Quantum Infrared Instabilities of gauge and gravity coupled Higgs Fields

By

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Srijit Bhattacharjee

DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

Srijit Bhattacharjee

To My Late Grandfather......

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Contents

List of Figures

List of publications associated with the thesis

1. Gauge-free Coleman-Weinberg Potential; *Srijit Bhattacharjee* and Parthasarathi Majumdar, **EPJ C 73: 2348, 2013**.

2. "Gauge-free Electroweak theory: Radiative effects"; *Srijit Bhattacharjee* and Parthasarathi Majumdar, **Phys. Rev. D 83:085019, 2011**.

3. Gauge invariant couplings of fields to torsions: a string inspired model; *Srijit Bhattacharjee* and Ayan Chatterjee , **Phys. Rev. D 83:106007, 2011**.

4. Infrared instability in Graviton-Higgs theory, arXiv:1210.0497 [hep-th], [gr-qc]; *Srijit Bhattacharjee* and Parthasarathi Majumdar.

Chapter 1

Introduction

We have by now, an almost complete understanding of physics upto the TeV scale, adequately described by the Standard Model (SM) of particle interactions. The discovery of what appears to be a fundamental scalar particle at the LHC last year has boosted our confidence that we may be on our way towards a fuller understanding of the fundamental constituents of nature. This journey of great achievements has continued ever since the birth of the quantum theory of interactions between charged particles with light by Feynman, Schwinger and Tomonaga [1–6]. The predictions of quantum electrodynamics match with experimental data with unprecedented precision and has put the theory of quantized fields on a firm footing.

Following along these lines, Standard electroweak theory was developed by Sheldon Glashow [7], Abdus Salam [8] and Steven Weinberg [9] during the 1960s. A better understanding of strong interactions emerged with the discovery of quantum chromodynamics which describes the interactions among quarks living inside neutrons, protons etc. The backbone of all these theories describing strong, weak and electromagnetic interactions is the principle of gauge invariance. In gauge theories the classical Lagrangian is invariant under a set of local gauge transformations. This invariance is not a symmetry in usual sense like Lorentz invariance in special relativistic quantum field theory since it does not possess conserved Nöther charges which are infinitesimal generators of global Lie group of internal symmetries [71]. When these global symmetries are gauged, the Nöther charges become first class constraints in the Hamiltonian description of the theory.

Unfortunately, gauge invariance prevents electroweak vector bosons (W, Z) to acquire masses although enough hints were coming from experimental results that the weak vector bosons should be massive. This problem was resolved by the introduction of *Higgs mechanism* put forward by three groups of people [11–13] almost simultaneously employing the relativistic version of the method introduced by Anderson [14] for explaining BCS superconductivity. In these works it is emphasized that the weak vector bosons become massive by eating up the would be Goldstone [15, 16] modes. Although the Higgs mechanism is usually referred to as an application of *spontaneous breaking of gauge symmetry* in the literature but this to me is a misnomer. Practically, gauge invariance is always maintained in Higgs phenomenon. Interacting massive vector bosons must have longitudinal polarizations and it is the transmutation of a scalar field mode into the longitudinal mode of a massless vector boson that is at the origin of mass of SM vector bosons. However, there is a priori no link between gauge invariance and mass generation of the vector bosons for all coupling strengths as remarked by Schwinger [17]. Nevertheless, at weak coupling, gauge invariance does imply masslessness for self- or fermion-coupled gauge theories, because of lack of the extra degree of freedom to provide masses to free transverse vector (gauge) bosons. Another interesting fact related to this subject is the puzzling aspect of Elitzur's theorem [18], which states that, strictly speaking, gauge symmetries are not spontaneously broken. This means, of course, that, for gauge theories, the name 'spontaneously broken symmetry' is particularly inappropriate, and should be replaced by some alternative such as 'hidden symmetry'. In fact, the generation of gauge-field masses is completely a gauge invariant phenomenon. In this thesis this point has been established with recasting the gauge theories in a manner where all field degrees of freedom are manifestly gauge-inert. The Higgs boson is introduced into the theory to provide a mechanism for generating masses of gauge bosons and fermions. Although, almost all constituents of SM particles have been detected with the development of high energy particle detectors, the only particle eluding detection experimentally until the last year is the Higgs boson. This was due for more than a decade after the last significant detection of SM particles was made in the Fermilab, namely the discovery of top quark. This detection is important for completeness of the SM. The confirmation of the observation of a Higgs like particle at LHC [19, 20] will certainly demystify the the issue of the origin of particle masses.

However, the question of the origin of the Higgs potential still remains open. The procedure of generating mass in SM (in the Higgs mechanism) involves a tachyonic or wrong sign mass term in the Lagrangian for the Higgs field. How this particular form of the Higgs potential arises, has not been understood satisfactorily yet. Introduction of a tachyonic term in a theory certainly triggers instability in the system which gets stabilized via production of the masses to the vector bosons and Higgs bosons. It would have been better to get the same effect without invoking such an unphysical mass term in the potential.

Shortly after the development of the SM, S. Coleman and E. Weinberg [21] showed that radiative corrections to the scalar potential in a theory with no scalar mass-squared parameter could still radiatively induce the Higgs mechanism and generate masses of scalar and vector bosons. Here, when one-particle irreducible vacuum graphs are summed, an instability is developed at the classical minimum of the potential triggering the Higgs effect. In the functional formalism of quantum field theory this is known as *effective potential.* The functional methods to study these was introduced by Schwinger [22] and developed by Jona-Lasinio [23]. After the seminal paper by Coleman and Weinberg, and the development of the the loop expansion [24], effective potential has become almost indispensable in the discussion of vacuum instability.

There have been many applications of radiative corrections to the tree level scalar potential. In electroweak models, radiative corrections to the scalar potential can have significant consequences. In the standard model, they can destabilize the standard model vacuum; the requirement of vacuum stability leads to severe bounds on Higgs and fermion masses. In supersymmetric models, they lead to the generation of the electroweak scale in terms of the unification scale. In the late seventies Linde and Weinberg [25, 26] put a lower bound on Higgs mass considering vacuum stability. For light fermions, ignoring the top-mass term one gets the lower bound of Higgs mass to be ~ 7 GeV. However, there is a possibility of instability when the beta function becomes negative for sufficiently large value of ϕ but one has to remember that perturbation theory is no longer valid for large value of ϕ [27]. Thus, a new constraint $V(\phi_{max}) > V(v/\sqrt{2})$ was imposed [28], where ϕ_{max} was an estimate of the point at which perturbation theory breaks down. This led to bounds on the minimum of Higgs mass as a function of mass of the top quark. Although later it was showed, the minimum Higgs mass should be calculated via Coleman-Weinberg mechanism to have the result consistent with the standard hot Big Bang model and the bound was precisely $\sqrt{2}$ times the Weinberg-Linde bound [29].

Different methods of calculation of effective potentials especially the renormalization group improved version can be found in [30]. In [30], finite temperature corrections to the effective potential, calculation of tunneling rates and the nature of cosmological phase transitions are also discussed. These results are then applied to the standard model to derive stringent bounds on Higgs and fermion passes. Models involving several Higgs fields, scalar potential in supersymmetric models, including dimensional transmutation and no-scale models, can also be found in [30]. Another noteworthy application of CW theory is in the inflationary phase of early Universe. It was shown that a straightforward extension of the minimal $SU(5)$ Higgs system yields a satisfactory inflationary scenario where the inflation starts at the top of the CW potential. A scalar field that transforms as a singlet under the unifying gauge symmetry is all that needs to be added [31]. Recently it has been reported that predictions for a class of realistic inflation models based on a quartic CW potential for a gauge singlet inflaton field agrees very well with the WMAP data [32].

Since effective potential is the generating functional of zero momentum one-particle irreducible Green's functions, in gauge theories it is plagued by gauge ambiguities. This raises question on the physicality of the results obtained directly from the effective potential. The applications of effective potential, described above will not be reliable if the gauge ambiguities not been removed from it. This motivates us to look into methods which will render the conclusions drawn from effective potential unambiguous. In this thesis this issue is addressed and a proposal has been made to remove these gauge ambiguities. The problem of gauge dependence is two fold. First, in the process of quantization, we have to introduce gauge-fixing terms, this makes the resulting effective action gauge dependent. The gauge dependence of the effective potential was first pointed out by Jackiw [24] in an explicit computation of effective potential for scalar electrodynamics. This finding raised concerns on the physical significance of the effective potential. In a later work by Dolan and Jackiw [33], the effective potential of scalar QED was calculated in a set of R_ξ gauges. It was concluded that only the unitary gauge corresponding to a limiting value of the gauge parameter ξ gives sensible results for radiatively induced masses.

This difficulty was partially resolved by the work of Nielsen [35]. The observables of a theory with radiatively induced symmetry breaking are found to be invariant, if a change in the gauge parameter is accompanied by a suitable change in the ground-state expectation value of the scalar field. In his paper, Nielsen derived a simple identity characterizing the mean-field and the gauge-fixing-parameter dependences of the effective potential, namely,

$$(\xi\frac{\partial}{\partial\xi} + C(\phi,\xi)\frac{\partial}{\partial\phi})V(\phi,\xi) = 0, \tag{1.1}$$

where ξ is the parameter appearing in the gauge-fixing term $L_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2$. $C(\phi,\xi)$ is the Greens function for certain composite operators containing a ghost field.

The above identity implies that the local extrema of V for different ξ are located along the same characteristic curve on (ϕ, ξ) plane, which satisfies $d\xi = \frac{d\phi}{C(\phi,\xi)/\xi}$. Hence covariant gauges with different ξ are equally good for computing V. Later, this work was generalized by Kobes et al. [36] to include the gauge dependence of the full effective action at zero and finite temperature. On the other hand, a choice of the multi-parameter gauge [33] $L_{gf} = -\frac{1}{2\xi}(\partial_\mu A^\mu + \sigma\phi_1 + \rho\phi_2)^2$ would break the homogeneity of Eq. (1) [35]. Hence effective potential calculated in this gauge has no physical significance. The implication of Nielsen's identity [35, 36] in the vacuum stability analysis of standard model sometimes gives ambiguous pedictions. In general the analysis performed in the literature is interested in a particular radius in field space, ϕ_{max}. Since effective potential is gauge dependent, it is possible that for one choice of gauge the potential may satisfy the stability requirement below ϕ_{max}, but for another choice of gauge the potential may become unstable [37]. Thus only for a class of gauges, the on-shell value of effective potential is gauge fixing independent when calculated in a self consistent approximation scheme.

Thereafter, this problem of gauge-dependence of effective potential was addressed with the introduction of the background-field method [38,39]. In this method the fields present in a particular theory are split into background and quantum parts. A background field gauge is chosen so that the gauge of the quantum field is fixed, while the gauge invariance of the effective action with respect to the background field is still maintained.

The second problem regarding gauge ambiguity of effective potential is related to the fact, even though the effective action, using the background-field method, is background-gauge invariant, it will still depend on the choice of the gauge-fixing condition on the quantum fields. This dependence will render the conclusions drawn directly from the effective potential in the studies of vacuum instability ambiguous. This problem has been successfully addressed by Vilkovisky [40]. His modified definition of the effective action is manifestly invariant under a reparametrization of the fields. Since the change of gauge choice can be viewed as a reparametrization of the physical field, the Vilkovisky effective action is believed to be independent of the gauge choice. Later, DeWitt [41], following the idea of Vilkovisky, derived a similar effective action, which coincides with Vilkovisky version in the one-loop approximation. For Yang-Mills theories, these two effective actions have been compared up to two-loop level [34], and it has been found that DeWitt's effective action preserves renormalizability, while Vilkovisky's does not.

The effective action defined by DeWitt is now known as the Vilkovisky-DeWitt effective action. This effective action has been applied to scalar QED [42, 43], Yang-Mills theories, and Kaluza-Klein theories [44]. The Vilkovisky-DeWitt effective-action formalism has also been extended to supersymmetric theories [42]. The gauge choice independence of the one-loop Vilkovisky-DeWitt effective potential has been explicitly demonstrated for the cases of scalar QED and Einstein gravity with a positive cosmological constant [42–44]. The case where Einstein gravity is minimally coupled to a scalar was discussed in [45, 46] and higher derivative gravity coupled to scalars in different backgrounds are also discussed in [47–54] (see also [155] for a recent comprehensive review). Towards the end of last decade it was pointed out [56] that the Higgs mass bound as derived from the effective potential is gauge-dependent due to the reasons described above. In an attempt to resolve this issue Boyanovsky, Loinaz and Willey had proposed a resolution [57] to the gauge dependence effective potential of the Higgs mass bound. Their approach is based upon the *physical effective potential* constructed as the expectation value of the Hamiltonian in physical states [58]. The effective potential of the abelian Higgs model is computed explicitly as an illustration.

However, this formalism requires the identification of first-class constraints in the theory and their implementation towards a projection to the physical states. Such a procedure necessarily breaks the manifest Lorentz invariance of the theory. Consequently it is highly non-trivial to apply this formalism to the SM. Later, in [59] this problem has been attacked in a different way. They have introduced the formalism of Vilkovisky and DeWitt [40, 41] for constructing a gauge-independent effective potential, and therefore obtaining a gauge-independent lower bound for the Higgs mass. However, this was presented with a toy model [60] which corresponds to neglecting all charged boson fields in the SM. The generalization to the full SM should be straightforward. The applicability of Vilkovisky-DeWitt formulation to non-abelian gauge theories has been extensively demonstrated in the literature [34]. A gauge-invariant recalculation of the Higgs mass bound for the full SM, to see whether it matches with LHC result is in order. This is especially because any mismatch with the experimental value will raise questions on the stability of Electroweak vacuum – a crucial issue not only in high energy physics but also for the physics of the early universe.

In this thesis a *gauge-free* approach has been adopted to compute the one-loop effective potential for abelian and non-abelian gauge theories, without having to do any gauge fixing whatsoever. Formulation of gauge theories sans the encumbrances entailing gauge fixing has been an ongoing programme since Dirac [61] proposed to define a static physical electron field using nonlocal prefactors to absorb the $U(1)$ phase transformation of the bare field, thereby endowing the electron with its own Coulomb field. The use of holonomies of the gauge potential $h_C[A](x) = \exp i \int_{C[\infty,x]} A \cdot dx$ instead of local gauge potentials themselves has been taken to perfection by Mandelstam [62]. This has paved the way for Wilson [63] to generalize these holonomy operators to Yang Mills theories, resulting in the famous Wilson loop variables which underlie lattice QCD. I may also mention that there have been many efforts in the past towards identifying gauge invariant variables and formulating gauge theories in terms of those. See e.g. the recent paper by Ilderton et. al. [64] which provides a definitive guide to the literature of the mid-1990s on these efforts, including the authoritative contribution of Lavelle and McMullan [65]. Related to these earlier works, recently Niemi et. al. [66] and Faddeev [67] have proposed a gauge invariant description of the Higgs-gauge sector of standard electroweak theory. Although similar in spirit to some of the assays stated above in a broad sense, the approach which is taken up in this thesis is distinct in that it is formulated in terms of a *physical* vector potential (instead of field strengths) as a *fundamental field variable*. In other words, here in this thesis, alternative action/field equations are proposed as a new starting point rather than attempt to express the standard gauge theory action in terms of new variables. It is thus a 'gauge-free' approach, rather than one which is based on gauge invariant functionals of the standard vector potential, including electric/magnetic fields and Wilson loops. This framework is discussed in detail in (3) of this thesis and applied for the case of scalar electrodynamics. The non-abelian extensions to this framework particularly for the case of the $SU(2) \times U(1)$ gauge-invariant electroweak theory is demonstrated in (4). CW mechanism in a gauge-free electroweak theory produces masses for the weak vector bosons and Higgs boson without any symmetry breaking at all! This again is an example of stabilization of quantum infrared instabilities via CW mechanism. There is another kind of instability one encounters in SM called 'naturalness'. Parameters of SM gauge theories receive large quantum corrections at mass scales larger than O(1 TeV) which are unstable under small perturbations of the parameters. This can also be termed as a kind of instability in a theory which can be avoided by unnatural adjustments of parameters. However supersymmetry guarantees cancellations of primitively quadratic divergent one-loop contributions to the masses of various physical particles [68]. This enables the supersymmetric theories to accommodate elementary scalar particles like Higgs and avoids the problem of unnatural fine tuning of parameters. In (4) it is shown that in gauge theories the Higgs mass can be generated radiatively without any tree level Higgs potential thus evading the issue of 'naturalness' in a different way.

In the CW mechanism, as applied to the SM, infrared instabilities in the perturbatively generated bosonic loops get stabilized by giving masses to the bosons. In the case of gravity coupled to Higgs fields we encounter another type of gauge interactions. In this case it is the linearized coordinate transformations of the spin-two graviton fields which play the analogous role of gauge transformations. An obvious question which arises, is what might be the gravitational analogue of a CW effective potential? In this situation, one finds that massless graviton and Higgs fields develop an additional instability on top of the usual instability in the Coleman-Weinberg potential at the origin of field space. This instability is manifested in the effective potential by the appearance of an imaginary term. We know that gravity couples with all kinds of matter and energy with a universal coupling which is always attractive. This unique feature of gravity makes itself distinct from other fundamental interactions. This very nature of gravity in turn is a source of many instabilities. In Newtonian gravity an application of this feature of gravity was put forward by Jeans [69] in his model of structure formation in the early universe. Later, this has been modified for the relativistic case by Lifshitz [70, 71].

A similar instability occurs when a spherical ball of perfect gas is in thermal equilibrium with its own gravitational field [72]. The stars condense via this mechanism out of interstellar gas. On the other hand gravitational collapse of matter produces space-time singularities in general relativity whenever the mass of a star is greater than the Chandrasekhar-Oppenheimer-Volkoff limit [73, 74]. If this happens then there will be formation of event horizon and ultimately, a singularity.

There is another kind of instability leading to a spacelike singularity in the future if the matter density is greater than the critical density 2×10^{-29} g cm^{-3} in Friedman-Robertson-Walker model.

All these examples suggest that gravitational interaction has an anti-screening effect – a phenomenon which was first hinted at by Anderson [14] in his classic paper on plasmons and gauge invariance. In another seminal paper about three decades ago, Gross et. al [75] had shown that for a system where a gas of gravitons are in thermal contact with a reservoir, an instability occurs due to graviton self interaction. Instability also occurs when gravitons are in thermal contact at finite spatial volume and interacting with thermally excited fermions. They established this fact by computing the thermal graviton self energy which lead to a negative induced mass. Employing Euclidean path-integral techniques and using a saddle-point approximation, it has been shown that Jeans instability arises as a tachyonic pole in the graviton propagator when small perturbations about hot flat space are considered. It was Lee Smolin [45] who probably first reported an instability in the effective potential where gravity is minimally coupled to a scalar field even at zero temperature. Later, working with Brans-Dicke theories, scalar fields minimally and non-minimally coupled to gravity and higher derivative gravity theories [46, 47] have been considered. It is perhaps not inappropriate to say that the issue of appearance of an imaginary part in one-loop effective potential has not yet been resolved satisfactorily. In this thesis, a new interpretation of the imaginary term is offered, from an analysis of the infrared limit of the effective one-loop graviton propagator. The finite temperature version of effective potential for Higgs-graviton theory has also been studied to examine the effect of non-zero temperature on the zero temperature instability.

The organisation of this thesis is as follows. In chapter 2, I will introduce the effective potential via functional formalism and review the Coleman-Weinberg mechanism for the case of scalar electrodynamics. I will discuss the issues of gauge dependence of effective potential in the chapter 3. Here I will introduce the "gauge-free" framework for vacuum electrodynamics first, then go on to introducing this same framework for interacting gauge fields. Need for parametrization invariance would require Vilkovisky-DeWitt method which is discussed in this chapter. Finally, a gauge-free parametrization invariant CW potential is obtained by combining "gauge free" and Vilkovisky-DeWitt method. In chapter 4, Electroweak theory is rewritten in terms of dynamical variables which are inert under $SU(2) \times U(1)$ gauge transformation. It will be shown that radiative effects generate a potential whose minimum is away from origin thereby implementing the Higgs mechanism for mass generation in a manifestly gauge-free manner. In chapter 5, I start looking into a theory where Einstein gravity is minimally coupled to a scalar field. The theory is quantized using functional integral technique with the background space time taken to be Minkowskian (or Euclidean). I investigate the one-loop infrared behaviour of the effective potential in Minkowski background. The gravitational analogue of one loop Coleman Weinberg effective potential turns out to be complex, the imaginary part indicating an infrared instability. This instability is traced to a tachyonic pole in the graviton propagator for a constant Higgs background. Physical implications of this behaviour are studied. I also discuss physical differences between gauge theories coupled to Higgs fields and graviton Higgs theory. The constant scalar background here is acting like a heat bath which is backreacting to the system to induce a Jeans-like instability. A finite temperature analysis of one loop effective potential for this theory is also done. In chapter 6, I will discuss quantum instability of a theory where massless scalar fields (dilaton, axion) are coupled to higher derivative gravity theory. The motivation behind studying these nonminimal interactions has different origin and will be discussed in detail in this chapter. I shall also discuss some astrophysical and cosmological implications of such higher derivative interactions. Finally in chapter 7, I conclude, touching upon some future prospects of the analyses performed in this thesis.

Chapter 2

Effective Potential and Coleman-Weinberg Mechanism

In this chapter, I review the functional formalism leading to Coleman-Weinberg potential. The framework described in this chapter will be followed in the subsequent chapters. I start with the path integral representation of generating functionals and introduce Effective action via functional Legendre transformation. Effective potential is introduced next as the sum of all vacuum one-particle irreducible graphs with vanishing external momenta. Then, Coleman-Weinberg mechanism is demonstrated for a theory containing single massless self-interacting scalar field and massless scalar electrodynamics.

In the path integral representation of quantum field theory the vacuum-vacuum transition amplitude for a scalar field, in presence of an external c-number source $J(x)$ is given by,

$$< 0^+|0^- >_J = Z[J] = \int \mathcal{D}\phi\mu[\phi] \exp i \left[\int d^4x S[\phi] + \int d^4x J(x)\phi(x) \right] \tag{2.1}$$

$Z[J]$ is called the generating functional of the Green's functions and is represented by functional integral over configuration space . $\mu[\phi]$ is called the functional measure. In the simplest cases like scalar field theory or in $O(N)$ symmetric theory $\mu[\phi] = 1$ but in case of nonlinear theories like nonlinear sigma model or gravity functional measure may be non trivial. In the non trivial cases we must add a term $-iTrln\mu[\phi]$ in the exponent of functional integral. The symbol $'Tr'$ denotes functional trace to be described later. $[\phi]$ is the classical action including interactions. The classical equation of motion is given by,

$$\frac{\delta S[\phi]}{\delta \phi} = -J \tag{2.2}$$

$Z[J]$ can be expressed as a functional Taylor expansion

$$Z[J] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 d^4x_2 \cdots d^4x_n \, G^n(x_1, x_2, ...x_n) \, J(x_1)J(x_2)\cdots J(x_n) \tag{2.3}$$

The Green's functions or n-point correlators are defined as

$$\left. \frac{\delta^n Z[J]}{\delta iJ(x_1)\delta iJ(x_2)\cdots\delta J(x_n)} \right|_{J=0} = G^n(x_1, x_2, ...x_n) \tag{2.4}$$

The generating functional of all connected Green's functions is related to Z via exponentiation,

$$Z[J] = e^{iW[J]} \tag{2.5}$$

The connected Green's function is obtained from eqn. (2.5) by functionally differentiating both sides with respect to Js.

$$G^n(x_1, x_2, ..., x_n|J) = \sum^{(n)} \prod G^c_k(x_1, ..., x_k|J) + iG^c_n(x_1, ..., x_n|J) \tag{2.6}$$

where

$$G^c_n(x_1, ..., x_n|J) = \frac{\delta^n W[J]}{\delta iJ(x_1)\delta iJ(x_2)\cdots\delta J(x_n)} \tag{2.7}$$

The expression (2.6) means that Green's functions obtained from Z (without setting J to be zero) are equal to sum of all possible products of connected Green's function with all types of splitting of n arguments $x_1, ..., x_n$ plus the connected n-point function. In the language of Feynman diagrams a connected Green's function is represented by a connected diagram. A connected diagram is one which can't be subdivided into parts which are not joined by the lines.

2.1 Effective Action

The mean field or 'classical' field is defined by,

$$\frac{\delta W[J]}{\delta J(x)} = \frac{1}{Z[J]}\frac{\delta Z[J]}{\delta i J(x)} = \left[\frac{< 0^+|\phi(x)|0^- >}{< 0^+|0^- >}\right]_J =< \phi(x) >_J = \Phi(x) \tag{2.8}$$

The mean field Φ is a function of space-time and a functional of source via above relation. In fact, using the definition

$$\frac{\delta W[J]}{\delta J(x)} = \Phi(x|J] \tag{2.9}$$

one can solve for the source J to express it in terms of Φ. Thus we can treat mean field as an independent variable and perform a Legendre transform using it.

The effective action is obtained from W by the following Legendre transformation

$$\Gamma[\Phi(x)] = W[J] - \int d^4x J(x)\Phi(x) \tag{2.10}$$

Here the source $J(x)$ has to be expressed as a functional of Φ via (2.9). Effective action satisfies a similar equation of motion as the classical action satisfies except the source in this case is a functional of Φ.

$$\frac{\delta\Gamma}{\delta\Phi} = -J(x|\Phi) \tag{2.11}$$

Comparing this with (2.2) we can see that effective action is the quantum counterpart of the classical action. The mean field being a solution of eqn. (2.11), incorporates quantum corrections within itself. The connected two-point function and the Hessian of the effective action satisfy the following convolution relation

$$\int d^4y \frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} \frac{\delta^2 \Gamma[\Phi]}{\delta\Phi(y)\delta\Phi(z)} = -\int d^4y \frac{\delta\Phi(x)}{\delta J(y)} \frac{\delta J(y)}{\delta\Phi(z)} = -\delta(x-z) \tag{2.12}$$

The two point vertex function or 2-point "proper vertex" is defined as

$$i^2 G_2^c(x,z|J) = -\int d^4y_1 d^4y_2 \frac{\delta^2 W[J]}{\delta J(x)\delta J(y_1)} \frac{\delta^2 \Gamma[\Phi]}{\delta\Phi(y_1)\delta\Phi(y_2)} \frac{\delta^2 W[J]}{\delta J(z)\delta J(y_2)} \tag{2.13}$$

Thus n^{th} derivative of Γ with respect to Φs give rise n-point vertex functions. This allows us to expand the effective action into a Taylor series

$$\Gamma[\Phi] = \sum_n \frac{1}{n!} \int d^4x_1 ... d^4x_n \Gamma^{(n)}(x_1...x_n)\Phi(x_1)...\Phi(x_n) \tag{2.14}$$

where $\Gamma^{(n)}(x_1...x_n)$ is the sum of all 1PI diagrams with n external lines. An alternative expansion of $\Gamma[\Phi]$ can be made in powers of momenta, about the point where all external momenta vanish. In configuration space this takes the following form:

$$\Gamma[\Phi] = \int d^4x[-V_{eff}(\Phi) + \frac{1}{2}(\partial_\mu\Phi)^2 Z(\Phi) + ...] \tag{2.15}$$

The first term in the expansion $V_{eff}(\Phi)$ is called the *effective potential* which is sum of all 1PI diagrams with zero external momenta. Using the above expansion of effective action we can express effective potential in terms of 1PI Green's functions. This is achieved by expressing n-point proper vertices in the momentum space,

$$\begin{aligned} \Gamma^{(n)}(x_1,...,x_n) = & \int \frac{d^4p_1}{(2\pi)^4} ... \frac{d^4p_n}{(2\pi)^4} (2\pi)^4 \delta^4(p_1 + ... + p_n) \\ & \times e^{i(p_1.x_1 + ..p_n.x_n)} \Gamma^{(n)}(p_1,...,p_n) \end{aligned} \tag{2.16}$$

Putting this into (2.14) and expanding $\Gamma^{(n)}$ in powers of momenta we get the following relation between effective action and n-point vertex functions.

$$\begin{aligned}\Gamma[\Phi] = & \sum_n \frac{1}{n!} \int d^4x_1 ... d^4x_n \int \frac{d^4p_1}{(2\pi)^4} ... \frac{d^4p_n}{(2\pi)^4} \\ & \times \int d^4x e^{i(p_1+\cdots+p_n)x} e^{i(p_1.x_1+..p_n.x_n)} \\ & \times [\Gamma^{(n)}(0,...0)\Phi(x_1)\Phi(x_2)....\Phi(x_n) \ + \ ...] \\ = & \int d^4x \sum_n \frac{1}{n!} [\Gamma^{(n)}(0,...0)[\Phi(x)]^n \ + \ ...] \end{aligned} \tag{2.17}$$

The second equality follows from the first after performing consecutive integrations over all momentum and configuration space coordinates in presence of delta functions. Comparing the above expression with (2.15) we see that effective potential is indeed effective action with zero external momenta. The n-th derivative of effective potential is the sum of all 1PI diagrams with n external lines carrying zero momenta. Diagrams without loops correspond to the interactions in the Lagrangian, while those with loops correspond to the quantum corrections.

$$V_{eff}(\Phi) = -\sum_n \frac{1}{n!} \Gamma^{(n)}(0,...0)[\Phi(x)]^n \tag{2.18}$$

Renormalization conditions can be easily expressed in terms of effective potential. The mass renormalization condition is defined by setting the squared mass of the scalar field to be equal to the value of inverse propagator at zero external momenta,

$$\left. \frac{d^2V_{eff}(\Phi)}{d\Phi^2} \right|_0 = m_R^2 \tag{2.19}$$

Similarly the renormalized coupling constant is defined as the value of the four-point function at zero external momenta,

$$\left. \frac{d^4V_{eff}(\Phi)}{d\Phi^4} \right|_0 = \lambda_R^2 \tag{2.20}$$

The wave function renormalization condition is similarly given by,

$$Z[0] = 1 \tag{2.21}$$

However, we will see that the renormalization points will not remain the same for theories possessing massless particles. Duo to infrared divergences we have to modify renormalization conditions in those cases due to occurrence of logarithmic singularities in the field space.

2.1.1 Effective Potential and Vacuum Instability

In this subsection I will briefly review how knowledge of the effective potential enables us to study the structure of vacuum of a theory. *Spontaneous symmetry breaking* occurs when some field in the Lagrangian develops a non-zero vacuum expectation value. The condition for this to happen can be easily derived from eqn. (2.11) when the source is switched off

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi} = 0\,, \quad \text{for } \Phi \neq 0 \tag{2.22}$$

If one considers only translational invariant vacuum state then the above condition reduces to

$$\frac{\partial V_{eff}(\Phi)}{\partial\Phi} = 0\,, \quad \text{for } \Phi \neq 0 \tag{2.23}$$

If the point at which the condition (2.23) holds is a minimum of the effective potential then the asymmetric vacuum is considered to be stable under small perturbations. Otherwise there will be instability in the vacuum structure of the theory.

