) \rightarrow Q(p') + q/\bar{q}/g(q'). \tag{2.32}$$

In the expression 2.32, p, q, p' and q' withiin the brackets are the momenta of the heavy quark and light quark/anti-quark/gluon of the medium before and after the collision respectively. The Feynman diagrams are depicted in Figs. 2.1and 2.2:

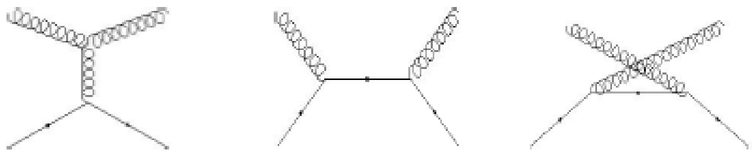

Figure 2.1: $Qg \to Qg$

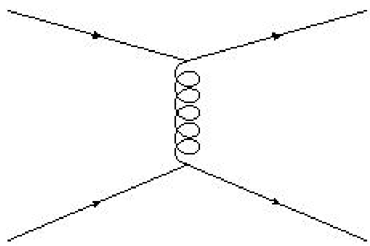

Figure 2.2: $Qq \to Qq$

The expressions of the invariant amplitudes for the $2 \to 2$ processes have been taken from the Ref. [9]. Once the matrix elements are known, one can proceed to calculate the transport coefficients. By looking at the expressions 2.30 and 2.31, we can write the generic transport coefficient (X) in the following way:

$$X = \int \text{phase space} \times \text{matrix element squared} \times \text{transport part} \tag{2.33}$$

Here, transport part is either momentum transfer (drag) or square of the momentum transfer (diffusions) between the HQ and the medium light particles. The specific expressions for A_i and B_{ij} are [1]:

$$\begin{aligned} A_i &= \frac{1}{2E_p} \int \Pi_{i=p',q,q'} D_i \frac{1}{\gamma_Q} \sum |M|^2 (2\pi)^4 \delta^4(p+q-p'-q') \\ &\times \; f(\vec{q})[1+f(\vec{q'})](p-p')_i = << (p-p')_i >>, \end{aligned} \tag{2.34}$$

When, $\Pi_{i=p',q,q'} D_i = \frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3q'}{(2\pi)^3 2E_{q'}}\frac{d^3p'}{(2\pi)^3 2E_{p'}}$. Likewise,

$$B_{ij} = \frac{1}{2} << (p-p')_i (p-p')_j >> \tag{2.35}$$

It would be constructive to decompose the drag and diffusion coefficients according to:

$$\begin{aligned} A_i &= A(p)p_i, \\ B_{ij} &= B_\perp(p)(\delta_{ij} - \frac{p_i p_j}{p^2}) + B_\parallel(p)\frac{p_i p_j}{p^2} \end{aligned} \tag{2.36}$$

In the above equations, A, $B_\perp$ and $B_\parallel$ are the drag, transverse and longitudinal diffusion coefficients correspondingly. It can be noticed from the expressions 2.36 that the transport coefficients, in general, depends on the momentum of the fast moving heavy quarks. It is also worth mentioning that this momentum dependence will play significant role in solving the Fokker Planck Equation, later. Again,

$$A = A_i p_i/p^2 = << 1 - \frac{\vec{p}.\vec{p'}}{p^2} >> \tag{2.37}$$

$$\begin{aligned} B_\perp &= \frac{1}{2}(\delta_{ij} - \frac{p_i p_j}{p^2})B_{ij} \\ &= \frac{1}{4} << [p'^2 - (\vec{p}.\vec{p'})^2/p^2] >> \end{aligned} \tag{2.38}$$

$$\begin{aligned} B_\parallel &= \frac{p_i p_j}{p^2} B_{ij} \\ &= \frac{1}{2} << [(\vec{p}.\vec{p'})^2/p^2 - 2\vec{p}.\vec{p'} + p^2 \times 1] >> \end{aligned} \tag{2.39}$$

In order to the perform the above integrations, we have to choose a frame of reference. Here, $\vec{q'}$ integration is trivially performed with the help of the delta function. While doing the $\vec{q}$ integral, it is useful to propagate the heavy quark along the polar axis of this integral as the integrand does not depend on the azimuth of $\vec{q}$. The $\vec{p'}$ integration has been implemented in the centre of mass frame. Consequently, for the collisional transport coefficient, $X_{\rm coll}$, the algebraically simplified version of the integration becomes:

$$\begin{aligned} X_{\rm coll}(\vec{p},T) &= \frac{1}{512\pi^4}\frac{1}{E_p}\int_0^\infty qdqd(cos\chi)\frac{s-M^2}{s}f(\vec{q})[1 \pm f(\vec{q'})] \\ & \int_{-1}^{1} d(cos\theta_{c.m})\frac{1}{\gamma_Q}\sum |M|^2_{2\to 2}\int_0^{2\pi} d\phi_{c.m}X(\vec{p'}). \end{aligned} \tag{2.40}$$

$X(\vec{p'})$ is either A or $B_\perp$ or $B_\parallel$ from Eq. 2.39. For the detailed steps of the integration, one can refer to [1]. In this work, this integration has been performed and results have been prepared for each of the transport coefficients in case of charm and bottom quarks propagating inside QGP.

2.2.1 Relation between drag and diffusion coefficients:

Before presenting the results for the transport coefficients, we will discuss briefly the physical significance of these coefficients and also the relationship among them.

Landau-Kinetic Equation 3.32, in general, does not need the background medium to be in thermal equilibrium. It is also evident from the expressions for the drag and diffusion coefficients 2.36 that there is no information of the temperature of the surrounding medium unless one assumes the medium particles to be thermal (Bose-Einstein or Fermi-Dirac). In our case, we have presumed that the Brownian particle is immersed inside a thermally equilibrated Quark Gluon Plasma having temperature,T, i.e. they follow Fokker Planck Equation. Therefore, theoretically, if allowed to evolve for infinite time (practically, for a long time compared to any other time scales), the heavy quark will attain thermal distribution at the end.

For a non-relativistic HQ, the Fokker Planck Equation 3.32 transforms into the following form:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}} \cdot (\vec{p} f) + D \frac{\partial^2 f}{\partial \vec{p}^2}, \tag{2.41}$$

where, $A_i = \gamma p_i$ and $B_{ij} = D\delta_{ij}$ (only one diffusion coefficient for non-relativistic case) and γ and D are independent of the momentum of the heavy quark. It is constructive to explore the effects of drag and diffusion coefficients on the Brownian particle from the one-dimensional form of Eq. 2.41:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial p}(pf) + D \frac{\partial^2 f}{\partial p^2} \tag{2.42}$$

The solution of the above Eq. 2.42 with a delta function initial condition $[f(p,0) = \delta(p - p_0)]$ for HQ distribution is [10],

$$f(p,t) = \left[\frac{\gamma}{2\pi D}(1 - e^{-2\gamma t})\right]^{-1/2} exp\left[-\frac{\gamma}{2D}\frac{(p - p_0 e^{-\gamma t})^2}{1 - e^{-2\gamma t}}\right] \tag{2.43}$$

We can infer from the above solution that the effect of drag is to shift the mean momentum of the Brownian particle, $< p >= p_0 e^{-\gamma t}$, along the momentum axis and diffusion makes the width of the distribution larger with a variance, $\sigma =< p^2 > - < p >^2= \frac{D}{\gamma}(1 - e^{-2\gamma t})$, reducing the value of the maximum.

A non-relativistic heavy particle, in the process of being dragged and diffused, will, eventually, reach thermal equilibrium in the medium. Therefore, the drag, γ and the diffusion, D must have to share a definite relation in order that the above condition is reached. This is the celebrated Einstein relation for non-relativistic case:

$$D = \gamma MT, \tag{2.44}$$

which tells us that the dissipation(drag) in the medium can be connected to the random fluctuating motion causing the diffusion. It can be shown that under certain circumstances, the momentum space diffusion coefficient, D can be related to the position space diffusion constant, D_x,

$$D_x = \frac{2dD}{M^2\gamma^2} \tag{2.45}$$

where d is the dimensionality. This is valid for time, $t >> \gamma^{-1}$, while γ^{-1} is the order for the relaxation time of the non-relativistic heavy quark.

Now, if we suppose that the heavy quark is relativistic, i.e. fast moving, the above relation 2.44 will not hold true. In this case also, there is a fare possibility that the heavy quark becomes thermalised at the end of its journey through the medium with a temperature (it may not be thermalised due to the short lifetime of QGP and the high momentum of the probe, heavy quark), T, the Einstein relation must have to be generalised so that it involves all the three momentum dependent transport coefficients, A, $B_\perp$ and $B_\parallel$.

Here, a short discussion on the derivation of the generalised Einstein relation for heavy quark is in order [11]. Let us revisit the FPE in the following way:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f + \frac{\partial}{\partial p_j}(B_{ij}(\vec{p})f)\right] = -\vec{\nabla}_p.\vec{\wp} \tag{2.46}$$

where $\vec{\wp}$ is the probability current. A relationship among the transport coefficients might be derived by demanding that $\partial f/\partial t$ is zero, i.e. $\vec{\wp}$ vanishes (due to the detailed balance) when Eq. 2.46 is satisfied by the equilibrium distribution function, f_{eq}^{HQ}. We will assume a particular form of the equilibrium distribution function for the heavy quark which may not be a regular Boltzmann type thermal distribution:

$$f_{eq}^{HQ}(p; T_T, q) = Nexp[-\Phi(p; T_T, q)], \tag{2.47}$$

where N is the normalisation factor and T_T, q are the parameters required to specify the shape of the distribution. Putting $\partial f_{eq}^{HQ}/\partial t = 0$, we get the following relation:

$$A_i(\vec{p}, T) = B_{ij}(\vec{p}, T)\frac{\partial \Phi(\vec{p})}{\partial p_j} - \frac{\partial B_{ij}(\vec{p}, T)}{\partial p_j} \tag{2.48}$$

Using the expressions for A_i and B_{ij}[Eq. 2.36] and the fact that f_{eq}^{HQ} depends only on the magnitude of the momentum for a spatially homogeneous case, a generalised Einstein relation involving the three drag/diffusion coefficients can be arrived at:

$$A(p, T) = \frac{1}{p}\frac{d\Phi}{dp}B_{\parallel}(p, T) - \frac{1}{p}\frac{dB_{\parallel}}{dp} - \frac{2}{p^2}[B_{\parallel}(p, T) - B_{\perp}(p, T)] \tag{2.49}$$

This relation is valid for any momentum of HQ. One can check that this reduces to the well-known non-relativistic Einstein relation 2.44 when $\Phi = p^2/(2MT)$, $A = \gamma$ and $B_{\perp} = B_{\parallel} = D$.

In view of the previous discussions, there are certain subtle questions which can be addressed here. We have mentioned a number of times that the generalised Einstein relation 2.49 must have to be obeyed in order that the heavy quark reaches thermal equilibrium at a time, $t \to \infty$ in a static infinite medium, theoretically. But, in practice, heavy flavours do not evolve for an infinite time and also the background QGP medium is not at all static: its temperature is a function of space and time, both and after a critical time, the medium of QGP ceases to exist. As a consequence, whether the heavy quark will thermalise or not is not a very settled and simple problem. Even if the equilibrium state is attained, the resulting distribution function may have a shape different from a thermal one. This issue of the shape of the equilibrium distribution will be elaborated in a later chapter of this treatise.

2.2.2 Collisional transport coefficients in Hard Thermal Loop (HTL) approach:

So far, while calculating the collisional transport coefficients using Eq. 2.40, we have essentially, used the vacuum matrix elements. In order to tackle the infra-red divergence due to soft intermediary gluon exchange and to incorporate the fact that the Heavy Quark-light parton interaction occurs in a thermal medium, we have shielded the t-channel divergence by replacing t by $t - m_D^2$ in an ad-hoc manner.

To this end, we will introduce Hard Thermal Loop (HTL) re-summed gluon propagator to regulate the t-channel divergence in a more self-consistent way and to calculate the transport coefficients. In the weak coupling regime, i.e. when $g \ll 1$, the HTL corrections due to the three-gluon vertex can be neglected in our case. This statement can be accounted for by adopting a simple power counting method [1]. It can be shown that the order of magnitude of the N-gluon vertex is $g^N T^2 p^{-N+2}$, where T is the temperature of the medium and p is the external momentum of the Feynman diagram. Therefore, it is easily followed that if $p \sim T$, i.e. hard, then the order of magnitude of the 3-gluon vertex is $g^3 T$ which is less in order than the bare vertex if we suppose $g \ll 1$.

Henceforth, the transport coefficients evaluated using HTL re-summed or "effective "gluon propagator (see Appendix-A) will be called "effective "as opposed to "bare "ones obtained by vacuum pQCD shielded by Debye mass (which can be obtained in the static limit from the HTL gluon self energy). We use zero-temperature fermionic propagator for heavy quarks because they do not thermalize with the medium. For $Qq \to Qq$ scattering the t-channel contribution is calculated in the Appendix-A. No other process is possible in this case. But $Qg \to Qg$ process contains all the channels among which we use bare perturbation results for s-channel, u-channel and their interference terms because of the involvement of heavy quarks in the internal line. For rest of the terms, i.e. the pure t-channel diagram, the interference of t-channel with s,and u-channels, we use the effective results (see Appendix-A).

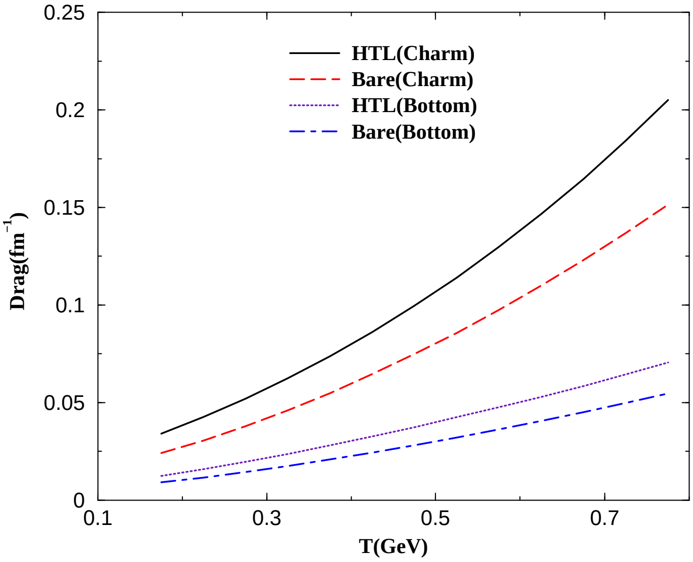

Figure 2.3: (Color online) Variation of drag of heavy quarks of momentum 1 GeV with temperature.[13]

Fig.2.3 depicts the variation of the drag coefficients of charm and bottom of a particular momentum with respect to temperature of the surrounding medium. It is observed that drag of charm and bottom increase with the increasing temperature for all the cases, i.e. when calculated using bare(shielded by Debye mass) gluon propagator as well as using HTL gluon propagator. This is because of the fact that as the temperature of the medium increases, the random thermal motion of the medium particles also increases. Therefore, the heavy quark encounters more and more interactions as the temperature is larger and lose more and more momentum which results into larger drag. It can be noticed from this plot that for both, charm and bottom, the values of the drag coefficients are larger when the divergence has been shielded with effective gluon propagator instead of the Debye mass shielding. We see that at 400 MeV temperature the HTL drag for charm quark is $\sim$33% more than the bare one, whereas, the corresponding difference is $\sim$25% for bottom quarks. We also observe that this difference increases with the increase in temperature. As the temperature attainable in the heavy ion collisions at RHIC

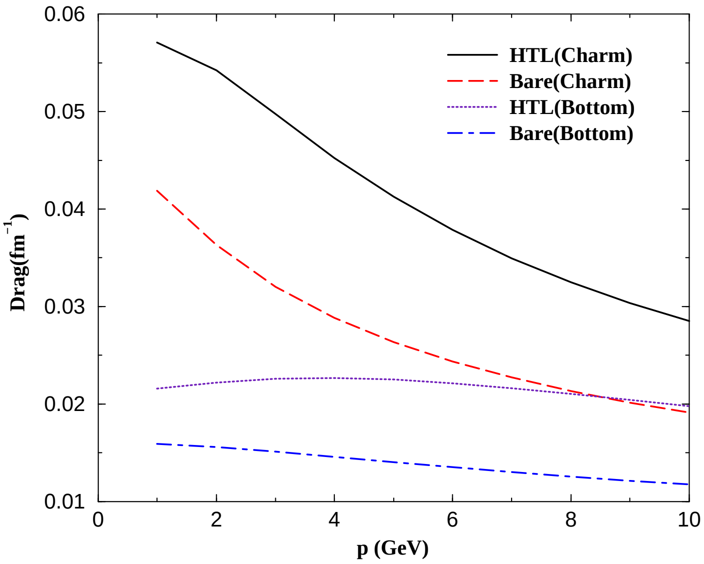

Figure 2.4: (Color online) Variation of drag of heavy quarks with momentum in a QGP bath of temperature 300 MeV.[13]

and LHC are quite high, the enhancement of drag at high temperatures will have significant effect in that regime.

In the Fig. 2.4, the drag is shown to be dependent on the momentum of the heavy quark propagating inside QGP. We can see from the plot that the profile of the drag coefficient of charm and bottom is decreasing with larger momentum. Physically, one can account for this behaviour by noting that it is harder to stop or to degrade the momentum of the heavy quark when it is really moving faster and faster. In that case, faster the heavy quark, lesser will be the exchange of its momentum with the bath particles. The momentum dependence of drag is distinctly affected if we consider the HTL re-summed gluon propagator. For a 5 GeV charm the corresponding difference is $\sim$50%. For higher momenta (10 GeV) the difference dies down to $\sim$45%. But, it must be emphasized that this kind of difference in momentum variation , if found in radiative processes too, will lead to an improvement in earlier works [1] where bare perturbation theory variation has been used.

The variation of the diffusion(transverse) coefficient with the bath temperature is shown in fig. 2.5. Diffusion shows the same profile as the drag coefficient, i.e. it also increases with

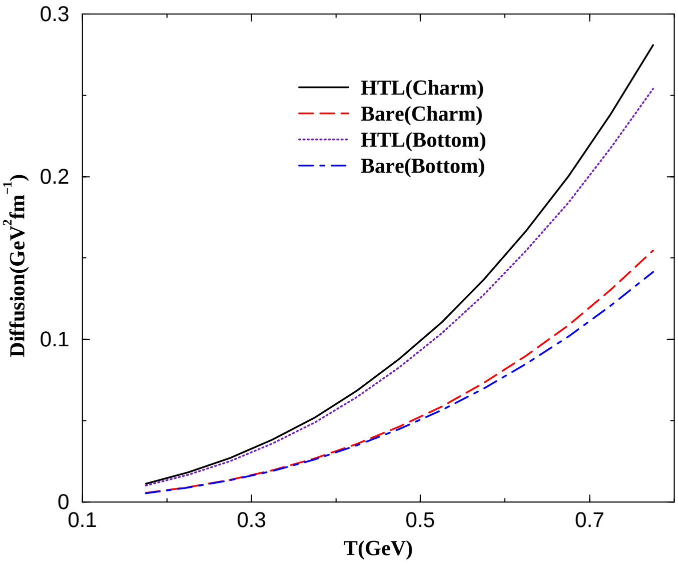

Figure 2.5: (Color online) Variation of diffusion of heavy quarks of momentum 1 GeV with temperature.[13]

the increasing temperature. The reason for this kind of behaviour has already been discussed while explaining fig. 2.3. Diffusion co-efficients seem to be more sensitive to the use of effective propagator in a sense that we observe $\sim$100% change between that effective and bare values at 400 MeV temperature and this difference increases with temperature. Though unlike drag, this difference is not much ($\sim$3.5%) for a difference in quark mass.

The momentum dependence of diffusion is just the opposite to that of the drag. Diffusion increases as the momentum of the heavy quark takes higher values(see fig. 2.6). The momentum dependence is also affected by the use of HTL gluon propagator, a 5 GeV charm diffuses $\sim$80% more when the exchanged gluon passes through all the possible processes in medium. This difference increases with the momentum of the external probe, i.e. the heavy quark.

