STUDY ON SOME ASPECTS OF NON-SUPERSYMMETRIC BRANE SOLUTIONS IN STRING THEORY

By

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I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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List of Publications

Published in Journal

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2. "Decoupling of gravity on non-susy Dp branes", K. Nayek and S. Roy, JHEP 1603, (**2016**) 102 [arXiv:1506.08583]
3. "Decoupling limit and throat geometry of non-susy D3 brane", K. Nayek and S. Roy, Phys.Lett. B766, (**2017**) 192-195 [arXiv:1608.05036]
4. "Anisotropic SD2 brane: accelerating cosmology and Kasner-like space-time from compactification", K. Nayek and S. Roy, European Physical Journal C 77 (**2017**) 462 [arXiv:1701.09158]

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2. "Gauge/Gravity Duality in Non-Susy Solutions of Type II String Theory", on 25th July, 2017 in the departmental seminar of the Department of Physics, Indian Institute of Technology Kharagpur.

KUNTAL NAYEK

To my family…
and my elder brother Late Uday Nayek…

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List of Abbreviations

AdS	**A**nti-**d**e **S**itter
BPS	**B**ogomol'nyi-**P**rasad-**S**ommerfield
CFT	**C**onformal **F**ield **T**heory
D	**D**irichlet
dS	**d**e **S**itter
KG	**K**lein **G**ordon
NS	**N**eveu-**S**chwarz
RR	**R**amond-**R**amond
SuGra	**Su**per**Gr**avity
SuSy	**Su**per-**Sy**mmetry
SYM	**S**uper **Y**ang-**M**ills
YM	**Y**ang-**M**ills

Synopsis

Superstring theory in the low energy limit is well-known to admit certain Bogomolnyi-Prasad-Sommerfield (BPS) p-brane solutions (both extremal as well as black) which are the higher dimensional analog of Schwarzschild or Riessner-Nordstrom solution in general theory of relativity. p-branes are spatially extended objects, i.e., when $p = 0$ it represents a point, when $p = 1$, it represents a string, when $p = 2$ it represents a membrane and so on and for general p it represents a spatially extended p-dimensional object. Therefore, a p-brane has a $(p+1)$-dimensional world-volume and depending on the boundary conditions p-branes can be of NSNS type or RR type. RR p-branes are called Dirichlet p-brane or Dp brane for short and we will be mainly interested in these Dp branes. In recent years these BPS Dp branes as well as their bound states have been found to play crucial roles both in the microscopic interpretation of Bekenstein-Hawking entropy in certain string theory black holes and also in the AdS/CFT correspondence or in general a gauge/gravity duality also known as holography where string theory in certain gravity background is related via strong-weak duality symmetry to a large N gauge theory (without gravity) in one less dimension. This latter duality has been proved to be very useful to extract information about the strongly coupled field theory using the weakly coupled string theory or supergravity.

It is also well-known by now that low energy superstring theory admits apart from these BPS Dp branes, their non-supersymmetric cousins known as non-susy Dp branes. Under certain double scaling limit these non-susy Dp branes can be shown to reduce to the standard BPS Dp branes. Certain aspects of these non-susy Dp branes have been studied quite extensively in the past, however, it is not quite known whether an AdS/CFT type correspondences exist for these non-susy solutions. Since nature as we know is non-supersymmetric in the low energy, so, it will be very useful if a holographic correspondence holds for these solutions. We have taken a first step in this direction in this thesis. Moreover, because the non-susy solutions have more adjustable parameters, these non-susy Dp branes can be converted into real cosmological solutions under a double Wick rotation. This is not possible for the BPS Dp branes as there are no adjustable parameters. In this thesis we have also studied some of these cosmological solutions.

In the first part of the thesis we have studied the decoupling of gravity from the non-susy Dp branes. For this we have studied the minimally coupled scalar as well as the graviton scattering in the background of non-susy Dp branes. We find that the dynamics of the scalar and also the graviton are given by a Schrödinger-like equation with certain potential. We also find that because of the complicated form of the potential, it is not easy to solve the scattering equation, in general, analytically. However, we study the potential numerically. We find that as the scalar or the graviton coming from infinity approaches the non-susy Dp brane the potential rises sharply in the low energy limit and eventually becomes infinite near the location of the brane and thus showing that they will never be able to reach the brane. This gives a clear indication that bulk gravity indeed decouples from the non-susy Dp branes. To support our claim we solve the scattering equation analytically in a special case and obtain the form of the graviton absorption cross-section. We find that the graviton absorption cross-section vanishes in the low energy or decoupling limit. We further study the decoupling limit in detail for non-susy D3 brane and obtain its throat geometry. We observe that this throat geometry has also been studied before by Constable and Myers and also by Csaki and Reece. However, how it came from the decoupling limit of a non-susy D3 brane was not clear at that time. Since the geometry is obtained from a decoupling limit of non-susy D3-brane, by gauge/gravity duality it must correspond to a non-susy gauge theory. Indeed it has been shown before that this gauge theory is both non-conformal and non-supersymmetric. It has a running coupling constant and shows the confinement property in certain range of its parameters, like QCD. We have taken a step further and by introducing a fundamental string probe we have obtained the screening length and the velocity dependent quark-antiquark potential in this theory. We find that other than certain details they have a very similar behavior as those obtained from a supersymmetric theory, thus showing that these features are quite robust and does not really depend on the supersymmetry of the underlying theory.

In the second part of this thesis we have studied certain cosmological solutions in string theory. As we mentioned non-susy Dp branes can be doubly Wick rotated to get some cosmological solutions. These solutions are termed as space-like Dp branes or SDp branes and have $(p+1)$-dimensional Euclidean "world-volume". We have performed hyperbolic space compactifications of these SDp branes and by writing the resulting space-time in the Einstein frame we obtain a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric. We have shown how this metric leads to an accelerating cosmology in any dimensions starting from (2+1) upto (7+1). These accelerations are transient with e-folding of the order of 1. We have also shown how the accelerations change with the variations of various parameters of the theory. At early times, we find that the resultant space-time can be cast into a (p+1)+1 dimensional de Sitter space upto a conformal transformation. Here we get a decelerating cosmology, but only in a particular conformal frame we get eternal acceleration. We have also studied cosmology resulting from certain anisotropic SD2 brane solution of string theory upon six dimensional hyperbolic space compactification. Here we find that again by writing the resulting four dimensional space-time in the Einstein frame leads to four dimensional metric of flat FLRW-like form, with a different scale factor associated with different spatial directions. This space-time again leads to transient accelerating cosmology, but here both the expansions and the accelerations are different for different directions. However, at early times we find that compactifications on six dimensional hyperbolic space leads to a Kasner-like geometry which can give expansions in all the spatial directions unlike in the standard Kasner cosmology obtained from vacuum Einstein equation, where expansions in all directions are not possible.

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Chapter 1

Introduction

In Physics, it is little hard to find such events where the experimental data fits exactly with theoretical formula. But a magic happened in 1900 when Max Planck wrote down his revolutionary formula for black body radiation. Generally, the fundamentals of theory match with experimental data by more or less the series of precise trial-and-error calculations. The *Planck's radiation law* was an exception, luckily, it explained the experimental data exactly and that formulation led to the quantum theory. The theory gave the idea of wave-particle duality.

Now in 1960s, one of the challenges in strong interaction physics was to explain the *Regge trajectory*. For particles like hadrons and mesons, the square of mass of the particle is directly proportional to the angular momentum of the particle i.e. $M^2 = J/\alpha'$, where $\alpha' \approx 0.9(\text{GeV})^{-2}$ is the constant in Particle Physics, called *Regge slope*. This relation indicates the linear variation of mass square with angular momentum. At that time, while *the quark model* of particle was not entirely known, the Quantum Field Theory was unable to explain that trajectory. Now instead of zero dimensional point, if we assume that the fundamental objects are string-like, then it can give a consistent derivation of the Regge trajectory. Another incompleteness of high energy physics was found in the calculation of the scattering amplitude (pion scattering). In that scattering process, one could obtain the total amplitude as the sum over the contributions from only *s-channel* poles or as the sum over the contributions from only *t-channel* poles. This was a quite contradiction to the field theory where we need to sum over the contributions from *both s- and t-channel* poles to get the total amplitude. Around the period 1968, various high energy experiments in SLAC demanded a new sort of theory of strong interaction. At that time, the Italian physicist Gabriele Veneziano proposed a *duality* model between the *s-channel* and *t-channel* tree level amplitudes, known as *Veneziano amplitude*. The rich structure of this model attracted the attentions of a large group of physicists which yielded many surprises in modern theoretical physics. In fact this beautiful model was the origin of *the String theory*. The Veneziano model was itself a model of relativistic string which gave the idea of the spin-2 graviton as a part of the closed string's spectrum. The model was accepted at that time and gave a consistent description of the Regge trajectories. On other hand, the high energy fixed angle (centre-of-mass scattering angle) scattering was understood in terms of the *parton model* (proposed by Richard Feynman in 1969) and the SLAC's experiments also verified the existence of the parton in high energy strong interactions. The Veneziano model was unable to describe those partons. This failure spoiled the initial motivation for Veneziano amplitude.

In around 1974, the *quark-gluon* model along with the concept of *color charges* came in front as *the Quantum Chromodynamics* (QCD) to describe the high energy experimental data. Then the Veneziano model was of no use. At the same time, the quantization of gravitational interaction was a blooming topic. The massless gravitational field has spin-2. Its quantized form is called the *graviton*, a massless particle of spin-2. The non-linear interactions in quantum gravity can be explained by a local symmetry group, *the group of diffeomorphisms of space-time*. An analogous group is also used to explain the non-linearities in the *Yang-Mills' gauge theory*. In spite of such similarity, the renormalizabilities of the scattering amplitudes of the spin-1 and spin-2 particles hugely differ in four dimensions.

The closed-string solutions of the Veneziano model mentioned before has some massless spin-2 excitations. Do those excitations represent the graviton in quantum field theory? To search a proper way to answer it, a group of physicists like John Schwarz, Edward Witten, Michael Green started to look at the theory of quantum gravity with a new assumption. They assumed

all the fundamental constituents of matter are one dimensional string-like object.

It gives a new avenue to solve the renormalizability problem of the quantized gravitational field theory. But such an assumption is valid in the Planck's scale $\sqrt{\hbar c/G} = 10^{19}$GeV i.e. 10^{16}TeV, which is very far from the energy scale achieved in various accelerators. In fact, this large energy scale depicts a picture of the moment just after the *Big Bang*. So it is very hard to think of an experimental verification of such theory.

Basic string theory

The most exciting proposal of theoretical physics in the 1960's was the concept of string theory. Like the zero dimensional point particle of known quantum field theory, string theory describes the quantum theory of a one dimensional fundamental object (known as string). There are two types of fundamental strings in this theory – the open-string (two end points are free) and the closed-string (two end points are connected with each other). As a point particle moves in $(3+1)$ dimensional space-time, it draws an one dimensional the world-line. Similarly, when a open-string (closed-string) moves in some d-dimensional space- time, it sweeps out a $(1+1)$ dimensional world-sheet of structure $\mathbb{R}^1 \otimes \mathbb{R}^1$ ($\mathbb{R}^1 \otimes \mathbb{S}^1$). The various string interactions are controlled by the energy per unit length of a string, i.e. the string tension T, the string length ℓ_s and the dimensionless coupling constant g_s, where

$$T = \frac{1}{2\pi\alpha'} \quad \text{and} \quad \sqrt{\alpha'} = \ell_s \tag{1.1}$$

The fundamental string length ℓ_s is of the order of Planck's length $\ell_p = 1.6 \times 10^{-35}\,\mathrm{m}$ *i.e.* $\sim 10^{-20}$*fm*. Initially it was found that the string theory is consistent only in 26 dimensions. The theory included only the bosonic excitations. It was named as bosonic string theory. But the bosonic string theory has some flaws. It is unstable due the presence of the tachyon (a state with negative mass-square). Also the extra twenty two dimensions has to be compactified to get back our real four dimensional world. In real world, there are both the bosons and the fermions. Imposing supersymmetry between them five consistent superstring theories in $(9+1)$ dimensions were constructed. Superstring theories do not have tachyons in their spectrum and are, therefore, stable. The five superstring theories are: (i) Type II A, (ii) Type II B, (iii) Type I, (iv) Heterotic $SO(32)$ and (v) Heterotic $E_8 \times E_8$.

Assuming supersymmetry on the two dimensional string world-sheet, we can write the action corresponding to an open superstring as

$$S = -\frac{1}{2\pi} \int d^2\sigma \left(\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right) \tag{1.2}$$

where, $\psi^\mu = \begin{pmatrix} \psi^\mu_- \\ \psi^\mu_+ \end{pmatrix}$ are the world-sheet spinors. For sake of convention, ψ_- is considered as left-moving spinor and ψ_+ is considered as right-moving one. The X^μ is the space-time coordinate. X^μ satisfies the massless Klein Gordon equation, as in case of bosonic string theory. Here we are interested in the fermionic part. It can be derived that the open-superstring's end points satisfy $\psi_-\delta\psi_- - \psi_+\delta\psi_+ = 0$. Now choosing $\psi_+ = \pm\psi_-$ the above equality can be satisfied. Without loss of generality one can fix

$$\psi^\mu_+(0,\tau) = \psi^\mu_-(0,\tau) \tag{1.3}$$

Now the relative sign between two mode at other end can be taken in two ways:
(i) The Ramond (R) condition $\psi^\mu_+(\pi,\tau) = \psi^\mu_-(\pi,\tau)$, and
(ii) The Neveu-Schwarz (NS) condition $\psi^\mu_+(\pi,\tau) = -\psi^\mu_-(\pi,\tau)$.
Here we see the left-moving and right-moving modes are related to each other. Thus we have only one free fermionic mode in open-superstring. In Ramond sector the fermion modes are periodic, but they are anti-periodic in NS sector.

