ASPECTS OF INFLATIONARY MODELS IN SUPERGRAVITY

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I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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Kumar Das

Dedicated to my most beloved

Father and Mother

To you my infinite gratitude

and

countless salutations

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CONTENTS

SYNOPSIS

The overwhelming curiosity for understanding our universe at its deepest and fundamental level has given birth to the laws of physics. These laws are primarily divided into the laws of physics at small and at large length scales. While the former belongs to the realm of particle physics, cosmology usually deals with the physics at very large length scales above the sizes of galaxies. But, according to the present understanding the cosmological structures that we see today originated when the energy scale of the Universe was very high. Therefore, the study of cosmological structures probes the physics at very high energy much above the TeV energy currently accessible by the collider experiments.

The Big Bang theory is a well established cosmological model for the Universe from the earliest known periods of MeV temperature through its subsequent evolution upto the present epoch. However the observations of Cosmic Microwave Background (CMB) radiation points out that the classical Big Bang picture is incomplete. It fails to explain why the Universe is so smooth at the level of 10^{-5} on scales that according to the standard Big Bang picture had never been in causal contact. Also in the standard Big Bang theory a spatially flat universe, favored by the current observations is unstable. These two issues are known in the literature as the flatness problem and the horizon problem respectively [1].

The paradigm of cosmic inflation is a way to dynamically resolve these issues by introducing a phase of very rapid expansion in the early Universe [6, 5, 7]. Apart from solving the flatness and horizon problem, the idea of cosmic inflation also explains the origin of large scale structures in our universe due to the quantum fluctuations of the inflaton field responsible for driving inflation [9, 10]. These fluctuations led their imprints in the CMB temperature anisotropy that we see today [15].

In the standard approach where inflation is driven by an unknown scalar field, the potential energy of the field dominates over its kinetic energy leading to an exponential expansion of the universe. Inflation stops when the kinetic energy of this field becomes comparable to its potential energy. This picture is referred to as slow roll inflation [7, 159]. Typically inflation happens much above the energy scales to be probed by the present day or future accelerators. Therefore, to unravel the mystery of early universe it is inevitable to consider particle physics and cosmology in a common framework.

At present the Standard Model of particle physics summarizes our current knowledge of physics upto the sub-atomic scale. The predictions of this model have been tested to a very high precision upto the scale of TeV. However, the Standard Model still leaves some issues unanswered. One of the most important of them is that why the mass of the Higgs particle, which is an elementary scalar of the theory, is not protected form receiving quantum corrections? Additionally we must also explain the existence of Dark Matter (DM) and how baryon number violation leads us to our existence. These are important motivations to consider the physics beyond the standard model (BSM). One very promising candidate in that direction is supersymmetry (SUSY). However since the idea of inflation necessarily incorporates the physics of gravitation, the natural BSM framework to describe inflation is supergravity which is a gauged (local) version of supersymmetry. The main aim of this dissertation is to study inflation from the standpoint of supergravity (SUGRA) [21, 22] as an effective theory. Here we have studied a few models of inflation based on the theoretical framework of supergravity. We have also explored phenomenological consequences of these models.

In the simplest supergravity set-up, a model with chiral superfields is specified by a Kähler potential term which is a real function of the chiral superfields and a holomorphic superpotential term [21, 34]. In terms of these two functions one can compute the supergravity F-term scalar potential of the inflaton field. In one work we took the idea of how monomial inflation is embedded in supergravity and proposed a model of N-flation [25]. In the case of N-flation model, there are N fields with sub-Planckian vevs that collectively drive inflation [109]. In our case also N number of fields contribute to drive inflation where each field having a quadratic mass term potential of chaotic inflation. We considered inflaton to be a member of the complex chiral superfield and for simplicity we have assumed it to be a singlet under the relevant gauge group. This is a generalisation of single-field chaotic inflation scenario in supergravity [85]. Considering that the individual field range is sub-Planckian, the effective description in supergravity is well under control as long as we demand that the imposed symmetry is not broken by the ultraviolet degrees of freedom. An interesting feature of our construction is that despite of the presence of field interactions among themselves, in the cosmological background they collectively behave like a single degree of freedom without any interactions. This is true only because of the particular nature of the interactions dictated by the proposed form of the model. This conclusion was first established analytically and numerically with a system of two fields and then considering its subsequent generalization to N number of fields analytically.

Although the above study is an example amongst many attempts to embed inflation in an effective theory description, there are various challenges on the theoretical front. One such issue is that inflationary slow-roll conditions are sensitive to the Planck-suppressed interactions. This sensitivity of inflation to the Planck-scale details demands its treatment in a UV complete theory [34]. String theory is considered to be a quantum theory of gravity where one can have precise control of these interactions. Now in string theory constructions moduli fields appear generically and their decay constant is suppressed by the Planck mass. Therefore, in another work we have studied the sensitivity of inflationary observables in the context of late time non-thermal history of the universe dominated by moduli fields. Due to vacuum misalignment the minima of the moduli fields during inflation are different from the minima after inflation. The presence of moduli implies non-standard post-inflationary cosmological time-line [167, 168, 170]. The consequence of this modified post-inflationary history is that the preferred range in the central value of the number of e-folds during observable inflation is dependent upon the moduli masses [46]. In this work we analyzed four representative models of inflation, namely chaotic inflation, axion monodromy, natural inflation and Starobinsky inflation by taking the mass of the lightest modulus as a free parameter [47]. We kept the displacement of the modulus from its post-inflationary minimum to a generic value. Finally we found out the observable parameters for those models in terms of this new preferred range of e-folds. The results, following Planck 2015 data [15], imply significant alteration in the predictions for these well known models. Future experiments, which are supposed to bring down the uncertainties in the measurement of observables, will make our analysis more relevant.

While concerning inflationary model building in supergravity framework, the main obstacle is that any scalar field including inflaton receives large supergravity contributions to its effective mass. The η-problem of supergravity F-term inflation with minimal coupling to gravity is a well-known example in this regard [83, 84]. In recent years there has been a lot of interests in connection to the supergravity models based on conformal symmetry where this problem is made explicit in a suitable Jordan frame. This particular Jordan frame is characterized by the canonical kinetic terms for the fields. In another work we have studied the formulation of inflation models in this Jordan Frame supergravity framework. Together with the superpotential and the Kähler potential functions, a supergravity model in this Jordan frame is specified by a frame function that is related to the Kähler potential of the theory. The scalar potential in this set-up separates into a globally supersymmetric part and a part that contains supergravity corrections. The later may pose serious threat to the desired flatness of the inflaton potential. Here we showed that if the F-term of an auxiliary field (not inflaton) is dominating the vacuum energy during inflation then the supergravity corrections to the Jordan frame scalar potential are generically suppressed. Moreover these corrections vanish if the superpotenial vanishes along the inflationary trajectory. However if the F-term of the inflaton field dominates the vacuum energy during inflation, these corrections are comparable to the global supersymmetric part of the potential. In addition the phenomenology of some representative models are discussed and relation to the recently much practised cosmological attractor models are made.

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ABBREVIATIONS

BBN	Big Bang Nucleosynthesis
BSM	Beyond the Standard Model
BICEP	Background Imaging of Cosmic Extra-galactic Polarization
CDM	Cold Dark Matter
CL	Confidence Label
CMB	Cosmic Microwave Background
CSS	Canonical Superconformal Supergravity
dS	de-Sitter
FI	Fayet-Iliopoulos
FLRW	Friedmann-Lemaître-Robertson-Walker
GeV	Giga Electron Volt
GUT	Grand Unified Theory
KKLT	Kachru Kallosh Linde Trivedi
KL	Kallosh Linde
LIGO	Laser Interferometer Gravitational-Wave Observatory
MeV	Mega Electron Volt
MSSM	Minimally Supersymmetric Standard Model
pNGB	pseudo-Nambu Goldstone Boson
SDSS	Sloan Digital Sky Survey
SEC	Strong Energy Condition
SUGRA	Supergravity
SUSY	Supersymmetry
TeV	Tera Electron Volt
UV	Ultra-Violet
VEV	Vacuum Expectation Value
WMAP	Wilkinson Microwave Anisotropy Probe

CHAPTER 1

INTRODUCTION

We start by presenting a very colloquial overview of the thesis. This introductory discourse is aimed at a compact description of the background ideas and motivations as regard to the main subject matter. The details and related attributes are treated in a regulated fashion for completeness. Finally we also gave a gist of the subsequent chapters.

1.1 General background

Human race was always curious to understand what fulfills their surroundings. From the dawn of civilization, there had been a constant endeavour to unravel the deepest secrets of nature. Little did we know then of nature's grand design. But with great perseverance and primitive tools our ancestors dreamt of breaking the code of creation. Modern mind is now ready to invite the challenges of penetrating one of the biggest mystery — *How the forces and laws of nature gave birth to this wonderful creation of stars and galaxies we see today?* The rapid growth in science and technology enables us to elucidate many things concerning the complexities of the cosmos, which at times past, thought to just had been manifested through accidental ways. Cosmology is that branch of science which systematically deals with our quests regarding the Universe and its diverse aspects. The streamline agendum of this field of study is to satisfy us by offering answers to the questions relating to the creation of celestial structures around us.

In the simplest picture of Big Bang cosmology, the Universe contains a cosmological constant (Λ) along with cold dark matter (CDM) and dark energy. This description is known as ΛCDM or concordance model, which provides theoretical narration of most of the evolutionary phases of our Universe [1, 2]. It is also frequently referred to as the standard model of cosmology. So far this model is in remarkable agreement with recent observations. Its credibility lies in the fact that with the aid of only six parameters (see Tab. 1.1), it describes in detail the synthesis of light elements during Big Bang Nucleosynthesis (BBN) [3] up until the decoupling of Cosmic Microwave Background (CMB) radiation at recombination when the first hydrogen atom was formed. Besides that it also explains the large scale structure formation through gravitational collapse.

While cosmological principles fit well for a description at large length scales typical

Parameters of ΛCDM	Symbol	Planck 2015 (68%) limits TT + Low polarization
Baryon density	$\Omega_b h^2$	0.02222 ± 0.00023
Dark matter density	$\Omega_c h^2$	0.1197 ± 0.0022
Sound horizon at last scattering	$100\theta_{MC}$	1.04085 ± 0.00047
Reionization optical depth	τ	0.078 ± 0.019
Amplitude of curvature perturbation	$\ln(10^{10} A_s)$	3.089 ± 0.036
Scalar spectral index	n_s	0.9655 ± 0.0062

Table 1.1: Constraints on the cosmological parameters of ΛCDM model. Here h is the dimensionless Hubble parameter of magnitude ~ 0.7 (defined in Section. 2.1). The angular scale (θ_{MC}) is an approximation to the angular size of sound horizon at last scattering. The reionization optical depth (τ) parametrizes the probability of a given CMB photon to scatter once with an ionized electron at low redshifts. The curvature perturbation amplitude and scalar spectral index are defined in Section. 2.3.4 and in this thesis we will be mostly concerned with these two parameters. They are calculated at the pivot scale $k_\star = 0.05$ Mpc^{-1} (See ref. [4]).

to the sizes of the galaxies, phenomena occurring at subatomic scales are governed by the laws of particle physics. From the perspective of physics these two seemingly different pictures are adhered to each other. In collider experiments we resolve structures at small length scales or equivalently at high energies. This is also the case when we are looking into the past history of our Universe. Therefore any attempt to building a model of the early Universe demands its simultaneous consistency with the understandings of both cosmology and particle physics.

A most impressive theory of twentieth century physics is the Standard Model. It describes the behaviour of known elementary particles and their interactions. Also this theory unifies three out of the four fundamental forces of nature which are the Electromagnetic interaction, the Strong interaction and the Weak interaction. One of its remarkable accomplishments is its stunningly accurate predictions that have been tested to unprecedented accuracies upto the Electroweak scale (TeV). However the Standard Model is supposed to be an effective description as it poses several issues in the form of a number of theoretical assumptions without any satisfactory explanation. Firstly, it fails to explain the fourth fundamental force of nature — gravity at the quantum mechanical level. Secondly, the particle content of the standard model make up only five percent of the matter we know today. Remaining things are dark matter and dark energy. Presently the standard model is yet to come up with necessary attributes to account for these objects. Thirdly and perhaps the most important issue is the fact that masses of the elementary scalars are not protected from receiving large quantum corrections of the order of cut-off scale of the Effective Field Theory. Therefore, despite of incredible success it turns out that Standard Model is not the ultimate end of the story. Rather it hints towards some new physics that might solve the problems therein.

Meanwhile the standard cosmological model also suffers some deficiencies. It fails to explain the spatial flatness of the Universe and the temperature homogeneity of the CMB at cosmological scales, which had never been in casual contact at earlier times. These two issues are referred to as the Flatness problem and Horizon problem respectively. Solving them in the existing cosmological framework requires a high degree of fine tuning of the initial conditions. The theory of cosmic inflation elegantly solves both the problems by postulating a period of accelerated expansion in the early Universe. A brief outline consisting of some of its basic treatments will be provided later.

1.2 Main subject

Back around 1980s it started with the work of Alan Guth when the theory of Cosmic Inflation was first proposed [5]. Its subsequent developments were then carried forward by

Alexei Starobinsky, Andrei Linde, Slava Mukhanov and many others [6, 7, 8]. Inflationary scenario revolutionized the traditional picture of cosmology and achieved major developments over the next three decades. It naturally solves the flatness and horizon problem of the standard Big Bang cosmology. Moreover it comes with an additional surprise that inflation also sets the seed for the large scale structure formation [9, 10, 11]. So far its generic predictions were independently confirmed through experiments like WMAP (Wilkinson Microwave Anisotropy Probe) [12, 13], PLANCK [4, 14, 15], and SDSS (Sloan Digital Sky Survey) [16].

There are two direct consequences of inflation. Firstly, inflation induces density perturbations in the primordial soup containing baryons and photons. As a result the temperature of this primordial soup undergoes characteristic fluctuations. These temperature fluctuations are none else but the temperature anisotropy, $\delta T/T \sim 10^{-5}$, in Cosmic Microwave Background (CMB). Therefore through cosmological perturbations, inflationary phase connects the physics of very small scales with that of the large ones. Secondly, inflation produces ripples in the space-time fabric known as primordial gravitational waves and these in turn distort the CMB pattern in a characteristic way. These primordial waves are supposed to be a key signature of inflation. Currently there exists an upper-bound on the strength of these waves. After LIGO [17] observation the future missions (*e.g.* BICEP3, KEK, POLARBEAR *etc.*) aim at detecting these primordial waves, as well as providing tightest constraints on the current existing bound.

To account for the temperature anisotropies in the CMB radiation, inflation requires an extremely flat potential stable against the radiative corrections [18]. Supersymmetry (SUSY) is believed to be one of the most attractive candidate from that consideration as it stabilizes the electroweak scale against quantum corrections in the Standard Model [19, 20]. But since inflation generically incorporates gravity we require local extensions of supersymmetry which naturally contains Einstein gravity. This gauged version of supersymmetry is known as supergravity (SUGRA) [21]. Moreover, inflation typically happened at very high energies close to the Planck scale where the effects of quantum gravity can not be neglected. String theory is a theory of quantum gravity where this sensitivity to Planck scale informations can be handled systematically. However, for the sake of investigation and comparison with experiments, low energy effective limit of string theory has proved to be very useful and supergravity arises as an effective limit of string theory. Therefore, in this work we intend to study inflationary models employing the tools of supergravity.

1.3 Structure of the thesis

This thesis focuses on the aspects of inflationary models along the line of the ideas presented above. It aims to explore phenomenological features of some inflationary models from the standpoint of the effective theory of supergravity. The structure of the thesis is as follows:

- ☞ Chapter 2 starts with an overview of the ideas and shortcomings of the Standard Big Bang cosmology. Then it introduces how the idea of inflation alleviates the problems of the standard theory. In summary this chapter gives a theoretical background of the basics of inflationary cosmology and its predictions to fundamental cosmological observables. For a detailed and much involved discussion on this topic the reader is referred to the review articles cited therein.

- ☞ Chapter 3 is devoted to the notion of elementary supersymmetry and supergravity. More particularly it intends to describe brief treatises of supergravity in Einstein frame and in Jordan frame. We here discuss the problem of embedding inflation in four dimensional supergravity theory. This is particularly relevant for the models we have depicted in the subsequent chapters.

☞ In Chapter 4, we discuss the embedding of a simple inflationary scenario in Einstein frame supergravity. In this case inflation is collectively driven by many interacting fields with sub-Planckian VEVs. This is an extension of the chaotic inflationary model in supergravity. Here the important aspect is the fact that despite of field interactions the effective behaviour of background dynamics is governed by a single field.

☞ In Chapter 5, we discuss inflationary models in non-minimal Jordan frame supergravity. When embedded in supergravity, typically all inflation models are plagued by dangerous supergravity contributions. In a special class of Jordan frame those contributions can be handled in an efficient way. Here we showed that whether the field responsible for driving inflation is inflaton or some auxiliary field, the supergravity contributions become of the same order as that of the global supersymmetric contributions or are generically suppressed. We illustrate these points through some representative models of inflation. In addition, relation to the recently much studied cosmological attractors are made.

☞ Chapter 6 deals with inflation in the context of moduli fields. In any realistic theory of inflation like string or supergravity moduli fields appear generically. Their role in modifying the post-inflationary history is very important. The minimum in the potential for a modulus changes during and after inflation. This fact is utilized to study how their presence can affect the inflationary predictions. Specifically we computed the change in the observable predictions for four well known models and discussed their relevance in this context.

- ☞ Chapter 7 aims at understanding the role of moduli fields upon the inflationary sector. Any realistic model of inflation finds its possibility to realize within string theory set up. In that context moduli stabilization plays a vital role. However the potential of a stabilized modulus may offer non-trivial effects upon the inflaton dynamics. We made a partial effort to understand this effect through studying the standard F-term hybrid inflation along with a KL stabilized modulus. The findings as revealed in the study of this model require more things to explore in future.

- ☞ Finally Chapter 8 discusses the conclusions of the materials presented in this dissertation and also an overall summary of the main results thereby obtained.

CHAPTER 2

INFLATIONARY COSMOLOGY

The contents of this Chapter concern an exposition of inflationary cosmology, in particular of slow roll inflation. We mentioned in the introduction that the shortcomings of Hot Big Bang theory imply classical picture is incomplete. Here we would like to introduce the idea of inflation and how this idea solves the problems of the standard scenario. For a much more comprehensive review the reader can go through [1, 5, 6, 7, 23, 24, 26, 27].

2.1 Basics of standard cosmology

In the standard Big Bang theory Big Bang happened at time $t = 0$. At that time our Universe was a very hot and dense plasma. We can not say much about our Universe before that initial phase since currently we don't have any theory available to explain the phenomena at extremely high energies. Another interesting feature of the Universe is its large scale homogeneity and isotropy. In 1929 the astronomical observation by Edwin Hubble [28] made a remarkable breakthrough concerning our understanding of the Universe. He found that the most distant galaxies are receding away from us with much greater velocity. Mathematically one can write this result as,

$$v = H(t)r, \tag{2.1}$$

where v is the recession velocity of the distant object, r is the physical distance from the point of observation and H is a proportionality constant known as the Hubble's constant. But in general H is time dependent and thereby regarded as the Hubble parameter. This empirical law is known as Hubble's law. The essence of this law is that the physical distance (r_{12}) between two bodies scales with their comoving distance (d_{12}) as

$$r_{12}(t) = a(t)d_{12} \qquad \& \qquad a(t) = exp \int H(t)dt, \tag{2.2}$$

where $a(t)$ is a quantity known as the scale factor. In terms of the scale factor the Hubble parameter encodes its time dependency as,

$$H(t) = \frac{\dot{a}}{a} \quad \text{where} \quad \dot{a} = \frac{da(t)}{dt}. \tag{2.3}$$

At present time (t_0) the measured value of Hubble parameter is $H_0 \equiv (67.8 \pm 0.9)$ Km Sec^{-1} Mpc^{-1} [4]. Sometimes it is convenient to define the dimensionless Hubble parameter as $h = H_0/100$ Km Sec^{-1} Mpc^{-1} $\simeq 0.7$ [29]. In summary Hubble's discovery together with the idea of expanding Universe, originated nearly 14 billion years ago with homogeneous and isotropic matter distribution, led to the development of standard Big Bang Cosmology.

2.1.1 FRW metric for the Universe

The assumption of homogeneity and isotropy of our Universe at scales above 100 Mpc $(\approx 10^{24}\text{m})$ is often regarded as the Cosmological principle [30, 31]. This principle simply means that we do not occupy any privileged position and direction in space. This is certainly not true at small scales — scales below 100 Mpc. At that scale the Universe is highly inhomogeneous. In order to understand these inhomogeneities we need perturbations to the homogeneous background and study them separately. However, the background evolution is also important as it gives the general behavior of the Universe, while all the structures of the visible Universe can be generated only by small perturbations around the background.

Since we are interested in the cosmological evolution at large length scales we need a metric that obeys the cosmological principles of homogeneity and isotropy. Moreover we have to ensure the expansion of the Universe. From general relativity it turns out that the most general class of metric satisfying the said criteria is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. In spherical polar co-ordinate it is written as

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{2.4}$$

where $a(t)$ is the cosmological scale factor taking into account the expansion of the Universe. The (r, θ, ϕ) co-ordinates depict the assumed symmetries in the metric. The parameter k is known as the curvature parameter of the Universe and is related to the constant curvature of the three dimensional spatial slices. k can take values $-1, 0$ and $+1$ corresponding to open, flat and closed Universe respectively. The following figure shows a schematic diagram for various geometries of the Universe.

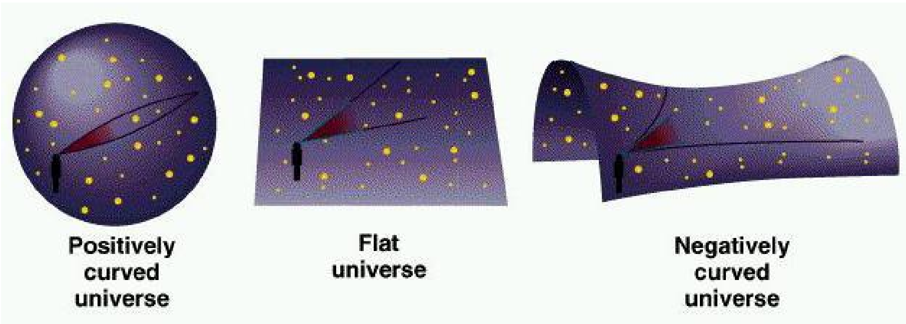


Figure 2.1: Picture of the Universe with $k = +1, 0$ and -1 respectively. Figure from [32]

The causal structure can alternately be described by introducing the conformal time (τ) defined as

$$\tau \equiv \int \frac{dt}{a(t)} \tag{2.5}$$

Redefining the variable r as $d\chi = dr/\sqrt{1-kr^2}$, the FRW metric becomes

$$ds^2 = a(\tau)^2\big(-d\tau^2 + d\chi^2 + \Phi_k(\chi^2)(d\theta^2 + \sin^2\theta d\phi^2)\big)$$

$$\text{where,} \qquad \Phi_k(\chi) \equiv \begin{cases} \sinh\chi & \text{if } k = -1, \\ \chi & \text{if } k = 0, \\ \sin\chi & \text{if } k = +1. \end{cases}$$

In this conformal time the radial null geodesics of light in the FLRW spacetime correspond to straight lines at angles $45°$ in the $\chi - \tau$ plane.

To obtain the dynamics of the Universe we need to get the Einstein equation for the FLRW metric. The Einstein equation relates space-time geometry with energy-momentum tensor associated with all forms of matter in the Universe. It is given by the following equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{2.6}$$

where $g_{\mu\nu}$ is the space-time metric tensor and G is Newton's constant. $R_{\mu\nu}$ is the Ricci tensor, $R = g_{\mu\nu} R^{\mu\nu} = R^{\mu}_{\mu}$ is the Ricci scalar and $G_{\mu\nu}$ is the Einstein tensor. They encode the curvature of the space-time fabric. Also, $T_{\mu\nu}$ is a symmetric stress-energy tensor containing all information about the energy content of the Universe. For the FLRW metric the non-vanishing components of the Ricci scalar and Ricci tensor are

$$R_{00} = -3\frac{\dot{a}}{a}, \tag{2.7}$$

$$R_{ij} = \delta_{ij}(2\dot{a}^2 + a\ddot{a}), \tag{2.8}$$

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right). \tag{2.9}$$

Treating the matter in the Universe as a perfect fluid consistent with the said homogeneity and isotropy, the energy-momentum tensor can be written as

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - p g^{\mu\nu}, \tag{2.10}$$

where ρ is the energy density of the perfect fluid, p is the pressure of the fluid and u_{μ} is the four-velocity vector of the fluid. We will work in the rest frame where $u^{\mu} = (1, 0, 0, 0)$. In this frame the stress-energy tensor becomes

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{2.11}$$

Now we have obtained the full information concerning the components of both sides of the Einstein eqn (2.6). Therefore we are ready to write down those equations in a more convenient form.

2.1.2 Friedmann equation

Friedmann realized a convenient way of writing the Einsteins equations in FLRW background. To see that we substitute eqns. (2.7), (2.8), (2.9) and (2.11) into the Einstein eqn. (2.6). The Einstein equation then breaks into two non-linear ordinary differential equations. They are called Friedmann equations for the Universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}, \tag{2.12}$$

$$H^2 + \dot{H} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{2.13}$$

To have the time evolution of the scale factor we need to specify the kind of matter that contribute to the energy density and pressure. Therefore, in the realistic case of our Universe ρ and p in eqns. (2.12), (2.13) should be replaced by

$$\rho = \sum_i \rho_i \quad \text{and} \quad p = \sum_i p_i, \tag{2.14}$$

where the summation index i extends over all kinds of matter. In addition to Friedmann equations we also have matter conservation equation

$$\nabla_\mu T^{\mu\nu} = 0. \tag{2.15}$$

For the FLRW metric this yields (taking $\mu = 0$ component of eqn. (2.15))

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{2.16}$$

Alternately, one can also derive this equation by combining the two Friedmann equations. Now assuming flat Universe $(k = 0)$, if we define the equation of state parameter, w as

$$p \equiv w\rho, \tag{2.17}$$

then from eqn. (2.16) we get for the scaling of energy density

$$\rho \propto a^{-3(1+w)} \tag{2.18}$$

Along with the Friedmann eqn. (2.12) and neglecting the curvature term, eqn. (2.18) yields

$$a(t) \propto \begin{cases} t^{2/3(1+w)}, & \text{if } w \neq -1 \\ e^{Ht}, & \text{if } w = -1 \end{cases} \tag{2.19}$$

Here the parameter w can vary depending upon the matter species. For example if we have,

Non-relativistic matter

Here $\rho \propto a^{-3}$ so, $\dot{a}^2 \propto \frac{1}{a} \implies \frac{d}{dt}(a^{3/2}) \propto 1 \implies a \propto t^{3/2}$.

Radiation

Here $\rho \propto a^{-4}$ so, $\dot{a}^2 \propto \frac{1}{a^2} \implies \frac{d}{dt}(a^2) \propto 1 \implies a \propto t^{1/2}$.

Vacuum Energy

Here $\rho =$ constant so, $a \propto e^{H_0 t}$.

The Following table summarizes the behaviour of the scale factor and other associated

Era in cosmic history	w	$\rho(a)$	$a(t)$	$H(t)$
Matter Dominated	0	a^{-3}	$t^{2/3}$	$\frac{2}{3t}$
Radiation Dominated	$\frac{1}{3}$	a^{-4}	$t^{1/2}$	$\frac{1}{2t}$
Cosmological Constant	-1	a^0	e^{Ht}	Constant

Table 2.1: Behaviour of the scale factor(a), the equation of state parameter(w), the energy density (ρ) and the Hubble parameter H at different era for FLRW solution with a spatially flat Universe $k = 0$.

quantities with respect to the type of energy content present in the Universe.

Based upon these various types of energy contents, the history of the Universe is often divided into a number of phases, where the duration of each phase depends on what kind of constituent consists of the energy density of the Universe at that phase. Therefore, there are three distinct kinds of ages — radiation dominated, matter dominated and vacuum energy dominated phases. It is often convenient to express the energy density of these phases in terms of the density parameter. From the Friedmann eqn. (2.12) one can define the critical energy density as

$$\rho_c(t) = \frac{3H(t)^2}{8\pi G} \tag{2.20}$$

at a given time corresponding to a flat Universe $(k = 0)$. Using it the density parameter is defined as the ratio of the absolute energy density to the critical energy density

$$\Omega_i = \frac{\rho_i}{\rho_c}, \tag{2.21}$$

where i corresponds to each different species. In terms of this normalized energy density Ω_i, eqn. (2.12) can be written as

$$\sum_i \Omega_i + \Omega_k = 1, \tag{2.22}$$

where $\Omega_k = -k/(a^2H^2)$ is attributed to the density parameter associated with curvature. The Large Scale Structure and the CMB observations tell us that at present $\Omega_k \simeq 0$ [4, 13, 33].

2.2 Problems with standard scenario

The previous section provided a short overview of the aspects of standard cosmology. However within this standard framework cosmologists are unable to explain certain puzzles. These puzzles worsen upto a point when extrapolating back towards the big bang (for a detailed discussion see refs.[1, 34, 35]). We will briefly summarize the two most important of them.

2.2.1 Flatness problem

To understand what is meant by the flatness problem let us once again consider the Friedmann eqn. (2.12)

$$1 - \Omega(a) = -\frac{k}{a^2 H^2}, \tag{2.23}$$

where $\Omega(a) = \sum_i \Omega_i(a)$. In standard scenario the quantity $(aH)^{-1}$ called comoving Hubble radius increases with time. Hence $|\Omega - 1|$ is divergent as time proceeds. Therefore $\Omega = 1$ turns to be an unstable fixed point. Another way of looking at the problem is by differentiating eqn. (2.23) which gives

$$\frac{d|\Omega - 1|}{d \ln a} = (1 + 3w)\Omega(\Omega - 1), \tag{2.24}$$

which again reflects that

$$\frac{d|\Omega - 1|}{d \ln a} > 0. \tag{2.25}$$

because in standard cosmology non-relativistic matter and radiation satisfies the strong energy condition $1 + 3w > 0$. Specifically at the Planck scale and during the BBN epoch these numbers are $|\Omega - 1| \leq \mathcal{O}(10^{-64})$ and $|\Omega - 1| \leq \mathcal{O}(10^{-16})$ respectively [24]. Therefore, it requires tremendous fine tuning of the initial conditions in the early Universe to get Ω close to 1 which poses the issue of naturalness.

2.2.2 Horizon problem

To describe the horizon problem let us introduce an important concept known as particle horizon. Starting from the initial singularity, the maximum distance that a particle can travel during some finite time $t > 0$ is known as the comoving particle horizon. Mathematically it is given by

$$\tau = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{d\ln a}{aH}, \tag{2.26}$$

where τ is the comoving particle horizon. If the Universe is dominated by a perfect fluid of eqn. (2.17) then $(aH)^{-1} \sim a^{(1+3w)/2}$. According to standard big bang cosmology, this perfect fluid (consisting of both non-relativistic matter and radiation) satisfies the strong energy condition (SEC) $1+3w > 0$. This means comoving Hubble radius $(aH)^{-1} = (\dot{a})^{-1}$ grows monotonically with time or the conformal time passed between initial singularity and CMB formation is smaller than that between recombination and today. That is to say the comoving scales entering the horizon today have never been in causal contact at CMB decoupling. Their past light cones did not overlap before the point of big bang singularity. Following that line of reasoning, the extreme homogeneity of CMB indicates that the Universe was highly homogeneous at the time of last scattering on scales which were acausal. This is certainly incompatible with the casual description provided by the standard cosmology.

Finally the standard cosmological model also faces an additional challenge that at extremely high energies during big bang, the model must come up with UV-completion. Depending upon various UV-description scenarios different objects (*e.g.* magnetic monopoles [36, 37], domain walls, topological defects etc.) might survive in the cosmic evolution.

However, we don't currently have any direct effect from the production of these objects.

2.3 Inflationary cosmology

The theory of cosmic inflation was originally proposed to solve all these shortcomings of the standard cosmology. From our earlier descriptions we can see that the most important parameter in this context is the comoving Hubble radius. The idea that cosmic inflation proposes is that instead of having an increasing comoving Hubble radius, let us consider a scenario where the comoving Hubble radius decreases sufficiently with time in the early Universe. This fundamental proposal changes the causal structure of the background space-time and introduces a finite phase of quasi-exponential expansion (see footnote on this page). Moreover, we get additional benefit that the presence of very small inhomogeneities in the CMB radiation can now be explained as quantum fluctuations in the very early Universe. These fluctuations represent the seeds for the large scale structures we observe in the sky.

2.3.1 Basic ideas

Inflationary epoch postulates a phase of accelerated expansion which is often regarded as a quasi-de Sitter phase[1]. In such an expansion the scale factor grows as $a \sim e^{Ht}$. Through Friedmann equations an accelerating phase can be related to shrinking comoving Hubble radius

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \Leftrightarrow \quad -\frac{\ddot{a}}{(aH)^2} < 0 \quad \Leftrightarrow \quad \ddot{a} > 0. \tag{2.27}$$

[1]In a pure de-sitter phase pressure and density are related as $P = -\rho$. This will make H to be a constant. But in a quasi-de Sitter phase H does vary slightly with time so $P \simeq -\rho$.