2.3 Highlights:

Therefore, at the end of this chapter, we hope to have developed a basic idea about the propagation of an open heavy quark in Quark Gluon Plasma. Also, the procedure of the calculations

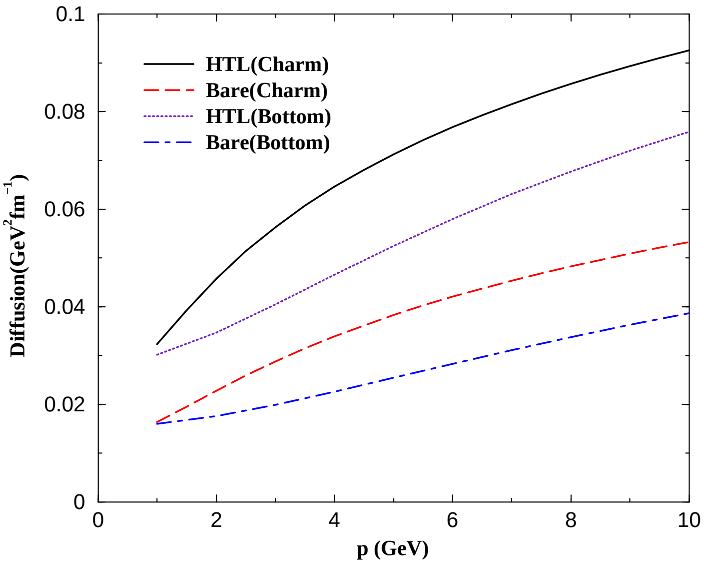

Figure 2.6: (Color online) Variation of diffusion of heavy quarks with momentum in a QGP bath of temperature 300 MeV.[13]

of drag and diffusion coefficients has been discussed with relevant plots for the case when the heavy quark scatters elastically with the medium particles. Let us write down the gist of this chapter combined into some important points:

1. The motion of the heavy quark in the background of the Quark Gluon Plasma has been elaborated within the framework of the Brownian motion. The evolution of the heavy quark inside the medium can be described by either Langevin Equation [1, 2, 3, 6] or Fokker Planck Equation(in the present work) each of which is basically the approximated version of the Boltzmann Transport Equation. Both these equivalent formalisms are discussed theoretically.

2. This dissertation mainly deals with the formulation and solution of the Fokker Planck Equation. So, the inputs to the equation, such as drag and diffusion coefficients, are computed when the heavy quark undergoes elastic collision with the medium particles.

3. While calculating the transport coefficients, the infra-red divergence due to the exchange of soft gluons has been shielded, first, by Debye mass in an ad-hoc manner. Results for a more self-consistent treatment of the Hard Thermal Loop(HTL) perturbation theory on

drag and diffusion coefficients have also been presented. It has been noted that the magnitude of the transport coefficients are enhanced in case of HTL due to the introduction of processes like Landau damping etc. which are typical manifestation of the existence of the various processes occurring inside the medium.

4. Qualitatively, we see that values of effective transport co-efficients are larger than bare ones. Actually, earlier calculations used thermal mass of gluon which is $\Pi_L(x \to 0)$ (see Appendix-A) as the regularizing agent. As evident from the expressions of gluon self-energies, taking HTL approximated (but no $x \to 0$ approximation) Π_L and Π_T we consider new processes like Landau damping ascertained by the imaginary parts of self-energies. Now, consideration of new processes will decrease the equilibration time of gluon. Since inverse of equilibration time is the imaginary part of self-energy, the previous statement implies that the denominator of effective propagator is smaller than the bare counterpart. Hence the amplitude of the process increases. We could have extrapolated our argument for radiative processes also. Since the radiative processes include processes like $Qq \to Qqg$ or $Qg \to Qgg$ we have two propagators to deal with. The radiative amplitude $M_{Qq \to Qqg}$ will involve only one effective (gluon) propagator and $M_{Qg \to Qgg}$ will involve two. So, we expect effective radiative drag to increase, too.

5. Inverse of drag is the measure of equilibration time. Hence, inclusion of effective thermal gluon propagator increases the likelihood of charm or bottom being equilibrated with the medium. Again, drag being the measure of energy loss, increase in effective drag results in increase in suppression of heavy flavours. That implies an increase in initial temperatures of QGP considered so far.

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Chapter 3

Gluon Radiation off Heavy Quarks

3.1 Radiation from a point charge in classical electrodynamics:

In the previous chapter, we have made an effort to analyse the process of binary elastic scattering suffered by the heavy quarks with the bulk particles and also have evaluated the relevant transport coefficients within the Fokker Planck formalism. But, besides being scattered elastically, heavy quark also emit bremsstrahlung gluons while traversing the thermal medium. In the present chapter, gluon radiation from heavy quark(HQ) will be discussed in details in the context of the transport coefficients. At later chapters, we will endeavour to explore the effect of radiation on other physical quantities of interest. Also, there are references of many well-known articles which discuss the radtion of gluons from heavy quark [4, 5, 6]

Throughout the entire treatment of the radiation, we will try to be pedagogic in most of the cases,but will become intricate in some particular topics. Let us start our discussion with a moving point massive charged particle which can be described by the simple Classical Electro Dynamics(CED). From the point of view of CED, it is known that the accelerated charge emits electromagnetic radiation (i.e. photons when the fields are quantised). Very briefly, we will first develop an idea of the angular distribution of radiation emitted by a point charge moving non-relativistically as well as relativistically within the framework of CED. After having done that we will step into the discussions of the radiation of gluons from heavy quark in the context of Quantum Chromo Dynamics (QCD).

We know that the fields for a moving charge is, in general, described by the well-known Liénard Wiechert potentials [1]:

$$A^{\mu}(x) = \left[\frac{eV^{\mu}}{V^{\nu}(x - r(\tau))_{\nu}}\right]_{\tau=\tau_0}, \tag{3.1}$$

where e is the electric charge, V^{μ} is the four velocity and $r^{\mu}(\tau)$ is the position at a proper time τ of the moving point charge, τ_0 is the retarded proper time defined by the light cone condition. $[x - r(\tau_0)]^2 = 0$ and the retarded requirement, $x_0 > r_0(\tau_0)$. Eq. 3.1 indicates that the potentials

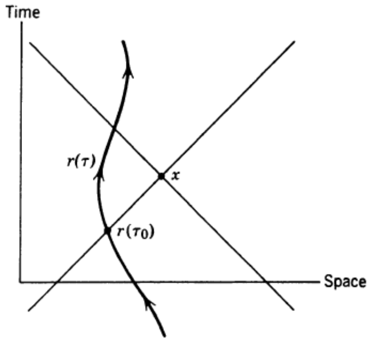

Figure 3.1: World line of the charged particle[1]

are to be calculated at the retarded time, τ_0. It can be explained nicely with the help of Fig. 3.1 from which it is quite clear that the world line of the charged particle intersects the light cone at two points and it is only the earliest point, $r^{\mu}(\tau_0)$ which contributes to the fields at x^{μ}.

Sometimes, for familiarity, Eq. 3.1 is written in a non-covariant form, where the scalar potential, $\Phi(\vec{x}, t)$ and the vector potential, $\vec{A}(\vec{x}, t)$ can be written as:

$$\Phi(\vec{x}, t) = \left[\frac{e}{(1 - \vec{\beta}.\hat{n})R}\right]_{\tau=\tau_0} \tag{3.2}$$

$$\vec{A}(\vec{x}, t) = \left[\frac{e\vec{\beta}}{(1 - \vec{\beta}.\hat{n})R}\right]_{\tau=\tau_0} \tag{3.3}$$

where, $\vec{\beta} = \vec{v}(\tau)/c$, $\vec{v}$ being the three-velocity of the particle, $R = |\vec{x} - \vec{r}(\tau)|$ which is the light cone condition in three dimensions and $\hat{n}$ is the unit vector in the direction of $\vec{x} - \vec{r}(\tau)$.

Here, our main aim is to look into the angular distribution of the radiation. For that we need to know the electric field which can be constructed out of the field strength tensor, $F^{\mu\nu}$ which is given by:

$$F^{\mu\nu} = \frac{e}{V.(x-r)} \frac{d}{d\tau} \left[\frac{(x-r)^{\mu} V^{\nu} - (x-r)^{\nu} V^{\mu}}{V.(x-r)} \right] \tag{3.4}$$

This field strength tensor is manifestly covariant, but for our purpose, we need the expression for the electric field:

$$\vec{E}(\vec{x}, t) = e \left[\frac{\hat{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \hat{n})^3 R^2} \right]_{\tau=\tau_0} + \frac{e}{c} \left[\frac{\hat{n} \times ((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}})}{(1 - \vec{\beta}.\hat{n})^3 R} \right]_{\tau=\tau_0}, \tag{3.5}$$

where, $\dot{\vec{\beta}} = d\vec{\beta}/dt$. The first term in Eq. 3.5 is called the velocity field and the second term is called the acceleration field due to the obvious reason that they depend on velocity and accelerations respectively. Also, the second term is responsible for radiation, whereas the first term is static.

Now, if we first consider the case of a non-relativistic accelerated charge, then the radiation term of the electric field becomes:

$$\vec{E_a} = \frac{e}{c} \left[\frac{\hat{n} \times (\hat{n} \times \dot{\vec{\beta}})}{R} \right]_{\tau=\tau_0} \tag{3.6}$$

Therefore, the power radiated per unit solid angle or the angular distribution of radiation will be

$$\frac{dP}{d\Omega} = \frac{c}{4\pi} |R\vec{E_a}|^2 = \frac{e^2}{4\pi c} |\dot{\vec{\beta}}|^2 sin^2\theta \tag{3.7}$$

where, θ is the agle between acceleration and $\hat{n}$. Eq. 3.7 is the famous Larmor result for non-relativistic accelerated point charge.

If the motion of the accelerated charged particle is relativistic, then the angular distribution will look like the following:

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \frac{|\hat{n} \times ((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}})|^2}{(1 - \hat{n} \cdot \vec{\beta})^5} \tag{3.8}$$

We can derive the expression for the angular distribution from the above equation when acceleration and velocity are parallel to each other:

$$\frac{dP}{d\Omega} = \frac{e^2\dot{\vec{\beta}}^2}{4\pi c}\frac{sin^2\theta}{(1-\beta cos\theta)^5} \tag{3.9}$$

From the Fig. 3.4 it is clear that as $\beta \rightarrow 1$, the angular distribution is tipped forward more and

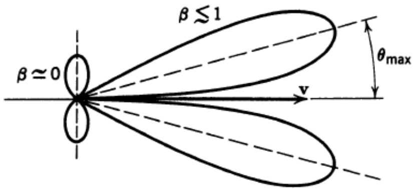

Figure 3.2: Radiation pattern when velocity is parallel to acceleration[1]

more and its magnitude is also increased. We can also see that there exists a small cone around the direction of propagation of the particle, where there is no radiation. This region is called the dead-cone region which does not exist for a highly or ultra relativistic particle for which mass has no significance at all. i.e. for a massless relativistic particle, there is no dead-cone zone.

If we now consider the scenario when the velocity and acceleration of the particle are orthogonal to each other, we will find no such phenomenon like dead-cone effect, rather the radiation will be always sharply peaked in the direction of the propagation of the particle. The expression

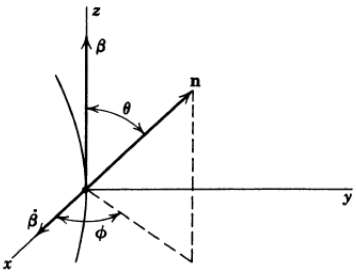

Figure 3.3: System of coordinates when $\vec{\beta}$ and $\dot{\vec{\beta}}$ are perpendicular to each other [2]

for the differential power will be:

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\frac{|\dot{\vec{\beta}}|^2}{(1-\beta cos\theta)^3}\left[1 - \frac{sin^2\theta cos^2\phi}{\gamma^2(1-\beta cos\theta)^2}\right] \tag{3.10}$$

As, here, velocity is perpendicular to the acceleration, it is an example of the instantaneous circular motion. This pattern of radiation is a typical of synchrotron radiation [2].

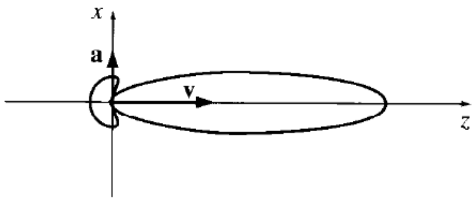

Figure 3.4: Radiation pattern when velocity, $\vec{v}$ is perpendicular to acceleration, $\vec{a}$[2]

3.2 Radiation from a heavy quark:

In the previous section, we have recapitulated some basic ideas of the classical radiation from an accelerated point charge. But, in this dissertation, as can be seen, we have dealt with the radiation of gluon (which is the mediatory particle of Quantum Chromo Dynamics) from the heavy quark propagating inside the deconfined medium of quarks and gluons. Now, the obvious question which arises here is, ″in which way the radiation in Quantum Field Theory(here QCD) different from or similar to that in Classical Field Theory?‟To this end, let us specify that from now on the discussions will border on the emission of radiation of ‘soft’gluon from the heavy quark. Here, soft means that the four momentum of the emitted gluon will be very small compared to other momentum scales. In this regime of soft radiation the results and the pattern of the radiation will be very similar to that of the classical. For detailed discussions one can refer to [3].

There are mainly two reasons for a heavy quark to radiate gluon:

i) When a heavy quark is produced after Heavy Ion Collision(HIC), it carries a large virtuality along with it. In order to reduce this virtuality it emits radiation until it becomes real.

ii) Heavy quarks also radiate gluons when they interact or scatter with another particle (suppose another quark, anti-quark or gluon).

This discussion will concentrate on the second kind of radiation which results from the elastic scattering of heavy quark.

For this purpose, the relevant Feynman diagrams will be evaluated to arrive at the spectrum of the radiated gluon and various approximations which have been used throughout the entire process, will be explained. The radiation of gluons will ultimately lead to the measure of the energy loss of heavy quarks by considering that the gluons which are emitted as radiation will be absorbed by the surrounding medium. We will first discuss the interaction of Heavy Quark with the light quark, their corresponding anti-quark of the medium. Later, we shall see that, within the ambit of the approximations to be used in the present formalism, the dominant interaction, i.e. the interaction of heavy quark with the gluons, will differ only in the colour factor with the previously calculated Feynman amplitude of the scattering of heavy quark and light quarks.

Feynman diagrams and the approximations:

The relevant Feynman diagrams for the scattering of Heavy Quark(Q) with light quark(q) where a single gluon is emitted as radiation are given in Fig. 3.5. In order to calculate $\sum |M|^2_{2\to3}$, the invariant amplitude squared for the process, the necessary Mandelstam variables are [7]:

$$\begin{aligned} s &= (p+q)^2, \quad s' = (p'+q')^2, && (3.11)\\ t &= (p-p')^2, \quad t' = (q-q')^2, && (3.12)\\ u &= (p-q')^2, \quad u' = (q-p')^2, && (3.13) \end{aligned}$$

with

$$s + t + u + s' + t' + u' = 4M^2. \quad (3.14)$$

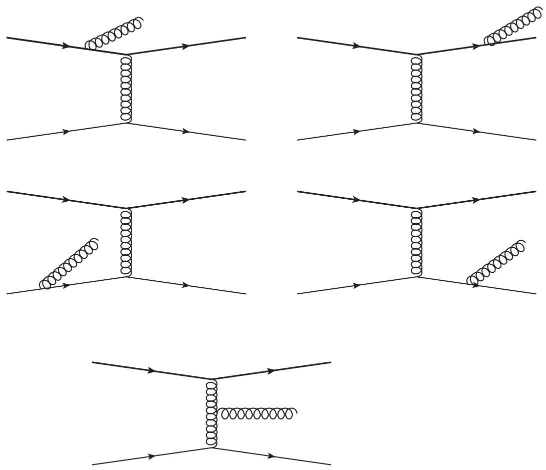

Figure 3.5: Feynman diagrams for $Qq \rightarrow Qqg$ process

Most of the treatments regarding the gluon radiation mainly concentrate on the 'soft-eikonal-collinear'regime of the radiation. Before explaining the above phrase within the inverted commas let us write down the four momenta of particles specified in the centre of momentum frame:

$$p = (E_p, 0, 0, p_z) \quad , q = (E_q, 0, 0, -p_z) \tag{3.15}$$

$$p' = (E_{p'}, \vec{q_\perp}, p'_z) \quad , q' = (E_{q'}, -\vec{q_\perp}, -p'_z) \tag{3.16}$$

$$k_5 = (E_5 = k_\perp cosec\theta, \vec{k_\perp}, k_\perp cot\theta). \tag{3.17}$$

Here, θ is the angle at which the gluon is radiated. We can now get back to the discussion of the usual approximations considered in the 'soft-eikonal-collinear'regime.

1. *Soft gluon emission*: The energy(E_p) of the parent parton, the heavy quark in this occasion, is quite large compared to the energy of the emitted gluon, E_5: $E_p \gg E_5$ or $k_5 \rightarrow 0$.

2. *Eikonal trajectory of HQ*: There are two types of eikonal trajectory of heavy quark possible:(a) when there is no recoil of the heavy parton due to the elastic scattering with the light quarks and gluons: $E_p \gg q_\perp$, (b) when the heavy quark does not recoil due to the emission of gluon radiation: $E_p \gg k_\perp$. This approximation is, in effect, the extrapolation

of the soft gluon approximation, i.e. the reason behind neglecting the recoil effect of HQ is the softness of the emitted gluon.

3. *Collinear gluon emission*: In this regime, the emitted gluon sweeps almost grazing angle with the direction of propagation of the heavy quark. Symbolically, this can be written as $E_5 \gg k_\perp$. Later, it will be observed that whereas a typical singularity arises in the expression of the emitted gluon spectrum due to this collinear emission of soft gluons, the mass of the heavy quark has a deeper consequence to remove this divergence up to certain limit.

The above approximations having the following hierarchy

$$E_p \gg E_5 \gg k_\perp, q_\perp \gg m_{TH} \gg \Lambda_{QCD}. \tag{3.18}$$

are the usual practices in the domain of the radiative energy loss. As we are dealing with the heavy quark as the probe particle, the gluons are no longer considered to emit collinearly with the parent parton. Later, this statement will be clear from the 'dead-cone'pattern of the radiation spectrum of heavy quark. Previously, the term 'dead-cone'has been introduced in the context of classical electrodynamics. Further discussion on this will follow. Consequently, the resulting hierarchy [7, 8] among the different scales of the analysis are:

$$E_p \gg q_\perp \gg m_{TH} \gg \Lambda_{QCD} \tag{3.19}$$

$$E_p \gg E_5 \geq k_\perp \gg m_{TH} \gg \Lambda_{QCD}. \tag{3.20}$$

The principal part of the present work is based on the second hierarchy(Eq. 3.20). But, we also endeavoured to remove another limitation by providing the heavy quark with a finite recoil after its being scattered elastically by the medium particles. In this way, we let go off the first eikonal trajectory approximation. Physically, this signifies that instead of continuing in the same direction, this time the heavy quark bents or changes direction after suffering elastic scattering. Therefore, yet another hierarchy has come into the picture: $E_p > q_\perp$ is considered instead of $E_p \gg q_\perp$. Every other relations are similar to hierarchy 3.20. This new concept of non-eikonal trajectory of HQ has been studied carefully and will be discussed very briefly in the upcoming section.