Now in closed superstring, the total derivative terms in the variation of world-sheet action vanish when each of the ψ_- and ψ_+ is periodic (R) or anti-periodic (NS) separately. According to the type of boundary conditions imposed on fermionic modes, four classes of closed superstring states are possible :

- R-R : where both ψ_- and ψ_+ are periodic.
- NS-NS : where both ψ_- and ψ_+ are anti-periodic.
- NS-R : where ψ_- is anti-periodic and ψ_+ is periodic.
- R-NS : where ψ_- is periodic and ψ_+ is anti-periodic.

The first two combinations give the bosons where the last two include fermions.

In ten dimensional space-time supersymmetry, the fermionic coordinates θ^A ($A = 1, 2 \cdots \mathcal{N}$) are taken to be the Majorana-Weyl spinors. For $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superstring formulation we have one and two θ respectively. Here we start with $\mathcal{N} = 2$. Now θ has two components θ^1 and θ^2. Majorana-Weyl condition ensures the definite handedness on θ^1 and θ^2. We have two options – either both θ^1 and θ^2 have same handedness or they have mutually opposite handedness. In the superstring theory with open superstring, these two spinors have to have same handedness at the two ends of the open superstring. Thus space-time supersymmetry reduces to $\mathcal{N} = 1$. This type of superstring theory is named as 'type I'. The open superstring excitations give a gauge field, which defines the Yang-Mills group theory. The consistency with quantum theory of such open superstring gives an unique choice of $SO(32)$ gauge group in this theory. Interacting open string also produces closed strings. The handedness of the θ in such closed string is same as the handedness relations implemented already in open string. So the type I superstring theory includes the open superstrings and also a special class of closed superstrings. The superstring theory based on only closed superstrings has independent θ^1 and θ^2. In type IIA theory two theta have opposite handedness i.e. θ^1 is periodic and θ^2 is anti-periodic or vice verse. The two conserved supercharge have opposite chirality. This theory is found to be left-right symmetric or non-chiral. The another possible choice of the handedness gives type IIB superstring theory. Here the left-moving and right-moving have same handedness i.e. both the θ^1 and θ^2 are periodic or both are anti-periodic. Now we have two space-time supersymmetries with the same handedness. This is a chiral theory. There are also some consistent superstring theories constructed with a single θ spinor, known as heterotic string theory. The heterotic theories are actually the combinations of 10 dimensional superstrings and 26 dimensional bosonic strings. According to symmetry groups there are two heterotic theories in ten dimensional superstring theory – heterotic

$SO(32)$ (in short HO) and heterotic $E_8 \times E_8$ (in short HE). Both the HO and HE have ten dimensional $\mathcal{N} = 1$ space-time supersymmetry.

In type II theories, the bosonic sector includes a rank-two metric field $g_{\mu\nu}$ (symmetric), a scalar *dilaton* field ϕ, a rank two tensor field $B_{\mu\nu}$ (antisymmetric) of NS-NS sector and a $(p+1)$ form-field A_{p+1} of R-R sector. Here $\mu,\ \nu = 0, 1, \cdots 9$ and p is even in type II A and odd in type II B theory. The NS-R and R-NS sectors are fermionic sector of space-time fermions. The type II theories have $\mathcal{N} = 2$ supersymmetry with 32 component spinors.

D-branes

Dynamics of open strings has some beautiful features in string theory. It is because of the two free end points. Suppose the temporal and spatial coordinates of the string world-sheet are parameterized by τ and σ. The string is evolving from the initial configuration at $\tau = \tau_i$ to the final configuration at $\tau = \tau_f$. The spatial length of the string is π, i.e. $\sigma \in [0, \pi]$. The open string dynamics is governed by the Polyakov action

$$\mathcal{S} = -\frac{1}{4\pi\alpha'} \int d^2\sigma\, \partial_\alpha X^\mu(\tau, \sigma) \cdot \partial^\alpha X_\mu(\tau, \sigma) \tag{1.4}$$

where, $\alpha = 0, 1$. $X^\mu(\tau, \sigma)$ is the location of (τ, σ) point of the world-sheet in the $(9+1)$ dimensional space-time i.e. $\mu = 0, 1, \cdots 9$. Or, mathematically, (τ, σ) point of world-sheet is mapped to X^μ point of the space-time through the function $X^\mu(\tau, \sigma)$. The extremality condition on the above action gives the equation of motion.

$$\begin{aligned} \delta\mathcal{S} &= -\frac{1}{2\pi\alpha'} \int d^2\sigma\, \partial_\alpha X \cdot \partial^\alpha \delta X \\ &= \frac{1}{2\pi\alpha'} \int d^2\sigma\ [\partial^\alpha \partial_\alpha X \cdot \delta X - \partial^\alpha(\partial_\alpha X \cdot \delta X)] \end{aligned}$$

The equation of motion of X^μ is simply the massless KG equation, $\Box X^\mu = 0$ i.e. $\partial^a \partial_a X^\mu = 0$. So, the second term of the variation remains as

$$\begin{aligned} \delta \mathcal{S} &= \frac{1}{2\pi\alpha'} \left[\int_0^\pi d\sigma \int_{\tau_i}^{\tau_f} d\tau \, \partial_\tau (\partial_\tau X \cdot \delta X) - \int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma \, \partial_\sigma (\partial_\sigma X \cdot \delta X) \right] \\ &= \frac{1}{2\pi\alpha'} \left[\int_0^\pi d\sigma (\partial_\tau X \cdot \delta X) \Big|_{\tau=\tau_i}^{\tau_f} - \int_{\tau_i}^{\tau_f} d\tau (\partial_\sigma X \cdot \delta X) \Big|_{\sigma=0}^{\pi} \right] \end{aligned}$$

Now the first term is zero as δX^μ explicitly goes to zero in the both initial and final states of propagation. So we need to impose some suitable conditions such that the second term also vanishes to keep the action unchanged.

$$(\partial_\sigma X \cdot \delta X) \Big|_{\sigma=0}^{\pi} = 0 \tag{1.5}$$

This condition can be achieved in two ways.

- Either $\partial_\sigma X^\mu = 0$ at the two end points of the string. It means that the derivatives of the field vanish at the end points. This is called *Neumann boundary condition*.
- Or $\delta X^\mu = 0$ at the end points of the string. Here the field is constrained to a constant value. This constant can even be zero. It is known as *Dirichlet boundary condition*.

In superstring theory suppose an open string moves with Dirichlet condition on some of its coordinates and Neumann condition on others.

$$\partial_\sigma X^a = 0, \quad \text{for } a = 0, 1, \dots p$$

and

$$\delta X^m = 0, \quad \text{for } m = p+1,\, p+2 \dots 9$$

These conditions fix the two ends of the string to lie on a $(p+1)$-dimensional surface and the surface is localized by $(9-p)$ constant coordinates. Implementation of such conditions breaks the space-time Lorentz group $SO(1,9)$ into,

$$SO(1,p) \times SO(9-p)$$

This hypersurface is called a *D-brane*. Here D stands for Dirichlet. Initially the Dirichlet conditions on a string were considered to be trivial. Polchinski first introduced this concept of brane in around mid-1990. When the hypersurface has p spatial dimensions, it is called a *Dp-brane*. So, a D0-brane is a particle; a D1-brane is an one dimensional D-string; a D2-brane a membrane and so on. In gravitational system, as there is no object at rest, the D-branes also move in space-time. So the dynamics of D-brane is an essential part to study. Brane dynamics is governed by the action which is a higher dimensional extension of the Nambu-Goto action. Dirac first studied this type of action for two dimensional membrane. Because of that the action is generally called Dirac action, which is expressed as

$$\mathcal{S}_{Dp} = -T_p \int d^{p+1}\zeta \sqrt{-\text{det}\gamma_{ab}} \tag{1.6}$$

where T_{D_p} is tension or energy density of the p-dimensional hypersurface. As Dp brane moves trough the space-time it makes an hyper-volume in space-time, known as the *worldvolume*. γ_{ab} is the pull back of the background metric on the wordvolume of the Dp brane. So, evolving with time, a Dp brane makes a $(p+1)$ dimensional worldvolume with p spatial directions and one time direction, the remaining $(9-p)$ coordinates are transverse to the brane. In some situations, all of these $(p+1)$ coordinates of the brane worldvolume are spatial while time coordinate belongs to the transverse coordinates. Such brane is called space-like Dp brane or SDp brane in short. As p determines the dimension of the higher dimensional brane, different p values generate different gravity theories of the space-time. The size of the D brane is not always infinite. It can also be finite, which is called the finite D brane. The general convention is that our present Universe is actually a D3 brane. Its $(3 + 1)$ worldvolume is the currently observed space-time. The energy of Universe is too small to unfold other six transverse dimensions of it. As $p = -1$, the worldvolume is zero dimensional, i.e. a point in space-time. In this case, Dirichlet boundary condition is applied also in time direction that means the time is fixed to a constant value. Such D(-1) brane solutions are called the *D-instanton*. Instantons are very unstable solutions. On other hand, for $p = 9$, the worldvolume is $(9+1)$ dimensional i.e. the dimension of the space-time in superstring theory. Here no direction satisfies the Dirichlet BC, the end points are free to move in all directions. The brane is existing at each and every points of the space-time. Because of this, D9 branes are called the *space-filling brane*. As there is no Dirichlet conditions in case of space-filling brane, the open string can exist inside the brane.

Dualities in String theory

In general, *duality* gives an equivalence between two apparently different theories. Such two theories are called dual to each other. In some cases, this concept of duality offers a simpler description of a complicated system. The physics on those two dual theories or systems are found to be same. In superstring theory there are various kinds of dualities. Among them, the *T-duality* is the simplest and we discuss it first in this section. The keypoint of T-duality is the compactification of a spatial dimension of the space-time on a circle. Say, a compactified dimension has length $2\pi R$, where R is the radius of that circle. According to the T-duality, the underlying physics is same for the two theories one compactified with radius R and the other compactified with radius α'/R. When T-duality acts on a direction parallel or perpendicular to a D-brane, the resultant brane becomes one dimensional lower/higher than the original brane. In fact T-duality is the exchange of the Neumann and Dirichlet boundary conditions along the direction of compactification. Here, in superstring theory, type IIA and IIB theories are related with each other under T-duality. To clarify this statement, suppose we compactify both the theories on a circle. Then one theory with large radius of compactification is identical to another theory with small radius.

Another duality in superstring theory is *S-duality*. Interaction strength in string theory is measured by the string coupling g_s. The string coupling constant is the vacuum expectation value of e^{ϕ}, where ϕ is the scalar dilaton field. In simple terms, the S-duality is about the duality between two theories under the inversion of the string coupling $g_s \rightarrow g_s^{-1}$. It is equivalent to $\phi \rightarrow -\phi$. This duality under the inversion of the coupling constant g_s is called the S-duality. S-duality indicates the correspondence between the strongly coupled theory and the weakly coupled theory. When two theories are S-dual to each other, the perturbative study of the weakly coupled theory gives the equivalent result of the strongly coupled theory. Under S-duality, some superstring theories are related to another superstring theories. The type I theory is S-dual to the heterotic $SO(32)$ theory. The type IIA theory under strong coupling gives the eleven dimensional M-theory where the dilaton field behaves as the eleventh dimension. This duality maps the type IIB theory into itself. As we mentioned type IIA and type IIB are also equivalent under T-duality with one dimension compactified on S^1. Thus, with multiple operations of S and T dualities it is found that all of those five superstring theories are related to each other. The unification among all five superstring string theories is also an important milestone in the development of string theory.

AdS/CFT correspondence

The AdS/CFT correspondence [2–5] relates a quantum field theory (QFT) with a gravitational theory. The idea of AdS/CFT comes from superstring theory. The original AdS/CFT duality was proposed by Maldacena in 1997. It claims the correspondence between the $(1+3)$ dimensional $\mathcal{N}=4$ super-Yang-Mills theory and the $(1+4)$ dimensional gravitational theory in anti-de Sitter geometry [2, 5]. In low energy limit, as the fundamental string length ($\sqrt{\alpha'}=\ell_s$) becomes sufficiently small the space-time of N no. of coincident BPS D3 brane reduces to a product space $AdS_5\times S^5$. The $(1+4)$ AdS spacetime has an isometry $SO(4,2)$. On the other hand, the $\mathcal{N}=4$, $SU(N)$ supersymmetric Yang-Mills theory (SYM) lives on the boundary of that AdS also has $SO(4,2)$ conformal symmetry. These two theories are equivalent to each other in the large N according to the AdS/CFT conjecture. It is also a holographic duality [6, 7] similar to an optical hologram which encodes the information of a three dimensional object in a two dimensional image. Here the gravity theory lives in a bulk, where as the gauge theory lives on the boundary of that bulk. So, naturally the gauge theory is one dimensional lower than the gravity theory. AdS/CFT duality is also a strong/weak coupling duality. If the gravity theory is weakly coupled then its corresponding gauge theory must be strongly coupled and vice-versa. One can get information of a strongly coupled gauge theory by studying the corresponding weakly coupled gravity theory. This is a very useful tool to deal with strongly coupled theory.