To satisfy the above condition is to assume that the Universe was filled with some kind of matter with negative pressure

$$p \leq -\frac{1}{3}\rho \tag{2.28}$$

This precise relationship between p and ρ is very crucial during inflation. In the standard inflationary scenario there exists a scalar field usually called inflaton, whose potential energy dominates the total energy of the Universe. It can be shown that if the potential energy of that inflaton field is much higher than its kinetic energy then its pressure and energy density satisfy the precise relationship mentioned above. The Universe then starts with a vacuum dominated phase and consequently undergoes the inflationary expansion. Therefore, in summary the two essential ingredients of inflation are — (a) decreasing comoving Hubble radius $(aH)^{-1} < 0$ and (b) violation of SEC *i.e.* $1 + 3w < 0$.

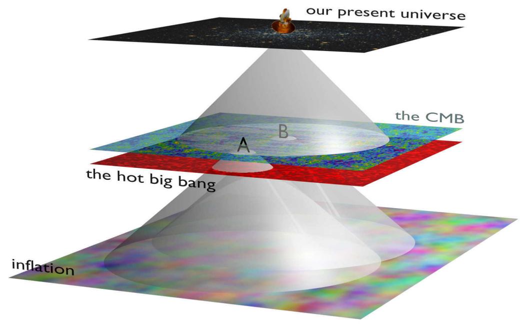
2.3.2 Solution to puzzles

Let us now look back again to our earlier problems of the standard big bang cosmology. We mentioned earlier that these problems arises because the comoving Hubble radius was increasing.

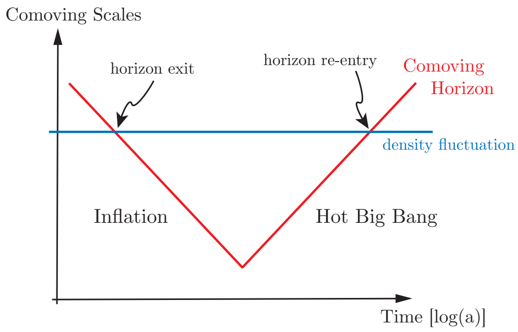
☛ Flatness problem: Now from the Friedmann equation we have,

$$\Omega - 1 = -\frac{k}{a^2 H^2}. \tag{2.29}$$

Since during inflation we have $(aH)^{-1} < 0$ for a considerable amount of time, therefore the quantity in the right hand side of the equation decays exponentially to zero. This means that the density of the Universe is approaching towards the critical den-



(a) Inflation provides more conformal time for past-light cones to get overlapped. Hence causality is automatically established.



(b) The behaviour of comoving Hubble radius (red) with respect to time in both the standard scenario (Hot Big Bang) and inflationary scenario. Scales of interest (blue) was inside the Hubble horizon at inflation and so causally connected.

Figure 2.2: Figures depicting how cosmic inflation elegantly bypass the horizon problem. Left figure is from [38] and right figure is from [24]

sity. Hence the Universe is naturally driven towards spatial flat geometry. Thus flatness problem is resolved.

☛ Horizon problem: Horizon problem is also resolved because a decreasing comoving Hubble radius means large scales entering the Universe now were inside the horizon at sufficiently early times before inflation. Therefore, during inflation the smooth patch of radius H^{-1} was casually coherent[2]. This would mean there were more conformal time between the initial singularity and the decoupling than envisaged by the classical big bang scenario. Thus the past light cones of widely separated points in CMB overlapped and hence homogeneity in CMB is automatically established. From Fig. 2.2 we can see how the decreasing comoving Hubble radius gradually lets those scales, initially inside the horizon, exit the horizon and once again re-entering into it in the future, after inflation has ended.

[2]Here H^{-1} is the physical Hubble radius that is related to comoving Hubble radius as $H^{-1} = a(aH)^{-1}$.

Before moving to the description of the dynamics of the scalar field driving inflation, let us see how cosmologists quantify the amount of relevant inflation necessary to solve the problems of standard cosmology. From Friedmann eqn. (2.13) we may write

$$\frac{\ddot{a}}{a} = H^2(1-\varepsilon) \quad \text{where,} \quad \varepsilon \equiv -\frac{\dot{H}}{H^2}. \tag{2.30}$$

Thus an accelerated expansion corresponds to,

$$\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1, \tag{2.31}$$

where the quantity $dN = Hdt$ is defined as the number of e-folds, N of inflationary expansion. More explicitly,

$$N = \int_{t_i}^{t_f} \frac{\dot{a}}{a}\,dt = \int_{t_i}^{t_f} \frac{da}{a} = \ln\left(\frac{a(t_i)}{a(t_f)}\right). \tag{2.32}$$

The number of e-folds essentially means how much exponentiation of the scale factor is needed during inflation. The resolution for the cosmological problems requires this number to be around $N \sim 50 - 60$.

2.3.3 Scalar field dynamics and slow-roll conditions

We have said earlier that during inflationary expansion spacetime is approximately de-Sitter *i.e.* $a \sim e^{Ht}$. To realize such an expansion requires breaking the strong energy condition which means equation of state parameter $w \approx -1$. In the simplest picture this is achieved if we consider the dynamics of a slowly rolling scalar field known as the inflaton field. This is one of the earliest and the most influential model of inflation.

The Lagrangian for an inflaton field minimally coupled to gravity is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{2.33}$$

If we restrict the scalar field to be homogeneous $\phi(\mathbf{x}, t) \equiv \phi(t)$, then from computing the energy-momentum tensor of this field we obtain

$$\rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{2.34}$$

So far we have not made any approximation. Now if we assume that the potential energy of $\phi(t)$ dominates over the kinetic energy i.e.

$$\frac{1}{2}\dot{\phi}^2 << V(\phi), \tag{2.35}$$

then the equation of state parameter becomes

$$w \equiv \frac{p(\phi)}{\rho(\phi)} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1. \tag{2.36}$$

Thus the scalar field can exert negative pressure and leads to an exponential expansion. The dynamics of the field in the FLRW background is given by the Klein-Gordon equation

$$\Box\phi + V_\phi = 0, \quad \text{where} \quad \Box\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi), \tag{2.37}$$

$$\text{or,} \quad \ddot{\phi} + 3H\dot{\phi} + V_\phi = 0, \tag{2.38}$$

$$\text{where} \quad H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{2.39}$$

Here ‘ dot (.)’ denotes derivative with respect to time and $V_\phi = \partial V/\partial\phi$. The second term in eqn. (2.38) is called the Hubble friction term since it is proportional to the velocity of the inflaton field. For large values of the potential, the field experiences high frictional force as it rolls down in its potential.

We knew that accelerated expansion requires

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{M_{pl}^2 H^2} < 1. \tag{2.40}$$

The above condition is none else than the requirement of eqn. (2.35). It is customary to name ε as the first slow roll parameter. Now to sustain a sufficiently long period of inflation we also require small acceleration of the field *i.e.* $|\ddot{\phi}| << |H\dot{\phi}|, |V_\phi|$. This statement is quantified by introducing a second slow roll parameter

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\epsilon}{dN} < 1. \tag{2.41}$$

The above two conditions in eqn. (2.40) and in eqn. (2.41) are often regarded as the slow roll conditions since the field evolves very slowly with respect to the quasi-de Sitter growth of the scale factor. These conditions can also equivalently be expressed in terms of the potential function as

$$\epsilon_v = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_v = M_{pl}^2 \frac{V''}{V}. \tag{2.42}$$

Here ϵ_v, η_v are called the potential slow roll parameters while the slow roll parameters introduced earlier in eqn. (2.40) and in eqn. (2.41) are sometimes called the Hubble slow roll parameters. During inflation they are related as

$$\varepsilon \approx \epsilon_v \quad \text{and} \quad \eta \approx \eta_v - \epsilon_v. \tag{2.43}$$

Now within the slow roll regime $\epsilon_v, |\eta_v| << 1$; the eqns. (2.38) and (2.39) are then simplified to

$$\ddot{\phi} \approx -V_\phi, \tag{2.44}$$

$$H^2 \approx \frac{V}{3M_{pl}^2} \approx constant. \tag{2.45}$$

Thus the spacetime is de Sitter. Inflation stops when the slow roll conditions are violated

$$\varepsilon|_{\phi=\phi_{end}} \equiv 1, \qquad \epsilon_v|_{\phi=\phi_{end}} = 1. \tag{2.46}$$

The slow roll conditions $\epsilon_v, |\eta_v| << 1$ constrain the shape of the inflaton potential. The relevant number of efolds is

$$\begin{aligned} N = \int_{t_i}^{t_f} H(t)dt = \int_{\phi_i}^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi \\ \approx \frac{1}{M_{pl}} \int_{\phi_i}^{\phi_{end}} \frac{d\phi}{\sqrt{2\epsilon_v}} \approx 40 - 60. \end{aligned} \tag{2.47}$$

After the end of inflation, the inflaton field begins to oscillate around the minimum of its potential. During such oscillations energy of the inflaton field is converted to ordinary particles within a process called reheating. The value of N actually depends on the inflationary model and on the details of this reheating process (see review [39, 40, 41, 42]). Below we give a concise presentation of the consistency condition one obtains when the cosmic fluid in reheating era is parametrized by an effective equation-of-state parameter (see also [43, 44]).

Consistency relation for reheating epoch:

We begin with a diagrammatic sketch of the cosmic history of the Universe in different stages in Fig. 2.3. Initially during inflation, the comoving Hubble radius $(aH)^{-1}$ decreases. At the end of inflation reheating phase begins and the comoving horizon starts to increase. This corresponds to $N_{\rm re}$ e-folds. In this stage the energy in the inflaton field has been com-

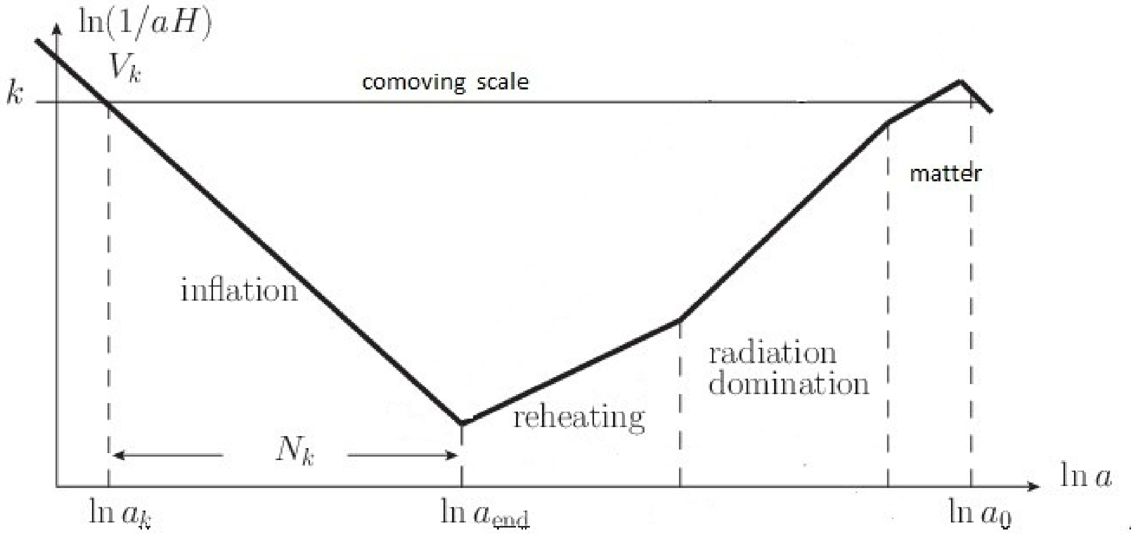


Figure 2.3: Evolution of comoving Hubble radius $(aH)^{-1}$ vs. the scale factor a at various epochs that are involved in getting the consistency condition

pletely dissipated into a hot plasma with an average temperature $T_{\rm re}$. Thereafter, the Universe further expands additional N_{RD} e-folds under radiation domination. Subsequently it makes a transition to matter dominated era.

The reheating phase is characterized by two parameters — efoldings during reheating $N_{\rm re}$ and equation of state parameter during reheating $w_{\rm re}$. Let us examine the evolution of energy density of the Universe from the time of horizon exit of a pivot mode to the present day. For a mode of comoving wavenumber k exiting the horizon we can write

$$\frac{k}{a_0 H_0} = \frac{a_k H_k}{a_0 H_0} = \frac{a_k}{a_{\rm end}} \frac{a_{\rm end}}{a_{\rm re}} \frac{a_{\rm re}}{a_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} \frac{H_k}{H_{\rm eq}}, \tag{2.48}$$

where quantities with subscript k are evaluated at horizon exit and a_0, H_0 denote the present scale factor and the Hubble parameter respectively. The other subscripts mean the scale factor at the end of inflation (end), at reheating (re) and at radiation-matter equality (eq). Taking logarithm on both sides of eqn. (2.48) we get

$$\ln \frac{k}{a_0 H_0} = -N_k - N_{\rm re} - N_{RD} + \ln \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0} + \ln \frac{H_k}{H_{\rm eq}}, \tag{2.49}$$

where $e^{N_k} = \frac{a_{\rm end}}{a_k}$, $e^{N_{\rm re}} = \frac{a_{\rm re}}{a_{\rm end}}$ and $e^{N_{RD}} = \frac{a_{\rm eq}}{a_{\rm re}}$. Here N_k is the number of inflationary e-foldings, $N_{\rm re}$ the number of e-foldings during the period of reheating and N_{RD} the number of e-foldings in the radiation dominated era. Now the relation between the energy density at the end of inflation $(\rho_{\rm end})$ and that at the end of reheating $(\rho_{\rm re})$ is

$$N_{\rm re} = \frac{1}{3(1+w_{\rm re})} \frac{\rho_{\rm end}}{\rho_{\rm re}}. \tag{2.50}$$

The energy density at reheating is related to the reheating temperature $(T_{\rm re})$ as

$$\rho_{\rm re} = \frac{\pi^2}{30} g_{\rm re} T_{\rm re}^4, \tag{2.51}$$

where $g_{\rm re}$ is the effective number of light species during reheating. The reheating temperature can also be related to the CMB temperature today (T_0) as

$$T_{\rm re} = \left(\frac{43}{11 g_{\rm s,re}}\right)^{1/3} \frac{a_0}{a_{\rm eq}} \frac{a_{\rm eq}}{a_{\rm re}}, \tag{2.52}$$

where $g_{\rm s,re}$ is the effective number of light species for entropy. Plugging eqn. (2.51) and eqn. (2.52) in eqn. (2.50) yields

$$N_{\rm re} = \frac{1}{4} \ln \rho_{\rm end} + \frac{1}{4} \ln \left(\frac{30}{\pi^2 g_{\rm re}}\right) + \frac{1}{3} \ln \left(\frac{11 g_{\rm s,re}}{43}\right) - \frac{1}{4} \ln T_0^4 + \ln \frac{a_{\rm eq}}{a_0} + N_{RD}. \tag{2.53}$$

Putting the above expression of N_{re} into eqn. (2.49) and using the expression of the Hubble parameter during inflation $H_k = \pi M_{pl}(r\Delta_s)^{1/2}/\sqrt{2}$, we obtain after simplifications

$$N_k + \frac{1}{4}(1-3w_{\text{re}})N_{\text{re}} \approx 55.43 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\text{end}}}\right), \tag{2.54}$$

where in the above expression we have taken $k = 0.05\text{Mpc}^{-1}, g_{\text{re}} \approx g_{\text{s,re}} \approx 100, T_0 = 2.725K$, and $\ln(10^{10}\Delta_s) = 3.089$. Eqn. (2.54) is the familiar consistency relation generalizing the number of e-folds between horizon exit for the modes relevant for CMB observations and the end of inflation. It accounts for the uncertainty in the reheating stage by considering $50 < N_k < 60$. Now in a more realistic study of inflation through BSM framework (*e.g.* string/supergravity constructions), light scalar fields have important impact on the post inflationary history. The presence of these fields modify the consistency relation such that the number of e-foldings become dependent upon their masses [46, 47]. More details on the phenomenological implications in response to this modification is explored in Chapter 6.

2.3.4 Quantum fluctuations

Upto now we have discussed the classical evolution of the background. Now we would like to talk about the perturbations in the homogeneous background $\phi(t)$. A detailed treatment of the cosmological perturbations however goes beyond the scope of the present thesis. The interested reader is referred to the reviews [1, 23, 24, 38, 48, 49, 50]. During inflation the quantum fluctuations of the inflaton and the metric field are given by

$$\phi(t,x) = \overline{\phi(t)} + \delta\phi(t,x), \qquad g_{\mu\nu}(t,x) = \overline{g_{\mu\nu}} + \delta g_{\mu\nu}(t,x) \tag{2.55}$$

Here the barred quantities represent the homogeneous background part of the field variable. The quantum fluctuation of the inflaton $\delta\phi(t)$ source the metric perturbations $\delta g_{\mu\nu}$ through Einstein equation. Therefore,

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j. \end{aligned} \tag{2.56}$$

In the above equation Φ, Ψ, B_i and E_{ij} parametrize the perturbations around the FLRW background. In real space they are typically decomposed into scalar, vector and tensor perturbations as

$$B_i = \partial_i B - S_i, \qquad \text{with} \quad \partial^i S_i = 0, \tag{2.57}$$

$$\text{also,} \quad E_{ij} = 2\partial_i\partial_j E + 2(\partial_i F_j + \partial_j F_i) + h_{ij}, \tag{2.58}$$

$$\text{with} \quad \partial^i F_i = 0, \quad h^i_i = \partial^i h_{ij} = 0. \tag{2.59}$$

It turns out that during inflation the vector perturbations S_i, F_i quickly decay with the expansion of the Universe. So we are mainly interested in the scalar and the tensor fluctuations that are observed as density fluctuations and gravitational waves in the late Universe. A better way to deal with these perturbations is that they do not change under a particular choice of coordinates. That means the fluctuations are gauge invariant [51, 52]. Unlike tensor fluctuations, scalar fluctuations in this case are not gauge invariant. Thus we need suitable gauge choice for the scalar perturbations. Two such gauge invariant combinations are used in practice [53]. One of them is the comoving curvature perturbation on uniform density hypersurfaces

$$-\zeta \equiv \Psi + \frac{H}{\dot{\bar{\rho}}}\delta\rho. \tag{2.60}$$

For adiabatic matter perturbations it remains constant outside the horizon. During inflation it is given as

$$-\zeta \approx \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi. \tag{2.61}$$

Another gauge invariant combination in use is the comoving curvature perturbation

$$\mathcal{R} \equiv \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi. \tag{2.62}$$

On superhorizon scales $(k << aH)$ and during slow-roll inflation we get approximately upto a minus sign [24]

$$\zeta \approx -\mathcal{R}. \tag{2.63}$$

Since ζ and $\mathcal{R}$ are frozen after inflation, this equality still holds. Therefore, the co-relation functions of ζ and $\mathcal{R}$ are same at horizon crossing. They do not evolve on superhorizon scales. This is the reason why during slow-roll inflation one computes ζ on horizon crossing and ignores the superhorizon contributions.

Now the scalar power-spectrum is defined as

$$\langle \mathcal{R}_{\vec{k}} \mathcal{R}_{\vec{k'}} \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k'}) P_{\mathcal{R}}(k), \tag{2.64}$$

$$\Delta_s^2 \equiv \Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k). \tag{2.65}$$

Scale-invariant spectrum corresponds to $\Delta_{\mathcal{R}}^2 \simeq constant$. The deviation from scale-invariant power-spectrum is parametrized by the spectral tilt (n_s)

$$n_s \equiv 1 + \frac{d \ln \Delta_{\mathcal{R}}^2}{d \ln k}. \tag{2.66}$$

For the tensor fluctuations we have two independent polarization states h^+, h^-. Their tensor spectral index and tensor power-spectrum are given as

$$\langle h_{\vec{k}} h_{\vec{k'}} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_h(k), \tag{2.67}$$

$$\Delta_h^2 = \frac{k^3}{2\pi^2} P_h(k) \quad \text{and} \quad \Delta_t^2 \equiv 2\Delta_h^2. \tag{2.68}$$

The scale-dependence of the tensor modes is determined by the relation

$$n_t = \frac{d \ln \Delta_t^2}{d \ln k}. \tag{2.69}$$

In a single field inflation model under slow-roll approximation the results for the primordial spectra, *i.e.* Δ_s^2 and Δ_t^2, are fully specified in terms of $V(\phi)$ and ϵ_v

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2} \frac{V}{M_{pl}^4} \frac{1}{\epsilon_v} \bigg|_{k=aH}, \tag{2.70}$$

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{pl}^4} \bigg|_{k=aH}. \tag{2.71}$$

Finally the scalar and tensor spectral index at leading order in the slow-roll parameters are

$$n_s \approx 1 - 6\epsilon_v + 2\eta_v \quad \text{and} \quad n_t \approx -2\epsilon_v. \tag{2.72}$$

and the tensor-to-scalar ratio is

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_v. \tag{2.73}$$

For all models of single field slow-roll inflation, eqns. (2.72) and (2.73) suggest a consistency condition $r = -8n_t$. The constraints upon these inflationary observables are discussed in the next subsection.

2.3.5 CMB power spectrum and Planck results

The discovery of the CMB in 1965 [54] have opened a new direction where our speculative ideas have found empirical verification. The temperature fluctuations observed in the CMB pattern provides a valuable insight into the early Universe cosmology. Its study led to

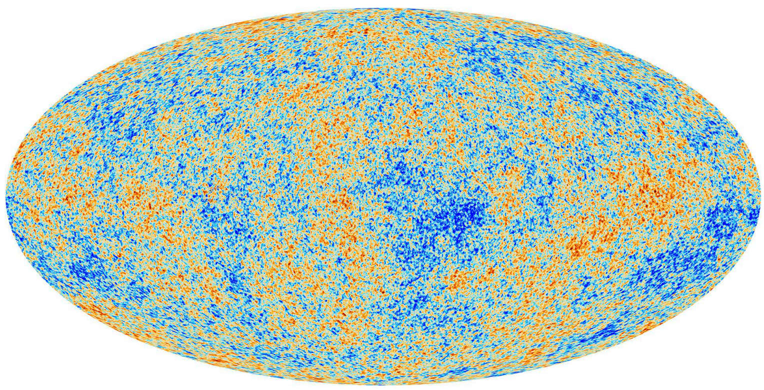


Figure 2.4: Temperature anisotropies in CMB as observed by Planck space telescope. $\delta T/T \sim 10^{-5}$ around the mean background $T^{av}_{CMB} = 2.73K$. Red spots are hotter than blue ones showing the density fluctuations at recombination.

the determination of almost all the cosmological parameters we know today. When the

Universe had cooled to a temperature of about 0.3 eV, the neutral hydrogen formation became entropically favoured. The free electron density dropped rapidly and the photons decoupled. These photons became the CMB upon which density perturbations of early Universe get imprinted. Fig 2.4 shows the characteristic sky map of the measured CMB temperature, taken from the Planck results [14]. (A proper investigation of the CMB physics is necessary to understand the functional form. However, this goes beyond the scope of the present work, for details see refs [55, 56, 23]). The CMB reflects a mean

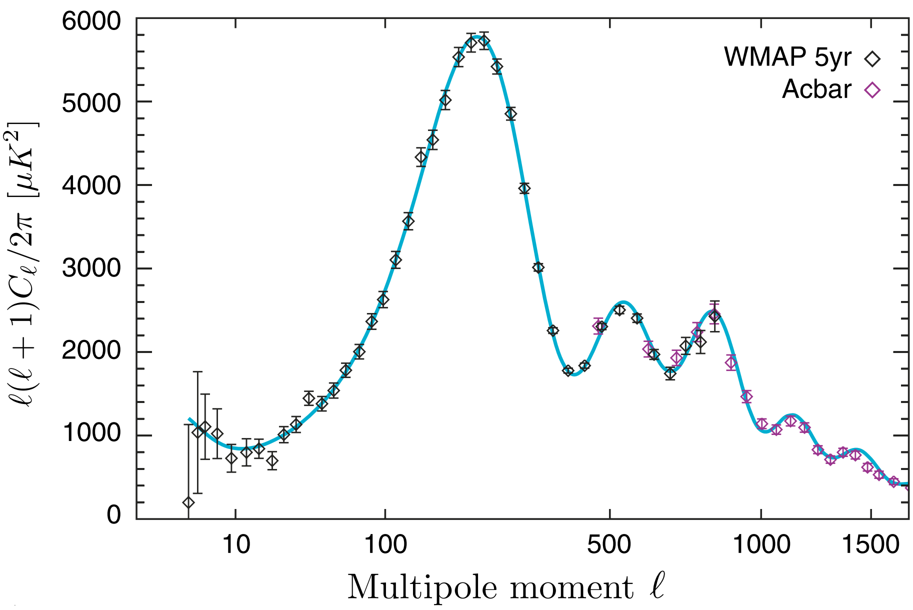


Figure 2.5: Angular power spectrum of CMB temperature anisotropy as measured by the Planck satellite (picture taken from [24]).

background temperature along with tiny fluctuations on top of that almost homogeneous and isotropic background — $T_{CMB} = 2.73 \pm 10^{-4} K$. These anisotropies represent density fluctuations in the primordial plasma that can be traced back to the curvature perturbations produced during inflation. The complete information of this characteristic shape is simply encoded in the two point co-relation function defined as

$$C(\theta) = \left\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \right\rangle, \tag{2.74}$$

where angle brackets denote ensemble average and $\cos\theta \equiv \mathbf{n} \cdot \mathbf{n}'$ with $\mathbf{n}, \mathbf{n}'$ being two distinct directions in the sky from which CMB photons enter the telescope. It is convenient to decompose the temperature fluctuations into spherical harmonics as

$$\frac{\Delta T(\mathbf{n})}{T} = \sum_{l=1}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\mathbf{n}), \tag{2.75}$$

where l and m are the eigenvalues of the differential operators on the sphere. If the fluctuations follow a Gaussian distribution then $\langle a_{lm} a^{\star}_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$. Here the collection C_l is called the angular power spectrum. It contains all information about the Gaussian temperature fluctuations. The measured angular power spectrum of the CMB temperature fluctuations is shown in Fig. 2.5. With respect to the present bound $r < 0.07$, CMB fluctuations are mostly dominated by scalar modes. In practice there is a quantity called the transfer function that relates the initial fluctuations from the moment of horizon re-entry to the time of recombination as well as their projection in sky today [55, 24]. The final result is the presence of Doppler peaks seen in Fig. 2.5. For the first pick on left, the associated mode had just time to compress once before decoupling. But the non-oscillating modes on left of the fist pick are superhorizon at the time of decoupling. The other peaks on small angular scales correspond more oscillations and therefore are damped.

Planck results:

We will now very briefly summarize the constraints on the parameters related to the inflationary dynamics as obtained from the combined data of Planck TT + low l polarization. The pivot scale is $k_\star = 0.05$ Mpc^{-1}. At this scale the scale dependence of the scalar power spectrum is [4, 15]

$$n_s = 0.9655 \pm 0.0062 \tag{2.76}$$

at 68% confidence level (CL). Thus the spectrum is nearly scale-invariant as consistent with the slow-roll inflation model. For the tensor spectral index we just know $n_t < 0.014$ at 95% CL. The best fit value of the amplitude of scalar perturbation is measured to be

$$\ln(10^{10}\Delta_s) = 3.089 \pm 0.036 \tag{2.77}$$

at the 68% CL. Therefore, the current observation fixes $\Delta_s^2 \sim 10^{-9}$. Under the approximation $\Delta_h^2 \sim H^2 \sim V$ we get an upper bound on the energy scale of inflation

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16} \;\; \text{GeV}. \tag{2.78}$$

Finally, the observational bound on the tensor-to-scalar ratio is $r < 0.07(95\%\text{CL})$ [15, 57, 58]. Thus with $r > 0.01$ inflation must happened at scales close to 10^{16} GeV.

The discussions made so far in this Chapter summarizes the basic framework of inflationary cosmology. This will be of use for the later part concerning the study of some example inflation models and their observable implications.

CHAPTER 3

AN ELEMENTARY TREATISE OF SUSY AND SUGRA

We have already mentioned in the introduction why it is important to embed inflationary models in the supergravity framework. In this chapter we will present in brief the key ideas and concepts of supersymmetry and supergravity. This elementary discussion is not intended to cover the subtleties and details of the different aspects of supergravity. We will collect the most essential features that will be explicitly needed to relate with the presentations given in the later part of the thesis (For a review on supergravity aspects of inflation see [59, 60])

3.1 Problems of the Standard Model

Standard model has been very promising in explaining three out of the four fundamental interactions of nature — namely the strong, weak and electromagnetic interaction. The theory was developed in stages by various scientists and was accomplished practically in the mid-1970s upon experimental confirmation of the existence of quarks. Thereafter, the success of standard model lies in the prediction for the existence of various particles, namely the **W** and **Z** bosons as well as the Higgs boson, which has been found very recently. Also, the magnetic moment of the electron has been confirmed up to 13 significant digits. All these successes added further credence to this model.

However despite being the most successful theory of particle physics to date, the Standard Model is not inherently complete.

- The theory is inconsistent with that of general relativity, to the point that one or both theories break down under certain conditions.

- Secondly, the particle content of Standard Model only accounts for nearly 4% of the known matter in the Universe. The rest are Dark matter (existence confirmed from its gravitational effects on visible matter [61]) and Dark energy (hypothesized to accommodate for recent observations that the Universe appears expanding at an accelerating rate [62].), by proportion 27% and 68% respectively. The Standard Model does not provide any fundamental particle that can serve as a good candidate for these objects.

- Thirdly, from the theoretical point of view hierarchy problem (see [63, 64, 65]) is by far the most compelling argument in favour of a new theory. In Standard Model the particles acquire masses through a process known as spontaneous symmetry breaking caused by the Higgs field. But the mass of the Higgs gets very large quantum corrections due to the presence of virtual particles (mostly from top quarks). These corrections are much larger than the actual mass of the Higgs. This means that the bare mass parameter of the Higgs within the Standard Model must be fine tuned to cancel against the radiative corrections. This does not explain the origin of weak scale.

For a much elaborate and thorough treatment concerning various problems with the Standard Model, including in particular the metastable electroweak vacuum, the strong CP problem and the matter-antimatter asymmetry, see for example [66, 67].

3.1.1 SUSY as a candidate of BSM

BSM refers to the theoretical developments to explain the deficiencies as well as to offer potential solutions to the incompleteness of the Standard Model. There exists several theories [68, 69, 70, 71] that provide potential solutions to the said discrepancies of the Standard Model. Supersymmetry is the most appealing among them. In the context of inflation, SUSY provides us with large number of fundamental scalar fields. Moreover, SUSY extensions of the Standard Model can account for the large amount of dark matter in our Universe. Supersymmetric theories typically predicts the existence of supersymmetric partner, or s-particles in short, corresponding to each particle in the Standard Model. The spin of s-particles differs by 1/2 from its non supersymmetric partner. Also their mass is typically much larger than their Standard Model counter parts.

The way SUSY solves the hierarchy problem is that unbroken supersymmetry generates extra radiative corrections from the sparticle loops to the Higgs mass that cancel its dependence on the cut-off Λ [72, 73]. However such a solution comes at the cost of introducing many new degrees of freedom in the theory. Therefore, supersymmetry has to be broken at least $\mathcal{O}(1)$ TeV scale [74] to explain why so many extremely intensive searches have not yet resulted in the detection of s-particles [75, 76].

Another convincing argument in favour of supersymmertry is that in certain theories like the Minimally Supersymmetric Standard Model (MSSM) [72], the gauge couplings of the three fundamental interactions seem to intersect quite accurately (within experimental bounds) at a certain energy scale. This gauge unification scale can be interpreted as the energy scale of some Grand Unified Theory (GUT).

3.1.2 SUSY algebra

Supersymmetry is an extension of the usual Poincaré space-time symmetry with new generators that transform bosonic fields into fermionic fields [77, 78, 21]. Since SUSY transformation changes the spin of a state, its generator Q has to be a spinor operator. Schematically, the transformation is represented as

$$\hat{Q}|\text{Boson}\rangle \equiv |\text{Fermion}\rangle, \qquad \hat{Q}|\text{Fermion}\rangle = |\text{Boson}\rangle, \tag{3.1}$$

where $|\ \rangle$ depicts the state undergoing supersymmetry transformation. To be precise let Q_α^A be the generator of SUSY transformations, where A is the number of supersymmetries — $A = (1, ..., \mathcal{N})$ (often called supercharges) and α is a Weyl-spinor index. In general SUSYs with $\mathcal{N} > 1$ are called extended supersymmetries and contain different supercharges, while for $\mathcal{N} = 1$ or unextended susy only one type of supercharge is present. In this thesis, we will be concerned only with $\mathcal{N} = 1$ SUSY as this has chiral representations. The generators satisfy $Q_\alpha = Q_{\dot{\alpha}}^\dagger$. Now the Poincaré algebra, when extended with supersymmetry generators Q_a obey the (anti-)commutation relations [21]

$$\{Q_\alpha, \bar{Q}^\dagger_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \tag{3.2}$$

$$\{Q_\alpha, M_{\mu\nu}\} = (\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta, \tag{3.3}$$

$$\{Q_\alpha, P_\mu\} = 0, \tag{3.4}$$

$$\{Q_\alpha, Q_\beta\} = 0, \tag{3.5}$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0. \tag{3.6}$$

where $P^\mu, M^{\mu\nu}$ are the generators of Poincaré algebra and $\sigma^\mu = (1, \sigma^i)$, σ^i being the Pauli spin matrices.