2.2 Loop Expansion

In the last section we have noticed that the study of spontaneous symmetry breaking is reduced to the calculation of the effective potential. Unfortunately, this is not so simply done; even if one accepts that perturbative expansion will remain valid to all orders, such a calculation requires summing an infinite number of Feynman diagrams, which is not so easy in most of the cases. It is therefore necessary to find an approximation scheme for the effective potential which will require the summation of only some subset of the relevant Feynman diagrams.

Expansion in powers of the coupling constants will not be that advantageous for two reasons. First, in theories with spontaneous symmetry breaking one commonly defines shifted fields, with a corresponding redefinition of the coupling constants. This can lead to confusion in comparing the order of diagrams calculated using different shifts. But this can be cured if one is careful; it certainly does not invalidate the method. The second point is more basic: We will often be considering theories which appear to contain massless particles, for example, massless scalar electrodynamics. An expansion in powers of coupling constants assumes that the successive powers of coupling constant have smaller contribution to the scattering amplitude, but for massless theories diagrams having more loops and external lines will be much more infrared divergent, and not at all negligible.

A better approximation is the expansion by the number of loops in a diagram. Consider the generating functional of Green's functions given by eqn. (2.1). We can write it as following:

$$\exp\left(\frac{iW[J]}{\hbar}\right) = \int \mathcal{D}\phi \exp\left[\frac{i}{\hbar}\left(S[\phi] + \int d^4x \phi(x) J(x)\right)\right] \tag{2.24}$$

Here I have introduced the Planck constant $\hbar$ in the exponential. This is justified as the exponent should be dimensionless and $\hbar$ has the correct dimension to match the action and other term. Previously I have just chosen $\hbar$ to be equal to one. The purpose of introducing $\hbar$ in the functional integral is to show that the loop expansion is equivalent to a power-series expansion in $\hbar$. The power, P, of $\hbar$ associated with a particular Feynman diagram is

$$P = I - V \tag{2.25}$$

where I is the number of internal lines and V is the number of vertices, since the vertices are obtained from the interaction Lagrangian while the propagators are obtained from the inverse of the free Lagrangian. The number of loops, L, is equal to the number of independent internal momenta. This is equal to the number of momenta (I), minus the number of energy-momentum delta functions (V) without counting the delta function corresponding to overall energy-momentum conservation. In other words

$$L = I - V + 1 = P + 1 \tag{2.26}$$

We see that the number of loops is determined by the power of a quantity which multiplies the whole Lagrangian, and does not depend on the details of how the Lagrangian was written; thus the loop expansion is unaffected by a redefinition of the fields. To sum the diagrams with increasing number of loops one can employ diagrammatic method as depicted in [21]. This method is useful for simpler field theories but not so for the case of non-linear sigma model or gravity. However, after Jackiw [24] introduced a functional technique to calculate the effective potential for arbitrary loops, things became manageable.

Let us see how loop expansion is achieved via functional methods. We start with the relation (2.24):

$$\exp\left[\frac{i}{\hbar}\left(\Gamma[\Phi] + \int d^4x J(x)\Phi(x)\right)\right] = \int \mathcal{D}\phi \exp\left[\frac{i}{\hbar}\left(S[\phi] - \int d^4x\phi(x)\frac{\delta\Gamma[\Phi]}{\delta\Phi}\right)\right] \tag{2.27}$$

Where I have used eqn. (2.10) in the exponent of left hand side. I have also replaced $J(x)$ by the equation of motion $J(x) = -\delta\Gamma[\Phi]/\delta\Phi$. The effective action $\Gamma[\Phi]$ satisfies the above integro-differential equation. Now, let us make a change of variable $\phi \to \phi + \Phi$, where Φ is the mean field. Then (2.27) reduces to

$$\exp\left(\frac{i}{\hbar}\Gamma[\Phi]\right) = \int \mathcal{D}\phi \exp\left[\frac{i}{\hbar}\left(S[\phi + \Phi] - \int d^4x\phi(x)\frac{\delta\Gamma[\Phi]}{\delta\Phi}\right)\right] \tag{2.28}$$

Now the action functional $S[\phi + \Phi]$ can be expanded in a functional Taylor series in ϕ

$$S[\phi + \Phi] = S[\Phi] + \sum_{n=1}^{\infty}\frac{1}{n!}\int d^4x_1...d^4x_n\, S_n(x_1, ..., x_n|\Phi)\phi(x_1)....\phi(x_n) \tag{2.29}$$

where

$$S_n(x_1, ..., x_n|\Phi) = \frac{\delta^n S[\Phi]}{\delta\Phi(x)...\Phi(x)} \tag{2.30}$$

are classical vertex functions depending on the mean fields. For notational simplicity we introduce the following:

$$\int d^4x_1...d^4x_n\, S_n(x_1, ..., x_n|\Phi)\phi(x_1)....\phi(x_n) = S_n[\Phi]\phi^n$$

and

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi} \equiv \Gamma_1[\Phi]\,, \quad \int d^4x\phi\frac{\delta\Gamma[\Phi]}{\delta\Phi} \equiv \phi\Gamma_1[\Phi] \tag{2.31}$$

With these definitions we can now rewrite relation (2.32) as

$$\exp\left(\frac{i}{\hbar}\Gamma[\Phi]\right) = \int \mathcal{D}\phi \exp\left[\frac{i}{\hbar}\left(S[\Phi] + \frac{1}{2}S_2\phi^2 + \sum_{n=3}^{\infty}\frac{1}{n!}S_n\phi^n - \phi(\Gamma_1[\Phi] - S_1[\Phi])\right)\right] \tag{2.32}$$

Now we make the following change of variable $\phi = \hbar^{1/2}\phi$ and get the following expression

$$\exp\left(\frac{i}{\hbar}(\Gamma[\Phi] - S[\Phi])\right) = \int \mathcal{D}\phi \exp\left[i\left(\frac{1}{2}S_2\phi^2 + \sum_{n=3}^{\infty}\frac{\hbar^{n/2-1}}{n!}S_n\phi^n - \hbar^{-1/2}\phi(\Gamma_1[\Phi] - S_1[\Phi])\right)\right] \tag{2.33}$$

If we look into the expression (2.33) carefully we can see that $\Gamma[\Phi]$ appears only in the combination $\Gamma[\Phi] - S[\Phi] \equiv \mathit{\Gamma}[\Phi]$. We can solve for $\mathit{\Gamma}$ iteratively using (2.33). Thus $\mathit{\Gamma}$ can be expressed as a power series in $\hbar$,

$$\mathit{\Gamma} = \sum_{n=1}^{\infty} \hbar^n \mathit{\Gamma}^{(n)}[\Phi] \tag{2.34}$$

This series is analogous to semi-classical expansion like WKB and it represents all the quantum corrections to the classical action $S[\Phi]$. We can rewrite (2.33) using the relation (2.34)

$$\exp\left(i\sum_{n=1}^{\infty}\hbar^{n-1}\mathit{\Gamma}^{(n)}[\Phi]\right) = \int \mathcal{D}\phi \exp\left[i\left(\frac{1}{2}S_2\phi^2 + \sum_{n=1}^{\infty}\frac{\hbar^{n/2-1}}{n!}S_n\phi^n - \hbar^{-1/2+n}\phi\mathit{\Gamma}_1^{(n)}[\Phi]\right)\right] \tag{2.35}$$

The above expression can be regarded as a perturbative expansion in the parameter $\hbar$. Each term in the series represents some Feynman diagram where the propagator and the vertices depend on the mean field $\Phi(x)$. One may think that the expansion in (2.35) contains non-integer powers of $\hbar$. However, the diagrams corresponding to the terms involved in the series $\hbar^{-1/2+n}\phi\mathit{\Gamma}_1^{(n)}[\Phi]$ are one-particle reducible ones. Those diagrams cancel by the other one-particle reducible Feynman graphs coming from the other terms of the expansion.

From (2.26), it is obvious that one-loop corresponds to the power of $\hbar$ equal to 0. Thus in one-loop approximation the relation (2.35) gives,

$$\exp i(\mathit{\Gamma}^{(1)}) = \int \mathcal{D}\phi \exp\left[\frac{i}{2}S_2\phi^2\right] \tag{2.36}$$

Then

$$\exp i(\mathit{\Gamma}^{(1)}) = det^{-1/2} S_2[\Phi] \tag{2.37}$$

Thus one-loop effective action is given by

$$\Gamma^{(1)}[\Phi] = S[\Phi] + \frac{i}{2}\hbar Tr ln S_2[\Phi] \qquad (2.38)$$

The symbol Tr denotes functional trace. If A is any operator in the function space then,

$$Tr A = \int d^4x A(x, x) \qquad (2.39)$$

The one-loop effective potential is obtained from (2.38) by setting the mean field $\Phi(x)$ to a space-time constant value Φ_c. From (2.15), it is clear that a space-time volume element will always has to be factored out from effective action for constant background field. The expression for effective potential in one-loop approximation reduces to,

$$V^{(1)}_{eff}[\Phi_c] = V_0[\Phi_c] - \frac{i}{2}\hbar Tr ln S_2[\Phi_c] \qquad (2.40)$$

Where V_0 is the tree level potential. When the source is switched off the equation of motion (2.11) for a constant mean field now leads to,

$$\frac{\partial V^{(1)}_{eff}[\Phi_c]}{\partial \Phi_c} = 0 \qquad (2.41)$$

Study of minima of effective potential directly gives us an idea of the vacuum structure of quantum field theory. It is thus very useful to study the minima of effective action to examine the possibility of symmetry breaking. We also can investigate whether radiative corrections change the qualitative nature of the extrema of classical potential.

2.3 Coleman-Weinberg Mechanism

2.3.1 Massless ϕ^4 Theory

In the case of massless theories the one-loop effective potential gives many interesting phenomena. In the case of single scalar field a massless particle with quartic self-coupling develops mass via loop corrections. Using diagrammatic technique this was first demonstrated by Coleman and Weinberg in [21]. In Coleman-Weinberg mechanism the infrared singularities in the one-loop diagrams get transformed into logarithmic divergence in the field space after the diagram sum is performed. I here briefly review this simple model to illustrate some salient features of Coleman-Weinberg mechanism using loop-expansion method. I will set $\hbar = 1$ in the functional integrals for the rest of this thesis.

The Lagrangian for the theory is given by,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4 + \frac{A}{2}(\partial_\mu \phi)^2 - \frac{B}{2}\phi^2 - \frac{C}{4!}\phi^4 \tag{2.42}$$

where the last three terms are usual wave-function, mass and coupling-constant renormalization counter-terms to be determined, order by order in the loop parameter, using renormalization conditions.

The classical theory is invariant under a discrete symmetry, namely $\phi \to -\phi$. Let us see what the radiative corrections do with this symmetry. In one-loop approximation we have to compute sum of vacuum graphs (Fig.[2.1]) including the counter terms with increasing number of internal lines. Thus the sum contains increasingly infrared divergent terms.

If we turn this some into integral or apply eqn. (2.40) we get the following expression,

$$V_{eff}^{(1)} = \frac{\lambda}{4!}\Phi_c^4 - \frac{B^{(1)}}{2}\phi^2 - \frac{C^{(1)}}{4!}\Phi_c^4 - \frac{i}{2}\int \frac{d^4k}{(2\pi^4)}\left[-k^2 + \frac{\lambda\Phi_c^2}{2}\right] \tag{2.43}$$

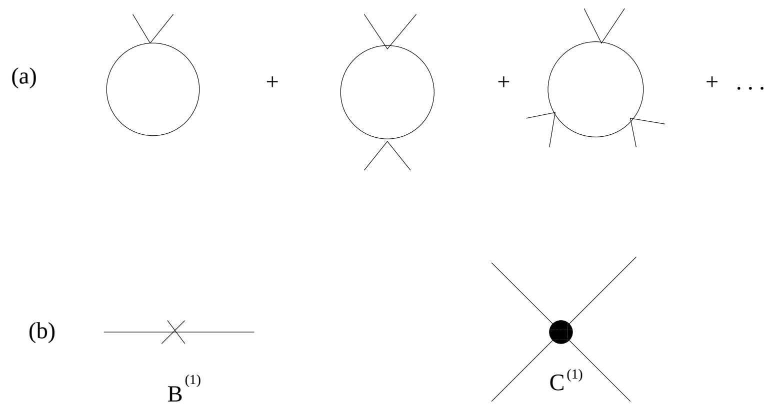

Figure 2.1: One-loop diagrams for massless ϕ^4 theory

Turning the momentum integral into Euclidean space we can rewrite it as,

$$V_{eff}^{(1)} = \frac{\lambda}{4!}\Phi_c^4 - \frac{B^{(1)}}{2}\Phi_c^2 - \frac{C^{(1)}}{4!}\Phi_c^4 + \frac{1}{2}\int \frac{d^4k_E}{(2\pi^4)} \ln \left[k_E^2 + \frac{\lambda\Phi_c^2}{2}\right] \tag{2.44}$$

It is clear from the above expression that the infrared divergences in the loop sum now has turned into a logarithmic singularity at the origin of classical field space. It is not too surprising that this should occur, since we expect infrared divergences only if the vacuum is at $\Phi_c = 0$. If spontaneous symmetry breaking occurs, then ϕ does not remain massless, and there is no reason to expect infrared problems.

Performing the integral by cutting off the momentum integral at $k^2 = \Lambda^2$ one obtains

$$\begin{aligned} V_{eff}^{(1)} &= \tfrac{\lambda}{4!}\Phi_c^4 + \tfrac{1}{64\pi^2}\left\{\lambda\Phi_c^2\Lambda^2 + \tfrac{\lambda\Phi_c^4}{4}\left[\log\tfrac{\lambda\Phi_c^2}{2\Lambda^2} - \tfrac{1}{2}\right]\right\} \\ &\quad + \tfrac{B^{(1)}}{2}\Phi_c^2 + \tfrac{C^{(1)}}{4!}\Phi_c^4 \end{aligned} \tag{2.45}$$

One has to determine the value of the renormalization counter-terms. The mass renormalization condition can be obtained using (2.19). This determines B,

$$B^{(1)} = -\frac{\lambda\Lambda^2}{32\pi^2} \tag{2.46}$$

Now for coupling constant renormalization condition, we now have a problem. Since the fourth derivative of V at the origin doesn't exist. Thus we have to define the coupling constant away from this singularity. Thus in this case we must change the eqn (2.47) to

$$\left.\frac{d^4 V_{eff}^{(1)}(\Phi_c)}{d\Phi_c^4}\right|_M = \lambda_R^2 \tag{2.47}$$

Where M is any nonzero arbitrary point of field space. This enables us to compute C.

$$C^{(1)} = -\frac{3\lambda^2}{32\pi^2}\left(\log\frac{\lambda M^2}{2\Lambda^2} + \frac{11}{3}\right) \tag{2.48}$$

also we set $Z[M] = 1$ and putting all the coefficients of counter terms together we obtain,

$$V_{eff}^{(1)} = \frac{\lambda}{4!}\Phi_c^4 + \frac{\lambda^2\Phi_c^4}{256\pi^2}\left[\log\frac{\Phi_c^2}{M^2} - \frac{25}{6}\right] \tag{2.49}$$

If we calculate the minimum of this potential it shows that apparently a spontaneous breaking of symmetry has occurred. Although this conclusion is illusory because the apparent new minimum occurs at a value of the field space where perturbation theory doesn't hold. The condition from which the new apparent minimum is determined is given by,

$$\lambda \log\frac{\Phi_{cm}^2}{M^2} = \frac{11\lambda}{3} - \frac{32\pi^2}{3} \tag{2.50}$$

This clearly shows that the value of the point at which the minimum occurs will be given by an inverse power series in coupling constant. For arbitrary small coupling constant this will render the result outside the range of validity of one-loop approximation. However, there are theories where this unaccountability doesn't hold. We will see in the next example that we do get physically interesting situation where spontaneous symmetry is broken and we can get an unambiguous minimum of the effective potential.

2.3.2 Massless Scalar Electrodynamics

In massless scalar electrodynamics a neutral complex scalar boson (or two real scalars) is minimally coupled to a vector boson. The Lagrangian is given by,

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\left(\partial_\mu\phi_1 - eA_\mu\phi_2\right)^2 + \tfrac{1}{2}\left(\partial_\mu\phi_2 + eA_\mu\phi_1\right)^2 - \tfrac{\lambda}{4!}(\phi_1^2+\phi_2^2)^2 \\ &\qquad + \text{counter}-\text{terms}\end{aligned} \tag{2.51}$$

In this case radiative corrections induce spontaneous symmetry breaking generating a double well type potential in one-loop order. The infrared divergences in the one-loop diagrams get stabilized generating masses to the vector boson and the neutral boson. This is an analogue of abelian Higgs mechanism triggered by purely quantum effects. The renormalized one-loop effective potential in Landau gauge is given by,

$$V^{(1)}_{eff} = \frac{\lambda}{4!}\Phi_c^4 + \frac{1}{64\pi^2}\left(\frac{5\lambda^2}{18} + 3e^4\right)\Phi_c^4\left[\log\frac{\Phi_c^2}{M^2} - \frac{25}{6}\right] \tag{2.52}$$

This effective potential has the shape shown in (Fig.[2.2]); it appears to have a minimum at a non-zero value of Φ_c. We again must determine whether the minimum occurs within the range of validity of the one-loop approximation. We can simplify our equations if we recall that M is an arbitrary parameter which sets the renormalization scale of the theory; we are certainly allowed to choose M to be the location of the minimum of the effective potential, $\langle\phi\rangle$. It can be argued using renormalization group analysis that for very small e and λ we can take λ of the order of e^4. If we do this, we should ignore the λ^2 terms, since they are of the same order of magnitude as the e^8 terms we expect from two-loop diagrams. Having done these changes We find

$$V^{(1)}_{eff} = \frac{\lambda}{4!}\Phi_c^4 + \frac{3e^4}{64\pi^2}\Phi_c^4\left[\log\frac{\Phi_c^2}{\langle\phi\rangle^2} - \frac{1}{2}\right] \tag{2.53}$$

Thus, in the one-loop approximation we find that if

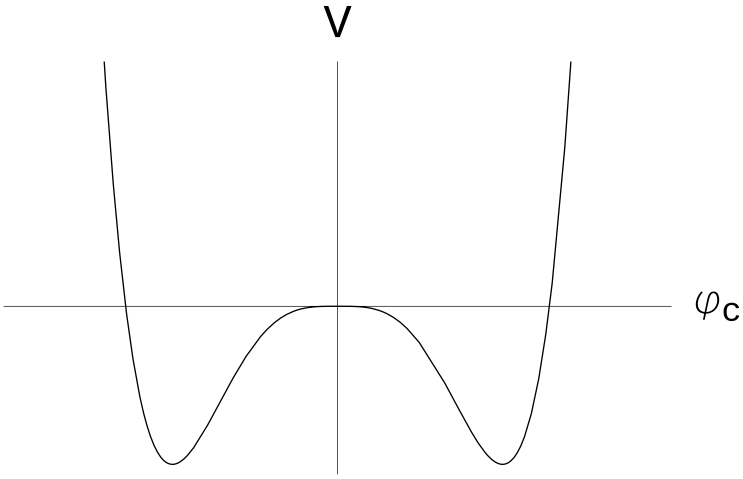

Figure 2.2: Coleman-Weinberg potential for massless scalar electrodynamics

$$\lambda = \frac{33e^4}{8\pi^2}$$

there will be a minimum in the effective potential which is within the region where the approximation is valid. Substituting the value of λ into eqn. (2.53), we obtain

$$V_{eff}^{(1)} = \frac{3e^4}{64\pi^2}\Phi_c^4 \left[\log\frac{\Phi_c^2}{\langle\phi\rangle^2} - \frac{1}{2}\right] \tag{2.54}$$

The final expression of effective potential (2.54) contains only one coupling constant e and all reference to the parameter λ has disappeared from the effective potential. This phenomena is known as *dimensional transmutation*. The original theory had two dimensionless couplings λ and e, after choosing renormalization scale M equal to the location of the minimum $\langle\phi\rangle$ of the effective potential we have a new dimensional parameter $\langle\phi\rangle$ in place of λ. The mass spectrum of the theory now can be determined by computing curvature of the effective potential in (2.54). The mass of the scalar boson is given by,

$$m^2(S) = \left.\frac{d^2V}{d\Phi_c^2}\right|_{\Phi_c=\langle\phi\rangle} = \frac{3e^4}{8\pi^2}\langle\phi\rangle^2 \tag{2.55}$$

The mass of the abelian vector field is given by,

$$m^2(V) = e^2\langle\phi\rangle^2 \tag{2.56}$$

From above two equations the scalar to vector mass ratio is obtained

$$\frac{m^2(S)}{m^2(V)} = \frac{3e^2}{8\pi^2} \tag{2.57}$$

This method can easily be generalized for theories containing fermions or non-abelian gauge bosons. In Landau gauge, we have to consider only polygon graphs with either gauge bosons or fermions are running around the loops. We illustrate the case of non-abelian gauge bosons here. We first define a mass matrix (the Hessian of the Lagrangian expanded around a constant background) of gauge bosons:

$$M_{ab}(\vec{\Phi}_c) = \frac{\partial^2 V_0}{\partial A^a_\mu \partial A^{\mu b}} \tag{2.58}$$

where $\vec{\Phi}_c$ denotes a complex scalar multiplet which couples to the vector bosons. V_0 is the tree potential and M is a quadratic function of $\vec{\Phi}_c$ and is a real symmetric matrix. Now the loop sum can be performed in the usual way to get the following result:

$$V_g^{(1)} = \frac{3}{64\pi^2} Tr[M^4(\vec{\Phi}_c)\ln M^2(\vec{\Phi}_c)] \tag{2.59}$$

Factor 3 comes from the trace of the numerator of the Landau-gauge propagator. A simple physical interpretation of the effective potential can be found in [76]. It can be shown that $V_{eff}(\Phi_c)$ is the expectation value of the energy density in the state $|\psi\rangle$ which minimizes $\langle\psi|H|\psi\rangle$ subject to the condition that $\langle\psi|\Phi|\psi\rangle = \Phi_c$. Although, there is a loophole in the above argument. It is noted in [77, 78] that the effective action, defined as the Legendre transform of the generating functional of connected diagrams, is the generating functional of 1PI diagrams only if the effective potential is convex. However, this problem has to a large extent been resolved by Kobes et al. calculating the effective action via a self-consistent approximation scheme [**?**].

2.4 Summary

In this chapter I have introduced the effective action via functional techniques and defined Coleman-Weinberg effective potential. The formula and computation scheme used in this chapter are quite general and will be followed in the subsequent chapters. Sample calculations of Coleman-Weinberg potential are also demonstrated to illustrate essential features of CW mechanism. In the next chapter I will address the issue of gauge-dependency of CW potential and will elucidate how it can be tackled using the *gauge-free* proposal.

Chapter 3

Abelian gauge fields coupled to Higgs field: Gauge-free framework

In this chapter I will demonstrate instances where Coleman-Weinberg potential depends on gauge fixing parameter and gauge fixing conditions. Then I'll briefly mention attempts made by other authors to resolve these issues. Next, I'll introduce *gauge-free* prescription for any abelian gauge theory which obviates any need of gauge fixing to quantize it. First, the gauge-free framework will be illustrated for vacuum electrodynamics and then the prescription will be extended for Lagrangians containing matter fields also and it will be established that abelian Higgs model is a perfectly gauge invariant (free of gauge artifacts) mechanism. Further, the problem of gauge dependence of Coleman-Weinberg for scalar electrodynamics is shown to be resolved using gauge-free prescription not requiring any gauge-fixing of quantum fluctuations of the photon degrees of freedom. This leads to a unique dynamical ratio at one loop of the Higgs mass to the photon mass. I next compare gauge-free approach and results with those obtained in geometric framework of DeWitt and Vilkovisky, which maintains invariance under field redefinitions as well as invariance under background gauge transformations, but *requires*, in contrast to gauge-free approach, gauge fixing of *fluctuating* photon fields. However, to yield a parametrization invariant

CW potential gauge-free approach has to be combined with Vilkovisky-DeWitt approach. This is also done in this chapter for scalar electrodynamics after introducing very briefly the VD framework for computing effective potential. The last section of this chapter is devoted to discuss possibility of gauge-free formulation of symmetric and antisymmetric rank two tensor fields. This chapter is mostly based on ref. [79, 80].

3.1 Gauge dependence of effective potential

In this section I will briefly discuss instances where CW effective potential suffers from various gauge ambiguities. The dependence of effective potential on choice of gauge was first shown by Jackiw [24] in the context of scalar-QED. The one loop effective potential for scalar QED obtained by Jackiw in a general gauge $-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$ is gauge dependent:

$$V_{eff}(\phi_c) = \frac{\phi_c^4}{4!}\left[\lambda + \frac{1}{8\pi^2}\left(\frac{5}{6}\lambda^2 + 9e^4 - \alpha e^2\lambda\right)\ln\phi_c^2\right] \tag{3.1}$$

It is thus possible to gauge away the one loop contribution to the effective potential by choosing α:

$$\alpha = \frac{5\lambda}{6e^2} + \frac{9e^2}{\lambda}$$

This is a serious problem because this raises the question of physicality of the effective potential itself. Soon after this result, Dolan and Jackiw [33] calculated the same effective potential in the so-called unitarity gauge and asserted that the theory in this gauge has no unphysical degrees of freedom, and hence the effective potential is *physical*. It is given by

$$\begin{aligned} V_U &= \frac{1}{2}dm^2\left(1 - \frac{\lambda}{64\pi^2}\right)\rho_c^2 \;+\; \frac{\lambda}{4!}\rho_c^2 \\ &+ \frac{\lambda}{64\pi^2}\left[3e^4\rho_c^4\ln\frac{\rho_c^2}{m^2} \;+\; (m^2 + \lambda/2\rho_c^2)^2\ln\left(1 + \frac{\lambda\rho_c^2}{2m^2}\right)\right], \end{aligned} \tag{3.2}$$

and is clearly different from the (3.1) in the limit when the scalar field is massless.

The problem of a non-unique one-loop effective potential had been discussed in several papers [43], [81]. It has been shown that the one loop effective potential depends not only upon the choice of gauge but also on the reparametrization of the fields. The reparametrization invariance means that the effective action coincides with the original one under a field redefinition $\phi \to \phi'$. It was shown that if two different field parametrizations are made then the effective potential does not agree with each other. The Coleman-Weinberg potential for scalar QED gives different results for different field parametrizations in the same family of gauges [43]. The CW potential calculated in Cartesian basis is given by

$$V(\phi_1, \phi_2) = \frac{\lambda}{4!}\rho_0^4 \; + \; \frac{1}{64\pi^2}\left(3e^2 + \frac{5}{18}\lambda^2 - \frac{2}{3}\lambda e^2\right)\rho_0^4\left[\log\frac{\rho_0^2}{M^2} - \frac{25}{6}\right] .$$

where $\rho_0^2 = \phi_1^2 + \phi_2^2$. On the other hand, in polar parametrization it takes the following form

$$V_{eff}(\rho_c) \;=\; \frac{\lambda}{4!}\rho_c^4 + \frac{1}{64\pi^2}\left(3e^2 + \frac{\lambda^2}{4}\right)\rho_c^4\left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right] .$$

Vilkovisky-DeWitt [40, 82] had introduced an effective action formalism which addresses both the problems (dependence of effective action on gauge fixing condition and on reparametrization of the fields) and provides a reasonable solution! A lot of work have been done on the issue of gauge and parameterization dependence of the effective potential and notable amongst those is the result obtained by Kunstatter [43] in VD approach.

$$V_{eff}(\rho_c) \;=\; \frac{\lambda}{4!}\rho_c^4 + \frac{1}{64\pi^2}\left(3e^4 + \frac{5}{18}\lambda^2 + \frac{2}{3}\lambda e^2\right)\rho_c^4\left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right] . \qquad (3.3)$$

However, Vilkovisky-DeWitt method does indeed need gauge fixing of the fluctuating gauge degrees of freedom which are being integrated over in the partition function. This of course is easily obviated by the use of the gauge free approach adopted here. Thus, the calculation of the effective potential becomes much easier in the gauge-free VD approach since the unphysical electrodynamic degrees of freedom are absent from the outset, and all fields are manifestly inert under $U(1)$ gauge transformations. Later I shall explicitly calculate the one-loop effective potential for scalar QED in the gauge-free VD approach and show that the potential does get modified from the earlier result and it also differs from the one obtained by Kunstatter [43].

3.2 Gauge-free Prescription

Aim of this chapter is to recalculate the effective potential for scalar QED at one loop, using an approach which will be called *gauge-free*. In this framework, quantum fluctuating dynamical variables are *manifestly inert* under (abelian) gauge transformations [79, 80]. In contrast to the usual treatment of functional quantization of gauge theories involving the Faddeev-Popov ansatz, here we propose a reformulation of electrodynamics in terms of a *physical* vector potential entirely free of gauge ambiguities right from the outset [79], **and which is spacetime divergenceless** : $\partial \cdot \mathbf{A}_{\mathcal{P}} = 0$. It is important to note that this last property of the vector potential is *not* the Lorentz gauge condition but a *physical restriction* on a physical vector potential. It is merely a restatement of the fact that in the standard formulation of pure electrodynamics, gauge transformations act only on the unphysical degrees of freedom of the gauge potential, with the physical, gauge invariant part of the gauge potential being divergenceless by definition, not as a matter of choice. The gauge free approach, by virtue of being based on a physical, divergenceless vector potential, evades the entire issue of gauge redundancy. Quantizing the theory with this prescription leads to a propagator that is gauge invariant by construction, in contrast to the standard photon propagator.

3.2.1 Gauge Free Vacuum electrodynamics

Here I start with the Maxwell Electrodynamics to develop the idea of gauge-free quantization. In vacuum electrodynamics, gauge ambiguities first appear when one solves the Maxwell-Bianchi identity

$$\partial_{[\mu}F_{\nu\rho]} = 0 \tag{3.4}$$

in terms of the vector potential $A_\mu \ : \ F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$; this solution is unique only upto $U(1)$ gauge transformations: $A_\mu \to A_\mu^{(\omega)} = A_\mu + \partial_\mu \omega$. This infinite ambiguity implies that the other Maxwell equation, written out in terms of A_μ

$$\Box A_\mu - \partial_\mu \partial \cdot A = J_\mu \tag{3.5}$$

does not yield a unique solution for A_μ, for conserved current sources $\partial \cdot J = 0$ *without a gauge fixing* $G(A) = 0$, where the function G may be arbitrary but must *not* be gauge invariant. Clearly, for every choice of G, one has $A = A[G]$, and one can scarcely call any of these As physical. Conventional wisdom relegates the status of A to that of a subsidiary tool, because of this gauge ambiguity, and reminds us that only gauge *invariant* quantities like $F_{\mu\nu}$ or Wilson loops $W[C] \equiv \exp i \oint_C A \cdot dx$ associated with the Ehrenberg-Siday-Aharonov-Bohm [83] phase in quantum mechanics, are truly physical and measurable. Here, I will show vacuum electrodynamics can be reformulated in terms of a physical space-time transverse vector potential.