In the presence of a D brane, the space-time contains both the open and closed string excitations. Consider an N numbers of coincident D3 branes in the 10-dimensional Minkowski space-time. The total space-time Lagrangian contains three parts, (i) the theory on the brane – the open string excitations, (ii) the theory in the bulk – the closed string excitations and (iii) the interaction theory between the two – mutual interactions between open and closed strings. In the low energy limit ($\ell_s \to 0$), only the massless states get excited. The massless open string excitations are $\mathcal{N} = 4$ vector super-multiplet. Thus the D3 brane world volume theory is mainly the $\mathcal{N} = 4,\ U(N)$ super-Yang-Mills theory living in $(1+3)$ dimensions. In the bulk, the low energy massless closed string excitations introduce gravity super-multiplet in ten dimensions. Finally all of these massless modes give an effective action in low energy limit.

$$\mathcal{S}_{total} = \mathcal{S}_{brane} + \mathcal{S}_{bulk} + \mathcal{S}_{int} \tag{1.7}$$

$\mathcal{S}_{brane}$ is the effective action in the brane world-volume. It is the worldvolume integration of the $\mathcal{N} = 4$ super-Yang-Mills Lagrangian plus some higher order derivatives of the gauge field (known as the correction terms). For example the corrections are like $\alpha'^2\text{Tr}(F^2)$. $\mathcal{S}_{bulk}$ is the effective supergravity action in ten dimensions. It also includes higher derivative terms $\mathcal{O}(\alpha'^2)$. The interactions between brane and bulk modes are included in $\mathcal{S}_{int}$, proportional to ten dimensional Newton's constant $G_N \sim \kappa_{10}^2$.

In the presence of the D branes, the massless modes of closed string (including graviton) get excited. This results a perturbation in the background Minkowski metric, $\eta_{\mu\nu} \to g_{\mu\nu} = \eta_{\mu\nu} + \kappa_{10}h_{\mu\nu}$. $\kappa_{10} = g_s\alpha'^2$ is the Newton's constant in ten dimensions. This gravitational excitation interacts with the whole space-time. It effects on both the bulk and brane theories. Let's look at the expansion of bulk action.

$$\mathcal{S}_{bulk} \sim \frac{1}{2\kappa_{10}^2}\int \sqrt{-g}\mathcal{R} \sim \int (\partial h)^2 + \kappa_{10}(\partial h)^2 h + \cdots \tag{1.8}$$

The first term of above expansion gives the gravity theory in the bulk, where the second term introduces the coupling between the bulk and brane. In the low energy limit, as

$\kappa_{10} \to 0$ in low energy limit, the interaction term vanishes. Again, after the covariantization of brane action the leading quadratic term is

$$\mathcal{S}_{brane} \sim \frac{1}{g_{YM}^2} \int \sqrt{-g} d^4x g_{ab} g_{cd} F^{ac} F^{bd} \tag{1.9}$$

$$\sim \frac{1}{g_{YM}^2} \int d^4x F_{ab} F^{ab} + \frac{1}{g_{YM}^2} \int d^4x (\kappa_{10} \eta_{ab} h_{cd} F^{ac} F^{bd} + \mathcal{O}(\kappa_{10}^2 h^2)) \tag{1.10}$$

where g_{YM}^2 is the Yang-Mills gauge coupling. The first term is purely the gauge field's contribution, but in the remaining higher order terms the gauge field couples with the metric perturbation. In low energy limit, only the first term survives where all the interaction terms vanish. Thus the low energy limit where these interactions vanish, is known as *decoupling limit*. In the decoupling limit, we get two decoupled theories – $(1+3)$-dimensional $\mathcal{N} = 4$ SYM theory on the D3 brane and the $(1+9)$-dimensional type IIB supergravity theory in the bulk.

Now look at the bulk geometry in low energy limit. Previously D branes are discussed. BPS D3 branes act as the source for the various supergravity fields. In the presence of BPS D3 brane, the ten dimensional supergravity background is given as

$$\begin{aligned} ds^2 &= f^{-1/2} \left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + f^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right) \\ F_5 &= (1 + *) dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge df^{-1} \\ f &= 1 + \frac{R^4}{r^4}, \;\; R^4 = 4\pi g_s \alpha'^2 N \end{aligned} \tag{1.11}$$

Since g_{tt} depends on the radial distance from the brane, the energy E of a particle measured by an observer sitting at infinity and the energy E_p of the same particle measured by an observer located at finite distance are related by the redshift factor

$$E = \sqrt{g_{tt}}E_p = f^{-1/4}E_p, \tag{1.12}$$

This means that the energy of an object measured by the observer at infinity becomes lower and lower if the object moves towards the brane. Now in low energy limit, the background given in (1.11) contains two kinds of low energy excitations (as observed by an observer at infinity). One is the massless excitations propagating in the bulk with large wavelength (i.e. small energy) and the other is the excitations which are being brought closer and closer to the brane. In this limit, these two types of excitation decouple from each other at the near horizon region (here, $r = 0$) because the absorption cross section in this region varies with energy as $\sigma \sim \omega^3 R^8$ (ω is the energy of the excitations)[]. The absorption becomes trivially small. The bulk excitations moving towards the brane face an infinite potential barrier. Thus the excitations are unable to reach the brane. On the other hand the low energy excitations that live in the near horizon region can not climb the infinite potential barrier to escape to the asymptotic bulk region. As a result, we have two decoupled regions – the ten dimensional supergravity in the bulk and the near horizon geometry. In the near horizon region $(r \ll R)$, $f \sim \frac{R^4}{r^4}$,

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + R^2\frac{dr^2}{r^2} + R^2 d\Omega_5^2 \tag{1.13}$$

So in this low energy limit, the ten dimensional space-time near the horizon has the form $AdS_5 \times S^5$.

In both the above cases, i.e., the points of view of the brane theory and the bulk geometry, we have arrived at two decoupled theories. The ten dimensional bulk supergravity is common in both the cases. Hence one can easily conclude that the other two theories are equivalent to each other.

$\mathcal{N} = 4$ super-Yang-Mills theory in $(1+3)$ dimensions $\Longleftrightarrow$ type IIB super-string theory on $AdS_5 \times S^5$

To be more precise about the decoupling limit, let us take $\alpha'(= \ell_s^2) \to 0$ i.e. the low energy limit. We want to consider arbitrary excited string states in the throat (the near-horizon region). The string excitations have to have the fixed energies in string unit in this region, i.e., $\sqrt{\alpha'}E_p$ is fixed. In this limit, (1.12) gives $E \sim \frac{r}{\sqrt{\alpha'}}E_p$. So, E remains fixed only if r/α' remains fixed. It means that $\alpha' \to 0$ and $r \to 0$ but $U = r/\alpha'$ is fixed. Here U has the dimension of energy. This scaling is also called the decoupling limit of BPS D3 brane. Using this decoupling limit the background metric takes the following form.

$$ds^2 = \alpha' \left[\frac{U^2}{\sqrt{4\pi g_s N}}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \sqrt{4\pi g_s N}\frac{dU^2}{U^2} + \sqrt{4\pi g_s N}d\Omega_5^2\right] \tag{1.14}$$

The same decoupled geometry $AdS_5 \times S^5$ has been found in (1.13). Although the AdS/CFT duality was first proposed for BPS D3 branes, the AdS/CFT-like gauge/gravity duality is quite general in type II superstring theory. The other BPS Dp branes of type II theory also lead to such gauge/gravity duality. This duality also holds for black Dp branes. There are many applications [8–11] of this duality and is quite useful to study the properties of strongly coupled field theories.

Outline of the thesis

This thesis has been arranged as follows. In chapter 2 the non-supersymmetric brane solutions of type II superstring theory have been discussed. We have derived the static isotropic non-susy Dp brane solutions from the ten dimensional supergravity action in the presence of a dilaton and a $p + 1$ form-field. The anisotrpoic non-susy branes, the space-like Dp brane, BPS limit have been discussed here. In chapter 3 we have shown a possibility of the gauge/gravity duality in non-susy Dp branes. Here we have studied the graviton scattering on non-susy Dp branes and have found numerically that the graviton never reaches these branes (for $p = 1, \cdots 4$) due to an infinite potential barrier. For stronger argument, the absorption cross-section σ of the gravitational wave has been calculated in near brane region. Chapter 4 is dedicated to derive the throat geometry of non-susy D3 brane. Here we have proposed the decoupling limit for the isotropic, non-susy D3 brane. Using that limit, we have found that the consistent near horizon geometry of this brane is non-AdS with the energy dependent g_{eff}. in chapter 5, this duality has been used to study some properties of QGP in fully non-supersymmetric background. We have considered a 'black' version of the non-susy D3 brane solution in the decoupling limit. By boosting that gravity solution along one of the brane directions and placing a Q-$\bar{Q}$ pair at an arbitrary orientation with this direction, we have numerically obtained various properties of QGP. In chapter 6 and chapter 7 we have discussed some cosmological solutions in string theory. The space-like Dp brane solutions of type II string theories having isometries $ISO(p+1) \times SO(8-p, 1)$ have been considered in chapter 6. At $\tau \sim \tau_0$, we have found $(p+1)+1$ dimensional flat FLRW metrics upon compactification on a $(8-p)$ dimensional hyperbolic space with time dependent radii. These resultant $(p+1)+1$ dimensional metrics describe transient accelerating cosmologies for all p from 1 to 6. For $\tau \ll \tau_0$, after compactification on $(8-p)$ dimensional hyperbolic space, the resultant metrics have been shown to take the form of $(p+1)+1$ dimensional de Sitter spaces upto a conformal transformation. In chapter 7, starting from an anisotropic non-susy D2 brane solution of type IIA string theory we have constructed an anisotropic SD2 brane solution. The compactification on six dimensional hyperbolic space (H_6) of time dependent volume of this SD2 brane solution has led to the accelerating cosmologies where both the expansions and the accelerations are different in three spatial directions of the resultant four dimensional universe. On the other hand at early times ($t \ll t_0$) this four dimensional space, in certain situations, has given the four dimensional Kasner-like cosmology. Finally, in chapter 8, we have summarised the results of our projects and have discussed some future scopes of them.

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Chapter 2

Some Basics Of The Non-Susy Dp Brane Solutions

Before proceeding with the main theme of this thesis, we like to take a short tour through the non-supersymmetric regime of String theory. In the last chapter we have given a brief introduction of AdS/CFT. It is strong/weak coupling duality and vice versa. People have holographically studied various properties of the strongly coupled gauge theory from the decoupling limit of supersymmetric BPS D3 brane solution [11]. In those cases, the near brane gravity theory has an isometry group $SO(4,2)$. Thus the corresponding super-Yang-Mills gauge theory belongs to a special class known as conformal field theory (CFT) [12, 13]. The CFT is invariant under the special conformal transformation. Now the physical interactions in our Universe can be understood with QCD which has no supersymmetry and is not invariant under the conformal transformation. So its corresponding gravity theories can not be derived from the supersymmetric BPS D3 brane solutions. Again, one of the recent topics in cosmology is the inflation. Many physicists are keen to understand this from String theory solutions. For this purpose one needs the time-dependent brane solutions. The easiest way to get it is the application of the Wick rotation on the space-dependent Dp brane solutions. The type II supergravity admits static, supersymmetric BPS Dp-branes with isometries ISO(p, 1) $\times$ SO($9 - p$) in 10-dimensions [14, 15]. Now using Wick rotation on supersymmetric brane, one can not construct an analogous real time-dependent Euclidean p-brane (or S-brane) solution with isometries ISO($p + 1$) $\times$ SO($8 - p$, 1) [16]. The other way to get time-dependent brane solutions is to solve the field equations of supergravity action with a time-dependent ansatz. In this case, if we relax the supersymmetry constraint the resultant solutions are non-supersymmetric and real time-dependent with aforementioned isometries. Therefore a class of non-supersymmetric, static solutions is quite expected to exist which gives the time-dependent brane solutions with above isometries under Wick rotations only.

The ten dimensional $N = 2$ supergravity is the low energy approximation of type-II superstring theory. The action is supersymmetric, but all of its solutions are not supersymmetric. It also admits the non-supersymmetric brane solutions. The zero temperature supersymmetric Dp brane solutions are extremal solutions i.e., its mass and charge are equal upto a constant. This restricts its charge to be non-zero. Unlike BPS solutions, the non-susy brane solutions are non-extremal solutions even at zero temperature. Its mass and charge are independent. Because of this it has more free parameters than susy brane. There are some articles where people have shown the existence of such solutions [17, 18]. In [18], the non-susy brane solutions have been derived from the background field equations, imposing a particular condition that breaks the supersymmetry of the space-time.

Here, in this chapter, to present a brief and useful review of non-susy brane we will follow those articles. The supersymmetry condition is violated here with a non-extremality function which introduces some extra parameters in the theory. It gives the static non-supersymmetric (non-susy, in short) Dp ($p = 0, \cdots 6$) brane solutions. $p = -1$ produces non-susy instanton with same kind of non-extremality function. The ADM masses of these Dp branes show the non-extremality of these solutions. To break supersymmetry conditions, there are three extra parameters in the static non-susy solutions. Fixing the two extra parameters in a certain limit the one-parameter BPS solutions can be recovered. In fact, due to these extra parameters, the non-susy branes are much more preferable to study cosmology from these brane solutions. If a certain parameter (among those extra parameters) is made imaginary then the double Wick rotation on these solutions gives real time-dependent SDp (i.e., space-like Dp) brane. In the following section the static non-susy brane has been discussed. After that we have given a short note on the anisotropic non-susy brane. Later in this thesis the anisotropic non-susy solutions will play a crucial role in string cosmology. In this chapter, we have also briefly discussed some relevant topics like the ADM mass, BPS limit and Wick rotation.