The particle content of a supersymmetric theory involves supermultiplet. A supermultiplet consists of a set of boson and fermion fields which transform among themselves under SUSY transformation. In other words it is the smallest irreducible representation of $\mathcal{N} = 1$ super-Poincaré algebra. Within each supermultiplet, we have exactly the same number of bosonic and fermionic degree of freedoms. In this work we will not introduce the supersymmetry transformations of these multiplets[1], instead we will list them for the $\mathcal{N} = 1$ SUSY theory and they are

Chiral Supermultiplet: It consists of a two-component left Weyl fermion χ_α in combination with one complex scalar Φ which transform into each others under SUSY transformations. In addition there is also an auxiliary complex scalar field F. F is auxiliary field in the sense that it has no kinetic term in the Lagrangian density and hence can be eliminated by its equation of motion. A chiral supermultiplet is denoted as $\Xi = (\Phi, \chi_\alpha, F)$

Vector Supermultiplet: This multiplet consists of a spin-1 gauge boson A^a_μ, a spin-1/2

[1]For details on supermultiplets, their SUSY transformation and construction of Lagrangian density the reader is referred to [21, 72, 79, 80]

fermion λ^a, called a gaugino, and a real auxiliary scalar field D^a. Here the index a runs over the adjoint representation of the gauge group under consideration. A vector supermultiplet is denoted as $V^a = (B^a_\mu, \lambda^a, D^a)$

We will be mainly interested in the chiral multiplet since they contain scalar fields, *i. e.* candidates for inflaton. The other multiplet is employed to have a supersymmetric versions of gauge interactions and gravity.

3.2 Supergravity in Einstein frame

Since we aim at incorporating inflation in a supersymmetric theory, therefore it is inevitable to combine supersymmetry with gravity. This means supersymmetry must become a local symmetry or a gauge symmetry. As with any gauge theory, one has to introduce a gauge field in order to sustain invariance of the Lagrangian under gauged SUSY transformation. This gauge field is a spin-3/2 fermion known as gravitino and appears together with a spin-2 graviton in a gravity multiplet. The details of gauging SUSY is beyond the scope of this thesis. We will merely pick up the essential ingredients important for later part of this dissertation.

The field content of $4D, \mathcal{N} = 1$ supergravity is composed of chiral multiplet, gauge multiplet and gravity multiplet. For us gauge-gravity interactions are not relevant. Also from now on we will discuss just the bosonic sector that will be of use in later chapters. With only a set of chiral fields the effective supergravity action is determined by the following functions [79, 21, 74]

- ▶ The Kähler potential $K(\Phi_i, \bar{\Phi}_{\bar{i}})$, a real function of the chiral field Φ_i and its hermitian conjugate $\bar{\Phi}_{\bar{i}}$

► The Superpotential $W(\Phi_i)$, an holomorphic function in Φ_i's

The action for the scalar part of the chiral field in Einstein frame is given by,

$$\mathcal{S}_E = \mathcal{S}_E^{grav} + \mathcal{S}_E^{scalar}. \tag{3.7}$$

The gravity part is Einstein-Hilbert *i.e.*

$$\mathcal{S}_E^{grav} = \int \mathcal{L}_E^{grav}\, d^4x = \int d^4x \sqrt{-g_E}\, \frac{1}{2} R(g_E) M_{pl}^2, \tag{3.8}$$

while the action for scalar part minimally coupled to gravity is given by

$$\mathcal{S}_E^{scalar} = \int \mathcal{L}_E^{scalar} d^4x = \int d^4x \sqrt{-g_E} \left[\frac{1}{\sqrt{-g_E}} \mathcal{L}_E^{kin} - V_E(\Phi_i, \bar{\Phi}_{\bar{i}}) \right], \tag{3.9}$$

$$\text{with} \quad \frac{\mathcal{L}_E^{kin}}{\sqrt{-g_E}} = g_E^{\mu\nu} K_{i\bar{j}} (\partial_\mu \Phi^i)(\partial_\nu \bar{\Phi}^{\bar{j}}). \tag{3.10}$$

where $K_{i\bar{j}} = \frac{\partial^2 K}{\partial \Phi_i \partial \bar{\Phi}_{\bar{j}}}$ is the metric on the Kähler manifold [81] and V_E is called the Einstein frame scalar potential. This V_E is actually a sum of supergravity F-term and D-term contributions.

The only result from supergravity needed for us is the expression for this F-term scalar potential; while D-term potential will be mentioned later very briefly. For the F-term it is given by

$$V_E^F = e^{\frac{K}{M_{pl}^2}} \left(K^{i\bar{j}} \mathcal{D}_i W \mathcal{D}_{\bar{j}} \bar{W} - 3 \frac{|W|^2}{M_{pl}^2} \right), \tag{3.11}$$

where $K^{i\bar{j}} = (K_{i\bar{j}})^{-1}$ is the inverse Kähler metric and $\mathcal{D}_i W$[2] is the Kähler covariant deriva-

[2]In global SUSY there is no Kähler potential. So, $\mathcal{D}_i W \equiv F_i = \partial W / \partial \Phi_i$

tive

$$\mathcal{D}_i W = W_i + W \frac{K_i}{M_{pl}^2}, \tag{3.12}$$

with the subscripts denoting partial derivatives with respect to Φ_i, $\partial_i Y = Y_i$.

Although a supergravity theory in the absence of gauge interaction is uniquely determined by $W(\Phi)$ and $K(\Phi, \bar{\Phi})$ there is a certain degeneracy in the definitions of these two functions. An important consequence of this is the invariance of the action under K$\ddot{a}$hler transformations of the form

$$K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + \bar{f}(\bar{\Phi}), \tag{3.13}$$

$$W(\Phi) \to e^{-f(\Phi)} W(\Phi), \tag{3.14}$$

where $f(\Phi)$ is a holomorphic function. If we define a quantity G, invariant under K$\ddot{a}$hler transformation, as

$$G \equiv K + \ln W + \ln \bar{W}, \tag{3.15}$$

then the F-term scalar potential can be written as

$$V_E^F = e^G (G_i G^{i\bar{j}} G_{\bar{j}} - W), \tag{3.16}$$

where $G^{i\bar{j}} = K^{i\bar{j}} = (K_{i\bar{j}})^{-1}$.

The chiral fields Φ^i introduced earlier are not in general charged. However, if they are charged under some gauge group $\mathcal{G}_a$ then the the space-time derivatives ∂_μ in eqn. (3.10) should be replaced by the gauge covariant derivatives $D_\mu = \partial_\mu - igB_\mu^a T^a$, where B_μ^a being the gauge field of the theory, g is the coupling constant and T^a are group generators.

Now apart from the F-term contribution, the full scalar potential also has a D-term contribution. However, the D-term models are not of interest to the present thesis. For the sake of entirety we quote only the main result. The D-term contribution to the scalar potential is given by,

$$V_E^D = \sum_a \frac{1}{2}(\text{Re} f_{ab})^{-1} D_a D_b, \tag{3.17}$$

with,

$$D_a = K_i (T_a)^{ij} \Phi_j + \xi_a, \tag{3.18}$$

where the subscript a represents gauge symmetry and T^a is the associated generator. ξ_a is the Fayet-Iliopoulos (FI) term [82] that is non-zero only for the Abelian gauge symmetry, like an $U(1)$ theory.

3.3 The supergravity η-problem

The main difficulty for successfully realizing supergravity inflation models is the so-called η -problem [83, 84, 34]. It appears generically in almost every SUGRA embeddings. To understand the origin and nature of the problem let us again recall the expression for the F-term scalar potential given in eqn. (3.11). For the simple choice of a canonical Kähler potential like

$$K(\Phi, \bar{\Phi}) = |\Phi|^2, \tag{3.19}$$

one obtains for the scalar potential

$$V = e^{|\Phi|^2/M_{pl}^2}\left(\left|\frac{\partial W}{\partial \Phi} + \frac{W\bar{\Phi}}{M_{pl}^2}\right|^2 - 3\frac{|W|^2}{M_{pl}^2}\right) \tag{3.20}$$

$$\sim e^{|\Phi|^2/M_{pl}^2}\left(\left|\frac{\partial W}{\partial \Phi}\right|^2 + \left|\frac{\partial W}{\partial \Phi}\right|^2\left|\frac{\Phi}{M_{pl}^2}\right|^2 + \cdots\right) \tag{3.21}$$

$$= \left(1 + \frac{|\Phi|^2}{M_{pl}^2} + \cdots\right)V_0, \tag{3.22}$$

where V_0 is the vacuum energy. For inflaton this gives a contribution to the second slow-roll parameter

$$\eta = M_{pl}^2 \frac{V''}{V} M_{pl}^2 \sim M_{pl}^2 \frac{V_0}{M_{pl}^2 V_0} = 1 + \cdots \tag{3.23}$$

Therefore, $\Delta\eta \sim 1$ which breaks the slow roll condition $\eta << 1$. Thus the exponential term in eqn. (3.11) invites serious trouble in protecting the flatness of the inflaton potential. This is the famous η-problem in F-term supergravity[3]. To realize inflation then requires either cancellation of order one terms by fine-tuning or employing some symmetry. In SUGRA, this problem is first eliminated for chaotic inflation through the shift symmetry in Kähler potential [85]. For more in this regard we refer to [115, 116, 117] and Section 4.2 of next Chapter.

3.4 Supergravity in Jordan frame

The formulation of supergravity theory in non-minimal Jordan frame is derived in ref by a gauge fixing of the $SU(2,2|1)$ superconformal theory [86, 87]. Unlike the Einstein frame, the Jordan frame is characterized by a direct coupling between the scalar fields and the

[3] D-term model does not suffer from η-problem [84]

Ricci scalar curvature. The locally supersymmetric action in this frame is defined by the choice of following functions — a real Kähler potential $K(z,\bar{z})$, an holomorphic superpotential $W(z)$ and a frame function $\Omega(z,\bar{z})$, determining the scalar-curvature coupling. Here z is the chiral field of the theory.

The pure scalar-gravity part of $4D, \mathcal{N}=1$ supergravity in Jordan frame is described by the action

$$\mathcal{S}_J = \int d^4x\sqrt{-g_J}\left[\frac{1}{2}\,\Omega^{-2}R(g_J)M_{pl}^2 + \Phi\mathcal{A}_\mu^2 + \frac{\mathcal{L}_J^{kin}}{\sqrt{-g_J}} - V_J\right], \tag{3.24}$$

where, the non-minimal factor and $\mathcal{L}_J^{kin}$ are given by

$$\Omega^{-2}(z,\bar{z}) = \frac{\Phi(z,\bar{z})}{3M_{pl}^2}, \tag{3.25}$$

$$\text{and} \qquad \frac{\mathcal{L}_J^{kin}}{\sqrt{-g_J}} = \left(\frac{\Phi K_{\alpha\bar{\beta}}}{3M_{pl}^2} - \frac{\Phi_\alpha\Phi_{\bar{\beta}}}{\Phi}\right)g_J^{\mu\nu}(\partial_\mu z^\alpha)(\partial_\nu \bar{z}^{\bar{\beta}}). \tag{3.26}$$

Here $\Phi_\alpha = \partial\Phi/\partial z^\alpha$, $\Phi_{\bar{\beta}} = \partial\Phi/\partial\bar{z}^{\bar{\beta}}$ and $K_{\alpha\bar{\beta}}$ is the Kähler metric in $(z,\bar{z})$ field space. $\mathcal{A}_\mu$ is the bosonic part of the auxiliary field of supergravity and on shell it is given by

$$\mathcal{A}_\mu = -\frac{i}{2\Phi}[(\partial_\mu z^\alpha)\partial_\alpha\Phi - (\partial_\mu\bar{z}^{\bar{\alpha}})\partial_{\bar{\alpha}}\Phi]. \tag{3.27}$$

The Jordan frame scalar potential is given in terms of the Einstein frame scalar potential

$$V_J = \frac{\Phi^2}{3M_{pl}^4}(V_E^F + V_E^D), \tag{3.28}$$

where V_E^F and V_E^D are already defined in eqns. (3.11) and (3.17) respectively. In this thesis we will consider only those configurations for which the contribution from the bosonic part of the auxiliary vector field vanishes i.e. $\mathcal{A}_\mu = 0$. For such configurations it turns out that picking up a suitable Jordan frame, where the frame function and Kähler potential are logarithmically related, can make the description of a class of models amazingly simple. Also this particular structure of the theory apparently hides the η-problem. In this connection a systematic study of SUGRA contributions and the phenomenology of some example models will be described in Chapter 5.

3.5 Spontaneous breaking of SUSY

The absence of any superpartner of the standard model particles upto the electroweak scale imply that supersymmetry cannot be an exact symmetry in nature but it has to be broken at the low scales observed so far. We will consider only spontaneous breaking of supersymmetry as will be relevant later in the thesis.

Now during inflation a large positive definite potential also breaks SUSY, however now at high scale. Therefore, breaking of supersymmetry technically means that SUSY generators fail to annihilate the true vacuum of the theory $Q_\alpha|0\rangle \neq 0$.

In global SUSY the Hamiltonian can be written in terms of the generators as [88, 89]

$$H = \frac{1}{2}(Q_1^\dagger Q_1 + Q_1 Q_1^\dagger + Q_2^\dagger Q_2 + Q_2 Q_2^\dagger). \tag{3.29}$$

If SUSY is unbroken then the supercharges annihilate the vacuum — $Q_\alpha|0\rangle = Q^\dagger_{\dot\alpha} = 0$. Thus H also annihilates a supersymmetric vacuum $H|0\rangle = 0$. In other words this means the scalar potential of a supersymmetric theory with a supersymmetric ground state has to vanish at its minimum. Now the full scalar potential in a SUSY theory in Einstein frame is

$$V_E = V_E^F + V_E^D \approx |F_i|^2 + |D_a|^2. \tag{3.30}$$

Therefore in a supersymmetric ground state $\langle F_i \rangle = \langle D_a \rangle = 0$. Conversely if SUSY has to be broken then

$$\langle F_i \rangle \neq 0 \quad \text{or} \quad \langle D_a \rangle \neq 0 \quad \Rightarrow \quad V_E|_{minima} > 0 \quad \Rightarrow \quad Q_\alpha |0\rangle = 0. \tag{3.31}$$

Thus $\langle F_i \rangle$ and $\langle D_a \rangle$ serve as the order parameters of spontaneous supersymmetry breaking. The same argument also holds for SUGRA. However, the important difference between global SUSY and SUGRA is that in an unbroken SUSY phase $V_E^F \sim -3e^{K/M_{pl}^2} \frac{|W|^2}{M_{pl}^2}$. So, from eqn. (3.11) it is evident that during inflation we need $\mathcal{D}_i W \neq 0$. This is important, for it takes care of adjusting the cosmological constant to a small value. Now we will briefly discuss the SUSY breaking mediated through F-term and also the SUSY breaking at early universe. The later will be particularly related to the cosmological moduli problem in early universe.

F-term SUSY breaking

We will put an example to demonstrate the F-term supersymmetry breaking. In general we can also break supersymmetry spontaneously through non-vanishing D-term [82, 90, 91]. However, we will ignore this detail in our simplified treatment of SUSY. To illustrate the point let us consider the *O'Raifeartaigh model* described by the superpotential [92]

$$W = \lambda \Phi_0 + m\Phi_1\Phi_2 + Y\Phi_0\Phi_1^2, \tag{3.32}$$

where λ, m, Y are the model parameters and Φ_0, Φ_1, Φ_3 are chiral superfields of the theory.

The scalar potential reads,

$$V_F = |F_0|^2 + |F_1|^2 + |F_2|^2 = |\lambda + Y\Phi_1|^2 + |m\Phi_2 + 2Y\Phi_0\Phi_1|^2 + |m\Phi_1|^2. \qquad (3.33)$$

The potential is minimized by $\Phi_1 = \Phi_2 = 0$, with Φ_1 arbitrary. This is satisfied if $m^2 > \lambda Y$. At the minimum SUSY is broken *i.e.* $F_1 = m^2 \neq 0$

SUSY breaking in the early Universe

Flat directions generically occur in supersymmetric field theories. However such flat directions can be lifted by supersymmetry breaking and non-renormalizable terms in the superpotential. These directions are often thought of as the VEV of some scalar fields commonly known as the moduli fields. In early Universe the coupling between the inflaton (Φ) and flat directions (Z) arise from Planck scale operators. Typically this results in the scalar potential, an induced term [93, 94, 95]

$$V(\Phi) \approx e^{K(Z,\bar{Z})} V_0(Z,\Phi) = H^2 M_{pl}^2 \mathcal{F}(\Phi/M_{pl}), \qquad (3.34)$$

where $\mathcal{F}$ is some function. During inflation, the moduli evolve under this induced potential with $H \sim const$. Since the fields are parametrically close to critically damped, they are driven to a local minimum of the potential (up to quantum de Sitter fluctuations) within a few e-foldings. The minimum of this induced potential is in general displaced by $\mathcal{O}(M_{pl})$ from the true minimum arising from hidden sector supersymmetry breaking [94]. From cosmological standpoint this has profound consequences in altering the phases after inflation (for details see Chapter 6).

The materials covered so far is all we need to know for the purpose of this thesis. In the next two subsequent chapters we have studied some examples to illustrate the features of how inflationary models are embedded in Einstein frame and also in Jordan frame supergravity.

CHAPTER 4

INFLATION IN EINSTEIN FRAME SUPERGRAVITY

We have proposed a simple model of N-flation in SUGRA where chaotic inflation has been realized. In this simple set-up, N fields collectively drive inflation where each field traverses sub-Planckian field values. The fields do interact among themselves, but we have shown that the dynamics can be described in terms of an effective single field.The predictions of the model is akin to the single field quadratic chaotic inflation.

4.1 Introduction

In the present Chapter we move on to describe a simple realization of primordial inflation in Einstein frame supergravity. In particular we have constructed a large field N-flation model in the supergravity framework. The basic inputs relevant in this context have already been reviewed earlier (Ref. Chapter 3). Here we first briefly discussed the key points of chaotic inflation in SUGRA in Section 4.2, and then N-flation in Section 4.3. After that we worked out our proposal for the two-field case in Section 4.4. Subsequently the generalization to N-fields is also made in Section 4.4, followed by the conclusion and discussion at the end.

We knew that during inflation the scalar perturbations responsible for the large scale structures of the Universe are generated due to the quantum fluctuations of the inflaton field, and the tensor perturbations originate from the fluctuations of the spin-2 graviton field. This tensor amplitude induces B-mode polarization in the CMB temperature anisotropy, which is considered to be an unique signature of inflation. Assuming that the source of the B-mode polarization is primordial, the value of r points to a scale of inflation that is close to the GUT scale of around 10^{16} GeV. Following Lyth bound, it also indicates super-Planckian field excursion ($\Delta\phi > M_{Pl}$) during inflation [96]. First of all, having super-Planckian field range with a cut-off scale of Planck mass is difficult to accommodate in the usual notion of effective quantum field theory. Secondly, it is turning out very difficult, if not impossible to arrange large field range in the ultraviolet complete theory like string theory. Many notable attempts have been made though [97], [98], [99], [100], [101], [102].

Another complimentary approach is to find special direction in the multi-dimens-ional field space where a particular combination of the field directions is flat enough to allow super-Planckian VEV. For two field case, this has been realized in 'aligned inflation' where two axion fields have been used [103], [104], [105], [106], [107], [108]. In the case of N-flation, many fields contribute to drive inflation where each field moves over sub-Planckian VEV [109]. Recently, attempts have been made in incorporating N-flation in the string theory set-up [110], [111], [112].

In this work, we propose a model of N-flation in the supergravity (SUGRA) frame work, where each field has a quadratic mass term potential of chaotic inflation. This is a simple generalization of single field chaotic inflation scenario in SUGRA [85]. Considering that the individual field range is sub-Planckian, the effective description in SUGRA is well under control as long as we demand that the imposed symmetry is not broken by the ultraviolet degrees of freedoms. One important aspect of our construction would be that even though the fields have interactions among themselves, in the cosmological background they collectively behave like single degree of freedom without any interactions. This is true only because of the particular nature of the interactions dictated by the proposed form of the model.

4.2 Chaotic inflation in SUGRA

In this section we will discuss inflationary scalar potential in the SUGRA framework, and as an example, we will outline how chaotic inflation can be realized in SUGRA [85]. Our N-flation construction is crucially dependent on this elegant proposal, and in fact it is a simple generalization to N fields.

Being very simplistic in the form of a potential with a mass term, chaotic inflation is an attractive model amongst the zoo of inflationary models. It had gained some tentative observational support after the release of BICEP II data that hinted towards a tensor to scalar ratio $r \sim 0.1$ [113]. Using Lyth bound [96]

$$\Delta\phi = \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2}, \tag{4.1}$$

the data immediately requires super-Planckian field excursion during inflation. Now, the super-Planckian field excursion is a natural requirement for chaotic inflationary potential $V(\phi) = \frac{1}{2}m^2\phi^2$ for slow-roll parameters being small. Thus embedding chaotic inflation in any particle physics set-up is of paramount importance. Here we briefly review how chaotic inflation potential emerges naturally in SUGRA.

The main feature of chaotic inflation models is that inflation occurs for field values larger that M_{pl} For the Canonical choice of $K = \Phi\bar{\Phi}$, the F-term potential has the common factor $V \propto e^{|\Phi|^2}$, and its slope is too steep to sustain the flatness required for chaotic inflation. This is the so called η-problem in F-term inflation [83], [59], [114]. An elegant solution to this problem was proposed in [85] where the authors introduced a shift symmetry to the Kähler potential of the complex chiral superfield Φ, under which $\Phi \to \Phi + iC$, where C is some real constant. This restricts the form of K to be a function of Re(Φ) only. In this case, Im(Φ) does not appear in the exponential, and it can be identified as inflaton free of η-problem[1].

In [85] the following superpotential and Kähler potential for chaotic inflation was proposed

$$W = mX\Phi, \quad K = X\bar{X} - \gamma(X\bar{X})^2 - \frac{1}{2}(\Phi \pm \bar{\Phi})^2, \tag{4.2}$$

where X is an auxiliary chiral superfield that remains at zero VEV during inflation. Inflaton ϕ is a member of the complex chiral superfield Φ, and for simplicity we are assuming

[1] Heisenberg symmetry can also be imposed in solving η-problem [115, 116, 117].

inflaton to be singlet under relevant gauge group. This allows us not to worry about D-term contribution to the scalar potential. The superpotential breaks the shift symmetry imposed in the Kähler potential, and thus gives rise to the tree-level mass term via the F-term of the auxiliary field X. In other words supersymmetry is broken along the X-direction ($\mathcal{D}_X W \neq 0$) and it supplies the necessary potential energy to drive inflation. Here $-\gamma(X\bar{X})^2$ term is added to render the mass of the X field being greater than the Hubble scale. For $\gamma = 0$, the mass of the X field is comparable to the mass of the inflaton. So there will be inflationary fluctuations of X during inflation and hence the dynamics can not be regulated with one field. The above construction is characterized by

$$W_{inf} = 0, \quad \mathcal{D}_X W_{inf} \neq 0, \quad \mathcal{D}_\Phi W = 0 \tag{4.3}$$

during inflation, and it has been generalized in the case of hybrid inflation scenario in the framework of tribrid inflation [118, 119, 120]

From the point of effective SUGRA theory, this set-up is complete in a sense that with the assumption of shift symmetry breaking term in the superpotential is small, its corrections (potentially shift symmetry breaking) to the Kähler potential are going to be also parametrically small. Now the smallness of the symmetry breaking parameter m is ensured by the scalar amplitude of density fluctuations. This fixes the value of $m \sim 10^{-5} M_{pl}$. Whether the symmetry breaking is under control in any UV complete theory like string theory is an open issue, and it requires understanding of the dynamics of stringy degrees of freedom.

Even though the construction is elegant, the difficulties behind the description of large field inflation with one single field is problematic from the point of view of purely effective field theories. In the context of effective field theory, the inflation potential can be written in the following form

$$V_{eff} = V(\phi) + \sum_{n=0}^{\infty} c_n V(\phi) \frac{\phi^{n+1}}{M_{Pl}^{n+1}}, \tag{4.4}$$

where c_n's are dimensionless coefficients of order one. Since the inflaton has to traverse over a trans-Planckian distance in the field space during the time inflation takes place, *i.e.* $\Delta\phi > M_{pl}$, each term in the summation contributes equally well to the potential unless its coefficients c_n's are finely tuned. So large field inflation becomes sensitive to an infinite number of such terms. If we want to predict the dynamics, we necessarily need to know all these terms. Note that in the setup of Eq. (4.2), these higher dimensional operators are under control due to the imposed symmetry and its soft breaking in the superpotential. In the next section, we will discuss how the problem of super-Planckian VEV of a single field can be evaded in the set-up of N-flation by distributing the job of driving inflation in N fields [109].

4.3 N-flation

Even though the formulation of chaotic inflation in SUGRA is well understood, the tentative observations require the field associated with the single inflaton to be super-Planckian. Thus constructing a model of inflation with more than one field is worth formulating where the job of driving inflation is distributed among many ϕ_i fields. Each ϕ_i satisfies $\Delta\phi_i < M_{Pl}$ Under this condition the potential for an individual field can be expanded in the effective field theory framework. As we will see, in the proposed N-flation scenario, the total field displacement is now

$$\Delta\phi^2_{total} = \sum_{i=1}^{N} \Delta\phi_i^2 > M_{Pl}. \tag{4.5}$$

The actual idea of N-flation was first proposed by Dimopoulos et.al [109], and the basic point was inflaton is a collection of N number of fields that drives inflation through assisted inflation mechanism [121, 122, 123], rather than a single field. Here each field ϕ has a field excursion smaller than Planck scale. Individual fields are not capable of producing the slow-roll for an appreciable number of e-folds, but a collection of such fields produces sufficient e-folds to solve the cosmological problem. So the dynamics is determined collectively by N such fields. The motivation was from the standpoint of particle physics where the existence of scalar field is ubiquitous. In the original work of [109], the individual field was axion having periodic potential. Around the bottom of the potential where inflation happens, the potential was written as the sum of potential for each individual field *i.e.*

$$V(\phi_i) = \sum_{i=1}^{N} V_i(\phi_i) = \sum_{i=1}^{N} \frac{1}{2} m^2 \phi_i^2 \tag{4.6}$$

So here each field ϕ_i is moving under the potential $m^2\phi_i^2/2$. It was assumed that the cross-couplings between the fields are negligible. Considering an initial configuration where each field is displaced from the minimum of the potential by a sub-planckian displacement $\langle\phi_{n0}\rangle = \alpha_n M_{Pl}$, the total displacement in field space in polar coordinate is

$$\rho^2 = \sum_i \phi_i^2 = \sqrt{N}\alpha M_{Pl}. \tag{4.7}$$

In term of the variable ρ the effective Lagrangian density is,

$$\mathcal{L} \simeq (\partial\rho)^2 + \rho^2(\partial\Omega)^2 - \frac{1}{2}m^2\rho^2. \tag{4.8}$$

Now the the angular degree of freedom Ω has no potential energy, and its equation of state parameter $\omega_\Omega = 1$. Thus its energy density falls as a^{-6}, and its contribution becomes negligible soon compared to the radial field ρ. Effectively, the radial variable ρ will act as an inflaton with its single field dynamics.

Our objective is to construct a model of N-flation in the framework of SUGRA. In our simple frame-work, the fields are going to have cross couplings among themselves. But because of the particular nature of the coupling that automatically arises in the set-up, the effective field dynamics in the cosmological background is similar to the single field. In our case the potential will be like

$$V(\phi_1, ..., \phi_N) = \sum_{i=1}^{N} V_i(\phi_i) + interactions. \tag{4.9}$$

We will first discuss a simple case where the inflaton is a collection of a pair of fields. In this case, we will solve the dynamics numerically to show that the field trajectory in the slow-roll attractor is a straight line. Then we will go for the generalization with N fields.

4.4 N-flation in SUGRA

We present our main result in this section. As a toy example we first analyze the two-field case, where trivial redefinition of fields can make the dynamics effectively single field. We also analyze the background dynamics numerically to show how the attractor solution emerges for the effective single field case. This shows that the dynamics is governed effectively by one degree of freedom. Subsequently, we do the generalization for the case of N number of fields, where we have analytically proved how it can be reduced to the case of a single field model.

4.4.1 Two field case

Let us begin with the following choice of the superpotential

$$W = mX(\Phi_1 + \Phi_2) \tag{4.10}$$

Here each Φ is a chiral superfield which contains one singlet inflaton field. The masses of the fields are taken to be degenerate for simplicity. X is another chiral superfield that is needed to provide the vacuum energy via its non-zero F-term. The Kähler potential is taken to be

$$K = X\overline{X} - \gamma(X\overline{X})^2 - \frac{1}{2}(\Phi_1 - \overline{\Phi}_1)^2 - \frac{1}{2}(\Phi_2 - \overline{\Phi}_2)^2 \tag{4.11}$$

Here $-\gamma(X\overline{X})^2$ term is added for the stabilization of X field as mentioned in detail in section (4.2). This will ensure that the X field gains a mass larger than the inflaton mass during inflation and hence it will not disturb the inflationary dynamics. The Kähler potential respects the shift symmetry for the inflaton fields: $\Phi_i \to \Phi_i + i\alpha_i$. The real components of those fields can be identified as inflaton fields. This is to avoid the usual η-problem. The

F-term scalar potential is

$$
\begin{aligned}
V_F = m^2 e^K \Bigg[& |X|^2 \{ (1 - (\Phi_1 + \Phi_2)(\Phi_1 - \bar{\Phi}_1))(1 + (\Phi_1 - \bar{\Phi}_1)(\bar{\Phi}_1 + \bar{\Phi}_2)) \\
& + (1 - (\Phi_1 + \Phi_2)(\Phi_2 - \bar{\Phi}_2))(1 + (\Phi_2 - \bar{\Phi}_2)(\bar{\Phi}_1 + \bar{\Phi}_2)) \} + \\
& \frac{|\Phi_1 + \Phi_2|^2 (1 + |X|^2 (1 - 2\gamma |X|^2))^2}{1 - 4\gamma |X|^2} - 3|X|^2 |\Phi_1 + \Phi_2|^2 \Bigg]
\end{aligned}
\tag{4.12}
$$

Let us now decompose the complex superfields Φ_1 and Φ_2 into a pair of real scalar fields

$$\Phi_1 \equiv \frac{1}{\sqrt{2}}(\phi + i\beta), \quad \Phi_2 \equiv \frac{1}{\sqrt{2}}(\chi + i\sigma). \tag{4.13}$$

The masses for these fields can be calculated using this F-term SUGRA expression given in Eq. (4.12). The masses-squared of the field X (both real and imaginary parts) is given by

$$m_X^2 = 12\gamma H^2 + 2m^2 \tag{4.14}$$

So for positive $\gamma \neq 0$, $m_X > \mathcal{O}(H)$, and it decouples from the inflationary dynamics in settling to its minima at $X = 0$. Now in the trajectory of $X = 0$, the mass squared of the fields Im Φ_1 and Im Φ_2 are

$$m_\beta^2 = m^2(1 + 2\sigma^2) + 6H^2, \quad m_\sigma^2 = m^2(1 + 2\beta^2) + 6H^2. \tag{4.15}$$

Therefore during inflation they are also stabilized at $\beta = \sigma = 0$. We have checked the stabilization numerically by solving the dynamics.

There are two inflaton fields ϕ and χ whose potential do not contain the e^K factor of Eq. (4.12) and thereby evade the η-problem. Along the inflationary trajectory $X = \beta =$

$\sigma = 0$, the scalar potential as computed from Eq.(4.12) looks like

$$V(\phi, \chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m^2\chi^2 + m^2\phi\chi. \tag{4.16}$$

Clearly, this potential is a sum of two chaotic inflation potential together with an interaction term as written earlier in Eq. (4.9). At this point, we draw particular attention to the nature of the coupling which depends on each field linearly. Including the kinetic terms, the Lagrangian density for the inflaton fields is given by

$$\mathcal{L} = (\partial_\mu\phi)(\partial^\mu\phi) + (\partial_\mu\chi)(\partial^\mu\chi) - V(\phi, \chi), \tag{4.17}$$

where $V(\phi, \chi)$ is given by Eq.(4.16).

The above Lagrangian can be easily casted in a more convenient form by defining two new fields

$$\varphi_1 = \frac{1}{\sqrt{2}}(\phi + \chi), \quad \varphi_2 = \frac{1}{\sqrt{2}}(\phi - \chi), \tag{4.18}$$

and in terms of these two fields the Lagrangian density can be written as,

$$\mathcal{L} = (\partial\varphi_1)^2 + (\partial\varphi_2)^2 - \frac{1}{2}(\sqrt{2}m)^2\varphi_1^2. \tag{4.19}$$

Here one degree of freedom φ_1 is massive while the other one φ_2 is massless. In the two dimensional field space, there is a flat direction φ_2 along which the potential vanishes. So the equation of state parameter $\omega = 1$ for φ_2. Thus its associated energy density redshifts very quickly and becomes cosmologically irrelevant,

$$\rho_{\varphi_2} \propto a^{-3(1+\omega_{\varphi_2})} \propto a^{-6}. \tag{4.20}$$

On the other hand φ_1 direction has a chaotic inflation potential that can drive inflation. The total field variation $\Delta\varphi_1 \sim \Delta\phi + \Delta\chi$. It is important to note that this is crucially different from the original N-flation set-up, where for two field case the effective displacement of the radial field direction is always positive $\Delta\varphi_1^2 \sim \Delta\phi^2 + \Delta\chi^2$ [109]. In our case, this is not necessarily true, and we consider this point as one drawback of our set-up. In our setup, the effective displacement of the field is dependent on the initial field configurations. At this point, we note that reduction of the potential with coupling term into an effective single-field case is possible only for the $m^2\phi\chi$ coupling. This is no longer true for $\phi^2\chi^2$ coupling where it is evident that field trajectory is curved in general, and can not be described by one degree of freedom [124].