For the standard gauge potential (abelian) one form A and semi-infinite curve C from spatial infinity to x, construct

$$\begin{aligned} \mathbf{A}_C(x) &\equiv h_{C(\infty,x)}[A](A+d)(h_{C(\infty,x)}[A])^{-1} \\ &= A - d\int_{C(\infty,x)} A \end{aligned} \tag{3.6}$$

For another semi-infinite oriented curve $C^{'}$ from ∞ to the point, it is easy to see that

$$\begin{aligned}\mathbf{A}_C - \mathbf{A}_{C'} &= d\left[\int_{C'} - \int_{C}\right] A \\ &= d\int_{S_{CC'}} dA = 0\end{aligned} \tag{3.7}$$

where, the second line follows from the first by Stokes theorem with $S_{CC'}$ being the surface bounded by the two semi-infinite curves C and C from spatial infinity to x. In the third line we have used the identity $d^2 = 0$. In other words, even though A_C formally depends on the curve C and is expected to be non-local, actually it is independent of C and hence local. This also agrees with the fact the A_C is gauge-independent $\forall$ C. Hence we drop the subscript C in what follows. Now, observe that

$$\begin{aligned}\mathbf{A} &= A - d\int_C A \\ &= A - d\int d^4x^{'} dy_a A^a(x^{'})\delta^4(y - x^{'})\end{aligned} \tag{3.8}$$

Define the d'Alembert Green function $G(x, x^{'})$ as $\Box G(x, x^{'}) = \delta^4(x-x^{'})$. This is consistent with

$$\partial_a G(x, x^{'}) = \int_{C(\infty,x)} dy_a \delta^4(x - y) \tag{3.9}$$

as can be seen by taking partial derivatives on both sides. Substituting eq (6) in (5), and performing a partial integration, we get

$$\begin{aligned}\mathbf{A} &= A - d\int d^4x^{'} G(x, x^{'})\partial^{'} A(x^{'}) \\ &= \mathbf{A}_{\mathcal{P}}\end{aligned} \tag{3.10}$$

Here $\mathbf{A}_{\mathcal{P}}$ denotes a spacetime transverse physical vector which can easily be proved as follows.

$$
\begin{aligned}
\mathbf{A}_{\mathcal{P}} &= A - d\int d^4x' G(x,x')\partial' A(x') \\
\partial.\mathbf{A}_{\mathcal{P}} &= \partial.A \, - \Box \int d^4x' \partial'.A(x') G(x-x') \\
&= 0
\end{aligned}
\tag{3.11}
$$

With this physical field $\mathbf{A}_{\mathcal{P}}$ we now write an action for Maxwell Electrodynamics. We emphasize the fact that no gauge fixing needs to be employed here; but since the functional integral describing the vacuum-to-vacuum amplitude is over all configurations of the vector field $\mathbf{A}_{\mathcal{P}}$, the transversality constraint must be directly inserted into the integral to ensure that the integral is only over transverse field configurations. The gauge-free formulation for vacuum electrodynamics starts with the action

$$
\begin{aligned}
S[\mathbf{A}_{\mathcal{P}}, \Lambda; \tilde{\mathbf{J}}] &= \int \left[-\frac{1}{2}\, \partial_\mu A_{\mathcal{P}\nu}\, \partial^\mu A^\nu_{\mathcal{P}} + \tilde{J}\cdot A_{\mathcal{P}} + \Lambda \partial \cdot A_{\mathcal{P}} \right] \\
&= \int \left[-\frac{1}{2}\, \partial_\mu A_{\mathcal{P}\nu}\, \partial^\mu A^\nu_{\mathcal{P}} + J\cdot A_{\mathcal{P}} \right] \, .
\end{aligned}
\tag{3.12}
$$

The second line above follows from the first by eliminating the Lagrange multiplier field Λ through its equation of motion and defining J_μ such that $\partial \cdot J = 0$ [79]. The relevant vacuum-to-vacuum amplitude (in presence of a transverse source) is given by

$$
\begin{aligned}
Z[\mathbf{J}] \quad &= \quad \int \mathcal{D}\mathbf{A}_{\mathcal{P}}\; \exp i \left(\frac{1}{2}\, A^\mu_{\mathcal{P}} \,\Box\, A_{\mathcal{P}\mu} \; + \; \int d^4x\; \mathbf{J}\cdot\mathbf{A}_{\mathcal{P}} \right) \; \delta[\partial_\mu\; \mathbf{A}^\mu_{\mathcal{P}}] \\
&= \quad \int \mathcal{D}\mathcal{S}\; \mathcal{D}\mathbf{A}_{\mathcal{P}}\; \exp i \int d^4x\; \left[\frac{1}{2}\, A^\mu_{\mathcal{P}} \,\Box\, A_{\mathcal{P}\mu} \; + \; (J_\mu \; - \; \partial_\mu\, \mathcal{S})\; A^\mu_{\mathcal{P}} \right] .
\end{aligned}
\tag{3.13}
$$

In the second line of (3.13) we have introduced an auxiliary scalar field $\mathcal{S}$ which acts as the Lagrange multiplier for the physical constraint (3.11). After integrating over $\mathbf{A}_{\mathcal{P}}$ and auxiliary field one gets,

$$Z[\mathbf{J}] \quad = \quad N \exp \frac{-i}{2} \left[J^{\mu} \left(\frac{\eta_{\mu\nu}}{k^2} - \frac{k_{\mu}k_{\nu}}{k^4} \right) J^{\nu} \right] \tag{3.14}$$

A series of partial integrations and using the transversality of the current density $\mathbf{J}$, and also identities like $\partial_x \mathcal{G}(x-y) = -\partial_y \mathcal{D}(x-y)$ leads to this simple expression

It is now straightforward to extract the free photon propagator from eq. (3.14) after introducing the generating functional for connected Green's function via $W[\mathbf{J}] = -iLogZ[\mathbf{J}]$:

$$\begin{aligned} \mathcal{D}_{\mu\nu}(k) \quad &\equiv \quad \frac{1}{2} \left. \frac{\delta^2 W[\mathbf{J}]}{\delta J^{\mu}(k)\ \delta J^{\nu}(-k)} \right|_{\mathbf{J}=0} \\ &= \quad \frac{1}{k^2 + i\epsilon} \left(\eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2 \ + \ i\epsilon} \right) \end{aligned} \tag{3.15}$$

Clearly, this propagator does not possess any gauge artifacts [79].

In gauge-free formulation, spacetime divergencelessness is *not* a matter of choice, it is a defining feature of what we mean by electromagnetism. Finally, note also that the free photon propagator falls off as $1/k^2$ for large momentum, as is expected for a *local* field.

3.2.2 Gauge-free electrodynamics with sources

The charged matter fields can be coupled with this *physical* photon field in a gauge-free fashion if we rewrite the fields in polar representation, so as to 'separate' charge and spin degrees of freedom. The modulus of the matter fields carries the spin (scalar) degrees of freedom and the phase part carries only the charge of it. This separation of spin and charge actually enables us to represent the theory in terms of manifestly gauge-inert variables.

One thing I would like to mention here that the radial decomposition of charged matter fields sometimes been referred to as imposing 'unitarity gauge', in the literature

[33,84]. I do not agree with this notion because "unitarity gauge" is not really a choice of gauge in the sense of other gauge choices, but a unique representation of a gauge theory with redefined fields which do not transform under gauge transformations. In gauge-free approach for the case of scalar QED, the action has an apparent similarity to the one obtained by Dolan-Jackiw by employing the so-called unitarity gauge. However, there is a crucial difference between their approach and the approach that has been adopted here in the functional integral. For gauge-free approach one has to include the physical constraint on the vector potential, $\partial\cdot\mathbf{A}_{\mathcal{P}} = 0$, in the functional integration which is absent in [33].

3.2.3 Charged matter fields

All charged matter fields are complex fields Φ such that they can be 'radially' decomposed : $\Phi = \phi \exp i\theta$ where ϕ carries all the spin degrees of freedom of Φ and the phase field θ is a scalar field which appears in the action only through its first order derivative $\partial\theta$: $S[\Phi] = S[\phi, \partial\theta]$. The gauge-free prescription for coupling the gauge-free vector potential $\mathbf{A}_{\mathcal{P}}$ to Φ is exceedingly simple : leaving ϕ as it is in the action, simply replace $\partial\theta \to \partial\theta - eA_{\mathcal{P}}$, so that $S[\Phi] \to S[\phi, \partial\theta - eA_{\mathcal{P}}] + S_{free}[A_{\mathcal{P}}]$. Recall of course that the gauge-free $\mathbf{A}_{\mathcal{P}}$ is subject to the 4-divergencelessness constraint (3.11). The interaction with matter for this vector potential is merely to add a *physical longitudinal* part to it so that potentially it can now turn massive even in the weak coupling limit, depending upon the form of $S[\Phi]$. An example of this is the Abelian Higgs model of scalar electrodynamics.

3.2.4 Abelian Higgs Model

A charged scalar admits the radial decomposition $\phi = (\rho/\sqrt{2}) \exp i\theta$ where ρ and θ are both to be treated as physical fields. With this decomposition, the action of the complex scalar field appears as (suppressing obvious indices)

$$S_0[\rho,\theta] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}\rho^2(\partial\theta)^2 - V(\rho)\right] . \tag{3.16}$$

This action (3.16) is invariant under the global $U(1)$ transformations $\rho \to \rho$, $\theta \to \theta + \omega$ where ω is a real constant.

Observe now that one can define $\Theta \equiv \theta - ea$ where a is introduced as part of the standard $U(1)$ gauge potential which carries the entire gauge transformation $a \to a + e^{-1}\omega$ when one couples the scalar theory to the standard gauge field A_μ. It is obvious that Θ is invariant under gauge transformations. Following our prescription above, coupling to the physical electromagnetic vector potential is obtained through the action (dropping obvious indices)

$$S[\rho,\Theta,\mathbf{A}_\mathcal{P}] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(\mathbf{A}_\mathcal{P} - e^{-1}\partial\Theta)^2 \;\; - \frac{1}{2}(\partial A_\mathcal{P})^2 - V(\rho)\right] \tag{3.17}$$

where $V(\rho)$ is the scalar potential, and $\mathbf{A}_\mathcal{P}$ obeys the divergenceless constraint (3.11). It is interesting that the phase field Θ occurs in the action only through the combination $\mathbf{A}_\mathcal{P} - e^{-1}d\Theta$; this implies that the shift $\Theta \to \Theta + const.$ is still a symmetry of the action. However, since there is no canonical kinetic energy term for Θ, it is hard to associate a propagating degree of freedom with Θ. Indeed, if one first makes a field redefinition

$$Y_\mu \;\equiv\; A_{\mathcal{P}\mu} \;-\; e^{-1}\,\partial_\mu\,\Theta\ . \tag{3.18}$$

the Θ can be completely absorbed into the new vector field $\mathbf{Y}_\mu$, appearing only in the constraint which replaces (3.11)

$$\partial\cdot Y = -\Box\Theta\ . \tag{3.19}$$

This implies that $\mathbf{Y}$ has three *physical* polarizations rather than the two that $\mathbf{A}_{\mathcal{P}}$ had. However, this does not immediately imply that $\mathbf{Y}$ has acquired a mass. Upon eliminating Θ through the constraint (3.2.4), eq. (3.17) assumes the form

$$S[\rho, \mathbf{Y}] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2} Y^a \left((\Box + e^2\rho^2)\eta_{ab} - \partial_a \partial_b \right) Y^b - V(\rho) \right], \tag{3.20}$$

This is the gauge-free Abelian Higgs model [79] with the field $\mathbf{Y}$ satisfying the constraint ().

One can now think of two kinds of scalar potentials $V(\rho)$: one for which the minimum of the potential $\langle\rho\rangle = 0$ and the other for which the minimum lies away from the origin $\langle\rho\rangle = \rho_c \neq 0$. It is this second case which is of interest to us. If $V(\rho)$ has a minimum at $\rho = \rho_c \neq 0$ one now also defines $\rho \to \rho + \rho_c$, it is easy to see that the $\mathbf{Y}$ acquires a mass $m_{\mathbf{Y}}^2 = e^2\rho_c^2$ while the ρ also acquires a mass $m_\rho^2 = V''(\rho_c)$. This is precisely the manner in which a *physical* longitudinal degree of freedom conjoins the photon field to produce a massive vector boson. In doing so, the new vector potential $\mathbf{Y}$ is no longer subject to the transversality constraint (3.11). It thus has one degree of freedom more than the $\mathbf{A}_{\mathcal{P}}$. Observe that the Higgs phenomenon of mass generation **did not involve any symmetry breaking at all**, reminding us of Elitzur's theorem [18] proved for QED on a cubic lattice. The vacuum expectation value $\rho_c \equiv \langle\rho\rangle$ does not break any continuous symmetry at all. *In our interpretation, the Higgs mechanism is a gauge-free mechanism of mass generation, involving neither symmetry breaking of any sort, nor unphysical particles in the spectrum.*

Before closing this subsection, let me point out that this aspect of the phase field attaching itself to the photon field as a *physical* longitudinal piece, is not confined to charged scalar fields. Consider for instance a free charged Dirac field given by the action

$$S[\psi] = \int d^4x \; \bar{\psi}(i\gamma \cdot \partial - m)\psi \; . \tag{3.21}$$

Performing the 'radial decomposition' $\psi = \chi \exp i\theta$ this reduces to

$$S[\chi, \theta] = \int d^4x \, (\, \bar{\chi}(i\gamma \cdot \partial - m)\chi - \bar{\chi}\gamma \cdot \partial\theta\chi \,) \; . \tag{3.22}$$

This action is of course invariant under the global $U(1)$ transformations $\chi \to \chi$, $\theta \to \theta + \omega$ for a constant ω. Employing the prescription above for coupling this field to the physical electromagnetic vector potential, the action now reads

$$S[\chi, \theta] = \int d^4x \, (\, \bar{\chi}(i\gamma \cdot \partial - m)\chi - \bar{\chi}\gamma\chi \cdot (\partial\theta - eA_{\mathcal{P}}) \,) \; . \tag{3.23}$$

It is obvious from the above that under any interaction, the vector potential is *poised* to pick up a physical longitudinal piece ($\partial\theta$) corresponding to the 'charge mode'. However, in this case there is no mechanism (at tree level) of mass generation due to the absence of a 'seagull' term. But this could be an artifact of weak coupling [79]. In the 1+1 dimensional quantum electrodynamics model analyzed half a century ago by Schwinger [17], the photon field does pick up a manifestly gauge invariant mass as an exact dynamical result.

3.3 Gauge-free scalar QED: Coleman-Weinberg Mechanism

In this section I will show that functional quantization of scalar quantum electrodynamics leads at the quantum level to a one loop effective potential which realizes the Coleman-Weinberg mechanism of mass generation in a gauge-free framework, thus resolving the issue of its gauge dependence. However, since the reparametrization invariance can only be ensured by treating the theory in the Vilkovisky-DeWitt (VD) approach, the CW potential is calculated with the gauge-free theory according to the VD approach. The new method which can be called a *gauge-free Vilkovisky-DeWitt* method gives a different result from the one calculated earlier by Kunstatter [43]. This difference is an indication that by eschewing redundant field degrees of freedom from the outset, it is possible to obtain a unique result for the vector potential.

The action for the theory is already given above eq.(3.17), with the choice $V(\rho) = (\lambda/4!)\rho^4$. Following [21], the theory is quantized using the functional integral formalism. In the standard formulation of QED, one needs to resort to the Faddeev-Popov technique of gauge fixing and extracting the infinite volume factor associated with the group of gauge transformations, from the vacuum persistence amplitude (generating functional for all Green's functions), in order that this amplitude does not diverge upon integrating over gauge equivalent copies of the gauge potential. In the gauge free approach here, this technique is not necessary. The integration over the transverse gauge potential is, of course, restricted to configurations that obey the spacetime transversality condition (3.11). Since the integration variables are unambiguous, the task, at least at the one loop level, is simpler.

The generating functional is thus given by

$$\begin{aligned} Z[J, J', \mathbf{J}] &= e^{iW[J,J',\mathbf{J}]} = \int \mathcal{D}\rho \, \mathcal{D}\Theta \, \mathcal{D}\mathbf{A}_{\mathcal{P}} \, \exp i \left[S[\rho, \Theta, \mathbf{A}_{\mathcal{P}}] + \int d^4x (J\rho + J'\Theta + \mathbf{J} \cdot \mathbf{A}_{\mathcal{P}}) \right] \\ & \quad \cdot \delta[\, \partial_\mu \, A^\mu_{\mathcal{P}} \,] \, . \end{aligned} \tag{3.24}$$

Here, the integration measures $\mathcal{D}\rho = \Pi_x d\rho(x)$, $\mathcal{D}\mathbf{A}_{\mathcal{P}} = \Pi d\mathbf{A}_{\mathcal{P}}$, but the remaining measure $\mathcal{D}\Theta = Det\rho \Pi_x d\Theta(x)$. The extra factor of $Det\rho$ can be seen to arise if one begins with the generating functional first expressed as functional integrals over a complex scalar field and its complex conjugate. Alternatively, one can obtain the configuration space functional integral starting with the functional integral over phase space. Integration over the momentum conjugate to Θ produces the same factor [85].

The effective potential is computed using the method described in (2). Even though the scalar potential is classically scale invariant, a mass scale is generated through renormalization in the quantum theory, which breaks this scale invariance. The effective potential may thus have a minimum away from the origin in ρ-space, defined in terms of the renormalization mass scale, which, in turn, relates to values of the dimensionless physical parameters of the theory (dimensional *transmutation* [21]).

Instead of evaluation of the functional integral over the Θ and $\mathbf{A}_{\mathcal{P}}$ fields, one makes a change of basis to Θ and $\mathbf{Y}$ via (3.18) and make use of the action (3.20) which is independent of Θ. The latter appears only in the constraint which now becomes a statement of non-transversality in spacetime of the $\mathbf{Y}$ field. Θ can be simply integrated out, leaving behind a field-independent normalization which we set to unity. The integration over ρ involves a saddle-point approximation around a field ρ_c which may be called a 'quantum' field, since it is the solution of the classical ρ-equation of motion augmented by $\mathcal{O}(\hbar)$ corrections. With no gauge ambiguities anywhere, there is no question of gauge fixing; functional integration over the physical vector potential $\mathbf{Y}$ can be performed straightforwardly.

Following (2.38), the one loop effective action is given schematically by

$$\Gamma^{(1)}\ [\rho_c] \ = \ S[\rho_c, 0, 0] \ + \ \frac{i}{2}\ TrlnS_2[\rho_c]\ , \tag{3.25}$$

where,

$$Tr\, ln\, S_2[\rho_c] \equiv \int \mathcal{D}\rho \mathcal{D}\mathbf{Y}\ \exp \frac{i}{2} \left[\ \int d^4x d^4y\ \rho(x) \mathcal{M}_{\rho\rho}(x,y)\ \rho(y) + Y^\mu(x) \mathcal{M}_{Y_\mu Y_\nu}(x,y) Y^\nu(y)\ \right] \tag{3.26}$$

with, generically,

$$\mathcal{M}_{AB}(x,y) \ \equiv \ \left(\frac{\delta^2 S[\Phi]}{\delta\Phi_A(x)\ \delta\Phi_B(y)} \right)_{\Phi=\rho_c,0,0} \ . \tag{3.27}$$

Since our object of interest is the one loop effective potential, we restrict ourselves to a saddle point ρ_c which is spacetime independent. The matrices $\mathcal{M}$ turn out to be diagonal in field space for the purpose of a one loop computation, with entries

$$
\begin{aligned}
\mathcal{M}_{\rho\rho} &= -\left(\Box + \frac{\lambda}{2}\rho_c^2 \right) \delta^{(4)}(x-y) \\
\mathcal{M}_{Y_\mu Y_\nu} &= \left[\eta_{\mu\nu} \; \left(\Box + e^2 \; \rho_c^2 \right) - \partial_\mu \partial_\nu \right] \delta^{(4)}(x-y) \; .
\end{aligned} \tag{3.28}
$$

One obtains easily

$$
\begin{aligned}
V_{eff}(\rho_c)^{(1)} &= \frac{1}{4!}\lambda\rho_c^4 + i \int d^4k \; \log \left[\left(-k^2 + e^2\rho_c^2\right)^{3/2} \left(-k^2 + \lambda\rho_c^2\right)^{1/2} \right] \\
&+ \frac{1}{2}B\rho_c^2 + \frac{1}{4!}C\rho_c^4 \; ,
\end{aligned} \tag{3.29}
$$

where B and C are respectively the mass and coupling constant counterterms. The momentum integral is performed with a Lorentz-invariant cut-off $k^2 = \Lambda^2$, yielding

$$
\begin{aligned}
V_{eff}(\rho_c)^{(1)} &= \frac{1}{4!}\rho_c^4 + \frac{1}{2}B\rho_c^2 + \frac{1}{4!}C\rho_c^4 \\
&+ \frac{\rho_c^2\Lambda^2}{32\pi^2}\left(\frac{1}{2}\lambda + 3e^2\right) \\
&+ \frac{\rho_c^4}{64\pi^2}\left[\frac{1}{4}\lambda^2 \left(\log\frac{\lambda\rho_c^2}{2\Lambda^2} - \frac{1}{2} \right) + 3e^4 \left(\log\frac{e^2\rho_c^2}{\Lambda^2} - \frac{1}{2} \right) \right]
\end{aligned} \tag{3.30}
$$

I remark here that in these manipulations, a $\exp(-\log\rho)$ term is generated in the one loop partition function, which cancels *exactly* against an identical term $Det\rho$ arising in the formal measure as discussed after eq. (3.24). This is precisely the point that was made earlier: the interpretation of that extra local factor in the formal functional measure as some sort of conformal mode in a conformally flat background is subject to some modification at the one loop level, since that factor is *eliminated* by a one loop contribution to the partition function [87]. This has been anticipated in ref. [86] where an attempt has been made to give an alternate interpretation in terms of a 'gauge-dependent gravity'.

The mass and coupling constant renormalizations B and C are fixed through the renormalization conditions

$$\left.\frac{d^2V}{d\rho_c^2}\right|_{\rho_c=M} = 0 \tag{3.31}$$

$$\left.\frac{d^4V}{d\rho_c^4}\right|_{\rho_c=M} = \lambda \tag{3.32}$$

leading to the renormalized one loop effective potential

$$\begin{aligned} V_{eff}(\rho_c) &= \frac{\lambda}{4!}\rho_c^4 + \rho_c^2 M^2 \left[-\frac{\lambda}{4} + \frac{9}{32\pi^2}(3e^4 + \frac{1}{2}\lambda^2)\right] \\ &+ \left(\frac{3e^4}{64\pi^2} + \frac{\lambda^2}{256\pi^2}\right)\rho_c^4 \left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right] . \end{aligned} \tag{3.33}$$

The potential has an extremum at $\rho_c = \langle\rho\rangle(M)$ leading eventually to the ratio of the squared masses of the Higgs boson to the photon

$$\frac{m_H^2}{m_A^2} = \frac{1}{e^2}\left[\frac{1}{3}\lambda - \left(\frac{3e^4}{8\pi^2} + \frac{\lambda^2}{32\pi^2}\right)\log\frac{\langle\rho\rangle^2}{M^2} - \frac{e^4}{\pi^2} - \frac{\lambda^2}{12\pi^2}\right] \tag{3.34}$$

The derivation of the mass ratio of the Higgs mass to the photon seemingly went through without any gauge fixing, since all fields being functionally integrated over are *physical* fields without any gauge ambiguity. The result (3.34) is thus a 'physical' result in this toy model where the photon acquires a mass. Notice that unlike in the original Coleman-Weinberg paper, no approximation has been made like choosing $\lambda \sim e^4$, to drop terms of $O(\lambda^2)$. Thus, even though the result agrees with the earlier papers qualitatively, there are significant quantitative differences. However, the point in this section is not so much the result of the computation of the mass ratio, but the observation that the effect is physical and not a gauge artifact.

3.4 Vilkovisky-DeWitt Effective Action

I will give a brief outline of the Vilkovisky-DeWitt unique effective action here. The dependence of effective action on choice of gauge conditions in gauge theories may be looked upon as dependence on the parametrization of the quantum fields. Vilkovisky was first to realize that there is a problem in the conventional definition of standard effective action and proposed a new definition of effective action which is gauge invariant and gauge-fixing independent using geometric consideration [40]. Later, DeWitt [41] analysed the Vilkovisky's modified effective action and proposed some modifications although it was realized that two definitions coincides in the one-loop level. It was pointed out that we must evaluate the effective action more carefully by treating the space of field configurations ϕ^i as a manifold $\mathcal{M}$. Consider the integro-differential equation satisfied by effective action (2.27) introduced earlier (for a single scalar field). It can be expressed alternatively as

$$\exp\left(i\Gamma[\Phi]\right) \;=\; \int D\phi\;\exp\left(iS[\phi] + i\int d^4x\;(\phi - \Phi)\,\frac{\delta\Gamma}{\delta\Phi}\right) \tag{3.35}$$

The problem with conventional formalism of effective action lies in the fact that the eqn. (3.35) does not have a correct geometrical interpretation because the difference of two points in the configuration space $\mathcal{M}$, namely $\phi-\Phi$, in general, is not a vector on that space. This spoils the covariance of the expression $(\phi-\Phi)\frac{\delta\Gamma}{\delta\Phi}$ under field reparametrizations. Thus the effective action fails to be a scalar function on the configuration space $\mathcal{M}$. To get rid of this problem Vilkovisky proposed that $\phi - \Phi$ should be replaced by a two-point function $\sigma^i(\Phi,\phi)$, which is a vector tangent to the curve joining the points Φ and ϕ. It is a vector with respect to the point Φ and a scalar w.r.t. the point ϕ. We adopt DeWitt's condensed notation here [88], so that the index i stands for continuous (i.e. spacetime) labels as well as ordinary indices. Repeated indices indicate a summation over ordinary indices as well as an integration over continuous labels. The properties of the bi-vector

$\sigma^i(\Phi,\phi)$ is also discussed in detail in [88]. With this definition of $\sigma^i(\Phi,\phi)$ the effective action now been derived from

$$\exp\left(i\Gamma[\Phi]\right) \quad = \quad \int D\phi\, d\mu[\phi]\;\exp\left(iS[\phi] + i\int d^4x\;\sigma^i(\Phi,\phi)\;\frac{\delta\Gamma}{\delta\Phi}\right) \tag{3.36}$$

Now the r.h.s. of eqn. (3.36) becomes a scalar function of ϕ and the functional integral is independent of reparametrization of ϕ. We can expand $\sigma^i(\Phi,\phi)$ in powers of $\Phi-\phi$ with co-efficients evaluated at Φ.

$$\sigma^i(\Phi,\phi) = (\Phi-\phi)^i - \frac{1}{2}\mathit{\Gamma}^i_{mn}(\Phi)(\phi-\Phi)^m(\phi-\Phi)^n + \cdots \tag{3.37}$$

$\mathit{\Gamma}^i_{mn}[\Phi]$ are the connections on $\mathcal{M}$. The connection is to be determined following several criteria. These are imposed by Vilkovisky in [40]

1. The connection should be determined from the classical action by a universal rule.

2. Connections for free field theories are identically zero.

3. The connection should be ultralocal i.e. it should not contain any derivative of delta functions.

Following this idea of Vilkovisky, DeWitt proposed another definition of effective action

$$\exp\left(i\Gamma_D[\Phi]\right) = \int D\phi\, d\mu[\phi]\;\exp\left(iS[\phi] + i\int d^4x\; C^i_j[\Phi]\sigma^j(\Phi,\phi)\;\frac{\delta\Gamma_D}{\delta\Phi}\right) \tag{3.38}$$

Where $C^i_j[\Phi]$ can be expressed as the expectation value of a power series of $\sigma^i(\Phi,\phi)$,

$$C^i_j = \langle\delta^i_j\; + \;\frac{1}{3}R^i_{jkl}[\Phi]\sigma^k(\Phi,\phi)\sigma^l(\Phi,\phi)\; + \cdots\rangle, \tag{3.39}$$

R^i_{jkl} is the Riemann tensor of $\mathcal{M}$. For calculational details can be found in [41,43].

In one-loop approximations $\mathit{\Gamma}$ and Γ_D coincide. Thus one gets

$$\Gamma^{(1)}_{VD}[\Phi] = S(\Phi) + \frac{1}{i}\ \ln\mu[\phi] + \frac{1}{2i} Tr\ln[\nabla_m\nabla_n S(\Phi)] + O(\hbar^2) \tag{3.40}$$

where ∇_m is the covariant derivative associated with connection $\mathit{\Gamma}^i_{mn}$. The connection is Christoffel and completely described by the metric on the manifold $\mathcal{M}$.

$$\mathit{\Gamma}^i_{mn} = \frac{1}{2}G^{ik}(G_{mk,n} + G_{nk,m} - G_{mn,k}) \tag{3.41}$$

and

$$\frac{\delta^2 S}{\delta\phi^m\delta\phi^n} \ \rightarrow\ \nabla_m\nabla_n S = \frac{\delta^2 S}{\delta\phi^m\delta\phi^n} - \mathit{\Gamma}^i_{mn}(\phi)\frac{\delta S}{\delta\phi^i}\,. \tag{3.42}$$

For gauge theories we must evaluate the effective action in physical configuration space i.e. the space of all the gauge fields modulo the possible gauge transformations. First consider the infinitesimal gauge transformation,

$$\delta\phi^i \ = \ K^i_\alpha[\phi]\epsilon^\alpha\,, \tag{3.43}$$

with $K^i_\alpha[\phi]$ being the generators of gauge transformation and ϵ^α the infinitesimal gauge-group parameters.