This chapter is organized as follows. In the first section we give the construction of static, non-susy Dp brane solution from the type II supergravity action. We discuss the BPS limit and ADM mass for these non-susy branes in section 2 and 3 respectively. In section 4, we briefely review the anisotropic non-susy brane solutions. In section 5, we show how the space-like Dp brane can be obtained from time-like non-susy branes under Wick rotations. Finally, we give a summary of this review in section 6.

General static, non-SUSY p-branes

In this section we derive the static, non-supersymmetric p-branes [18] from type II supergravity action. Here we directly solve the equations of motion of different background fields – space-time metric $g_{\mu\nu}$, dilaton ϕ and a $(7-p)$-form magnetic gauge field. The 10-dimensional supergravity action has the following form,

$$S = \int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2\cdot(8-p)!}e^{a\phi}F^2_{[8-p]}\right] \quad (2.1)$$

where $a = \frac{p-3}{2}$ is a constant of the dilaton and gauge field coupling. R is the Ricci scalar of $g_{\mu\nu}$ or the scalar curvature of the background. In $F_{[8-p]}$, p can run over 0 to 6. It is even and odd in type IIA and IIB superstring theories respectively. The action (2.1) has three background fields. So, taking the variation of those fields we have three field equations as follows,

$$R_{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{e^{a\phi}}{2\cdot(7-p)!}\left[F_{\mu\alpha_2\ldots\alpha_{8-p}}F_\nu{}^{\alpha_2\ldots\alpha_{8-p}} - \frac{7-p}{8(8-p)}F^2_{[8-p]}g_{\mu\nu}\right] = 0 \quad (2.2)$$

$$\partial_\mu\left(\sqrt{-g}e^{a\phi}F^{\mu\alpha_2\ldots\alpha_{8-p}}\right) = 0 \quad (2.3)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) - \frac{a}{2\cdot(8-p)!}e^{a\phi}F^2_{[8-p]} = 0 \quad (2.4)$$

The numbers of Dirichlet and Neumann boundary conditions at the open string end points define the geometry of the background space-time. Here we have a Dp brane associated with the magnetic form-field strength $F_{[8-p]}$ i.e., the p no. of Neumann conditions and the $(9-p)$ no. of Dirichlet conditions. Thus the background metric can be expressed as the linear combination of two spaces – a $(p,1)$ dimensional Lorentzian world-volume plus a $(9-p)$ dimensional Euclidean space. This suggests the initial assumptions for the background metric and fields as,

$$\begin{aligned} ds^2 &= e^{2A(r)}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) + e^{2B(r)}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) && (2.5)\\ F_{[8-p]} &= b \ \mathrm{Vol}(\Omega_{8-p}) && (2.6)\end{aligned}$$

Here the metric and the form field are assumed. The world-volume is in Cartesian coordinates and the transverse space is in polar coordinates. $r = (x_{p+1}^2 + \cdots + x_9^2)^{1/2}$ is the radial coordinate of transverse space where the Dp brane is located at some fixed radius r. The field strength is related to the volume-form Vol(Ω_{8-p}) with a real constant b known as the magnetic charge parameter. The above space-time in (2.5) has the isometries SO($9-p$) $\times$SO(p, 1). Thus it represents a magnetically charged p-brane in ten dimensions. The space-time is asymptotically flat. So $A(r)$ and $B(r)$ must go to zero at infinity. From the above ansatz, we get the supersymmetric brane solutions saturating the BPS bound if the functions $A(r)$ and $B(r)$ satisfy the following condition.

$$(p+1)B(r) + (7-p)A(r) = 0 \tag{2.7}$$

Actually the above condition insists the background metric to preserve some fraction of supersymmetry and it was shown in [19], with $p+1 = d$ and $7-p = \tilde{d}$ in their notation. It is well-known that the solutions of the equations of motion with (2.5) – (2.7) lead to the usual supersymmetric BPS p-branes [14, 15, 19]. Now we will not restrict the background to obey the supersymmetry. We extend the above condition (2.7) as,

$$(p+1)B(r) + (7-p)A(r) = \ln E(r) \quad \text{where, } E(r) \neq 1 \tag{2.8}$$

The supersymmetries of the solution are broken by the non-extremality function $E(r)$. This gives the real, non-susy, magnetically charged Dp brane solutions. We can show that in a certain limit this non-extremality function goes to one and consequently the supersymmetries get restored in the brane solutions and they turn into the susy BPS solutions (it will be discussed later).

The non-vanishing Ricci tensor components can be obtained from the background metric (2.5) as,

$$R_{rr} = (p+1)\left[B'' + B'^2 - A'B'\right] + (8-p)\left[A'' + \frac{A'}{r}\right] \tag{2.9}$$

$$R_{xx} = -R_{tt} = e^{2B-2A}\left[B'' + (7-p)A'B' + (p+1)B'^2 + (8-p)\frac{B'}{r}\right] \tag{2.10}$$

$$R_{mn} = r^2\left[A'' + (7-p)A'^2 + (15-2p)\frac{A'}{r} + (p+1)B'(A' + \frac{1}{r})\right]\bar{g}_{mn} \tag{2.11}$$

Here we have used different indices. t and x represent the coordinates on the brane or the longitudinal directions and r is the radial coordinate in the transverse space. m, n are the indices for the transverse spherical (angular) coordinates. $\bar{g}_{mn}$ is the metric on the $(8-p)$-dimensional unit sphere Ω^{8-p}. Here *prime* denotes the derivative with respect to r. In (2.6) the gauge form-field depends on the angular coordinates (θ's) of Ω_{8-p}, and this automatically satisfies (2.3). Now we use (2.9) – (2.11), (2.5), (2.6) and (2.8) into the other two field equations (2.2) and (2.4) to get

$$A'' + \frac{E''}{E} - \frac{E'^2}{E^2} + \frac{1}{p+1}\left(\frac{E'}{E} - (7-p)A'\right)^2 + (7-p)A'^2 - \frac{E'}{E}A' + \frac{8-p}{r}A' + \frac{1}{2}\phi'^2 - \frac{b^2(7-p)}{16}\frac{e^{2(p+1)B+a\phi}}{E^2 r^{16-2p}} = 0 \tag{2.12}$$

$$B'' + \frac{8-p}{r}B' + \frac{E'}{E}B' - \frac{b^2(7-p)}{16}\frac{e^{2(p+1)B+a\phi}}{E^2 r^{16-2p}} = 0 \tag{2.13}$$

$$A'' + \frac{8-p}{r}A' + \frac{E'}{E}(A' + \frac{1}{r}) + \frac{b^2(p+1)}{16}\frac{e^{2(p+1)B+a\phi}}{E^2 r^{16-2p}} = 0 \tag{2.14}$$

$$\phi'' + \frac{8-p}{r}\phi' + \frac{E'}{E}\phi' - \frac{ab^2}{2}\frac{e^{2(p+1)B+a\phi}}{E^2 r^{16-2p}} = 0 \tag{2.15}$$

Now we substract (2.13) from (2.14) using (2.8). This gives a second order differential equation of $E(r)$ as,

$$E'' + \frac{15-2p}{r}E' = 0 \tag{2.16}$$

Assuming the asymptotic flatness of the background we find two solutions for $E(r)$ as,

$$E_-(r) = 1 - \frac{r_p^{2(7-p)}}{r^{2(7-p)}}, \qquad E_+(r) = 1 + \frac{\tilde{r}_p^{2(7-p)}}{r^{2(7-p)}} \tag{2.17}$$

These are two independent solutions of (2.16) where both r_p and $\tilde{r}_p$ are real. For the next part of the calculation $E_-(r)$ and $E_+(r)$ have been factorized as follows,

$$E_-(r) = 1 - \frac{r_p^{2(7-p)}}{r^{2(7-p)}} = \left(1 + \frac{r_p^{7-p}}{r^{7-p}}\right)\left(1 - \frac{r_p^{7-p}}{r^{7-p}}\right) = H(r)\tilde{H}(r) \tag{2.18}$$

$$E_+(r) = 1 + \frac{\tilde{r}_p^{2(7-p)}}{r^{2(7-p)}} = \left(1 + \frac{i\tilde{r}_p^{7-p}}{r^{7-p}}\right)\left(1 - \frac{i\tilde{r}_p^{7-p}}{r^{7-p}}\right) = \mathcal{H}(r)\tilde{\mathcal{H}}(r) \tag{2.19}$$

where $H(r) = 1 + r_p^{7-p}/r^{7-p}$, $\tilde{H}(r) = 1 - r_p^{7-p}/r^{7-p}$, $\mathcal{H}(r) = 1 + i\tilde{r}_p^{7-p}/r^{7-p}$ and $\tilde{\mathcal{H}}(r) = 1 - i\tilde{r}_p^{7-p}/r^{7-p}$. These functions are called the harmonic functions of solutions. $\mathcal{H}$ and $\tilde{\mathcal{H}}$ are the imaginary functions. They lead to the imaginary non-susy brane solution. As we are interested in the real non-susy brane solutions, we discard $E_+(r)$. Substituting $E_-(r)$ in (2.13) and (2.15), we get

$$\left(\phi - \frac{8a}{7-p}B\right)'' + \frac{8-p}{r}\left(\phi - \frac{8a}{7-p}B\right)' + \frac{E_-'}{E_-}\left(\phi - \frac{8a}{7-p}B\right)' = 0 \tag{2.20}$$

Solving this equation, we get the functional form of ϕ,

$$\phi = \frac{8a}{7-p}B + \delta \ln \frac{H}{\tilde{H}} \tag{2.21}$$

where δ is an arbitrary real constant of integration. So,

$$e^{2(p+1)B+a\phi} = \left(\frac{H_1}{\tilde{H}_1}\right)^{a\delta} e^{\frac{32B}{7-p}} \tag{2.22}$$

The second order differential equation of $B(r)$ in (2.13) takes the following form,

$$B'' + \frac{8-p}{r}B' + \frac{E'_-}{E_-}B' - \frac{b^2(7-p)}{16}\frac{e^{\frac{32B}{7-p}}H^{a\delta-2}}{r^{2q}\tilde{H}^{a\delta+2}} = 0 \tag{2.23}$$

At this point, we take another ansatz for $B(r)$,

$$e^B = \left[\cosh^2\theta\left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\beta}\right]^{\gamma} = F^{\gamma} \tag{2.24}$$

where $F = \left[\cosh^2\theta\left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta\left(\frac{\tilde{H}}{H}\right)^{\beta}\right]$. Here, α and β are the real parameters and θ and γ are constant of integration. In (2.23), the sign of the last term of this second order differential equation suggests the hyperbolic function of θ in F. This new parameter θ is also real and it is supposed to indicate the number of branes. Substituting (2.24) in (2.23) we obtain,

$$\left[\gamma(7-p)r_p^{2(7-p)}(\alpha+\beta)^2\sinh^2 2\theta\right]\frac{H^{\alpha-\beta}\tilde{H}^{\beta-\alpha}}{F^2} + \frac{b^2}{16}H^{a\delta}\tilde{H}^{-a\delta}F^{\frac{32\gamma}{7-p}} = 0 \tag{2.25}$$

The left hand side of the equation has to be explicitly zero. But the harmonic functions H, $\tilde{H}$ and their combination F can not be zero at finite point. To satisfy the above equation we need to match the power of the various harmonic functions by comparing the two terms in the lhs of (2.25). This matching gives two conditions on α, β, δ and γ. Using these two parametric conditions on the above equality we express the constant b in terms of mass parameter r_p and other parameters of the solution.

$$\begin{aligned} \gamma &= -\frac{7-p}{16}, \qquad\qquad \alpha - \beta \;=\; a\delta \\ b &= (7-p)(\alpha+\beta) r_p^{7-p} \sinh 2\theta \end{aligned} \tag{2.26}$$

As we are mainly interested in the positive charged solution here, $\alpha+\beta$ and θ can be taken to be positive without any loss of generality. In (2.26), the first condition shows a fixed value of γ. In the second condition we have only two independent parameters among α, β and δ. Using these parametric conditions (2.26) in (2.12), we get

$$\frac{1}{2}\delta^2 + \frac{1}{2}\alpha(\alpha - a\delta) = \frac{8-p}{7-p} \tag{2.27}$$

Using (2.26) and (2.27), α and β can be written as,

$$\begin{aligned} \alpha &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}\delta^2} + \frac{p-3}{4}\delta \\ \beta &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}\delta} - \frac{p-3}{4}\delta \end{aligned} \tag{2.28}$$

So, we have only one independent parameter δ, where α and β can be determined from δ. As α, β and δ all are real here, the value of δ is bounded by following inequality.

$$|\delta| \;\leq\; \frac{4}{7-p}\sqrt{\frac{2(8-p)}{(p+1)}} \tag{2.29}$$

Now we can get the final form of $A(r)$ and $B(r)$, using (2.8) and (2.26).

$$\begin{aligned} e^{2B} &= F^{-\frac{7-p}{8}} \\ e^{2A} &= (H\tilde{H})^{\frac{2}{7-p}} F^{\frac{p+1}{8}} \end{aligned}$$

The complete non-supersymmetric, static, magnetically charged Dp-brane solutions can be expressed in Einstein frame as,

$$\begin{aligned} ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})^{\frac{2}{7-p}}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) + F^{-\frac{7-p}{8}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= F^{-a}\left(\frac{H}{\tilde{H}}\right)^{2\delta} \\ F_{[8-p]} &= b \ \mathrm{Vol}(\Omega_{8-p}) \end{aligned} \tag{2.30}$$

This is the non-susy Dp brane solutions of type II superstring theories where $p = 0, \cdots, 6$. The brane is located at $r = r_p$ giving a naked singularity in the above background. The location of source always gives a δ-function singularity, but here the singularity is more severe. So our physical space is given by $r > r_p$. It can be shown that in the near extremal limit (large θ limit), the above instability goes away. These final solutions are characterised by three independent parameters δ, r_p and θ. Here, r_p is not exactly similar to the mass parameter of BPS brane, but it has direct relation with the ADM mass of non-susy brane. The solution has two more parameters than BPS brane. θ is generally identified as the charge parameter or the number of branes. δ is called the dilaton parameter because of the existence of non-constant dilaton field in this background. Later in this thesis we will discuss about the temperature of these non-susy solutions and will come to know that (2.30) is a zero-temperature solution.