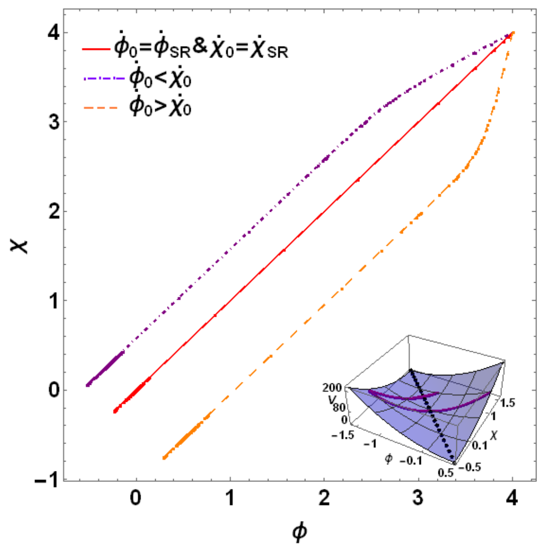


Figure 4.1: Field space plots for three different boundary conditions. In the inset, we show the evolution of the field for single trajectory near its minima. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Fig (4.1), we show the field space plot of this simple set-up. We have analyzed three cases that corresponds to three different initial boundary conditions to elucidate the nature of the attractor behaviour in this case. As we see the attractor solutions in the field space are straight lines. Here each field satisfies the same Friedmann equation. During slow roll of ϕ and χ the field displacements for ϕ will be proportional to the field displacement for χ in the phase space *i.e.* $\Delta\phi \propto \Delta\chi$ in same time interval. So $\phi = \chi + c$ will be the attractor solution, where c is a constant. In the field space this will be reflected in straight line behaviour which is what we obtained in Fig.(4.1).

Now the potential can be written as $V(\phi,\chi) = \frac{1}{2}m^2(\phi+\chi)^2$. The minima of this

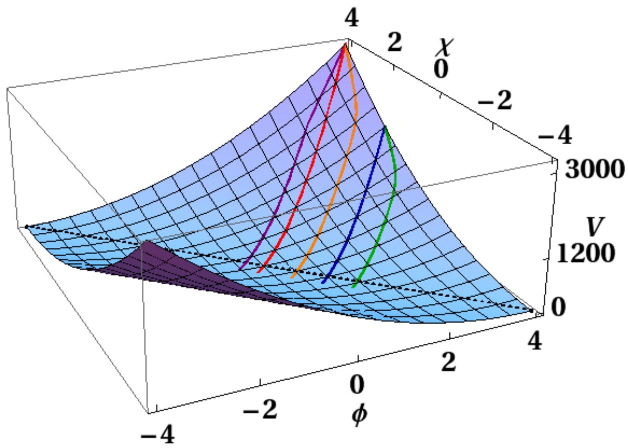


Figure 4.2: Effective potential showing the joint evolution of fields ϕ and χ. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

potential is not a single point but a line whose equation is $\phi + \chi = 0$. This equation is satisfied by many field points (ϕ,χ). The central plot (red) in Fig.(4.1) corresponds to the initial conditions $\phi_0 = \chi_0 = 4M_{pl}$ and $\dot{\phi}_0 = \dot{\phi}_{SR}$ and $\dot{\chi}_0 = \dot{\chi}_{SR}$. In this case, equation of minima is $2\phi = 0$ or $2\chi = 0$. So minimum of $V(\phi,\chi)$ is at the origin. That is why the central plot passes through the origin. Now we keep the former initial condition intact and change the later to $\dot{\phi}_0 \neq \dot{\phi}_{SR}$ and $\dot{\chi}_0 \neq \dot{\chi}_{SR}$. Then two cases may appear one is $\dot{\phi}_0 < \dot{\chi}_0$ (purple) and the other is $\dot{\phi}_0 > \dot{\chi}_0$ (orange). For the first case (shown in purple) the trajectory is initially curved and then parallel to the central straight line (red). The reason for this is that as we have started with a different $\dot{\phi}$ so it will first meet the attractor when it slow rolls and then its subsequent evolution in the phase space is similar to the case of a single-field chaotic inflation. So is the case with $\dot{\chi}$. But as at $t = 0$ we have given different initial values to $\dot{\phi}$ and $\dot{\chi}$ the manner in which both $\dot{\phi}$ and $\dot{\chi}$ converge to their respective attractor solution is not the same. This is manifested in the curvature of the purple coloured plot. The explanation is same for the second case $\dot{\phi}_0 > \dot{\chi}_0$ (orange) too as the initial condition is just the opposite of the first. Near the lower left corner of Fig.(4.1)there is a little thick portion in each of these lines. This is where the fields oscillate around the minima of V. The minimum is different for three lines as explained earlier. The behaviour of the fields near the minima of the potential is shown in the small box at the lower right corner in this figure. This represents the oscillatory part of the field evolution.

The Fig. (4.2) is a 3D plot that shows the combined evolution of the fields ϕ and χ in the effective potential $V(\phi, \chi)$. The field trajectories of Fig.(4.1) is shown on the potential. Basically Fig.(4.1) is a two dimensional projection of Fig.(4.2) in $\phi - \chi$ plane. The dotted black line represents the minima of the potential. Two additional field trajectories are shown in Fig.(4.2) for $\phi_0 > \chi_0$. Blue line corresponds to the initial condition $\dot{\phi}_0 = \dot{\phi}_{SR}$ and $\dot{\chi}_0 = \dot{\chi}_{SR}$ and green line corresponds to the condition $\dot{\phi}_0 > \dot{\chi}_0$. All these family of straight lines depicts the attractor behaviour. In summary, we note that $\phi = \chi + constant$ are attractor solutions, and even if the field has non-slow-roll initial conditions, it reaches to the attractor solution quickly and follow the straight line trajectory in showing its effective one degree of freedom.

In the next step we will consider N number of fields where the fields will have interactions among themselves .

4.4.2 N-Field generalization

Now we are going to extend our formalism for the case of N-field configuration in $\mathcal{N}=1$ SUGRA. Here the involvement of N-fields share among themselves the complete task of producing super-Planckian field excursion during inflation. We propose the following superpotential

$$W = mX\sum_{i=1}^{N}\Phi_i, \tag{4.21}$$

where $\{\Phi_i\}$'s are the set of complex chiral superfields and $i \in \mathbb{I}$. Each Φ_i contributes one inflaton. The Kähler potential is

$$K = (X\bar{X}) - \zeta(X\bar{X})^2 - \frac{1}{2}\sum_{i=1}^{N}(\Phi_i - \bar{\Phi}_i)^2 \tag{4.22}$$

Shift symmetry is satisfied by every chiral field *i.e.* $\Phi_i \to \Phi_i + iC,\ \ \forall\, i$, C is real constant. Now each complex Φ_i can be decomposed into a real and imaginary parts like two-field case. Then along the direction of inflation $X = \text{Im}\ \Phi_1 = \text{Im}\ \Phi_2 = ... = \text{Im}\ \Phi_N = 0$. The scalar potential in this case takes the form

$$\begin{aligned} V &= \frac{1}{2}m^2\left(\sum_{i=1}^{N}\phi_i\right)^2 & (4.23)\\ &= \frac{1}{2}m^2\sum_{i=1}^{N}\phi_i^2 + \text{two field interaction}, & (4.24)\end{aligned}$$

where $\phi_i = \text{Re}\ \Phi_i,\ \forall\, i$. Masses of the fields are degenerate. Now we will find an orthogonal combination of Φ_i's such that the potential becomes a single-field potential.

Let us define a vector,

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} \tag{4.25}$$

Here the set of basis vectors $\{\mathbf{e_i}\}$ are

$$|e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad |e_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

$$|e_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad \ldots \quad , |e_N\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

Let us move to a different set of basis vectors $\{\mathbf{e'_i}\}$. With respect to this new set of basis vectors the components of the chiral field $\mathbf{\Phi}$ will also change accordingly. Thus we can write,

$$\mathbf{\Phi} = \phi_{\mathbf{i}}|\mathbf{e_i}\rangle = \psi_{\mathbf{j}}|\mathbf{e'_j}\rangle \tag{4.26}$$

The old basis is related to new basis by

$$|e_i'\rangle = \mathbf{T_{ij}}|\mathbf{e_j}\rangle \tag{4.27}$$

$$\mathbf{T_{ij}} = \langle e_j|e_i'\rangle \quad \text{so} \quad \mathbf{\Phi} = \underbrace{\phi_i T_{ij}^{-1}}|\mathbf{e_j'}\rangle \tag{4.28}$$

Comparing eqns.(4.26) and (4.28) we find

$$\psi_j = \phi_i \mathbf{T_{ij}^{-1}} \tag{4.29}$$

As we aim at reducing the N-field Lagrangian density into a single field Lagrangian density so we define,

$$\psi_1 = \frac{1}{\sqrt{N}}(\phi_1 + \phi_2 + \ldots + \phi_N) \tag{4.30}$$

From eqn.(4.29)

$$\psi_1 = T_{11}^{-1}\phi_1 + T_{21}^{-1}\phi_2 + \ldots + T_{N1}^{-1}\phi_N \tag{4.31}$$

$$\text{So,} \qquad T_{11}^{-1} = T_{21}^{-1} = T_{31}^{-1} \ldots\ldots = T_{N1}^{-1} = \frac{1}{\sqrt{N}} \tag{4.32}$$

But as T_{ij} is an orthogonal matrix *i.e.* $T_{ij}^{-1} = T_{ji} = T_{ij}^T$, so, $T_{11} = T_{12} = \ldots = T_{1N} = \frac{1}{\sqrt{N}}$. Now we obtain from eqn.(4.27)

$$|e_1'\rangle = \frac{1}{\sqrt{N}}(|e_1\rangle + |e_2\rangle + \ldots + e_N\rangle) = \frac{1}{\sqrt{N}}\begin{pmatrix}1\\1\\\vdots\\1\end{pmatrix} \tag{4.33}$$

Now by Gram-Schmidt process the other orthonormal basis vectors can be found out and they are,

$$
\begin{aligned}
|e_2'\rangle &= \frac{1}{\sqrt{\langle e_2'|e_2'\rangle}}\left(|e_2\rangle - \frac{\langle e_1'|e_2\rangle}{\langle e_1'|e_1'\rangle}|e_1'\rangle\right)\\
|e_3'\rangle &= \frac{1}{\sqrt{\langle e_3'|e_3'\rangle}}\left(|e_3\rangle - \frac{\langle e_1'|e_3\rangle}{\langle e_1'|e_1'\rangle}|e_1'\rangle - \frac{\langle e_2'|e_3\rangle}{\langle e_2'|e_2'\rangle}|e_2'\rangle\right)
\end{aligned}
$$

$\dots \quad \dots \quad \dots \quad \dots \quad \dots$

In general,

$$
|e_N'\rangle = \frac{1}{\sqrt{\langle e_N'|e_N'\rangle}}\left(|e_N\rangle - \sum_{i=1}^{N-1}\frac{\langle e_i'|e_N\rangle}{\langle e_i'|e_i'\rangle}|e_i'\rangle\right)
$$

With respect to these new basis $\{\mathrm{e_i'}\}$, the new components ψ_i's can be found out using eqns.(4.29) and (4.27). The Lagrangian density is therefore,

$$
\mathcal{L} = \sum_{i=1}^{N}(\partial\psi_i)^2 - \frac{1}{2}(\sqrt{N}m)^2{\psi_1}^2 \tag{4.34}
$$

Thus our method also worked successfully for N-field configuration. Here we see from eqn.(4.34) all fields ψ_i, where $i \in [2, N]$, other than ψ_1 has no potential energy term. So, they will be easily overthrown from the dynamics since their energy density falls faster than that of ψ_1. So, again

$$
\mathcal{L}_{eff} = (\partial\psi_1)^2 - \frac{1}{2}(\sqrt{N}m)^2{\psi_1}^2 \tag{4.35}
$$

The interaction term which initially appeared in the expression of the F-term scalar potential does not affect the evolution of ψ_1. Only the mass of ψ_1 field is enhanced by a factor of $\sqrt{N}$. Thus the effective dynamics in the attractor solution is governed by one degree of freedom ψ_1. Here the total field variation in the field space $\Delta\psi_1 \sim \sum_{i=1}^{N} \Delta\phi_i$. It is indeed true that the total field variation is dependent on the initial field configuration, i.e initial conditions for the individual field. But in the multidimensional field space, it is expected that the sum can be easily super-Planckian even with some cancellations with sub-Planckian field ranges for individual field.

4.5 Conclusion and Discussions

Inflation amplifies the vacuum fluctuations of the metric. This leads to a nearly scale-invariant tensor power spectrum, well characterized by a single parameter called the tensor-to-scalar ratio defining the (relative) amplitude of tensor fluctuations. The net effort to detect gravitational waves from PLANCK and BICEP II experiments so far has been transformed into a lower bound on this parameter, $r < 0.07$. In accordance with this bound, a large r even close to $r > 10^{-2}$ means that field excursion is super-Planckian. In the single field set-up, this is problematic from the point of view of effective field theory. Therefore, one option is to use the effects of multiple fields. With tuned parameters, two fields can achieve this large field excursion in the context of natural inflation [103, 104, 105]. Another possibility is the use of multiple fields having collective dynamics. In the context of many axions in string theory, N-flation was proposed [109]. Even though each axion has a periodic potential protected by a perturbative shift symmetry, the inflation happens at the bottom of the potential where the potential can be safely approximated by the chaotic form of quadratic potential.

In this work, we propose a simple realization of N-flation in SUGRA. This is based on the generalization of the set-up of chaotic inflation in SUGRA where η-problem is solved by the shift symmetries of each individual field. Even though the effective potential has couplings between all fields, but the nature of the couplings allows us to reduce the potential in effective one degree of freedom in the cosmological background. The model has only one free parameter m that controls the breaking of the shift symmetry. Its value is fixed by the normalization of scalar density perturbations. For the case of two field case, we have solved the background dynamics numerically to show that the attractor behaviour of the solutions, and we have found that the field trajectory is straight line on the attractor solution. For the case of N-fields, reduction to single field has been done analytically. The model has similar predictions to the single-field chaotic inflation case: tensor to scalar ratio $r \sim 0.1$ and $n_s \sim 0.96$. Obviously, the produced density perturbations is of adiabatic type. As we have noted earlier, in our set-up $\Delta\phi_{eff} = \sum_i^N \Delta\phi_i$, showing that the effective field range in the multi-dimensional field space is dependent on the initial field configurations. We would also like to note that the produced density perturbations would be adiabatic in nature. This is true because the effective dynamics can be well described by one degree of freedom in the attractor solution.

Our set-up involves N number of fields, and therefore it is natural to think that there would be isocurvature modes which are highly constrained by Planck data. It is indeed true that the possibility of existence of isocurvature mode arises only when there are multiple degrees of freedom that carry energy density during inflation and are at the same time lighter than the Hubble constant during inflation. But isocurvature mode can be best understood by analyzing the field dynamics in the multi-dimensional field space[125]. In this case, the isocurvarure mode can be associated with the curvature of the field trajectory. In other way, if the field trajectory is one parameter family of lines, the only relevant perturbations are adiabatic. This is clear for the simplest two-dimensional case. In our case, as we have shown, the effective field dynamics is one dimensional in the slow-roll trajectory. Other than analytical proof, it has been shown with the numerical analysis for two fields. Therefore, the only source of isocurvature modes is when the field has not reached the attractor solution. Assuming long enough epoch of inflation, this initial phase can be neglected. This argument is similar to the original work of N-flation[109] where the perturbations are only of adiabatic type. In summary, we also conclude that in our set-up, only the adiabatic mode is relevant to density perturbations, and for all practical purposes isocurvature perturbations can be neglected.

At the end of inflation, the potential is going to have many massless degrees of freedom along which the potential is flat. But it is expected that the flat directions are going to be lifted with the soft masses related to low energy SUSY breaking. Explicit nature of these masses can be understood only when N-flation is considered in conjunction with the SUSY breaking sector along the line of [126]. We note that the large number of light species in a given theory typically makes corrections to the Planck mass proportional to $\sqrt{N}$, and this can potentially increase the effective slow-roll parameters spoiling N-flation [109]. To answer this question in a concrete manner, we need a complete ultraviolet theory where these corrections can be reliably computed [112]. In the effective SUGRA set-up, this question can not be addressed. Possible resolutions those that are protected from UV physics have been discussed in [127, 128].

In our set-up, we have considered the most simple generalization of the single field chaotic inflation set-up in SUGRA. Other generalization of Eq. (4.21) are possible. For example, instead of having a common auxiliary field X, we may have X_i for each Φ_i, or the mass parameter m can be different. It would be interesting to explore the outcome of those constructions. But in this case, it is expected that the simple analytical understanding as we have done is not possible, and a statistical approach would be more suitable [129].

CHAPTER 5

INFLATION IN JORDAN FRAME SUPERGRAVITY

We presented a systematic study of SUGRA contributions relevant for inflationary models in Jordan frame supergravity. In a special class of Jordan frame, the scalar potential separates into a tree-level term and a SUGRA contribution term which is potentially dangerous for sustaining inflation. If during inflation the vacuum energy is mainly due to the F-term of an auxiliary non-inflaton field, the SUGRA corrections to the scalar potential are generically suppressed or may even vanish if the superpotential vanishes along the inflationary trajectory. However, if the F-term of inflaton dominates the vacuum energy, SUGRA contributions $\approx$ global SUSY contributions. In addition, the non-minimal coupling significantly impacts inflationary models depending on the size and sign of this coupling.

5.1 Introduction

In this Chapter we discuss the phenomenological aspects of inflationary models within non-minimal Jordan frame supergravity. The overall descriptions given here also relies on our primary treatments covered in Chapter 3.

We start the presentation of our work introducing the structure of CSS models in the Jordan frame in Section 5.2. Then we turn to the supergravity contributions to the Jordan frame scalar potential in Section 5.3. In Section 5.4 we derive the conditions under which these generic contributions vanish. We demonstrate that for (effective) single-field inflation these conditions are equivalent to the requirement of a vanishing superpotential along the inflationary trajectory, and illustrate these results by applying them to some well-known inflation models: Monomial inflation, hybrid inflation and tribrid inflation. In Section 5.5 we turn to supergravity contributions in the kinetic term in the Einstein frame, shedding light on the feature of 'attractor' models in this set-up. Section 5.6 discusses two explicit examples in which all the above effects are illustrated. The details of the tribrid model are left in Section 5.7. Finally we conclude in Section 5.8.

Supergravity [21], arising as the low energy limit of string compactifications, is a promising theoretical framework to describe inflation: providing numerous (complex) scalar fields potentially suitable for inflation, it also consistently accounts for the Planck-suppressed corrections to global supersymmetry, which can no longer be simply neglected at the high energy scales of inflation.

On the one hand, these supergravity contributions represent a challenge for inflationary model building, potentially spoiling the flatness of the scalar potential required for slow-roll inflation. The infamous η-problem of F-term inflation (with minimal coupling to gravity) is a well-known example of this problem [83, 93] (see also Section 3.3 of Chapter 3). On the other hand, inflation models with a non-minimal coupling to gravity have recently received a lot of interest, e.g. in the context of Higgs inflation [130] and so-called attractor models [131]. A key observation is that including the non-minimal coupling to gravity, many inflation models with very different scalar potentials asymptotically approach the same unique predictions for the spectral index n_s and the tensor to scalar ratio r, which moreover lie right in the sweet spot of the recent Planck data [132, 86, 133, 134, 135, 136, 137, 138, 139, 140]. Such a non-minimal coupling to gravity is a characteristic feature of supergravity in the Jordan frame [141, 142, 143, 131, 144, 146].

Here we systematically classify the supergravity contributions to Jordan frame inflation models. Starting from the framework of canonical superconformal supergravity (CSS) models suggested in [87], characterized by canonical kinetic terms in the Jordan frame and an approximate conformal symmetry, we determine the generic properties of supergravity contributions to the Jordan frame scalar potential, as well as the resulting contributions to the Einstein frame Lagrangian. This enables us to disentangle two important supergravity effects: contributions to the (Jordan frame) scalar potential which typically come in powers of ϕ/M_{pl} yielding dangerous corrections at large field values and contributions to the kinetic term in the Einstein frame, which in many cases lead to a flattening of the potential, favourable for slow-roll inflation.

Our analysis reveals that the CSS models provide a powerful model building framework to control supergravity contributions. If the vacuum energy driving inflation is dominated by the F-term associated with the chiral multiplet of the inflaton, the supergravity contributions become comparable to the globally supersymmetric contributions but do not necessarily dominate at large field values. If the F-term of the inflaton field is subdominant, the dangerous supergravity contributions to the scalar potential are generically suppressed at large field values compared to the contributions from global supersymmetry. We further investigate under which conditions these generic supergravity contributions to the (Jordan frame) scalar potential vanish, finding (for single-field inflation) that this can only be achieved if the F-term of the inflaton is subdominant and if the superpotential vanishes along the inflationary trajectory. Turning to the kinetic term in the Einstein frame, we generalize the results obtained in the context of α-attractors [144], demonstrating that non-canonical kinetic terms lead to an exponential flattening of the potential if the functional dependence of the Jordan frame scalar potential and the non-minimal coupling to the Ricci scalar on the inflaton field are adjusted accordingly. The combination of the (mildly broken) conformal symmetry inherent to CSS models in combination with specific choices of superpotentials allows for inflation models in agreement with current observations.

5.2 Non-minimal SUGRA and CSS model

We have already seen that a supergravity model in $\mathcal{N} = 1$ and $D = 4$ is most commonly described in the Einstein frame, where the matter part is minimally coupled to gravity and the scalar and gravitational parts of the Lagrangian density relevant for inflation are determined by the superpotential W and the Kähler potential K [21]. On the other hand, a $\mathcal{N} = 1$, $D = 4$ supergravity model may also be considered in a Jordan frame characterized by a frame function $\Phi(z^\alpha, \bar{z}^{\bar{\alpha}})$. In this case, the gravitational and the scalar parts of the

Lagrangian density read [147, 86, 87]

$$\mathcal{L}_J^{grav} = -\sqrt{-g_J}\,\frac{1}{6}\,\Phi(z,\bar{z})\,R(g_J)\,, \tag{5.1}$$
$$\mathcal{L}_J^{scalar} = \sqrt{-g_J}\left[\left(\frac{\Phi K_{\alpha\bar{\beta}}}{3M_{pl}^2} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi}\right) g_J^{\mu\nu}(\partial_\mu z^\alpha)(\partial_\nu \bar{z}^{\bar{\beta}}) - V_J\right]. \tag{5.2}$$

We note from Eq. (5.1) that the matter fields z^α are non-minimally coupled to gravity. A conformal transformation allows us to switch from a Jordan frame to the Einstein frame Lagrangian as

$$g_{\mu\nu}^J = \Omega^2\, g_{\mu\nu}^E \quad \text{where,} \quad \Omega^2 = -\frac{3M_{pl}^2}{\Phi} > 0\,. \tag{5.3}$$

In particular, the Jordan frame scalar potential (V_J) is related to the scalar potential (V_E) in the Einstein frame as

$$V_J = \frac{\Phi^2}{9M_{pl}^4} V_E\,. \tag{5.4}$$

If $\Phi = -3M_{pl}^2$ the transformation is trivial; this particular Jordan frame represents the Einstein frame. However, for our purpose we will focus on the F-term contribution to the Einstein frame scalar potential. So for convenience let us recall its structure which is

$$V_E^F = e^{K/M_{pl}^2}\left[K^{\alpha\bar{\beta}}\mathcal{D}_\alpha W \mathcal{D}_{\bar{\beta}}\bar{W} - \frac{3|W|^2}{M_{pl}^2}\right]. \tag{5.5}$$

Here the exponential factor in Eq. (5.5) is the source of the usual η-problem in supergravity inflation, which needs to be overcome when constructing inflation models in supergravity [83, 93]. This may be achieved by either imposing a symmetry in the Kähler potential [85, 116], or tuning the model parameters [148].[1] Additional contributions to the η-parameter arise from the other terms of the F-term scalar potential, and these typically depend on the form of the superpotential during inflation.

Non-trivial Jordan frame inflation models are characterized by a non-minimal coupling of gravity to the inflaton field, a feature which has recently received a lot of interest in the context of Higgs inflation [130, 132, 86]. Moreover, the Jordan frame has intriguing properties from a more conceptual point of view. As was demonstrated in Ref. [147], the usual formulation of supergravity can be obtained by starting from a larger symmetry group, namely the superconformal group, and gauge-fixing the additional degrees of freedom. This approach naturally leads to Jordan frame supergravity models, which can then be translated to the Einstein frame by a conformal transformation. All inflationary observable quantities are frame independent, and can be calculated in either the Jordan or the Einstein frame [150, 151]. However, the simplicity of a given model may be obscured depending on the frame used.

A class of models inspired by this approach are the canonical superconformal supergravity (CSS) models, cf. Ref. [87], which feature an intriguingly simple structure in the Jordan frame. They are characterized by the choice

$$\Phi(z,\bar{z}) = |z^0|^2 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} \mapsto -3M_{pl}^2 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}}\,, \tag{5.6}$$

$$K(z,\bar{z}) = -3M_{pl}^2 \ln\left(-\frac{\Phi(z,\bar{z})}{3M_{pl}^2}\right), \tag{5.7}$$

with $z_0 \mapsto \sqrt{3}M_{pl}$ denoting the gauge-fixing of the conformal compensator field. Eqs. (5.6) and (5.7) lead to

$$K_{\alpha\bar{\beta}} = -\frac{3M_{pl}^2}{\Phi}\left(\delta_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi}\right), \tag{5.8}$$

[1] As an alternative solution to η-problem, please see [149].

and hence Eqs. (5.1) and (5.2) become

$$\frac{1}{\sqrt{-g_J}}\mathcal{L}_J^{\text{grav + scal}} = \underbrace{\frac{1}{2}M_{pl}^2\,R(g_J)}_{\text{pure SUGRA}} - \underbrace{\frac{1}{6}M_{pl}^2\,R(g_J)\,|z|^2 - \delta_{\alpha\bar{\beta}}\,g_J^{\mu\nu}(\partial_\mu z^\alpha)(\partial_\nu \bar{z}^{\bar{\beta}}) - V_J}_{\text{superconformal matter}}\,. \quad (5.9)$$

This Jordan frame Lagrangian shows several remarkable features. For a suitable (scale-invariant) scalar potential, the matter sector features a superconformal symmetry. This in particular includes invariance under local conformal transformations,

$$g_{\mu\nu} \to e^{-2\alpha(x)}g_{\mu\nu}\,, \qquad z \to e^{\alpha(x)}z\,, \qquad \bar{z} \to e^{\alpha(x)}\bar{z}\,. \quad (5.10)$$

The first term in Eq. (5.9) breaks this symmetry [152], which can be traced back to the fixing of the conformal compensator. The kinetic terms of the matter fields z^α are canonical[2] and the F-term scalar potential can be expressed as [153, 154]

$$V_J^F = V^{\text{glob}} + \Delta V_J \quad (5.11)$$

$$= \delta^{\alpha\bar{\beta}}\,W_\alpha \bar{W}_{\bar{\beta}} + \frac{1}{\Delta_K}|\delta^{\alpha\bar{\beta}}\,W_\alpha \Phi_{\bar{\beta}} - 3W|^2 \quad (5.12)$$

where,

$$\Delta_K = \Phi - \delta^{\alpha\bar{\beta}}\,\Phi_\alpha \Phi_{\bar{\beta}}\,. \quad (5.13)$$

Here the first term in Eq. (5.12) corresponds to the scalar potential obtained in global supersymmetry and the second term represents the supergravity contributions in the Jordan frame.

[2] In general, there are Planck-suppressed corrections to this, proportional to the bosonic part of the auxiliary field of the supergravity Weyl multiplet, these however vanish on inflationary trajectories along the purely real or imaginary part of any single z^α.

Couplings between the conformal compensator and the matter fields of the theory lead to additional operators in the frame function which break the conformal symmetry. Following Ref. [87], we will consider the following two operators,

$$\delta\Phi = \chi \left(a_{ab} \frac{z^a z^b \bar{z}^{\bar{0}}}{z^0} + h.c. \right) - 3\zeta \frac{|z^n \bar{z}^{\bar{n}}|^2}{z^0 \bar{z}^{\bar{0}}} , \tag{5.14}$$

with $\{a, b\}$ and n running over distinct subsets of $\{1, 2, ...\}$. In particular, we will be interested in the case where the first term provides an additional parameter to the potential of the inflaton (which we will refer to as ϕ in following), $a_{ab} = \delta_{a\phi}\delta_{b\phi}/2$, whereas the second term (with ζ being positive) may stabilize orthogonal directions to inflaton in the field space [85, 87]. We will in particular be interested in employing this term for so-called stabilizer fields, commonly denoted by X. After gauge-fixing the conformal compensator, the frame function then finally reads

$$\Phi(z, \bar{z}) = -3M_{pl}^2 + |z^\alpha|^2 + \frac{\chi}{2}(\phi^2 + \bar{\phi}^2) - \zeta |X\bar{X}|^2/M_{pl}^2 . \tag{5.15}$$

Note that the first term in Eq. (5.14) does not modify Eq. (5.8) (canonical kinetic terms in the Jordan frame) and Eq. (5.12), since this holds for all frame functions with $\Phi_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}$. The second term in Eq. (5.14) on the other hand modifies Eq. (5.12), leading to

$$V_J^F = \delta^{\alpha\bar{\rho}}\delta^{\gamma\bar{\beta}}\Phi_{\alpha\bar{\beta}} W_\gamma \bar{W}_{\bar{\rho}} + V'_{\Delta\text{sugra}} , \tag{5.16}$$

with $V'_{\Delta\text{sugra}}$ denoting a lengthy expression, whose explicit form is little enlightening. In the limit $X \to 0$, Eq. (5.16) however reduces to Eq. (5.12) with $V'_{\Delta\text{sugra}} \overset{X\to 0}{\longmapsto} V_{\Delta\text{sugra}}$. This may be verified by considering the Kähler metric

$$K_{\alpha\bar{\beta}} = -\frac{3M_{pl}^2}{\Phi}\left(\Phi_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi}\right), \tag{5.17}$$

and the Kähler derivatives

$$\mathcal{D}_\alpha W = W_\alpha - 3W\frac{\Phi_\alpha}{\Phi}, \tag{5.18}$$

confirming that all contributions stemming from the second term in Eq. (5.14) vanish when the X-field is successfully stabilized at zero VEV along the inflationary trajectory. The same conclusion holds for the corrections to the canonical kinetic terms in the Jordan frame. Consequently, Eq. (5.12) will prove to be a powerful guide to find models unspoiled by supergravity corrections, even when including higher-dimensional operators in the frame function which break the conformal symmetry. In the following we will work in units of the reduced Planck mass, $M_{pl} = 1$.

5.3 Supergravity corrections to the scalar potential

In the previous section, we noted that with the choice of the Kähler potential of Eq. (5.7), where the frame function Φ is given by Eq. (5.15), there is a clear separation between the globally supersymmetric and the supergravity contribution to the scalar potential in the Jordan frame. In this section we determine the importance of this supergravity contribution in generic inflation models. In particular we will focus on single field inflation models, in which $\chi < 0$, the inflaton direction is given by $\varphi = \sqrt{2}\text{Re}(\phi)$ and all stabilizer fields X_i are stabilized at zero. See Sec. 5.6 for explicit examples of this type. Introducing

$$f(\varphi) \equiv \langle \delta^{\alpha\bar{\beta}} W_\alpha \Phi_{\bar{\beta}} - 3W \rangle, \quad g^2(\varphi) \equiv V^{\text{glob}}, \tag{5.19}$$

where the expectation value $\langle .. \rangle$ indicates that the corresponding terms are to be evaluated along the inflationary trajectory, the Jordan frame potential (with the frame function (5.15)) for the inflaton reads

$$V_J(\varphi) = V^{\text{glob}} + \Delta V_J \tag{5.20}$$

$$= g^2(\varphi) - \frac{|f(\varphi)|^2/3}{1+\chi\,(1+\chi)\,\varphi^2/6}\,, \tag{5.21}$$

where we have employed the frame function (5.15). We recall that in the Jordan frame the kinetic term of the scalar field φ is canonically normalized. Let us assume that $f(\varphi)$ and $g(\varphi)$ can be expressed as polynomials of order p_f and p_g in φ, respectively,

$$g(\varphi) = \sum_{n=1}^{p_g} a_n \varphi^n\,, \qquad |f(\varphi)| = \sum_{n=1}^{p_f} b_n \varphi^n\,, \tag{5.22}$$

with generic coefficients $\{a_n, b_n\} = \mathcal{O}(1)$. In the large field limit $\varphi^2|\chi(1+\chi)|/6 \gg 1$, $|\varphi| \gg 1$, the above potential simplifies to

$$V_J(\varphi) \simeq (a_{p_g}\varphi^{p_g})^2 - \frac{2\,(b_{p_f}\varphi^{p_f})^2}{\chi\,(1+\chi)\,\varphi^2}\,. \tag{5.23}$$

The supergravity correction term appears with a suppression factor of $1/(\varphi^2\chi(1+\chi))$, which will suppress the supergravity corrections iff

$$p_f \leq p_g\,, \tag{5.24}$$

i.e. if the maximal power of φ in $f(\varphi)$ is not larger than the maximal power of φ in $g(\varphi)$.

What is the relation between p_f and p_g in generic inflation models? To answer this question, let us first make some simplifying assumptions: (i) we will consider only single-field inflation models, i.e. models in which the inflaton field φ is the only dynamical field and (ii) all other fields are hence stabilized at fixed value in field-space during inflation, without loss of generality we can take this value to be zero. We will denote these fields by X_i. We will consider two limiting cases, $|F_\phi| \gg \sum_i |F_{X^i}|^2$ and $|F_\phi| \ll \sum_i |F_{X^i}|^2$.

In the first case, $V^{\rm glob} \simeq |F_\phi|^2 \sim |W/\phi|^2$.[3] With Φ as in Eq. (5.15), the power p_f of $f(\varphi)$ is given by the power of ϕ in $W(\phi)$ - in the absence of cancellations among the terms in Eq. (5.19). We will return in detail to the possibility of such cancellations in the next section. For now we assume that they are absent, leading to $p_f = p_g + 1$. Thus, for large values of φ, the second term in Eq. (5.23) is generically of the same size as the globally supersymmetric term.