Let $G_{ij}[\Phi]$ be the metric in the naive field space;

$$ds^2 = G_{ij}\delta\phi^i\delta\phi^j \tag{3.44}$$

The metric of the physical field space is given by,

$$ds^2_P = \gamma_{ij}\delta\phi^i\delta\phi^j = G_{ij}\delta\phi^i_P\delta^j_P \tag{3.45}$$

where the physical field is defined as

$$\delta\phi^i_P = \Pi^i_j \delta\phi^j \tag{3.46}$$

The projector Π^i_j projects the vectors of naive field space onto a subspace of vectors which are perpendicular to the space of tangent vectors to the orbits generated by K^i_α.

$$\Pi^i_j = \delta^i_j - K^i_\alpha N^{\alpha\beta} K^l_\beta G_{lj} \tag{3.47}$$

The projection operator satisfies

$$\Pi^i_j \Pi^j_k = \Pi^i_k, \quad \Pi^i_j k^j_\alpha = 0 \tag{3.48}$$

$N^{\alpha\beta}$ is the inverse of

$$N_{\alpha\beta} = G_{ij} k^i_\alpha k^j \beta \tag{3.49}$$

and

$$N^{\alpha\beta} = \gamma^{\xi\alpha} \gamma^{\chi\beta} K^i_\xi K^j_\chi G_{ij} \tag{3.50}$$

Using the metric γ_{ij} a new connection is constructed . The modification of the connection due to gauge field gives

$$\Gamma^i_{jk} = \mathit{\Gamma}^i_{jk} \ + \ T^i_{jk} \tag{3.51}$$

where we have ignored a piece proportional to K^i_α since it will annihilate the action and will not contribute to the one-loop order. The gauge-part of connection reads,

$$T^m_{ji} = K^\alpha_{(j} K^\beta_{i)} K^l_\alpha K^m_{\beta;l} \ - \ K^\alpha_j K^m_{\alpha;i} \ - \ K^\alpha_i K^m_{\alpha;j} \tag{3.52}$$

The convention for symmetrization is

$$A_{(i}B_{j)} = \frac{1}{2}(A_iB_j + A_jB_i)$$

3.4.1 Scalar QED in Gauge-free VD approach

The detailed calculation of one-loop effective potential for scalar QED is given in [43]. We do not repeat it here but merely restate essential results of that work. The action for the scalar QED is

$$S[\rho, \theta, A] = \int d^4x \left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(A - e^{-1}\partial\theta)^2 \; - \frac{1}{2}(\partial A)^2 - \frac{\lambda}{4!}\rho^4\right], \tag{3.53}$$

The metrics in field space are given by

$$G_{\rho(x)\rho(y)} = \delta^4(x-y) \tag{3.54}$$

$$G_{\theta(x)\theta(y)} = \rho^2\delta^4(x-y) \tag{3.55}$$

$$G_{A_\mu(x)A_\nu(y)} = -\eta_{\mu\nu}\delta^4(x-y) \tag{3.56}$$

For scalar QED the correct measure which is invariant under general co-ordinate transformations in $\mathcal{M}$ will contain the determinant of the metric. Thus here,

$$d\mu[\phi] = \sqrt{detG} = Det|\rho(x)\delta^4(x-y)|$$

The only contribution to the one-loop effective potential from the Christoffel symbol Γ^i_{jk} is

$$\Gamma^{\rho(z)}_{\theta(x)(\theta(y)} = -\frac{\lambda\rho_c^4}{6}\delta^4(x-y) \tag{3.57}$$

To get the contribution from gauge part of the connection T^i_{jk} identify the generators of gauge transformations

$$\begin{aligned} K_y^{\rho(x)} &= 0 \\ K_y^{\theta(x)} &= e\delta^4(x-y) \\ K_y^{A_\mu(x)} &= -\partial_\mu\delta^4(x-y) \end{aligned} \tag{3.58}$$

with the partial derivative with respect to the first argument of the δ-function. Here I write down the non-trivial Γ^i_{jk}s. The calculation details can be found in [43].

$$\begin{aligned} T^{\rho(z)}_{\theta(x)\theta(y)} &= e^2\rho_c^3[\delta^4(x-y)N^{zx} + \delta^4(z-x)N^{zy} - e^2\rho_c^2 N^{xy}N^{xz}] \\ T^{\rho(z)}_{A_\mu(x)A_\nu(y)} &= -e^4\rho_c^5\partial^\mu N^{yx}\partial^\nu N^{zx} \\ T^{\rho(z)}_{A_\mu(x)\theta(y)} &= e\rho_c[\partial^\mu N^{xz}\delta^4(x-y) - e^2\rho_c^2\partial^\mu N^{xy}N^{xz}] \end{aligned} \tag{3.59}$$

The one-loop effective potential calculated in this formalism turns out to be independent of the gauge parameter. In fact it is equal to the one calculated by Jackiw with $\alpha = -1$ [43].

$$V_{eff}(\rho_c) = \frac{\lambda}{4!}\rho_c^4 + \frac{1}{64\pi^2}\left(3e^4 + \frac{5}{18}\lambda^2 + \frac{2}{3}\lambda e^2\right)\rho_c^4\left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right]. \tag{3.60}$$

Now, lets turn to the case of gauge-free scalar QED. In gauge-free approach the action from which the effective potential was calculated is given by eqn. (3.17),

$$S[\rho,\Theta,\mathbf{A}_\mathcal{P}] = \int d^4x\left[\frac{1}{2}(\partial\rho)^2 + \frac{1}{2}e^2\rho^2(\mathbf{A}_\mathcal{P} - e^{-1}\partial\Theta)^2 - \frac{1}{2}(\partial A_\mathcal{P})^2 - \frac{\lambda}{4!}\rho^4\right],$$

Now for reparametrization invariance I apply VD technique to calculate the effective potential for this action. The metrics of the field space are

$$G_{\rho(x)\rho(y)} = \delta^4(x-y) \tag{3.61}$$

$$G_{\Theta(x)\Theta(y)} = \rho^2\delta^4(x-y) \tag{3.62}$$

$$G_{A_\mu(x)A_\nu(y)} = -\eta_{\mu\nu}\delta^4(x-y). \tag{3.63}$$

However, the only non-trivial contribution to the one-loop effective potential will be from $\Gamma^{\rho}_{\Theta\ \Theta}$ and an additional contribution occurs from our gauge-free conventional calculation.

$$\mathcal{M}_{\Theta(x)\Theta(y)} = -\rho_c^2\left[-k^2 + \frac{\lambda\rho_c^2}{6}\right] \tag{3.64}$$

Since again the theory doesn't posses any non-vanishing gauge generators ($K_y^{\Theta(x)} = 0; K_y^{\mathbf{A}_P(x)} = 0$) one doesn't have any gauge part of the connection. The one-loop effective potential in this gauge-free framework becomes:

$$\begin{aligned} V_{eff} = \quad & \frac{\lambda\rho_c^4}{4!} - i\int\frac{d^4k}{(2\pi)^4}\log\rho_c + \frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\log[-\rho_c^2k^2] \\ + \quad & \frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\log(-k^2+\frac{\lambda\rho_c^2}{6}) \\ + \quad & i\int d^4k\ \log\left[\left(-k^2+e^2\rho_c^2\right)^{4/2}\left(-k^2+\lambda\rho_c^2\right)^{1/2}\right] \\ + \quad & \frac{i}{2}Tr\int d^4k\log\left[\frac{k^2}{k^2-e^2\rho_c^2}\right] \end{aligned} \tag{3.65}$$

The first integral comes from the integration measure which exactly cancels a divergent part coming from the inverse propagator of Θ(the second integral). The last term is the contribution from the transversality constraint on $\mathbf{A_P}$, as already included in eqn. (3.24). This result clearly differs from the earlier result calculated by Kunstatter in [43]. It also doesn't agree with the result obtained by gauge-free approach (5.37) due to the third integral in the r. h. s. of V_{eff}. This is due to the fact that I have ignored the reparametrization invariance of the gauge-theories which was not captured in the gauge-free non geometric approach. The renormalized one-loop effective potential becomes

$$V_{eff}(\rho_c) \;=\; \frac{\lambda}{4!}\rho_c^4 + \frac{1}{64\pi^2}\left(3e^4 + \frac{5\lambda^2}{18}\right)\rho_c^4\left[\log\frac{\rho_c^2}{M^2} - \frac{25}{6}\right] \;. \qquad (3.66)$$

This is the unique gauge-free Coleman-Weinberg potential for scalar QED [79] and surprisingly this coincides with the result of Coleman-Weinberg's original paper!

3.5 Generalization

The gauge-free prescription for free photon field can be extended to formulate a similar gauge-free version of the spin 2 linearized graviton field and the Kalb-Ramond antisymmetric second rank gauge potential [89], as we now proceed to show. This leads us immediately to formulating the theory of these tensor fields in terms of gauge-free graviton and antisymmetric tensor fields.

3.5.1 Graviton field

The graviton field is defined in terms of spin two fluctuations about Minkowski spacetime

$$g_{\mu\nu} \;=\; \eta_{\mu\nu} \;+\; h_{\mu\nu} \;. \qquad (3.67)$$

If the Einstein-Hilbert action is expanded in powers of the spin 2 fluctuations $h_{\mu\nu}$ upto bilinear terms, the effective action is invariant under linearized infinitesimal coordinate (gauge) transformations

$$h_{\mu\nu} \;\rightarrow\; h_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)} \;. \qquad (3.68)$$

Consider now the double projection on these spin 2 fluctuations

$$h^T_{\mu\nu} \equiv \mathcal{P}^\lambda_\mu \, \mathcal{P}^\rho_\nu \, h_{\lambda\rho} \qquad (3.69)$$

with the projection operator, $\mathcal{P}$ defined as

$$\mathcal{P}^\mu_\nu = (\delta^\mu_\nu - \partial^\mu \Box^{-1} \partial_\nu) \qquad (3.70)$$

It is easy to verify that under the linearized coordinate transformation (3.68), $h^T_{\mu\nu}$ is *invariant*. Further, it satisfies the spacetime transversality condition

$$\partial^\mu \, h^T_{\mu\nu} \; = \; 0 \; . \qquad (3.71)$$

The linearized equation of motion for the graviton field is given by

$$\begin{aligned} \mathcal{G}_{\mu\nu} \; &\equiv \; \frac{1}{2}\left(\partial_\mu \partial_\nu h \; + \; \Box h_{\mu\nu}\right) - \partial_\rho \partial_{(\mu} h^\rho_{\nu)} \\ &- \; \frac{1}{2}\eta_{\mu\nu}\left(\Box h - \partial_\rho \partial_\lambda h^{\rho\lambda}\right) \\ &= \; 8\pi G T_{\mu\nu} \; , \end{aligned} \qquad (3.72)$$

where, $h \equiv h^\mu_\mu$. In terms of the projected tensor field $h^T_{\mu\nu}$, this equation reduces to

$$\mathcal{G}_{\mu\nu} \; \equiv \; \frac{1}{2}\Box\left(h^T_{\mu\nu} \; - \; \mathcal{P}_{\mu\nu} h^T\right) \; . \qquad (3.73)$$

Defining

$$\bar{h}^T_{\mu\nu} \equiv h^T_{\mu\nu} - \mathcal{P}_{\mu\nu} h^T \; , \qquad (3.74)$$

the linearized equation reduces to the inhomogeneous d'Alembert wave equation

$$\mathcal{G}_{\mu\nu} \;=\; \frac{1}{2}\Box\bar{h}^T_{\mu\nu} = 8\pi G T_{\mu\nu}\ . \tag{3.75}$$

The field $\bar{h}^T_{\mu\nu}$ is also spacetime transverse and manifestly gauge invariant just like $h^T_{\mu\nu}$. However, note that it is *not* traceless : $\bar{h}^T \neq 0$.

With this as motivation, it is now possible to define the physical graviton field $h_{\mathcal{P}\mu\nu}$ which obeys

$$\begin{aligned} \partial_\mu\, h^{\mu\nu}_{\mathcal{P}} &= 0 \\ \Box h_{\mathcal{P}\mu\nu} &= 8\pi G T_{\mu\nu}\ . \end{aligned} \tag{3.76}$$

How unique is this physical graviton field ? If we make the standard linear coordinate gauge transformation for graviton fields discussed above, i.e., $\delta h_{\mathcal{P}\mu\nu} = 2\partial_{(\mu}\xi_{\nu)}$, we find, from eqn. (3.76) that the gauge function ξ^μ must satisfy the equation $\Box\xi_\mu + \partial_\mu\partial\cdot\xi = 0$ which does not appear to have any nontrivial solution ! Our physical graviton field is thus unique.

3.5.2 Kalb-Ramond two form potential

The Kalb-Ramond two form potential $\mathbf{B}$ has a field strength $\mathbf{H} = d\mathbf{B}$ which is clearly invariant under the gauge transformation $\mathbf{B} \to \mathbf{B} + d\Lambda$ for any one form field Λ. Construct now the projected two form field $\mathbf{B}^T \equiv \mathcal{P}\otimes\mathcal{P}\mathbf{B}$. Since $\mathcal{P}df = 0\ \forall f$, under the gauge transformation of $\mathbf{B}$, $\mathbf{B}^T \to \mathbf{B}^T + \mathcal{P}\otimes\mathcal{P}d\Lambda = \mathbf{B}^T$. Further, in a coordinate system,

$$\partial_\mu\, B^{T\mu\nu} \;=\; 0 \tag{3.77}$$

implying that it is indeed transverse. Finally, it is clear that $\mathbf{H} = d\mathbf{B} = d\mathbf{B}^T$, which means that $\mathbf{B}^T$ is indeed the physical part of the two form potential.

As in the case of gauge free electrodynamics, one can formulate the theory of Kalb-Ramond fields purely in terms of a *physical* antisymmetric tensor potential $B_{\mu\nu}$ defined by the action

$$S_{KR} = \int d^4x \left(-\frac{1}{2} B_{\mathcal{P}\nu\rho} \, \Box B^{\nu\rho}_{\mathcal{P}} + J_{\nu\rho} \, B^{\nu\rho}_{\mathcal{P}} \right) , \tag{3.78}$$

where, $\partial^\mu B_{\mathcal{P}\mu\nu} = 0 = \partial^\mu J_{\mu\nu}$.

We once again ask how unique the potential $B_{\mathcal{P}\mu\nu}$ is. Observe that both the field equation and the divergenceless condition remain invariant under a gauge transformation $B_{\mathcal{P}\mu\nu} \to (B_{\mathcal{P}\mu\nu})^\Lambda = B_{\mathcal{P}\mu\nu} + 2\partial_{[\mu}\Lambda_{\nu]}$ where Λ_μ satisfies the equation $\Box\Lambda_\mu - \partial_\mu \partial \cdot \Lambda = 0$. In contrast to the case of the graviton field, it is obvious that this equation has an infinity of *gauge equivalent* solutions, the equivalence being under $\Lambda_\mu \to \Lambda_\mu + \partial_\mu \omega$ for an arbitrary function ω. Restricting $\Lambda_\mu(\infty) = 0$ is not enough to make it vanish everywhere. We need to additionally restrict $\partial \cdot \Lambda = 0$ everywhere with the requirement that $\omega(\infty) = const.$ This additional restriction appears necessary in this preliminary investigation to make the two form potential unique.

The reason why an identical procedure as for the photon or graviton field does not suffice to yield a gauge-free formulation of antisymmetric tensor potentials is because of the aspect of *reducibility* of these potentials: the vectorial gauge parameter of the two form potential itself has a gauge invariance. Perhaps our approach will need to be somewhat modified to produce a gauge-free theory of potentials that have a reducible gauge invariance.

3.6 Discussions

For pure Yang Mills theories, the construction of a gauge-free alternative has not yet been attempted, even though lattice gauge theories represent an explicitly gauge invariant formulation. A local, gauge-free formulation of Yang Mills theories is not obviously in contradiction with extant ideas about colour confinement of quarks and gluons. This gives us the opportunity to attempt a construction of a physical *non-Abelian* one form in terms of the usual Yang-Mills gauge one form $\mathbf{A}$ (which takes values in the Lie algebra of the gauge group $\mathcal{G}$).

Defining the holonomy along the curve C from y to x as $h_{C[y,x]}[\mathbf{A}] \equiv \mathbf{P} \exp \int_{C(y,x)} \mathbf{A}$, with $\mathbf{P}$ denoting path ordering, we note that under local gauge transformations of the gauge potential $[\mathbf{A}(x)]^{\Omega(x)} = \Omega(x)^{-1}[\mathbf{A}(x) + d]\Omega(x)$, where $\Omega \in \mathcal{G}$ the holonomy variables transform as

$$h_{C(y,x)}[\mathbf{A}^{\Omega}] = \Omega^{-1}(y)\ h_{C(y,x)}[\mathbf{A}]\ \Omega(x)\ . \tag{3.79}$$

If one chooses the point $y \to \infty$ and require $\Omega(\infty) = \mathcal{I}$, eqn. (3.79) now takes the form

$$h_{C(\infty,x)}[\mathbf{A}^{\Omega}] = h_{C(\infty,x)}[\mathbf{A}]\ \Omega(x)\ . \tag{3.80}$$

One can now formally define a *local* one form potential $\mathcal{A}(x)$ as

$$\mathcal{A}(x) \equiv \int \mathcal{D}C\ \bar{\mathbf{A}}_{C(\infty,x)} \tag{3.81}$$

where,

$$\bar{\mathbf{A}}_{C(\infty,x)} \equiv h_{C(\infty,x)}[\mathbf{A}]\,(\mathbf{A} + d)\,(h_{C(\infty,x)}[\mathbf{A}])^{-1}. \tag{3.82}$$

The path integral symbol at this point is formal, and is meant to stand for some sort of averaging over all paths originating at asymptopia and extending upto the field point x. It is then easy to see that, under gauge transformations of $\mathbf{A}$ and using eqn. (3.80),

$$\mathcal{A}^{\Omega}(x)\ =\ \mathcal{A}(x)\ . \tag{3.83}$$

What has not been determined yet is what constraint replaces the divergencefree condition (3.11) for the Yang Mills one form $\mathcal{A}$, so that the physics of these local gauge-free one forms can be explored more thoroughly without gauge encumbrances. One also envisages application of these ideas to general relativity formulated as a gauge theory of Lorentz (or Poincaré) connection.

3.7 Summary

In this chapter, it has been shown that abelian gauge theory can be cast as a gauge-free theory using the *gauge-free* formalism. Then this formalism is used to resolve the issue of gauge dependence of CW potential for scalar QED and the Higgs to Vector mass ratio is explicitly calculated. Moreover, if one further demands reparametrization independence of CW potential then one should employ VD method to calculate CW potential in a gauge-free theory. In the next chapter an extension of gauge-free formalism for non-abelian gauge theory will be described and CW effective potential will be calculated for a reduced electroweak theory where all the dynamical fields are gauge-free.

Chapter 4

Gauge-free Electroweak Theory

In this chapter, I will describe an extension of gauge-free approach for the non-abelian gauge theory namely $SU(2) \times U(1)$ theory. This is done following a recent reformulation of the scalar-vector sector of standard electroweak theory (without any Higgs potential) in terms of manifestly $SU(2)_W$ gauge invariant variables proposed by Faddeev et. al. [66,67]. I will briefly describe this new variable approach first, then it will be shown following the gauge-free prescription for abelian case in the earlier chapter the action of electroweak theory can be made manifestly free of the residual $U(1)_{em}$ gauge transformations as well. Functional evaluation of the one loop gauge-free effective Higgs potential is shown to precisely cancel effects due to the local functional measure of the Higgs field found earlier. The implications for the classical interpretation of the Higgs scalar as the dilaton field in a background conformal gravity theory are discussed. The Higgs scalar is shown to radiatively acquire a one loop vacuum expectation value which gives masses to the W and Z bosons but not to photons, without any notion of 'spontaneous gauge symmetry breaking' appearing anywhere. This chapter is based on [66, 67, 87].

4.1 Gauge-free reformulation of $SU(2) \times U(1)$ theory

Recently a reformulation of the the Higgs-vector boson sector of standard electroweak gauge theory has been proposed [66, 67, 90, 91], together with a novel interpretation of the Higgs scalar. Beginning with a standard $SU(2) \times U(1)$ gauge theory augmented by a Higgs action, but *without* a Higgs potential, field redefinitions are performed whereby all fields are rendered completely *inert* under $SU(2)$ gauge transformations. As a result, the theory has a residual gauge invariance under $U(1)_{em}$ with the photon being left over as the only massless gauge potential transforming as a gauge connection.

This section has two parts: the first subsection summarizes the field redefinitions proposed in [66], making them local $SU(2)$ gauge-free. In the second subsection, we introduce further redefinitions so that all field variables are manifestly $SU(2) \times U(1)$ gauge-free.

4.1.1 Local $SU(2)$ Gauge-free fields

The standard electroweak theory contains the $SU(2)$ gauge field $\mathbf{B}_\mu = B_\mu^a t^a, a = 1, 2, 3$, the $U(1)$ gauge field Y_μ and the Higgs $SU(2)$ doublet Φ, transforming under $SU(2) \times U(1)$ gauge tranformations as

$$\begin{aligned} \mathbf{B}_\mu \to \mathbf{B}_\mu^{(\Omega)} &= \Omega \mathbf{B}_\mu \Omega^{-1} - \partial_\mu \Omega \Omega^{-1} \\ Y_\mu \to Y_\mu^{(\omega)} &= Y_\mu + \partial_\mu \omega \\ \Phi \to \Phi^{(\Omega)} = \Phi\Omega \quad &, \quad \Phi \to \Phi^{(\omega)} = \Phi \exp i\omega. \end{aligned} \tag{4.1}$$

The Lagrange density for this sector of the electroweak theory is

$$\mathcal{L} = (\nabla_\mu \Phi \nabla^\mu \Phi) - \frac{1}{4g^2} tr \mathbf{B}_{\mu\nu}^2 - \frac{1}{4g'^2} Y_{\mu\nu}^2, \tag{4.2}$$

where

$$\nabla_\mu \Phi = \partial_\mu \Phi + \frac{i}{2} Y_\mu \Phi + B^a_\mu t^a \Phi \tag{4.3}$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + \epsilon_{abc} B^b_\mu B^c_\nu \tag{4.4}$$

$$Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu \tag{4.5}$$

with

$t^a = \frac{i}{2}\tau^a$, τ^a Pauli matrices, g and g' are the coupling constants.

The essential feature of this approach is the 'polar decomposition' of the complex scalar doublet into two parts.

$$\Phi = \frac{1}{\sqrt{2}} \rho \chi, \tag{4.6}$$

where, ρ is a real positive scalar (modulus) field is completely gauge-inert, while the 'phase' part χ carries all the gauge transformation properties of Φ. Now, one introduces the matrix

$$s = \begin{pmatrix} \chi_1 & -\bar{\chi}_2 \\ \chi_2 & \bar{\chi}_1 \end{pmatrix} \tag{4.7}$$

with a normalisation $(\chi, \chi) = \bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2 = 1$ (which defines the group manifold of $SU(2)$), it is easy to verify [66] that s is unimodular and unitary. It stands to reason that $s \in SU(2)$ so that under an $SU(2)$ gauge transformation

$$s \to s^{(\Omega)} = \Omega s. \tag{4.8}$$

However, since χ_i and $\bar{\chi}_i$ have different weak hypercharges, under a $U(1)$ gauge transformation, $s \to s^{(\omega)} = s e^{i\omega\tau_3}$

The covariant derivative of s is given by

$$\nabla_\mu s = \partial_\mu s + \frac{i}{2} Y_\mu s \tau_3 + \mathbf{B}_\mu s \ . \tag{4.9}$$

Defining the new Yang Mills triplet $\mathbf{W}_\mu = W^a_\mu t^a$ as

$$\mathbf{W}_\mu \equiv s^\dagger \left(\mathbf{B}_\mu + \partial_\mu\right) s \tag{4.10}$$

it is easy to see that under an $SU(2) \times U(1)$ gauge transformations,

$$\begin{aligned} \mathbf{W}^{(\Omega)}_\mu &= \mathbf{W}_\mu \\ \mathbf{W}^{(\omega)}_\mu &= e^{-i\omega\tau_3} \mathbf{W}_\mu e^{i\omega\tau_3} + i\tau_3 \partial_\mu \omega. \end{aligned} \tag{4.11}$$

These fields $\mathbf{W}_\mu$ are thus explicitly $SU(2)$ gauge invariant, even though they have non-trivial $U(1)$ gauge transformations. One defines the linear combinations

$$\begin{aligned} Z_\mu &\equiv Y_\mu + W^3_\mu \\ A_\mu &\equiv \frac{1}{g^2 + g'^2} \left(g'^2 W^3_\mu - g^2 Y_\mu\right) , \end{aligned} \tag{4.12}$$

where, the vector field Z_μ is manifestly $SU(2) \times U(1)$ invariant, and the A_μ field transforms under $U(1)$ as $A^{(\omega)} = A_\mu - 2\partial_\mu \omega$. The charged combinations $W^\pm_\mu \equiv W^1_\mu \tau_1 \pm W^2_\mu \tau_2$ are $SU(2)$ gauge invariant, but carry indicated charges under $U(1)_{em}$ gauge transformations under which A_μ transforms as the photon field, with the electronic charge being $e^{-2} \equiv g^{-2} + g'^{-2}$.

The entire gamut of field redefinitions leave only a $U(1)_{em}$ gauge theory with the photon field being the sole gauge connection; the Yang Mills connections have been rendered entirely gauge free under $SU(2)$ gauge transformations, and behave as charged (or neutral) vectorial matter fields under electromagnetism. The theory is described by the

Lagrange density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{\rho^2}{8}(Z_\mu^2 + W_\mu^+ W^{\mu,-}) - \frac{1}{4g^2}(\nabla_\mu W_\nu^+ - \nabla_\nu W_\mu^+)(\nabla^\mu W^{\nu,-} - \nabla^\nu W^{\mu,-})$$
$$- \frac{1}{4(g^2+g'^2)}Z_{\mu\nu}^2 - \frac{1}{4e^2}A_{\mu\nu}^2 - \frac{2}{4g^2}\, H_{\mu\nu}(A^{\mu\nu} + e^2 Z^{\mu\nu}) - \frac{1}{4g^2}H_{\mu\nu}^2, \quad (4.13)$$

where

$$\begin{aligned} Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 \\ H_{\mu\nu} &= \frac{1}{2i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^-) \\ \nabla_\mu W_\nu^\pm &= \partial_\mu W_\nu^\pm \pm i W_\mu^3 W_\nu^\pm. \end{aligned} \quad (4.14)$$

The functional measure for path integral now becomes,

$$d\mu = \prod_x \rho^3 d\rho\, dZ_\mu dW_\mu^+\, dW_\mu^-\, dA\, ds, \quad (4.15)$$

where

$\prod_x ds$ is volume of the gauge group, which is completely separated from measure without any gauge fixing.

The question is : does this theory generate a mass for the $W_\mu^\pm$, the Z_μ and the ρ fields, as is achieved in the standard Higgs mechanism by means of a Higgs potential with degenerate minima ? In other words, what is the scale of the vacuum expectation value ρ here, since there is no Higgs potential to generate that scale ? In [66] the Higgs modulus field ρ is interpreted, because of the appearance of the local $\rho^3\mathcal{D}\rho$ factor that appears in the partition functional integral, as the dilaton of a background conformally flat spacetime. The vacuum value of ρ is related to its asymptotic value in this spacetime, and is supposed to be determined cosmologically. Excitations around this vacuum value are of course to be interpreted as a new massless scalar field. So, a new perturbative mechanism to produce vector boson masses becomes available, as an alternative to the standard Higgs mechanism. The issue is: does this mechanism survive quantization ? Before addressing this question let us consider the residual gauge invariance of the theory.

4.1.2 $U(1)$ Gauge-free fields

In this subsection we will see even though the theory has a residual $U(1)_{em}$ gauge invariance, one can rewrite it explicitly in terms of entirely *gauge free* variables [79, 80], so that no gauge fixing is at all necessary to evaluate the partition function. Again, begin by a radial decomposition of the charged weak vector boson fields

$$W^{\pm}_{\mu} = w_{\mu} \exp \pm i\theta^{(\mu)} \; , \textit{no sum on } \mu \tag{4.16}$$

which implies that under $U(1)$ gauge transformations

$$[w_{\mu}]^{(\omega)} = w_{\mu} \; , \; [\theta^{(\mu)}]^{(\omega)} = \theta^{(\mu)} + 2\omega \; . \tag{4.17}$$

One can think of w^{μ} as the component of the charged vector boson carrying only the *spin* while $\theta^{(\mu)}$ is the *charge* mode, thus affecting a 'separation of the charge and spin modes'. It follows that

$$\nabla_{\mu} W^{\pm}_{\nu} = \left[\partial_{\mu} w_{\nu} \pm i w_{\nu} \left(\partial_{\mu}\theta^{(\nu)} - A_{\mu} - \frac{e^2}{g'^2} Z_{\mu} \right) \right] e^{i\theta^{(\nu)}} \tag{4.18}$$

The only quantity sensitive to $U(1)_{em}$ gauge transformations is the phase factor; the gauge transformation parameter cancels between the θ and A fields within the parentheses. However, all fields can now be expressed explicitly in terms of entirely gauge free fields except the phase factor which indeed must carry the full burden of gauge transformations, through the field redefinitions [79, 80].

$$\begin{aligned}\Theta^{(\mu)} &\equiv \theta^{(\mu)} - 2\mathrm{a} \\ \mathbf{A}_\mu &\equiv A_\mu - 2\partial_\mu \mathrm{a} \ ,\end{aligned} \tag{4.19}$$

where, $\mathrm{a}(x) \equiv \int d^4x' G(x-x')\partial' \cdot A(x')$ is a scalar field giving the longitudinal mode of $A_\mu(x)$ with $G(x-x')$ being the Green's function for the d'Alembertian. As a result of these field redefinitions, the kinetic energy of the charged vector bosons (and indeed the entire Lagrange density) is rendered free of *all* local gauge transformations. The former assumes the form [87]

$$\begin{aligned}\nabla_{[\mu} W^+_{\nu]} \nabla^{[\mu} W^{\nu]-} &= \frac{1}{2}\sum_{\mu,\nu=0}^{3} \{[\partial_\mu w_\nu \partial^\mu w^\nu + w_\nu(\tilde{A}^{(\nu)}_\mu + \frac{e^2}{g'^2} Z_\mu) w^\nu (\tilde{A}^{\mu(\nu)} + \frac{e^2}{g'^2} Z^\mu)] \\ &- \cos\Theta^{(\mu\nu)} [\partial_\mu w_\nu \partial^\nu w^\mu + w_\nu (\tilde{A}^{(\nu)}_\mu + \frac{e^2}{g'^2} Z_\mu) w^\mu (\tilde{A}^{(\mu)\nu} + \frac{e^2}{g'^2} Z^\nu)] \\ &- \sin\Theta^{(\nu\mu)} [\partial_\nu w_\mu w^\nu (\tilde{A}^{(\nu)\mu} + \frac{e^2}{g'^2} Z^\mu)]\}\end{aligned} \tag{4.20}$$

where, $\tilde{A}^{(\mu)}_\nu \equiv \mathbf{A}_\nu - \partial_\nu \Theta^{(\mu)}$ and $\Theta^{(\mu\nu)} \equiv \Theta^{(\mu)} - \Theta^{(\nu)}$

Since the $W^\pm$ carry electric charge ± 1, one can in fact choose the phases $\Theta^{(\mu)}$ to be the same, independent of the μ, without any loss of generality. With this choice, eq. (4.20) simplifies considerably

$$\begin{aligned}\nabla_{[\mu} W^+_{\nu]} \nabla^{[\mu} W^{\nu]-} &= w^2_{\mu\nu} + \frac{1}{2} w^2 \left(\mathbf{A} - \partial\Theta + \frac{e^2}{g'^2} Z\right)^2 \\ &- \frac{1}{2}\left[w \cdot \left(\mathbf{A} - \partial\Theta + \frac{e^2}{g'^2} Z\right)\right]^2 \ ,\end{aligned} \tag{4.21}$$

where, $w_{\mu\nu} \equiv 2\partial_{[\mu}w_{\nu]}$. This equation exhibits the $U(1)_{em}$ gauge freedom of the fields manifestly, and also shows explicitly the coupling of the charged vector boson modes to the physical $U(1)$ photon vector potential [87].