Here we have derived the magnetically charged non-susy branes given in (2.30). $F_{[8-p]}$ is the magnetic form-field strength. The corresponding electric form-field strength can be found by applying the Hodge duality ($*$) on magnetic field strength. Thus the electrically charged brane solutions can be obtained from (2.30) by using the transformation $g_{\mu\nu} \to g_{\mu\nu}$, $\phi \to -\phi$ and $F_{\text{magnetic}} \to F_{\text{electric}} = e^{-a\phi} * F_{\text{magnetic}}$. It gives the electric form-field strength as,

$$F_{[p+2]} = e^{a\phi} * F_{[8-p]} = dA_{[p+1]} \tag{2.31}$$

where $A_{[p+1]}$ is $(p+1)$-form gauge field of electrically charged brane and the dilaton ϕ is given in eq.(2.30). The $A_{(p+1)}$ gauge field can be calculated from (2.31) as,

$$A_{[p+1]} = \left(\frac{C}{F}\right) \sinh\theta \cosh\theta dt \wedge \cdots \wedge dx_p \tag{2.32}$$

where,

$$C = \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \left(\frac{\tilde{H}}{H}\right)^{\beta}$$

Although we have explicitly discussed the derivation of static non-susy Dp-branes here, the type II superstring theory also includes non-susy D-Instanton solution. By the same method one can also derive the non-susy instanton solutions by putting $p = -1$. The non-susy instanton in Einstein frame

$$\begin{aligned}
ds^2 &= \left(H\tilde{H}\right)^{\frac{1}{4}} \left(dr^2 + r^2 d\Omega_9^2\right) \\
e^{2\phi} &= \left[\cosh^2\theta \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta \left(\frac{H}{\tilde{H}}\right)^{-\alpha}\right]^2 \\
F_{[9]} &= b\mathrm{Vol}(\Omega_9)
\end{aligned} \tag{2.33}$$

where,

$$H = 1 + \frac{r_-^8}{r^8}, \quad \tilde{H} = 1 - \frac{r_-^8}{r^8} \tag{2.34}$$

and the constant parameters α and b have been found to be

$$\alpha = -\frac{3}{2}, \quad \text{and} \quad b = -24 r_-^8 \sinh 2\theta \tag{2.35}$$

The scalar field for the corresponding electrically charged solution has the form

$$A_{[0]} = \frac{2}{a} \sinh\theta \cosh\theta \left(\frac{C}{F}\right) \tag{2.36}$$

where

$$C = \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \left(\frac{H}{\tilde{H}}\right)^{-\alpha}, \qquad F = \cosh^2\theta \left(\frac{H}{\tilde{H}}\right)^{\alpha} - \sinh^2\theta \left(\frac{H}{\tilde{H}}\right)^{-\alpha}. \tag{2.37}$$

The D-instanton is a point in background space-time and has a zero dimensional Euclidean world-volume.

BPS limit

The Bogomol'nyi-Prasad-Sommerfeld (in short BPS) branes are the extremal susy Dp brane solutions. Here the word 'extremal' refers to the proportionality between the values of mass and charge of the brane. For susy Dp brane solutions there is a bound on the charge $\mathcal{Q}_p$ in terms of the mass $\mathcal{M}_p$. This bound is known as BPS bound. The BPS bound for susy Dp brane is mathematically given as,

$$\mathcal{Q}_p \leq \sqrt{2}\kappa\mathcal{M}_p \tag{2.38}$$

For physical solutions (satisfying cosmic censorship hypothesis), it can be shown that the maximum allowed value of the charge of that brane can be equal to $\sqrt{2}\kappa$ times its mass. This is all that has been stated in the above inequality relation. In BPS brane this charge is extremal (maximum) and its value is equal to the mass. Now in case of our non-susy solutions, we can achieve this extremal limit by simultaneously increasing the charge parameter and decreasing the mass parameter. In type II superstring theory, the BPS limit is the limiting point where the non-susy Dp brane transforms into the BPS Dp brane. The non-susy branes discussed in last section have three free parameters: the mass parameter r_p, the charge parameter θ and the dilaton parameter δ. Mathematically the BPS limit, in this case, can be expressed as [20],

$$
\begin{aligned}
& r_p \to 0 \\
& \theta \to \infty \\
& \text{where ``}(\alpha+\beta) r_p^{7-p} \sinh 2\theta = \bar{R}^{7-p}\text{'' is fixed at a finite value.}
\end{aligned}
\tag{2.39}
$$

Now we apply this limit into the non-susy brane background. At first we check the non-susy condition (2.8). According to the first limit of (2.39), $E(r) \to 1$. It transforms the non-susy condition (2.8) into susy condition (2.7). In this limit, $H(r),\ \tilde{H}(r) \to 1$ and the function F is modified as,

$$
F(r) \to 1 + \frac{(\alpha+\beta) r_p^{7-p} \sinh 2\theta}{r^{7-p}} \equiv 1 + \frac{\bar{R}^{7-p}}{r^{7-p}} = \bar{H}(r) \tag{2.40}
$$

In this limit, the magnetically charged static non-susy Dp brane (2.30) takes the following form

$$
\begin{aligned}
ds^2 &= \bar{H}^{\frac{p+1}{8}} \left(dr^2 + r^2 d\Omega_{8-p}^2\right) + \bar{H}^{-\frac{7-p}{8}} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\
e^{2\phi} &= \bar{H}^{-a} \\
F_{[p+2]} &= b \mathrm{Vol}(\Omega_{8-p})
\end{aligned}
\tag{2.41}
$$

This is the well-known BPS Dp brane solution in Einstein frame. Not only the background, the various properties of non-susy branes exactly merge to those properties of the BPS brane in this limit.

ADM Mass

ADM mass formula is a special way to define the mass of a gravitational system in general relativity [21, 22]. The standard definition of ADM mass holds for the space-time that goes to a well-defined geometry in asymptotic limit. In our case, the non-susy Dp brane solution is asymptotically flat solution – the metric asymptotically goes to the Minkowski metric at infinity. So without any restriction one can use ADM mass formula to derive the mass of the non-susy brane solutions [23]. The ADM mass and total charge of the non-susy Dp brane (2.30) is found to have the following form [20]

$$\begin{aligned}\mathcal{M}_p &= \frac{\Omega_{8-p}}{2\kappa_{10}^2} 2(7-p)\left(\alpha\cosh^2\theta + \beta\sinh^2\theta\right) r_p^{7-p} \\ \mathcal{Q}_p &= \frac{\Omega_{8-p}}{\sqrt{2}\kappa_{10}} b = \frac{\Omega_{8-p}}{\sqrt{2}\kappa}(7-p)(\alpha+\beta) r_p^{7-p}\sinh 2\theta\end{aligned}$$

The ratio of ADM mass and total charge is

$$\sqrt{2}\kappa_{10}\frac{\mathcal{M}_p}{\mathcal{Q}_p} = \frac{\alpha\cosh^2\theta + \beta\sinh^2\theta}{(\alpha+\beta)\sinh\theta\cosh\theta} \tag{2.42}$$

The numerical calculation shows that the rhs in (2.42) is greater than or equal to one for all values of θ. So for finite θ, $m_p > Q$. We can fix θ value to get extremal solution. For $\theta = \tanh^{-1}\left(\frac{\alpha}{\beta}\right)$ and $\theta = \infty$ the rhs of (2.42) is equal to one, i.e. the solutions are extremal.

In BPS limit, as θ is very large, $\sinh\theta \approx \cosh\theta$. So, in BPS limit the mass and charge ratio

$$\frac{\mathcal{M}_p}{\mathcal{Q}_p} \to \frac{1}{\sqrt{2}\kappa_{10}} \tag{2.43}$$

This is exactly same as the BPS brane solutions. For finite (comparatively small) value of

θ, the mass-to-charge ratio given in (2.42) is always greater than $1/\sqrt{2}\kappa_{10}$. It says that the static non-susy brane solutions are generally non-extremal brane. But it is also possible to have extremal non-susy brane solutions.

Anisotropic non-susy branes

Till now we have seen the static non-susy solutions with $ISO(p, 1) \times SO(9-p)$ isometries. Those solutions are little ideal in a sense that all of the brane's coordinates are treated equally. Usually the space-time geometries are not always so regular, it can have anisotropy in $(p+1)$ longitudinal coordinates of the brane. In this section, we will discuss about the static, anisotropic, non-susy Dp brane solutions. This type of non-susy brane solutions was first discussed by J X Lu, Shibaji Roy and their other collaborations in a number of publications [24–26]. Here we are not going into the detailed derivation. It is available in [24]. To give a minimum idea of these anisotropic solutions we just explain the method how to get static anisotropic non-susy brane here. The starting point is a special kind of supergravity background with a static, isotropic Dq brane where among $9-q$ transverse coordinates the brane is delocalised in $p-q$ transverse directions. These type of brane solutions can be easily found from the field equations of the type II supergravity with proper assumptions. Here, according to the open string dynamics, these $p-q$ coordinates satisfy the Dirichlet conditions. Then we apply T-duality along those $p-q$ delocalised directions. The T-duality transforms the Dirichlet conditions into Neumann conditions. Thus our background is reduced to a Dp brane solution. But in this solution, all of the $p+1$ longitudinal coordinates do not have the same factors multiplying them. It means there is an anisotropy among the brane's coordinates. The anisotropic non-susy

Dp brane solution is given bellow.

$$\begin{aligned}
ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})^{\frac{2}{7-p}}\left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)}{8}\delta_1+\frac{(3-p)}{2(7-p)}\sum_{i=2}^{p-q+1}\delta_i}\left(dr^2+r^2d\Omega_{8-p}^2\right)\\
&+F^{-\frac{7-p}{8}}\left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q)}{8}\delta_1+\sum_{i=2}^{p-q+1}\frac{\delta_i}{2}}\left(-dt^2+\sum_{i=1}^{q}(dx^i)^2\right)\\
&+F^{-\frac{7-p}{8}}\sum_{i=2}^{p-q+1}\left(\frac{H}{\tilde{H}}\right)^{\frac{(p-q-8)}{8}\delta_1-\frac{3\delta_i}{2}+\sum_{j(\neq i)=2}^{p-q+1}\frac{\delta_j}{2}}(dx^{q+i-1})^2\\
e^{2\phi} &= F^{\frac{3-p}{2}}\left(\frac{H}{\tilde{H}}\right)^{\frac{(4-p+q)}{2}\delta_1-\sum_{i=2}^{p-q+1}2\delta_i}\\
F_{[8-p]} &= b\text{Vol}(\Omega_{8-p})
\end{aligned} \tag{2.44}$$

where the harmonic functions are same as defined before, $H,\ \tilde{H}$ in (2.18) and F in (2.24). But the parametric relations have been modified as follows,

$$\begin{aligned}
&\alpha-\beta=\frac{q-3}{2}\delta_1\\
&\frac{1}{2}\delta_1^2+\frac{1}{2}\alpha\beta+\frac{2}{7-p}\sum_{i>j=2}^{p-q+1}\delta_i\delta_j=\frac{8-p}{7-p}\left(1-\sum_{i=2}^{p-q+1}\delta_i^2\right)\\
&b=(7-p)r_p^{7-p}(\alpha+\beta)\sinh 2\theta
\end{aligned} \tag{2.45}$$

This is a static, anisotropic, non-susy Dp brane. Here, in the world-volume, some of the brane's coordinates $t, x_1, \cdots x_q$ have same factors, giving the $SO(q,\,1)$ isometry. But for the remaining brane's coordinates $x_{q+1}\cdots x_p$, each of them has different factors which breaks the isotropy of the worldvolume. These type of anisotropic solutions can be considered also as the intersecting solutions of different branes [27]. The above solution can be interpreted the charged non-susy Dp brane intersecting with charge-less q-brane, $(q+1)$-brane , . . . , up to $(p-1)$-brane in Einstein frame. This is actually more general non-susy brane solution, because, by choosing appropriate value of p and q we can get all type of non-susy solutions. For $p=q$ in (2.44) we get static, isotropic, non-susy Dp brane with zero temperature as given in (2.30) and for $q = 0$ (2.44) gives static, anisotropic, non-susy Dp branes with finite temperature. Again modifying different δ parameters we can make different kinds of non-susy solution.