Gluon radiation spectrum:

In this section, we will elaborate on the method of calculation of the invariant amplitude squared of the Feynman diagrams drawn in the previous section 3.5 using the hierarchy 3.20 as explained in Ref.[7]. Therefore, we will skip all the intricacies of the calculations, rather quote the result obtained in the above reference. The square of the invariant amplitude, $|M|^2_{Qq\to Qqg}$ or $|M|^2_{Qg\to Qgg}$ can be expressed in terms of the respective binary scattering amplitudes. This factorisation is only possible because of the soft gluon emission,

$$|M|^2_{Qq\to Qqg} = 12g^2|M|^2_{Qq\to Qq}\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2\eta}\right)^{-2}, \tag{3.21}$$

$$|M|^2_{Qg\to Qgg} = \frac{C_A}{C_F}|M|^2_{Qq\to Qqg}. \tag{3.22}$$

Here, M is the mass of the heavy quark, $C_A/C_F = 9/4$ is the Casimir factor, $\eta = -ln(tan(\theta/2))$ is the rapidity of the emitted gluon. The factor, $D = \left(1+\frac{M^2}{s}e^{2\eta}\right)^{-2}$ is the suppression factor called the dead-cone factor. The binary scattering amplitude is,

$$|M|^2_{Qq\to Qq} = \frac{8}{9}g^4\frac{s^2}{t^2}\left(1-\frac{M^2}{s}\right)^2. \tag{3.23}$$

The gluon radiation spectrum is defined as the ratio of the invariant amplitude squared of the radiative/inelastic(two particle going to three particle) gluon bremsstrahlung process to that of the elastic(two particle going to two particle) process:

$$\frac{|M|^2_{2\to3}}{|M|^2_{2\to2}} = 12g^2\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2\eta}\right)^{-2} \tag{3.24}$$

This relation is valid in the full rapidity domain of the emitted gluon and in the full range of the variable $M/\sqrt{s}$. In this note, it is constructive to recall the notion of the dead-cone as explained in the beginning of this chapter in the context of Classical Electro Dynamics (CED). The region of 'no radiation'along the forward direction of the radiation spectrum of heavy quark was termed as dead-cone region. This region is typically due to the heavy mass of the probe and this is absent in case of a light emitter of gluon radiation as is evident in the Gunion-Bertsch [9] formula for the radiation spectrum of light quark. If we take the limit $M \to 0$ we can arrive at the Gunion-Bertsch approximated dead-cone factor $\frac{q_\perp^2}{(q_\perp - k_\perp)^2}$ with an extra approximation $q_\perp \gg k_\perp$ which is attributed to the singularity when $q_\perp = k_\perp$. This approximation or limit is called the Gribov limit. The deeper consequence of the Gribov limit lies in the fact that one can neglect the Feynman diagram involving the three-gluon vertex if one stays within this regime. Again, if we go back to the problem of heavy quark energy loss, we may note that in the soft gluon radiation limit, the concept from CED is extended and in the limit $M \ll \sqrt{s}$ and $\theta \ll 1$ it has already been shown in Ref. [10] that a region of dead-cone exists. In that case, the dead-cone factor is simply the approximated version of the generalised factor derived in Ref. [7]:

$$\left(1 + \frac{M^2}{s\ tan^2(\theta/2)}\right)^{-2} \xrightarrow{M \ll \sqrt{s},\ \theta \ll 1} \left(1 + \frac{M^2}{E_p^2 \theta^2}\right)^{-2}. \tag{3.25}$$

All the approximations and re-derivation of different known formulae are discussed in greater detail in Ref. [7]. It is imperative to note another kind of singularity called the infra-red singularity caused by the soft gluon exchange in the binary scattering processes. If the propagator gluon is soft, i.e. the exchanged momentum is quite small, the term appearing in the invariant amplitude squared, $|M|^2$ will start to diverge when we try to extract any physical observable by integrating $|M|^2$ over the appropriate phase space. This infra-red divergence or t-channel divergence can be shielded by the Debye mass (m_D) by replacing $\frac{1}{t^2}$ with $\frac{1}{(t-m_D^2)^2}$. This ad-hoc shielding can be done away with a more self-consistent method of shielding following Hard Thermal Loop (HTL) approximations. Though this is already discussed in Chapter-2 of the dissertation in case of binary elastic scattering, still it will directly translate into the radiative regime because of the factorisation limit. Therefore, it is very much evident that the existence of the thermal QGP is responsible in lifting the infra-red divergence.

The notion of the existence of the thermal medium can also have effect on the gluon radiation spectrum in a rather different way: the gluons that are emitted as radiation can very well thermalise and can be part of the surrounding medium. Then only one can claim that the energy lost by the heavy quark, due to gluon bremsstrahlung, has been transfered to the medium. Subsequently, it is a constructive exercise to consider the emitted gluon as thermal and calculate the radiation spectrum again taking this 'Ter-Mikayelian Effect'into account. In this case, the modified or re-defined momenta of the particles taking part in the scattering processes are:

$$p = (E_p, 0, 0, p_z),\ \ q = (E_q, 0, 0, -p_z). \tag{3.26}$$

In the eikonal limit, $p = p'$ and $q = q'$. The four momenta of the emitted gluon will be:

$$k_5 = (\sqrt{k_\perp^2 + m_g^2} cosec\theta, \vec{k_\perp}, \sqrt{k_\perp^2 + m_g^2} cot\theta). \tag{3.27}$$

The virtuality condition satisfied by the emitted gluon is $k_5^2 = m_g^2$. It is important to mention that although gluons(i.e. the mediatory boson of QCD) do not carry mass in vacuum, yet they attain an effective mass due to the presence of the thermal medium. This concept is similar to the effective mass gained by the electron while traveling inside a lattice. After calculating the invariant amplitude squared and adding them all, we get the following expression:

$$|M|^2_{Qq\to Qqg} = 12g^2 |M|^2_{Qq\to Qq} \frac{1}{(k_\perp^2 + m_g^2)}\ D_{TM}, \tag{3.28}$$

where, D_{TM} is the new dead-cone factor due to the Ter-Mikayelian effect given by:

$$D_{TM} = D + \frac{m_g^2}{s}\left[\frac{(1+\frac{M^2}{s}e^{2\eta})^{-1}}{6(1-\Delta_M^2)} - \frac{4(1+\frac{M^2}{s}e^{2\eta})^{-1}}{9\Delta_M^2(1-\Delta_M^2)} + \frac{4(1+\frac{M^2}{s}e^{2\eta})^{-2})}{9\Delta_M^2}\right], \tag{3.29}$$

and $\Delta_M^2 = M^2/s$. Therefore, in this case, the gluon radiation spectrum can be written as:

$$\frac{|M|^2_{2\to 3}}{|M|^2_{2\to 2}} = 12g^2 \frac{1}{(k_\perp^2 + m_g^2)} D_{TM}. \tag{3.30}$$

If we compare the expression of the gluon radiation spectrum from heavy quark without Ter-Mikayelian effect(Eq. 3.24) to that with this effect(Eq. 3.30), we will be able to notice that the soft divergence in Eq. 3.24 appearing due to the smaller values of $k_\perp$ disappears in the case where Ter-Mikayelian effect is considered, i.e. the divergence is shielded by the thermal mass of the gluon.

3.3 Radiative transport coefficients of heavy quark:

Having calculated the gluon radiation spectrum in various approximations, we are now in a position to evaluate drag and diffusion coefficients of heavy quarks suffering gluon bremsstrahlung while propagating through Quark Gluon Plasma(QGP). In this context, the radiation spectrum with soft- eikonal approximation(as obtained in Ref. [7]) has been used in the present work.

Let us begin by recalling the Eq.2.31 of Chapter-2 which gives the expression for the generic transport coefficient in case of elastic scattering of heavy quark with the medium particles:

$$X(\vec{p}) = \int \text{ phase space } \times \text{ interaction } \times \text{ transport part} \tag{3.31}$$

Also, we might recall that the transport coefficients like drag and diffusion can be calculated from the Eq. 3.31 and they are the inputs to the Fokker Planck Equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\vec{p})f]\right] \quad , \tag{3.32}$$

Here, we are skipping the detailed description of the terms involved in Eq. 3.32 and the mathematical expressions for drag, A_i and diffusion, B_{ij}(details are provided in Chapter-2). This same expression 3.31 for $X(\vec{p})$ will also hold for the determination of drag(A_i or $A(\vec{p})$) and diffusion(B_{ij}, i.e. longitudinal, $B_\parallel$ and transverse, $B_\perp$ diffusion coefficients) when HQ undergoes gluon radiation. According to the present approximation, the soft gluons will carry negligible amount of momenta along with them so that the magnitude of momentum transfer will become exactly same as that in case of elastic scattering. Therefore, the expressions for the transport part of drag and diffusion remain same for both elastic and radiative interactions of heavy quark. The things which will distinguish radiative process from the elastic case are the phase space factor and the invariant amplitude squared for the processes as is clear from the expression below:

$$\begin{aligned}
X_{\text{rad}} &= \frac{1}{2E_p}\int \frac{d^3q}{(2\pi)^3 2E_q}\int \frac{d^3q'}{(2\pi)^3 2E_{q'}}\int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \\
&\times \int \frac{d^3k_5}{(2\pi)^3 2E_5}\frac{1}{\gamma}\sum |M|^2_{2\to 3}(2\pi)^4\delta^4(p+q-p'-q'-k_5) \\
&\times \hat{f}(E_q)(1\pm \hat{f}(E_{q'}))(1+\hat{f}(E_5)) \\
&\times \theta(\tau-\tau_F)\theta(E_p-E_5)
\end{aligned} \tag{3.33}$$

The inelastic or three body process($2\to 3$) described here is symbolised as:

$$Q(p)+q/\bar{q}/g(q)\to Q(p')+q/\bar{q}/g(q')+g(k_5). \tag{3.34}$$

In comparison to the expression for the collisional transport coefficient, the extra phase space integration is appearing due to the emission of the radiated gluon. Therefore, in the above Eq. 3.33, there are four integrations over the three momenta of all the particles participating in the radiative process except over the momenta of the probe particle. The invariant amplitude squared used here is that explained in the previous section in Eq. 3.22. The delta function signifies the four momentum conservation. The factor $[1+\hat{f}(E_5)]$, where $\hat{f}(E_5)$ is Bose-Einstein distribution function for thermal gluons, is taking care of the absorption of the emitted gluon by the surrounding background medium. This physically will signify that unless the emitted gluon is not absorbed by the medium, the energy of the heavy quark will not be considered to be lost by radiation which goes into the medium. There are also other two theta functions: (i) $\theta(\tau-\tau_F)$ keeps the kinematics of the process strictly within the additive or 'Bethe-Heitler'regime where the scattering centres are separated enough to have the additive gluon radiation. In this way, the incoherent multiple gluon emission due to Landau-Pomenronchuk-Migdal(LPM) effect is excluded. (ii) $\theta(E_p-E_5)$ prohibits the emission of gluons with energy greater than that of the incoming parent heavy quark.

It can be shown using Eq. 3.22 that the radiative transport coefficient might be expressed in terms of the elastic/collisional transport coefficient when the radiated gluon carries soft momentum. Earlier also this limit of factorisation has been mentioned and it may be noted that this is only valid in the soft radiation approximation.

$$\begin{aligned} X_{\text{rad}} &= X_{\text{coll}} \times \int \frac{d^3k_5}{(2\pi)^3 2E_5} 12 g_s^2 \frac{1}{k_\perp^2} \\ &\times \left(1 + \frac{M^2}{s} e^{2y}\right)^{-2} [1 + \hat{f}(E_5)] \theta(\tau - \tau_F) \theta(E_p - E_5). \end{aligned} \tag{3.35}$$

After doing some simple steps and rewriting the integration in terms of $k_\perp$ and η, we get the following expression:

$$X_{\text{rad}} = X_{\text{coll}} \times \int_{k_{\perp,\min}}^{k_{\perp,\max}} \frac{1 + f(E_5)}{k_\perp} dk_\perp (1 + \frac{M^2}{s} e^{2\eta})^{-2} d\eta \tag{3.36}$$

Here, we have removed θ functions because the limits of $k_\perp$ will be determined by them:

(a) $\theta_1(\tau - \tau_F) = \theta_1(\Lambda^{-1} - \tau_F)$, where Λ is the elastic scattering interaction rate and $\tau_F = cosh\eta/k_\perp$ is the formation time of the emitted gluon. The formation time is the time required for the radiated gluon to be separated from the parent heavy quark [11]. The interaction rate for the elastic scattering of heavy quark with the medium particle is given by:

$$\begin{aligned} \Lambda &= \frac{1}{2E_p} \int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q} \int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{q'}} \int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{p'}} \frac{1}{\gamma_Q} \sum |M|^2 \\ &\times (2\pi)^4 \delta^4(p + q - p' - q') \hat{f}(E_q) \end{aligned} \tag{3.37}$$

According to the first θ function in Eq. 3.33:

$$\begin{aligned} \Lambda^{-1} &> \frac{cosh\eta}{k_\perp} \\ k_\perp &> \Lambda\ cosh\eta \\ \therefore k_{\perp,\min} &= \Lambda\ cosh\eta. \end{aligned} \tag{3.38}$$

(b) The second θ function in Eq. 3.33 tells:

$$\begin{aligned} E_p &> E_5 \\ E_p &> k_\perp cosh\eta \\ k_\perp &< \frac{E_p}{cosh\eta} \\ \therefore k_{\perp,\max} &= \frac{E_p}{cosh\eta}. \end{aligned} \tag{3.39}$$

In order to simplify algebra and to perform the $k_\perp$ integration in the Eq. 3.36, we have considered further approximation in calculating the term $[1 + \hat{f}(E_5)]$:

$$\begin{aligned} 1 + \hat{f}(E_5) &= 1 + \frac{1}{e^{k_\perp cosh\eta/T} - 1} \\ &= \frac{e^{k_\perp cosh\eta/T}}{e^{k_\perp cosh\eta/T} - 1}. \end{aligned} \tag{3.40}$$

Now, if we assume $E_5 \ll T$ then the above equation becomes:

$$\begin{aligned} 1 + \hat{f}(E_5) &= \frac{1 + \frac{k_\perp cosh\eta}{T}}{\frac{k_\perp cosh\eta}{T}} \\ &= 1 + \frac{T}{k_\perp cosh\eta} \\ &\approx \frac{T}{k_\perp cosh\eta}. \end{aligned} \tag{3.41}$$

With the above assumptions and after having performed the $k_\perp$ integration,

$$X_{\text{rad}} = X_{\text{coll}} \times \frac{6\alpha_s T}{\pi} \int d\eta \left[\frac{1}{k_{\perp,\min}} - \frac{1}{k_{\perp,\max}}\right] \frac{1}{cosh\eta}(1 + \frac{M^2}{s} e^{2\eta})^{-2}. \tag{3.42}$$

In the previous chapter the transport coefficients for the elastic scattering have already been calculated and in this chapter their radiative counterparts are computed. Once we have the transport coefficients for both the elastic and inelastic/radiative processes, we add them to get the effective/total transport coefficients because we assume that the elastic and radiative processes are occurring independent of each other in the medium:

$$X_{\text{eff}} = X_{\text{coll}} + X_{\text{rad}} \tag{3.43}$$

All the transport coefficients are functions of momentum of HQ and temperature of the heat bath. Therefore, effective and collisional transport coefficients are plotted once with temperature keeping momentum of charm quark as a parameter and then with momentum of charm holding temperature as parameter [12].

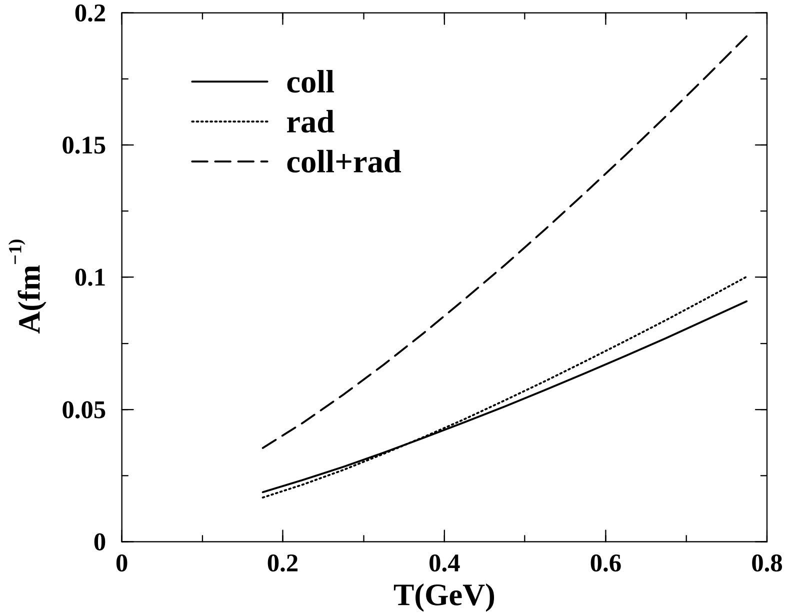

Figure 3.6: Drag of charm carrying mometum 5 GeV vs T[12]

In Fig. 3.6, we display the temperature dependence of the drag of Charm Quarks(CQ) with momentum $p = 5$GeV. At low T, although the drag for radiative loss is comparable to that for collisional loss, at high temperatures the radiative drag tends to dominate. The difference between total and collisional transport coefficients broadens with increasing temperature. Even at temperatures attainable at RHIC, this distinction is significant enough to have a pronounced effect on certain experimental observables like nuclear modification factor, elliptic flow of CQs, etc. In the temperature range that may be achieved at LHC collision conditions, the radiative contributions to the drag may surpass the elastic contributions. Therefore, radiative processes will play a more dominant role at LHC than at RHIC. For a CQ (mass $M = 1.3$ GeV) with $p = 5$ GeV and $T = 300$ MeV, the drag coefficient attains a value almost double the value for the collisional case when radiation is included. At a temperature of 600 MeV, total drag

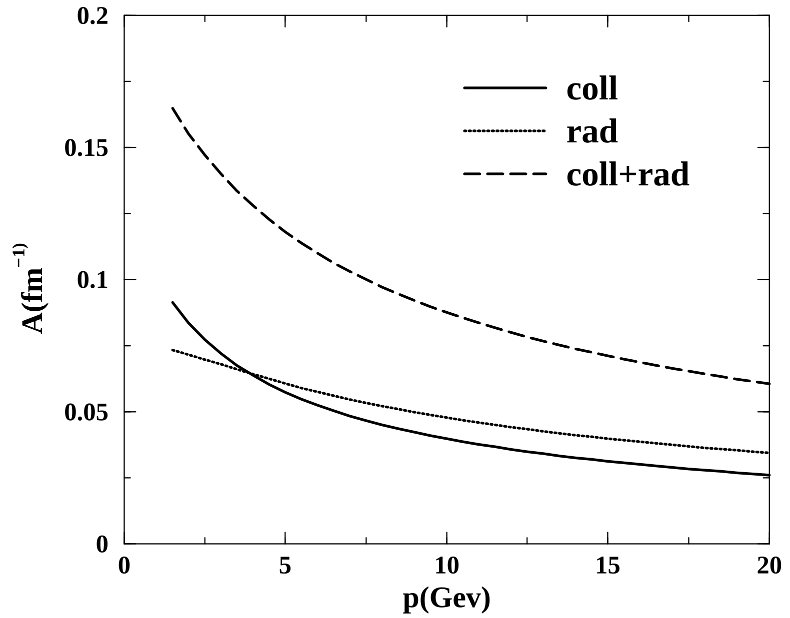

Figure 3.7: Drag of charm vs momentum at bath temperature, T=525 MeV[12]

becomes 2.12 times the collisional drag. The variation of drag with p at $T = 525$ MeV is depicted in Fig. 3.7. The dominance of radiative processes, in spite of dead cone suppression, is evident from the results for p beyond 5 GeV.

In Figs. 3.8 and 3.9, the variations of longitudinal diffusion coefficients with temperature and momentum, respectively, are displayed. Similar to drag, the contributions from radiative processes dominate over the collisional processes for higher T and p. For $T = 300$ MeV, the radiative and collisional losses have similar contributions to $B_\perp$, but for T beyond 500 MeV, the radiative part exceeds the collisional part(Fig. 3.10). It is interesting to note the qualitative change in the momentum dependence of $B_\perp$ from $B_\parallel$ at fixed T (Fig. 3.11). The variation of $B_\perp$ with p is slower than that of $B_\parallel$. In this case, again the domination of the radiative transport coefficient over its collisional counterpart is evident. Though the nature of the momentum dependence of the diffusion coefficients is different from that of drag, it is always true that, save at very low momentum of the CQ, the radiative contribution is more than the elastic contribution at $T = 525$ MeV. Accordingly, for a relativistic CQ, inclusion of the radiative effects becomes imperative for the analysis of experimental data from nuclear collisions at RHIC and LHC.