On the other hand, for $|F_\phi| \ll \sum_i |F_{X^i}|^2$, the globally supersymmetric scalar potential is given by $V_J^{\rm glob} \simeq \sum_i |F_{X^i}|^2$, whereas the supergravity contributions are controlled (in the large field limit) by

$$\Delta V_J = -\frac{2\,|f(\varphi)|^2}{\chi\,(1+\chi)\,\varphi^2} \sim \frac{\langle |W|\rangle^2}{\varphi^2} \sim |F_\phi|^2 \ll \sum_i |F_{X^i}|^2 \simeq V_J^{\rm glob}\,, \tag{5.25}$$

where we have used $\langle W\rangle \sim F_\phi \phi$ since the terms in W proportional to X^i vanish along the inflationary trajectory. We have also dropped factors of χ and $(\chi+1)$, assuming that in the large field limit, $\chi(1+\chi)\varphi^2/6 \gg 1$, these are roughly order one factors. With this, we conclude that for $|F_\phi| \ll |F_{X^i}|$, the supergravity contributions to the Jordan frame scalar potential are always suppressed.

[3]The latter relation assumes that the term in the superpotential responsible for the dominant contribution to the vacuum energy also yields the dominant contribution to W. In the limit of large ϕ, this assumption is justified.

To illustrate this point, consider e.g. the superpotential

$$W = \lambda X \phi^2 + \epsilon \phi^n \,, \tag{5.26}$$

where we assume for the moment $\langle X \rangle = 0$ and $\langle \text{Im}(\phi) \rangle = 0$, with the inflaton given by $\varphi = \sqrt{2}\,\text{Re}(\phi)$. The F-terms are give by $\langle F_X \rangle = \lambda \langle \phi^2 \rangle$ and $\langle F_\phi \rangle = n\epsilon \langle \phi^{n-1} \rangle$, respectively. The first term in Eq. (5.26) is the usual superpotential of monomial inflation, which we will return to in Sec. 5.6. The second term adds a non-vanishing F_ϕ-term, tunable with the parameter ϵ. With the Kähler potential (5.7) and the frame function (5.15), the scalar potential for the inflaton in the Jordan frame reads

$$V_J = \frac{1}{2}\lambda^2 \varphi^4 + \epsilon^2 \frac{n^2}{2^{n-1}} \varphi^{2n-2} - \epsilon^2 \frac{((n-3)+\chi n)^2}{2^n \left[1 + \varphi^2 \chi (1+\chi)/6\right]} \varphi^{2n} \,, \tag{5.27}$$

The first two terms here correspond to the bare potential obtained in global supersymmetry and the last term stems from the supergravity contribution. If $|F_\phi| \gg |F_X|$, the globally supersymmetric scalar potential is dominated by the second term (proportional to $\epsilon^2 \varphi^{2n-2}$), and is thus of the same order as the supergravity contribution in the large field limit. On the other hand, if $|F_\phi| \ll |F_X|$, the first term will dominate the scalar potential and the supergravity contribution is suppressed. Note that for $n > 3$, the condition $|F_\phi| \ll |F_X|$ will be violated at very large field values, bringing us back to the scenario discussed above. We will return to this toy-model at the end of Sec. 5.5, where we will in particular focus on the case $n = 3$.

The considerations above focused on large field values for φ. Of course, for small field values, the supergravity contributions become parametrically suppressed by φ / M_{pl}. The question of supergravity contributions is thus a question of large field inflation models.

In summary, we conclude that by construction, the CSS framework prevents excessive supergravity contributions to the Jordan frame scalar potential. The supergravity contributions can be at most of the same power in the inflaton field as the globally supersymmetric contribution, and in inflation models whose vacuum energy is generated by F-terms of fields other than the inflation, they are even subdominant compared to the globally supersymmetric contribution in the large field limit.

5.4 Criteria for vanishing supergravity contributions in V_J

In the previous section we discussed the generic size of the supergravity contributions to the scalar potential in the Jordan frame, and for that purpose, we did not assume any specific form of the superpotential. In this section we investigate under which conditions the above generic conclusions can be evaded, more specifically under which conditions the supergravity contribution to the Jordan frame scalar potential vanishes. This will bring us to the questions of a possible cancellation in Eq. (5.19), as mentioned in the previous section.

Let us start with writing the general form of the superpotential $W(z)$ as

$$W = W^{(0)} + W^{(1)} + W^{(2)} + W^{(3)} + W^{(4)} + \dots\,, \tag{5.28}$$

with the superscript $i = \{1, 2, ..\}$ denoting the number of superfields in the respective term, i.e.

$$W = W^{(0)} + a^1_\alpha z^\alpha + a^2_{\alpha\beta} z^\alpha z^\beta + a^3_{\alpha\beta\gamma} z^\alpha z^\beta z^\gamma + a^4_{\alpha\beta\gamma\delta} z^\alpha z^\beta z^\gamma z^\delta + \dots\,, \tag{5.29}$$

with $W^{(0)}$ and a^i are constants of the theory. Denoting the inflaton field as ϕ, the frame function of Eq. (5.15) reads

$$\Phi = -3 + |z^\alpha|^2 + \frac{\chi}{2}(\phi^2 + \bar{\phi}^2) - \zeta |X^i|^4 \,, \tag{5.30}$$

with $\alpha = \{1, 2, ..\}$ running over all the fields of the theory, and X^i denoting a subset of fields not containing the inflaton field.

The second term of Eq. (5.12) containing the supergravity contributions vanishes if

$$|\delta^{\alpha\bar{\beta}}\, W_\alpha \Phi_{\bar{\beta}} - 3W|^2 = 0 \,, \tag{5.31}$$

which along the inflationary trajectory can be rewritten using Eq. (5.28) as

$$\langle W^{(4)} - W^{(2)} - 2W^{(1)} - 3W^{(0)} - 2\zeta\, W_{X^i} X^i |X^i|^2 + \chi W_\phi \bar{\phi} \rangle = 0 \,. \tag{5.32}$$

Note that both χ and ζ are parameters of the Kähler potential. Barring fine tuning between the parameters of the Kähler potential and the parameters of the superpotential, Eq. (5.32) can be expressed as

$$\mathcal{C} : \begin{cases} \langle W^{(4)} - W^{(2)} - 2W^{(1)} - 3W^{(0)} \rangle = 0 \,, \\ \langle \chi W_\phi \bar{\phi} - 2\zeta\, W_{X^i} X^i |X^i|^2 \rangle = 0 \,. \end{cases} \tag{5.33}$$

Note the special role of the trilinear term in W, which, invariant under the conformal symmetry, is not constrained by the first part of this condition. The conditions (5.32) and (5.33) can easily be generalized if several fields receive holomorphic contributions in the frame function, i.e. for $\chi/2(\phi^2 + \bar{\phi}^2) \mapsto \chi a_{ab}(z^a z^b + \bar{z}^a \bar{z}^b)$ in Eq. (5.30). In this case the we find

$$\chi W_\phi \bar{\phi} \mapsto \chi W_a (a_{ab} + a_{ba})\, \bar{z}^b \tag{5.34}$$

in Eqs. (5.32) and (5.33). The first line of Eq. (5.33) remains unchanged.

Now, a comment on the strength of these conditions is in order. The conditions $\mathcal{C}$ protect the direction of the (complex) inflaton field ϕ from large supergravity corrections, however other, orthogonal directions in the field space may still receive large (possibly tachyonic) masses, see eg. [86, 87, 155]. Tachyonic directions orthogonal to the inflationary trajectory render the latter unstable. For 'stabilizer'-type fields, which feature a vanishing VEV during and after inflation, this may technically be remedied by adding the above mentioned $-\zeta|X^i|^4$ term to the Kähler potential. However for hybrid-inflation models which contain a 'Higgs'-type field whose dynamics is responsible for ending inflation, the problem is more severe. In this case, a $-\zeta|X^i|^4$ term in the frame function stabilizing this field during inflation will generically also do so after inflation, thus preventing the desired phase transition ending inflation.

5.4.1 Connection to $\langle W \rangle = 0$ in single field inflation

We now highlight how in single field inflation, under some generic conditions, the conditions for vanishing supergravity corrections (5.33) are equivalent to the requirement of a vanishing superpotential along the inflationary trajectory, $\langle W \rangle = 0$. Considering the expression for the F-term scalar potential in the Einstein frame, Eq. (5.5), it is well known that models with $\langle W \rangle \neq 0$ obtain dangerous supergravity corrections due to the $-3|W|^2/M_{pl}^2$ term. Here we extend this understanding, showing how in the class of CSS models, $\langle W \rangle = 0$ is equivalent to the exact vanishing of the supergravity contribution in the Jordan frame scalar potential, $\Delta V_J = 0$.

Let us first consider an inflation model in which Eq. (5.33) is fulfilled. Then for $\chi \neq 0$ the second line of Eq. (5.33) requires $\partial W/\partial\phi = 0$ along the inflationary trajectory (assuming that all X^i's are stabilized at zero VEV by the ζ_i-terms). Hence, assuming that during inflation there is only one[4] dynamical field, which is the inflaton ϕ, we can write

$$\langle W \rangle = \langle c_0 \rangle + \langle c_1 \rangle \phi + \langle c_2 \rangle \phi^2 + \ldots \; . \tag{5.35}$$

When we require $\partial W/\partial\phi = 0$ along the inflationary trajectory over a finite range of ϕ, the terms of different order in ϕ have to vanish independently, i.e $\langle c_1 \rangle = \langle c_2 \rangle = \cdots = 0$. We hence obtain $\langle W \rangle = \langle c_0 \rangle$, with $\langle c_0 \rangle$ being a function of the fields other than the inflaton. Due to our assumption that the inflaton is the only dynamical field during inflation, all other fields which might enter into $\langle c_0 \rangle$ are characterized by a constant VEV during inflation, and without loss of generality, we may set these VEVs to zero.[5] Hence a non-zero $\langle c_0 \rangle$ can only be sourced by a true constant in the superpotential, $W_{\text{inf}} = W^{(0)} \neq 0$. This however is excluded by the first line of Eq. (5.33) (again taking into account that this condition must hold over a finite range for ϕ). In summary, under the assumption of (effectively) single field inflation, the condition (5.33), i.e. the vanishing of supergravity contributions to the Jordan frame scalar potential, implies $\langle W \rangle = 0$. This is one of the main results discussed in this article.

Conversely, under the assumption that the fields other than the inflaton are stabilized at constant VEV during inflation, $\langle W \rangle = 0$ guarantees that the conditions in Eq. (5.33) are also satisfied. To prove this statement, we explicitly express the terms $W^{(i)}$ introduced in

[4]this includes models of hybrid-like inflation, where a second field becomes dynamical at the end of inflation.

[5]More generally, we may allow any constant VEV for these fields during inflation: Starting from $\langle X \rangle = c$, we define $X' = X - c$ and perform the above analysis for the redefined field X'.

Eq. (5.28) as a power expansion in ϕ,

$$W^{(i)} = \langle c_{i\,0} \rangle + \sum_{j=1}^{i} c_{ij} \phi^j = \langle c_{i\,0} \rangle + \langle c_{i\,1} \rangle \phi + \cdots + \langle c_{i\,i-1} \rangle \phi^{i-1} + c_{ii} \phi^i \,. \tag{5.36}$$

Here $W^{(0)}$ and the c_{ii}'s are pure constant terms and $\langle c_{ij} \rangle$ for $i \neq j$ are the coefficients obtained from the VEVs of all fields except inflaton ϕ. Now if during inflation all fields other than the inflaton are stabilized at zero VEV, then $\langle c_{\substack{ij \\ i \neq j}} \rangle = 0$. We are then left with $W = W^{(0)} + \sum_i c_{ii} \phi^i$. Now, if we demand that during inflation $W_{\text{inf}} = 0$ is satisfied for all inflaton field values, this requires $W^{(0)}$ and c_{ii} to be zero independently, and hence $W^{(0)} = W^{(1)} = W^{(2)} = W^{(3)} = W^{(4)} = \ldots = 0$. So the first part of the condition in Eq. (5.33) is immediately satisfied. A little bit of more algebra shows that the second part of the condition too is fulfilled. In the context of usual Einstein frame supergravity, the advantage of $W_{\text{inf}} = 0$ for inflation model building has been noted earlier in [156, 119, 157].

In summary, the generic supergravity contributions to the Jordan frame scalar potential identified in Sec. 5.3 can be avoided by a suitable construction of the superpotential. This leads in a first step to the condition of Eq. (5.32), which may be re-expressed as the two conditions of Eq. (5.33) in the absence of correlations between the parameters of the Kähler and superpotential. For single-field inflation models, this further simplifies to the condition of a vanishing superpotential along the inflationary trajectory. Returning to the two cases $|F_\phi| \ll \sum_i |F_{X^i}|$ and $|F_\phi| \gg \sum_i |F_{X^i}|$ discussed in Sec. 5.3, we note that the second line of Eq. (5.33) immediately implies $F_\phi = \partial W / \partial \phi = 0$. Hence a cancellation of the supergravity contributions to the Jordan frame scalar potential by a suitable choice of superpotential is only possible for inflation models driven by a vacuum energy stemming from F-terms of fields other than the inflaton. Models with $|F_\phi| \gg \sum_i |F_{X^i}|$, which were found to generically obtain larger supergravity contributions in Sec. 5.3, cannot be protected in this way.

5.4.2 Illustrative examples

Let us apply the above mentioned derived condition for vanishing supergravity contributions in the Jordan frame to some well-known classes of inflation models.

Hybrid inflation

The superpotential of F-term hybrid inflation [202, 148, 83] is given by

$$W = \lambda\phi(H_+H_- - M^2)\,, \tag{5.37}$$

with coupling constant λ, mass parameter M and $H_\pm$ denote so-called waterfall fields which obtain non-zero VEVs in a phase transition ending inflation. During inflation, $\langle H_\pm\rangle = 0$, and hence $\langle W\rangle \neq 0$. The resulting supergravity contributions induce a large tachyonic mass to the imaginary part of ϕ, thus spoiling inflation [154].

Monomial inflation

Monomial chaotic inflation is characterized by the following choice of the superpotential [159, 85, 120],

$$W = mXf(\phi)\,. \tag{5.38}$$

Here ϕ is the inflaton field and X is an auxiliary field, often called a stabilizer field, which has a vanishing vacuum expectation value (VEV) during inflation. Hence $W_{\text{inf}} = 0$ and the supergravity contributions vanish in the Jordan frame inflaton potential. We will return to this example in more detail in Sec. 5.6.

Tribrid inflation

The superpotential of tribrid inflation involves the interplay of three fields, i.e. the inflaton field ϕ, the auxiliary field X that is stabilized at zero VEV, and the waterfall field H which triggers the phase transition ending inflation [116, 118, 160, 161]. The tribrid inflation superpotential can be defined by

$$W = \kappa X(H^l - M^2) + \lambda\phi^n H^m \, , \tag{5.39}$$

and for simplicity, we will take $l = 2$. We will restrict ourselves to $n, m \leq 2$ in accordance with the maximum number of fields in W given by Eq. (5.28). Here κ, λ are positive constants. In contrast to the standard hybrid inflation case of Eq. (5.37), there is an additional stabilizer field X which drives inflation by providing a large vacuum energy through its F-term. During inflation $\langle X\rangle = \langle H\rangle = 0$ and after inflation $\langle H\rangle = M$. With $W_{\text{inf}} = 0$ during inflation, the Jordan frame potential for the inflaton $V_J(\phi)$ is protected from supergravity contributions.

However, the waterfall field H is not protected in this way and may obtain large supergravity contributions. An indication of this can be obtained from evaluating Eq. (5.33) slightly away from the inflationary trajectory, $H \to 0 + \epsilon$, in which case the first line of Eq. (5.33) no longer vanishes. As mentioned in Section 5.4, Eq. (5.33) does not guarantee the vanishing of supergravity corrections orthogonal to the inflaton trajectory - which is precisely the trouble we are running into here: the mass of the H field is not protected from supergravity contributions. In Section 5.7 we explicitly calculate this contributions for $n = 2, m = 1, 2$, showing that for $n = 2, m = 1$ they can destabilize the inflationary trajectory whereas for $n = m = 2$, the model is more robust and could be a very interesting case for further study.

5.5 Further supergravity effects

In sections 5.3 and 5.4, we have discussed supergravity contributions to the Jordan frame scalar potential. However, a full analysis of the supergravity effects must also include the effects stemming from the non-minimal coupling to gravity (in the Jordan frame) or correspondingly from the conversion $V_E = \Omega^4 V_J$ and from the non-canonical kinetic terms (in the Einstein frame). An instructive analysis which directly applies to the set-up we discuss in the paper was given in Ref. [144] in the context of so-called universal ξ-attractors [162, 142, 144] (for a recent study on attractors in scalar-tensor theories see also [145]). These are characterized by a non-minimal coupling to the Ricci-scalar and the Lagrangian density is given by[6]

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{1}{2}\,\Omega^{-2}(\varphi)\,R(g_J) - \frac{1}{2}\,K_J(\varphi)\,(\partial\varphi)^2 - V_J(\varphi)\right]\,, \tag{5.40}$$

with $\Omega^{-2}(\varphi) = 1 + \xi\varphi^2$. For $K_J = 1$, $\xi = -(1 - |\chi|)/6$ this corresponds precisely to the setup discussed in this paper (once the real scalar inflaton field φ has been identified), as will be illustrated in the explicit examples of Sec. 5.6.

With a suitable conformal transformation, the Lagrangian density above can be ex-

[6]Note the slightly different notation compared to [144]: $\Omega_{[144]} = \Omega^{-2}$. For supergravity embeddings of this model see e.g. [114, 120, 132, 141, 142].

pressed in the Einstein frame,

$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{1}{2}R(g_E) - \frac{1}{2}\underbrace{\left(\frac{K_J}{\Omega^{-2}} + 6\,\frac{\Omega'^2}{\Omega^2}\right)}_{K_E(\varphi)}(\partial\varphi)^2 - \Omega^4 V_J(\varphi)\right]\,. \tag{5.41}$$

If $K_J\Omega^2 \ll 6\Omega'^2/\Omega^2$ and $\xi > 0$ (i.e $|\chi| > 1$), the above Lagrangian can be re-casted (in terms of the dynamical variable Ω) with a kinetic term having a second order pole at $\Omega = 0$ (corresponding to the large field limit $\varphi \to \infty$) [144],

$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{1}{2}R(g_E) - 3\left(\frac{\partial\Omega}{\Omega}\right)^2 - V_E(\Omega)\right]\,. \tag{5.42}$$

From Eq. (5.42), we see that the canonically normalized field $\hat{\varphi}$ is related to the dynamical variable $\Omega(\varphi)$ as $\Omega = \exp(-\hat{\varphi}/\sqrt{6})$.[7]

Let us now consider a Jordan frame scalar potential $V_J(\varphi) = c_V\varphi^{2l}$ (the situation is easily generalized to any polynomial scalar potential, whose maximum power of φ is $2l$). With $V_E = \Omega^4 V_J$ and $\varphi^2 = (\Omega^{-2} - 1)/\xi$ we find

$$V_E = c_V\xi^{-l}\Omega^4\left(\Omega^{-2} - 1\right)^l \tag{5.43}$$

$$= c_V\xi^{-l}\left(\Omega^{4-2l} + \cdots + \Omega^4\right)\,. \tag{5.44}$$

We can thus identify three qualitative different situations. (i) For $l < 2$, all powers of Ω appearing in Eq. (5.44) are positive and hence $V_E(\hat{\varphi}) \to 0$ for $\hat{\varphi} \to \infty$. Together with $V_E \simeq V_J \simeq c_V\hat{\varphi}^{2l}$ at small field values, this yields (for $c_V > 0$) a hilltop type potential

[7] We note that $\Omega(\varphi \to \pm\infty) = 0$. The canonical normalization of Eq. (5.42) leaves the sign ambiguity $\Omega = \exp(\pm\hat{\varphi}/\sqrt{6})$. We choose here the solution $\Omega = \exp(-\hat{\varphi}/\sqrt{6})$, obtaining the desired asymptotic behaviour $\Omega(\hat{\varphi} \to +\infty) = 0$ for positive field values.

(see e.g Fig. 5.2(a)). (ii) For $l = 2$, the leading order term in Ω is constant, and hence these models approach an exponentially flat plateau for large field values (see eg Fig. 5.4). For $c_V > 0$ this reproduces the characteristic feature of the Starobinsky-type inflation models [6], which are favoured by the current CMB data. (iii) For $l > 2$, Eq. (5.44) contains negative powers of Ω, leading to an exponential growth of the potential at large field values, unsuitable for slow-roll inflation. We will discuss explicit realizations of these different scenarios, in particular of the cases (i) and (ii), in the next section.

Note that the phenomenologically very promising case (ii) can more generally be obtained for any $V_J \propto f(\varphi)^2$ and $\Omega^{-2} = 1 + \xi f(\varphi)$, which has coined the name 'attractor'-models [130, 131, 133, 134, 135, 136, 137, 138, 139], since different models are 'attracted' to the sweet spot of the Planck data as the parameter ξ is increased. These models might have very different potentials at small field values, but asymptotically they all feature an exponentially flat potential. The predictions in the n_s - r plane are described by a one-parameter region (here ξ) which converges to $(1 - 2/N, 12/N^2)$ at leading order in $1/N$ and for large ξ. The above analysis underlines that this mechanism requires the leading order power of φ in $V_J^{1/2}$ and Ω^{-2} to be identical, which, as an ad hoc assumptions, requires some degree of tuning.

We now discuss the case of $\xi < 0$ in $\Omega^{-2}(\varphi) = 1 + \xi\varphi^2$. For $0 < |\chi| < 1$, ξ is bounded between $-1/6$ and 0. In this case there is a pole in Ω for $\varphi \to 1/\sqrt{|\xi|}$. We thus make a change of variable to $\tilde{\Omega} = 1/\Omega$, and the Lagrangian of Eq. (5.41) becomes

$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{1}{2}R(g_E) - 3\left(\frac{\partial\tilde{\Omega}}{\tilde{\Omega}}\right)^2 - V_E(\tilde{\Omega})\right] . \tag{5.45}$$

In this case, the canonically normalized field $\hat{\varphi}$ is related to the dynamical variable $\tilde{\Omega}(\varphi)$

as $\tilde{\Omega} = \exp(-\hat{\varphi}/\sqrt{6})$.[8] Similar to the case for $\xi > 0$, the kinetic term now has a pole at $\tilde{\Omega} = 0$. As before, in terms of the canonically normalized field $\hat{\varphi}$, this pole is located at infinite field values. For $V_J = c_V \varphi^{2l}$, substituting φ by $\tilde{\Omega}$ yields

$$V_E = c_V \xi^{-l} \left(\tilde{\Omega}^{-4} + \cdots + \tilde{\Omega}^{(2l-4)} \right) . \tag{5.46}$$

For $l > 0$, we see that the first term always dominates the form the potential, and it is exponentially steep in terms of the canonical field $\hat{\varphi}$. Hence even if we get the required amount of inflation, it does not lead to the attractor prediction in the n_s-r plane. We return to explicit examples of this type in the next subsection.

Returning to our discussion of the Jordan frame scalar potential V_J in Secs. 5.3 and 5.4, we note that the supergravity contribution always comes with a negative sign, i.e. $c_V < 0$ in the above parametrization. This implies that a viable inflation model will always require this contribution to be subdominant. This is easily achieved if the supergravity contribution comes with a power of $p_f < 3$ (see Eq. (5.23)), corresponding to case (i) above. In this case the supergravity contribution will vanish in the large field limit. On the other hand, if $p_f > 3$ (case (iii)), the potential becomes unbounded from below at large field values. In the intermediate case (ii), the viability of the inflation model will depend on the relative size of the supergravity contribution, see example below.

Let us illustrate these points by returning to the model introduced in Eq. (5.26) with $n = 3$. The scalar potential for the canonically normalized inflaton field in the Einstein frame is depicted in Fig. 5.1. From Eq. (5.27), we note that both the globally supersymmetric and the supergravity contribution come with a power of φ^4. For small values of ϵ, the globally supersymmetric contribution dominates, leading to a positive plateau in the large

[8] Note that if we consider negative field values, the corresponding canonically normalized field is given by $\tilde{\Omega} = \exp(\hat{\varphi}/\sqrt{6})$.

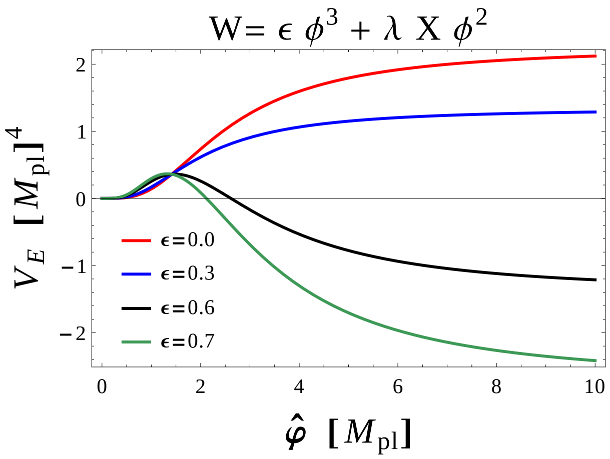


Figure 5.1: Behaviour of supergravity corrections for $W = \lambda X\phi^2 + \epsilon\phi^3$. This plot shows the behaviour of the Einstein frame potential with incresing ϵ for $\lambda = 1$. As we tune ϵ to higher values supergravity corrections become $\mathcal{O}(1)$.

field regime. For increasing values of ϵ, the supergravity contribution becomes to dominate, and the scalar potential is found to take negative values in large field regime.

5.6 Examples of inflation models

To illustrate the analysis of the previous sections, we present two representative examples, based on $W = \lambda X\phi^2$ and $W = mX\phi$ as well as the frame function (5.15),

$$\Phi = -3 + \phi\bar{\phi} + X\bar{X} + \frac{\chi}{2}(\phi^2 + \bar{\phi}^2) - \zeta|X\bar{X}|^2\,. \tag{5.47}$$

Contrary to the example discussed at the end of the previous section, both examples here feature $\langle W \rangle = 0$ during inflation, hence the supergravity contributions to the Jordan frame scalar potential vanish and we are left with the type of supergravity effects described in Sec. 5.5. The phenomenology of these models has been previously studied in Ref. [163].

5.6.1 Hilltop inflation from $W = m\phi X$

For sufficiently large ζ, the stabilizer field X can be taken to be fixed at $\langle X\rangle = 0$, and hence following Eq. (5.4), the F-term scalar potential in Einstein frame is given by

$$V_E|_{X\to 0} = \frac{m^2(\varphi^2+\tau^2)}{2\left(1-\frac{\varphi^2}{6}(1+\chi)+\frac{\tau^2}{6}(-1+\chi)\right)^2}\,, \tag{5.48}$$

where we have decomposed $\phi = (\varphi + i\,\tau)/\sqrt{2}$. We first note that the resulting Lagrangian is invariant under the simultaneous transformation of $\varphi \leftrightarrow \tau$ and $\chi \to -\chi$. We thus restrict our analysis to $\chi \leq 0$, for which the real part of ϕ will play the role of the inflaton.

In the Einstein frame, the kinetic term of the complex scalars ϕ and X is not canonically normalized. We can however extract information about the dynamics of these fields, in particular the values of their masses and the inflationary slow-roll parameters, by exploiting

$$\frac{da}{d\hat{a}} = \frac{1}{\sqrt{K_{z\bar{z}}}} \quad \text{for} \quad a = \mathrm{Re}(z), \mathrm{Im}(z) \tag{5.49}$$

$$\to \quad \frac{\partial V}{\partial \hat{a}} = \frac{1}{\sqrt{K_{z\bar{z}}}}\frac{\partial V}{\partial a}\,, \tag{5.50}$$

where $\hat{z}$ (and correspondingly $\hat{a}$) denote the canonically normalized field. With this, we verify that both τ (the imaginary part of the complex inflaton field ϕ) and the complex stabilizer field X are stabilized at zero VEV with $m^2_{\tau,X} \geq \mathcal{O}(\mathcal{H}^2)$. For the stabilizer field X, this is achieved if $\zeta \gtrsim 0.7$ in the frame function. For the real scalar τ, one may at first glance worry about about a pole in Eq. (5.48) for large values of τ. However since the Kähler metric features the same pole structure, this pole is not reached for any finite value of the canonically normalized field $\hat{\tau}$.

We now turn in more detail to the inflationary dynamics. Along the inflationary trajectory, the scalar potential in the Einstein frame reduces to

$$V_E = \frac{m^2\varphi^2}{2\left(1 - \frac{\varphi^2}{6}(1+\chi)\right)^2} \, . \tag{5.51}$$

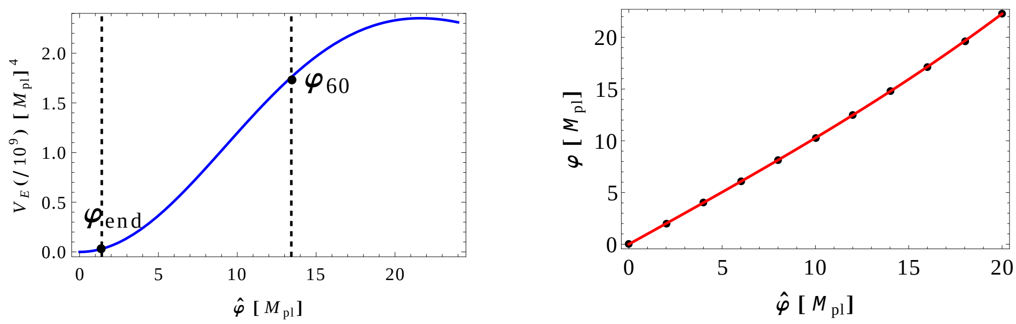
We can distinguish two qualitatively different regimes: For $\chi < -1$, the denominator of Eq. (5.51) is always strictly positive, whereas for $-1 < \chi < 0$ the potential features a pole for large values of φ (which is however not reached for any finite value of the canonically normalized field $\hat{\varphi}$). Note that for $\chi = -1$, the transformation to the Jordan frame becomes trivial, $\Omega^2(X = 0) = 1$. Hence in this case $V_E = V_J$ and we reproduce the predictions of standard chaotic inflation with a quadratic potential.

We shall first consider the case $\chi < -1$. The inflationary potential in terms of canonical inflation field $\hat{\varphi}$ is shown in Fig. 5.2(a). The potential has a minimum at $\varphi = 0$, and it also vanishes for infinitely large field values. Inflation is possible around its maximum when the field rolls towards the minimum at $\varphi = 0$. Note that this potential has a serious initial value problem for the inflation field. If the initial field value of the inflaton is larger than the position of the maximum of the potential φ_{max}, the field rolls towards the wrong post-inflationary vacua. For our considerations, we assume that the field starts to roll from any field value between φ_{max} and φ_{60} - see Fig. 5.2(a). The evolution of the inflaton field is governed by the slow-roll equation of motion,

$$3K_{\phi\bar{\phi}}\mathcal{H}\dot{\varphi} + V_E'(\varphi) = 0 \, , \tag{5.52}$$

where $\mathcal{H}$ denotes the Hubble parameter during inflation. This can be expressed as

$$\frac{d\varphi}{dN} - \frac{1}{V_E}\left(\frac{V_E'(\varphi)}{K_{\phi\bar{\phi}}}\right) = 0 \, , \tag{5.53}$$



(a) Einstein frame scalar potential V_E in terms of the canonical field $\hat{\varphi}$, for $\chi = -1.01$ and $m = 5.6 \times 10^{-6}$ (the latter fixed by A_s^0). The vertical dashed black lines represent the field value at the end of inflation and 60-efolds earlier, respectively.

(b) Inflaton field φ vs. its canonically normalized counterpart $\hat{\varphi}$ plot for $\chi = -1.01$. Along the inflationary trajectory, $\varphi \approx \hat{\varphi}$ is a good approximation.

Figure 5.2: Inflationary dynamics for $W = m\phi X$.

allowing the evaluation of $\varphi(N)$ without explicitly normalizing the inflaton field φ.

The predictions for the amplitude of scalar perturbations, tilt of the scalar perturbations, and the ratio of tensor and scalar perturbations amplitudes in the Einstein frame are given respectively by

$$A_s = \frac{V_E}{24\,\pi^2\,\epsilon_E}\,, \qquad n_s = 1 - 6\,\epsilon_E + 2\,\eta_E\,, \qquad r = 16\,\epsilon_E\,, \tag{5.54}$$

evaluated when the CMB pivot scale exited the horizon at $N_* = 60$ e-folds before the end of inflation. The slow-roll parameters ϵ_E and η_E are given by

$$\epsilon_E = \frac{1}{2}\left(\frac{V_E'(\hat{\varphi})}{V_E}\right)^2, \quad \eta_E = \frac{V_E''(\hat{\varphi})}{V_E}\,, \tag{5.55}$$

where the derivatives with respect to the canonically normalized field $\hat{\varphi}$ may be evaluated using Eq. (5.50). For the scalar potential (5.51), this leads to

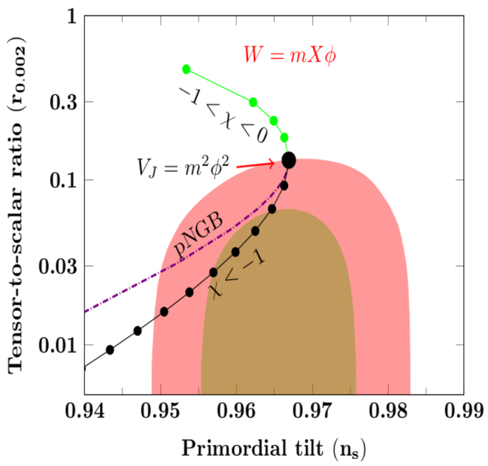
$$\epsilon_E = \frac{2\big(1+\frac{\varphi^2}{6}(1+\chi)\big)^2}{\varphi^2\big(1+\frac{\varphi^2}{6}\chi(1+\chi)\big)}\,, \tag{5.56}$$

$$\eta_E = \frac{2(1+\varphi^2(1+\chi)(1+\frac{1}{6}\varphi^2(1+\chi)(\frac{1}{6}+\chi(1+\frac{1}{18}\varphi^2(1+\chi)))))}{\varphi^2(1+\frac{1}{6}\varphi^2\chi(1+\chi))^2}\,. \tag{5.57}$$

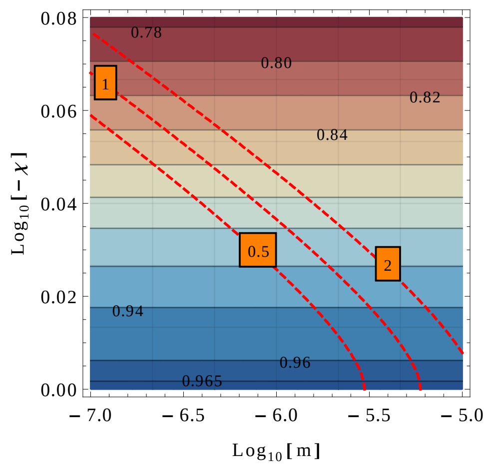
We note that the slow-roll parameters are functions of χ and N only, and do not depend on the parameters m and ζ. Note moreover that while the above calculation has been performed in the Einstein frame, identical expressions for n_s and r can be obtained directly in the Jordan frame [151].