Wick rotation and S-branes

Understanding cosmology from the string theory solutions is an important problem. The cosmology involves real time dynamics of space-time geometry. In this respect the time-dependent brane solutions play important roles [28–34]. The Wick rotation of static solutions also gives dynamical solutions. It rotates a pair of coordinates with the angle $\pm\pi/2$ which is a transformation from the real plane to the imaginary plane. Under this transformation the supergravity action remains invariant except the static quantities are transformed into dynamic quantities. Unlike the BPS Dp branes the non-susy brane solutions give the real time-dependent brane solutions under Wick rotation. We have this particular advantage with non-susy branes because of the more no. of free parameters. Details will be discussed in this section.

We will apply Wick rotation on the non-susy Dp branes (2.30) in this section. The static branes depend on the radial coordinate r of the $9 - p$ dimensional transverse space. The Wick rotation can be defined here as

$$\begin{aligned} t &\to -ix_{p+1} \\ r &\to i\tau \end{aligned} \tag{2.46}$$

Here x_{p+1} and τ are space-like and time-like coordinates respectively. So, the Wick rotation is mainly a conversion of the space-like and the time-like coordinates. The volume-element of total background geometry must be invariant under these rotations to follow the same structure of supergravity action. Let's look at the volume element first. As only t and r have been rotated, $dtd^pxdr \to dx_{p+1}d^pxd\tau = d\tau d^{p+1}x$ is invariant. But another part of the volume element $r^{8-p}d\Omega_{8-p} \to (i)^{8-p}\tau^{8-p}d\Omega_{8-p}$ is not real. This change can be nullified using one more rotation in one of the angular coordinates of Ω_{8-p}, say, $\psi \to -i\psi$. Now the line element on the sphere,

$$d\Omega_{8-p}^2 = d\psi^2 + \sin^2\psi d\Omega_{7-p}^2 \to -d\psi^2 - \sinh^2\psi d\Omega_{7-p}^2 = -dH_{8-p}^2 \tag{2.47}$$

and the spherical volume element changes to

$$\begin{aligned} d\Omega_{8-p} &= (\sin\psi)^{7-p}d\psi\wedge\cdots \\ &\to (-i)^{8-p}(\sinh\psi)^{7-p}d\psi\wedge\cdots \\ &= (-i)^{8-p}dH_{8-p} \end{aligned} \tag{2.48}$$

The volume element $dtd^pxdrd\Omega_{8-p}r^{8-p} \to d\tau d^{p+1}xdH_{8-p}\tau^{8-p}$ is real. Now in (2.18) H and $\tilde{H}$ takes the following forms

$$\begin{aligned} H(r) &= 1 + \frac{r_p^{7-p}}{r^{7-p}} \to 1 + \frac{\tau_p^{7-p}}{\tau^{7-p}} = H(\tau) \\ \tilde{H}(r) &= 1 - \frac{r_p^{7-p}}{r^{7-p}} \to 1 - \frac{\tau_p^{7-p}}{\tau^{7-p}} = \tilde{H}(\tau) \end{aligned} \tag{2.49}$$

where we have used the fourth rotation as $r_p \to i\tau_p$. In order to explain the transformation of the gauge form-field $F_{[8-p]}$ in (2.30), we have seen in (2.48) that the spherical volume element have been changed to a hyperbolic volume element with an imaginary coefficient. This implies that the field $F_{[8-p]}$ will be real only if $b \to (i)^{8-p}b$. From (2.26) it is clear that the suggested transformation of b is possible if θ gets the rotation $\theta \rightarrow i\theta$. Now the new form of charge is trigonometric function of θ and it also changes the harmonic function F of (2.24).

$$\begin{aligned} F(r) \rightarrow F(\tau) &= \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{\alpha} \cos^2\theta + \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{-\beta} \sin^2\theta \\ b &= (7-p)\tau_p^{7-p} \sin 2\theta \end{aligned} \tag{2.50}$$

Finally for non-susy Dp brane solutions, the complete set of Wick rotations is

$$t \rightarrow -x_{p+1} \qquad r \rightarrow i\tau$$
$$\psi \rightarrow -i\psi \quad r_p \rightarrow i\tau_p \quad \theta \rightarrow i\theta$$

Now we come up with completely real, time-dependent version of (2.30).

$$\begin{aligned} ds^2 &= F^{\frac{p+1}{8}}(H\tilde{H})^{\frac{2}{7-p}}\left(-d\tau^2 + \tau^2 dH_{8-p}^2\right) + F^{-\frac{7-p}{8}}\left(dx_1^2 + \cdots + dx_p^2 + dx_{p+1}^2\right) \\ e^{2\phi} &= F^{-a}\left(\frac{H}{\tilde{H}}\right)^{2\delta} \\ F_{[8-p]} &= b\ \mathrm{Vol}(H_{8-p}) \end{aligned} \tag{2.51}$$

where the other parameters like $\alpha,\ \beta$ and δ are related as the same relations given in (2.26) where the new form of b and F is given in (2.50) and $H,\ \tilde{H}$ are given in (2.49). Here the time coordinate is transverse to the brane and brane world-volume is purely Euclidean or space-like. The backgrond isometry can be expressed as $ISO(p+1)\times SO(8-p,\ 1)$. Such time-dependent brane solutions are known as space-like D branes or SD-branes [35, 36]. The above metric (2.51) defines an SDp-brane with a time dependent scalar ϕ and a form-field $F_{[8-p]}$. These SD-brane solutions can be used to study the cosmological solutions from string theory background. By the compactification of $(8-p)$ dimensional transverse hyperboloid these time dependent solutions (2.51) gives $(p+1,\ 1)$ dimensional FLRW-like cosmologies with an extra scalar field due to dimensional reduction. Rescaling the time coordinate the exact FLRW model can be found along with the scale factor $a^2(t)$. Later in this thesis certain aspects of cosmologies are studied with these SDp brane solutions.

Conclusion

In this chapter we have given a brief review on some important topic related to the non-susy Dp brane solutions. The static, isotropic, magnetically charged non-susy Dp branes (2.30) of zero temperature have been derived from ten dimensional supergravity action of type II superstring theory. These solutions are found to be very analogous in their forms with BPS solutions. Both the susy and the non-susy brane solutions come from the same supergravity action and both of them have isometry $SO(p, 1) \times SO(9-p)$. The isotropic non-susy brane has three independent parameters r_p, θ and δ. Among them r_p and θ can be identified as the mass parameter and charge parameter respectively. In BPS D3 brane, the dilaton field is constant. But for non-susy D3 brane, the dilaton varies with the spatial coordinates for non-zero δ. But for $\delta = 0$, the dilaton is constant for non-susy D3 brane. So one can identify δ as the dilaton parameter. We have discussed the BPS limit which is mainly a parametric limit. The BPS limit transforms the non-susy brane into susy brane. We have calculated the ADM mass of the non-susy branes and it is found to be greater than the charge of the brane. We get two values of θ: $\tanh^{-1}(\alpha/\beta)$ and ∞ for which the solutions are extremal i.e., $\sqrt{2}\kappa_{10}$ mass $=$ charge. The first value gives the non-susy extremal brane solution where as the other one is like a BPS limit. In BPS limit, the charge and mass of the non-susy brane are proportional i.e. an extremality condition. Here we have also mentioned the anisotropic non-susy Dp brane solution (2.44). This anisotropic brane solution is a general class of the non-susy brane solution. For $q = 0$ and $p = 3$ (2.44) gives the 'black brane-like' anisotropic non-susy D3 brane solutions. With

$\delta_2 = \delta_3 = \delta_4$ it gives the 'black brane-like' isotropic non-susy D3 brane. At a certain parametric value this non-susy D3 brane gives the black D3 brane. In fact the isotropic branes are zero-temperature solutions where as certain anisotropic solutions are finite temperature solutions. Its temperature depends on the mass parameter, charge parameters and the parameter defining the anisotropy in g_{tt}. The anisotropic solutions can be made isotropic by vanishing certain parameters which also makes the temperature zero. The Wick rotations on non-susy brane solutions have been discussed where we have found the Euclidean brane solutions which have been termed as 'space-like' D brane or SD brane. In these brane the time is a transverse direction and the other directions here some time-dependent factors. So the background is dynamic which motivates to investigate the cosmological properties in these solutions. The SDp brane has isometries ISO$(p + 1) \times$ SO$(8-p, 1)$). After the compactification of the $8-p$ dimensional hyperboloid this solution gives a $(p+1,\ 1)$ dimensional cosmological space-time with some background fields and constant parameters (like $\tau_p,\ \delta,\ \theta$). Some interesting cosmological properties of such SD brane will be discussed in chapter 6 and 7. At the end of the day our goal is to understand the real world from these various string theory solutions. In superstring theory, the static Dp brane has $(p,\ 1)$ dimensional world-volume. So D3 brane is more important to study real $(3,\ 1)$ dimensional world. The gauge/gravity duality is very useful tool to connect the real world with the brane solutions. Naturally the next step of this project should be the investigation of gauge/gravity duality for these non-susy Dp branes. The initial clue of gauge gravity duality is the decoupling of gravity from the brane world-volume theory. In the following chapter we will discuss the graviton scattering on the non-susy brane.

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Chapter 3

Graviton Scattering

In the context of the original AdS/CFT correspondence the precise way to see that the gravity decouples from the BPS D3 brane is to study the graviton scattering in the background of D3-brane [37–39]. Both from studying the scattering potential and also from the calculation of graviton absorption cross-section [37–42] one can see, that the scattering potential takes the form of an infinite barrier and the absorption cross-section vanishes in the decoupling limit. Therefore, a graviton propagating in the bulk and approaching the brane is never able to reach the brane in the decoupling limit and thus the bulk gravity gets decoupled from the brane. This decoupling is the primary structure of AdS/CFT correspondence [2–5].

The correspondence holds good even when there are less number of supersymmetries and with [43] or without conformal symmetries [44, 45] . So, for example, BPS Dp

branes (for $p \leq 5$ and $\neq 3$) of type II string theories has a decoupling limit in which the field theories on the brane get decoupled from the bulk gravity [46]. The field theories in these cases have 16 supercharges and no conformal symmetries giving rise to a non-AdS/non-CFT correspondence. One can explicitly check that decoupling of gravity on the brane indeed occurs by studying the graviton scattering on these branes. As in BPS D3 brane case, here also one can see that the graviton equation of motion takes the form of a Schrödinger-like equation, where the potential becomes infinite in the decoupling limit, indicating that gravity gets decoupled from the brane. One can further check the graviton absorption cross-section on the brane and indeed one finds that it vanishes in the decoupling limit which clearly indicates that the graviton does not reach the brane and the bulk gravity gets decoupled from the brane world-volume theory [1]. Decoupling of gravity does not occur for BPS D6 brane as the scattering potential in this case does not have a barrier. BPS D5 brane is a border-line between D6 brane and other Dp branes. Here, the scattering potential indicates that gravity decouples if the energy carried by the graviton is below certain critical value, and above that value gravity couples to BPS D5 brane [1].

It is generally believed that AdS/CFT-like correspondence must hold good for more general backgrounds and even for the non-supersymmetric backgrounds. However, there is no explicit calculation in the literature showing the decoupling of gravity for the non-supersymmetric gravitational systems[1]. In this chapter, we study the graviton scattering on the known non-supersymmetric (non-susy) Dp-brane solutions of type II string theories [17, 18]. As a simple exercise we first look at the dynamics of a scalar coupled minimally to this background [49–52]. We write the equation of motion in the string frame and show that the scalar satisfies, in this background, a Schrödinger-like equation with certain

[1] Some earlier comments and calculation about the non-decoupling of gravity for non-susy branes can be found in [47] and [48], however, we differ with their conclusion.

potential. The purpose of studying the minimally coupled scalar is that when we study the graviton scattering next, we will see that it will essentially (upto a multiplicative function) satisfy the same Schrödinger-like equation with the same potential in the background of non-susy Dp brane solutions. We study the scattering potential numerically and show that in the decoupling limit the potentials act like an infinite barriers for the graviton to reach the brane for all $p \leq 5$. However, for $p = 6$, there is no barrier for the potential and it always goes to negative infinity indicating that the gravity in this case does not decouple from the brane. To further strengthen our claim we compute the graviton absorption cross-sections for the non-susy Dp branes. For this we need to solve the Schrödinger-like scattering equation. It is in general difficult to solve the equation, but, the equation can be solved both at the far region and at the near region and this is what is needed for obtaining the expression for the absorption cross-sections. However, unlike the BPS brane case [1,37,38] the near region solution for the non-susy case can be obtained only if the parameters of the solutions satisfy certain conditions discussed in section 4. Then the solutions in the far region and the near region can be matched in the overlapping region and that fixes the arbitrary constant in the solution[2]. From the form of the solution, we can straightforwardly obtain the expressions for the graviton absorption cross-sections on the brane. Then we show that the cross-sections vanish for all non-susy Dp branes with $p \leq 4$ in the decoupling limit. We, therefore, conclude that the bulk gravity indeed decouples for all non-susy Dp branes for $p \leq 4$. The calculation does not work for $p = 5$, however, by studying the scattering potential we conclude that here depending on one of the parameters of the solution the decoupling occurs without any restrictions, otherwise, it occurs only when the energy of the graviton is below certain critical value, and above that value it couples. Even for $p = 6$, the similar calculation does not go through and so, we conclude from the scattering potential that the gravity in this case couples as the

[2]Similar calculation has been done for BPS D3 brane in [37]. See also [40–42] for some earlier calculation of similar type for D1/D5 black holes.

potential here is attractive.

This chapter is organized as follows. In the section 1 we briefly review the structure of non-susy Dp branes in the string frame and their BPS limits. Then in section 2, we study the scattering of a minimally coupled scalar and obtain the form of the scattering potential. In section 3, we study the graviton scattering and study the scattering potential numerically. In section 4, the absorption cross-section of the graviton is obtained when the parameters of the solutions satisfy certain conditions. Finally we conclude in section 5.