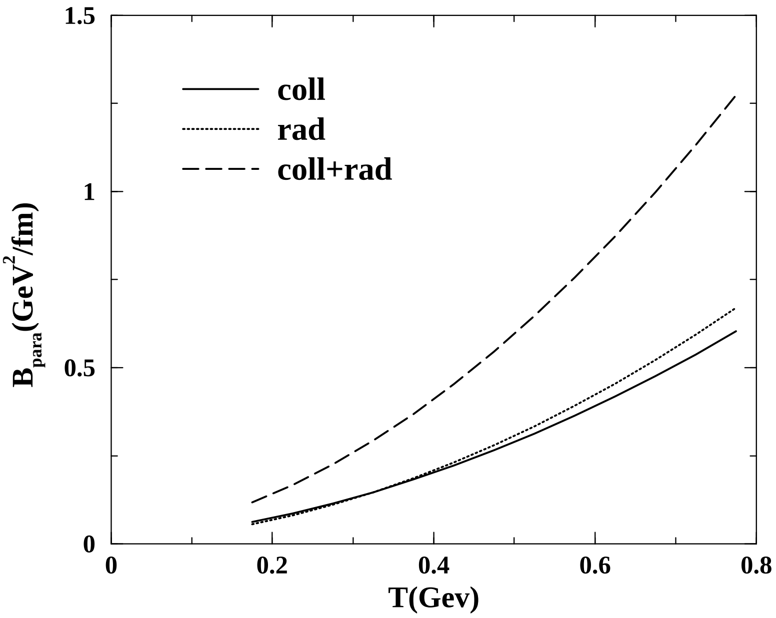

Figure 3.8: Longitudinal diffusion coefficient of charm with momentum 5 GeV vs T[12]

This statement can be put on a firmer ground if we quote some quantitative results comparing radiative and collisional contributions to the transport coefficients. The drag coefficient of a CQ having a momentum of 10 GeV is 0.038 fm^{-1} in case of elastic loss, whereas the radiative contribution is 0.047 fm^{-1} . Radiative $B_{\perp}$ is about 1.33 times its collisional counterpart. In the case of longitudinal diffusion coefficient, the radiative contribution is 1.2 times the elastic one.

3.4 Highlights:

The main findings of this chapter is summarised below:

1. This chapter has been started by discussing radiation from an accelerating point charge from the point of view of Classical Electro Dynamics (CED). The pattern of radiation has been discussed for two cases: when velocity and acceleration of the particle are, respectively, parallel and perpendicular to each other. In this context, it has been observed

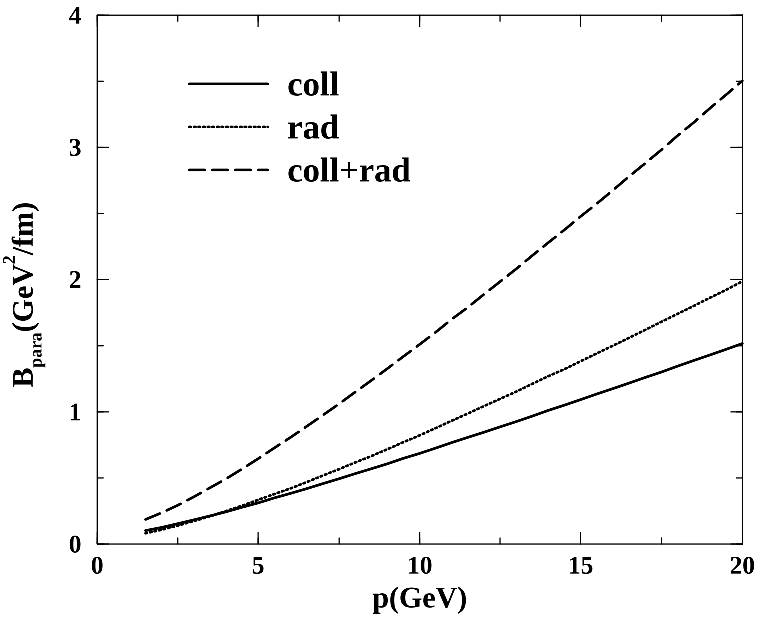

Figure 3.9: Longitudinal diffusion coefficient of charm vs momentum at bath temperature, T=525 MeV[12]

that for massive particles, the radiation in the forward direction is restricted and this region of no radiation is called dead–cone region.

2. Now, when the heavy quark is traversing through the QGP, the emission of soft gluons as radiation has been pointed out to be similar in construction just like the case in CED. Therefore, the phenomenon of dead-cone also happens in case of gluon bremsstrahlung off heavy quarks. The gluon radiation spectrum has been discussed in details mentioning various approximations.

3. The transport coefficients like drag and diffusion have been estimated for the radiative case. it has been noted that the radiative transport coefficients can be factorised into that for elastic interaction multiplied by an integration over the relevant phase space weighted by the spectrum of gluon radtion in the soft gluon emission limit.

4. Drag, transverse and longitudinal diffusion coefficients are plotted against temperature of the bath as well as the momentum of the charm quark. In later chapters, we will see that

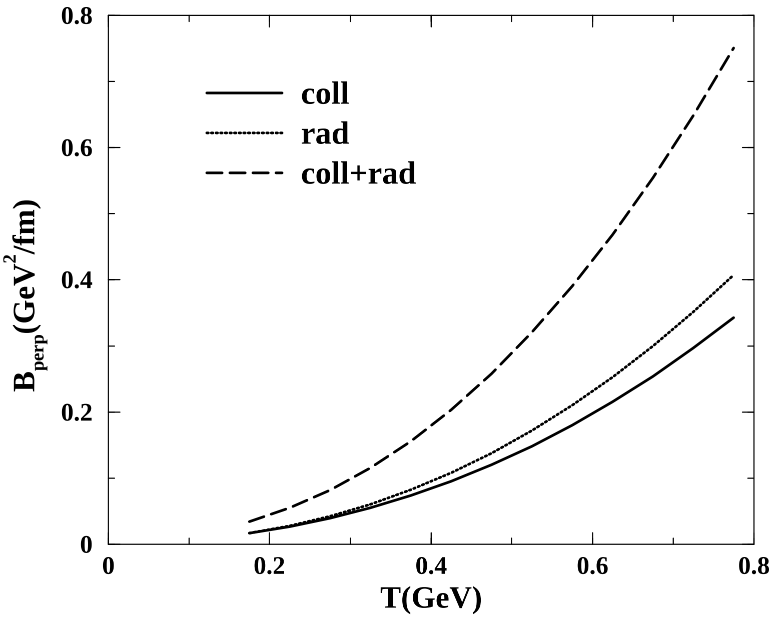

Figure 3.10: Transverse diffusion coefficient of charm with momentum 5 GeV vs T[12]

these radiative transport coefficients have considerable effects on different observables like the nuclear modification factor, R_{AA}, the shear viscosity of QGP, η/s etc.

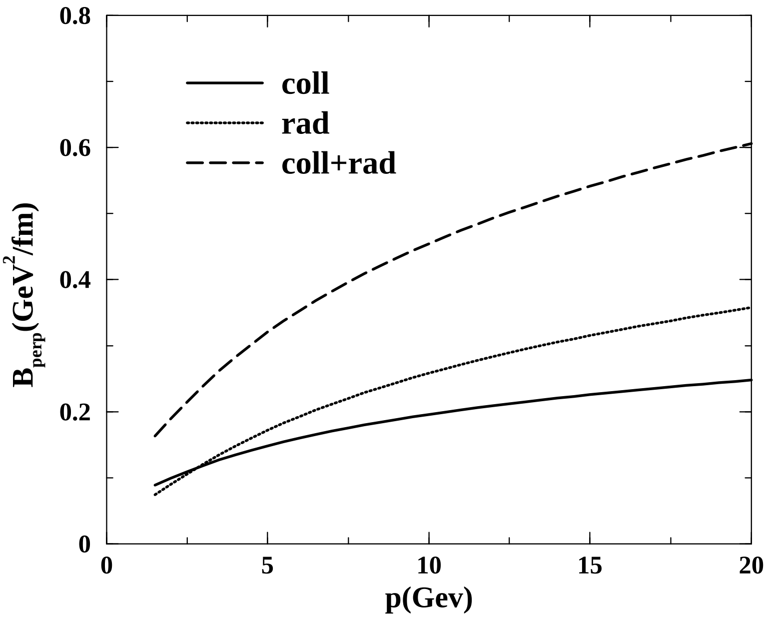

Figure 3.11: Transverse diffusion coefficient of charm vs momentum at bath temperature, T=525 MeV[12]

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Chapter 4

Solution of Fokker Planck Equation and nuclear suppression factor

4.1 Solution of Fokker Planck Eqquation and nuclear modification factor:

In the previous two chapters, we have discussed how to compute the drag and diffusion coefficients of heavy quarks traversing the QGP medium for both the cases, when the heavy quark scatter only elastically with the medium particles as well as when gluons are radiated from them. As those coefficients are the inputs to the Fokker Planck equation(FPE), one can now be ready to solve FPE once the initial conditions are considered properly(which will also be discussed in this chapter).

4.1.1 Background assumptions:

Before mentioning the underlying assumptions of the Fokker Planck Equation, let us write down this equation in momentum space:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\vec{p})f + \frac{\partial}{\partial p_j}(B_{ij}(\vec{p})f)\right], \tag{4.1}$$

where, $f(\vec{p}, t)$ is the single particle distribution function of an ensemble of heavy quarks immersed in the deconfined medium of light partons and A_i and B_{ij} are the drag/diffusion coefficients of heavy quark. The previous chapters were devoted to calculate these transport coefficients from pQCD when heavy quarks scatter elastically from the medium particles as well as when they suffer gluon bremsstrahlung. We have also discussed about the fluctuation-dissipation theorem which relates the three transport coefficients considering the ultimate equilibration or thermalisation of the heavy quarks. We will have a more critical into this in later chapters.

Before proceeding towards solving FPE, we can discuss about the underlying assumptions taken into account:

i) As the magnitude of the longitudinal diffusion coefficient, $B_{\|}$ is even smaller than that of the transverse diffusion coefficient, $B_{\perp}$, we neglected $B_{\|}$ while solving Fokker Planck Equation.

ii) We have calculated the transverse component of diffusion, $B_{\perp}$ from the expressions of the transport coefficients as elaborated in the previous chapters. Also, one can estimate $B_{\perp}$ from the fluctuation dissipation relation of the form $B_{\perp} = AE_pT$, where E_p is the energy of HQ and T is the temperature of the bath. This relation holds when the heavy quark is near to the thermal equilibrium(yet not in equilibrium) and is called the well known Einstein relation.

4.1.2 Method of solution of FPE:

In this formalism of solving FPE, we have considered the full momentum dependence of the transport coefficients in such a way that only the first derivatives of drag/diffusion coefficients survive. Fokker-Planck equation, under this approximation, in Cartesian coordinate system becomes, [1]:

$$\begin{aligned}
\frac{\partial f}{\partial t} &= C_1(p_x,p_y,t)\frac{\partial^2 f}{\partial p_x^2} + C_2(p_x,p_y,t)\frac{\partial^2 f}{\partial p_y^2} \\
&+ C_3(p_x,p_y,t)\frac{\partial f}{\partial p_x} + C_4(p_x,p_y,t)\frac{\partial f}{\partial p_y} \\
&+ C_5(p_x,p_y,t) f + C_6(p_x,p_y,t) \qquad (4.2)
\end{aligned}$$

where,

$$C_1 = B_\perp \qquad (4.3)$$

$$C_2 = B_\perp \qquad (4.4)$$

$$C_3 = A\, p_x + 2\,\frac{\partial B_\perp}{\partial p_T}\,\frac{p_x}{p_T} \qquad (4.5)$$

$$C_4 = A\, p_y + 2\,\frac{\partial B_\perp}{\partial p_T}\,\frac{p_y}{p_T} \qquad (4.6)$$

$$C_5 = 2\,A + \frac{\partial A}{\partial p_T}\,\frac{p_x^2}{p_T} + \frac{\partial A}{\partial p_T}\,\frac{p_y^2}{p_T} \qquad (4.7)$$

$$C_6 = 0\ . \qquad (4.8)$$

where the momentum, $\mathbf{p} = (\mathbf{p}_T, p_z) = (p_x, p_y, p_z)$. We numerically solve Eq. 4.2 [2] with the boundary conditions: $f(p_x, p_y, t) \to 0$ for p_x,$p_y \to \infty$ and the initial (at time $t = \tau_i$) momentum distribution of charm and bottom quarks are taken from MNR code [3]. It is evident from Eq. 4.2 that with the momentum dependent transport co-efficients the FP equation becomes complicated and needs to be solved numerically. It is possible to write down the solution of the FP equation in closed analytical form [4] in the special case of momentum independent drag and diffusion co-efficients.

4.1.3 Initial conditions and definition of nuclear modification factor:

The initial temperature,T_i and the initial thermalization time, τ_i for the background QGP expected to be formed at RHIC and LHC can be constrained to the total multiplicity as follows:

$$T_i^3\tau_i \approx \frac{2\pi^4}{45\zeta(3)}\,\frac{1}{4a_{eff}}\,\frac{1}{\pi R_A^2}\,\frac{dN}{dy} \quad , \tag{4.9}$$

where R_A is the radius of the system, $\zeta(3)$ is the Riemann zeta function, $a_{eff} = \pi^2 g_{eff}/90$, and $g_{eff}(= 2\times 8+7\times 2\times 2\times 3\times N_F/8)$ is the degeneracy of quarks and gluons in QGP and N_F is the number of flavours. The value of the transition temperature, T_c has been taken to be 175 MeV. We have used the boost invariant model of relativistic hydrodynamics proposed by Bjorken [5] for the space time evolution of the expanding QGP back ground. We will have a more detailed discussion on this in the next chapter. The value of T_i and τ_i for the QGP fireball are taken as $T_i = 300$ MeV and $\tau_i = 0.5$ fm/c. The corresponding quantities for LHC are $T_i = 550$ MeV and $\tau_i = 0.1$ fm/c. The pressure (P)-energy density(ϵ) relation for the QGP has been taken as $P = \epsilon/3$.

The ratio of the solution of the Fokker Planck Equation for charm and bottom at the critical temperature, T_c, $f(\vec{p}, T_c)$ (this is the distribution function of charm/bottom when they emerge from QGP) to the initial distribution, $f_{in}(\vec{p}, T_i, \tau_i)$ taken from MNR code (this is the charm/bottom distribution function just before entering QGP) is the theoretical definition of the nuclear modification factor for heavy quark:

$$R_{AA}^{c/b}(p_T) = \frac{f^{c/b}(p_T, T_c)}{f_{in}^{c/b}(p_T, T_i)} \tag{4.10}$$

The suppression of both charm and bottom quarks (before fragmentation to hadrons) are plotted against p_T in Figs. 4.1 and 4.2 respectively. We note from these plots that if one takes the drag to be momentum independent (or more precisely takes the value of A at low p and extends it upto very high p) then the drag due to collisional process causes about 50% suppression (dashed line). However, if we take into account the variation of A with p obtained from pQCD calculation then about 20% of suppression can be achieved, *i.e.* the contribution from the collisional loss becomes smaller when we consider momentum dependent drag because drag reduces with the increasing momentum of the HQ. This means that without the momentum dependence of the drag coefficient(which is the realistic scenario for the fast moving heavy quark) one could have overestimated the nuclear suppression factor. In this way, the consideration of momentum dependence enables us to estimate the suppression factor more accurately. Again, it is important to notice that the observed large suppression of the heavy quarks at RHIC is predominantly due to radiative loss of heavy quarks. In fact, the inclusion of the radiative processes increases the suppression to about 75%. This can be understood from the fact that the drag due to the radiative loss is large. The suppression of the bottom quark is much less because of the smaller values of drag and initial harder momentum distribution (Fig. 4.2).

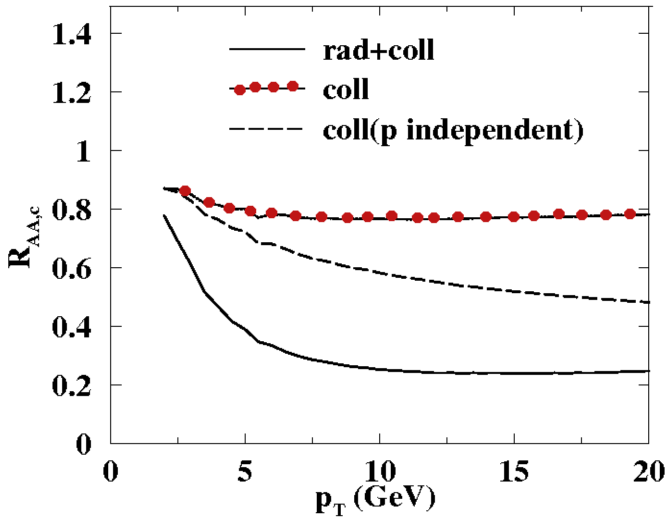

Figure 4.1: Suppression factor of charm quarks in QGP as a function p_T

Hadronisation or fragmentation of heavy quark into heavy mesons

The deconfined QGP medium formed in the heavy ion collisions lasts only for a short time span. After a very short time(within 2-3 fm), a phase transition occurs and the deconfined quarks and gluons again combine to form hadrons. Therefore, one cannot really measure the energy loss or the suppression of deconfined heavy quarks, rather the suppression of the heavy mesons like D or B mesons can be measured. So, it is also necessary to hadronise the heavy quarks into D or B mesons, theoretically. This is done by using Peterson fragmentation function [6]:

$$f(z) \propto \frac{1}{z\left[1-\frac{1}{z}-(\frac{\epsilon_Q}{1-z})\right]^2} \tag{4.11}$$

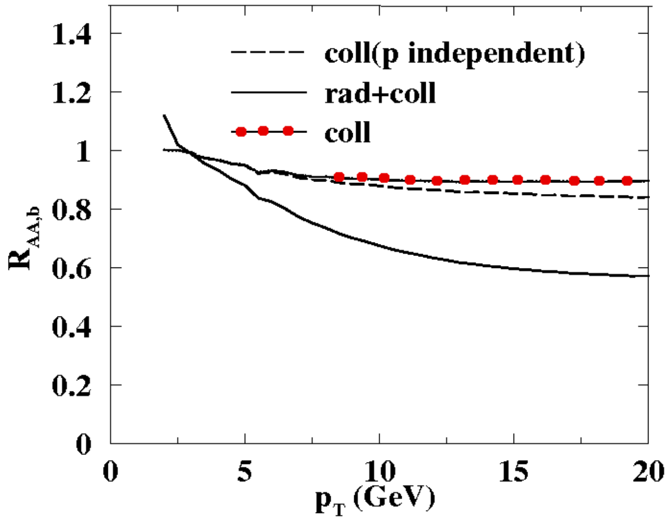

Figure 4.2: Suppression factor of bottom quarks in QGP as a function p_T

where, z is the fraction of momentum of the heavy quark carried by the hadrons, ϵ_Q for charm and bottom are $\epsilon_c = 0.05$ and $\epsilon_b = \epsilon_c(\frac{M_c}{M_b})^2$ respectively. We have convoluted the solution of the Fokker Planck equation at T_c(at the end of the QGP phase) and the initial distribution function (at the beginning of the QGP phase) for heavy quark with this fragmentation function and have got the nuclear modification factor for D and B mesons by taking their ratios. These are plotted in Figs. 4.3 and 4.4 at both RHIC and LHC energies respectively. There are also other fragmentation functions available in the literatures. The sensitivity of the results on the type of fragmentation function has been studied in details in Ref. [7].