One may observe that the physical photon field $\mathbf{A}$ is invariably accompanied in eq. (4.21) by the *physical* longitudinal mode $\partial\Theta$, so that a further redefinition of the type $\mathbf{A}' \equiv \mathbf{A} + \partial\Theta$ may be contemplated. This removes the physical phase field Θ completely from the action, in lieu of the field $\mathbf{A}'$ which is no longer spacetime transverse, but now has three independent physical degrees of freedom. However, $\mathbf{A}'$ remains massless classically.

4.2 Higgs as a Dilaton

In this section I will briefly highlight the dilaton aspect of Higgs given by the authors [66, 67,90,91]. In the last section we have expressed the bosonic sector of standard electroweak Lagrangian in terms of manifestly gauge-inert variables. We have seen in eq. (4.13) the gauge invariant modulus of the Higgs field couples to the W and Z bosons in the standard manner and can indeed provide masses to them if it picks up a vacuum expectation value. Unlike in the standard Higgs mechanism which employs a(n) (unstable) Higgs potential for this purpose, the Higgs degree of freedom here is given a novel interpretation: it is the dilaton in a conformally flat background gravity theory, such that its vacuum value is fixed by its large distance (cosmological) behaviour. This interpretation derives from the altered functional measure (4.15) for the Higgs modulus field because of field redefinitions: it is *local* in nature. Similar ideas of interpreting the Higgs field as a dilaton in a model which resembles Nambu-Jona Lasinio models of dynamical chiral symmetry breaking have been advanced by Foot et. al. [92]. In [66] the authors proposed to interpret ρ^2 as conformal factor of the metric in space-time

$$g_{\mu\nu} = \rho^2 \eta_{\mu\nu}. \tag{4.22}$$

Then it was shown that the Lagrangian (4.13) can be rewritten in the following manifestly generally covariant form,

$$\mathcal{L}_{\mathrm{WS}} = \sqrt{-\mathcal{G}}\left\{\frac{1}{16\pi G}(R-2\Lambda)+\mathcal{L}_M\right\} \tag{4.23}$$

with the matter Lagrangian $\mathcal{L}_M$

$$\mathcal{L}_M = -\frac{1}{4}\mathcal{G}^{\mu\rho}\mathcal{G}^{\nu\sigma}\vec{G}_{\mu\nu}\vec{G}_{\rho\sigma} - \frac{1}{4}\mathcal{G}^{\mu\rho}\mathcal{G}^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \kappa^2(g^2+g'^2)\mathcal{G}^{\mu\nu}Z_\mu Z_\nu - \kappa^2 g^2\mathcal{G}^{\mu\nu}W_\mu^+W_\nu^- \tag{4.24}$$

The parameters which are introduced are given by, $G = 3/(8\pi\kappa^2)$ and $\Lambda = (9\lambda)/(8\pi G)$. The result (4.23), (4.24) re-interprets the electroweak theory as a generally covariant gravity theory with massive vector fields Z and $W^\pm$ and the (massless) photon A_μ.

The scalar curvature is given by

$$R = \frac{1}{6}\frac{\partial_\mu\rho\partial_\mu\rho}{\rho^4} + \text{divergence}. \tag{4.25}$$

Finally, the Christoffel's symbols, entering the definition of the field strengths$A_{\mu\nu}$, $Z_{\mu\nu}$, $W_{\mu\nu}^\pm$ via covariant derivatives, cancel due to antisymmetry. Thus the Lagrangian can be rewritten in manifestly covariant form. In the ground state, the massive vector fields Z_μ and $W_\mu^\pm$ and the photon field A_μ all must vanish. Consequently the ground state is determined by minimizing the gravitational contribution (4.23). The choice of particular value of Λ^2 corresponds to the particular choice of the vacuum.

It was also argued in [67] that ρ^2 must acquire a non-zero vev to maintain asymptotic flatness.

$$\rho^2|_{r\to\infty} \to \Lambda^2 \tag{4.26}$$

and the parameter Λ^2 enters as a new parameter of the model.

Thus, in this new interpretation the nonzero expectation value for the ρ^2 can be invoked without the Higgs potential. The fundamental question which remains, is the origin of the excitations for the field ρ. In [67], two possible answers had been given. In the first case where there is a degenerate vacuum it is interpreted as a kind of Goldstone mode and in the equivalent conformal gravity model it is interpreted as a dilaton. Indeed, these ideas are a fascinating alternative to the standard interpretation of the Higgs boson in electroweak theory. The issue is : what happens under quantization ? The possibility that the dilaton interpretation may *not* remain unaltered has already been hinted at in [86]. Further, Ilderton et. al. [64] have pointed out that the gauge invariance of the theory under $SU(2)$ may suffer from the existence of Gribov copies non-perturbatively, unless the background conformally flat spacetime is asymptotically flat.

In the next section I'll consider the perturbative loop expansion of the Higgs sector of the theory by explicitly computing radiative effects without any gauge fixing. In the calculation of the one loop effective potential of the theory, it is found that the local terms in the functional measure in fact do *not* survive quantization : they are exactly cancelled by genuine radiative terms arising from the functional determinants in the one loop effective potential [87]. In other words, classical scale invariance is explicitly broken at one loop due to renormalization effects, resulting in a nontrivial one loop trace anomaly.

4.3 Effective Potential and Higgs Mass

I now turn to the question of the perturbative quantum behaviour of the theory. To study this question, let's consider the one loop effective potential of the theory given by (4.13), and investigate if it has a nontrivial minimum driven by infrared instabilities [21]. Lets do not use the new interpretation given in [66]. The issue then amounts to investigating the possibility of radiative generation of a Higgs potential (not just a mass term), with a self-coupling *determined* in terms of the gauge couplings. Since we are interested in only one-loop calculations we drop all the cubic and quartic interaction terms from the Lagrangian in (4.13). This is easy to justify by simply drawing all possible one loop graphs with ρ external lines: none of them have the vertices that are being discarded here. One is thus dealing with the truncated Lagrangian relevant for one-loop calculations,

$$\begin{aligned} \mathcal{L}_{trun} &= \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{\rho^2}{8}(Z_\mu^2 + w_\mu^2) - \frac{1}{4g^2}w_{\mu\nu}^2 \\ &- \frac{1}{4e^2}\mathbf{A}_{\mu\nu}^2 - \frac{1}{4(g^2+g'^2)}Z_{\mu\nu}^2 \ , \end{aligned} \tag{4.27}$$

where, the physical photon field is divergenceless $\partial \cdot \mathbf{A} = 0$ as discussed in [79, 80].

The generating functional of all Feynman graphs is given by

$$\begin{aligned} Z[J, \mathbf{J}_A, \mathbf{J}_Z, \mathbf{J}^+, \mathbf{J}^-] = & \int d\mu \ \delta[\partial \cdot \mathbf{A}] \ \exp i \int d^4x \, \mathcal{L}_{trun} \\ & . \ \exp i \int d^4x (J\rho + \mathbf{J}_{\mathbf{A}}.\mathbf{A} + \mathbf{J}_Z.Z + \mathbf{J}_w.w + J_\Theta \Theta) \end{aligned} \tag{4.28}$$

where the measure

$$d\mu = Det\rho^3 \mathcal{D}\rho\mathcal{D}\mathbf{A}\mathcal{D}a\mathcal{D}Z\mathcal{D}w\mathcal{D}\Theta\mathcal{D}s \tag{4.29}$$

where $\mathcal{D}s$ is the $SU(2)$ group volume and $\mathcal{D}a$ is that of the $U(1)_{em}$. Since the action is manifestly independent of the fields characterizing these group volumes, they can be factored out of the functional integral and discarded as irrelevant multiplicative factors. Note however the *local* measure associated with the Higgs field ρ; this implies that ρ is not an usual scalar field, as pointed out in [66]. It appears to behave like a *dilaton* field which might acquire a vacuum value from cosmological sources. However, under perturbative quantization at the one loop level, this local measure undergoes a precise cancellation with terms appearing in the quantum effective action.

The one loop effective potential is calculated functionally from the generating functional using saddle-point method shifting the positive real scalar field

$$\rho \to \rho_0 + \rho$$

where ρ_0 is a spacetime constant chosen to be the saddle point.the one-loop effective potential becomes

$$\begin{aligned}
V_{eff}(\rho_0) &= 3i \int \frac{d^4k}{(2\pi)^4} \ln(\rho_0) - \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \ln(-k^2) \\
&- \frac{i}{2g^2} \int \frac{d^4k}{(2\pi)^4} \ln det \left[\eta^{\mu\nu} \left(-k^2 + g^2 \frac{\rho_0^2}{4} \right) + k^\mu k^\nu \right] \\
&- \frac{i}{2(g^2+g'^2)} \int \frac{d^4k}{(2\pi)^4} \ln det \left[\eta^{\mu\nu} \left(-k^2 + (g^2+g'^2) \frac{\rho_0^2}{4} \right) + k^\mu k^\nu \right] \\
&- \frac{i}{2e^2} \int \frac{d^4k}{(2\pi)^4} \ln det \left[-\eta^{\mu\nu} k^2 \right] \ , \qquad (4.30)
\end{aligned}$$

The determinants inside the integrals are really in Minkowski space. They are easily evaluated to obtain

$$\begin{aligned}
V_{eff}(\rho_0) &= 3i \int \frac{d^4k}{(2\pi)^4} \ln(\rho_0) - \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \ln(-k^2) \\
&- 3i \int \frac{d^4k}{(2\pi)^4} \ln \left(k^2 - g^2 \frac{\rho_0^2}{4} \right) - i \int \frac{d^4k}{(2\pi)^4} \ln(\rho_0) \\
&- \frac{3i}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left(k^2 - (g^2+g'^2) \frac{\rho_0^2}{4} \right) - 2i \int \frac{d^4k}{(2\pi)^4} \ln \rho_0 \ . \qquad (4.31)
\end{aligned}$$

It is not difficult to see that the Jacobian contribution from the functional measure discerned in [66] (the 1st term in (4.31)) is exactly cancelled by two terms coming from the neutral and charged vector boson operators (the fourth and seventh terms respectively in (4.31)). One way to understand this is that, in the quantum effective action, scale invariance is violated explicitly because in the quantum theory, renormalization invariably introduces a scale parameter. Because of this conformal anomaly, the 'dilaton' interpretation of the Higgs field needs to be reexamined. From a phenomenological standpoint this is not necessarily undesirable, since the theory must acquire a mass scale that sets the range of the weak interactions. However, whether or not the dilaton interpretation of the original proposal can be retained is not obvious at this point.

Introducing renormalizing counterterms and Wick rotating the contour of integration, the effective potential becomes

$$\begin{aligned} V_{eff}(\rho_0) &= \frac{B}{2}\rho_0^2 + \frac{C}{4!}\rho_0^4 \\ &+ \frac{3}{2}\int \frac{d^4k_E}{(2\pi)^4} \ln\left(k_E^2 + (g^2+g'^2)\frac{\rho_0^2}{4}\right) + 3\int \frac{d^4k_E}{(2\pi)^4} \ln\left(k_E^2 + g^2\frac{\rho_0^2}{4}\right) \end{aligned} \quad (4.32)$$

where B, C are the usual mass and coupling-constant renormalization counter terms. The counterterms are determined using the same renormalization scheme used in the last chapter using eqns (3.32).

The renormalized effective one loop Higgs potential is now given by

$$\begin{aligned} V_{eff}(\rho_0) &= \frac{27(g^2+g'^2)^2M^2\rho_0^2}{512\pi^2} + \frac{27g^4M^2\rho_0^2}{256\pi^2} \\ &+ \left(\frac{3(g^2+g'^2)^2\rho_0^4}{1024\pi^2} + \frac{3g^4\rho_0^4}{512\pi^2}\right)\left(\ln\frac{\rho_0^2}{M^2} - \frac{25}{6}\right) , \end{aligned} \quad (4.33)$$

where the parameter M serves as a scale of the theory.

Henceforth we drop the subscript 0 on ρ_0. The plot of effective potential (Fig. [4.1]) shows that it has three extrema in the physically interesting region i.e. in the region where $\langle\rho\rangle > 0$. Apart from a local minimum at the origin the potential possesses a maximum at about $\langle\rho\rangle \simeq 1.98M$. The true minimum is around $\langle\rho\rangle \simeq 5.34M$ for which $\log(\langle\rho\rangle/M) \simeq 1.67$.

The mass generated for the $W^{\pm}$ bosons is $(1/2)g\langle\rho\rangle$, while for the Z bosons is $(1/2)(g^2+g'^2)^{1/2}\langle\rho\rangle$. Thus, since $\langle\rho\rangle = 246$ Gev reproduces the observed W and Z boson mass spectrum, this implies that one must make the choice $M \sim 46$ Gev This also corresponds to a ρ boson mass, computed as the curvature of the effective potential at the absolute

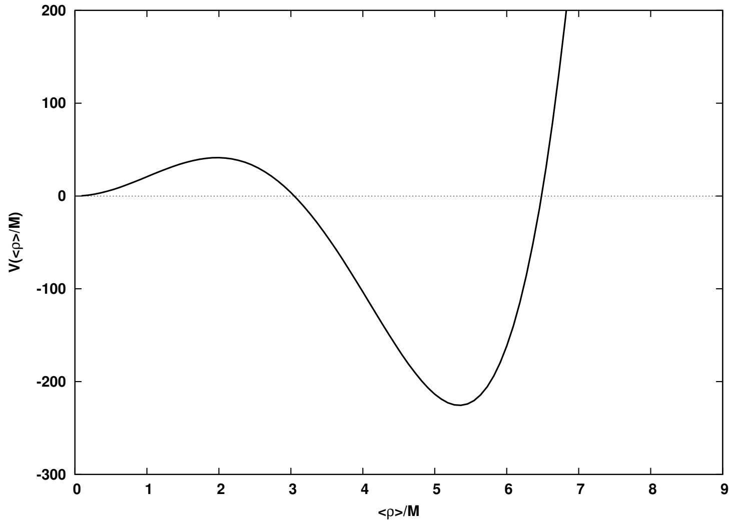

Figure 4.1: Plot of the Effective Potential as a function of $\frac{\langle\rho\rangle}{M}$.

minimum [87],

$$m_H^2 = 9\frac{(g^2+g'^2)^2+2g^4}{256\pi^2}\left(3M^2+\langle\rho\rangle^2\, ln\frac{(\langle\rho\rangle)^2}{M^2}-3\langle\rho\rangle^2\right) \tag{4.34}$$

The mass of the ρ field is computed to be $\sim$ 6.9 Gev : perhaps too light to be phenomenologically relevant as a standard Higgs field. Especially after the discovery of Higgs like boson at the LHC in 2012 it would be hard to identify ρ with a physical Higgs boson. This mechanism of mass generation must therefore be thought of as a toy model at this stage. Another interesting observation is, the Higgs mass computed here surprisingly very close to the Linde-Weinberg lower bound [25, 26] found earlier excluding the fermion contribution to the renormalization group improved potential. The Higgs mass is however lower than the estimated bound computed earlier using CW potential [29]. This is probably due to the fact no Higgs potential is included here to compute the CW potential.

The Vilkovisky-DeWitt version of gauge-free electroweak theory does not give any new contribution in this case since Lagrangian relevant for one-loop effective potential has only fields which are gauge-inert. Thus all the metrics of the configuration space are independent of any field leading to vanishing Christoffel connections with the gauge part of the connections identically zero. Thus, this formalism will be extremely useful to find out lower bound on Higgs mass in a completely gauge-free way including the fermionic sector of standard electroweak theory which has only been done for a toy model so far [60] neglecting all charged bosons.

4.4 Discussions

Even though it was not phrased in this language, the emergence of a one-loop scalar potential from the gauge sector of a classically scale invariant theory signals a conformal (trace) anomaly : $\langle T^{\mu}_{\mu} \rangle \neq 0$, originating in the renormalization scale M which is needed to renormalize the parameters of the theory. The conformal anomaly arises here as a result of infrared quantum instabilities. In the interpretation of ref. [66], the dilaton field ρ has acquired a potential which explicitly breaks scale invariance. The issue that arises immediately is : how does this modify the background conformally flat gravity theory discerned in [66] ?

This was first discovered by Coleman and Weinberg [21] four decades ago. The effective potential (4.33) coincides with the corresponding Coleman-Weinberg result for vanishing scalar self-coupling. (See also [93, 94] for more recent work on this mechanism in the present context.) The basic *difference* in this work from the one done before is this is a gauge-free formulation of a gauge theory. Consequently, the understanding of the Higgs mechanism, not as a 'spontaneous breakdown of gauge symmetry' which is anyway impossible within this gauge-free paradigm, but as a *gauge invariant* mechanism using instabilities (classical or quantum) in the theory to generate gauge boson masses, is new. The only symmetry that is violated in our case explicitly, due to radiative corrections, is global scale invariance. I think this is a better description of the physics involved.

An important issue is that of 'naturalness' of the scalar sector of the theory. Apart from the lacuna discussed above, this does *not* appear to be an issue, since there is no scalar self coupling. The renormalization scale M cannot arbitrarily slide to the GUT or the Planck scale without ruining the vector boson mass spectrum which is extremely well determined experimentally. The scalar mass is then *constrained* to be comparatively lower than the vector boson masses, and thus never requires fine tuning of dimensionless parameters.

The one loop effective Higgs potential generated in the theory has the prospect of supplying the observed spectrum of weak vector boson masses and also a Higgs mass. However, with only one tunable parameter, it is impossible to reproduce both a phenomenologically viable weak vector boson spectrum and also meet the stringent LEP lower bound on the Higgs mass. On the positive side, the theory in this paper has no Higgs self-coupling parameter, and the mass spectrum is completely determined by the gauge couplings with appropriate choice of the renormalization scale. It is unfortunate that a theory with such a high predictive power is not more useful phenomenologically.

The key question not addressed in this work is of course the issue of fermion masses. One could add to the Lagrange density (4.2) fermion-gauge and Yukawa coupling terms where the Higgs vacuum expectation value produces fermion masses as in the standard electroweak theory. Gauge-free prescription for fermions has already been considered in [79]. Incorporating Yukawa couplings within a somewhat different gauge-free scheme has already been explored [95]. The radiatively generated Higgs vacuum expectation value then can be a source of fermion masses in the standard manner. The real challenge is to produce the fermion masses dynamically from gauge interactions at the quantum level. Chiral symmetry prevents this from happening radiatively, so that the source of fermion masses might have to be nonperturbative.

4.5 Summary

In this chapter the field redefinitions employed in [66] which render all fields free of $SU(2)$ gauge transformations are discussed briefly. Then these gauge-inert fields are also made free of the residual $U(1)_{em}$ gauge invariance, following the earlier works [79, 80]. The new theory is thus also manifestly $U(1)$ invariant and does not need to fix any gauge. This is followed with a computation of the one loop effective Higgs potential, using the same local measure discerned in [66]. It is shown how the effects of the local measure are cancelled by radiative terms in the effective potential. The effective Higgs potential develops an absolute minimum away from the origin in Higgs field space, and the spectrum of particles around this minumum are given. The vacuum again gets stabilized by generating masses to the scalar and vector bosons. The concluding section contains discussions of main results and possibility of accommodating fermions in this picture. In the next chapter radiative effects of a theory where spin-2 graviton field is minimally coupled to a Higgs field are studied.

Chapter 5

Infrared Instabilities in Graviton-Higgs Theory

In this chapter quantum vacuum instability of a theory containing spin-2 gravitons coupled to Higgs field is considered. Gravitons minimally coupled to a massless scalar field in a background Minkowski spacetime is shown to develop an instability in their propagators in presence of a spacetime-independent Higgs field background. The instability is indicated in the graviton propagator due to appearance of a tachyonic pole. The one loop effective potential for this theory is shown to develop an infrared instability in the form of acquiring an imaginary part, which can be traced to the tachyonic pole in the graviton propagator. This instability is analogous to the finite temperature infrared instability of a gas of gravitons coupled to fermions found by Gross et. al. [75], even though it already exists at zero temperature; it is thus reminiscent of the Jeans instability thought to be at the heart of structure formation in the early Universe. A finite temperature analysis of the effective potential at one loop shows that in the high temperature limit, the zero-temperature instability is in fact *reinforced* by finite temperature effects. In the low temperature limit, the finite temperature contribution to the imaginary part of the effective potential exhibits a damped oscillatory behaviour; all thermal effects are damped out as the temperature vanishes, consistent with the zero-temperature result. This chapter is based on ref. [96].

5.1 Jeans instability and tachyons at finite temperature

Stability of flat spacetime under quantum gravitational fluctuations has been studied extensively since the incipient work on the Euclidean path integral formulation of gravity [97, 98]. Employing a saddle-point approximation in the Euclidean partition function, Gross et. al. [75] show that flat space is stable at zero temperature both classically and quantum mechanically under perturbative qantum fluctuations of Euclidean 4-space. However, when the system is kept in contact with a heat bath, the self-gravitating system becomes unstable, both by itself (vacuum) and in the presence of massless spinor fields. This is unlike in the case of an electrical plasma where charge carriers produce a screening effect over the fluid (Debye screening). This distinct feature of gravity is the source of several instabilities. In classical Newtonian gravity one such instability occurs when we treat the Universe as being filled with a static, homogeneous nonrelativistic fluid. For long-wavelength gravitational perturbations the system develops an instability. This instability is very often be related to classical Jeans instability [69]. Jeans' Universe is filled with a non viscous fluid with mass density ρ, pressure p, and velocity $\vec{v}$ satisfying the usual continuity and Navier-Stokes equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \qquad Eqn.\, of\, continuity \tag{5.1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla \mathbf{p} + \mathbf{g} \qquad Euler\, equation \tag{5.2}$$

The gravitational field **g** satisfies the following equations

$$\begin{aligned} \nabla \times g &= 0 && (5.3) \\ \nabla . g &= -4\pi G\rho \qquad gravitational\, field\, eqn. && (5.4) \end{aligned}$$

To analyze the dynamics of the system one considers small perturbations ρ_1, p_1, v_1, g_1 around an equilibrium configuration which is taken to be a homogeneous, static fluid. The effect of gravitation are also ignored in the unperturbed solution. The density perturbation satisfies the following equation of motion

$$\frac{\partial^2 \rho_1}{\partial t^2} - V_s^2 \nabla^2 \rho_1 = 4\pi G \rho \rho_1 \tag{5.5}$$

where V_s is the speed of sound in the fluid. The solution of the equation for the density perturbation gives the usual plane waveform except the fact the right hand side of eq. (5.5) has a mass like term with wrong sign.

$$\rho_1 \propto \exp\{i\mathbf{k}.\mathbf{x} - i\omega t\} \tag{5.6}$$

leading to a dispersion relation,

$$\omega^2 = v_s^2 \mathbf{k^2} - 4\pi G\rho \tag{5.7}$$

where ω is imaginary for wave numbers below the critical value

$$k_J = \left(\frac{4\pi G\rho}{v_s^2} \right)^{\frac{1}{2}} \tag{5.8}$$

So the perturbation ρ_1 has a runway mode below this value and can result in an exponential growth or decay of the disturbances [99].

In a classic work, Gross et. al. [75] consider a gas of gravitons in thermal contact at finite spatial volume and interacting with thermally excited fermions. Integrating over the fermionic degrees of freedom, the graviton is shown to acquire an imaginary mass leading to a tachyonic instability. The presence of a heat bath as a source for inducing thermal fluctuations is crucial in this work, as is evident from the fact that the induced masses have power law dependence on the temperature.

In ref. [75] it is shown that flat space is stable both quantum mechanically and classically under small perturbations due to gravitons and spinors at zero temperature. They also showed that when a gas of gravitons is kept at finite temperature, an instability, stemming from thermally generated graviton modes, appears. This induces a Jeans-like instability since the thermally excited modes interact with gravitons. In a theory with gravitons coupled to thermally excited fermions, the one-loop graviton propagator contains a tachyonic term [75] which can be interpreted as a mass term for the *longitudinal* mode of the graviton h_{00}; this mass is of magnitude

$$m_g^2 = -14/15\pi^3 G T^4 \tag{5.9}$$

The generation of an imaginary mass term when gravitons couple with thermally excited matter field is a generic feature. This also holds for the case of scalar fields at finite temperature. In fact the mass induced for the case of scalar field can again be traced from the self energy component $\Pi_{00,00}$. The longitudinal part of graviton h_{00} here again develops a mass term due to thermal fluctuations [100]. The value in this case is

$$m_g^2 = -\frac{4}{5}\pi^3 G T^4 \tag{5.10}$$

All these effects are purely thermal, implying that hot flat space is unstable and leads to Jeans instability.

5.2 Tachyonic mode at zero temperature propagator

In this section it will be shown that even at zero temperature, when gravitons couple to massless scalar field backgrounds which are spacetime independent, a similar instability appears, with the effective one-loop graviton propagator acquiring a tachyonic pole. This, in turn, leads to the appearance of an imaginary contribution in the one-loop effective action for a wide class of theories involving graviton and scalars, when evaluated using the Euclidean path integral saturated at a saddle point characterized by a flat Euclidean metric and a constant scalar background. This implies that graviton fluctuations coupled to constant scalar field background at $T = 0$ in flat spacetime plays a role similar to gravitons in a finite temperature heat-bath inducing an instability in flat spacetime. It is perhaps not inappropriate to state that this phenomenon has been an issue not particularly well-understood [45, 46, 48] as to how the instability resolves itself. It is not unlikely that the instability will involve decay to a de Sitter spacetime, but the actual proof of this has not been addressed here. Let us now go back to the case without an external heat bath, and consider a minimally coupled graviton-Higgs theory at $T = 0$. Assuming that the theory has a vacuum characterised by a constant value of the Higgs field which plays the role of an external background for the gravitons, an instability similar to the finite temperature case is discerned. In other words, the scalar field background plays the role of a heat bath which induces a tachyonic instability in the graviton modes even at zero temperature.

To evaluate the effective propagator for Higgs-graviton theory let us start from the Einstein-Hilbert action and expand it around the flat space i.e. employ weak field approximation.

$$\sqrt{-g}\mathcal{L}_g = \sqrt{-g}\left[\frac{1}{\kappa^2}R\right] \tag{5.11}$$

where $\kappa^2 = 16\pi G$; $g = det\, g_{\mu\nu}$ and $R = g^{\mu\nu}R_{\mu\nu}$

The Lagrangian for Gravity coupled to a massless scalar field,

$$\sqrt{-g}\mathcal{L} = \frac{1}{\kappa^2}R + \sqrt{-g}\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \sqrt{-g}V(\phi) \tag{5.12}$$

Expanding the metric around a flat background we get,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}, \tag{5.13}$$

where the fluctuations $h_{\mu\nu}$ are small, $|h_{\mu\nu}| < 1$ and $\eta_{\mu\nu} = diag(-1,1,1,1)$. For the decomposition (6.45), the inverse of the metric is

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa\, h^{\mu\nu} + \kappa^2\, h^{\mu}{}_{\lambda}\, h^{\lambda\nu} + \dots \tag{5.14}$$

Furthermore, the determinant of the metric, which will be needed in the following, will be given by:

$$(-g)^{\frac{1}{2}} = 1 + \kappa\frac{1}{2}h^{\alpha}{}_{\alpha} - \kappa^2\frac{1}{4}h^{\alpha}{}_{\beta}\, h^{\beta}{}_{\alpha} + \kappa^2\frac{1}{8}\left(h^{\alpha}{}_{\alpha}\right)^2 + \dots \tag{5.15}$$

The quadratic part of the Lagrangian from pure gravity sector is given by

$$\mathcal{L}_g = -\frac{1}{4}\partial_\lambda h_{\mu\nu}\partial^\lambda h_{\mu\nu} - \frac{1}{2}\partial_\mu h\partial_\nu h^{\mu\nu} + \frac{1}{2}\partial_\mu h^{\mu}_{\nu}\partial_\alpha h^{\nu\alpha} + \frac{1}{4}\partial_\mu h\partial^\mu h\ . \tag{5.16}$$

This Lagrangian is invariant under linear gauge transformations

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu\ . \tag{5.17}$$

A gauge fixing term has to be added here to break this gauge invariance in order to get the propagator. The following gauge fixing Lagrangian is used here,

$$\mathcal{L}_{gf} = -\frac{1}{2}\left[\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial^\nu h\right]^2 \tag{5.18}$$

In this gauge the graviton propagator is finally determined from the surviving quadratic part of the pure gravity Lagrangian, which is

$$\mathcal{L}_g = -\frac{1}{4}\,\partial_\mu h_{\alpha\beta}\,\partial^\mu h^{\alpha\beta} + \frac{1}{8}\,(\partial_\mu h^\alpha{}_\alpha)^2 \tag{5.19}$$

The latter can be conveniently re-written in terms of a matrix O

$$\mathcal{L}_g = -\frac{1}{2}\,\partial_\lambda h_{\alpha\beta}\,O^{\alpha\beta\mu\nu}\,\partial^\lambda h_{\mu\nu} \tag{5.20}$$

with

$$O_{\alpha\beta\mu\nu} = \frac{1}{2}\,\eta_{\alpha\mu}\eta_{\beta\nu} - \frac{1}{4}\,\eta_{\alpha\beta}\eta_{\mu\nu} \tag{5.21}$$

The matter sector is also expanded around a space-time constant background $\phi = \phi_0 + \Phi$

$$\sqrt{-g}\mathcal{L}_m = \frac{1}{2}\Phi(\Box - V^{''}(\phi_0))\Phi - \frac{\kappa}{2}hV^{'}(\phi_0)\Phi - \Phi V^{'}(\phi_0) - \frac{\kappa}{2}hV(\phi_0) + \frac{1}{2}\kappa^2\,h_{\alpha\beta}O^{\alpha\beta\mu\nu}h_{\mu\nu}V(\phi_0) \tag{5.22}$$

If we write down an effective linearized equation of motion for the graviton field from the quadratic part of the Lagrangian, we get an equation

$$\mathcal{I}^{\alpha\beta,\mu\nu}h_{\mu\nu} = \kappa T^{\alpha\beta}\ , \tag{5.23}$$

where $T^{\mu\nu}$ contains interaction terms containing appropriately contracted products of terms *linear* in the scalar and graviton fluctuation fields. The operator $\mathcal{I}^{\mu\nu,\alpha\beta}$ can be extracted from the bilinear effective Lagrangian. In Fourier space this takes the following form:

$$\mathcal{I}^{\mu\nu\alpha\beta} = (-k^2 + \kappa^2 V)\mathcal{O}^{\mu\nu,\alpha\beta} \tag{5.24}$$

Now this operator can be inverted to get the propagator,

$$D_{\mu\nu\alpha\beta}(k) = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\,\eta_{\alpha\beta}}{k^2 - \kappa^2 V(\phi_0)} \tag{5.25}$$

Clearly, the poles of the propagator are at $k_0 = \pm\sqrt{\mathbf{k}^2 - \kappa^2 V}$ which give rise tachyon in the infrared limit for positive definite potential terms [96]!