Non-susy Dp branes and their BPS limits

In this section we recall the static, non-susy Dp brane solutions of type II string theories (2.30). There the solution is given in Einstein frame. We can write it in string frame with a simple transformation: *metric in string frame* $=\ e^{-\phi/2}\times$ *metric in Einstein frame*, where ϕ is the dilaton filed. So, the non-susy Dp brane solution in string frame is

$$
\begin{aligned}
ds^2 &= F(r)^{-\frac{1}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta}{2}}\left(-dt^2+\sum_{i=1}^{p}(dx^i)^2\right)\\
&\qquad +F(r)^{\frac{1}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta}{2}}\left(H(r)\tilde{H}(r)\right)^{\frac{2}{7-p}}\left(dr^2+r^2d\Omega_{8-p}^2\right)\\
e^{2\phi} &= F(r)^{\frac{3-p}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{2\delta}, \qquad F_{[8-p]} = Q\mathrm{Vol}(\Omega_{8-p})
\end{aligned}
\tag{3.1}
$$

The various functions appearing in the solution are defined as,

$$
\begin{aligned}
H(r) &= 1+\frac{r_p^{7-p}}{r^{7-p}}, \qquad \tilde{H}(r) = 1-\frac{r_p^{7-p}}{r^{7-p}}\\
F(r) &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\alpha}\cosh^2\theta-\left(\frac{\tilde{H}(r)}{H(r)}\right)^{\beta}\sinh^2\theta
\end{aligned}
\tag{3.2}
$$

In the above we have suppressed the string coupling constant g_s which is assumed to be small, Q is the RR charge and $\mathrm{Vol}(\Omega_{8-p})$ is the volume form of the transverse $(8-p)$-dimensional unit sphere. Also, α, β, δ, θ, Q and r_p (r_p has the dimension of length, but its actual form is different for different p branes) are six parameters characterizing the solution. However, not all of them are independent. In fact, there are three relations among them following from the consistency of the equations of motion and they are,

$$\begin{aligned}
&\alpha - \beta = \frac{p-3}{2}\delta \\
&\frac{1}{2}\delta^2 + \frac{1}{2}\alpha\left(\alpha - \frac{p-3}{2}\delta\right) = \frac{8-p}{7-p} \\
&Q = (7-p)r_p^{7-p}(\alpha+\beta)\sinh 2\theta
\end{aligned} \tag{3.3}$$

So, we can use these relations to eliminate three of the above six parameters and therefore, we take δ, θ and r_p as the independent parameters[3] of the solution. In fact one can obtain α, β in terms of δ from the second relation in (3.3) as,

$$\begin{aligned}
\alpha &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}\delta^2} + \frac{p-3}{4}\delta \\
\beta &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16}\delta^2} - \frac{p-3}{4}\delta
\end{aligned} \tag{3.4}$$

where for the reality of the parameters δ must be constrained as,

$$|\delta| \leq \frac{4}{7-p}\sqrt{\frac{2(8-p)}{p+1}} \tag{3.5}$$

One may think that there are too many parameters in the solution which may violate Birkhoff's theorem. However, note that because of the form of $\tilde{H}(r)$ given in (**??**), the solution has a singularity at $r = r_p$, and therefore, Birkhoff's theorem does not apply

[3] Attempts have been made to give physical meaning to these three parameters in terms of number of branes, number of anti-branes and a tachyon vev or tachyon parameter in [20, 47].

here. So, $r > r_p$ is the physical region. We have also taken $(\alpha+\beta)$, θ and Q to be positive semi-definite without any loss of generality. Note that the non-susy Dp brane solutions given in (3.1) is asymptotically flat and is magnetically charged. One can obtain the BPS Dp brane from the non-susy Dp brane solutions given in (3.1) as discussed in the last chapter. We notice that, in BPS limit, if we take $r_p \to 0$, then both $H(r)$ and $\tilde{H}(r)$ go to 1, but the function

$$F(r) \to 1 + 2(\alpha\cosh^2\theta + \beta\sinh^2\theta)\frac{r_p^{7-p}}{r^{7-p}} \tag{3.6}$$

Now if we further take $\theta \to \infty$ such that the product $2(\alpha+\beta)\sinh^2\theta r_p^{7-p} = \bar{r}_p^{7-p} =$ fixed, then $F(r) \to \bar{H}(r)$, where $\bar{H}(r) = 1 + \bar{r}_p^{7-p}/r^{7-p}$ is the standard Harmonic function and the solution (3.1) reduces exactly to the standard BPS Dp brane solutions. Note that now all the extra paramaters from the non-susy Dp brane solutions disappear and the BPS solutions are characterized by a single parameter $\bar{r}_p$ as expected.

In order to study the graviton scattering we first rewrite it in a more convenient form by introducing a new radial coordinate given by,

$$r = \rho\left(\frac{1+\sqrt{G(\rho)}}{2}\right)^{\frac{2}{7-p}}, \qquad \text{where, } G(\rho) = 1 + \frac{4r_p^{7-p}}{\rho^{7-p}} \equiv 1 + \frac{\rho_p^{7-p}}{\rho^{7-p}} \tag{3.7}$$

In this new coordinate we have

$$\begin{aligned} H &= \frac{2\sqrt{G(\rho)}}{1+\sqrt{G(\rho)}}, \qquad \tilde{H} = \frac{2}{1+\sqrt{G(\rho)}} \\ dr^2 + r^2 d\Omega_{8-p}^2 &= \left(\frac{1+\sqrt{G(\rho)}}{2}\right)^{\frac{4}{7-p}}\left[\frac{d\rho^2}{G(\rho)} + d\Omega_{8-p}^2\right] \end{aligned} \tag{3.8}$$

Using (3.8), we can rewrite the non-susy Dp brane solution (3.1) as,

$$ds^2 = F(\rho)^{-\frac{1}{2}} G(\rho)^{\frac{\delta}{4}} \left(-dt^2 + \sum_{i=1}^{p} (dx^i)^2\right) + F(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1}{7-p}+\frac{\delta}{4}} \left(\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_{8-p}^2\right)$$
$$e^{2\phi} = F(\rho)^{\frac{3-p}{2}} G(\rho)^{\delta}, \qquad F_{[8-p]} = Q\text{Vol}(\Omega_{8-p}) \tag{3.9}$$

where $G(\rho)$ is as given in (3.7) and

$$F(\rho) = G(\rho)^{\frac{\alpha}{2}} \cosh^2\theta - G(\rho)^{-\frac{\beta}{2}} \sinh^2\theta \tag{3.10}$$

The parameter relations remain exactly the same as given in (3.3). The BPS limit now would be given as $\rho_p \to 0$, $\theta \to \infty$, such that $(1/2)(\alpha+\beta)\rho_p^{7-p}\sinh^2\theta = \bar{\rho}_p^{7-p} = \text{fixed}$ (where $\bar{\rho}_p^{7-p} = 4\bar{r}_p^{7-p}$). Note that in this case $G(\rho) \to 1$ and $F(\rho) \to \bar{H}(\rho)$, where $\bar{H}$ is the standard harmonic function of a BPS Dp brane and thus (3.9) reduces precisely to the BPS Dp brane solution. We will use the solution (3.9) to study both the minmally coupled scalar scattering and the graviton scattering in the following.

Minimally coupled scalar scattering

In this section we will study the scattering of a massless scalar (φ) coupled minimally to the non-susy Dp brane [49,53] background and obtain the form of the scattering potential it experiences while moving in the background. The reason for studying this, as we will see, is that the graviton also experiences the same scattering potential (studied in the next section) as the scalar. The relevant part of the action in this case is

$$S_{\text{scalar}} = \frac{1}{4\pi G_{10}} \int d^{10}x \sqrt{-g} e^{-2\phi} \partial_\mu \varphi \partial^\mu \varphi \tag{3.11}$$

where $g_{\mu\nu}$ is the background metric in string frame and ϕ is the dilaton field. Note that we have omitted the Ricci scalar, the kinetic energy of the dilaton and the RR form-field terms from the action since they don't contribute to the scalar equation of motion. From (3.11) the scalar φ is found to satisfy the following equation of motion,

$$D_\mu \partial^\mu \varphi - 2D_\mu \phi \partial^\mu \varphi = 0 \tag{3.12}$$

Now we assume that φ has spherical symmetry in the transverse space (that means we are considering only scalar s-wave, since the higher partial waves will give an additional repulsive centrifugal term and will have greater chances of decoupling) and is independent of the spatial coordinates of the brane (assumed for simplicity). Thus φ has the form,

$$\varphi = \Phi(\rho) e^{i\omega t} \tag{3.13}$$

Now using (3.13), the scalar equation of motion (3.12) with the background given in (3.9) reduces to the following second order differential equation,

$$\partial_\rho^2 \Phi(\rho) + \left[\frac{8-p}{\rho} + \frac{\partial_\rho G(\rho)}{G(\rho)}\right] \partial_\rho \Phi(\rho) + \omega^2 F(\rho) G(\rho)^{-\frac{6-p}{7-p}} \Phi(\rho) = 0 \tag{3.14}$$

Redefining the radial coordinate as, $u = \omega\rho$ and introducing a function $g(u)$ by

$$\Phi(u) = k(u) g(u), \qquad \text{where,} \quad k(u) = \frac{1}{\sqrt{u^{8-p} G(u)}}, \tag{3.15}$$

(3.14) takes the form of a Schrödinger-like equation in terms of the new function as,

$$\left(\partial_u^2 - V(u)\right) g(u) = 0 \tag{3.16}$$

where,

$$V(u) = \frac{(8-p)(6-p)}{4u^2} - \frac{1}{4}\left(\frac{\partial_u G(u)}{G(u)}\right)^2 - F(u)G(u)^{-\frac{6-p}{7-p}} \tag{3.17}$$

and

$$G(u) = 1 + \frac{(\omega\rho_p)^{7-p}}{u^{7-p}}, \qquad F(u) = G(u)^{\frac{\alpha}{2}}\cosh^2\theta - G(u)^{-\frac{\beta}{2}}\sinh^2\theta. \tag{3.18}$$

Once we have the form of $g(u)$ by solving the equation (3.16), we can obtain the form of the scalar as

$$\varphi(t,\rho) = \frac{1}{(\omega\rho)^{\frac{8-p}{2}}}\frac{g(\omega\rho)}{\sqrt{G(\omega\rho)}}e^{i\omega t}. \tag{3.19}$$

Eq.(3.17) represents the scattering potentials the minimally coupled scalar experiences while moving in the background of non-susy Dp branes. We will analyze these potentials (3.17) in the next section.

Graviton scattering

Scattering Potentials

In this section we will study the graviton scattering on the non-susy Dp branes given in (3.9). We will obtain the form of the scattering potential (which will have the same form (3.17) as the scattering potential of a minimally coupled scalar obtained in the previous section) and analyze it numerically. To study the scattering we need the linearized equations of motion from the action (2.1) in the background (3.9). The equations of motion can be linearized by perturbing the background metric as, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, where background metric is denoted with a 'bar'. The linearized forms of the dilaton and the graviton equations of motion are [1],

$$\Gamma(h)^{\mu}_{\mu\nu}\partial^{\nu}\phi + h^{\mu\nu}D_{\mu}\partial_{\nu}\phi = -\frac{1}{4}\bar{R}_{\mu\nu}h^{\mu\nu} + \frac{1}{4}\left(D_{\mu}D_{\nu}h^{\mu\nu} - D^2 h^{\mu}_{\mu}\right) + h^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi \tag{3.20}$$

$$\begin{aligned} & D_{(\mu}D_{\rho}h^{\rho}_{\nu)} - \frac{1}{2}D^2 h_{\mu\nu} - \frac{1}{2}D_{\mu}D_{\nu}h^{\rho}_{\rho} + \bar{R}_{\rho(\mu}h^{\rho}_{\nu)} + \bar{R}_{\nu\rho\sigma\mu}h^{\rho\sigma} \\ & = 2\Gamma(h)^{\rho}_{\mu\nu}\partial_{\rho}\phi - \frac{e^{2\phi}}{2\cdot(8-p)!}\Big((8-p)(7-p)h^{\rho\sigma}F_{(\mu|\rho}{}^{\mu_1\cdots\mu_{6-p}}F_{\nu)\sigma\mu_1\cdots\mu_{6-p}} \\ & \qquad -\frac{8-p}{2}\bar{g}_{\mu\nu}h^{\rho\sigma}F_{\rho}{}^{\mu_1\cdots\mu_{7-p}}F_{\sigma\mu_1\cdots\mu_{7-p}} + \frac{1}{2}h_{\mu\nu}F^2\Big) \end{aligned} \tag{3.21}$$