In Fig. 5.3 we summarize the resulting CMB predictions, obtained by numerically solving the slow-roll equation of motion. Requiring A_s to lie within the 99.7% CL of the Planck data [15], we determine the spectral index n_s and the tensor-to-scalar ratio r by numerically solving Eq. (5.53). The results are depicted in Fig. 5.3(a), with the background contours corresponding to PLANCK 2015 TT + low l polarization [15]. The black dots indicate different values of χ parameter with χ varying from -1 to about -1.03 with decreasing r and n_s. For $\chi < -1.03$, the spectral index n_s lies well outside the Planck contour, see also Fig. 5.3(b) which shows the dependence of n_s on the pair of parameters $\{m, \chi\}$, together with the curvature perturbation A_s (dashed red lines) normalized to the Planck best-fit value A_s^0: For increasing values of $|\chi|$, the spectral index decreases until it reaches values well beyond the current observational bound. Some of these contour lines are marked in the figure.

In the vicinity of the limiting case of chaotic inflation ($\chi = -1$), the CMB predictions may be understood analytically. For $\delta \equiv \frac{\varphi^2}{6}\chi(1+\chi) \ll 1$ and $\varphi(N) \gg \varphi_{\text{end}}$, integrating



(a) Predictions for n_s and r for $W = m\phi X$, overlayed with the 68% and 95% CL contours from Planck. The black line represents variations w.r.t χ, where $\chi = -1$ corresponds to the predictions of chaotic inflation. For comparison, the dashed curve shows the predictions of natural inflation.



(b) Amplitude of the scalar perturbations A_s/A_s^0 (red dashed line) and its tilt n_s (horizontal contours) as a function of the model parameters m and χ.

Figure 5.3: Parameter space and predictions for $W = m\Phi X$.

Eq. (5.53) yields

$$4N \simeq \varphi(N)^2 \frac{(1-\chi)}{2} + \frac{(\varphi^2(\chi-1)+12)\delta}{4\chi} + \frac{(\varphi^2(1-\chi)+9)\delta^2}{6\chi^2} + \mathcal{O}(\delta^3)\,. \tag{5.58}$$

In the limit $\chi \to -1$, we recover the familiar expressions of quadratic chaotic inflation,

$$\varphi^2 \simeq 4N\,, \qquad \epsilon_E \simeq \frac{1}{2N}\,, \qquad \eta_E \simeq \frac{1}{2N}\,. \tag{5.59}$$

The χ-dependence in the vicinity of $\chi \to -1$ limit can be understood by expanding the spectral index and tensor-to-scalar ratio in terms of the expansion parameter δ. Employing

Eq. (5.58), we obtain

$$n_s \simeq 1 - \frac{2}{N} + \frac{2}{27}(\chi+1)^2(3\chi^2+2\chi-14)N + \mathcal{O}(\delta^3/N)\,, \tag{5.60}$$

$$r \simeq \frac{8}{N} - \frac{8}{3}(\chi+1)(\chi-4) + \mathcal{O}(\delta^2/N)\,, \tag{5.61}$$

where $\delta \simeq 2/3N\chi(1+\chi)$. Qualitatively the χ-dependence in the $n_s - r$ plane as shown in Fig. 5.3(a) is similar to the result found for the natural inflation (pNGB inflation) potential $V \sim \left(1-\cos\frac{\varphi}{f}\right)$ [164]. In the latter case, expanding in powers of $1/f \ll 1$, with f being the axion decay constant in Planck units, yields

$$n_s \simeq 1 - \frac{2}{N} - \frac{N}{6f^4} + \mathcal{O}(1/f^8)\,, \tag{5.62}$$

$$r \simeq \frac{8}{N} - \frac{4}{f^2} + \mathcal{O}(1/f^4)\,. \tag{5.63}$$

For a fixed value of n_s, comparing Eqs. (5.60) and (5.62) we find $1/f^4 = 4/9(\chi+1)^2(3\chi^2+2\chi-14)$. We then immediately see that the value of r from Eq. (5.61) is in fact smaller than the natural inflation counterpart from Eq. (5.63). Hence the two models yield similar, but not identical predictions in the $n_s - r$ plane. As the numerical analysis shows, this is true in the entire parameter range of interest, see Fig. 5.3(a).

Finally we turn our attention to the case $-1 < \chi < 0$. Again we can identify the real axis in the complex plane spanned by $\phi = (\varphi + i\tau)/\sqrt{2}$ as the only viable inflationary trajectory (the trajectory along the imaginary axis exhibits a tachyonic instability). We can achieve enough efolds of slow-roll inflation only if the parameter χ is not too different from $\chi = -1$ (otherwise the potential becomes too steep). However even in this case, the predicted tensor-to-scalar ratio becomes larger than the usual quadratic chaotic inflation model, and remains outside the 2-σ contour of PLANCK data - see Fig. 5.3(a).

The example discussed above explicitly demonstrates some of the effects discussed in Sec. 5.5. In the regime $\chi < -1$ (corresponding to $\xi > 0$), we find ourselves in the case (i) of Sec. 5.5. In terms of the canonically normalized field, the scalar potential in the Einstein frame vanishes for large field values. Together with $V \propto \varphi^2$ at small field values, this leads to a hilltop-type potential. Similarly for the case $0 > \chi > -1$ (i.e $\xi < 0$) the potential is exponentially steep and the model produces a too large tensor-to-scalar ratio (far away from the attractor point in the n_s-r plane).

5.6.2 Starobinsky inflation from $W = \lambda\phi^2 X$

We now turn to an example of the case (ii) of Sec. 5.5: $W = \lambda\phi^2 X$, see also Ref. [155]. The Jordan frame scalar potential following Eq. (5.12) is $V_J = \lambda^2\phi^4$. As in the previous section, we can restrict ourselves without loss of generality to $\chi < 0$, and as above we find that both X and $\hat{\tau}$ settle to zero VEVs due to a large masses compared to the Hubble scale during inflation. The F-term scalar potential along the inflationary direction in the Einstein frame is given by

$$V_E = \frac{\lambda^2\varphi^4}{4\left(1 - \frac{\varphi^2}{6}(1+\chi)\right)^2}\,, \tag{5.64}$$

where we have used $\varphi = \sqrt{2}\,\mathrm{Re}\,\{\phi\}$. In contrast to the previous example, the above potential asymptotically approaches a constant value, due to the quartic power of the inflaton field in the numerator.

The potential in the limit of $\chi = -1$ is a simple φ^4 potential, and in this case the kinetic term becomes canonical with the conformal factor being unity. Therefore, the potential in the Einstein frame is too steep, and is disfavoured by the PLANCK data [15]. We next turn

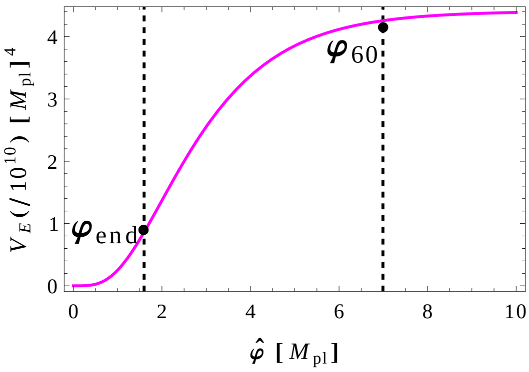
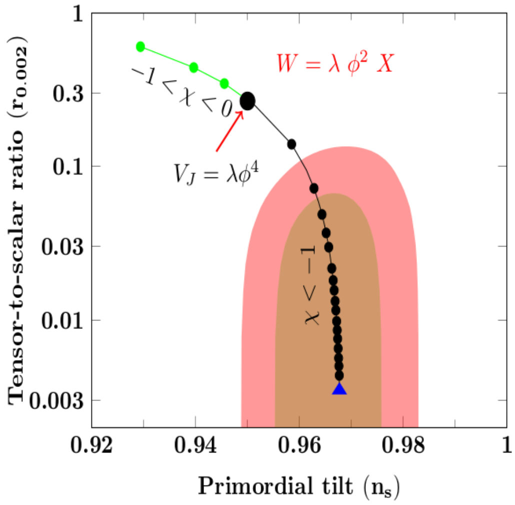


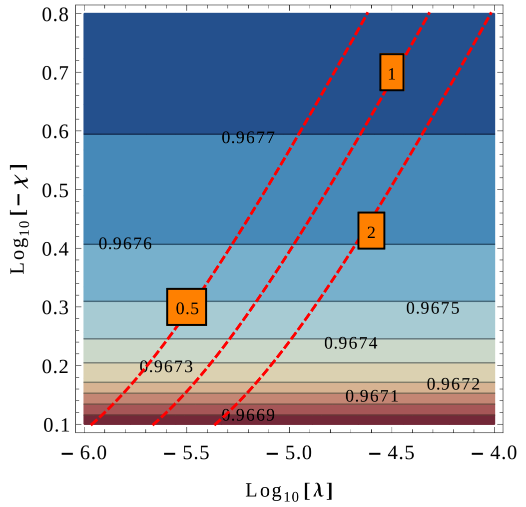
Figure 5.4: Plot of V_E in terms of the canonical field $\hat{\varphi}$ for $W = \lambda\phi^2 X$. The parameter values are $\chi = -3$ and $\lambda = 1.4 \times 10^{-5}$ (fixed by A_s^0). The dashed vertical lines represent the field value at the end of inflation and 60-efolds earlier, respectively.

to the case $\chi < -1$. A plot of this potential after canonical normalization of the kinetic term of φ is shown in Fig. 5.4, where $\hat{\varphi}$ is the canonical inflaton field. It is also a two parameter $\{\lambda, \chi\}$ potential. For a given χ and for large field values this potential has a long plateau type region with a slowly varying slope. In fact, this potential can accommodate successful inflation for wide range of values of the χ parameter. Increasing χ will add more flatness to the potential. The value of λ will be fixed by the amplitude of density perturbations. To demonstrate this feature we show the variation of χ from -1 to -5 (from top to bottom) in the usual n_s vs. r plot, cf. Fig. 5.5(a). In the limit $\chi \to -\infty$ the predictions asymptotically approach to those of the Starobinsky inflation model [6] (see next paragraph). Fig. 5.5(b) shows the variation of n_s as well as the normalized curvature perturbation A_s/A_s^0 (red dashed lines) with respect to the model parameters $\{\lambda, \chi\}$. Again n_s is independent of model parameter λ. The asymptotic behaviour of the spectral index as $|\chi|$ increases can be clearly seen in this plot.

This behaviour can be well understood by considering the following analytic expres-



(a) n_s - r plot for $W = \lambda\phi^2 X$ overlayed with the 68% and 95% CL contours from 2015 Planck data. For $|\chi| \gg 1$ predictions are asymptotically close to Starobinsky inflation model shown by blue triangle.



(b) Amplitude of the scalar perturbations A_s (red dashed line) and its tilt n_s (vertical contours) as a function of the model parameters λ and χ.

Figure 5.5: Parameter space and predictions for $W = \lambda\Phi^2 X$.

sions of inflationary slow-roll parameters given by

$$\epsilon_E = \frac{8}{\varphi^2(1+\frac{1}{6}\varphi^2\chi(1+\chi))}\,, \tag{5.65}$$

$$\eta_E = \frac{12+\frac{1}{6}\varphi^2(1+3\chi+2\chi^2)+\frac{2}{9}\varphi^4\chi(1+\chi)^2}{\varphi^2(1+\frac{1}{6}\varphi^2\chi(1+\chi))^2}\,, \tag{5.66}$$

with

$$8N = -\varphi^2\chi - 6\ln\left(1-\frac{1}{6}\varphi^2(1+\chi)\right), \tag{5.67}$$

where Eq. (5.67) is obtained by integrating the slow-roll Eq. (5.53) with the potential given by Eq. (5.64). For $\varphi^2(1+\chi)/6 \ll 1$, this yields $8N \simeq \varphi^2$ whereas for $\varphi^2(1+\chi)/6 \gg 1$ we find $8N \simeq -\varphi^2\chi$. So in the former case, we find

$$\epsilon_E \simeq \frac{1}{N}\,, \quad \eta_E \simeq \frac{3}{2N} \quad \to n_s \simeq 1-\frac{3}{N}\,, \quad r \simeq \frac{16}{N}\,, \tag{5.68}$$

which are just the results for chaotic inflation with a quartic potential. In the latter case, we find

$$\epsilon_E \simeq \frac{3\alpha}{4N^2}\,, \quad \eta_E \simeq -\frac{1}{N} \quad \rightarrow n_s \simeq 1 - \frac{2}{N}\,, \quad r \simeq \frac{12\alpha}{N^2}\,, \tag{5.69}$$

with $\alpha = \chi/(1+\chi)$, which are the predictions of the so-called α-attractors [141]. For $\alpha = 1$ ($|\chi| \rightarrow \infty$) we obtain the predictions of the Starobinsky model. We thus explicitly see the mechanism described in case (ii) of Sec. 5.5 at work here.

Finally, we consider the case $-1 < \chi < 0$ i.e $\xi < 0$. As in the example of Sec. 5.6.1, the φ direction is identified as the only possible inflationary trajectory, with the orthogonal direction stabilized during inflation. The prediction for tensor-to-scalar ratio exceeds the one for quartic inflation with a canonical kinetic term - see Fig. 5.5(a). As this is well outside the 2-σ contour of PLANCK data, we can conclude that $-1 < \chi < 0$ is not a viable parameter range for this model. This is in agreement with the general argument of Sec. 5.5.

5.7 Tribrid inflation in CSS

In this section, we will explore the supergravity effects for tribrid inflation models described by the following superpotential [116]

$$W = \kappa X(H^l - M^2) + \lambda\phi^n H^m\,, \tag{5.70}$$

in the context of Jordan frame supergravity. In this kind of models, inflation ends via a phase transition that is triggered by the mass of the waterfall field H becoming tachyonic as the inflaton field rolls towards smaller field values.

As in the main text, we will assume the following frame function

$$\Phi = -3 + |\phi|^2 + |X|^2 + |H|^2 - \zeta |X|^4 + \frac{\chi}{2}(\phi^2 + \bar{\phi}^2) \,, \tag{5.71}$$

and we will restrict the superpotential by our choice to $(n, m) \leq 2$ and $l = 2$. Constraints on the powers l, m and n have been discussed in the literature [?, 160]. We would like to emphasize that the tribrid inflation models satisfy the conditions of Eq. (5.33) of vanishing supergravity corrections along the inflationary trajectory in the Jordan frame. But the supergravity corrections to the fields orthogonal to the inflaton directions (e.g waterfall H field) are not protected from these corrections, and are potentially dangerous in spoiling the waterfall mechanism.

We start with the case $n = m = 2$, see also [119]. In the globally supersymmetric limit, this model leads to a tree-level flat potential, lifted by one-loop corrections. The spectral index is found to be $n_s \simeq 0.98$ and may be lowered by supegravity contributions [119]. These results are reproduced here for $\chi = -1$ (after taking into account the effective one-loop potential arising after integrating out the waterfall fields). Departing from $\chi = -1$, the supergravity contributions can result in a positive ($\chi > -1$) or negative ($\chi < -1$) slope of the tree-level potential (this is just the effect of the non-canonical kinetic term discussed in Sec. 5.5). For values of χ sufficiently close to $\chi = -1$, these small corrections may modify the globally supersymmetric predictions in an interesting way. We leave a detailed investigation to future work.

Next, we consider the case $n = 2$, $m = 1$. In this case, the globally supersymmetric tree-level scalar potential for the inflaton $\varphi = \sqrt{2}\,\mathrm{Re}\,\{\phi\}$ is given by

$$V^{\mathrm{glob}} = V_J = \kappa^2 M^4 + \frac{\lambda^2 \varphi^4}{4} \,, \tag{5.72}$$

where we have assumed $\langle X \rangle = 0$ and $\langle H \rangle = M$ during inflation[9]. In global supersymmetry, this potential is simply too steep to yield a viable inflation model in agreement with the current data. However, given our discussion in Sec. 5.5, we may hope to achieve a sufficient flattening of the scalar potential when transforming to the Einstein frame taking into account the canonical normalization of the inflaton field in the Einstein frame.

Switching to the Einstein frame, the complete potential including the waterfall fields becomes,

$$V_E(\varphi, H) = \frac{\big((H^2 - M^2)(\bar{H}^2 - M^2)\kappa^2 + \frac{\lambda^2\varphi^4}{4}\big)\big(1 + \frac{\varphi^2}{6}\chi(1+\chi)\big) + 2|H|^2\lambda^2\varphi^2(1 + \frac{\varphi^2}{6}\chi)}{\big(1 + \frac{\varphi^2}{6}\chi(1+\chi)\big)\big(1 - \frac{|H|^2}{3} - \frac{\varphi^2}{6}(1+\chi)\big)^2}. \tag{5.73}$$

Note that the H field is non-canonical in the Einstein frame ($\because K_{H\bar{H}} \neq 1$), and the masses of the canonical waterfall fields in the Einstein frame during inflation are given by

$$\begin{aligned} m^2_{\hat{H}_R} = &\frac{-2\kappa^2 M^2 + 2\lambda^2\varphi^2}{(1 - \frac{\varphi^2}{6}(1+\chi))^2} \\ &+ \frac{2\kappa^2 M^4}{3(1 - \frac{\varphi^2}{6}(1+\chi))^2} + \frac{\lambda^2\varphi^4}{6(1 - \frac{\varphi^2}{6}(1+\chi))^2}\left(1 + \frac{2\chi(1 - \frac{\varphi^2}{6}(1+\chi))(1 - \frac{\kappa^2 M^2}{\lambda^2\varphi^2}(1+\chi))}{(1 + \frac{\varphi^2}{6}(1+\chi))^2}\right), \end{aligned} \tag{5.74}$$

$$\begin{aligned} m^2_{\hat{H}_I} = &\frac{2\kappa^2 M^2 + 2\lambda^2\varphi^2}{(1 - \frac{\varphi^2}{6}(1+\chi))^2} \\ &+ \frac{2\kappa^2 M^4}{3(1 - \frac{\varphi^2}{6}(1+\chi))^2} + \frac{\lambda^2\varphi^4}{6(1 - \frac{\varphi^2}{6}(1+\chi))^2}\left(1 + \frac{2\chi(1 - \frac{\varphi^2}{6}(1+\chi))(1 + \frac{\kappa^2 M^2}{\lambda^2\varphi^2}(1+\chi))}{(1 + \frac{\varphi^2}{6}(1+\chi))^2}\right). \end{aligned} \tag{5.75}$$

[9]It can be shown that the canonically normalized imaginary part of the ϕ field has a mass larger than the Hubble scale during inflation, and it settles to zero VEV.

In the limit of $\chi \to -1$,

$$m^2_{\hat{H}_I,\hat{H}_R} = \pm 2\kappa^2 M^2 + 2\lambda^2\varphi^2 + \frac{2}{3}\kappa^2 M^4 - \frac{1}{6}\lambda^2\varphi^4 \,, \tag{5.76}$$

where the first two terms are the global SUSY mass terms for the waterfall fields. The last two terms provide the supergravity corrections suppressed by M_{pl}^2. In particular the last term in the above expression makes the waterfall mass tachyonic for large inflaton field values $\varphi \gtrsim M_{pl}$. On the other hand, the tachyonic instability for small field values, at $\varphi^2 < \varphi_c^2 \leq \kappa^2 M^2/\lambda^2$ indicates the usual waterfall instability which ends inflation. Thus the requirement of a viable waterfall mechanism limits the inflaton field range. As we move away from $\chi \simeq -1$, the field range for which the mass for the waterfall fields remain positive shrinks further, introducing the necessity to fine-tune the initial value of the waterfall field, until finally it even becomes impossible to account for 60 e-folds of inflation.

However, in order to achieve a sufficient flattening of the scalar potential in the Einstein frame, as discussed in Sec. 5.5, we need sufficiently large values of $\phi^2|1-\chi|$. In this sense, the observed destabilization of the waterfall field at large field values prevents us from constructing a viable inflation model.

One might hope to resolve the problem of the tachyonic instability in the waterfall field mass at large inflaton-values by adding a term $-\zeta_H|H|^4$ term in the frame function of Eq. (5.71), as for the stabilizer field X. However, this will also prevent the desired destabilization of the waterfall field at $\varphi < \varphi_c$, an essential feature of tribrid inflation. We point out that on the contrary, in the $n = m = 2$ case discussed above, the the waterfall field is not destabilized at large inflaton values. This can be understood by looking at the second line of Eq. (5.33), which yields a negative mass term for H for $n = 2$, $m = 1$ but not in the case $n = m = 2$.

In all tribrid models, once the auxiliary field is stabilized, it has nothing to do with the field dynamics. Now as the critical point is approached where the waterfall transition has to take place, the non-trivial dynamics is governed by two fields simultaneously. So in the two-dimensional (ϕ, H)-space the requirement of $V_{\Delta SUGRA}$ to be zero in ϕ-direction ($i.e$ satisfying the conditions of Eq. (5.33)) alone is insufficient to comply with the dynamics. We recall that the usual hybrid inflation model given by Eq. (5.37) is also difficult to implement in this framework as the imaginary component of the complex inflaton field has a tachyonic mass [154].

5.8 Conclusions and Outlook

Current CMB data allow for an energy scale of inflation as high as about 10^{16} GeV. At these energies, supergravity effects can no longer be neglected. They may spoil the flatness of the inflationary direction, destabilize the inflationary trajectory (as e.g. in F-term hybrid inflation [86, 87, 155]), or even also improve the flatness of the potential (as in models with non-minimal coupling to gravity such as Higgs inflation [130] or α-attractors [131]). In this paper we have systematically studied supergravity contributions to Jordan frame inflation models, i.e. inflation models characterized by a non-minimal coupling to gravity and canonical kinetic terms in the Jordan frame. Our focus here is on single-field inflation models driven by F-term potentials, for an example of D-term inflation in this setup see [154].

We disentangle two types of supergravity contributions in the Jordan frame, arising from contributions to the scalar potential and from the non-minimal coupling to gravity. We find that the former generically yields a contribution to the Jordan frame scalar potential which is at most of the same power in the inflaton field as the contribution from global supersymmetry. We moreover derive the condition on the superpotential for which this term vanishes identically (cf. Eq. (5.32)) and find that for single-field inflation models this corresponds to the condition of a vanishing superpotential during inflation.

In a second step, we turn to the effects of the non-minimal coupling to gravity, which translate to non-canonical kinetic terms in the Einstein frame. As observed e.g. in the context of α-attractors [131], this can lead to an exponential flattening of the scalar potential in the Einstein frame in terms of the canonically normalized field. However, this mechanism requires the powers of the inflaton field appearing in the non-minimal coupling to gravity and in the (Jordan frame) scalar potential to be adjusted accordingly.

The findings of this paper are illustrated in various examples. In particular we focus on two examples of the type $W = \lambda X \phi^n$, with $n = \{1, 2\}$. The CMB data can be reproduced for certain values of the parameter χ, which parametrizes the non-minimal coupling to gravity. In particular in the $n = 2$ case we asymptotically reproduce the Starobinsky inflation model for $|\chi| \gg 1$. Since in all these models the superpotential vanishes along the inflationary trajectory, the supergravity contributions to the Jordan frame potential are identically zero. We further comment on tribrid inflation models, which, in contrary to the more commonly discussed hybrid inflation models, are protected from supergravity contributions to the Jordan frame scalar potential. However, this protection does not encompass the second dynamical degree of freedom in these models, the waterfall field. Using two different realizations of tribrid inflation we illustrate how this may lead to a destabilization of the inflationary trajectory or to a potentially viable inflation model.

Our results may be used as guidelines to easily estimate the effect of supergravity contributions in a given Jordan frame inflation model. They may moreover be useful in inflationary model building to easily understand what type of terms in the superpotential or Kähler potential may help modify a given scalar potential in a desired way through supergravity contributions.

In this paper, we focused on a simple frame function motivated by approximate scale invariance and on superpotentials which can be expressed as polynomials in the inflaton field. It would be interesting to extend this work beyond these two assumptions. Moreover, the models studied in this paper should be considered as illustrative toy models, at this point without a deeper motivation from particle physics and also lacking a study of the subsequent cosmology after inflation. In particular, we have not addressed the question of (low-energy) supersymmetry breaking.

CHAPTER 6

MODULI AND COSMIC INFLATION

We discussed the role of moduli on the inflationary observables. The presence of moduli particles leads to an epoch in the post-inflationary history in which the energy density is dominated by cold moduli particles. This changes the post inflationary history and implies that the preferred range for the number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation depends on moduli masses. As a result the precision CMB observables become sensitive to moduli masses. In this Chapter we analyze this sensitivity for some representative models.

6.1 Introduction

We highlighted in Chapter 2 that precision measurements of the cosmic microwave background (CMB) have put the inflationary paradigm as the leading candidate for a theory of early Universe cosmology. On the theoretical front however, the paradigm of cosmic inflation faces many challenges. The inflationary slow roll conditions are ultraviolet sensitive; we should embed models of inflation in a quantum theory of gravity. In this light, an important direction of research is study of the effects that can arise as a result of ultraviolet completion of inflationary models. String theory provides a setting where one can hope to carry out a systematic study of such effects.

A generic feature of supergravity/string models are the moduli fields. The vacuum expectation value of moduli fields set the strength and form of the low energy effective action of string models, hence moduli fields play a central role in string phenomenology. There has been an extremely useful interplay between studies of moduli stabilization and inflationary model building in string theory, see for e.g. [165, 166, 34]. In the current Chapter, we examined the sensitivity of precision CMB observables – the spectral tilt (n_s) and the tensor-to-scalar ratio (r) to the mass of the lightest modulus field.

Given a model of inflation, one can express n_s and r in terms of the number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation (N_k). Predictions for n_s and r are then made by using the "preferred range" of N_k in these formulae. The preferred range for N_k is determined by tracking the history of the Universe for the time of horizon exit to the present epoch i.e. the computation is sensitive to the post-inflationary history of the Universe. For the standard cosmological timeline (which has the epochs inflation, reheating, radiation domination, matter domination, accelerated expansion) the preferred range for N_k is 50 to 60.

From the very early days of inflationary model building in supergravity, it was realized that a generic implication of having moduli fields is a non-standard post-inflationary cosmological timeline [167, 168, 169, 170, 171, 172] (often referred to as the modular cosmology timeline). The modular cosmology timeline sets in as a result of vacuum misalignment of moduli fields during the inflationary epoch. The associated production of moduli particles leads to an epoch in the post-inflationary history of the Universe in which the energy density is dominated by cold moduli particles. The history is thermal after the decay of the moduli particles[1]. Reference [46] derived the preferred range of N_k for the modular cosmology timeline[2] and found it to be

$$\left(55 - \frac{1}{4}N_{\rm mod}\right) \pm 5, \tag{6.1}$$

where $N_{\rm mod}$ is the number of e-foldings of the Universe during the epoch that the energy density is dominated by cold moduli particles. As we will see in Section 6.2.1, in generic models is $N_{\rm mod}$ essentially determined by the the post-inflationary mass[3] of the lightest modulus field (m_φ).

Our goal is to explore in detail the phenomenological implications of (6.1). After giving a brief review of modular cosmology in Section. 6.2.1, we discussed in Section 6.2.2 the dependence of the preferred range of N_k on the mass of the lightest modulus. It implies that *n_s and r are sensitive to the mass of the lightest modulus.* In Section. 6.3 we examined this sensitivity for some representative models of inflation ($m^2\chi^2$ [159], axion monodromy [97, 98], natural inflation [173] and the Starobinsky model [6]) and for those

[1]The successes of big bang nucleosynthesis imply that the decay of the modulus has to take place before nucleosynthesis.

[2]See [43] for a systematic discussion of various effects that can affect the preferred range for N_k.

[3]Curvature couplings imply that the mass of a modulus field can be significantly different during the inflationary and post-inflationary epochs.

models a bound on the modulus mass is also determined in Section 6.4. Motivated by the varied spectra of phenomenologically viable supergravity models we treated the mass of the lightest modulus (m_φ) as a parameter. We analyzed our results in the context of PLANCK 2015 data [15]. The implications are very interesting; the changes in inflationary predictions can significantly affect the scorecard for models. In Section 6.5 a brief account of the density perturbation in modular cosmology is provided.

6.2 Review

6.2.1 Modular cosmology

At tree level, string compactifications have massless scalar fields which interact via Planck suppressed interactions (the moduli). Moduli acquire masses from sub-leading effects, their masses are typically well below the string scale and hence moduli are part of the low energy effective action.

Moduli fields usually have curvature couplings; this makes their masses and potential dependent on the expectation value of the inflaton. As a result, the minimum of the potential for a modulus of post-inflationary mass less than Hubble during inflation $(m_\varphi < H_{\text{inf}})$ is different during the inflationary and post-inflationary epochs – such a modulus finds itself displaced from its post-inflationary minimum at the end of inflation. This "initial displacement" is typically of the order of M_{pl} [174, 175, 93, 94, 176].

As discussed in the introduction, this "misalignment" implies a non-standard cosmological timeline. We briefly review this timeline and refer the reader to [167, 168, 169, 170, 171, 172] for a more complete discussion. Let us begin by describing the case when there is a single modulus whose post-inflationary mass m_φ is below the Hubble scale during inflation. At the end of inflation the Universe reheats, the energy density associated with the inflaton gets converted to radiation. At this stage, the energy density of the Universe consists of two components — radiation, and the energy associated with the modulus displaced from its minimum[4]. Also, the high value of the Hubble friction keeps the modulus pinned at its initial displacement. As the Universe cools, the Hubble constant drops. When the Hubble friction falls below the mass of the modulus, the modulus begins to oscillate about its post-inflationary minimum. With this, the associated energy density dilutes as matter i.e. much slower than that of the radiation. Eventually the energy density associated with the modulus dominates the energy density of the Universe; the Universe enters into the epoch of modulus domination. This epoch lasts until the decay of the moduli particles. The Universe reheats for a second time after the decay of the modulus, after which the history is thermal. In summary, the modular cosmology timeline consists of the following epochs - inflation, reheating (associated with inflaton decay), radiation domination, modulus domination, reheating (associated with modulus decay), radiation domination, matter domination and finally the present epoch of acceleration.

In models with multiple moduli with post inflationary mass below Hubble during inflation, there are multiple epochs of modulus domination and reheating associated with the moduli. In cases where there is a separation of scale between the mass of the lightest modulus and the mass of other moduli the lightest modulus outlives the others and sets the time scale for the epoch of modulus domination. The dynamics of the system can be effectively described by a model with a single modulus; with the effect of the heavy moduli being incorporated in the reheating epoch after inflation[5]. In models in which there is no distinct

[4]Since $m_\varphi < H_{\rm inf}$, right after reheating the energy density associated with radiation dominates over the energy density associated with the displaced modulus.

[5]Moduli decay via Planck suppressed interactions. Hence the lifetime scales as m_φ^{-3}, this implies that this effective description can be useful even for a moderate separation between the mass of the lightest moduli and the heavier ones.

lightest modulus the dynamics is more complicated to analyse; this was discussed briefly in [46]. We will confine ourselves to situations in which there is a distinct lightest modulus in this paper.