This tachyonic mode in the propagator in the infrared limit can induce a Jeans Like instability. Recall that the dispersion relation in Jeans' treatment of the gravitational instability of a homogeneous fluid is $\omega^2 = v_s^2\mathbf{k^2} - 4\pi G\rho$. Hence

$$h_{\mu\nu}(k) = D_{\mu\nu,\alpha\beta}(k)\; T^{\alpha\beta}(k)$$

will produce a runway solution triggering a Jeans-like instability.

Thus in the infrared limit the constant scalar background induces an imaginary mass proportional to the potential of the field. If we choose $V(\phi)$ to be $\lambda\frac{\phi^4}{4!}$ then the induced mass is proportional to the fourth power of constant background ϕ_0. It is perhaps not a coincidence that in (5.10) the induced tachyonic mass is proportional to the fourth power of the *temperature* [96].

Let us now proceed to investigate how this effect is manifested in the quantum effective potential for this theory.

5.3 One-loop effective potential for graviton-Higgs theory

Here, the one-loop effective potential for a theory where gravity is coupled to a Higgs field minimally is computed. Let us start with the the Euclidean path integral,

$$Z = \int \mathcal{D}g\mathcal{D}\chi e^{-S_E} = e^{-W} \tag{5.26}$$

where S_E is the Euclidean action for the full theory and χ is any generic field. However, since the path integral has configurations which are identical under the diffeomorphic transformations we have to impose gauge condition to integrate over gauge inequivalent configurations.

The Lagrangian (for positive definite Euclidean metric) for gravity minimally coupled to scalar field is,

$$\sqrt{g}\mathcal{L} = -\frac{1}{\kappa^2}R + \sqrt{g}\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \sqrt{g}V(\phi) \tag{5.27}$$

The one-loop effective potential is obtained from (5.26) employing the loop expansion technique [24] and setting the background scalar field to be a constant. Remember, it is sufficient to expand the terms in the full Lagrangian upto quadratic in fluctuating fields to get the one-loop effective potential.

The full Lagrangian (up to quadratic order in fluctuating fields) is the sum of following terms,

$$\mathcal{L}_q = \left\{\mathcal{L}_m^{(0)} + \mathcal{L}_g^{(2)} + \mathcal{L}_{gf} + \mathcal{L}_{ghost} + \mathcal{L}_m^{(2)}\right\} \tag{5.28}$$

The gauge fixing and ghost Lagrangians are

$$\mathcal{L}_{gf} = \frac{1}{2}\left[\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial^\nu h\right]^2 \tag{5.29}$$

and

$$\mathcal{L}_{ghost} = \frac{1}{2}\partial_\alpha \zeta_\mu \partial^\alpha \bar{\zeta}^\mu \tag{5.30}$$

The ghosts decouple in this gauge and don't contribute to the effective action. Retaining terms upto quadratic in fluctuating field Φ we get the Lagrangian relevant for one-loop EP effective potential

$$\begin{aligned} \mathcal{L}^1 &= \frac{1}{2}\Phi(-\Box_E + V^{''})\Phi + \frac{1}{4}h_{\mu\nu}(-\Box_E - \kappa^2 V)h^{\mu\nu} \\ &- \frac{\kappa}{2}hV^{'}(\phi_0)\Phi - \frac{1}{8}h\left[(-\Box_E) - \kappa^2 V\right]h \end{aligned} \tag{5.31}$$

Inserting the above Lagrangian into the path integral (5.26) and integrating over scalar fluctuations Φ one gets an effective action which contains a propagator corresponding to the operator $(-\Box_E + V^{''})$, coming from the bilinear interaction term proportional to $h\Phi$. This effective action Γ, is now quadratic in $h_{\mu\nu}$.

$$e^{-\Gamma^1[\phi_0]} = e^{-V(\phi_0) - \frac{1}{2}TrLog[-\Box_E + V^{''}]} \int \mathcal{D}h_{\mu\nu}\, e^{\int -\frac{1}{2}h_{\mu\nu}M^{\mu\nu\alpha\beta}h_{\alpha\beta}} \tag{5.32}$$

The exponent inside the functional integral can be cast in a compact form as follows,

$$\frac{1}{2}\Psi_i M_{ij} \Psi_j$$

where Ψ_i $(i = 1, 2, .., 10)$ represents ten independent components of $h_{\mu\nu}$ [101, 102]. The components of the ten dimensional vector Ψ_i are related to the graviton field tensor components as follows:

$$\begin{aligned} \Psi_i &= h_{ii}\ ,\ i = 1, ...4 \\ \Psi_5 = h_{12}\ ,\ \Psi_6 &= h_{13}\ ,\ \Psi_7 = h_{14} \\ \Psi_8 = h_{23}\ ,\ \Psi_9 &= h_{24}\ ,\ \Psi_{10} = h_{34} \end{aligned} \tag{5.33}$$

The matrix elements of M can now be easily obtained from (5.33). The matrix takes a simple block diagonal form. The lower 6×6 part of the matrix is diagonal, with each of them have same entry $k^2-\kappa^2 V$. The upper 4×4 part is a symmetric matrix with diagonal entries $(-k^2+\kappa^2 V)/4+\frac{V'^2\kappa^2}{(k^2-V'')}$ and off-diagonal entries are $-(-k^2+\kappa^2 V)/4+\frac{V'^2\kappa^2}{(k^2-V``)}$.

The eigenvalues for the matrix M are,

$$\begin{aligned}
\lambda_i &= k^2-\kappa^2 V \ ; (1\le i\le 6) \\
\lambda_i &= \frac{1}{2}(k^2-\kappa^2 V) \ ; (7\le i\le 9) \\
\lambda_{10} &= -\frac{1}{2}\left[\frac{(k^2-\kappa^2 V)(k^2+V'')+2\kappa^2 V'^2}{k^2+V''}\right]
\end{aligned} \tag{5.34}$$

The eigenvalue for the operator coming from the quadratic part of the scalar field is given by,

$$\lambda_\phi = k^2+V''$$

The effective potential is given by

$$V^1_{eff} = V(\phi_0)+\frac{1}{2}LogDet[M]\ +\frac{1}{2}TrLog\lambda_\phi \tag{5.35}$$

In terms of the momentum integrals the effective potential is given by,

$$\begin{aligned}
V^1_{eff} &= V+\frac{9}{2}\int_0^\Lambda \frac{d^4k}{(2\pi)^4}\ln(k^2-\kappa^2 V) \\
&+ \frac{1}{2}\int_0^\Lambda \frac{d^4k}{(2\pi)^4}\ln\left[k^4+\,(V''-\kappa^2 V)k^2+\kappa^2(2V'^2-VV'')\right]
\end{aligned} \tag{5.36}$$

Evaluating the momentum integrals with a cut-off Λ the one loop effective potential is given by,

$$
\begin{aligned}
V^1_{eff}(\phi_0) = \quad V \quad & + \frac{9}{32\pi^2}\left[\frac{\kappa^4 V^2}{2}\left(\ln\frac{\kappa^2 V}{\Lambda^2} - \frac{1}{2}\right) - \kappa^2 V \Lambda^2\right] - \frac{i9\kappa^4 V^2}{64\pi} \\
& + \frac{1}{32\pi^2}\left[(V^{''} - \kappa^2 V)\Lambda^2 + \frac{a^2 - 2b}{4}\left(\ln\frac{b}{\Lambda^4} - 1\right)\right] \\
& + \frac{a\sqrt{a^2 - 2b}}{64\pi^2}\ln\left[\frac{a + \sqrt{a^2 - 4b}}{a - \sqrt{a^2 - 4b}}\right] \qquad (5.37)
\end{aligned}
$$

where

$$
\begin{aligned}
a &= V^{''} - \kappa^2 V \\
b &= \kappa^2 (2V^{'2} - VV^{''})
\end{aligned}
$$

The source of the imaginary part in the last term of the first line of eq. (5.37) is due to the non-linear nature of graviton-Higgs theory. It is clear from 5.36 in the infrared limit the momentum integrals (functional traces) become non-analytic due to negative logarithms.

One may think that the logarithmic terms in the second and third lines of the eq. (5.37) may give rise imaginary contributions also. Indeed this could happen in some cases. However, this is not possible for any monomial potential with positive coefficient. The most obvious example of this kind is $\lambda\phi^4$. It is easy to see that b is positive for this case and since the terms which are Planck-suppressed always dominated by unsuppressed ones both a and $a^2 - 4b$ are positive so long as $\phi_0 < M_{Planck}$. These conditions appropriately rule out the possibility of any imaginary contribution from other terms of eq. (5.37). However, if we don't restrict the potential to be of this particular form then for positive V the sufficient condition for not getting any additional imaginary part from the logarithmic terms in the effective potential reduces to $a, b > 0$ [96].

Similar results have been obtained in [45, 46] etc. In a related work, Fradkin et al [42] have shown that for a gauged supergravity theory one of the modes in the spectral decomposed one-loop operators in de Sitter background contains negative modes. The appearance of imaginary part in one loop effective action was also reported by Odintsov for $SU(5)$ GUT theory in de

Sitter background [103]. In some higher derivative gravity theories, with non-minimal coupling to scalar fields, similar imaginary terms in the effective potential have also been observed [47, 104].

An interesting feature of the one-loop effective potential is that the effect completely disappears if the classical Higgs potential is set to zero. This is in contrast to the flat spacetime gauge field theories where a minimally coupled Higgs field *generates* an effective potential perturbatively, even if the classical potential vanishes. This is because in Higgs-graviton theory the absence of a classical Higgs potential, the Higgs field has no other coupling to the graviton field when expanded around a constant vacuum value. In standard electroweak theory in flat spacetime, in contrast, the classical Lagrangian has Higgs-gauge field seagull terms which lead to the one loop effective potential [87, 105] even in the absence of a classical potential. This does not happen in perturbative quantum gravity since there is no such interaction for a constant Higgs background. In fact, this feature of scalar-graviton theory appears to persist to higher orders of perturbation theory for spacetime independent Higgs backgrounds. In the next section the finite temperature counterpart of the effective potential for graviton-Higgs theory has been taken into consideration to see the effect of nonzero temperature on the instability obtained in the zero temperature case.

5.4 Effect of finite temperature

In this section the effective potential for graviton-Higgs system is computed for finite temperature. The thermal contribution of the one loop effective potential is important to investigate, to ascertain whether the zero temperature instability is reinforced or weakened. Doubtlessly, the result of this assay will have implications for inflation and perhaps also for the electroweak phase transition in the early Universe. The recent discovery of a 125 Gev scalar boson at CERN lends special credence to theories with gravitons interacting with Higgs fields vis-a-vis their implication for various instabilities in the early Universe [106, 107]. The appearance of an imaginary part in the zero temperature one-loop effective potential prompts one to investigate the situation for the finite temperature counterpart of the theory. I have already cited the literature where a tachyonic pole in the one loop graviton self-energy has been discerned, leading to an instability in the theory. The issues addressed in this section are : (a) how this instability manifests in the one-loop effective potential, and (b) if there are additional *imaginary* temperature-dependent contributions at one loop, whether these contributions neutralize the zero-temperature imaginary part of the effective potential found in the last section, or *enhance* it. It is found that the effect of constant scalar background is being amplified in the high temperature limit of Higgs-graviton effective potential. The low temperature limit, on the other hand, shows a rather interesting behaviour : in the physically relevant region the temperature dependent imaginary part oscillates with a damping amplitude. This oscillation may be a reminiscence of the instability of flat background under perturbation in presence of interaction between gravitons and thermally excited matter fields [96].

From (5.37) one can write down the expression for the one-loop effective potential in momentum space in a slightly modified form,

$$V_{eff} = V + \frac{9}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 - \kappa^2 V\right) + \sum_{i=1}^{2} \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left[k^2 + A_i\right] \tag{5.38}$$

with A_i's are root of the quartic equation $k^4 + ak^2 + b = 0$ where $a = V'' - \kappa^2 V$ and $b = \kappa^2(2V'^2 - VV'')$.

To obtain finite temperature effective potential one has to shift the momentum integrals of (5.38) by

$$\begin{aligned} \int d^4k &\rightarrow T\sum_n \int d^3\mathbf{k} \\ k &\rightarrow (2\pi nT, \mathbf{k}) \end{aligned}$$

Thus now, the finite temperature counterpart of the effective potential, in terms of Euclidean momentum integrals becomes [108, 109],

$$V_{eff} = V_0 + \frac{1}{2}T\sum_{i=1}^{2}\sum_{n=-\infty}^{\infty}\int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln(\mathbf{k}^2 + 4\pi^2 n^2 T^2 + A_i) + \frac{9}{2}T\int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln(\mathbf{k}^2 + 4\pi^2 n^2 T^2 - \kappa^2 V) \tag{5.39}$$

The above integrals can be represented in a general form,

$$I(t,u) = \frac{t^{\frac{1}{2}}}{2\pi}\sum_{n=-\infty}^{n=\infty}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\ln(\mathbf{k}^2+tn^2+u) \tag{5.40}$$

Here $t = 4\pi^2T^2$.

Since $I(t,u)$ is a divergent quantity one has to regularize this integral. Dimensional regularization is most convenient to evaluate such integrals. Let us perform an integral transform to tackle the infinite sum in the expression. The basic integral is,

$$I(t,u,d) = \frac{t^{\frac{1}{2}}}{2\pi}\sum_{n=-\infty}^{n=\infty}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\ln(\mathbf{k}^2+tn^2+u) = -\frac{t^{\frac{1}{2}}}{2\pi}\sum_{n=-\infty}^{n=\infty}\frac{1}{(4\pi)^{d/2}}\int_0^\infty d\tau\tau^{-d/2-1}e^{-\tau(tn^2+u)}\ , \tag{5.41}$$

where I have used the relation

$$\int\frac{d^d\mathbf{k}}{(2\pi)^d}\ln(\mathbf{k}^2+tn^2+u) = -\frac{\partial}{\partial\alpha}\int\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{(\mathbf{k}^2+tn^2+u)^\alpha}\bigg|_{\alpha=0} = -\frac{\Gamma(-\frac{d}{2})}{(4\pi)^{\frac{d}{2}}}(tn^2+u)^{\frac{d}{2}}\ , \tag{5.42}$$

and also assumed that the τ integration has no singularities.

To evaluate the integral (5.41) a large temperature expansion of the integrand is performed. At high temperature limit i.e. for $\frac{u}{t} << 1$ One can write the sum over n as a binomial expansion in $\frac{u}{t}$

$$\begin{aligned}\sum_{n=1}^{\infty}(tn^2+u)^{\frac{d}{2}} &= \sum_{n=1}^{\infty}t^{\frac{d}{2}}\left[n^d+(\frac{d}{2})\left(\frac{u}{t}\right)\frac{1}{n^{2-d}}+\frac{1}{2}\left(\frac{d}{2}\right)\left(\frac{d}{2}-1\right)\left(\frac{u}{t}\right)^2\frac{1}{n^{4-d}}+O\left(\frac{u}{t}\right)^3\right] \\ &= t^{\frac{d}{2}}\left[\zeta(-d)+\left(\frac{d}{2}\right)\zeta(2-d)\left(\frac{u}{t}\right)+\zeta(4-d)\left(\frac{d}{4}\right)\left(\frac{d}{2}-1\right)+O\left(\frac{u}{t}\right)^3\right]\end{aligned} \tag{5.43}$$

where the definition of Riemann zeta function and its analytic continuation to the region $n < 1$ is used.

$$\zeta(n) = \sum_{k=1}^{\infty}\frac{1}{k^n}\ ,\ n > 1$$

One can now easily extract the pole part of the integral. Defining $\epsilon = 3-d$ the high temperature part of (5.41) becomes,

$$\begin{aligned} I(t,u,d-3) &= -\frac{u^2}{16\pi^2}\left(\frac{1}{\epsilon}\right) - \frac{1}{6\pi^2}\zeta(-3)t^2 - \frac{1}{4\pi^2}\zeta(-1)tu + \frac{u^2}{32\pi^2}\ln\frac{u^2}{M^2} \\ &= \frac{1}{12\pi^2}u^{\frac{3}{2}}t^{\frac{1}{2}} + \frac{1}{32\pi^2}u^2\ln u/t \\ &- \frac{u^2}{16\pi^2}\left(\gamma - 3/4 + \frac{1}{2}\psi(3) - \frac{1}{2}\ln\frac{M^2}{\pi}\right) + O(t^{-1}) \end{aligned} \tag{5.44}$$

with

$$\psi(x) = \frac{d}{dx}\Gamma[x]$$

Here we have also introduced M as an arbitrary scale of renormalization. From the above expression we can easily see that there will be imaginary contributions from some of the terms involved. If we closely inspect the possible u's from eq. (5.38) this becomes clear. Apart from the irrelevant constants and after getting rid of the pole term by a suitable counter term one can write the effective potential at high temperatures as,

$$\begin{aligned} V_{eff} &= V + \frac{1}{64\pi^2}\sum_{i=1}|A_i|^2\ln\left(\frac{|A_i|}{M^2}\right) + \frac{9\kappa^4V^2}{64\pi^2}\ln\left(\frac{\kappa^2V}{M^2}\right) \\ &+ V_{eff,im} + V_{eff,T} \end{aligned} \tag{5.45}$$

where, $V_{eff,im}$ consists of zero-temperature imaginary terms and $V_{eff,T}$ is the temperature-dependent part of V_{eff}.

It is easy to see that the imaginary part of effective potential in this limit is

$$V^{Im} = \frac{\kappa^4V^2}{16\pi} + \frac{\kappa^3V^{3/2}}{6\pi}T \tag{5.46}$$

This indicates, the temperature-dependent contribution to the imaginary part in fact *reinforces* the zero-temperature piece, thereby exacerbating the instability discussed in the last section. The plot (Fig.[5.1]) of temperature dependent imaginary contribution is simple and shows that it grows with the temperature. where $x = \frac{\kappa V^{\frac{1}{2}}}{T}$ It is clear that since the dimensional pole term is proportional to V^2 instead of V the theory is non-renormalizable. However, the main focus of this calculation is not on the ultraviolet behaviour of the theory, but rather its infrared

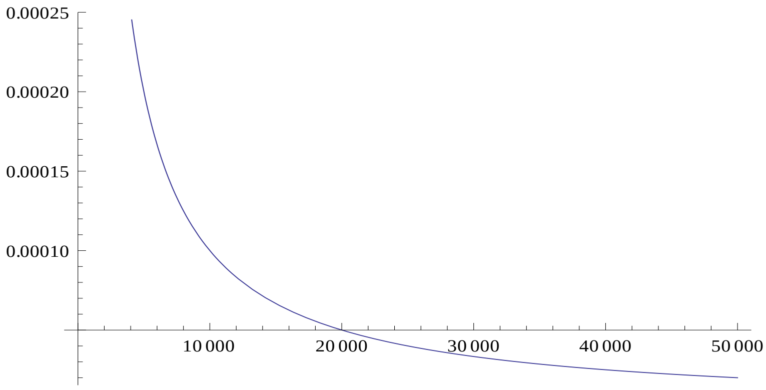

Figure 5.1: Plot of temperature dependent imaginary part of EP versus x, for $x << 1$

instabilities at zero and finite temperature. Thus, even if the ultraviolet divergences are tamed as conventional with appropriate counter-terms, it is obvious that the infrared instabilities will persist.

In order to ensure that the finite temperature treatment has the correct limit to vanishing temperature, one needs to consider the *low* temperature limit of the temperature-dependent part of V_{eff}. To obtain the low temperature limit of eq. (5.41) we now have to use the following identity

$$\sum_{n=-\infty}^{\infty} e^{-\tau t n^2} = \left(\frac{\pi}{t\tau}\right)^{1/2} \sum_{n=-\infty}^{\infty} e^{-\pi^2 n^2/t\tau} \tag{5.47}$$

With the help of this we can write eq. (5.41) as

$$I(t,u,d) = -\frac{1}{2\pi^{1/2}} \sum_{n=-\infty}^{n=\infty} \frac{1}{(4\pi)^{d/2}} \int_0^\infty d\tau \tau^{-(d+3)/2} e^{-\tau t} e^{-\pi^2 n^2/t\tau} \tag{5.48}$$

This decomposes into two parts, one being temperature dependent and other, zero temperature. Once again the pole term is independent of temperature. After separating out the $n = 0$ piece from the above expression one gets,

$$-\frac{1}{2\pi^{\frac{1}{2}}} \frac{1}{(4\pi)^{d/2}} u^{(d+1)/2} \Gamma\left(-\frac{d}{2} - \frac{1}{2}\right) - \frac{1}{\pi^{1/2}} \sum_{n=1}^{n=\infty} \frac{1}{(4\pi)^{d/2}} \int_0^\infty d\tau \tau^{-(d+3)/2} e^{-\tau t} e^{-\pi^2 n^2/t\tau} \ . \tag{5.49}$$

From the first term we easily extract the pole term

$$-\frac{u^2}{16\pi^2}\left(\frac{1}{\epsilon}\right)-\frac{u^2}{32\pi^2}\left(\psi(3)+\ln\frac{4\pi}{u}\right)+O(\epsilon) \tag{5.50}$$

One can perform the τ integration to get the temperature dependent part. The result is given in terms of a modified Bessel function [110],

$$\int_0^\infty d\tau\tau^{-(d+3)/2}e^{-\tau t}e^{-\pi^2n^2/t\tau}=\left(\frac{tu}{\pi^2n^2}\right)K_2(2\sqrt{\pi^2n^2u/t}) \tag{5.51}$$

The low-temperature behaviour of the integral eq. (5.48)

$$\begin{aligned} I(t,u,3-\epsilon) &= -\frac{u^2}{16\pi^2}\left(\frac{1}{\epsilon}\right)-\frac{u^2}{32\pi^2}\left(\psi(3)+\ln\frac{4\pi}{u}-\ln\frac{u}{M^2}\right) \\ &- \frac{u^2}{2^{\frac{1}{2}}}\sum_{n=1}^{\infty}\left(\frac{t}{4\pi^2n^2u}\right)^{\frac{5}{4}}e^{(-4\pi^2n^2u/t)^{1/2}}\ , \end{aligned} \tag{5.52}$$

where for large value of the argument I have taken an asymptotic expansion for the modified Bessel function.

To analyze the low temperature behaviour of the potential I have ignored any imaginary part coming from the A_i's and will only concentrate on $u=-\kappa^2V$ part. Then at Low temperature imaginary contribution for effective potential has the following form [96]

$$\frac{\kappa^4V^2}{32\pi}+\frac{\kappa^4V^2}{2}\left(\frac{T^2}{\kappa^2V}\right)^{\frac{5}{4}}\sum_{n=1}^{\infty}\frac{1}{n^{\frac{5}{2}}}(\cos nx-\sin nx) \tag{5.53}$$

The second term above is temperature dependent. One can approximate the sum as a integral over n as n goes upto infinity or we can compute the sum exactly. Performing both using MATHEMATICA we have found the behaviour of the second term of eq. (5.53) is oscillatory with damping amplitude (Fig.[5.2]) for values of $x>0.6$ approximately and for large value of x i.e. for $T\to 0$ the oscillations die away and temperature dependent imaginary part vanishes [96].

The analysis above is based on a one-loop effective action evaluated in a certain gauge. However as has mentioned earlier in this thesis, there is a gauge invariant way of calculating the one-loop effective potential due to Vilkovisky and De-Witt [?,41]. If one calculates the effective

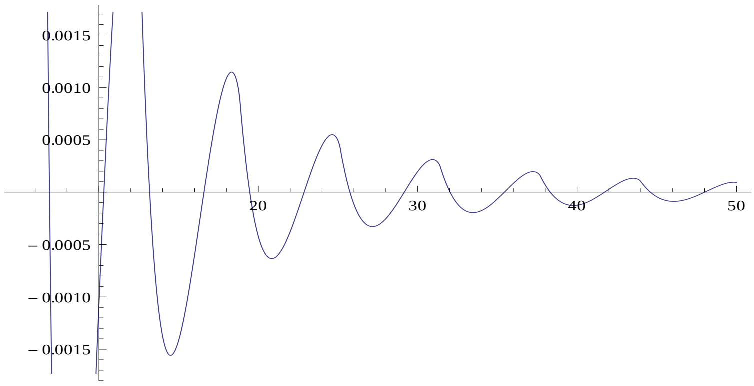

Figure 5.2: Plot of temperature dependent imaginary part of EP versus x, $x >> 1$

potential using this method, one finds that there is hardly any qualitative change in the effective potential particularly the imaginary part doesn't disappear. However, the numbers do change. I state below the important factors here which undergo a change, without going into details of the calculative scheme. Details can be found in [46].

The structure of the potential is almost the same for VD approach except the prefactor of the momentum integral changes to 5 in place of 9 in eq. (5.36). The constants a and b in this case are

$$a = V'' - \frac{3}{2}\kappa^2 V$$

and

$$b = \frac{1}{2}\kappa^4 V^2 - \kappa^2 V V'' + \frac{3}{4}\kappa^2 V'^2$$

The above analysis may be redone with these minor changes which clearly do not affect the qualitative nature of the solution.

5.5 Discussions

In this chapter, it is shown that for a theory in which the graviton field is minimally coupled to a Higgs scalar field, flat Minkowski spacetime is unstable. This instability is exhibited as a tachyonic mode in the one-loop propagator. A constant scalar field background resembles a thermal bath which backreacts to the gravitons to produce the instability in the system. This infrared instability in the effective propagator may be regarded as a graviton induced Jeans-like instability. This infrared instability is also manifested in the one-loop effective potential as an imaginary term, independent of the ultraviolet cut off. This term arises also from the infrared limit of the loop integrals.

I have also computed the effect of finite temperature for the graviton-Higgs theory and compared it with the zero temperature result. The high temperature sector involves temperature dependent terms which adds to the imaginary contribution obtained in the zero temperature case. The infrared sector exhibits an instability because of imaginary contribution from both zero temperature and temperature dependent part. Moreover it exhibits an oscillatory behaviour which eventually gets damped out as we lower the temperature, thereby ensuring that the temperature-dependent calculation smoothly interpolates to the previous zero-temperature one-loop effective potential.

The existence of a non-vanishing constant Higgs field background itself is known to signify a vacuum instability which, for the standard electroweak theory, resolves itself by producing masses for the gauge bosons via the Higgs mechanism, as also for fermionic fields Yukawa-coupled to the Higgs field, and the Higgs field itself. The additional instability of such fields coupled to gravitons may have originated from that vacuum instability itself, although it is obvious that this does not resolve itself by generating a mass for the gravitons. To this extent, this latter instability found in this study may be of a more serious nature than the electroweak vacuum instability, since it is far from clear what a system bound by this sort of behaviour will decay into. While the possibility of the role this instability in structure formation (aĺa the Jeans instability) is intriguing, I do not have a viable cosmological picture so far to claim that this is what this instability must do.

One possible explanation of the zero-temperature instability has been given by Smolin [45] where it has been claimed that it might disappear if one begins with a de Sitter background spacetime instead of a Minkowski spacetime. However, in a recent work by Polyakov [112], it has been pointed that even de Sitter space possesses various quantum instabilities. In any case, it is necessary to estimate the lifetime of any system subject to such an instability and compare that with the age of the Universe to ensure that it does play a significant role.

It is worthwhile to note here that what is being computed in this paper entails fluctuating gravitons coupled to fluctuating Higgs fields, in the absence of a cosmological constant term in the classical action. When one perturbs this theory around a constant scalar as well as a flat spacetime background and integrate out fluctuations around that, an instability develops in the infrared regime of the fluctuations, manifesting in the one loop effective potential. One might construe that in the one-loop approximation a cosmological term has been induced by the constant scalar background. This observation may provoke one to interpret the source of this instability as being due to the wrong choice of vacuum since flat space is not a solution of Einstein equation with a cosmological constant. However this argument has two possible pitfalls : first, there are quantum fluctuations around the constant classical potential in the full perturbative expansion which will contribute to the Higgs propagator and also at higher loops. Secondly, interpreting the one loop vacuum energy as a possible cosmological constant is incorrect since it is unacceptably large phenomenologically for any reasonable minimum of the one loop effective potential.

Although in chapter 3, gauge-free formulation for free massless gravitons has been discussed but calculations carried out in this chapter don't follow that way. Since free graviton has more than one unphysical degrees of freedom extracting the gauge-free part in an interacting theory is nontrivial. One possible way to achieve this might be to perform a Hodge-de Rham decomposition of gravitons and try to construct gauge-free couplings of Higgs with gravitons but due to highly non-linear nature of the interactions this procedure doesn't work. Therefore, this is still an open issue which is to be investigated with care.

5.6 Summary

In this chapter, the appearance of the tachyonic pole in the classical graviton propagator in minimally-coupled graviton-Higgs theory is explicitly exhibited. The one-loop effective potential of the graviton-Higgs theory also develops an instability. The presence of this instability is traced to the tachyonic pole in the graviton propagator. A comparison of the nature of the one-loop effect between gauge-Higgs and graviton-Higgs theory has been made. Then a detailed study on the one-loop effective potential at finite temperature is done. The infrared limit describes an interesting situation exhibiting an instability due to the temperature dependent contribution to the effective potential developing an additional imaginary part over and above the one in the $T = 0$ limit. Various aspects of this instability are discussed. In the next chapter couplings of massless scalar and pseudoscalar with higher derivative gravity are considered.

Chapter 6

Vacuum instability in higher derivative gravity coupled to Higgs fields

In this chapter, gauge invariant interactions of gauge fields (photons and gravitons) with torsion are studied. First, I'll motivate the problem of constructing gauge-invariant couplings in presence of torsion and review earlier attempts to address this issue. It is understood that the antisymmetric rank-2 tensor, Kalb-Ramond field, can act like a source of spacetime torsion. In a consistent heterotic string theory, the massless Kalb-Ramond field, present in the spectrum, is augmented by Yang-Mills and gravitational Chern-Simons terms. When compactified to 4-dimensions and in the field theory limit, such additional terms give rise to interactions with interesting astrophysical predictions. These interactions are of higher-derivative (in metric) in nature. I will briefly discuss the consequences of these interactions on electromagnetic and gravitational waves. If one is also interested in coupling 2 or 3-form (Abelian or non-Abelian) gauge fields to torsion, one needs another class of interaction. The possible source of this new interaction is investigated here. Then, behaviour of electromagnetic and gravitational waves due to these new interactions are also studied. Finally the CW potential for gravity coupled to higher derivative terms is computed and the result shows vacuum instability. The concluding section will have a discussion on the main results of this chapter. This chapter is based on [104].