Semi-leptonic decay

It can drawn in attention that whereas in the fig. 4.4, the theoretical results have been contrasted with the experimental data, there is no comparison with the experimental data in case of RHIC energies(fig. 4.3). This is because of the fact that in LHC, the direct measurement of the D-meson R_{AA} has been possible, but the same was not possible in RHIC. So, for comparing our results with the experimental data obtained at RHIC at the centre of mass energy, $\sqrt{s} =$ 200 GeV per nucleon, we have obtained the single electron spectra originated from the semi-leptonic decay of heavy-flavoured mesons. The result illustrated in Fig. 4.6 depicts the nuclear

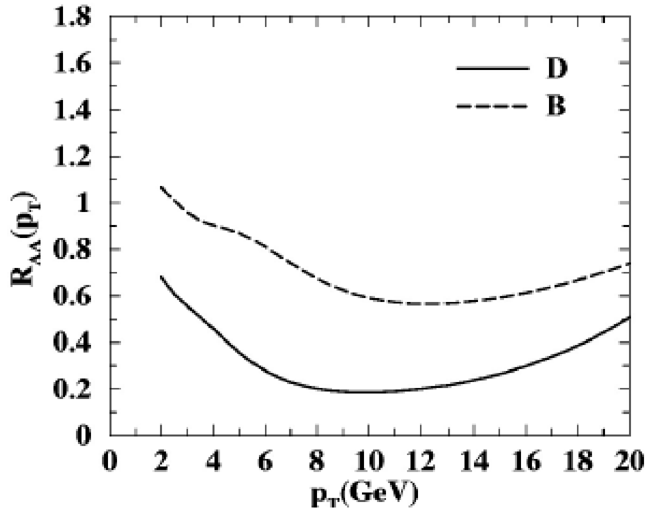

Figure 4.3: R_{AA} as a function of p_T for D and B mesons at RHIC

modification factor for the electrons from D and B mesons separately as well as the combined R_{AA} from both the sources. The relevant decay channels are as follows:

$$D \;\rightarrow\; Ke^-\bar{\nu}_e \tag{4.12}$$

$$B \;\rightarrow\; Ke^-\bar{\nu}_e \tag{4.13}$$

In fig. 4.5, the light anti-quark of the D-meson acts like a spectator whereas the electrons are being emitted from the weak decay of the charm quark residing inside the D-meson. The similar type of mechanism holds true for B-meson also. Consequently, one can conclude that measurement of the nuclear modification factor(R_{AA}) of the electrons from the semi-leptonic decay of D/B mesons reflects R_{AA} of the heavy mesons which in turn gives us the modification factor of the open heavy quarks from the QGP sector. Let, the transverse momentum carried by electron originated from semi-leptonic decay of D/B-meson is p_T and that carried by the parent D/B meson is q_T. If the p_T-spectrum of non-photonic electron is $dN^e/p_Tdp_T = f(p_T)$, then

$$\frac{dN^e}{p_Tdp_T}(p_T) = \int dq_T \frac{dN^D}{q_Tdq_T} F(p_T, q_T), \tag{4.14}$$

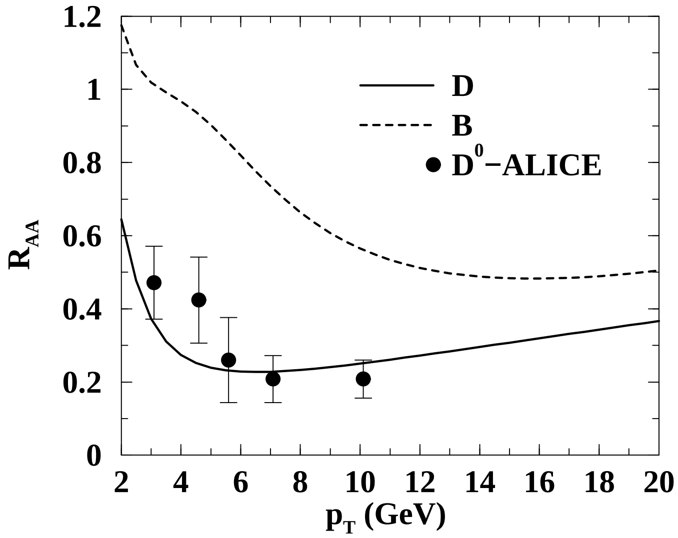

Figure 4.4: R_{AA} as a function of p_T for D and B mesons at LHC. Experimental data taken from [8].

where, a specific case of D-meson decay has been considered. $dN^D/q_T dq_T$ is the transverse momentum spectra of D-meson and the function $F(p_T, q_T)$ is

$$F(p_T, q_T) = \omega \int \frac{d(\vec{p_T}.\vec{q_T})}{2p_T(\vec{p_T}.\vec{q_T})} g\left(\frac{p_T q_T cos\theta}{M}\right). \tag{4.15}$$

Here,

$$\begin{aligned} \omega &= 96(1 - 8m^2 + 8m^6 - m^8 - 12m^4 ln\ m^2)^{-1} M^{-6}, \\ d(\vec{p_T}.\vec{q_T}) &= p_T q_T d(cos\theta) and \\ m = \frac{M_K}{M_D}. & \end{aligned} \tag{4.16}$$

Therefore, we can write for $F(p_T, q_T)$ and g as follows:

$$\begin{aligned} F(p_T, q_T) &= \frac{\omega}{2p_T} \int \frac{d(cos\theta)}{cos\theta} g\left(\frac{p_T q_T cos\theta}{M}\right) \\ g\left(\frac{p_T q_T cos\theta}{M}\right) &= \frac{p_T^2 q_T^2 cos^2\theta (M^2 - M_X^2 - 2p_T q_T cos\theta)^2}{M(M^2 - 2p_T q_T cos\theta)}. \end{aligned} \tag{4.17}$$

After having performed the integration 4.15 with the help of Eqs. 4.17, we can switch that into integration 4.14 which will result into the final semi-leptonic electron spectra. This process is repeated for the medium-modified D/B meson spectra(solution of the Fokker Planck Equation) as well as for that of the D/B meson spectra fragmented from the initially produced spectra for charm and bottom quark which is unmodified by the medium. The ratio will then be the nuclear modification factor, R_{AA} of the electrons from the semi-leptonic decay of D/B meson:

$$R_{AA}^{D/B \to e^-} = \frac{f_{final}^{D/B \to e^-}}{f_{in}^{D/B \to e^-}}. \tag{4.18}$$

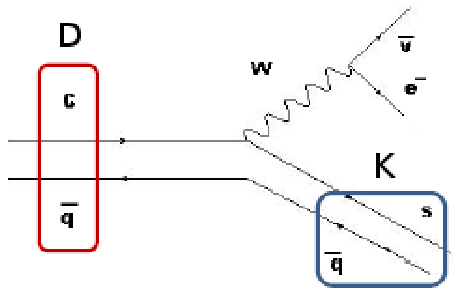

Figure 4.5: Decay from D meson to electron

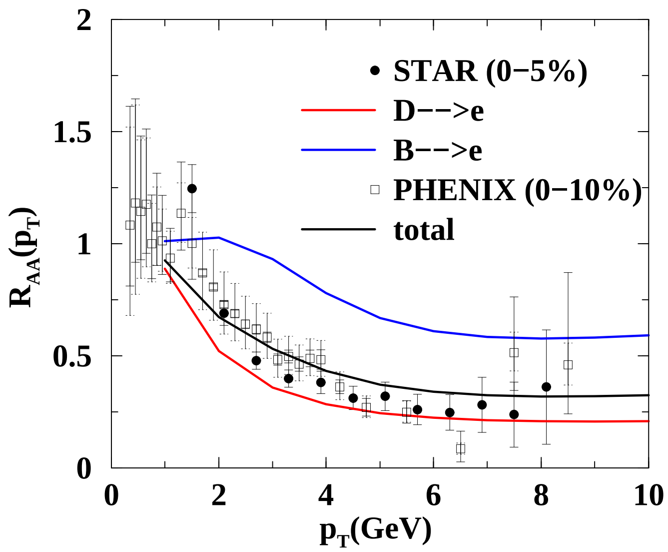

Figure 4.6: R_{AA} as a function of p_T for the electrons originated from the semi-leptonic decays of D and B mesons[9]

Fig. 4.6 depicts the comparison of the experimental data from RHIC at the centre of mass energy, $\sqrt{s}$ = 200 GeV/nucleon with the theoretically calculated results from the Eq. 4.18. We can say that this is a good agreement with the experiment with the initial temperature, $T_i = 300$ MeV and square of the velocity of sound is 1/3.

Highlights

i) The momentum dependence of the drag coefficient is found to be crucial in reproducing the trend in the the transverse momentum (p_T) dependence of the nuclear modification factor of heavy flavour extracted from the experiments.

ii) The radiative loss due to gluon bremsstrahlung off heavy quark plays more dominant role than elastic loss particularly at higher temperature of the bath and higher momentum of the heavy probe.

iii) The suppression of bottom is less than that of charm owing to the smaller magnitude of the drag and the harder initial transverse momentum distribution.

iv) The initial temperature, T_i and the initial thermalisation time, τ_i of the medium for RHIC ($\sqrt{s} = 200$ GeV/nucleon, $0-5\%$ centrality) are 300 MeV and 0.5 fm/c and for LHC ($\sqrt{s} = 2.76$ TeV/nucleon, $0-10\%$ centrality) are 550 MeV and 0.1 fm/c respectively.

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Chapter 5

Effect of Equation of State on the determination of the initial conditions

5.1 Initial conditions of Quark Gluon Plasma:

In the previous chapter, we have seen that different values of initial temperature, T_i and initial thermalisation time, τ_i can be extracted by contrasting the theoretical result on the nuclear modification factor, R_{AA} of the heavy quarks to the experimental data obtained at RHIC and LHC energies. Therefore, it has been quite clear that those two initial parameters have not been evaluated theoretically, rather they were adjusted/extracted phenomenologically. In the absence of any first principle methods of the estimation of the initial conditions, our aim, here, is to constrain those from the relevant experimental observables. Keeping in mind this inherent ambiguity regarding the accurate determination of these parameters, we will try to predict a particular range of values for T_i and τ_i in case of RHIC energy ($\sqrt{s}$ = 200 GeV/nucleon and $0-10\%$ centrality) in the current chapter. The same methodology is also applicable for LHC energies.

5.1.1 Procedure:

Lattice QCD (LQCD) calculations indicate that at a temperature $\sim$ 175 MeV the entropy density (s) of the hadronic matter rises significantly due to the release of colour degrees of freedom which are confined within the hadrons at zero temperature. Therefore, it is of foremost importance to have an estimate of the value of the initial entropy density (s_i) / initial temperature (T_i) for the system formed in nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) and assess whether the system is formed in colour deconfined phase or not.

One of the possible ways to estimate the value of the initial entropy density is the extrapolation of the measured (final) observables backward in time through a suitable dynamical model. In absence of viscous loss the time reversal symmetry of the system is valid, therefore, the measured multiplicity at the freeze-out of the system can be used to estimate s_i. The s_i and the thermalization time (τ_i) are constrained by the measured (final) hadron multiplicity (dN/dy) by the following relation [1]:

$$s_i\tau_i = \kappa \frac{1}{A_\perp}\frac{dN}{dy} \tag{5.1}$$

where $A_\perp$ is the transverse area of the system which can be determined from the collision geometry and κ is a known constant (=3.7 for massless Bosons). The value of dN/dy ($\sim$ $1.5 \times dN_{charge}/dy$) which is connected to s_i through Eq. 5.1 is readily available for different collision centralities [2]. In Eq. 5.1 there are two unknown quantities, τ_i and s_i both of which can not be determined from a single equation involving a single measured quantity, dN/dy. Therefore, we choose another experimentally measured quantity the nuclear suppression, R_{AA} of heavy quarks (HQ) [3, 4], which is sensitive to the initial condition and hence will be very useful to estimate s_i.

In the previous chapters, we have elaborated, in detail, the procedure of evaluating the nuclear modification factor, R_{AA} by solving the Fokker Planck Equation for the Heavy Quarks when they suffer elastic as well as gluon bremsstrahlung in the medium. A full account of the calculations of the drag/diffusion coefficient of HQ for both elastic and radiative cases has been presented in the earlier chapters. Therefore, here, we already know the method of estimation of R_{AA} of the electrons originated from the semi-leptonic decays of heavy flavour for RHIC ($\sqrt{s}$ = 200 GeV/nucleon and 0 − 10% centrality). We can now investigate the role of the Equation of state in deciding the initial entropy density of the medium of QGP.

5.1.2 Results:

The equation of state (EoS) plays a crucial role in describing the space time evolution of the expanding QGP from the initial state to the quark-hadron transition point. We use boost invariant hydrodynamic model [7] with the LQCD calculation EoS [5] for the space time description of the matter. The velocity of sound (c_s) as obtained in [5] from LQCD calculations shows a significant variation with temperature (Fig. 5.1). It starts with a very low value of c_s^2 at $T \sim T_c$ and then increases with T to reach a maximum value ($c_s^2 \sim 0.3$) but remains below the Stefan-Boltzmann limit [5] even at a temperature as high as 1000 MeV. The EoS for almost baryon free QGP expected at RHIC energy is taken as: $P = c_s^2\epsilon$. The EoS sets the expansion time scale for the system as $\tau_{\rm exp} \sim [(1/\epsilon)d\epsilon/d\tau]^{-1} \sim \tau/(1+c_s^2)$ indicating the fact that lower value of c_s makes the expansion time scale longer *i.e.* the rate of expansion slower. Therefore, for given values of T_i and T_c the life time of the QGP will be longer for smaller c_s. The value of T_c is fixed at 175 MeV. The nuclear modification factor for heavy quark electrons(Fig. 5.2) has been evaluated using this temperature dependent sound squared velocity, c_s^2 and it is observed that a value of $T_i = 250$ MeV and $\tau_i = 0.83$ fm/c reproduces the experimentally measured nuclear suppression [3, 4] reasonably well. The corresponding value of $s_i = 2\pi^2 g_{eff} T_i^3/45 \sim 34/\ \mathrm{fm}^3$ for $g_{eff} \sim 38$ extracted from the variation of s/T^3 with T provided by the LQCD calculations [5].

As discussed above, for larger c_s the expansion time scale is shorter *i.e.* the QGP life time is smaller, consequently the HQ spends less time in the QGP which ultimately leads to less suppression of the HQs. Therefore, we take the following strategy to obtain the possible range of initial temperature allowed by the experimental data. We take the highest possible value of $c_s^2(= 1/3)$ for the space-time description of the flowing QGP background, in the present

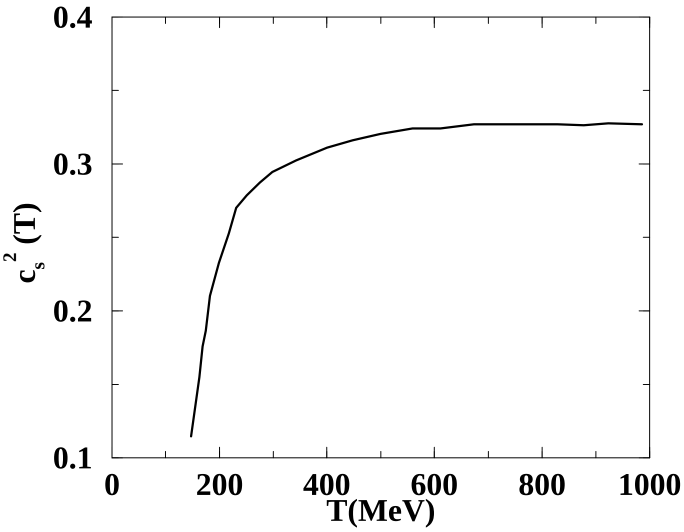

Figure 5.1: Velocity of sound squared as a function temperature [5][6]

approach this will lead to the maximum value of T_i. In this case the HQ will spend the lesser amount of time in the QGP. Therefore, to achieve the experimentally measured R_{AA} one will need larger drag or in other word larger initial temperature. The results for $c_s^2 = 1/3$ is displayed in Fig. 5.3. The value of T_i obtained from the analysis is 300 MeV, the corresponding value of $s_i \sim 59/\text{ fm}^3$, to be considered as the highest value of T_i or s_i admitted by the data.

For $c_s^2 = 1/5$ the HQs spend longer time in QGP (compared to the case when $c_s^2 = 1/3$). Therefore, the observed suppression (5.4) dictates to reduce the initial temperature. In this case the data is well reproduced with $T_i = 210$ MeV and $s_i \sim 19.66/\text{fm}^3$.

Further lowering of c_s will make the value of τ_i large (for given dN/dy) enough to contradict other results like the observation of large hadronic elliptic flow which requires small τ_i (see [8] for review). That will also result in lower T_i with which it will be difficult to explain other experimental results. For all the theoretical results displayed in Figs. 5.2, 5.3 and 5.4, we have kept the quantity dN/dy constant and consequently the value of τ_i changes to 0.83, 0.48 and 1.4 fm/c for $T_i = 250, 300$ and 210 MeV respectively. The changes in T_i is forced by the changes in the EoS. In Fig. 5.5 we show the variation of T_i with c_s^2 obtained by constraints imposed by the experimental data on R_{AA} and dN/dy. The value of T_i varies from 210 to 300 MeV depending

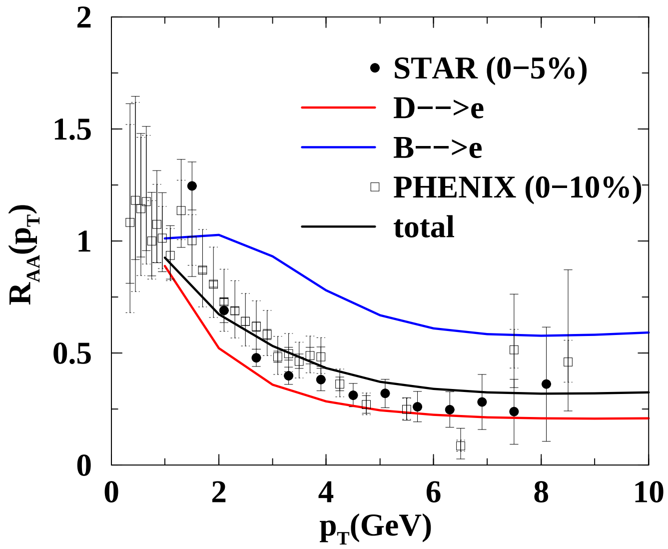

Figure 5.2: (colour online) Variation of R_{AA} with p_T for the space time evolution with initial condition $T_i = 250$ MeV and $\tau_i = 0.84$ fm/c and the EoS which includes the variation of c_s with T.[6]

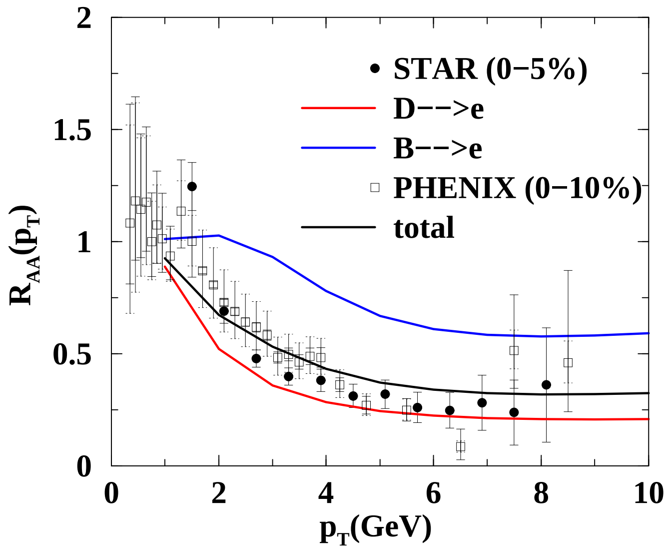

Figure 5.3: (colour online) Variation of R_{AA} with p_T for for $c_s^2 = 1/3$ and $T_i = 300$ MeV.[6]

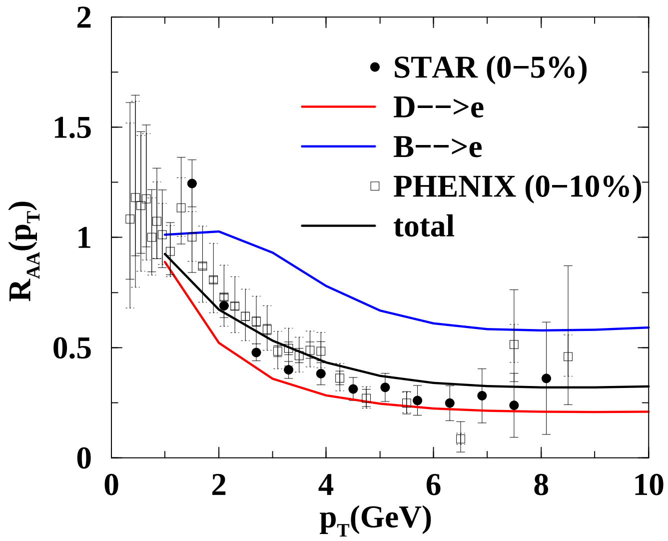

Figure 5.4: (colour online) Variation of R_{AA} with p_T for $c_s^2 = 1/5$ and $T_i = 210$ MeV.[6]

on the value of c_s. In this plot the abscissa corresponding to the ordinate, $T_i = 250$ MeV is taken as 0.275, which is the average value obtained from the variation of c_s^2 with T shown in Fig. 5.1. In this context we compare the value of T_i obtained in the work with some of those reported earlier. In Refs. [9] the value of T_i is obtained as ~ 375 MeV from the study of heavy quark suppression. From the simultaneous analysis of light and heavy quarks suppressions in Ref. [10] a value of $T_i = 400$ MeV is obtained. The authors in Ref. [11, 12, 13] mentioned the values of the initial gluon rapidity distribution, dN_g/dy, which may be converted to $T_i = 290, 270$ and 310 respectively. It is important to note that the lowest value of T_i obtained from the present analysis is well above the quark-hadron phase transition temperature, indicating the fact that the system formed in Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV might be formed in the partonic phase.