6.2.2 The preferred range of $\mathrm{N_k}$ in modular cosmology

In this section we briefly review the results of [46] relevant for our analysis. Our focus will be on models in which adiabatic perturbations are generated as a result of quantum fluctuations during the inflationary epoch. The strength of the inhomogeneities generated is given by

$$A_s = \frac{2}{3\pi^2 r}\left(\frac{\rho_k}{M_{\rm pl}^4}\right),$$

where A_s is the amplitude of the scalar perturbations, ρ_k the energy density of the Universe at the time of horizon exit and r the tensor to scalar ratio. We review the details of generation of density perturbations in the context of modular cosmology in Section. 6.5. The scalar amplitude A_s is constant to a very good approximation until the point of horizon re-entry. The strength of temperature fluctuations in the CMB can be obtained by tracking its subsequent evolution. Thus the measurement of the strength of temperature fluctuations gives us the value of the energy density of the Universe at the time of horizon exit (modulo r). CMB observations also give us the value of the energy density today (ρ_0) via determination of the Hubble constant. Thus any theoretical proposal for the history of the Universe between horizon exit and the present epoch must be such that ρ_k evolves to ρ_0. Reference [46] applied this consistency condition to the modular cosmology timeline described in section 2.1. This gave the relation[6]

$$N_k + \frac{1}{4}N_{\rm mod} + \frac{1}{4}(1-3w_{\rm re1})N_{\rm re1} + \frac{1}{4}(1-3w_{\rm re2})N_{\rm re2} \approx 55.43 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right) \quad (6.3)$$

where N_k is the number of e-foldings between horizon exit of the modes relevant for CMB observations and the end of inflation, $N_{\rm mod}$ is the number of e-foldings that the Universe undergoes during the epoch of modulus domination, $w_{\rm re1}$ and $w_{\rm re2}$ are the effective equation of state parameters during the two reheating epochs, $N_{\rm re1}$ and $N_{\rm re2}$ are the number of e-foldings during the two reheating epochs, ρ_k the energy density at the time of horizon exit and $\rho_{\rm end}$ the energy density at the end of inflation. The number of e-foldings of modulus domination was found to be

$$N_{\rm mod} \approx \frac{4}{3}\ln\left(\frac{\sqrt{16\pi}M_{\rm pl}Y^2}{m_\varphi}\right) \quad (6.4)$$

where Y is the initial displacement of the modulus from its post-inflationary minimum in Planck units. Eqn. (6.3) can be used to obtain the "preferred range" of N_k for modular cosmology. A discussion of the analogous analysis for the standard cosmological timeline can be found in [177]. Making the same generality assumptions regarding the reheating epoch, change in the energy density of the Universe during inflation and the scale of inflation as in Section 2.3 of [177], eqn. (6.3) gives the preferred range for N_k to be

$$\left(55 - \frac{1}{4}N_{\rm mod}\right) \pm 5. \quad (6.5)$$

[6] The analogous relation for the standard cosmological timeline is

$$N_k + \frac{1}{4}(1-3w_{\rm re})N_{\rm re} \approx 55.43 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_k}{\rho_{\rm end}}\right). \quad (6.2)$$

Note that this can be thought of as lowering of the central value of the preferred range of N_k by $N_{\mathrm{mod}}/4$. As mentioned earlier, there are general arguments [174, 175, 93, 94, 176] which imply Y is an $\mathcal{O}(1)$ quantity[7]. Thus the shift in the central value of N_k is essentially determined by m_φ.

Before ending this section we would like to emphasize that the relation (6.3) and expression (6.5) are valid only if the post inflationary mass of the modulus m_φ is below Hubble during inflation. If the post-inflationary mass of the lightest modulus is well above Hubble during inflation then the misalignment mechanism is not operational and the preferred range is 55 ± 5.

6.3 Implications for Inflationary Models

In this section, we will study the phenomenological implications of the results described in Section 6.2.2 for some representative models of inflation. Given the diverse spectra of phenomenologically viable supergravity models we will treat m_φ as a phenomenological parameter in our analysis. The central value of N_k also depends on Y. As discussed in Section 6.2.2 typically Y is $\mathcal{O}(1)$. We note that apart from the classical contributions to Y discussed in [93, 94], quantum contributions to the effective potential of the field can also be present (see for e.g. [1]), although if the mass of the modulus is of the order of Hubble during inflation one expects the classical contribution to be dominant. Exact determination of Y requires the knowledge of coupling between the inflaton and moduli fields; hence is sensitive to the embedding of a model of inflation in a compactification. For field displacement due to classical effects we will take $Y = 1/10$ (as is often taken in analysis of the cosmological moduli problem see for e.g. [169]). So our choice of $Y = 1/10$ can be

[7] These expectations have been borne out in explicit constructions of inflationary models in string compactifications, see for e.g. [178].

considered conservative; but this ensures better control over the effective field theory. We leave the exact computation of Y and its dependence on various parameters (such as the mass of the modulus) in specific compactifications for future work. The quantum effects become stronger as the field becomes lighter; for a modulus well below the Hubble scale the field displacement due to quantum effects can dominate over the classical contribution.

We will focus on four benchmark models of inflation - $V(\chi) = \frac{1}{2}m^2\chi^2$ [159] (we will denote the inflaton by χ), axion monodromy i.e $V(\chi) = \hat{m}^{10/3}\chi^{2/3}$ [97, 98], natural (pNGB) inflation [173] and the Starobinsky model [6]. Let us record n_s and r as a function of N_k for each of these models

- $m^2\chi^2$: $n_s = 1 - 2/N_k, \quad r = 8/N_k$
- Axion monodromy: $n_s = 1 - 4/(3N_k), \quad r = 8/(3N_k)$
- Natural inflation: $n_s = 1 - \left[\frac{M_{\rm pl}}{f}\right]^2 \left[\frac{1+\frac{e^{-x}}{p}}{1-\frac{e^{-x}}{p}}\right], \quad r = 8\left[\frac{M_{\rm pl}}{f}\right]^2 \left[\frac{\frac{e^{-x}}{p}}{1-\frac{e^{-x}}{p}}\right]$ with $p = 1 + \frac{M_{\rm pl}^2}{2f^2}$,

  $x = \frac{N_k M_{\rm pl}^2}{f^2}$ where f is the axion decay constant.
- Starobinsky model: $n_s = 1 - 2/N_k, \quad r = 12/N_k^2$

The change in the preferred range of N_k (6.5) occurs if m_φ is less than Hubble during inflation. We begin by implementing this condition for each of the models. The Hubble constant at the time of horizon exit is

$$H_k = \frac{\pi}{\sqrt{2}}(A_s r)^{1/2} M_{\rm pl} \tag{6.6}$$

Note that the right hand side of (6.6) depends on m_φ; since r is determined by N_k and the preferred range for N_k depends on m_φ. Also, r decreases with an increase in N_k.

Therefore, the condition can be implemented over the entire preferred range by requiring that it holds for the maximum value of N_k

$$N_{\max} = 60 - \frac{1}{3} \ln \left(\frac{\sqrt{16\pi} M_{\rm pl} Y^2}{m_\varphi} \right). \tag{6.7}$$

Thus we want to impose the condition

$$\frac{\pi}{\sqrt{2}} (A_s r[N_{\max}])^{1/2} M_{\rm pl} > m_\varphi \tag{6.8}$$

with $N_{\max}$ as given by (6.7). We solve for this condition numerically in the plot shown in Fig. 6.1. The condition is most stringent for the Starobinsky model, for which the right

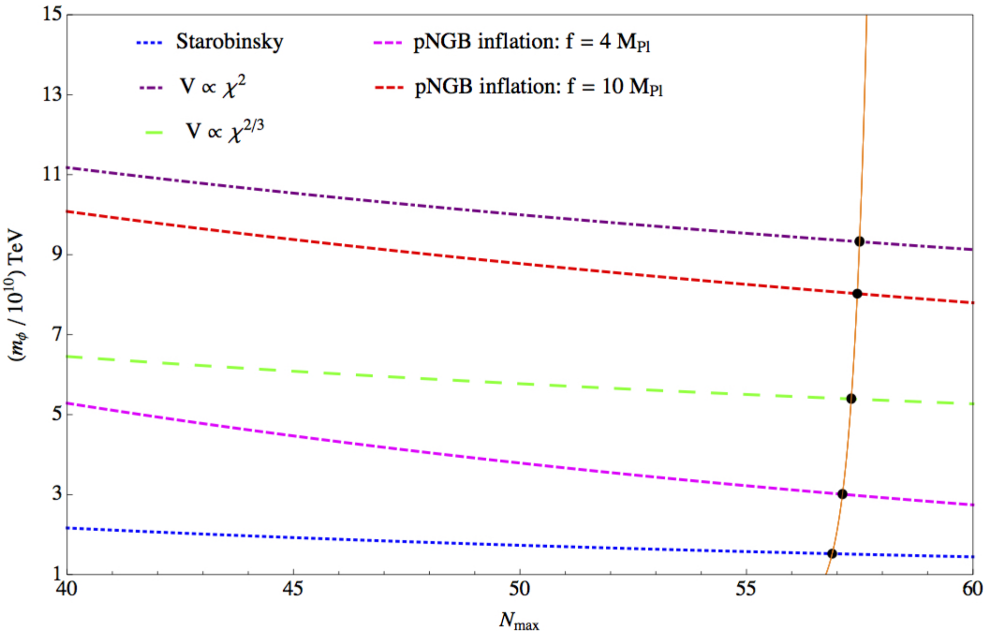


Figure 6.1: Numerical solution for the condition $H_{\rm infl} > m_\varphi$. The solid curve is a plot of m_φ as a function of $N_{\max}$ as given by (6.7). The dashed curves are plots of the left hand side of (6.8) as a function of $N_{\max}$ for various models.

hand side and left hand side of (6.8) are equal for $m_\varphi \approx 1.5 \times 10^{10}$ TeV. We will be conservative and study the implications of the shift in the central value of N_k if the mass of the modulus is at least two orders of magnitude below this i.e $m_\varphi < 10^8$ TeV (this value will be used for all models).

On the other hand, the cosmological moduli problem (CMP) bound, based on the requirement of successful nucleosynthesis requires $m_\varphi > 30$ TeV [167, 168, 169, 179]. We will use this consideration to set the lower value of m_φ in our analysis. In summary, we will use the range 10^2 TeV $< m_\varphi < 10^8$ TeV to study the effects of the epoch of modulus domination on inflationary predictions.

We now have all the ingredients necessary to compute the predictions for n_s and r. We compute the predictions for n_s and r for $m_\varphi = 10^3, 10^6$ and 10^8 TeV. We begin by taking $Y = 1/10$, appropriate for a classical displacement. We will study the case of displacement due to quantum effects later in the section. The results are shown in Fig. 6.2, the plot for the standard cosmological timeline (which is equivalent to $m_\varphi > H_{\text{inf}}$) is also included for reference. The shaded regions correspond to the $1-\sigma$ and $2-\sigma$ results for n_s and r from PLANCK 2015 analysis for TT modes and low P [15]. We find that for the $m^2\chi^2$ model even a very heavy modulus of mass 10^8 TeV implies predictions for n_s and r which are well outside the $2-\sigma$ region. The axion monodromy model moves inside the $1-\sigma$ region for m_φ below 10^5 TeV. The Starobinsky model remains in the $1-\sigma$ region for almost the entire mass range.

In the above analysis, we have taken $Y = 1/10$ even for a modulus mass of 10^3 TeV. For such a light field the quantum fluctuations can be large. The fluctuation squared is expected to be order of

$$\langle \varphi^2 \rangle = \frac{3H^4}{8\pi^2 m_\varphi^2},$$

see for e.g. [1]. For a modulus of mass 10^3 TeV and the inflationary scale at the GUT

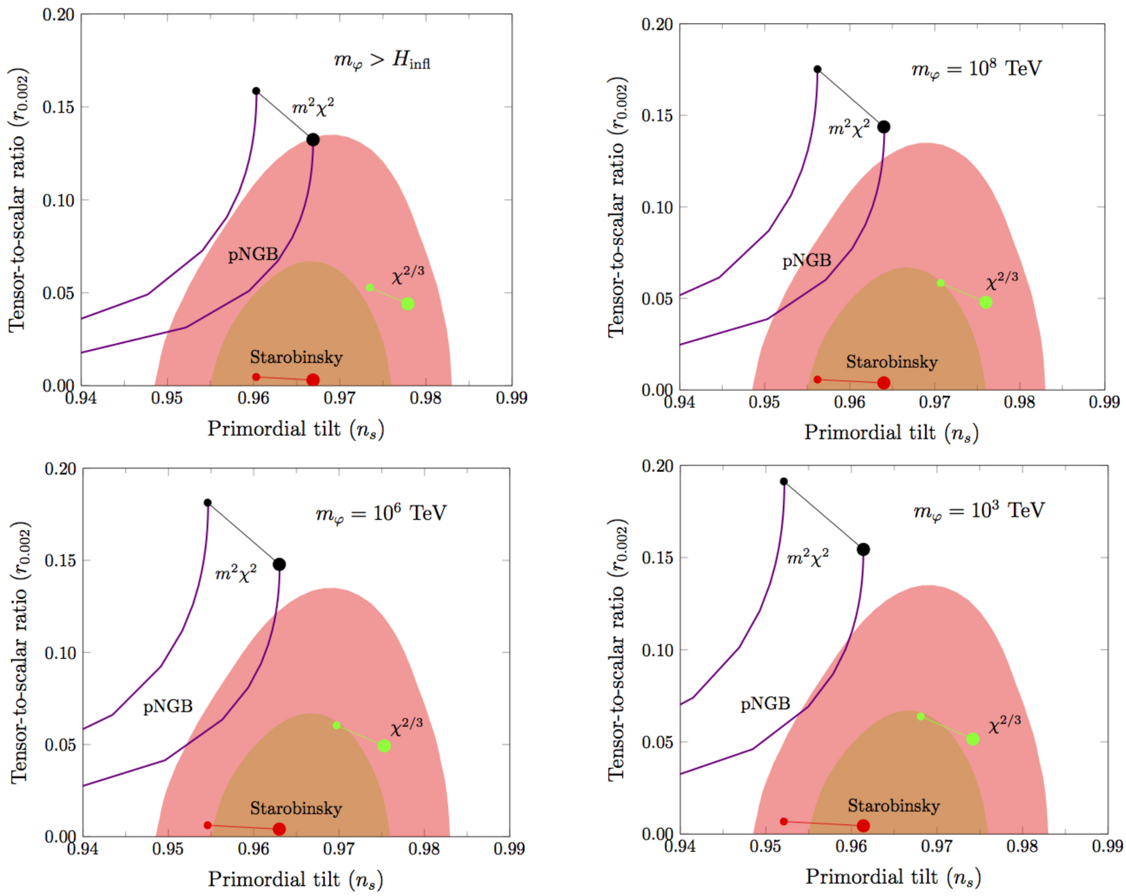


Figure 6.2: Inflationary predictions for $m^2\chi^2$ (black), Natural/pNGB inflation (purple), Axion monodromy (green), Starobinsky model (red). For the cases of no misalignment $(m_\varphi > H_{\rm infl})$, $m_\varphi = 10^3, 10^6, 10^8 \quad$ TeV.

scale this yields $Y \approx 10$, significantly larger that the value of Y used by us earlier. Thus for the modulus mass 10^3 TeV we compute the inflationary predictions taking $Y = 10$. The results are presented in Fig. 6.3.

Finally, we would like to mention a general implication. For gravity mediated models moduli masses are tied to the scale of supersymmetry breaking. Thus, for gravity mediated models our results correlate inflationary predictions with the scale of supersymmetry breaking. The effect is significant even for models with a high scale of supersymmetry

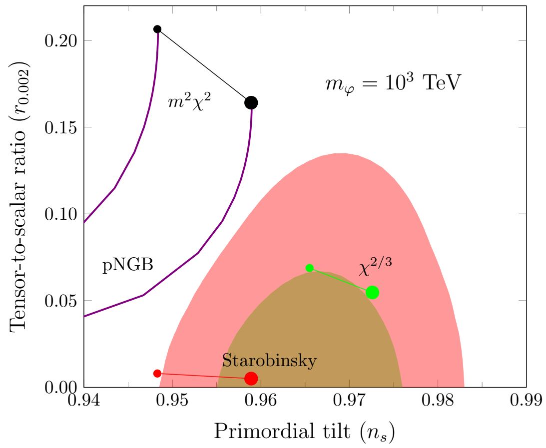


Figure 6.3: Inflationary predictions for $m^2\chi^2$ (black), Natural/pNGB inflation (purple), Axion monodromy (green), Starobinsky model (red) for $m_\varphi = 10^3$TeV, with the field displacement taken to be of quantum origin (this dominates over the classical effect).

breaking.

6.4 A bound on moduli masses

The consistency condition (6.3) can be used to obtain a bound on moduli masses given a model of inflation by taking input from observations on the value of n_s [46]. The approach can be considered complimentary to that of the previous section where we discussed inflationary predictions as a function of the mass of the late time decaying modulus. In this section, we analyse the bound for our representative models and update some of the discussion in [46] in light of the PLANCK 2015 data release [15].

The bound is obtained by combining the consistency condition (6.3) with expression for $N_{\rm mod}$ (6.4) and demanding that the reheating epochs are not exotic, i.e. $w_{\rm re1}, w_{\rm re2} < 1/3$ (see for e.g. [44, 42, 45, 177] for a discussion of on this condition on the effective equation of state during reheating). With this, one can arrive at a lower bound on m_φ

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \, e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\rho_{\rm k}/\rho_{\rm end}) + \frac{1}{4}\ln r\right)}. \tag{6.9}$$

The bound applies only if m_φ is less than Hubble during inflation (as equation (6.3) was derived under this assumption). Given a model of inflation and observational input on the value of n_s, one can explicitly compute the quantities in the exponent in the right hand side of (6.9). Typically, N_k is related to n_s by a relation of the form $N_k = \frac{\beta}{1-n_s}$, where β depends on the model of inflation. This makes the bound highly sensitive to the value of n_s. The PLANCK 2015 release [15] gives the central value of n_s to be 0.9680; there is a shift in the positive direction in comparison with the 2013 value of $n_s = 0.9603$ [177]. This implies an increase in the number of efolds, N_k for inflationary models and thereby a more stringent bound.

Let us now discuss the bound in the context of our representative models. For the $m^2\chi^2$ model, the PLANCK central value of n_s gives the right hand side of (6.9) to be well above Hubble during inflation (as obtained in Figure 1); modular cosmology is incompatible with this value of n_s. The lower end of the $1\!-\!\sigma$ value gives $m_\varphi > 10^{10}$ TeV. On the other hand, for the axion monodromy model (6.9) yields a value below the CMP bound based on nucleosynthesis considerations [167, 168, 169, 179], thus is not of phenomenological interest as a bound. The fact that the bound is not strong for the axion monodromy model is consistent with the results shown in figure 2 - the axion monodromy model is in the $1\!-\!\sigma$ region for $m_\varphi = 10^3$ TeV. Similarly, in the case of the Starobinsky model and pNGB inflation the value of the bound is in keeping with the results shown in figure 2.

For small field models, the second term in the exponent of the right hand side of (6.9) (the term involving the ratio of the energy densities at the time of horizon exit and end of

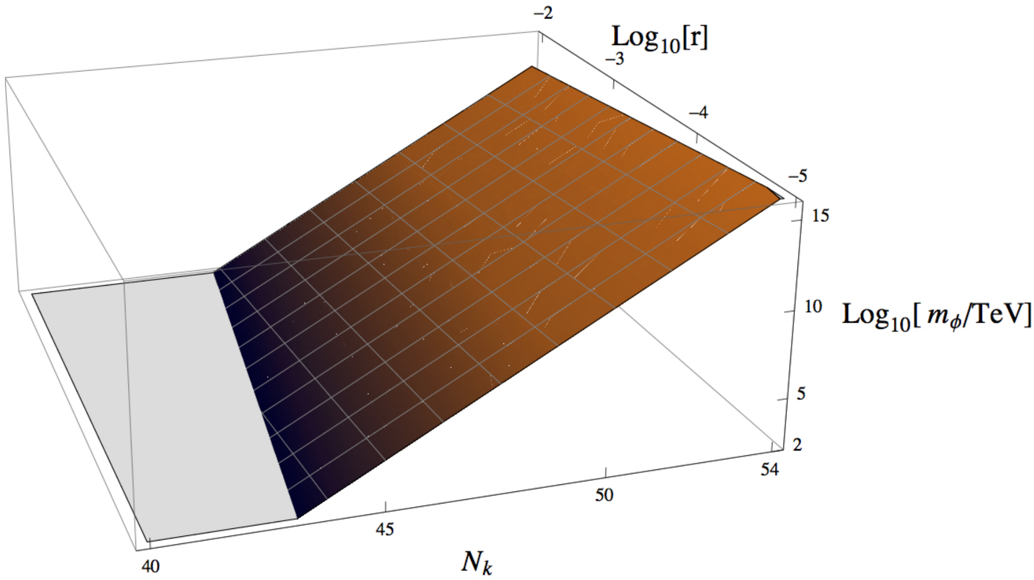


Figure 6.4: Bound on the modulus mass for small field models. The allowed values of m_φ are in the region above the shaded plane. We have chosen $Y = 1/10$.

inflation) makes a negligible contribution. In Fig. 6.4 we show the allowed range for m_φ as a function of N_k and r. The plot illustrates that the scale for the bound is essentially set by N_k. For $N_k \gtrsim 50$ the bound is very strong; $m_\varphi \gtrsim 10^7$ TeV. The bound is stronger than the CMP bound as long as $N_k \gtrsim 44.5$. The plot in Fig. 6.4 can be used to read off the implications of the bound for any small field model. It will be interesting to explore the implications of this bound for inflationary model building in moduli stabilised string compactifications.

6.5 Density perturbations in modular cosmology

In this section we review the generation of density perturbations in the context of modular cosmology. As discussed in section 2 the minimum of the potential of the late time decaying modulus depends on the inflaton expectation value; thus as the inflaton moves along its trajectory the expectation value of the late time decaying modulus (and potentially other moduli) necessarily changes. Thus, the trajectory in field space during inflation involves displacement along the inflaton direction, late time decaying modulus (and potentially other moduli). We will require the directions in field space orthogonal to the trajectory in field space during inflation to have mass of at least of the order of Hubble (this as we will see in what follows will ensure that isocurvature perturbations are suppressed). Infact, curvature couplings naturally lead to such mass terms of the order of Hubble (see for e.g.[93, 94]).

The perturbations generated are best understood in the formalism developed in [125] — coordinates in field space are chosen such that one of the coordinate directions is along the trajectory in field space (during the inflationary epoch) and the remaining are orthogonal to the trajectory in field space. The key result of [125] is that quantum fluctuations associated with the direction in field space parallel to the trajectory are adiabatic, while the ones orthogonal generate isocurvature perturbations. Thus, imposing the condition that the directions in field space orthogonal to the trajectory have mass at least of the order of Hubble ensures that isocurvature perturbations at the time of horizon exit are suppressed; the perturbations are to a very good approximation adiabatic at the time of horizon exit. We will denote the adiabatic perturbation at the time of horizon exit by $\mathcal{R}_*$ and the isocurvature perturbations by $\mathcal{S}^i_*$. These have to be evolved into the radiation epoch (after the decay of the modulus) to determine the strength of the temperature fluctuations they seed. The result of this evolution is given by a transfer matrix [180], which takes the general form (to keep the presentation simple we include one isocurvature direction, it is easily generalised to the case of multiple isocurvature perturbation directions)

$$\begin{bmatrix} \mathcal{R}_{\rm rad} \\ \mathcal{S}_{\rm rad} \end{bmatrix} = \begin{bmatrix} 1 & \mathcal{T}_{\rm RS} \\ 0 & \mathcal{T}_{\rm SS} \end{bmatrix} \begin{bmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{bmatrix}$$

where $\mathcal{R}_{\rm rad}$ and $\mathcal{S}_{\rm rad}$ are the isocurvature and adiabatic perturbations after the modulus decay. An important feature of the transfer matrix is that the entries in the first column are completely model independent [180] - they follow from the fact that a purely adiabatic perturbation is conserved and does not lead to any isocurvature perturbations. On the other hand, the transfer functions $\mathcal{T}_{\rm RS}$ and $\mathcal{T}_{\rm SS}$ are model dependent. But, the form of the transfer matrix implies that if $\mathcal{S}_* << \mathcal{R}_*$, then isocurvature perturbations remain suppressed and $\mathcal{R}_{\rm rad}$ is essentially determined by $\mathcal{R}_*$. Thus, for models in which the only light direction during the inflationary epoch is the trajectory in field space the density perturbations are adiabatic and determined by the curvature perturbation at the time of horizon exit.

Other scenarios to generate density perturbations are the curvaton scenario [181] and modulated fluctuations [182]. We shall not explore these possibilities here, see [183, 184] for their realisations in string models.

6.6 Conclusions and Discussions

In this Chapter, we have studied the sensitivity of n_s and r to the mass of the lightest modulus in the context of modular cosmology. The results of Section 6.3 clearly exhibit that it is important to explicitly incorporate the effect of the epoch of modulus domination in obtaining the preferred range of N_k. The effect can significantly alter the inflationary predictions for n_s and r of string/supergravity models; being relevant even for very heavy moduli $(m_\varphi \approx 10^8 \ \text{TeV})$. Furthermore, future experiments [185] are likely to bring down the uncertainties in the measurement of n_s by one order of magnitude; making our analysis all the more relevant. Given that modular cosmology is generic in string/supergravity models [167, 168, 169, 170, 171, 172] our results should have broad implications.

Our approach has been phenomenological; we have treated the mass of the lightest modulus as a free parameter and taken the initial displacement of the modulus (that results due to misalignment) to have a generic value. The results strongly motivate the study of specific models where the modulus mass takes a fixed value and it is possible to compute the value of the initial displacement explicitly. Some models worth exploring in this context are fibre inflation [178], Kahler moduli inflation [186], M-flation [127] and Gauged M-flation [187]. For a recent effort in explicit computation of the moduli displacement see [188].

Another important direction in the study of specific models is first principles analysis of the reheating epoch. This can reduce the uncertainty in N_k, allowing for more precise predictions of n_s and r. This question has received much attention recently [44, 42, 45, 189, 190]. The methods developed in [190] can be useful in analysing the decay of moduli particles.

More generally, modular cosmology can also have implications for dark matter, structure formation and the phenomenology of SUSY models [191]. It is natural to look for correlations between our results for CMB observables and other phenomenological signatures.

CHAPTER 7

SUPERGRAVITY HYBRID INFLATION WITH MODULUS

We have already computed the changes in the inflationary predictions due to the presence of a mudulus field. While the modulus in earlier case are supposed to be stabilized around the minimum of its potential, here we are going to investigate whether such stabilization is exact or any small displacement around the minimum of modulus potential can affects the inflationary dynamics in a non-trivial manner.

7.1 Introduction

Last chapter deals with the effects of moduli at the end stage of inflation. It showed that the presence of moduli fields in the late Universe will cause a non-trivial modifications in the relevant number of efoldings of inflation. The modified expression for the efolds is now a function of the moduli masses. The precise mathematical form of this expression was also highlighted in the previous chapter.

Moduli however do play an important role during inflation and also poses severe problems for cosmology [192, 193, 194, 167, 168, 169]. In this chapter our aim is to study how their presence affects the inflaton potential in a generic supergravity inflation model. Since supergravity is obtained as low energy effective version of the string theory, so it naturally contains moduli fields. But for simple models to realize in such a setup requires moduli being stabilized. It turns out that the moduli potential generically yields large contribution to the inflationary η-parameter [195, 196]. Moreover the inflationary sector of the theory may also destabilize the minimum of the moduli potential [197, 198, 34]. Therefore, moduli stabilization means to find out a mechanism (*i.e.* a non-trivial potential) that freezes the modulus during inflation. The details of moduli stabilization is beyond the scope of this work (See refs [199, 200, 201, 195] for details). So, here we have considered a particular version of the stabilized moduli often called the KL stabilized moduli [197]. For the inflationary sector of our theory we have considered F-term hybrid inflation in $4D$ $\mathcal{N} = 1$ SUGRA. It belongs to an interesting class of inflation models where inflation ends via a phase transition when the so-called waterfall fields acquire expectation values. In Sec. 7.2 we will briefly review some standard features of supergravity hybrid inflation. Thereafter, in Secs. 7.4 and 7.5 we will combine the same inflationary sector with KL stabilized moduli. Also for the sake of simplicity we have restricted ourselves to a single modulus field during inflation. In general the interplay between the inflaton sector and the modulus sector kills the prediction of an otherwise succesfull model. This simple model will elucidate the possibility of embedding the the SUGRA hybrid inflation in a more realistic framework by cultivating how modulus interfere with the background inflationary potential. We conclude in Section. 7.6.

7.2 Review of F-term hybrid inflation

In single field inflation models, it is the same field which is responsible for driving the inflationary dynamics as well as the end of inflation. In contrast hybrid inflation works through the coupling of inflaton with a second field [202]. This coupling makes the inflaton potential distinct from the exit from inflation.

The superpotential for F-term hybrid inflation is given by [203]

$$W = \kappa\Phi(M^2 - H^2) \tag{7.1}$$

where Φ is a gauge singlet chiral superfield representing inflaton. H is also a chiral superfield commonly known as the waterfall fields which triggers the halt of inflation. This theory has two free parameters — a dimensionless parameter κ and the symmetry breaking scale M which basically sets the scale of inflation and the VEV of H after inflation.

We note that during inflation $H = 0$ and $W \neq 0$. Taking the canonical Kähler potential

$$K = |\Phi|^2 + |H|^2 \tag{7.2}$$

the F-term scalar potential obtained by using the standard formula is

$$V_F = \kappa^2 e^{|\Phi|^2+|H|^2}\big[(1-|\Phi|^2+|\Phi|^4)|H^2-M^2|+|\Phi|^2\big(-2H^2M^2+ \\ 2|H|^2(2+|H|^2)+((H^2-M^2)|\bar{H}|^2+2|H|^2)(2+|H|^2)\big)\big] \tag{7.3}$$

Now if we expand the exponential factor in front of the potential then at leading order we get

$$V_F \approx \kappa^2 M^4\left(1+\frac{|\Phi|^4}{M_{pl}^4}\right) \tag{7.4}$$

Also the mass squared eigenvalues for the waterfall field come out to be

$$M_{\pm}^2 \simeq \kappa^2 M^2 \pm \kappa^2|\Phi|^2 \tag{7.5}$$

where the $'\pm'$ sign corresponds to the the real and pseudo-scalar part of the scalar component of H field. Thus during inflation H-direction also acquired a large positive mass squared contribution upto the critical value $|\Phi| > \Phi_c$, where $|\Phi_c| \approx M$. This stabilizes both $\langle H, \bar{H}\rangle$ to their respective VEVs at zero. However, when $|\Phi|$ becomes smaller than the critical value, M_-^2 changes sign and hence $|\Phi|$ quickly rolls down to the global minimum, which ends inflation. The above mass splitting happens due to the SUSY breaking during inflation. Such a splitting induces radiative correction to the tree level potential which is of the Colemann-Weinberg type and it is

$$V_{1loop} \simeq \frac{\kappa^2 M^4}{8\pi^2}\ln\left(\frac{\phi}{\phi_c}\right) \tag{7.6}$$

where $\phi = \sqrt{2}Re\Phi$. Now the effective scalar potential becomes

$$V_F \approx \kappa^2 M^4 \left(1 + \frac{\kappa^2}{8\pi^2} \ln\left(\frac{\phi}{\phi_c}\right) + \frac{\phi^4}{8M_{pl}^4}\right) \tag{7.7}$$

This effective potential is dominated by the false vacuum energy $\kappa^2 M^4$. Meanwhile, the inflationary dynamics is determined by the competition between the last two terms of the right hand side. From phenomenological point of view hybrid inflation can be embedded in a supersymmetric GUT theory. But this leads to the inescapable production of cosmic strings which severely constraints the parameters of this model [204, 205]. In the following section we aim to study whether the aspects of the hybrid inflation model is preserved when we treat the same model within a more sophisticated framework.

7.3 Review of KL stabilization

In this section we give an oversimplified discussion on the KL stabilization scheme which will be relevant later. In inflationary context moduli stabilization is a method to constrain the moduli sector in such a way that the gravition mass becomes smaller than the scales involved in the theory. A trick due to this is developed by KKLT [195], where the modulus superpotential and Kähler potential are taken as,

$$W = W_0 + Ae^{-aT}, \qquad K = -3\log(T + \bar{T}). \tag{7.8}$$

This theory provides the SUSY preserving AdS minimum *i.e.* $\mathcal{D}_i W = 0$. But to describe our Universe requires a dS vacuum with a positive and small value of the cosmological constant. This is achieved by adding an uplifting term of the form

$$V_{up} = \frac{C}{(T+\bar{T})^2}. \tag{7.9}$$

In this case, the full potential involves only one scale which is the gravitino mass $(m_{3/2})$. Again the height of the barrier, protecting the moduli from getting destabilized, also depends upon the gravitino mass in the present vacuum and must be larger than the scale of inflation $(H_\star)$. This sets the bound $m_{3/2} > H_\star$ which is at odd in the sense that low scale SUSY breaking compells low scale inflation.

Kallosh and Linde found a solution to this problem by considering an extra exponential term in the KKLT superpotential as [197]

$$W = W_0 + Ae^{-aT} + Be^{-bT}. \tag{7.10}$$

The theory now has two extra parameters to allow us for a metastable SUSY Minkowski vacuum *i.e.* $\mathcal{D}_T W = 0, W = 0$, at which the modulus is stabilized. Also the gravitino mass at the present vacuum can be disentangled from the height of the barrier protecting the stabilized modulus. Thus we get a hierarchy $m_{3/2} << H_\star << m_T$, where m_T denotes the modulus mass. This allows us to have low scale SUSY breaking without requiring high scale inflation.