6.1 Motivation

The low energy physics of particle interactions is satisfactorily described by the standard model and general relativity. At higher energies available at the early universe or at astrophysical processes, it is expected that new degrees of freedom will emerge to play important role. Otherwise inaccessible at the present energy scale, these fields might interact with degrees of freedom of the standard model leading to some interesting theoretical predictions and observational signatures. Since string theory is a candidate for a unified description of field interactions even upto the Planck scale, we envisage that nature and the specific form of interaction of new fields with known degrees of freedom can be extracted from this theory in an unambiguous way. In this chapter, gauge invariant interactions of gauge fields (electromagnetic, gravitational and 2 and 3-form gauge fields) to torsion are examined.

In string theory, since the Kalb-Ramond (KR) field acts as a source of torsion, one thus looks at possible gauge and gravitational interactions of a this KR field. The KR field is generic to any closed string spectrum but is *not* a degree of freedom of the standard model. One can anticipate that any observational effect involving the KR field, obtained using standard fields as probes, is then a window into the otherwise inaccessible world of very high energy physics supposedly predicted by string theories. On the other hand, loop quantum gravity (LQG) is also a candidate for quantum theory of gravity. In LQG, the Barbero-Immirzi parameter is a one-parameter ambiguity which describes various topological sectors. This parameter also comes up in the area spectrum and consequently in entropy of black holes wherefrom its value is ascertained by comparing with the Bekenstein-Hawking entropy formula. If the Barbero-Immirzi parameter is promoted to a field, it acts as a source for torsion. It is then interesting to compare and contrast various interactions of fields with these two sources of torsion that arise in these two theories of quantum gravity. Since there are observational implications, the issue is even more satisfying.

In the context of the heterotic string theory, electromagnetic and gravitational interactions of

KR fields arise quite naturally from the requirements of consistency. As is well known [113], the $(E_8 \otimes E_8)$ or $SO(32)$ heterotic strings are two anomaly free gauge groups which can be coupled to $N = 1$ supergravity in 10 dimensions. Anomaly cancellation (the Green-Schwarz mechanism) requires that the KR 3-form field strength is augmented by addition of $(E_8 \otimes E_8)$ Yang-Mills Chern-Simons 3-form and local Lorentz Chern-Simons 3-form [113]. This augmentation induces electromagnetic and gravitational interactions of the KR field which lead to potentially interesting physical effects showing up in the Maxwell and Einstein equations, when the theory is compactified to four dimensions. The electromagnetic effect mainly comprise a rotation of the polarization plane of electromagnetic waves from large redshift sources, upon scattering from a homogeneous KR background [114–118]. This rotation is independent of the wavelength of the electromagnetic wave and cannot be explained by Faraday effect where the plane of polarization of the electromagnetic wave rotates depending quadratically on the wavelength while passing through some magnetized plasma. Similarly, the gravitational interaction leads to the result that the plane of polarization of gravitational waves rotate through an angle that is proportional to (a power of) the KR field strength component [119]. Predictions of this kind can then be useful if some deviations from the traditional expectations are observed. For example, such interactions have been studied within the framework of the five dimensional Randall-Sundrum braneworld model. When compactified to four dimensions, they lead to huge deviations from the expected results [119–123] which can be used to put bounds on the various parameters in the theory [124–126]. On the other hand, if the predictions are non-observable, they lead to upper bounds on the presence of new fields which is important in our search for new theories and their couplings. To exemplify, in the present case of rotation of plane of polarization of electromagnetic waves, the magnitude of the effect is sensitive to the dimensional compactification of the underlying theory. For toroidal compactification (as well as for the Calabi-Yau compactification) of the theory (in the zero slope limit), the predicted rotation is proportional to the appropriate KR field strength component (scaled by the inverse scale factor in a Friedmann universe), so that bounds on the observed rotation translate into a stringent upper bound on the size of the KR field strength component. Moreover, if one uses the bounds on the KR field strength obtained from the cosmic optical activity, the order of magnitude of the similar effect for gravitational waves can be calculated.

The interactions which give rise to the above-mentioned predictions arise very naturally in string theory and they have been well studied. Interestingly, one can also perceive of another class of interactions which has not been discussed in this context except for in [127, 128], where only the electromagnetic interaction was considered. In this chapter, I shall extend the study to non-Abelian gauge fields and discuss the effects of these possible new interactions in detail.

The interest in LQG for such interactions and consequently it's relation or differences with string theory/supergravity is due some recent studies [129–135]. These papers deal with the consequences of promoting the Barbero-Immirzi (BI) parameter to a field. It turns out that the derivative of the BI field is the source for torsion. Moreover, since the BI field is pseudo-scalar[1], it is natural to compare and contrast this BI field with the axion [130]. If the BI field is an axion, it's derivative is dual to the $H_{\mu\nu\lambda}$ field alluded to above and such fields might have interactions with electromagnetic and gravitational fields in the way very similar to the one discussed above in the context of string theory. These issues will be discussed in detail below and some observational implications will be pointed out.

6.2 Gauge invariant interactions of fields with torsion

In this section let us discuss the construction of gauge invariant interactions of gauge fields with torsion. The issue originally arose during the study of Einstein-Cartan (EC) spacetime. The idea was to construct a gauge invariant coupling of electromagnetic field (A_μ) to torsion which is another geometrical property of the EC spacetime along with the metric. The field strength ($F_{\mu\nu}$) for such a spacetime also depends on torsion [136]. However, because the torsion does not have a transformation under $U(1)$ gauge transformation, the electromagnetic field strength is not gauge invariant. This is dissatisfactory since we expect that field strengths must be measurable even in spacetimes with torsion. This requirement on the field strength demands that the torsion must also stay invariant under $U(1)$ gauge transformation. This situation implies that there

[1]The expression for area spectrum in LQG depends on the BI parameter and as such must be a pseudo-scalar for a well-defined transformation property of the area element.

is a non-gravitational field, possibly massless, to function as the source of the torsion [114]. Since that field must be bosonic, one can opt for the KR antisymmetric second rank tensor field $B_{\mu\nu}$ as a possible candidate. $B_{\mu\nu}$, being a massless antisymmetric field, is expected to be a gauge connection, as indeed it is, with the following gauge transformation $\delta_\lambda B_{\mu\nu} = \partial_{[\mu}\lambda_{\nu]}$ and this leaves its field strength $H_{\mu\nu\lambda}$ gauge invariant. Moreover, for anomaly-free quantum theory, $H_{\mu\nu\lambda}$ must be modified with the addition of an electromagnetic Chern Simons three tensor and if $B_{\mu\nu}$ is endowed with a non-trivial electromagnetic gauge transformation along with Kalb-Ramond gauge transformations, the KR field strength remains invariant under $U(1)$ gauge transformation. This is precisely what was needed: the torsion field is gauge invariant. Interactions of this type gives rise to interactions in the form of rotation of plane of polarization of electromagnetic (and gravitational) waves as discussed in the previous paragraph.

What if one wants to couple a 2-form or a 3-form gauge field to torsion? Such fields arise in the perturbative and non-perturbative sector (D-branes) of string theory compactified to four dimensions and in supergravity. Again, field strengths for such higher rank tensor field are also not invariant under their respective gauge transformations in presence of spacetime torsion. Once we take the KR field as a source for torsion, there is a possible way out. Again demand the field strengths of $2-$ form or 3-form gauge fields to be observable so that one again has to modify $H_{\mu\nu\lambda}$, but is a peculiar way. This extra term, instead being of the form $A \wedge F$ for the $(U(1))$ case above, is $A \wedge {}^*F$, where $*$ denotes the Hodge dual and A is a one, two or a three form field. Once again, if the field $B_{\mu\nu}$ has a non-trivial transformation under the gauge transformation of the form fields, its field strength $(H_{\mu\nu\lambda})$ and hence torsion remains invariant under gauge transformations, as required (for this case, I shall work in order $O(\sqrt{G})$). It is also interesting to note that addition of such terms $(A \wedge {}^*F)$ not only works for 2 and 3 form fields, but also for a 1-form field. Moreover, one gets an additional set of interaction for the electromagnetic fields and $H_{\mu\nu\lambda}$ field with observable consequences. These issues were first discussed in [127] and a possible embedding of such terms in $N = 1$ supersymmetric theory was discussed in [128]. One thing should be pointed out here, no attempt to derive these new interactions from any string theory is being made here but merely possibility of such terms from the requirements of gauge invariance is illustrated. One finds more evidence for existence of such terms: the gravitational counterpart of this new interaction contributes the Euler invariant to the effective action in 4-dimensions, which is well known in gravity and supergravity theories to come from stress-tensor anomaly in curved spacetime [137–139]. In the sense of effective field theory [140, 141], which does not assume any precise details of the fundamental interactions or short distance degrees of freedom, such kind of terms are ubiquitous and leads to quantum corrections for GR. In the appendix of this paper one possible origin of such terms will be shown.

Interestingly, because of the presence of the Hodge dual, the new interactions of violate spatial parity. With the CMB data and the Planck data available, it might be interesting to look for such ideas now. Indeed, observational implications of such terms have already been discussed [142–146], though the possible origin of terms have not been discussed in these papers and the coupling constant for such interactions are usually not pinned down.

In the standard Einstein-Maxwell theory,the electromagnetic field-strength, reduces to the flat space expression on account of the symmetric nature of the Christoffel connection. However, in the theory of gravity described by Einstein-Cartan theory, *i.e.* in case where one has spacetime torsion, the situation changes quite drastically, because the electromagnetic field strength is no longer gauge invariant [136]. Indeed, it is easy to see that

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - T_{\mu\nu}{}^{\rho} A_{\rho}, \tag{6.1}$$

where, $T_{\mu\nu}{}^{\rho} A_{\rho}$ is the torsion (antisymmetric combination of the Christoffel connection), is obviously not invariant under $U(1)$ gauge transformation $\delta_{\lambda} A = d\lambda$, λ being the gauge function. Since $F_{\mu\nu}$ and any field strengths must be measurable quantities even in a curved spacetime with torsion, the torsion tensor, a purely geometric quantity like curvature must also be gauge invariant. However, this implies that one must also have another geometrical quantity which might compensate for the loss of gauge invariance due to torsion. In absence of such compensating fields, it is natural to look for non-gravitational fields to act as a source for torsion [114]. In the context of string theory, the Kalb-Ramond (KR) field seems to be an ideal candidate source [114]. Indeed, it also has all the desired gauge transformation properties required of torsion.

In this section, I shall first review the basic facts about the KR field as is known from string theory with special emphasis on it's gauge transformation properties. The KR field is characterized by a 2-form potential B which has a 3-form field strength $H \equiv dB$; the field strength is invariant under the KR gauge transformation $\delta_{\bar{\lambda}} B = d\bar{\lambda}$, where $\bar{\lambda}$ is a one-form gauge parameter. Immediately, one obtains the Bianchi identity for the KR field:

$$dH = 0 \tag{6.2}$$

In 4 dimensional spacetime, the free KR action is given by

$$S_H = \int_{\mathcal{M}_4} H \wedge {}^*H , \tag{6.3}$$

where, ${}^*H$ is the Hodge-dual of the field strength H. Varying this action w.r.t. B yields the KR field equation

$$d^*H = 0 \tag{6.4}$$

which has the local solution

$${}^*H = d\Phi_H, \tag{6.5}$$

where, Φ_H is a scalar. Substituting this in the one obtains for the field Φ_H

$$d^*d\Phi_H = 0 . \tag{6.6}$$

Thus, on-shell the Bianchi identity for the field B is the equation of motion for it's Hodge dual field. This is not surprising and is a feature of all Hodge-dual related fields.

Let us now point to the string theory connection. B occurs in the massless spectrum of the free string in ten dimensional heterotic string theory. In the zero slope limit, this theory reduces to ten dimensional $N = 1$ supergravity coupled to $N = 1$ $E_8 \otimes E_8$ super-Yang-Mills theory. The requirement of ten dimensional supersymmetry and that the quantum theory be free of all anomalies implies that the KR field strength H be augmented as [113]

$$H \;=\; dB \;-\; \frac{1}{M_P}\,(\Omega_{YM} \;-\; \Omega_L)\;, \tag{6.7}$$

where

$$\Omega_{YM} \;\equiv\; \mathrm{tr}(A\wedge dA \;+\; \frac{2}{3}g\,A\wedge A\wedge A) \tag{6.8}$$

is the Yang-Mills Chern-Simons 3-form with A the gauge connection 1-form and M_P is the Planck mass in 4- dimensional spacetime. Ω_L is the gravitational Chern-Simons 3-form obtained by replacing the Yang-Mills gauge connection A by the spin connection 1-form ω, and the trace is taken over the local Lorentz indices. The augmentation in eq. (6.7) has important consequences. The field H, being a field strength, must remain gauge invariant under both Yang-Mills gauge transformations and under local Lorentz transformations. This implies that B must now transform non-trivially under both gauge transformations inspite of B being neutral. To simplify and to set the notations for the remaining part of the paper, let us say that the gauge field A is $U(1)$ valued. Then, the transformation of A is given by

$$\delta_\lambda A \;=\; d\lambda, \tag{6.9}$$

where, λ is the gauge parameter. The Chern-Simons term now only contains $A\wedge dA$. Let us now denote Ω_{YM} by Ω_{EM} and this term varies as

$$\delta_\lambda\,\Omega_{EM} \;=\; d\lambda\;\wedge dA \tag{6.10}$$

Thus, to achieve gauge invariance for the H field, the transformation law for B should include the 2-form in (6.10) so that under Yang-Mills gauge transformation

$$\delta_\lambda\, B = -\frac{1}{M_P}(\lambda\; dA) \tag{6.11}$$

Also, the gravitational field in the vielbein formalism can be treated very similarly to the

Yang-Mills field. Specifically the Yang-Mills potential A is analogous to the spin connection 1-form ω_{AB}, where A, B are Lorentz indices. Under an infinitesimal Lorentz transformation with parameters given by an $SO(D-1,1)$ matrix Θ, the transformation of ω is

$$\delta_L \omega \;=\; d\Theta \;+\; [\omega, \Theta], \tag{6.12}$$

The Lorentz Chern-Simons term varies as

$$\delta_L \Omega_L \;=\; \mathrm{tr}(d\Theta \;\wedge d\omega) \tag{6.13}$$

Similar to the argument above, transformation law for B should include the 2-form in (6.13) so that under Lorentz transformation

$$\delta_L B = -\frac{1}{M_P}\mathrm{tr}(\Theta \; d\omega) \tag{6.14}$$

Retaining the form of the KR action (6.3), it follows that the KR field equation does not change. Therefore, ${}^*H$ still has the local solution (6.5). However, the KR Bianchi identity certainly changes, leading to

$$d^* d\Phi_H \;=\; \frac{1}{M_P}\; \mathrm{tr}(F \wedge F \;-\; R \wedge R) \;, \tag{6.15}$$

where $F(R)$ is the Yang-Mills (spacetime) curvature 2-form. The Yang-Mills and Einstein equations change non-trivially. Let us consider these below in special situations viz., the Maxwell part of the gauge interaction and linearized gravity.

This scenario works well for 1-form gauge fields. How about if we want a gauge invariant coupling of higher form fields to torsion? In [127], it was proposed that one needs additional terms to be augmented to the KR field strength. For $U(1)$ gauge fields, it was proposed that an additional augmentation to H in the form of ${M_P}^{-1}\,(A \wedge {}^*F)$ is needed. But again, such an addition is not $U(1)$ gauge invariant. One needs to go further

$$H \to H + \frac{1}{M_P}(A \wedge {}^*F + \lambda\, d^*F) \tag{6.16}$$

The argument is obviously not based on any requirements arising from string theory and it is not known if one can embedd such an interaction in any string theory. However, I will discuss this issue later how in effective field theories, such terms are generic and lead to macroscopically observable results. Since effective field theories do not assume any precise details of microscopic interaction, we expect such terms to exist in string theory or in it's low energy effective action. In the appendix (see section (6.7)), origin of such terms from a different perspective will be shown. It is also clear that in presence of such terms, the gauge transformation of B field changes from that obtained in equation (6.11) [2]:

$$\delta_\lambda\, B = -\frac{1}{M_P}(\lambda\, F + \lambda^* F) \tag{6.17}$$

We can also proceed further and add to equation (6.16) the spin-connection terms so that the augmentation takes the following form:

$$H \to H + \frac{\zeta}{M_P}(\,A \wedge {}^*F + \,\omega \wedge {}^*R), \tag{6.18}$$

where, ζ is a parameter which takes values $+1$ or -1. This parameter has been introduced here because the coefficient of the interactions are not quite fixed. Now, instead of equation (6.15), the result of such additional terms in equation (6.18) is (terms only upto order M_P^{-1} are considered here)

$$d^* d\Phi_H \;=\; \frac{1}{M_P}\,\mathrm{tr}(F \wedge F \;+ \zeta\, F \wedge {}^*F - \;R \wedge R - \zeta\, R \wedge {}^*R\,)\;, \tag{6.19}$$

In short, the upshot of the above analysis is that one can consider a gauge invariant action

[2]An immediate consequence of this gauge transformation is that the $H_{\mu\nu\lambda}$ now can no longer be thought of as a parity eigenstate, and thus neither is its dual Φ_H. In other words, one can decompose $\Phi_H = \Phi_H^{(+)} + \Phi_H^{(-)}$ where $+$ indicates even parity and $-$ is for odd parity. However, we shall continue to use the generic term Φ_H for this field.

of the following form [114, 115]:

$$S[g,T] = \int_{M_4} d^4x \, [\, R(g,T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + T_{\mu\nu\lambda} H^{\mu\nu\lambda} \,] \tag{6.20}$$

where $H_{\mu\nu\lambda}$ is defined through equation (6.7) and the torsion tensor $T_{\mu\nu\lambda}$ is an auxiliary field satisfying the constraint $T_{\mu\nu\lambda} = H_{\mu\nu\lambda}$. Putting the local solution $H = -{}^*d\Phi_H$ from equation (6.5) in the action (6.20), we get the effective equation for the field Φ_H:

$$\begin{aligned} S[g, A, \Phi_H] = \int_{M_4} d^4x \, [\, R(g,T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \; &- \; \frac{1}{2} \partial_\mu \Phi_H \, \partial^\mu \Phi_H \,] \\ &+ \; \frac{1}{M_P} \Phi_H (\, F \wedge F \; + \zeta \, F \wedge {}^*F - \; R \wedge R - \zeta \, R \wedge {}^*R \,) \end{aligned} \tag{6.21}$$

which is precisely the action for a pseudo-scalar (Φ_H) coupled to gravity[3]. Note that the extra interaction contributes to the action in case of electromagnetism while is a higher derivative term for gravity. Without the Φ_H term, the higher derivative gravity terms $R \wedge R$ and $R \wedge {}^*R$ are the Pontryagin and the Euler invariants. They are related to the gravitational axial current anomaly and stress-tensor anomaly respectively [137–139]. The equation of motion for this pseudo-scalar is however given by equation (6.19). If the Barbero-Immirzi parameter is promoted to a field, the torsion is dual to the derivative of that pseudo-scalar field (just like the equation (6.5)). In that case, one gets an effective action same as the first part of the action above [129, 134]. In the following sections, consequences of such interactions are studied.

6.3 Electromagnetic interactions of KR field

In this section, study of electromagnetic interactions of the KR field in four dimensional Minkowski spacetime will be made. Let us first restrict ourselves to the interaction of the type $\Phi_H F_{\mu\nu} {}^*F^{\mu\nu}$. Observe that since the field Φ_H is a pseudo-scalar, the interaction is parity conserving. The

[3]Now, because ϕ_H can be both parity violating as well as parity conserving, each interaction is both parity conserving and parity violating. In what follows, I will only consider the case where Φ_H is parity violating.

relevant four dimensional field equations are

$$\begin{aligned}\partial_\mu H^{\mu\nu\rho} &= 0 \\ \partial_\mu F^{\mu\nu} &= M_P^{-1}\ H^{\nu\rho\eta}\ F_{\rho\eta}\ .\end{aligned} \tag{6.22}$$

The corresponding Bianchi identities are

$$\begin{aligned}\Box\Phi_H &= M_P^{-1}\ F^{\mu\nu}\ {}^*F_{\mu\nu} \\ \partial_\mu^* F^{\mu\nu} &= 0\ .\end{aligned} \tag{6.23}$$

To simplify, let us assume that the 'axion' field Φ_H is *homogeneous* and provides a background with which the Maxwell field interacts. As already mentioned let restrict our attention to lowest order in the inverse Planck mass M_P, so that terms on the RHS of the axion field equation (6.23) are ignored to a first approximation. Consequently, $\dot{\Phi}_H \equiv d\Phi_H/dt = f_0$ where f_0 is a constant of dimensionality of $(mass)^2$. Under these conditions, the Maxwell equations can be combined to yield the inhomogeneous wave equation for the magnetic field $\mathbf{B}$

$$\Box\mathbf{B}\ =\ -\ \frac{2\ f_0}{M_P}\ \nabla\times\mathbf{B}\ . \tag{6.24}$$

With the ansätz for a plane wave traveling in the z-direction, $\mathbf{B}(\mathbf{x},t)\ =\ \mathbf{B}_0(t)\ \exp ikz$, we obtain, for the left and the right circular polarization states $B_{0\pm} \equiv B_{0x} \pm iB_{0y}$,

$$\frac{d^2B_{0\pm}}{dt^2}\ +\ (k^2\ \mp\ \frac{2f_0k}{M_P})\ B_{0\pm}\ =\ 0\ . \tag{6.25}$$

Here I concentrate on the equation for magnetic field as the conclusions will be same for that of electric field. The right and left circular polarization states have different angular frequencies (dispersion)

$$\omega_\pm^2 = k^2\ \mp\ \frac{2kf_0}{M_P} \tag{6.26}$$

so that over a time interval Δt, the plane of polarization undergoes a rotation (for large k)

$$\Delta\Psi_{op} \equiv |\omega_+ - \omega_-| \; \Delta t \; \simeq \; 2\frac{f_0}{M_P} \; \Delta t \; . \tag{6.27}$$

In FRW spacetime, the value of observed angle of rotation also incorporates the scale factor [115]. This means that $\Delta\Psi = \Delta\Psi(z)$, where z is the red-shift, and increases with red-shift. This rotation also differs from the better-understood Faraday rotation in that it is *achromatic* in the limit of high frequencies. Observationally, even for large redshift sources, the angle of rotation is less than a degree, which imposes the restriction on the dimensionless quantity $f_0/M_P^2 < 10^{-20}$. In regard to astrophysical observations of optical activity, it appears that there is no definite evidence that the rotation of the plane of polarization traveling over cosmologically large distance is not entirely attributable to Faraday rotation due to magnetic fields present in the galactic plasma [147]. However, it is therefore not unlikely that the axion field will endow observable effect in CMB.

In contrast, if one considers only the extra augmentation, *i.e* the interaction $\Phi_H \, F_{\mu\nu} F^{\mu\nu}$, the resulting wave equation for the **B** field lead to entirely different results. Observe that this interaction violates spatial parity. The wave equation is simple to determine:

$$\frac{d^2\mathbf{B}}{dt^2} - 2\nabla\mathbf{B} + \frac{\zeta}{M_P} f_0 \frac{d\mathbf{B}}{dt} = 0 \tag{6.28}$$

which eventually leads to the following equation for the left/right circularly polarised light [127]:

$$\frac{d^2 B_{+(-)}}{dt^2} + \frac{\bar{f}_0}{M_P}\frac{dB_{+(-)}}{dt} + k^2 dB_{+(-)} = 0, \tag{6.29}$$

where, $\bar{f}_0 = \zeta \, f_0$. The effect of parity violation is confined to the second term, which signifies either an enhancement or an attenuation, of the intensity of the observed electromagnetic wave, depending on the sign of $\bar{f}_0$ [127]. I shall not go into the details of this calculation. Instead, in the next section it will be shown that a similar effect also exists for gravity waves which might lead to some observational effects.

6.4 Behaviour of gravitational waves

First, let us discuss the gravitational analogue of the rotation of plane of polarisation (optical activity, equation (6.26)) discussed above [119]. This arises due to the parity conserving term of the form $\Phi_H \mathrm{tr}(R \wedge R)$ in equation (6.15). First note that the augmentation of H in (6.7) implies that the $\mathrm{tr}(R \wedge R)$ term contributes an additional term to the Einstein equation over and above the energy-momentum tensor of the KR field. Formally,

$$\mathcal{G}_{\mu\nu} \;=\; \frac{8\pi}{M_P^2} T_{\mu\nu} \;+\; \frac{16\pi}{M_P^3} \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x' \sqrt{-g}(x') \; \Phi_H(x') \; R_{\rho\lambda\sigma\eta}(x') \; {}^*R^{\rho\lambda\sigma\eta}(x') \; , \quad (6.30)$$

where,

$$T_{\mu\nu} \;=\; H_{(\mu|\tau\rho} \; H_{\nu)}{}^{\tau\rho} \;-\; \frac{1}{6} g_{\mu\nu} H^2 \; . \quad (6.31)$$

It has been established in [119] that the in the linearised approximation, the propagation of gravity waved in a homogeneous axion background is governed by (in large k limit, but in the Planckian regime $k < M_P$ with $16\pi k f_0/M_P^3 << 1$ as an expansion parameter):

$$\left[\frac{d^2}{dt^2} \;+\; k^2 \;+\; 8\pi f_0^2/M_P^2 \;\mp\; 1024\pi^2 k f_0^3/M_P^5\right] \varepsilon_\pm \;\simeq\; -\; 8\pi f_0^2 \; (1 \mp 16\pi k f_0/M_P^3)/M_P^2 \; . \quad (6.32)$$

We can now read off the dispersion relation

$$\omega_\pm^2 \;=\; k^2 \;+\; 4\pi f_0^2/M_P^2 \;\mp\; 1024\pi^2 k f_0^3/M_P^5 \quad (6.33)$$

whence the group velocity is $v_{g\pm} \;=\; 1 \;+\; O(k^{-2})$ and the phase velocity is given by $v_{p\pm} \;=\; 1 \;\mp\; 512\pi^2 (f_0^3/M_P^5 k)$ As in the electromagnetic case, the rotation of the polarization plane for gravitational waves is given by

$$\Delta\Psi_{grav} \;\simeq\; 1024\pi^2 \frac{f_0^3}{M_P^5} \; \Delta t \; . \quad (6.34)$$

With the limits on f_0 given in the previous subsection, it is very small $O(10^{-30})$. However, since the tensor perturbations characterizing the gravitational wave do not get randomized, so the effect is in principle observable.

Let us now restrict ourselves to the parity violating term of the form $\Phi_H \mathrm{tr}(R \wedge {}^*R)$. Quite striking differences are seen with the new term. The electromagnetic analogue of this term has been discussed in [127,128] and reviewed in equation (6.29). In contrast to the rotation of plane of polarisation for gravity waves as observed above, equation (6.33), new consequences may be expected. First, the effective action can be written as:

$$\mathcal{G}_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^3} \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x' \sqrt{-g}(x') \; \Phi_H(x') \; R_{\rho\lambda\sigma\eta}(x') \; R^{\rho\lambda\sigma\eta}(x') \; , \quad (6.35)$$

where,

$$T_{\mu\nu} = H_{(\mu|\tau\rho} \, H_{\nu)}{}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2 \; . \quad (6.36)$$

Let us consider Einstein equation in a linearized approximation. It is done decomposing the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with the fluctuation $h_{\mu\nu}$ being considered small so that one need only retain terms of $O(h)$ in the Einstein equation. Further the Lorenz gauge condition $h_{\mu\nu},{}^{\nu} = \frac{1}{2} h,_{\mu}$ is imposed on the fluctuations $h_{\mu\nu}$. The axion field Φ_H is treated as a homogeneous background satisfying eq. (6.23). In the whole analysis I will restrict to the lowest inverse power of the Planck mass for which a nontrivial effect is obtained. Next, ignoring terms on the RHS of the axion field equation and set

$$\Box \Phi_H = 0 \quad (6.37)$$

Since in Lorenz gauge and not all components of $h_{\mu\nu}$ are independent it remains to extract only physical modes. In fact, the only physical degrees of freedom of the spin 2 field are contained in h_{ij}, for which let choose a plane wave ansätz traveling in the z- direction,

$$h_{ij} = \varepsilon_{ij}(t) \; \exp \; -ikz \; . \quad (6.38)$$

The Latin indices above correspond to spatial directions. The other components of $h_{\mu\nu}$ can be gauged away, so that their field equation need not be considered. The only non-vanishing polarization components can be chosen to be $\varepsilon_{11} = -\varepsilon_{22}$, $\varepsilon_{12} = \varepsilon_{21}$; from these the circular polarization components can be constructed as in the Maxwell case: $\varepsilon_{\pm} \equiv \varepsilon_{11} \pm i\varepsilon_{12}$.

Again, I will assume that the scalar field is homogeneous and it has only time dependence so that $d\Phi/dt =: f_0$ is a constant. The equation of motion for the h_{ij} can be determined in a straightforward manner:

$$\Box h_{ij} = -\frac{16\pi}{M_P^2}[-(\eta_{ij} + h_{ij})f_0^2] - \frac{16\pi}{M_P^3}\zeta\,[\,f_0\,\Box h_{ij,\,t} + \Phi_H \Box\Box h_{ij}\,] \tag{6.39}$$

Now, to facilitate the calculation, let us make some simplified assumption and notations. First, as seen from the previous section, let us define the dimensionless quantity $\alpha := (f_0/M_P^2) << 1$. Secondly, we shall remain in the Planckian regime but the wave number k is such that the dimensionless quantity $\beta := k/M_P$ is small (let us say $O(10^{-5})$). The modulus of the field Φ_H is taken to be order 1. The previous equation now reduces to:

$$\frac{d^2\epsilon_{ij}}{dt^2} + 16\pi\,\alpha\zeta\,\beta\,k\frac{d\,\epsilon_{ij}}{dt} + k^2(1 - \frac{16\pi\alpha^2}{\beta})\,\epsilon_{ij} = \frac{16\pi\,f_0^2}{M_P^2}\,\eta_{ij} \tag{6.40}$$

This is an equation for a damped oscillator with a forcing term. The system can get damped or can sustain gravity waves. This depends on the value of the "$(b^2 - 4ac)$" term which here is given by:

$$2ik\,[1 - \frac{16\pi\alpha^2}{\beta} + \frac{(16\pi\,\alpha\,\zeta\,\beta)^2}{4}]^{1/2} \tag{6.41}$$

Let us list the various possible cases. First, when $\alpha^2/\beta\ \geq 1$, *i.e.* small values of k (note that the third term in (6.41) is very small, with the value of β, it is of the order of 10^{-15} smaller compared to the second term and will not contribute appreciably), one gets the scenario where the gravity waves dampen and is not observed:

$$h_{ij}(t,z) = \exp(\,-\frac{16\pi\alpha\,\zeta k}{M_P}\,)\,[A_{ij}\,e^{\bar{k}t - ikz} + B_{ij}\,e^{-\bar{k}t - ikz}] \tag{6.42}$$

Second, consider the case when $\alpha^2/\beta < 1$ (*i.e.* large values of k). Then, the solutions of the equation (6.40) are:

$$h_{ij}(t,z) = \exp(-\frac{16\pi\,\zeta\,\alpha\,k}{M_P})\,[\,A_{ij}\,e^{ikt-ikz} + B_{ij}\,e^{-ikt-ikz}] \tag{6.43}$$

This is the standard solution where the wave proceeds sinusoidally. It is clear that the solution to this equation can give attenuation/amplification of amplitude of gravity waves. To see this, choose $\zeta = +1$ then the equation (6.43) leads to attenuation of gravity waves whereas for $\zeta = -1$, one gets amplification of gravity waves [104]. In short, in this case we do not see any rotation of plane of polarization of gravity wave, rather the attenuation/amplification of the wave during propagation is the result of such an interaction. Such phenomena for gravity waves was suggested in [142] which however was largely phenomenological. If such effects are present, they have implications for CMB spectrum. They lead to non-zero cross-correlation in multipole moments C_l^{TB} and C_l^{EB}. Such effects cannot be induced by Faraday rotation (if there is any intervening magnetic fields). This is because it is an anisotropic effect which will also change l. With the Planck data coming up, one expects to see some of these effects or if these are not seen, the experiments can be used to put bounds on the coupling constants for these interactions.