The main conclusions drawn from this chapter:

i)The effects of the EoS on the suppression of single electrons originating from the decays of heavy flavours produced in Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV/nucleon has been studied.

ii)We found that the initial temperature may vary from 210 to 300 MeV depending on the magnitude of the velocity of sound, which sets the scale for the expansion of QGP.

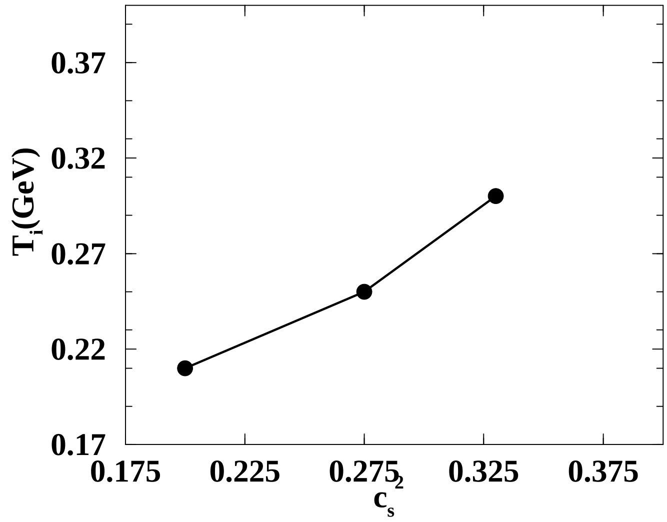

Figure 5.5: The variation of T_i with c_s^2 for fixed dN/dy.[6]

iii)We have used experimental data (charged particle multiplicity and $R_{\rm AA}$ of heavy flavours) and LQCD results (c_s, g_{eff} etc.) to keep the model dependence minimum.

iv)The effects of transverse expansion is neglected here. With the transverse expansion the HQ will (a) travel longer path (b) with diluted density. However, the two competing effects (a) and (b) might have some sort of cancellation due to which our final conclusion may not get altered.

5.1.3 Highlights and some critical comments:

1. Throughout the treatment in this chapter we have considered ideal longitudinal hydrodynamic expansion. We can develop some naive critical thoughts related to the dissipative hydro.

   In case of isentropic expansion of an ideal fluid, the conservation of entropy implies that dN/dy is a constant of motion. In such circumstances, the observed (final) multiplicity, dN/dy may be related to s_i or T_i and τ_i through Eq. 5.1. Assuming a value of τ_i one can estimate T_i. For dissipative systems, however, such an estimate is obviously inapplicable. Generation of entropy during the evolution invalidates the role of dN/dy as a constant of motion. Moreover, the irreversibility arising out of dissipative effects implies that estimation of the initial entropy (temperature) from the measured (final) dN/dy is no longer a trivial task. Nevertheless, one can relate the experimental

$$T_f^3\tau_f \propto \frac{1}{A_\perp^2}\frac{dN}{dy} \tag{5.2}$$

To estimate the initial temperature, T_i for the dissipative fluid is treated as a parameter; for each T_i, the system is evolved forward in time under the condition of Israel-Stewart dissipative fluid dynamics [14] till a given freeze-out temperature T_f is reached. Thus τ_f is determined. We then compute dN/dy at this τ_f from eq. 5.2 and compare it with the experimental dN/dy. The value of T_i for which the calculated dN/dy matches the experimental number is taken to be the value of the initial temperature. It is found [15] that the value of T_i may change by about 6% for $\eta/s = 1/(4\pi)$ for the entire evolution of the system from the QGP initial state to the final hadronic freeze-out state via an intermediary QGP to hadron transition. In the present work we are concerned with evolution of the QGP phase only. In view of this one expects that the change in the value of T_i due to viscous effects QGP phase will be even smaller than 6%.

2. It may be imperative to consider the effects of the pre-equilibrium evolution when the equilibration time is not small. We have estimated this effects for the case when $\tau_i = 1.4$ fm/c. We solved the Boltzmann transport equation with relaxation time approximations for the pre-equilibrium stage of the partonic system to obtain a solution of the form:

$$f_{\rm preq} = f_{\rm eq}(1 - e^{-\tau/\tau_R}) + f_0 e^{-\tau/\tau_R} \tag{5.3}$$

where τ_R is the relaxation time [16], f_0 is the initial distribution [17] and $f_{\rm eq}$ is the equilibrium momentum distribution, of the partons which interact with the HQ. The modification of the HQ spectra can be obtained as [18] $\sim (1 - e^{-\gamma_{\rm preq}\Delta\tau}) < 5\%$ with the duration of the pre-equilibrium phase $\sim$ 1 fm/c. This indicates that the effects of the pre-equilibrium stage in the present situation is not large and the value of the T_i obtained in the present analysis may not change significantly.

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Chapter 6

Gluon bremsstrahlung from Heavy Quark: revisited

All the preceding chapters have dealt with the elastic as well as radiative interactions of heavy quark with the light quarks, light anti-quarks and gluons of the medium of Quark Gluon Plasma (QGP). In this dissertation, one of the main aims is to explore the dominance of the gluon radiation off Heavy Quarks propagating through QGP on various observables. While the third and the fourth chapter discuss the method of calculating the relevant drag/diffusion coefficients of HQ emitting gluons due to the phenomenon of scattering with the medium particles and see its effect on the nuclear modification factor of heavy flavours, in the present chapter, the effect of this gluon bremsstrahlung on other physical quantities/observables will be investigated.

6.1 Distribution Function of Charm Quark:

The Heavy Quarks are produced very early in heavy ion collisions due to hard scattering processes. The single particle momentum distribution function of the HQ, it is going to follow is determined by the perturbative QCD calculations. It can be shown that the transverse momentum distribution function, $f_{in}(p_T)$ follows a power law:

$$f_{in}(p_T) \propto \frac{1}{(a+bp_T)^c}. \tag{6.1}$$

This is the distribution function of the heavy quark before it enters the medium. After that, FPE has been used to study this modification due to its interaction with the QGP medium. In our study, the main object will be to examine whether the distribution function of the external probe, the heavy quark, follows that of the medium particles, i.e. whether the HQ's are thermalised or not. In this work, we have studied the charm quarks as probes. However, the similar methodology applies for bottom quark as well, but being heavier than charm, it has lesser probability to thermalise in the medium, i.e. to follow the equivalent distribution function as followed by the bulk particles.

Let us first assume that charm quark equilibrates in the medium with the following class of distribution:

$$f_{eq}^{CQ}(p;T,q) = Ne^{-\Phi(p;T,q)} \tag{6.2}$$

where, N is the normalisation factor, p is the momentum of charm, T and q are two parameters specifying the distribution function. In the following discussion, we will analyse whether this f_{eq}^{CQ} is that of the Boltzmann type followed by the light partons of the medium of QGP. To proceed in that way, we will need to make use of the generalised Einstein relation, i.e. the fluctuation-dissipation relation, for the relativistic charm quark, already discussed in Chapter-2. For a spatially homogeneous case, the generalised Einstein relation reads:

$$A(p,T) = \frac{1}{p}\frac{d\Phi}{dp}B_{\parallel} - \frac{1}{p}\frac{dB_{\parallel}}{dp} - \frac{2}{p^2}(B_{\parallel}(p,T) - B_{\perp}(p,T)) \tag{6.3}$$

where these A, $B_{\perp}$ and $B_{\parallel}$ are the drag, transverse and longitudinal diffusion coefficients as appear in the Fokker Planck equation(for further details one can refer to Chapter-2).

From Eq. 6.3, it is clear that if the three transport coefficients are known, then one can infer the correct equilibrium distribution function obeyed by CQs and ascertain whether or not CQs will fall under the Boltzmann-Jüttner class of statistics. It is evident from the variation of

$d\Phi/dp$ (calculated from Eq. 6.3) with p (Fig. 6.1) that $d\Phi/dp$ deviates significantly from $d/dp(\sqrt{p^2+m^2}/T)$, charm quarks seem to be away from the Boltzmann like distribution. This figure also shows that radiative energy loss of heavy quarks does not affect the results significantly, which means that whether we include gluon bremsstrahlung into the description or not $d\Phi/dp$ will deviate from Boltzmann distribution. In principle, we should have ascertained the precise form of Φ from Eq. 6.3 had we been able to estimate the accurate drag/diffusion coefficients (like including non-perturbative effects in our pQCD calculations etc.). Therefore, to study the equilibrium distribution of charm and its deviation from Boltzmann like distribution, let us consider Tsallis distribution [1], where Φ is given by:

$$\Phi_{Ts} = \frac{1}{1-q} ln \left[1 - (1-q)E(p)/T_T\right] \tag{6.4}$$

where T_T (temperature-like) and q are parameters. Φ_{Ts} reduces to the Boltzmann distribution

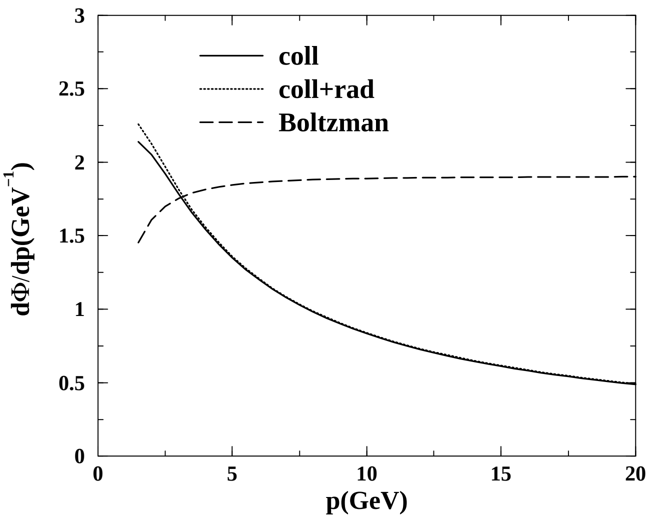

Figure 6.1: For charm propagating in QGP having temperature, $T = 525$ MeV.[2]

in the limit $q \to 0$ and $T_T \to T$ (where T is the temperature of the bath). The values of T_T and q will decide the form of the equilibrium distribution of charm quark. Putting Eq. 6.4 into Eq. 6.3, we get [3]:

$$T_T + (q-1)E = \frac{dE}{dp} \frac{B_\parallel}{pA + \frac{dB_\parallel}{dp} + \frac{2}{p}(B_\parallel - B_\perp)} \tag{6.5}$$

Our aim is to calculate the right hand side of the above equation(Eq. 6.5) and determine the values of T_T and q by studying the variation of $T_T + (q-1)E$ and parametrising the variation by a straight line. We will perform the same exercise with only elastic and after that with elastic combined with the radiative transport coefficients of charm. Let us first take the elastic processes only. The dependence of $T_T + (q-1)E$ on E for CQs of mass, $M = 1.3$ GeV propagating inside a heat bath of $T = 525$ MeV is plotted in Fig. 6.2, considering A, $B_\perp$ and $B_\parallel$ for collisional energy loss as well as including effect due to gluon bremsstrahlung. We

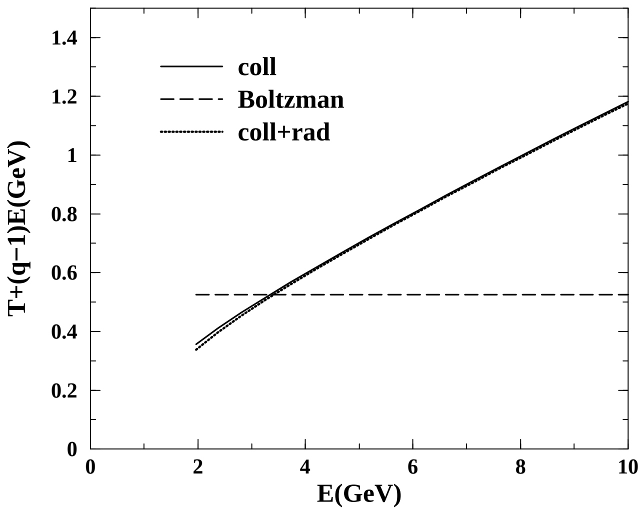

Figure 6.2: Plot of RHS of Eq. 6.5 vs E for collisional as well as total transport coefficients at $T = 525$ MeV. Long dashed line: expected for Boltzman-Jüttner distribution[2]

get $q = 1.101$ and $T_T = 184MeV$. Φ_{Ts} with these values of T_T and q is far from being that of Boltzmann-Juüttner statistics (shown by long-dashed line). Results displayed in Fig. 6.2 also indicate that the inclusion of radiative effects on the drag/diffusion coefficients does not make any noteworthy change on the shape of the equilibrium distribution function of charm. In Figs. 6.2 and 6.3, the long-dashed horizontal lines represent the Boltzmann distribution ($q = 1$ and $T_T = T$) which is obeyed by the bulk particles of the background Quark Gluon Plasma. As a matter of fact, this sort of effect is not quite expected if we look at Eq. 6.5. It is not the magnitude of the transport coefficients, but rather their ratios which decide the shape of f_{eq}^{CQ}. Therefore, it is not surprising that although the value of the relaxation time of CQs is dictated by the magnitude of the drag coefficient (in which the radiative contribution is significant), the

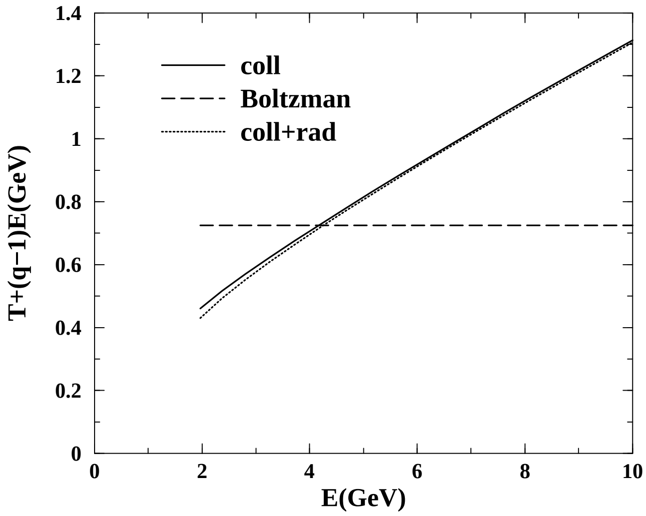

Figure 6.3: Plot of RHS of Eq. 6.5 vs E for collisional as well as total transport coefficients at $T = 725$ MeV. Long dashed line: expected for Boltzman-Jüttner distribution[2]

shape of the equilibrium distribution is largely independent of the magnitude of the transport coefficients. In turn, this means that the nature of the underlying interaction of the charm quark with the bath particles, i.e. whether it suffers only elastic collisions or undergoes gluon bremsstrahlung also, has very little to do with the ultimate fate of f_{eq}^{CQ}. This conclusion remains unaltered even when we increase the bath temperature, T to 725 MeV. At $T = 725$ MeV, the slope the straight line remains unchanged, i.e. the value of the parameter q comes out to be 1.095, which is close to the value obtained for $T = 525$ MeV. This study of $T + (q-1)E$ vs E at different temperatures is important because the medium we are talking about is not having a constant temperature all through, rather the medium evolves and the temperature changes. So, to this end, it is constructive to note that though at higher temperatures, radiative energy loss of HQ is more, however, that does not make the equilibrium distribution function of HQ like that of the medium particles. However, the value of the other parameter of the Tsallis distribution, T_T is found to be 335 MeV. At this temperature of the heat bath too (i.e. 725 MeV), the incorporation of the radiative drag/diffusion coefficients hardly has any bearing as far as the shape of the equilibrium distribution function of charm quark is concerned. For the charm quark to become a part of the background system, i.e. to follow the same statistics as that of the bath particles, both the parameters q and T_T/T need to be 1. Instead, we notice that q and T_T/T (this second ratio is 2.85 at $T = 525$ MeV and 2.164 at $T = 725$ MeV) are far from unity. Therefore, it might be concluded that although the CQ may equilibrate while propagating through QGP, it may not share the same distribution with the bath particles, i.e. with light quarks and gluons, for a wide range of CQ energies and bath temperatures.

6.2 Shear Viscosity to entropy density ratio of Quark Gluon Plasma:

The value of the shear viscosity (η) to entropy density (s) ratio, η/s, plays a pivotal role in deciding the nature of QGP, i.e., whether the medium behaves like a weakly coupled gas or a strongly coupled liquid. In this work we evaluate η/s by calculating the transport parameter, $\hat{q}$, which is a measure of the squared average momentum exchange between the probe and the bath particles per unit length [4, 5, 6]. The $\hat{q}$, which has been found to be1 GeV^2/fm in Ref. [6], can be related to the transverse diffusion coefficient of the CQ, which is calculated here. When a CQ with a certain momentum propagates in QGP, a transverse momentum exchange with the bath particles occurs. Hence, the momentum of the energetic CQ is shared by the low-momentum (on the average) bath particles, which is expressed through the transverse dif- fusion coefficients. The transverse diffusion coefficients cause the minimization of the momentum (or velocity) gradient in the system. Therefore, it must be related to the shear viscous coefficients of the system which drive the system toward a depleted velocity gradient. The transverse momentum diffusion coefficient $B_\perp$ can be written as

$$B_\perp = \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_{ij} \tag{6.6}$$

Here, we might recall the expression for the tensorial form of the diffusion coefficient:

$$B_{ij} = \frac{1}{2} \ll (p' - p)_i (p' - p)_j \gg, \tag{6.7}$$

where, $\ll\gg$ means an integration over the relevant phase space weighted by the invariant amplitude squared of the interaction of the heavy quark with the medium particles. Using Eq. 6.7 with $(p'-p)_i = q_i$, we get

$$\begin{aligned} B_\perp &= \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)\frac{1}{2} \ll q_i q_j \gg \\ &= \frac{1}{4} \ll \vec{q}^2 - \frac{(\vec{p}.\vec{q})^2}{\vec{p}^2} \gg \end{aligned}$$

If we take $\vec{p}$ to be along z-axis,

$$\begin{aligned} B_\perp &= \frac{1}{4} \ll \vec{q}^2 - q_z^2 \gg \\ &= \frac{1}{4} \ll q_\perp^2 \gg \\ &= \frac{1}{4}\hat{q} \end{aligned} \tag{6.8}$$

With this definition of $\hat{q}$, we calculate η/s of a weakly coupled QGP from the following expression[4]:

$$\frac{\eta}{s} \approx 1.25\frac{T^3}{\hat{q}} \tag{6.9}$$

Therefore,

$$4\pi\frac{\eta}{s} \approx 1.25\pi\frac{T^3}{B_\perp}. \tag{6.10}$$

Eq. 6.10 indicates that the η/s can be estimated from $B_\perp$. We display $4\pi\frac{\eta}{s}$ against the bath temperature, T when the CQ undergoes both collisional and radiative interactions in the medium. From the analysis of the experimental data [6], it was found that $4\pi\frac{\eta}{s} = 1.4 \pm 0.4$, which may be compared with the AdS/CFT bound $4\pi\frac{\eta}{s} \geq 1$ [7].