7.4 Hybrid inflation along with KL stabilization

Let us consider a standard F-term SUSY hybrid model, with minimal Kähler potential for the inflaton field Φ and the waterfall field H, *i.e.*

$$K_{inf} = |\Phi|^2 + |H|^2, \tag{7.11}$$

$$W_{inf} = \kappa\Phi(M^2 - H^2) \tag{7.12}$$

where Φ is a gauge singlet chiral multiplet containing the inflaton field. κ and M are respectively the dimensionless and dimensional parameters of this model. In the simplest KKLT scenario the resulting F-term potential for modulus has one minimum that occurs at large negative values of the effective potential. This leads to a tension between high-scale inflation and low-energy supersymmetry breaking. To stabilize the modulus in a supersymmetric Minkowski minimum the tree level Kähler potential is taken as [197]

$$K_M = -c\ln(T + \bar{T}) \tag{7.13}$$

with $c = 1$ for the dilaton in heterotic string theory and $c = 3$ for a Kähler modulus in type IIB string theory. Also the modulus supertpotential is

$$W_M = W_0 + Ae^{-aT} + Be^{-bT} \tag{7.14}$$

where W_0 is a tree level contribution and A, B, a, b are parameters of the superpotential. Let $\sigma_0 = \text{Re}(T_0)$ denote the minimum of the scalar potential. Now at the minimum the modulus superpotential and its first derivative vanish

$$W_M(\sigma_0) = 0, \qquad \mathcal{D}_T W_M(\sigma_0) = 0, \tag{7.15}$$

with $\mathcal{D}_T$ being the Kähler derivative with respect to the modulus field T. These conditions

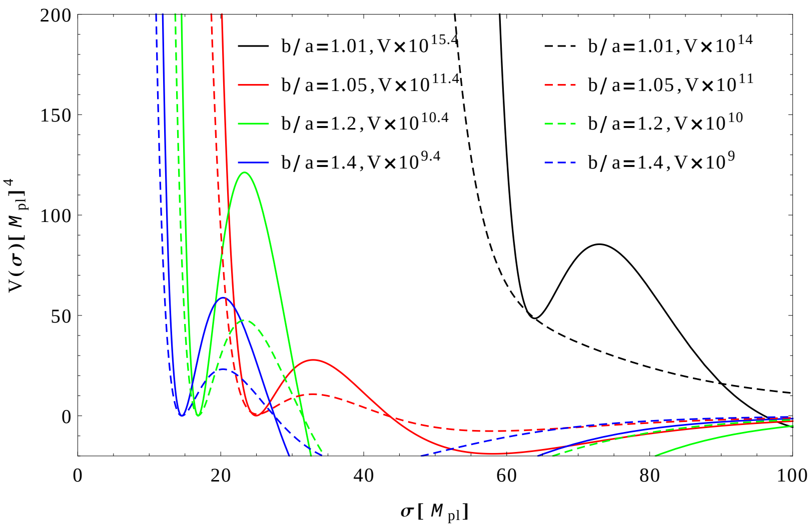


Figure 7.1: Modulus potential during inflation, for fixed value of the inflaton field, for different choices of the ratio b/a. We have taken $A = 1$, $B = 1.03$, and $c = 3$. Solid curves for $\kappa M^2 = 2 \times 10^{-4}$ and dashed curves for $\kappa M^2 = 10^{-3}$

fixes the value of W_0 and σ_0 at

$$\sigma_0 = \frac{1}{a-b} \ln \left| \frac{aA}{bB} \right| \tag{7.16}$$

$$W_0 = -Ae^{-a\sigma_0} - Be^{-b\sigma_0} \tag{7.17}$$

Therefore, the F-term scalar potential is given by

$$\begin{aligned} V = V_0 \frac{e^{K_{inf}}}{(T+\bar{T})^c} \big(1 + 2|\Phi|^2 + |\Phi|^4 + |\Phi^2||W_M|^2 + 2(1+|\Phi|^2) Re\Phi^* W_M \\ + \frac{(T+\bar{T})^2}{c} |W_T|^2 - 2(T+\bar{T}) Re\Phi^* W_T - 2(T+\bar{T}) Re W_M^* W_T - (c-3)|W|^2 \big) \end{aligned} \tag{7.18}$$

where $V_0 = \kappa^2 M^4$, the inflaton and modulus field are given in Planck units, and we have normalized the modulus superpotential and its derivative by $V_0^{1/2}$

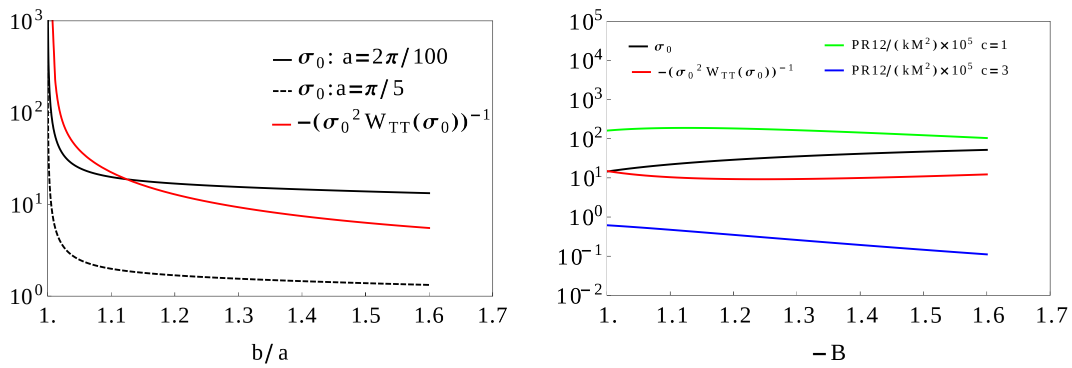


Figure 7.2: Left plot (a): modulus minimum σ_0, for different choices of the ratio b/a. We have taken $A = 1, B = 1.03$. Right plot (b): modulus minimum σ_0, for different choices of the parameter B. Other values parameters as indicated in the plot. Included is also the amplitude of the primordial spectrum, for $c = 1$ and $c = 3$.

$$\hat{W} = W/\kappa M^2, \quad \hat{W_T} = W_T/\kappa M^2 \quad \hat{W_M} = W_M/\kappa M^2 \tag{7.19}$$

Keeping the real components of the field $\phi = \text{Re}\ \Phi$ and $\sigma = \text{Re}\ T$ the potential can be written as

$$V = V_0\frac{e^{K_{inf}}}{(2\sigma)^c}(1 + (c-1)\phi^2 + \phi^4 + A_1(\sigma)\phi + A_2(\sigma)\phi^2 + A_3(\sigma)\phi^3 + B(\sigma)) \tag{7.20}$$

where,

$$A_1(\sigma) = 2((c-2)W_M - 2\sigma W_T) \tag{7.21}$$

$$A_2(\sigma) = W_M^2 \tag{7.22}$$

$$A_3(\sigma) = 2W_M \tag{7.23}$$

$$B(\sigma) = \frac{4\sigma^2}{c}W_T^2 - 4\sigma W_T W_M + (c-3)W_M^2 \tag{7.24}$$

Fig 7.1 shows the plot of the inflationary potential for fixed values of the inflaton field

ϕ, and different choices of the KL superpotential. As a reference we take the values given in [197]: $A = 1, B = 1.03, a = \pi/50, b = 2\pi/99 (b/a \simeq 1.01)$, for which $\sigma_0 \simeq 62.41$ in Planck units. Increasing the ratio b/a we lower the value of σ_0, but still there is a minimum during inflation for lower values of κM^2.

From eqn (7.20) one can see that unless $c = 1$ the presence of a modulus with a non-minimal Kahler potential spoils the cancellation of the quadratic term for the inflaton and reintroduces the η-problem. There is an additional contribution to the quadratic term from the modulus field, the A_2 term, but this term is always positive, and therefore it cannot alleviate the η problem of this model.

During inflation ϕ is a light field and all other fields are stabilized. For this to happen as ϕ rolls through its potential, the modulus field must be settled in its minimum at $\sigma = \sigma_0$. But since modulus is also a dynamical field it will not remain absolutely fixed at a constant value. The minimum of the modulus potential will be slightly shifted away from σ_0. The new minimum will track the value of the inflaton field during inflation. Thus

$$\delta\sigma = \sigma - \sigma_0 \propto \phi \tag{7.25}$$

Therefore, taking the dynamics of the modulus field into account we need to check whether it introduces large correction to the η parameter. To compute the shift in the modulus minimum during inflation we performed a Taylor expansion of the potential in eqn. (7.20) around σ_0 and obtain

$$\frac{dV}{d\sigma} = V_0 \frac{2e^{K_{inf}}}{(2\sigma)^{c+1}} (C_\phi + C_1\phi + C_2\phi^2 + C_3\phi^3 + C_B) \tag{7.26}$$

where, $\quad C_\phi = -cA_\phi = -c((c-1)\phi^2 + \phi^4) - c$

$$C_1 = -cA_1 + \sigma A_1' = 2c(c-2)W_M + 2(3c-4)\sigma W_T 4\sigma^2 W_{TT}, \tag{7.27}$$

$$C_2 = -cA_2 + \sigma A_2' = 2\sigma W_M W_T - cW_M^2, \tag{7.28}$$

$$C_3 = -cA_3 + \sigma A_3' = 2\sigma W_T - 2cW_M, \tag{7.29}$$

$$\begin{aligned} C_B = -cB + \sigma B' = \frac{8(1-c)\sigma^2 W_T^2}{c} + 2(3c-5)\sigma W_M W_T \\ + \frac{8\sigma^3 W_T W_{TT}}{c} - 4\sigma^2 W_{TT} W_M - c(c-3)W_M \end{aligned} \tag{7.30}$$

Now employing the minimalisation condition $dV/d\sigma = 0$ in eqn. (7.26) we obtain the following expression

$$\delta\sigma \simeq D_0 + D_1\phi + D_2\phi^2 + \cdots \tag{7.31}$$

where,

$$D_0 = \frac{c}{C_B'} = \frac{c^2}{8\sigma_0^3 W_{TT}^2}, \tag{7.32}$$

$$D_1 = -\frac{C_1}{C_B'} - \frac{cC_1'}{C_B'^2} = \frac{c}{2\sigma_0 W_{TT}} + \frac{c^3(6-5c)}{32\sigma_0^3 W_{TT}^3} + \frac{c^3 W_{TTT}}{16\sigma_0^4 W_{TT}^4}, \tag{7.33}$$

$$\begin{aligned} D_2 = -\frac{c(c-1)}{C_B'} + \frac{C_1 C_1'}{C_B'^2} + \frac{cC_1'^2}{C_B'^3} = \frac{c^2(5-4c)}{8\sigma_0^3 W_{TT}^3} + \\ \frac{c^4(38-60c+23c^2)}{128\sigma_0^7 W_{TT}^4} + \frac{c^4(6-5c)}{32\sigma_0^6 W_{TT}^5} + \frac{c^2 W_{TTT}}{4\sigma_0^2 W_{TT}^3} + \frac{c^4 W_{TTT}^2}{32\sigma_0^5 W_{TT}^6} \end{aligned} \tag{7.34}$$

Using the above expression for $\delta\sigma$ and expanding the exponential in front of the potential we finally get for the resulting inflaton potential

$$V(\phi) = \frac{V_0}{(2\sigma_0)^c}(A(\sigma_0) + B(\sigma_0)\phi + (C(\sigma_0) + A(\sigma_0)\phi^2 + \cdots) \tag{7.35}$$

where,

$$A(\sigma_0) \simeq 1 - \frac{c^3}{(2\sigma_0)^4 W_{TT}^2} + \cdots, \tag{7.36}$$

$$B(\sigma_0) \simeq - \frac{4c^2}{(2\sigma_0)^2 W_{TT}} + \cdots, \tag{7.37}$$

$$C(\sigma_0) \simeq -1 + \frac{c^3(10c-8)}{(2\sigma_0)^4 W_{TT}^2)} + \cdots \tag{7.38}$$

Thus we can see that when $(\sigma_0 W_{TT})^{-1} << 1$ there is no η problem in the potential function. This result confirms the findings in Buchmuller et.al [206].

7.5 Spectrum

During inflation the energy density is dominated by the constant potential energy density

$$V \simeq \frac{V_0}{(2\sigma_0)^c} \tag{7.39}$$

and the slope of the inflaton potential is dominated by the linear term

$$\frac{dV}{d\phi} \simeq \frac{V_0}{(2\sigma_0)^c} B(\sigma_0) \tag{7.40}$$

Hereform we can estimate the amplitude of the primordial spectrum as

$$P_R^{1/2} \simeq |\frac{H}{\dot{\phi}}| \frac{H}{2\pi} \simeq |\frac{3H^2}{V_\phi}| \frac{H}{2\pi} \simeq \frac{1}{\sqrt{6\pi}|B(\sigma_0)|} \frac{V_0^{1/2}}{(2\sigma_0)^{c/2}} \tag{7.41}$$

now replacing $|B(\sigma_0)| = c^2/\sigma_0^2/W_{TT}$ we obtain

$$P_R^{1/2} \simeq \frac{1}{\sqrt{6\pi}} \frac{\sigma_0^2 W_{TT}(\sigma_0)}{c^2 (2\sigma_0)^{c/2}}, \tag{7.42}$$

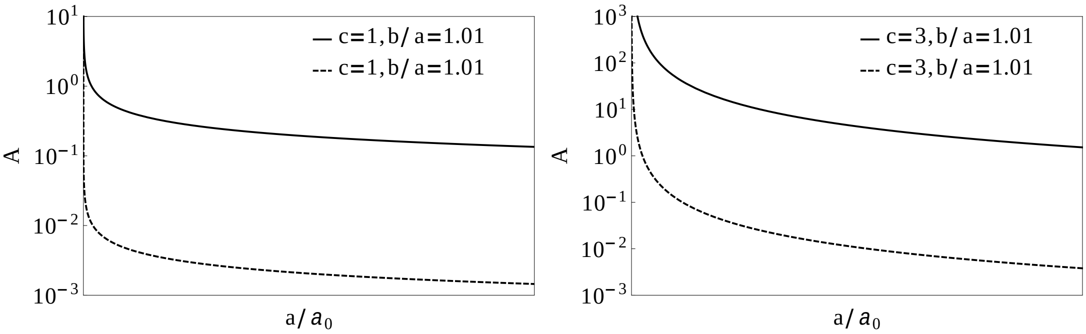


Figure 7.3: Value of the parameter A to match the amplitude of the spectrum (black line) versus a/a_0 . We have taken $\kappa = 1, a_0 = \pi/50, B = 1.03A, b/a = 1.01, 1.4$, and $c = 1$ on the RHS plot, while $c = 3$ on the LHS

where $W_{TT} = a^2 A e^{-a\sigma_0} + b^2 B e^{-b\sigma_0}$. The power spectrum is then suppressed by the value of the modulus at the minimum, which depends on the values of $a, b/a$ and the ratio B/A. And going linear with W_{TT} it depends linearly on the parameter A. Therefore, for a given set of values for the parameters $a, b/a, B/A$ we can always adjust the value of A to match the amplitude of the primordial spectrum.

On the other hand, to get enough number of e-folds, we need to ensure that the system is in the slow-roll regime, *i.e.*, that $\epsilon \sim |\eta| < 1$, where

$$\epsilon \simeq \frac{c^4}{(2\sigma_0^2 W_{TT}/M_{pl})^2}\Big(\frac{\kappa M^2}{M_{pl}^2}\Big)^2 \tag{7.43}$$

$$\eta \simeq \frac{c^3(10c-9)}{4(2\sigma_0^2 W_{TT}/M_{pl})^2}\Big(\frac{\kappa M^2}{M_{pl}^2}\Big)^2 \tag{7.44}$$

A low enough value of the combination κM^2 will ensure that we have enough inflation. To summarize, the amplitude of the spectrum will depend on the parameters of the KL superpotential, but not on the scale V_0, whereas for a given set of parameters of the KL superpotential we can choose the scale V_0 to get enough number of e-folds. In Fig. 7.2(a) we have plotted the value of σ_0 for the KL superpotential, when varying the ratio b/a. In

Fig. 7.2(b) we have plotted the value of σ_0 for the KL superpotential, but when varying the parameter B only. In Fig. 7.3 we have plotted the value of A needed to match the amplitude of the spectrum versus a/a_0 taking $a_0 = \pi/50, B = -1.03A$, and two different values of $b/a = 1.01, 1.4$ respectively.

Spectral index: Although in principle the model works in the sense that it does stabilize the modulus, and we can always find parameters to get the amplitude of the spectrum, it would predict a too close to scale-invariant spectrum. The expression for the spectral index would be given by the standard slow-roll one

$$n_s - 1 = 2\eta - 6\epsilon \; . \tag{7.45}$$

To evaluate this expression we first need the value of the field at $N_e = 50 - 60$ e-folds before the end of inflation. The number of e-folds, assuming that the last $50 - 60$ e-folds happen when the inflaton field is below M_{pl} and therefore the dynamics is controlled by the linear term in the potential, *i.e.*

$$N_e \simeq \left(\frac{\sigma_0^2 W_{TT}/M_{pl}^2}{c^2} \right) \left(\frac{\varphi_N}{M_{pl}} - \frac{M}{M_{pl}} \right) \tag{7.46}$$

where φ_N is the value of the inflaton field N_e e-folds before the end. However to get the η parameter, given by the second derivative of the potential, we have to include higher order terms in the potential upto order $\mathcal{O}(\varphi^4)$

$$V(\varphi) = \frac{V_0}{(2\sigma_0)^c} \left(A_0 + \frac{B_0}{\sqrt{2}}\varphi + \frac{C_0}{2}\varphi^2 + \frac{D_0}{2\sqrt{2}}\varphi^3 + \frac{E_0}{4}\varphi^4 \right) , \tag{7.47}$$

where, $$A_0 \simeq 1 - \frac{c^3}{(2\sigma^0)^4 W_{TT}^2} + \cdots \simeq 1, \tag{7.48}$$

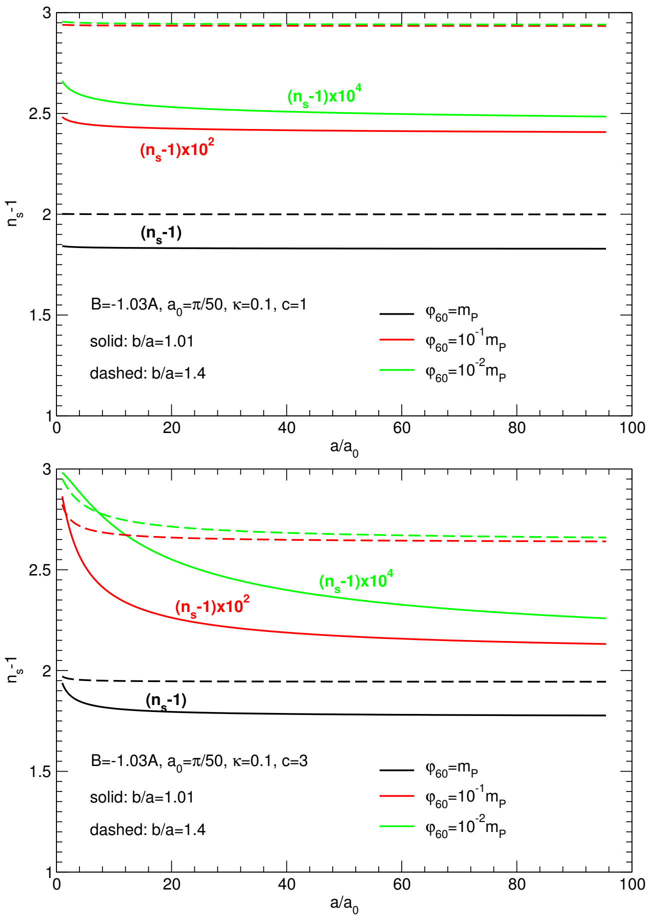


Figure 7.4: Values of $n_s - 1$ versus a/a_0 , for $a_0 = \pi/50, B = -1.03A, \kappa = 0.1$, and different values of the ratio b/a and φ_N as indicated in the plot, with $N_e = 60$. We have taken $c = 1$ (Upper plot) and $c = 3$ (Lower plot).

$$B_0 \simeq -\frac{2c^2}{(2\sigma_0)^2 W_{TT}} + \cdots , \tag{7.49}$$

$$C_0 \simeq \frac{c^3(10c-9)}{(2\sigma^0)^4 W_{TT}^2} - \frac{16c^3\sigma_0^3 W_{TTT}}{(2\sigma^0)^6 W_{TT}^3} + \cdots \sim \mathcal{O}(B_0^2), \tag{7.50}$$

$$D_0 \simeq \frac{3c^2(c-2)}{(2\sigma^0)^2 W_{TT}} - \frac{c^2\sigma_0^3 W_{TTT}}{2(\sigma^0)^4 W_{TT}^2} + \cdots \sim \mathcal{O}(B_0), \tag{7.51}$$

$$E_0 \simeq \frac{1}{2} - \frac{c^3(63-116c+20c^2)}{2(2\sigma^0)^4 W_{TT}^2} + \cdots \simeq \frac{1}{2} \, . \tag{7.52}$$

Therefore,

$$V_{\varphi\varphi} \simeq \frac{V_0}{(2\sigma_0)^c}\left(C_0 + \frac{3D_0}{\sqrt{2}}\varphi + 3E_0\varphi^2 + \cdots\right) \tag{7.53}$$

and given the scaling of the coefficients $C_0 \propto ((\kappa M^2)/A)^2, D_0 \propto (\kappa M^2)/A$, the second derivative is controlled by the sugra correction $E_0 \simeq 1/2$ and therefore

$$\eta \simeq \frac{3}{2}\left(\frac{\varphi_\star}{M_{pl}}\right)^2, \tag{7.54}$$

where $\varphi_\star$ is the value of the inflaton field at horizon crossing. This parameter is larger than ϵ, and therefore we get for the spectral index

$$n_s - 1 \simeq 2\eta \simeq 3\left(\frac{\varphi_\star}{M_{pl}}\right)^2 \tag{7.55}$$

which is blue-tilted, and anyway too small for $\varphi_\star << M_{pl}$ as can be seen in Fig. 7.4. Here we have estimated the spectral index taking into account all the terms in eqn. (7.53) when computing the slow-roll parameters, but only the linear term to get N_e .

The alternative is to have the inflationary dynamics controlled by the one-loop corrections, instead of the modulus field. However, we need to check first that one-loop corrections do not spoiled the stabilization of the modulus in the first place.

One-loop correction: In general, the radiative correction to the potential in a theory for one loop contribution is give by the formula

$$V_{loop} = V_{loop}^{(1st)} + V_{loop}^{(2nd)} \tag{7.56}$$

$$= \frac{1}{32\pi^2}\mathrm{Str}M^2\Lambda^2 + \frac{1}{64\pi^2}\mathrm{Str}M^4 \log\frac{M^2}{\Lambda^2} \tag{7.57}$$

where Λ is the cut-off scale of the theory. The one loop correction to the potential consists of two terms. In the case of standard hybrid inflation the first term of eqn. (7.57) vanishes. The logarithmic contribution provides the slope to the flat tree level potential. In contrast the tree level potential in our case is non-flat due to the interactions with the modulus field. Also, in this case both the terms in eqn. (7.57) are functions of the inflaton field and hence contribute to the bare inflaton potential.

Now the supertrace is $\mathrm{Str}M^2 = \mathrm{Tr}(M_B^2) - \mathrm{Tr}(M_F^2)$. Generally, calculating the supertrace involves explicit computations of the bosons and fermion masses of the theory. For the first term in one-loop potential, finding supertrace is however a bit tricky, as we only require the trace of the mass-squared matrix and not the explicit expression for the fermion masses. This is certainly not the case for the second term. In order to the extract the contribution of the second term we need individual masses. For the fermions, the mass matrix in this case involves too intricate expression to provide the mass eigenvalues. The detail inspection of them will be accomplished later as future work of this study. In the present work we will calculate the first term and focus on its effects. The trace remains invariant whether we are working in $(\Phi, \bar{\Phi})$ or $(\mathrm{Re}\Phi, \mathrm{Im}\Phi)$ basis. In our covension where we stick to the real part of the fields, the supertrace during inflation is given by the following expression,

$$\begin{aligned}\mathrm{Str}M^2 = V_0 \frac{2e^{\phi^2}}{(2\sigma)^c}\Bigg[\Big(1 + (c-2)\phi^2 + \phi^4 + 2\phi(c-3+\phi^2)W_M(\sigma) \\ + (c-4+\phi^2)W_M(\sigma)^2\Big) - 4\frac{(c-1)}{c}\big(\phi + W_M(\sigma)\big)\sigma W_T \\ + 4\frac{(c-1)}{c}\sigma^2 W_T^2\Bigg]\end{aligned} \tag{7.58}$$

Employing this expression into the first term of eqn. (7.57) we get for the effective total potential

$$
\begin{aligned}
V_{eff} &= V_{tree} + V^{(1st)}_{loop} &\quad (7.59)\\
&= V_0 \frac{e^{\phi^2}}{(2\sigma)^c}\Bigg[1 + \frac{1}{16\pi^2} + \left((c-1) + \frac{(c-2)}{16\pi^2}\right)\phi^2 + \left(1 + \frac{1}{16\pi^2}\right)\phi^4 \\
&\qquad + \tilde{A}_1(\sigma) + \tilde{A}_2(\sigma) + \tilde{A}_3(\sigma) + \tilde{B}(\sigma)\Bigg], &\quad (7.60)
\end{aligned}
$$

where,

$$\tilde{A}_1(\sigma) = 2((c-2)W_M - 2\sigma W_T) + \frac{(c-3)}{8\pi^2}W_M + \frac{(c-1)}{4c\pi^2}\sigma \quad (7.61)$$

$$\tilde{A}_2(\sigma) = W_M^2 + \frac{W_M^2}{16\pi^2} \quad (7.62)$$

$$\tilde{A}_3(\sigma) = 2W_M + \frac{W_M}{8\pi^2} \quad (7.63)$$

$$
\begin{aligned}
\tilde{B}(\sigma) &= \frac{4\sigma^2}{c}W_T^2 - 4\sigma W_T W_M + (c-3)W_M^2 + \\
&\frac{(c-4)}{8\pi^2}W_M - \frac{(c-1)}{4c\pi^2}\sigma W_M W_T + \frac{(c-1)}{4c^2\pi^2}\sigma^2 W_T^2 \quad (7.64)
\end{aligned}
$$

where we have used V_{tree} from the earlier eqn. (7.35) and the cut-off Λ is taken to be M_{pl} .

Now once again taking the first derivative of eqn. (7.60) and using the minimalization condition $\frac{dV_{eff}}{d\sigma}|_{\sigma\to\sigma_0} = 0$, we obtain $\delta\sigma \simeq \tilde{D}_0 + \tilde{D}_0\phi + \tilde{D}_0\phi^2 + \cdots$. Using this $\delta\sigma$ and expanding the exponential factor infront of the effective potential we get

$$V_{eff}(\phi) = \frac{V_0}{(2\sigma_0)^c}(\tilde{A}(\sigma_0) + \tilde{B}(\sigma_0)\phi + (\tilde{C}(\sigma_0) + \tilde{A}(\sigma_0))\phi^2 + \cdots) \quad (7.65)$$

where,

$$\tilde{A} \simeq 1 + \frac{1}{16\pi^2} - \frac{c^4(1+16\pi^2)^2}{256\pi^2(c-1+16\pi^2)\sigma_0^4 W_{TT}^2} + \cdots , \quad (7.66)$$

$$\tilde{B} \simeq -\frac{c^2(1+16\pi^2)}{32\pi^2\sigma_0^2 W_{TT}} + \cdots , \quad (7.67)$$

$$\tilde{C} \simeq -1 - \frac{1}{16\pi^2} + \frac{c^3(1+16\pi^2)\big(3 - c(9+64\pi^2) + c^2(5+80\pi^2)\big)}{128\pi^2(c-1+16\pi^2)\sigma_0^4 W_{TT}^2} + \cdots \tag{7.68}$$

So,

$$\tilde{C}(\sigma_0) + \tilde{A}(\sigma_0) \simeq \frac{c^3(1+16\pi^2)\big(6 - c(8+112\pi^2) + c^2(5+80\pi^2)\big)}{256\pi^2(c-1+16\pi^2)\sigma_0^4 W_{TT}^2} + \cdots \tag{7.69}$$

Thus $V_{loop}^{(1st)}$ term does not introduce any η-problem at the leading order. Only it modifies the earlier coefficients in eqn (7.35).

7.6 Summary and future intent

This work mainly aims at exploring the effects of moduli fields during inflationary epoch. In our set-up, since none of the moduli are the inflaton field, we would like to stabilize them with a mass at least of the order of the Hubble scale during inflation. There exists different mechanism to stabilize moduli during and after inflation. However when that procedure is achieved in conjunction with the inflationary sector, the flatness of the inflaton potential can get disrupted. For our purpose we considered here a single modulus field under KL-stabilization scheme [197]. When coupled with the standard F-term hybrid inflation model, the minimum of the modulus potential undergoes small displacement which has now become a function of the inflaton field. To start with, we first neglect the quantum corrections, but account the shift in the potential minimum. We have found that the spectral index in this case is blue-tilted, and thus can be excluded by observations. But at this point the analysis is certainly not complete. To make concrete statements in reference to the inflationary observables, we need to consider the full effects coming from the loop corrected inflaton potential, and require explicit computation of the field dynamics for both the inflaton and also the modulus field. This we have left for future work.

We have made partial progress in calculating the quantum loop corrections. With respect to the masses involved in the theory, the one loop correction to the tree level potential has a quadratic part, and a logarithmic contribution part. In the conventional F-term hybrid inflation without modulus, the loop corrections receive field dependent contributions from the logarithmic term while the quadratic term turns out to be a constant. In this work we have calculated the quadratic term and it is found to be dependant upon the inflaton field due the presence of moduli field. In addition, the potential would have field dependent logarithmic corrections. It is to be seen whether the full loop corrected potential can bring back the spectral index n_s being less than 1, which is otherwise blue-tilted. Additionally, we have to check the full dynamics of both the inflaton and the modulus field, and will have to make sure that the modulus is stabilised all the way when inflation happens.

CHAPTER 8

CONCLUSIONS

Finally we come to the epilogue of our work and here we will summarize the main results obtained from our study.

In this thesis our principle focus has been inflation and its description in conformity with the treatments of an effective field theory. In particular, we have studied a general outlook of different inflationary models in the context of supergravity.

Inflation is a paradigm which has received much of its support from current observations. At present, it is also an well established theory for the early Universe cosmology. Typically, inflation is assumed to have occurred at some very high energy scale and the simplest version of this scenario relies upon a single scalar field. The dynamics of this scalar field can induce acceleration of the scale factor, if the potential energy of the field is flat enough for the time period corresponding to the elimination of cosmological problems. The requirement of this flat potential is a non-trivial consideration when viewed from the effective description of supergravity theory. Usually a supergravity theory is defined in the Einstein frame, where the F-term scalar potential offers large contribution to the inflationary η parameter. This can be overcome by employing symmetries in the Kähler potential of the theory. Proceeding along this line of thought, in Chapter 4 we proposed a simple embedding of inflation model consisting of N sub-Planckian fields with a shift symmetric Kähler potential for each field. Though the fields are of interacting in nature, the key observation here is the reduction of the background dynamics to an effective single field. This model is a generalization of single field chaotic inflation in supergravity.

Another possibility with SUGRA inflation is to take into account the non-minimal coupling to gravity. The feature of non-minimal coupling is a characteristics of the Jordan frame. Recently supergravity models in Jordan frame have drawn much attraction, for they can accommodate a wide variety of scalar potentials with the same unique predictions for the inflationary observables that lie right so far in the most comfortable zone of Planck data. In this thesis (see Chapter 5) we focused on a special class of Jordan frames where the non-minimal function is related to the Kähler potential. For such class of Jodan frames, the F-term scalar potential disentangles into a global SUSY contribution and a SUGRA correction. This splitting makes the study of the effects stemming from SUGRA contribution particularly helpful. We showed that if the inflaton field sources the vacuum energy then the SUGRA corrections are of comparable magnitude to the global SUSY contribution, while they are subdominant or may even vanish for the vacuum energy driven by some auxiliary non-inflaton field. Moreover, the case for vanishing SUGRA corrections is tied to a vanishing superpotential during inflation. In this regard the phenomenological details of some models are also cultivated. This study will be helpfull to understand what sorts of terms in the superpotential/Kähler potential are desirable to weaken the effects of supergravity corrections.

The above studies are toy examples of inflation models in supergravity and does not weigh the aspects of UV completion of the underlying theory. However, as the dynamics of inflation being highly sensitive to the Planck scale physics, a more complete understanding calls for a quantum theory of gravity, for which string theory is believed to be one candidate. Now, in string theory moduli fields are generically abundant and they parametrize the shapes and sizes of the compactified extra dimension. Their presence have huge impact on the inflationary observables in the context of post-inflationary history. There exists two approach to deal with moduli during inflation. Either any of them can be inflaton or they can decouple from the inflationary sector. The latter approach belongs to the study of moduli stabilization during inflation. We did not pursue that option here. Instead in Chapter 6 of the thesis, we considered a single modulus field and this field is stabilized at a different minimum after inflation than that of during inflation. In this case, at the end of inflation when the Hubble constant becomes smaller than the mass of the modulus field, it starts to oscillate around it post-inflationary minimum. Coherent oscillations around this minimum produces cold moduli particles that starts to dominate the energy density of the Universe.

As a result, the Universe passes through an epoch of unusual matter domination before the relevant reheating from the decay of the modulus fields. The relevant inflationary efolds, as a result, became a function of the modulus mass. Treating the mass of the modulus field as a free parameter and assuming a generic value for the modulus displacement, we captured the sensitive of inflationary observables with respect to the modulus mass. The results obtained significantly alters the prediction of some well known models. Moreover, they firmly suggest to study inflation models where computation of the modulus displacement can be made explicit.

However, the modulus, being a dynamical field, its stabilization does not mean it is absolutely fixed during inflation. The assumption of a stabilized modulus during inflation is tempting to conclude that modulus sector can be coupled with the inflationary sector with no mutual interplay between them. In practice this is not so, because, the modulus minimum also shifts from its fixed position and became a function of the inflaton field. Therefore we need to explicitly see that the moduli fields, even if stabilized, do not spoil the flatness of the inflationary potential. An effort has been made in Chapter 7 to study the interplay between the modulus sector along with the hybrid inflationary sector. As a preliminary analysis, the spectral index is found to be blue tilted. However this does not take into account the full effect coming from the one loop correction to the potential and also of the complete field dynamics of the inflaton as well as the modulus. This motivates us to observe further if the spectral index, when subjected to the said consideration, can lie within the best fit of region Planck data. This example will be particularly helpful for gaining a general understanding of how an inflationary theory works in a more realistic framework.

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