6.5 Quantum gravity effects for the higher derivative Lagrangian

In this section I will study the effects of quantum fluctuations of different fields for a theory governed by the action (6.21) by calculating the one-loop effective potential using loop-expansion scheme [24]. I will concentrate on the gravitational part of the action only. Effective-potential serves as a useful tool to investigate the vacuum structure of such a theory where one can define the theory to be valid upto an energy scale (Planck energy) through cut-off and make predictions treating it as an effective theory. As has been argued in [148], in a theory with *anomolous* terms like that considered earlier, the cosmological constant may become a space-time dependent quantity. The quantum fluctuations also affect the CMB spectrum which differs significantly from simple inflationary models leading to constraints from observational data. Indeed, the bi-spectrum, tri-spectrum and the non-gaussianities of the calculated CMB spectrum can lead to newer understanding. For such a reason, we devote this section to the calculation of the effective potential which is the first step to the calculation of parameters in the inflatory models.

To keep the matters very general, I will consider a theory of gravitation coupled with three different kinds of matter fields. The Einstein term is minimally coupled with a massive/massless scalar field ϕ_S which has a self interacting potential. The action also contains an (axion) field ϕ_A coupled with a CP-odd term $R_{\mu\nu\alpha\beta}\,{}^*R^{\mu\nu\alpha\beta}$ and another field ϕ which is coupled to the CP-even term $R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta}$. In Euclidean signature, the Lagrangian of the theory is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{g1} + \mathcal{L}_{g2} + \mathcal{L}_{g3} + \mathcal{L}_m \\ &= -\frac{1}{\kappa^2}\,R + a\,\phi\,R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta} + b\,\phi_A\,R_{\mu\nu\alpha\beta}\,{}^*R^{\mu\nu\alpha\beta} \\ &+ \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi_A\,\partial_\nu\phi_A + \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi_S\,\partial_\nu\phi_S + V(\phi_S)\end{aligned} \tag{6.44}$$

where $\kappa^2 = 16\pi G$ and a, b are coupling constants which can be specified later (they are M_P^{-3}). Let us now turn to calculate the effective potential. For that purpose, we first expand the metric $g_{\mu\nu}$ around a flat background:

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa\,h_{\mu\nu}, \tag{6.45}$$

where $\delta_{\mu\nu}$ is a flat background and the fluctuations $h_{\mu\nu}$ are small, $|h_{\mu\nu}| < 1$. For the decomposition (6.45), the inverse of the metric is

$$g^{\mu\nu} = \delta^{\mu\nu} - \kappa\,h^{\mu\nu} + \kappa^2\,h^{\mu}{}_{\lambda}\,h^{\lambda\nu} + \ldots \tag{6.46}$$

Furthermore, the determinant of the metric, which will be needed in the following, will be given by:

$$(g)^{\frac{1}{2}} = 1 + \frac{1}{2}h^{\alpha}{}_{\alpha} - \frac{1}{4}h^{\alpha}{}_{\beta}\,h^{\beta}{}_{\alpha} + \frac{1}{8}\left(h^{\alpha}{}_{\alpha}\right)^2 + \ldots \tag{6.47}$$

To calculate one-loop effective potential we need to expand the Lagrangians only upto quadratic order in the $h_{\mu\nu}$. The expansions are listed below:

$$\sqrt{g}\,\mathcal{L}_{g1}=\sqrt{g}\,R \;=\; -\frac{1}{4}\partial_\alpha h_{\mu\nu}\,\partial^\alpha h^{\mu\nu}+\frac{1}{4}\partial_\alpha h\,\partial^\alpha h-\frac{1}{2}\partial_\alpha h\,\partial_\beta h^{\alpha\beta} +\frac{1}{2}\partial_\alpha h_{\mu\beta}\,\partial^\beta h^{\mu\alpha}+\text{total derivatives} \qquad (6.48)$$

The expressions for the other two terms are long. However, they are given below. First,

$$\begin{aligned}\sqrt{g}\,\mathcal{L}_{g2} &= \sqrt{g}\,a\,\phi\,R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta}\\ &= a\,\kappa^2\,(\partial_\nu\partial_\rho\phi\,h_{\mu\sigma}\,\partial^\nu\partial^\rho h^{\mu\sigma}+\partial_\rho\phi\,h_{\mu\sigma}\Box\,\partial^\rho h^{\mu\sigma}+\phi\,h_{\mu\sigma}\Box\Box\,h^{\mu\sigma}\\ &+ \partial_\nu\partial_\rho\phi\,h_{\mu\sigma}\partial^\mu\partial^\sigma h^{\nu\rho}+\partial_\rho\phi\,h_{\mu\sigma}\,\partial^\mu\partial^\sigma\partial_\nu h^{\nu\rho}+\phi\,h_{\mu\sigma}\,\partial^\mu\partial^\sigma\partial_\nu\partial_\rho h^{\nu\rho}-2\,\partial_\nu\partial_\rho\phi\,h_{\mu\sigma}\,\partial^\nu\partial^\sigma h^{\mu\rho}\\ &- 2\,\partial_\rho\phi\,h_{\mu\sigma}\Box\,\partial^\sigma h^{\mu\rho}-2\,\phi\,h_{\mu\sigma}\Box\,\partial^\sigma\partial_\rho h^{\mu\rho}) \qquad (6.49)\end{aligned}$$

and

$$\begin{aligned}\sqrt{g}\mathcal{L}_{g3} &= \sqrt{g}\,b\,\phi_A\,R_{\mu\nu\alpha\beta}\,{}^*R^{\mu\nu\alpha\beta}\\ &= 2b\,\kappa^2\left\{\partial_\lambda\partial_\sigma\phi_A\,\partial_\alpha\partial^\lambda\,h^\rho_\beta h_{\rho\eta}+\partial_\sigma\phi_A\,h_{\rho\eta}\Box\,\partial_\alpha h^\rho_\beta-\partial_\lambda\partial_\sigma\,h_{\rho\eta}\partial_\alpha\partial^\rho h^\lambda_\beta\right\}\epsilon^{\alpha\beta\sigma\eta} \qquad (6.50)\end{aligned}$$

Note that due to the presence of a Levi-civita tensor which is completely anti-symmetric in it's indices, only three terms will survive in the expansion of $\mathcal{L}_{g3}$, Since to calculate one-loop effective potential, one only needs terms of order 2 in fluctuations. To obtain one-loop effect, it is sufficient to choose spacetime independent saddle points for the scalar (and pseudo-scalar) fields;

$$\phi(x)=\phi_0+\Phi(x);\;\;\phi_A(x)=\phi_{A0}+\Phi_A(x);\;\;\phi_S(x)=\phi_{S0}+\Phi_S(x)$$

With these choices, the derivative terms of the scalar fields will not contribute to the resulting Lagrangian (expanded about the saddle points). The Lagrangian relevant for calculating one loop effective potential is by invoking the transverse-traceless gauge [45, 149]. With $\partial_\mu h^{\mu\nu}=0$

and $h = 0$, the relevant part of the Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_{rel} &= \frac{1}{4} h_{\mu\nu}(-\Box_E)\, h^{\mu\nu} + a\,\kappa^2\,\phi_0\, h_{\mu\nu}\,\Box_E\,\Box_E\, h^{\mu\nu} - \frac{1}{2}\Phi_S(-\Box_E + V''(\phi_{S0}))\Phi_S - V(\phi_{S0}) \\ &- \frac{1}{4}\,\kappa^2\, h_{\mu\nu} V h^{\mu\nu} + \frac{1}{2}\Phi\,(-\Box_E)\Phi + \frac{1}{2}\Phi_A\,(-\Box_E)\Phi_A, \end{aligned} \tag{6.51}$$

where $\Box_E$ is the operator in Euclidean space. Since the perturbation is around a flat background and a choice of linear gauge is being made, ghosts don't appear in this case [46,47,150]. However, although the higher derivative quantum gravity bare action contains massive negative norm states at tree level, whether they will spoil the unitarity of S matrix or not is inconclusive because quantum corrections may destabilize the ghosts [151,152]. Moreover, from the effective field theory description of gravity these issues can be sidelined [140,141]. The mass of the ghost fields are of the order of Planck mass, they will not be excited below the Planck scale [153,154] and here, we are dealing with a theory below that energy scale.

Note here that the (axion) field Φ_A has no contribution to the one-loop effective potential. Now, eqn (6.51) may be conveniently written as

$$\mathcal{L}_{rel} = \frac{1}{2} h_{\mu\nu}\mathcal{O}^{\mu\nu\alpha\beta} h_{\alpha\beta} + \frac{1}{2}\Phi_S(-\Box_E + V''(\phi_{S0}))\Phi_S + \frac{1}{2}\Phi\,(-\Box_E)\Phi + \frac{1}{2}\Phi_A\,(-\Box_E)\Phi_A \tag{6.52}$$

where the operator

$$\mathcal{O}^{\mu\nu\alpha\beta} = \frac{1}{2}\delta^{\mu\alpha}\delta^{\nu\beta}\left[-\Box_E + 2a\kappa^2\,\phi_0\,\Box_E\Box_E - \kappa^2\, V(\phi_{S0})\right] \tag{6.53}$$

Now, let me again rewrite the Lagrangian in terms Ψ_i where $i = 1, 2, ...10$ denotes ten independent components of $h_{\mu\nu}$ [102] and calculation of CW potential will proceed similarly as shown in the last chapter.

$$\mathcal{L}_{rel} = \frac{1}{2}\Phi(-\Box_E + V^{''}(\phi_{S0}))\Phi + \frac{1}{2}\Psi_i M_{ij}\Psi_j, \tag{6.54}$$

where the following index correspondence: $\mu\nu \to i$ and $\alpha\beta \to j$ is employed. The operator for scalar field is trivial. The eigenvalues of the matrix M are,

$$\begin{aligned} \lambda_i &= -\frac{1}{2}(k^2 + 4a\kappa^2\phi_0 k^4 - \kappa^2 V) \; ; (1 \le i \le 4) \\ \lambda_i &= \left(k^2 + 4a\kappa^2\phi_0 k^4 - \kappa^2 V\right) \; ; (5 \le i \le 10) \end{aligned} \tag{6.55}$$

The one-loop effective potential is given by

$$V^{(1)}_{\text{eff}} = V(\phi_{S0}) + \frac{1}{2}\text{Tr}\ln(k^2 + V'') + \sum_{i=1}^{10}\frac{1}{2}\text{Tr}\ln\lambda_i \;, \tag{6.56}$$

where Tr is the functional trace. Performing the momentum space integrals and introducing a cut-off we obtain the unrenormalized one-loop effective potential [104]

$$\begin{aligned} V_{\text{eff}}(\phi_{S0},\phi_0) &= \frac{5}{16\pi^2}\left[\left(\frac{\Lambda^4}{2} - \frac{1-2eg}{4e^2}\right)\ln\frac{e\Lambda^4}{g} + \frac{\Lambda^2}{2e} + \frac{g}{2e} - \frac{1}{4e^2} + \frac{\sqrt{1-4eg}}{4e^2}\ln\left(\frac{1+\sqrt{1-4eg}}{1-\sqrt{1-4eg}}\right)\right] \\ &+ \frac{\Lambda^2 V''}{32\pi^2} + \frac{V''^2}{64\pi^2}\left(\ln\frac{V''}{\Lambda^2} - \frac{1}{2}\right) + V(\phi_{S0}) \end{aligned} \tag{6.57}$$

where $e = 4\phi_0 a\kappa^2$ and $g = -\kappa^2 V$, Λ^2 is the momentum cutoff. If we put the expressions of e and g back into the above expression the effective potential is seen to have an imaginary part [104]:

$$\begin{aligned} V_{\text{eff}}(\phi_{S0},\phi_0) &= \frac{5}{16\pi^2}\left[\left(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2a^2} - \frac{\Lambda^4}{2}\right)\ln\frac{V}{\Lambda^4} + \frac{\Lambda^2}{8\kappa^4\phi_0 a^2} - \frac{V}{2} - \frac{1}{64\kappa^4\phi_0^2a^2}\right. \\ &+ \left.\frac{\sqrt{1+8\kappa^4\phi_0 aV}}{64\kappa^4\phi_0^2a^2}\ln\left(\frac{1+\sqrt{1+8\kappa^4\phi_0 aV}}{1-\sqrt{1+8\kappa^4\phi_0 aV}}\right)\right] + \frac{5i}{16\pi}\left(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2a^2} - \frac{\Lambda^4}{2}\right) \\ &+ \frac{\Lambda^2 V''}{32\pi^2} + \frac{V''^2}{64\pi^2}\left(\ln\frac{V''}{\Lambda^2} - \frac{1}{2}\right) + V(\phi_{S0}) \end{aligned} \tag{6.58}$$

It is interesting to see here again that an imaginary part is generated in the effective potential. This is very much similar to the one found in the case of where a single scalar field is coupled to gravity [45, 96]. The imaginary part of the effective potential signifies that we have chosen an unstable vacuum, in fact flat space is not a stable vacuum of this theory. It is perhaps due to the presence of a constant scalar(pseudo) field.

The calculation of effective potential here done in conventional approach which is not devoid of gauge ambiguities. However, as already mentioned that Vilkovisky-DeWitt (VD) [40, 41] approach of deriving effective potential is free from any ambiguities related to gauge-fixing condition or parameterization of the theory. We don't employ the method of VD here, although quite a number of papers have already been in the literature which calculate the effective potential in VD approach for ordinary and higher derivative gravity [47, 48, 155]. The key point is, imaginay part is still present in the VD effective potential [47] which indicates that it is not a gauge artifact. VD effective potential for the theory under consideration may be taken as a future project.

6.6 Discussions

In string theory, the Kalb-Ramond field acts as a source term for torsion which has various interactions with gauge fields. In order that the interactions are gauge invariant, the Kalb-Ramond field $B_{\mu\nu}$ must be endowed with non-trivial transformations under gauge fields. This leads to some interesting interactions with observable consequences. One of them is the rotation of plane of polarization for electromagnetic and gravity waves. These had been studied earlier and have been matched with experimental results. However, these interactions are not the only possible ones. One can have additional ones which arise from the gauge invariant coupling of higher form fields to torsion. Such interaction was proposed in [127] and that has been corrected here. Also the interaction term has been extended for non-abelian fields and gravity. Observational consequences of such interactions are all together different. They lead to amplification/attenuation of electromagnetic or gravity waves and have important implications for anisotropy of the Cosmic Microwave Background (CMB) by spatial parity violation [142]. For such parity breaking term, one can get certain non-vanishing multipole moment correlations between the temperature anisotropy and polarization of the CMB. In the CMB data, one usually observes correlations like $C_l^{TT}, C_l^{EE}, C_l^{BB}$ and C_l^{TE} which arise from parity conserving interactions. On the other hand, cross-correlations like C_l^{EB} and C_l^{TB} arise from parity violating interactions from which bounds on the strength of such parity violating terms can be ascertained. The Coleman-Weinberg mechanism for such extended theory leads to a potential which might have some significance in the early universe and inflation. Initial studies with this potential show that one can generate the requisite number of e-foldings from such a theory near the Planck scale. Other consequences from such a potential requires further study.

Let us now discuss the possible origin of the new augmented terms introduced in (6.16). The first and probably the most compelling one is that such terms are necessary to form gauge invariant coupling of higher form gauge fields to torsion or Kalb-Ramond fields. However in the literature, any explicit reference to such terms has not been found by anyone in any low energy string effective action. It has been shown that it is possible to embed such terms in a supersymmetric theory [128]. For a second argument to support the claim which has been made, in the appendix, a derivation of the requisite term from the boundary symplectic potential (associated to the standard first order Yang-Mills action) is done which shows that such terms can arise quite generically. The third argument is from the point of view of effective field theory. Since the distance scales which have been considered in this entire chapter are large or cosmological it is precisely in the realm of effective field theory . The length scales at which the quantum effects are studied here are much larger than the ultraviolet cut-off scale of gravity [141, 148, 156]. And thus, such effective theories, donot require the knowledge of precise details of the interaction of the newer degrees of freedom at the Planck scales. Inspite of that, semiclassical effective theories capture the universality of interactions. The object of the semiclassical theory is to consider the spactime to be classical but the matter fields to be quantum mechanical. If the Planck's constant is not vanishing, the stress-energy tensor which is now a quantum operator, has a quartic divergence. Upon renormalisation, it arises that GR is a effective quantum field theory if one augments the standard action by the the trace anomaly terms. Terms such as $R \wedge {}^{*}R$ are precisely the stress-tensor anomalies arising during quantization of massless scalar fields in curved spacetime [157]. As it turns out, the quantum effective action is actually non-local but can be made local through introduction of scalar fields. Interestingly, the scalar fields in equation (6.44) play this role. In the bottom-up scenario of effective field theory, these scalar fields, which were absent in the original action, arise when one goes up the energy scale. Conversely, when one comes down in the energy scale from a Planck scale, which is what is done in string theory, one also expects to recover this action at some energy scale in

4−dimensions. Precisely due to this reason, one expects some way to generate the full action (6.16) from any string theory.

As already mentioned in the previous chapter that a gauge-free formulation of interacting gravitons is due, in this chapter again, no attempts have been made to make the interactions inert under gauge transformations. In this chapter, emphasis is given on the importance of gauge invariance in determining the dynamics (gauge invariant couplings) of these class of theories.

6.7 Appendix

In this appendix, the existence of the extra term of the form $(A \wedge {}^*F)$ added to the KR field in equation will be deduced. The question is: where to look for such terms? To motivate, let us recall that the usual Chern-Simons term (Ω_{YM}) augmented to the KR field strength H in equation (6.7) is actually a boundary term. In the $U(1)$ version for example, the Chern-Simons term (Ω_{YM}) reduces to $(A \wedge F)$ which is precisely the contribution to boundary term corresponding to $(F \wedge F)$ in $U(1)$ gauge theory. In the same token, let us look for the boundary terms for the action itself. Moreover, the usual Chern-Simons is an anomaly cancelling contribution just like the new terms which arise due to stress-tensor anomaly. More precisely, the gravitational Chern-Simons arise from the axial gravitational anomaly whereas the gravitational analogue of the new term is related to stress-tensor anomaly. Thus, we expect to find a derivation of the new contribution in a similar way to that of the Chern-Simons term.

Consider the Lagrangian 4-form for the *free* Yang-Mills theory

$$L = \mathrm{tr}(F \wedge *F) \tag{6.59}$$

The on-shell variation of the Lagrangian gives

$$\delta L = 2\mathrm{tr}\, d(\delta\, A \wedge {}^*F) := d\Theta(\delta) \tag{6.60}$$

The term $\Theta(\delta)$ is a three form and is often called the symplectic potential. Now, consider the variation of the one-form A through a parameter μ, $0 \leq \mu \leq 1$ and define:

$$\delta_\mu A := A\,\delta\mu \quad \text{and} \quad A_{(\mu)} := \mu\, A \quad \text{so that} \tag{6.61}$$

$$^*F_{(\mu)} = \mu^* F + (\mu^2 - \mu)^*(A \wedge A) \tag{6.62}$$

This implies that

$$\Theta(\delta_\mu) = 2tr\,(A \wedge {}^*F_{(\mu)})\,\delta\mu \tag{6.63}$$

Thus, on-shell, the above equation (6.63) is equivalent to:

$$\frac{\delta}{\delta\mu}\text{tr}(F \wedge {}^*F) = 2\,d\,tr\,[\mu\, A \wedge {}^*F + (\mu^2 - \mu) A \wedge {}^*(A \wedge A)] \tag{6.64}$$

Integrating with respect to μ, we get

$$\begin{aligned} \text{tr}(F \wedge {}^*F) &= d\,tr\,[A \wedge {}^*F - \frac{1}{3}\,A \wedge {}^*(A \wedge A)] \\ &= d\,tr\,[A \wedge {}^*dA + \frac{2}{3}\,A \wedge {}^*(A \wedge A)] \end{aligned} \tag{6.65}$$

Note that this term arises from a boundary contribution and is valid only on-shell. In contrast, the usual Chern-Simons term, which can be derived in a similar fashion from the other boundary term $\text{tr}(F \wedge F)$ only requires the Bianchi identity. In standard treatments, the boundary term vanishes by the boundary conditions on the fields. The above derivation is merely to show the existence of such terms in general when the field has all possible configurations.

Two comments are in order. Firstly, in the equation above, only the *free* Yang-Mills theory is considered. Now suppose that the Yang-Mills field is also coupled to other fields as the KR field $H_{\mu\nu\lambda}$ in the present paper. In that case, the equation of motion for the Yang-Mills field is not merely $D^* F^i = 0$, but has contributions from the KR fields too. One then needs to look for the modification due to presence of such terms also. Secondly, as mentioned earlier, if one wants not only to couple 1-form field to H field but also 2 and 3-form fields. In those cases, the term $[A \wedge {}^*(A \wedge A)]$ does not arise (and in not a 3- form). For this reason, in what follows, that term is disregarded altogether. From above construction the result is the following:

$$\delta \mathrm{tr}(F \wedge {}^*F) = d\, tr\, [A \wedge {}^*F] + \mathrm{tr}(\delta A \wedge D^* F) \tag{6.66}$$

For $\delta A^i = d\lambda^i + [A, \lambda]^i$, another term needs to be added to the first term. Thus in total, the contribution to the total derivative is:

$$\delta \mathrm{tr}(F \wedge {}^*F) = d\, tr\, [A \wedge {}^*F + \lambda D^* F] \tag{6.67}$$

To understand the effect of this term, let us restrict to $U(1)$ gauge theory for simplicity. For $U(1)$ gauge fields, the effect of this augmentation leads to:

$$H \to H = dB + \frac{1}{M_P}(A \wedge {}^*F + \lambda d^* F) \tag{6.68}$$

If we want that H remains invariant under $U(1)$ gauge transformation then, B must transform non-trivially under $U(1)$ gauge transformation. This can be easily found from the above equation:

$$\delta_\lambda B = \frac{1}{M_P} \lambda\, {}^*F \tag{6.69}$$

In the whole set up, equation of motion have never been explicitly used. Note that I have never added the term $\lambda d^* F$ in the equation. That is because only effects of order M_P^{-1} are looked upon here while the contribution of second term is of order M_P^{-2}.

6.8 Summary

In this chapter, firstly I have discussed the gauge invariant couplings of various form fields to torsion and have shown how these are constructed with special reference to electromagnetism and gravity. Next, I have reviewed the consequences of such interactions for the Maxwell fields and have extended them to gravity in the next section. Then, quantum effective-potential (Coleman-Weinberg potential) for a theory of gravity by including the modified interactions is computed. It is seen that inclusion of a parity violating scalar field (axion) doesn't have any effect in the one-loop effective potential of a theory where higher curvature terms are present. The constant scalar background again triggers infrared instability as the CW potential again develops an imaginary part.

Chapter 7

Conclusion

In this thesis quantum instabilities in the gauge and gravity coupled Higgs fields have been studied. Realization of Coleman-Weinberg mechanism in gauge theories in a way stabilizes or destabilizes the vacuum which mimics Higgs mechanism where an unstable potential is invoked to generate the masses of the vector and scalar bosons. The key message of this thesis is to emphasize the role of quantum infrared instabilities in the case of generating masses of the matter particles. Conventionally, the Higgs potential is chosen to have an instability due to the presence of a tachyonic mass term in the Lagrangian. However, the origin of such a term is yet to be determined. There must be some physical process which may trigger such phenomena. In the case of structure formation at the early Universe Jeans instability plays an important role which has also been realized in the case of gravitons interacting with thermally excited matters(scalar or fermions) or with space-time constant Higgs field. There could be similar kind of mechanism, to be discovered yet, which develops the instability in Higgs potential to provide masses to all the particles in the Universe.

Let us briefly summarize the main results of this thesis and discuss possible extensions of the works in different directions.

Abelian and non-abelian gauge theories have been shown to be expressible in terms of gauge-free variables. This obviates any need of gauge fixing to quantize these theories. Coleman-Weinberg mechanism has been made free of any gauge ambiguity with the gauge-free prescription

[79,80,87]. This was done rewriting the theory in terms of manifestly gauge-inert variables. This approach resolved the issue of gauge dependence of Coleman-Weinberg potential for scalar QED. It was shown that this theory gives a unique scalar to vector mass ratio [79]. However, one can still ask about reparametrization invariance of the effective potential which was taken care of by recombining the gauge-free theory with the Vilkovisky-DeWitt geometric method [40, 41, 79]. $SU(2) \times U(1)$ theory has been successfully rewritten in terms of completely gauge inert variables. Here one decomposes the matter sector into a radial and a phase part and makes use of a $SU(2)$ valued matrix to write the $SU(2) \times U(1)$ theory with the help of variables which are manifestly inert under gauge transformations [67, 87]. Radiative corrections generate masses for the vector bosons and Higgs boson without requiring any Higgs self coupling! This avoids the issue of 'naturalness' problem. However, this wonderful idea is unfortunately not useful for phenomenology as the mass for Higgs is turning out to be rather low. On the positive side, the theory has no Higgs self-coupling parameter, and the mass spectrum is completely determined by the gauge couplings with the renormalization scale chosen to reproduce the observed gauge boson masses.

Wide applications of effective potential in particle physics and cosmology have already been mentioned in the introduction of this thesis. There are several directions in which the research conducted in this thesis can be extended.

The first way of extending the work on gauge-free formulation of standard model is to formulate a gauge-free version of $SU(3)$ gauge theory which should be useful for the study of QCD. Apart from the possible extension of gauge-free framework in terms of Wilson lines demonstrated at the end of chapter (3) it will be interesting if one could write down a Lagrangian of $SU(3)$ Yang-Mills theory in terms of gauge-free variables using the same trick shown in chapter (4).

The second direction of research which may be carried out is to calculate the lower bound of Higgs mass in a gauge invariant way and check whether it is compatible with the LHC and Planck data. For this, one must adopt a gauge-free approach as has been pointed out by several authors [59, 60]. To achieve this one has to accommodate fermions in the gauge-free version of electroweak theory and calculate the one-loop RG improved effective potential in the gauge-free formalism demonstrated in the chapter (4). The finite temperature correction of SM effective potential in the gauge-free approach will be another interesting thing to study as it will have implications in early universe inflationary scenario.

While the gauge-free proposal suits well for standard electroweak theory, it is not yet fully developed for models of gravitons interacting with matter fields. Due to non-linearity of Einstein GR it is perhaps not a trivial task. One possible way to formulate the gauge-free version of theory of linearized gravity with or without interacting matter fields is using the tetrad formalism. Then, one has a theory with spin connections and tetrads which transform under local Lorentz group identically as the abelian and non-abelian gauge potentials of SM. Thus embedding the tetrad version of gravity into gauge-free framework does not appear to be too optimistic.

In the case of gravity coupled Higgs theory it has been shown that a tachyonic pole appeared in the effective graviton propagator for constant Higgs background in the infrared limit [96]. This type of instability was reported by Gross et al. [75, 98] in the case of gravitons in contact with a thermal bath. This on the other hand is happening at zero temperature and possibly the constant scalar background is acting as a heat bath itself. The presence of a tachyonic mode is indeed supported by the quantum effective potential of this theory which also contain an imaginary part. Finite temperature counterpart of one-loop effective potential shows a reinforcement of the imaginary part at high temperature limit. At low temperature it shows an oscillatory behaviour which eventually gets damped as one goes towards zero temperature reproducing the earlier result.

In this connection a possible line of future study could be to explore the same issue for the case where the background is curved. A computation of one-loop graviton corrected effective potential for a scalar field coupled minimally (or non minimally) to gravity in background de Sitter space would be interesting to study. The nature of the quantum vacuum of this theory will be useful to study early Universe scenarios like inflationary era where de Sitter space is supposed to be the background spacetime.

In the last chapter of this thesis a theory where scalar fields are coupled with some higher derivative terms in gravity is studied. The source of such couplings had a different motivation. These had arisen to construct gauge invariant coupling of different fields with torsion. I have studied this interaction and offered some astrophysical and cosmological predictions [104]. In the case of gravitational wave due to these new kind of interactions one could have attenuation/amplification of the amplitude of the wave. I also calculated explicitly the quantum effective potential in one-loop with a theory which includes these kind of interactions. I have shown that in the case of flat background there is an imaginary part in the effective potential which indicates again the vacuum instability.

The task still remain to examine which modes of the gravitational waves, produced due to such kind of interactions, exit the Hubble horizon. We can measure the primordial power spectrum which is independent of η (the epoch of horizon exit) directly by making a map of the temperature and polarization of the CMB over the whole sky. This will be a test for existence of such kind of parity violating interactions (axion-graviton) in the early Universe. With Planck having already begun to send data presence of such interactions is immediately tasteable now.

Another line of research could be to study the axion-graviton scattering cross section on the basis of the model which have been demonstrated in chapter (6). Theory suggests that axions were created abundantly during the Big Bang. Because of a unique coupling to the instanton field of the primordial universe (the "misalignment mechanism"), an effective dynamical friction is created during the acquisition of mass following cosmic inflation. This robs all such primordial axions of their kinetic energy. On the other hand it would be very interesting to compute the cross-section for axion-graviton exchange interactions or scatterings in perturbative approach. Theoretical motivation behind this kind of interactions already has been there as stated in the chapter (6); in fact, the interaction which is considered for the gravitational wave case can be used to compute such cross sections. This model could also tell us about the fact why axions are so less abundant today. This may be due to the reason that in the early Universe (high energy scale) the cross-section for the process where axions got converted to gravitons was low and at low energy or recently it became very high to match the observed scenario of today.

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1. *Gauge-free Coleman-Weinberg Potential*

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6. *Infrared Issues in Graviton Higgs Theory*

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7. *Gauge-free Electrodynamics*

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