From the results shown in Fig. 6.4, it should be noted that the value of η/s changes substantially with the inclusion of the radiative effects. The inclusion of the radiative loss in the transverse diffusion, $B_\perp$ brings the theoretical values closer to the experimental findings [8]. This highlights the importance of the radiative loss of the charm quark in QGP. It is interesting to note that the value of $\hat{q}$ for $T = 300$ MeV is about 2 GeV^2/fm, which is close to the one obtained in the Gyulassy- Levai-Vitev approach [9] to energy loss but lower than the value extracted from Baier-Dokhshitzer-Mueller-Peigne-Schiff [10] or Armesto-Salgado-Wiedemann [11] approaches.

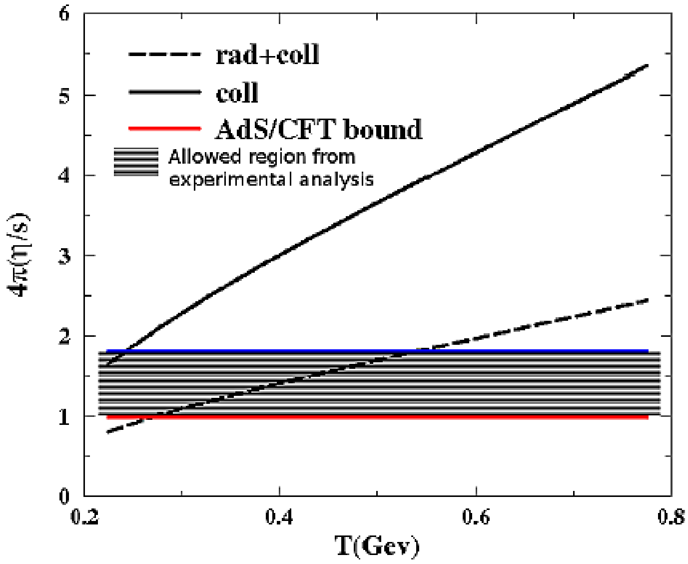

Figure 6.4: For a charm quark with momentum, $< p_T >= 5$ Gev propagating in QGP of temperature, T[2]

Highlights

We summarise the findings of this chapter below:

1. We have investigated the effect of the gluon bremsstrahlung off heavy quark discussed in the third chapter of this thesis on the shape of the equilibrium distribution function of heavy quark. The underlying assumption of this treatment is that the heavy quark, in the course of its journey in the QGP medium, equilibrates at the end. One may note that even if the heavy quark comes to equilibrium, it never shares the same distribution function like the bath particles, rather it follows a new class of distribution functions, called Tsallis distribution. It has been observed that the shape of the HQ equilibrium distribution depends on the ratios of the drag/diffusion coefficients and consequently, it does not change even with the inclusion of the gluon radiation. Yet, gluon radiation does have effect on the rate of equilibration of the heavy quark in the thermal medium as drag is enhanced when HQ suffers radiative energy loss.

2. The shear viscosity to the entropy density ratio(η/s) has been estimated using heavy quark as the probe by using a relation between the transverse diffusion, $B_\perp$ and η/s of the medium. The radiation of gluons has been seen to affect the magnitude of η/s considerably. It is noticed that when one takes into account the gluon bremsstrahlung, the value of η/s comes closer to the result obtained from the analysis of the experimental data.

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Chapter 7

A coda: Summary, conclusions and outlook

Our quest to understand the early universe a few microseconds after the Big Bang took us to a journey starting from re-creating the situation, similar to that existed in the early universe, by colliding two heavy ions in the laboratories. The new state of matter thus created is called "Quark Gluon Plasma"(QGP). We have also seen that this medium is composed of the quarks and gluons which are not bound in the form of hadrons. This happens due to the distinctive feature of Quantum Chromo Dynamics(QCD) governing the physics of deconfined quarks and gluons; the asymptotic freedom. This trait of QCD made it possible to realise this state of matter characterised by the coloured degrees of freedom.

In this dissertation, our central goal has been to study and to explore different properties of the QGP using heavy quarks, like charm and bottom, as probes. It has been explained that the heavy quarks, produced quite early(before the formation of QGP medium) in the heavy ion collisions due to the hard scatterings, can be described by the Brownian motion in the thermal bath of light quarks and gluonws. The equation describing the motion of heavy quark, i.e. the Fokker Planck Equation(FPE) has been discussed and solved in this work. In the way of doing so, we have calculated the transport coefficients appearing in FPE which has been solved with appropriate initial conditions. Let us summarise the main findings of this discourse and what we have learnt about various properties of QGP.

1. The medium of QGP created in the heavy ion collisions, is assumed to be in thermal equilibrium. HQs, which are produced in the early hard scatterings has been used to probe the properties of QGP. The equation of motion of heavy quark immersed in a QGP fluid can be described by the well-known Boltzmann Transport Equation (BTE). It has also been explained that in the limit of the soft gluon exchange, i.e. when the momentum transfer of the heavy quark is very small due to the elastic scattering with the medium particles, one can approximate BTE into either Langevin Equation or Fokker Planck Equation. Though, our work deals only with the parts and parcel of FPE, we also gave some account of the Langevin equation, with the appropriate references, for the sake of completeness of the discussion. Between the two equivalent formalisms we adopted Fokker Planck formalism.

2. We have evaluated the drag and longitudinal and transverse diffusion coefficients of the heavy quark traversing QGP within the ambit of perturbative QCD and kinetic theory. The full dependence of the transport coefficients on the temperature of the bath as well as on the momentum of the heavy quark is taken into account. In this way we have computed the inputs to the Fokker Planck Equation for two cases:(i) when the heavy quark encounters elastic scatterings with the medium particles, (ii) when the heavy quark suffers radiative loss in the medium.

3. Before calculating the transport coefficients in case of gluon bremsstrahlung off heavy quarks, a brief discussion about the angular distribution of radiation from a point charge in Classical Electro Dynamics (CED) was in order. Similar to what happens in CED, the spectrum of gluon radiation from a heavy particle includes a conical region of "no radiation", called the dead-cone region. This region is not present in case of a light particle. In the radiative transport coefficients of HQ, the effect due to the dead-cone has been taken into consideration. It has been noticed that as long as the radiated gluon is 'soft', the radiative transport coefficients can be factorised into the elastic transport

coefficients multiplied by the gluon radiation spectrum integrated over the relevant phase space. The radiative transport coefficients thus estimated have been seen to exceed the elastic transport coefficients in magnitude especially at higher bath temperature, T and higher momentum, p of the heavy quark. The method of calculations of the elastic and the radiative transport coefficients has been elaborated in Chapter-2 and 3.

4. The transport coefficients arising from the elastic scatterings of HQ with the medium particles has been evaluated shielding the infra-red divergence(due to the soft mediatory gluon) by the Debye mass in an ad-hoc manner. Later, an effort has been made to shield this divergence with the HTL re-summed gluon propagator which includes the imaginary as well as real part of the self energy of the gluon. It is observed that this self-consistent way of shielding the divergence led to enhanced values of the transport coefficients. This happens because of the inclusion of the typical processes occurring in medium, like Landau damping etc. into the calculations of the relevant Feynman matrix elements. This increase in the magnitudes of the drag/diffusion coefficients will have effect on the energy loss of HQ and also on the equilibration rate of the heavy quark in QGP.

5. After having calculated the transport coefficients of heavy quark, we have stepped forward to solve the FPE, with proper initial conditions which has been discussed in the fourth chapter. The solution of the Fokker Planck Equation has led us to the theoretical estimation of the nuclear modification factor, R_{AA} of the charm and bottom quarks. The ratio of the final solution of FPE at the critical temperature, T_c of QGP(at this temperature, QGP ceases to exist and hadrons phase starts) to the initial distribution of heavy quark(before entering into QGP medium) is defined as the nuclear suppression/modification factor, R_{AA} of the heavy quark. In order to match the theoretical results with the experimental data obtained in heavy ion collisions at RHIC and LHC, the distribution functions have been convoluted with the fragmentation functions to hadronise the open charm and bottom quarks into D and B mesons respectively. In this way, the R_{AA} of D and B mesons are also calculated. In case of RHIC, the nuclear modification factor has been estimated for the electrons originated from the semi-leptonic decays of the heavy mesons. The detailed

calculations have been given in Chapter-4. For LHC, the D-meson spectra is measured directly.

6. The determination of the initial conditions of QGP medium, i.e the initial temperature, T_i and the initial thermalisation time, τ_i is ambiguous due to the absence of any first principle methods of calculations of those quantities. It has been discussed in the fifth chapter how one can estimate the initial conditions, T_i and τ_i by constraining the experimental observables like total particle multiplicity and R_{AA} of the heavy flavour. In this way, it is possible to predict a range of the initial parameters depending upon the magnitude of the velocity of sound used in the present calculation. Though this procedure has been followed for RHIC energies, it is similarly applicable for LHC energies, too. A rough estimation shows that the pre-equilibrium stage does not affect the conclusion about the values of T_i and τ_i much.

7. Chapter-6 dealt with the effect of the radiation on the shape of the distribution function of the charm quark, if it equilibrates in the course of its journey through the medium of QGP. In this context, the generalised Einstein's relation has been discussed for a relativistically moving heavy quark. The shape of the equilibrium distribution has been seen to depend upon the three drag/diffusion coefficients of the HQ. It has been conjectured that the shape remains unaltered irrespective of the fact that whether the heavy quark interacts elastically with the medium particles or emits gluon radiation. This happens due to the fact that the shape depends on the ratios of the transport coefficients and not on their absolute values.

8. At the end of this treatise, we also endeavoured to estimate the shear viscosity to the entropy density ratio(η/s) of QGP using charm quark as a probe. A relation has been established connecting the transverse diffusion, $B_\perp$ of HQ to η/s of the surrounding medium. We observe that the value of η/s becomes close to the value obtained from the analysis of the experimental data when one adds the contribution due to the gluon bremsstrahlung on top of the elastic scatterings of the HQ with the medium particles.

The above mentioned points are the gist of the work which has been covered in this entire thesis. In view of these findings, we can conclude that, in this dissertation, we were able to illustrate a basic picture of the heavy quark travelling inside the medium of QGP and to develop an idea about various properties of QGP by studying the equation of motion of HQ. In this way the heavy quark successfully can be described as a good probe of the thermalised medium of Quark Gluon Plasma.

Outlook

Now that we have investigated the main features of the medium created in heavy ion collisions(HIC) using the heavy quark(formed at the early stage of HIC before the formation of QGP medium) as probe, we can look forward to concentrate on various other aspects, applications and improvement of the formalism presented in this thesis.

1. To this end, we have considered the evolution of the bulk medium(QGP) to be governed by boost invariant one dimensional Bjorken ideal hydrodynamics to calculate the experimental observables like the nuclear modification factor, R_{AA} of the open heavy flavour. The higher dimensional viscous hydrodynamics is on its way to be implemented in the present formalism.

2. With the higher dimensional viscous hydrodynamics incorporated, we can evaluate another important experimental observable like the elliptic flow, v_2 of heavy flavour and we can endeavour to reproduce experimental results on R_{AA} and v_2 of HQ, simultaneously.

3. The calculation of the radiative transport coefficients with the improved technique presented here might also affect the estimation of the elliptic flow and in turn the possibility of equilibration of heavy quark in the QGP medium. This issue of the heavy flavour equilibrium is of contemporary interest.

4. So far, the gluon radiation spectrum has been calculated keeping the heavy quark in a straight 'eikonal'trajectory after elastic scattering as well as after the emission of radiation.

We have made a recent effort to lift this eikonal approximation where a finite recoil of the heavy quark has been considered after it elastically scatters off the medium particles. This non-eikonality is yet to be explored on any relevant experimental observable.

Work is now in progress where we are implementing the above features to the presently discussed formalism.

The pursuit of knowledge led us to a pathway of illumination where we have tried to learn and discuss the most fundamental questions of all the decades: "How was the universe like at the beginning just after the Big Bang?" and "After the miniature universe has been created due to the heavy ion collisions in the laboratories, how can we describe the medium formed called Quark Gluon Plasma".

Appendix:

In this appendix we demonstrate the calculation of matrix elements of the processes $Qq \rightarrow Qq$ and $Qg \rightarrow Qg$ applying Hard Thermal Loop (HTL) approximation. Due to presence of medium QCD interaction is divided into longitudinal and transverse parts. That means, the gluon self energy is divided into longitudinal component and transverse component [1]. u_μ is the fluid four-velocity ($u_\mu u^\mu = 1$). Any four-vector can be decomposed into a part which is parallel to fluid velocity and another one perpendicular to fluid-flow. We can decompose the four-momentum transfer Q_μ such that

$$\begin{aligned} \omega &= Q.u \\ \tilde{Q}_\mu &= Q_\mu - u_\mu (Q.u) \end{aligned} \tag{7.1}$$

and

$$\begin{aligned} Q^2 &= \omega^2 - q^2 \\ \tilde{Q}^2 &= -q^2 \end{aligned} \tag{7.2}$$

Eqs. 7.1 and 7.2 are valid in the local rest frame of fluid, i.e. in a frame where $u = (1, \vec{0})$. Similarly a tensor orthogonal to u_μ can be defined,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \tag{7.3}$$

The full gluon propagator with momentum Q is obtained from the vacuum polarization by using Dyson-Schwinger equation ([2],[3])

$$\Delta^{\mu\nu} = \frac{\mathcal{P}_T^{\mu\nu}}{-Q^2 + \Pi_T} + \frac{\mathcal{P}_L^{\mu\nu}}{-Q^2 + \Pi_L} + (\alpha - 1)\frac{Q^\mu Q^\nu}{Q^2} \tag{7.4}$$

where α is a gauge-fixing parameter. The longitudinal tensor $\mathcal{P}_T^{\mu\nu}$ and the transverse tensor $\mathcal{P}_L^{\mu\nu}$ are defined as [4]

$$\mathcal{P}_L^{\mu\nu} = -\frac{1}{Q^2 q^2}(\omega Q^\mu - Q^2 u^\mu)(\omega Q^\nu - Q^2 u^\nu) \tag{7.5}$$

$$\mathcal{P}_T^{\mu\nu} = \tilde{g}_{\mu\nu} + \frac{\tilde{Q}_\mu \tilde{Q}_\nu}{q^2} \tag{7.6}$$

which are orthogonal to Q^μ as well as to each other

$$Q_\mu \mathcal{P}_L^{\mu\nu} = Q_\mu \mathcal{P}_T^{\mu\nu} = \mathcal{P}_{L\nu}^{\mu} \mathcal{P}_T^{\nu\rho} = 0 \tag{7.7}$$

But,

$$\mathcal{P}_i^{\mu\rho} \mathcal{P}_{i\nu\rho} = \mathcal{P}_{i\nu}^{\mu} \quad ,\mathrm{i = L/T} \tag{7.8}$$

The free gluon propagator in zero temperature is

$$D^{\mu\nu} = \left(-g^{\mu\nu} + \alpha\frac{Q^\mu Q^\nu}{Q^2}\right)\frac{1}{Q^2} \tag{7.9}$$

The transverse and longitudinal self-energies are

$$\Pi_L(Q) = (1 - x^2)\pi_L(x), \qquad \Pi_T(Q) = \pi_T(x) \tag{7.10}$$

where $x = \omega/q$ and scaled self-energies π_T and π_L are given by [1, 5],

$$\pi_T(x) = m_D^2\left[\frac{x^2}{2}+\frac{x}{4}(1-x^2)ln\left(\frac{1+x}{1-x}\right)-i\frac{\pi}{4}x(1-x^2)\right]$$
$$\pi_L(x) = m_D^2\left[1-\frac{x}{2}ln(\frac{1+x}{1-x})+i\frac{\pi}{2}x\right] \tag{7.11}$$

Non-zero imaginary parts of the self-energies signify the new processes (Landau damping) arising due to presence of medium [1].

With this introduction, we will calculate the matrix element of the process $Qq \to Qq$ in a self-consistent manner. We must point out at this point that all the earlier works introduced thermal mass in an ad hoc fashion to 'cure' the divergence due to very low-momentum intermediary gluon exchange. But here we make use of HTL approximation as a remedy.

7.0.1 $Qq \to Qq$ Matrix Element from HTL approximation:

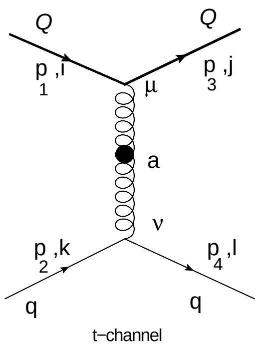

Figure 7.1: $Qq \to Qq$ Feynman diagram. Bold lines are for heavy quarks(Q).

From diagram fig.7.1 we can calculate the t-chennel matrix element for the process $Qq \to Qq$. We will use the effective gluon propagator obtained by HTL approximation[1]. Pictorially, an effective propagator will be denoted by a bare one with a solid circle on it. We can write the ampilitude in Feynman Gauge($\alpha = 1$) from fig. 7.1 as,

$$
\begin{aligned}
-iM_t = \overline{u}(p_3)(-ig\gamma^\mu t^a_{ji})u(p_1)\,[-i\Delta_{\mu\nu}] \\
\overline{u}(p_4)(-ig\gamma^\nu t^a_{lk})u(p_2)
\end{aligned}
\tag{7.12}
$$

where g is strong coupling and $g^2 = 4\pi\alpha_s$. $i, j, k, l \quad (i \neq j, \;\; k \neq l)$ are quark colours and ‘a’ is the colour of intermediary gluon with polarizations μ, ν. After squaring and averaging over spin and colour as well as using eq. 7.4 we get,

$$
\begin{aligned}
\frac{|M_{Qq}|^2}{4C_{Qq}g^4} &= 2\frac{p_4.\mathcal{P}_T.p_3 p_2.\mathcal{P}_T.p_1}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_3 p_2.\mathcal{P}_L.p_1}{(t-\Pi_L)^2} \\
&+ 2\frac{p_4.\mathcal{P}_T.p_1 p_2.\mathcal{P}_T.p_3}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_1 p_2.\mathcal{P}_L.p_3}{(t-\Pi_L)^2} \\
&+ 2A\frac{p_4.\mathcal{P}_L.p_3 p_2.\mathcal{P}_T.p_1 + p_4.\mathcal{P}_T.p_3 p_2.\mathcal{P}_L.p_1}{(t-\Pi_T)^2 (t-\Pi_L)^2} \\
&+ 2A\frac{p_4.\mathcal{P}_L.p_1 p_2.\mathcal{P}_T.p_3 + p_4.\mathcal{P}_T.p_1 p_2.\mathcal{P}_L.p_3}{(t-\Pi_T)^2 (t-\Pi_L)^2} \\
&- 2p_4.p_2\left[\frac{p_3.\mathcal{P}_T.p_1}{(t-\Pi_T)^2} + \frac{p_3.\mathcal{P}_L.p_1}{(t-\Pi_L)^2}\right] \\
&- 2p_3.p_1\left[\frac{p_4.\mathcal{P}_T.p_2}{(t-\Pi_T)^2} + \frac{p_4.\mathcal{P}_L.p_2}{(t-\Pi_L)^2}\right] \\
&+ p_3.p_1 p_4.p_2\left[\frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2}\right] \\
&+ m^2\left[2\frac{p_4.\mathcal{P}_T.p_2}{(t-\Pi_T)^2} + 2\frac{p_4.\mathcal{P}_L.p_2}{(t-\Pi_L)^2}\right] \\
&- m^2\left[2\frac{p_4.p_2}{(t-\Pi_T)^2} + \frac{p_4.p_2}{(t-\Pi_L)^2}\right]
\end{aligned}
\tag{7.13}
$$

where $C_{Qq} = \frac{2}{9}$ is the Color factor [6], $Q^2 \equiv t = (p_1 - p_3)^2$, $A = t^2 - t(Re\Pi_T + Re\Pi_L) + Re\Pi_T\Pi_L^*$ and we have used the following relations.

$$\Delta^{\mu\rho}\Delta^{*\nu}_{\rho} = \frac{\mathcal{P}^{\mu\nu}_T}{(t-\Pi_T)^2} + \frac{\mathcal{P}^{\mu\nu}_L}{(t-\Pi_L)^2}$$
$$|\Delta|^2 = \Delta^{\mu\nu}\Delta^{*}_{\nu_\mu} = \frac{2}{(t-\Pi_T)^2} + \frac{1}{(t-\Pi_L)^2} \tag{7.14}$$

Using eqs. 7.5, 7.6 we can show that

$$p_1.\mathcal{P}_L.p_2 = p_3.\mathcal{P}_L.p_4 = p_4.\mathcal{P}_L.p_1 = p_2.\mathcal{P}_L.p_3 \tag{7.15}$$

where all the calculations are done in the rest frame of fluid element.

7.0.2 $Qg \to Qg$ Matrix Element fron HTL Approximation

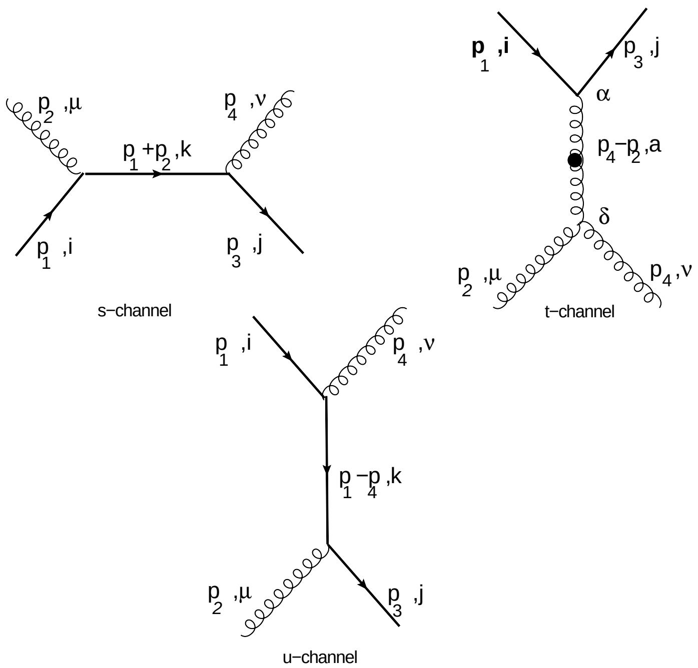

Figure 7.2: $Qg \to Qg$ Feynman diagrams

The Feynman diagram for this process contain all three channels s, t and u (fig. 7.2). Among them the t-channel diagram contains gluon propagator. Since heavy quarks are not thermalized we use bare fermion propagators for s channel and u channel diagrams. As a consequence, we use naive perturbation theory results [7, 9] for $|M_s|^2$, $|M_u|^2$ and cross-term $M_s M_u^*$. On the other hand, we have to use effective propagator for t channel diagram for consistent removal of divergence. Hence, $|M_t|^2$ and cross-terms $|M_s M_t^*|$ as well as $|M_u M_t^*|$ are drastically different from their bare counterpart. For sake of completeness we write down $M_s,\ M_t,\ and\ M_u$ for the process under discussion (fig. 7.2).

$$-iM_s = \overline{u}(p_3)(-ig\gamma^\nu t^b_{jk})i\frac{\not{p_1}+\not{p_2}}{s-m^2}(-ig\gamma^\mu t^c_{ki})u(p_1)\epsilon_\mu\epsilon^*_\nu \tag{7.16}$$

$$-iM_u = \overline{u}(p_3)(-ig\gamma^\mu t^c_{jk})i\frac{\not{p_1}-\not{p_4}}{u-m^2}(-ig\gamma^\nu t^b_{ki})u(p_1)\epsilon_\mu\epsilon^*_\nu \tag{7.17}$$

$$-iM_t = \overline{u}(p_3)(-ig\gamma^\alpha t^a_{ji})u(p_1)(-i\Delta_{\alpha\delta})gf_{abc}\mathcal{C}^{\mu\delta\nu}\epsilon_\mu\epsilon^*_\nu \tag{7.18}$$

where $\mathcal{C}^{\mu\delta\nu} = [(2p_4-p_2)^\mu g^{\delta\nu} + (-p_4-p_2)^\delta g^{\mu\nu} + (2p_2-p_4)^\nu g^{\mu\delta}]$ is the three-gluon vertex. Mandelstam variables s, t, u are defined as usual, $s=(p_1+p_2)^2,\ t=(p_1-p_3)^2$ and $u=(p_1-p_4)^2$. While summing over gluon polarization we must exclude unphysical degrees of freedom. We follow the trick used in ref. [9] which deletes terms like $p_4.\epsilon$ as well as $p_2.\epsilon$ and uses the substitution

$$\sum_{polarization}\epsilon_\mu\epsilon^*_\nu = -g_{\mu\nu} \tag{7.19}$$

So for all practical purposes we use

$$\mathcal{C}'^{\mu\delta\nu} = [2p_4^\mu g^{\delta\nu} + (-p_4-p_2)^\delta g^{\mu\nu} + 2p_2^\nu g^{\mu\delta}] \tag{7.20}$$

as three-gluon vertex along with eq. 7.19.

The matrix element square contains the following terms:

$$\frac{9}{4g^4}|\overline{M_s}|^2 = \frac{(s-m^2)(m^2-u)+2m^2(s+m^2)}{(s-m^2)^2} \tag{7.21}$$

$$\frac{9}{4g^4}|\overline{M_u}|^2 = \frac{(s-m^2)(m^2-u)+2m^2(u+m^2)}{(m^2-u)^2} \tag{7.22}$$

$$\frac{9}{g^4}Re\overline{M_s M_u^*} = \frac{m^2(4m^2-t)}{(s-m^2)(m^2-u)} \tag{7.23}$$

where overbar in l.h.s denotes spin as well as color sum and averge.

As already said, the above terms contain bare heavy-quark propagator. But for gluon propagator we have to use the HTL approximated effective one. The real part of effective cross-term $M_s M_t^*$ is as follows:

$$\frac{16}{g^4}(s-m^2)Re\overline{M_s M_t^*} = M_{st}^1 + M_{st}^2 \tag{7.24}$$

where,

$$
\begin{aligned}
M_{st}^{1} \;=\; & \frac{4m^2t\omega E_1\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4st\omega E_1\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{8m^2tE_1^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{8stE_1^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{8t^2E_1^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{4m^2t\omega E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{4st\omega E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{12m^2tE_1E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4stE_1E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{12t^2E_1E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4t^2E_2^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4m^2t\omega E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&+\frac{4st\omega E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{24m^2tE_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{24stE_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{8t^2E_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{20m^2tE_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{12stE_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{8t^2E_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4m^2t\omega E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{4st\omega E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&+\frac{12m^2tE_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4stE_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}-\frac{4t^2E_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{4t^2E_2E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{4m^2tE_3E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}+\frac{4stE_3E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^2}\\
&-\frac{8m^4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{8s^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{8st\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&-\frac{4m^2t\omega E_1\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4st\omega E_1\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{8m^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&-\frac{8sE_1^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{8tE_1^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{8m^2\omega^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&+\frac{8s\omega^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}
\end{aligned}
$$

and

$$
\begin{aligned}
M_{st}^2 \;=\;& \frac{8t\omega^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} - \frac{4m^2t\omega E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4st\omega E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&+\frac{12m^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{4sE_1E_2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{12tE_1E_2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} \\
&-\frac{12m^2\omega^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4s\omega^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{12t\omega^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&-\frac{4tE_2^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} + \frac{4t\omega^2E_2^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4m^2t\omega E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&-\frac{4st\omega E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{24m^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{24sE_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} \\
&-\frac{8tE_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{24m^2\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{24s\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&+\frac{8t\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{20m^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{12sE_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} \\
&-\frac{8tE_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{20m^2\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{12s\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&+\frac{8t\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4m^2t\omega E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} - \frac{4st\omega E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&+\frac{12m^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{4sE_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{4tE_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} \\
&-\frac{12m^2\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4s\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4t\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} \\
&-\frac{4tE_2E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} + \frac{4t\omega^2E_2E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} + \frac{4m^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} \\
&+\frac{4sE_3E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}} - \frac{4m^2\omega^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}} - \frac{4s\omega^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}
\end{aligned}
$$

Similarly, the real part of effective $M_uM_t^*$ after proper summing and avergaing becomes:

$$
\frac{16}{g^4}(m^2-u)Re\overline{M_uM_t^*} = M_{ut}^1 + M_{ut}^2 \tag{7.25}
$$

$$
\begin{aligned}
M_{ut}^{1} \;=\;& \frac{4m^2t\omega E_1\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4st\omega E_1\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4t^2\omega E_1\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&+\frac{8m^2tE_1^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{8stE_1^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4m^2t\omega E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&-\frac{4st\omega E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4t^2\omega E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4m^2tE_1E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&+\frac{4stE_1E_2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4m^2t\omega E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4st\omega E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&+\frac{4t^2\omega E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{24m^2tE_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{24stE_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&-\frac{16t^2E_1E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{12m^2tE_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4stE_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&-\frac{4t^2E_2E_3\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4m^2t\omega E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4st\omega E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&+\frac{4t^2\omega E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4m^2tE_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4stE_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&-\frac{8t^2E_1E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4t^2E_2E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}-\frac{4m^2tE_3E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&+\frac{12stE_3E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4t^2E_3E_4\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}+\frac{4t^2E_4^2\,(t-\mathrm{Re}\Pi_L)}{q^2\,(t-\Pi_L)^{\,2}}\\
&-\frac{24m^4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}+\frac{32m^2s\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}-\frac{8s^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}\\
&+\frac{16m^2t\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}-\frac{8st\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}-\frac{4m^2t\omega E_1\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}\\
&+\frac{4st\omega E_1\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}+\frac{4t^2\omega E_1\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}+\frac{8m^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}\\
&-\frac{8sE_1^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^{\,2}}-\frac{8m^2\omega^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}+\frac{8s\omega^2E_1^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}\\
&-\frac{4m^2t\omega E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}+\frac{4st\omega E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}+\frac{4t^2\omega E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^{\,2}}
\end{aligned}
$$

$$
\begin{aligned}
M_{ut}^2 \;=\;& \frac{4m^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{4sE_1E_2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{4m^2\omega^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&-\frac{4s\omega^2E_1E_2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4m^2t\omega E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}-\frac{4st\omega E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&-\frac{4t^2\omega E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{24m^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{24sE_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&-\frac{16tE_1E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{24m^2\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{24s\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&+\frac{16t\omega^2E_1E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{12m^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{4sE_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&-\frac{4tE_2E_3\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{12m^2\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4s\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&+\frac{4t\omega^2E_2E_3\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4m^2t\omega E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}-\frac{4st\omega E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&-\frac{4t^2\omega E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4m^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{4sE_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&-\frac{8tE_1E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{4m^2\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}-\frac{4s\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&+\frac{8t\omega^2E_1E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}+\frac{4tE_2E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{4t\omega^2E_2E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&-\frac{4m^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{12sE_3E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}+\frac{4tE_3E_4\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}\\
&+\frac{4m^2\omega^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}-\frac{12s\omega^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}-\frac{4t\omega^2E_3E_4\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}\\
&+\frac{4tE_4^2\,(t-\mathrm{Re}\Pi_T)}{(t-\Pi_T)^2}-\frac{4t\omega^2E_4^2\,(t-\mathrm{Re}\Pi_T)}{q^2\,(t-\Pi_T)^2}
\end{aligned}
$$

The properly summed and averaged effective $|M_t|^2$ term becomes.

$$\frac{8}{g^4}\overline{M_tM_t^*}=M_{tt}^1+M_{tt}^2+M_{tt}^3+M_{tt}^4+M_{tt}^5 \tag{7.26}$$

$$
\begin{aligned}
M_{tt}^{1} \;=\; & -\frac{4t^3\omega^2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{8t^2\omega^4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{8t^2\omega^3E_1}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& -\frac{8t^3\omega E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{16t^2\omega^3E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega^2E_1E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& +\frac{8t^2\omega^3E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega^2E_1E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{16t^2\omega^2E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& -\frac{32t^2\omega E_1E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{8t^3\omega E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega^3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& +\frac{16t^2\omega^2E_1E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{16t^3E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{32t^2\omega^2E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& +\frac{32t^2\omega E_1E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega^2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{32t^2\omega E_1E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}\\
& -\frac{32t^2\omega E_2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}+\frac{64t^2E_1E_2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_L^*\right)}-\frac{4t^2\omega^4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}\\
& +\frac{4t^2\omega E_1}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t^3\omega E_1}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t^2\omega^3E_1}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}\\
& +\frac{4t\omega^3E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t^2\omega^3E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t\omega^5E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}\\
& +\frac{8m^2tE_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8stE_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t^3E_1E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}\\
& -\frac{4t\omega^2E_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{8t^2\omega^2E_1E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t\omega^4E_1E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}\\
& -\frac{4t^2\omega E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t^3\omega E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}
\end{aligned}
$$

$$
\begin{aligned}
M_{tt}^2 \;=\; & -\frac{4t^2\omega^3 E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t^2\omega^2 E_1 E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{8m^2 t E_2 E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{8stE_2E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8t^2E_2E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t^3E_2E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{4t\omega^2E_2E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8t^2\omega^2E_2E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t\omega^4E_2E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{16t\omega E_1E_2E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t^2\omega E_1E_2E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t\omega^3E_1E_2E_3}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{4t\omega^3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t^2\omega^3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t\omega^5E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{8m^2tE_1E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8stE_1E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8t^2E_1E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{4t^3E_1E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t\omega^2E_1E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8t^2\omega^2E_1E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{4t\omega^4E_1E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{16t\omega^2E_2E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t\omega^4E_2E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{16t\omega E_1E_2E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega E_1E_2E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{16t\omega^3E_1E_2E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{8m^2tE_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{8stE_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t^3E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{4t\omega^2E_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{8t^2\omega^2E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{4t\omega^4E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{16t\omega E_1E_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{16t^2\omega E_1E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{16t\omega^3E_1E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& -\frac{16t\omega E_2E_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t^2\omega E_2E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}+\frac{16t\omega^3E_2E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)} \\
& +\frac{64tE_1E_2E_3E_4}{q^2\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{64t\omega^2E_1E_2E_3E_4}{q^4\left(t-\Pi_T\right)\left(t-\Pi_L^*\right)}-\frac{4t^2\omega^4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}
\end{aligned}
$$

$$
\begin{aligned}
M_{tt}^3 \quad = \quad & \frac{4t^2\omega E_1}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{4t^3\omega E_1}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{4t^2\omega^3 E_1}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& +\frac{4t\omega^3 E_2}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t^2\omega^3 E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t\omega^5 E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& +\frac{8m^2tE_1E_2}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{8stE_1E_2}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{4t^3E_1E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& -\frac{4t\omega^2E_1E_2}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{8t^2\omega^2E_1E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{4t\omega^4E_1E_2}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& -\frac{4t^2\omega E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t^3\omega E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t^2\omega^3 E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& +\frac{16t^2\omega^2E_1E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{8m^2tE_2E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{8stE_2E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& -\frac{8t^2E_2E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t^3E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{4t\omega^2E_2E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& -\frac{8t^2\omega^2E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t\omega^4E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{16t\omega E_1E_2E_3}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& +\frac{16t^2\omega E_1E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}+\frac{16t\omega^3E_1E_2E_3}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}-\frac{4t\omega^3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)}
\end{aligned}
$$

$$
\begin{aligned}
M_{tt}^4 \;=\;& \frac{4t^2\omega^3 E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{4t\omega^5 E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{8m^2 t E_1 E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{8stE_1E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{8t^2E_1E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{4t^3E_1E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& + \frac{4t\omega^2E_1E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{8t^2\omega^2E_1E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{4t\omega^4E_1E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{16t\omega^2E_2E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{16t\omega^4E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{16t\omega E_1E_2E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{16t^2\omega E_1E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{16t\omega^3E_1E_2E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{8m^2tE_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{8stE_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{4t^3E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{4t\omega^2E_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& + \frac{8t^2\omega^2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{4t\omega^4E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{16t\omega E_1E_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{16t^2\omega E_1E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{16t\omega^3E_1E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} - \frac{16t\omega E_2E_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& + \frac{16t^2\omega E_2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{16t\omega^3E_2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{64tE_1E_2E_3E_4}{q^2\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} \\
& - \frac{64t\omega^2E_1E_2E_3E_4}{q^4\left(t-\Pi_L\right)\left(t-\Pi_T^*\right)} + \frac{16m^4}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{32m^2s}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} \\
& + \frac{16s^2}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{16m^2t}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} + \frac{16st}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} \\
& + \frac{4t^3\omega^2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{8t^2\omega E_1}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{8t^3\omega E_1}{q^4\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} \\
& - \frac{8t^2\omega E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} + \frac{8t^2\omega^3E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} + \frac{16m^2E_1E_2}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} \\
& - \frac{16sE_1E_2}{\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} + \frac{8t^2E_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{16m^2\omega^2E_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} \\
& + \frac{16s\omega^2E_1E_2}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} - \frac{16t^2\omega^2E_1E_2}{q^4\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)} + \frac{8t^2\omega E_3}{q^2\left(t-\Pi_T\right)\left(t-\Pi_T^*\right)}
\end{aligned}
$$

$$
\begin{aligned}
M_{tt}^5 \;=\; & \frac{8t^3\omega E_3}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16t^2\omega^2 E_1 E_3}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16m^2 E_2 E_3}{(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16s E_2 E_3}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16t E_2 E_3}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{8t^2 E_2 E_3}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16m^2\omega^2 E_2 E_3}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16s\omega^2 E_2 E_3}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16t\omega^2 E_2 E_3}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& + \frac{16t^2\omega^2 E_2 E_3}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{32t\omega E_1 E_2 E_3}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{32t\omega^3 E_1 E_2 E_3}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& + \frac{8t^2\omega E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{8t^2\omega^3 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16m^2 E_1 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16s E_1 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16t E_1 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{8t^2 E_1 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16m^2\omega^2 E_1 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16s\omega^2 E_1 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16t\omega^2 E_1 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& + \frac{16t^2\omega^2 E_1 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16t E_2 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{32t\omega^2 E_2 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16t\omega^4 E_2 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{32t\omega E_1 E_2 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{32t\omega^3 E_1 E_2 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& + \frac{16m^2 E_3 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16s E_3 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{8t^2 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{16m^2\omega^2 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{16s\omega^2 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{16t^2\omega^2 E_3 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{32t\omega E_1 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{32t\omega^3 E_1 E_3 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{32t\omega E_2 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& - \frac{32t\omega^3 E_2 E_3 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)} + \frac{64 E_1 E_2 E_3 E_4}{(t-\Pi_T)\,(t-\Pi_T^*)} - \frac{128\omega^2 E_1 E_2 E_3 E_4}{q^2\,(t-\Pi_T)\,(t-\Pi_T^*)} \\
& + \frac{64\omega^4 E_1 E_2 E_3 E_4}{q^4\,(t-\Pi_T)\,(t-\Pi_T^*)}
\end{aligned}
$$

While going through terms in eqs. 7.24 and 7.25 we encounter those like $(t-\Pi_T)^2$ and $(t-\Pi_L)^2$. We define them in the following way:

$$
\begin{aligned}
(t-\Pi_T)^2 &= (t-\Pi_T)(t-\Pi_T^*) \\
&= t^2 - 2tRe\Pi_T + |\Pi_T|^2 \qquad (7.27)
\end{aligned}
$$

Similarly,

$$\begin{aligned}(t - \Pi_L)^2 &= (t - \Pi_L)(t - \Pi_L^*) \\ &= t^2 - 2tRe\Pi_L + |\Pi_L|^2 \qquad (7.28)\end{aligned}$$

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