Again, for the reasons given before for the scalar, we here consider only the graviton s-wave and assume (for simplicity) it to be independent of the spatial directions of the brane. So, the graviton takes the form,

$$h_{\mu\nu} = \epsilon_{\mu\nu}h(\rho)e^{i\omega t} \tag{3.22}$$

where $\epsilon_{\mu\nu}$ is the polarization tensor for the graviton. Further, we take gravitons to have the polarizations along the brane only and therefore, $\epsilon_{\mu\nu} = 0$, for $\mu, \nu = p+1, p+2, \ldots, 9$. The transversality conditions of the graviton further restrict the polarization tensor, namely, we must have $\epsilon_{\mu 0} = 0$ and for the consistency with (3.20) we take it to be traceless, i.e., $g^{ab}\epsilon_{ab} = 0$, where $a,\ b = 0, 1, \ldots, p$. So, altogether there are $[p(p+1)/2]-1$ number of possible choices of polarization tensor ϵ_{ab}. Among them $p(p-1)/2$ are off-diagonal and $p-1$ are diagonal. We choose the only non-zero off-diagonal components as $\epsilon_{12} = \epsilon_{21} = 1$, and the only non-zero diagonal components as $\epsilon_{11} = -\epsilon_{22} = 1$ satisfying all the restrictions on the polarization tensor. Since both types of polarizations give the same equation for $h(\rho)$, we give its form using (3.21) and (3.9) as,

$$\begin{aligned} & \partial_{\rho}^2 h(\rho) + \left[\frac{8-p}{\rho} + \frac{\partial_{\rho}F}{F} + \left(1-\frac{\delta}{2}\right)\frac{\partial_{\rho}G}{G}\right]\partial_{\rho}h(\rho) \\ & + \left[\omega^2 F G^{-\frac{6-p}{7-p}} + \frac{1}{4}\left(\frac{\partial_{\rho}F}{F} - \frac{\delta}{2}\frac{\partial_{\rho}G}{G}\right)^2 - \frac{1}{2}\left(\frac{Q}{\rho^{8-p}}\right)^2 F^{-2}G^{-\frac{3-p}{4}\delta-2}\right]h(\rho) = 0 \end{aligned} \tag{3.23}$$

Again redefining the radial coordinate by $u = \omega\rho$, and introducing the same function $g(u)$ as before, but now is related to $h(u)$ by

$$h(u) = \tilde{k}(u)g(u), \qquad \text{where,} \quad \tilde{k}(u) = \frac{1}{\sqrt{u^{8-p}F(u)G(u)^{1-\frac{\delta}{2}}}}, \tag{3.24}$$

we find, after some manipulation, that (3.23) reduces precisely to the same Schrödinger-like form in terms of $g(u)$ as (3.16) with the potential as given in (3.17). However, the graviton solution $h_{\mu\nu}(t,\rho)$, is obviously different (as can be seen from (3.24)) from that of the minimally coupled scalar (3.19). It is therefore clear that the graviton also experiences the same scattering potentials as a minimally coupled scalar when it moves in the background of non-susy Dp branes.

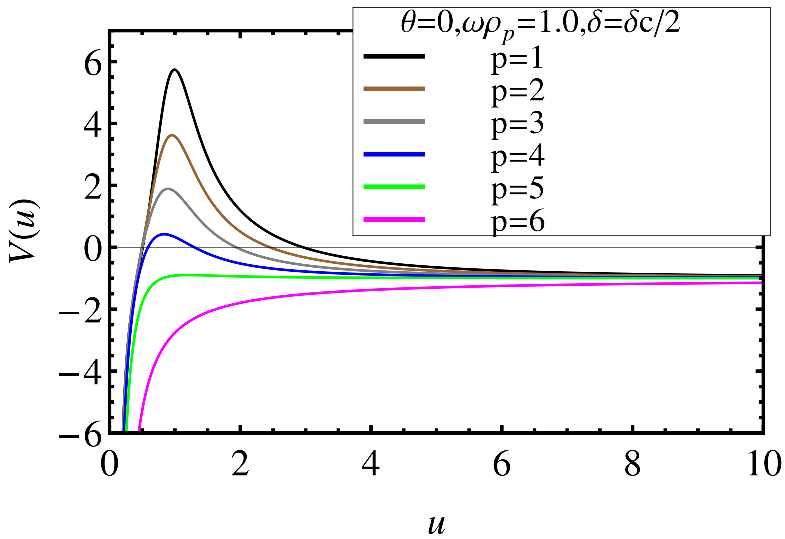

FIGURE 3.1: *Plot of the potential* $V(u)$ *given in* (3.17) *vs* u. *Here the potential is given for* $\theta = 0$, $\omega\rho_p = 1.0$ *and* $\delta = \delta_c/2 = \frac{2}{7-p}\sqrt{\frac{2(8-p)}{p+1}}$.

We now take a close look at the graviton scattering potential $V(u)$ given in (3.17). First we note that it has three terms of which the first one is positive for $p \leq 5$ and is zero for $p = 6$, and the second and third terms are always negative. So, the potential has both

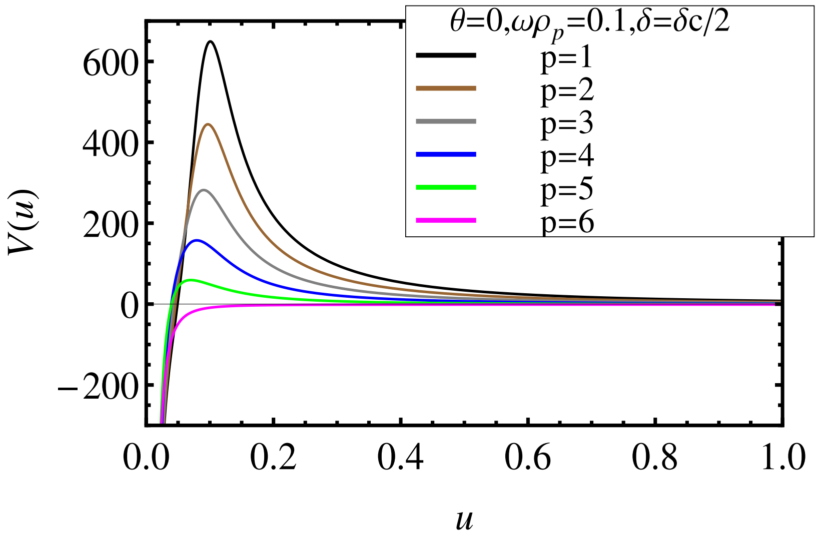

FIGURE 3.2: *Plot of the potential $V(u)$ given in* (3.17) *vs u. Here the potential is given for $\theta = 0$, $\omega\rho_p = 0.1$ and $\delta = \delta_c/2 = \frac{2}{7-p}\sqrt{\frac{2(8-p)}{p+1}}$.*

repulsive and attractive pieces for $p \leq 5$, but it is always (for all u) attractive for $p = 6$. In fact this is the reason that gravity does not decouple from the non-susy D6 branes. However, for all p, when $u \to \infty$, i.e., in the far region $G(u) = 1 + (\omega\rho_p)^{7-p}/u^{7-p} \to 1$ and as a result $F(u) = G(u)^{\alpha/2}\cosh^2\theta - G(u)^{-\beta/2}\sinh^2\theta \to 1$ and so, we find from (3.17) that the potential $V(u) \to -1$. On the other hand in the near region, i.e., when $u \to 0$, $V(u) \to -1/(4u^2) - F(u)G(u)^{-(6-p)/(7-p)}$ and since $F(u)$ is positive and the second term is negative, $V(u) \to -\infty$. So, one might think that since in the near region the potential is attractive, gravity will not decouple from the brane. But this is not true. The point is that in between $u = \infty$ and $u = 0$, there might exist some u, where the potential can have a maximum. We find that this is indeed the case, however, it is difficult to find the value of u analytically where the maximum of the potential occurs because of its complicated form. So, we have plotted the potentials $V(u)$ given in (3.17) versus u for different values of the parameters in Fig:3.1, Fig:3.2, and Fig:3.3. We have taken

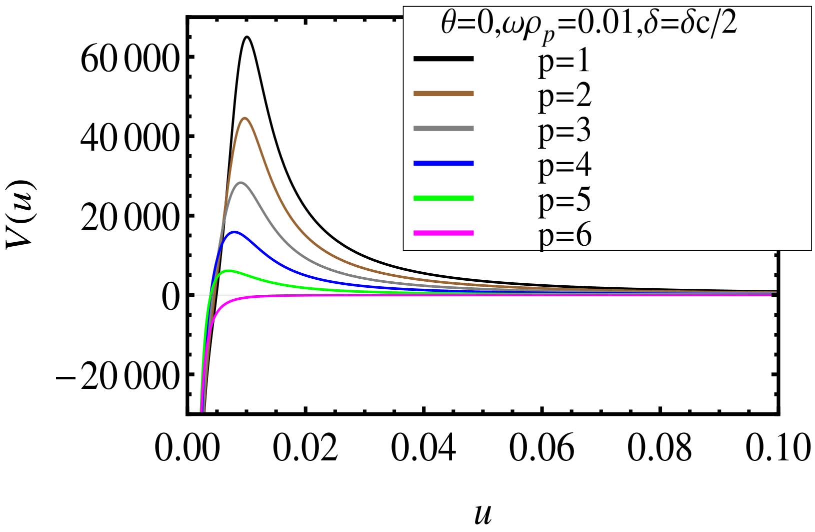

FIGURE 3.3: *Plot of the potential* $V(u)$ *given in* (3.17) *vs* u. *Here the potential is given for* $\theta = 0$, $\omega\rho_p = 0.01$ *and* $\delta = \delta_c/2 = \frac{2}{7-p}\sqrt{\frac{2(8-p)}{p+1}}$.

$\theta = 0$[4] and $\delta = \delta_c/2$, where $\delta_c = \frac{4}{7-p}\sqrt{\frac{2(8-p)}{p+1}}$ which is the maximum allowed value of $|\delta|$. These are some typical values of the parameters to see the variation of the potential $V(u)$. Actually we have seen that the variations of these two parameters have little effects on the potential. In Fig:3.1, Fig:3.2, and Fig:3.3, we have taken $\omega\rho_p = 1,\ 0.1,\ \text{and}\ 0.01$ and then plotted $V(u)$ for various values of p. We notice that for $\omega\rho_p = 1$, the potentials have maxima for each values of $p = 1,\ 2,\ 3,\ 4$ (also for $p = 5$ but it is not visible because of the scale chosen and will be clear in Fig:3.2, and Fig:3.3) except for $p = 6$. For $\omega\rho_p = 0.1$, the maxima shift towards the origin and rises sharply for all values of p except for $p = 6$. When $\omega\rho_p = 0.01$, the maxima of the potentials further shift towards the origin and take very large values for all values of p, but goes to large negative value for $p = 6$. Note that as we scale down $\omega\rho_p$ by a factor of 10, the potentials roughly rise by a factor of 100. So, the potentials are approaching to infinity much faster than $\omega\rho_p$ going to zero. Thus

[4]This is taken for simplicity. However, note that by the relation (3.3), this implies that the brane is chargeless and this is a characteristic of a non-susy brane as a BPS brane should always be charged. Therefore, when we vary $\omega\rho_p$ in the plots Fig:3.1, Fig:3.2, Fig:3.3, the branes always remain non-susy even when we take the decoupling limit, namely, $\omega\rho_p \to 0$.

eventually when we take $\omega\rho_p \to 0$, i.e., in the decoupling limit[5] the potentials will rise to positive infinity near the origin for all values of p except for $p = 6$. For $p = 6$ it will go to negative infinity without any maximum in between. Due to the infinite potential barriers, the graviton coming from infinity will not be able to overcome them and reach the brane and therefore bulk gravity gets decoupled on all the non-susy Dp branes for $p \leq 5$ and since for $p = 6$, there is no potential barrier, the graviton will reach the brane and therefore gravity does not decouple from the non-susy D6 brane. We will discuss more on $p = 5$ case towards the end of next section.

Now one can easily check that in the BPS limit ($\rho_p \to 0$, $\theta \to \infty$, such that $(1/2)(\alpha + \beta)\rho_p^{7-p}\sinh^2\theta = Q/(7-p) = \bar{\rho}_p^{7-p} = $ fixed), $G(\rho) \to 1$ and $F(\rho) \to 1 + Q/\{(7-p)\rho^{7-p}\}$, the standard harmonic function of a BPS Dp brane, the potentials $V(u)$ for the non-susy Dp branes as given in (3.17) reduce precisely to the scattering potentials of a graviton (or a minimally coupled scalar) for a BPS Dp branes given in Eq.(4.6) of [1].

To further support our claim that bulk gravity does decouple from the non-susy Dp brane world volumes we compute the graviton absorption cross-sections on the non-susy Dp branes in the next section and show that they vanish in the decoupling limit.

Absorption cross-section

To compute the graviton absorption cross section we will solve the Schrödinger-like scattering equation given in (3.16),

$$\left(\partial_u^2 - V(u)\right) g(u) = 0, \quad \text{with,} \quad V(u) = \frac{(8-p)(6-p)}{4u^2} - \frac{1}{4}\left(\frac{\partial_u G(u)}{G(u)}\right)^2 - F(u)G(u)^{-\frac{6-p}{7-p}} \tag{3.25}$$

[5] Note that $\rho_p \sim \ell_s$, where ℓ_s is the fundamental string length and so, in the decoupling limit $\ell_s \to 0$ implies $\rho_p \to 0$. $\omega\rho_p$ is the dimensionless parameter which also tends to zero in the decoupling limit.

where $G(u)$ and $F(u)$ are as given before in (3.18). It is difficult to solve this equation in general and so, as usual, we will solve it both in the far region and in the near region and then match the two solutions in the overlapping region to fix a constant in the solution. In the far region $u \gg \omega\rho_p$ and therefore $G(u) \approx 1$ and $F(u) \approx 1$ and then (3.25) reduces to

$$\left[\partial_u^2 + \left(1 + \frac{1 - 4\left(\frac{7-p}{2}\right)^2}{4u^2}\right)\right] g(u) = 0 \tag{3.26}$$

This is the Bessel equation of order $\nu = \frac{7-p}{2}$ and so, the solution is,

$$g(u) = c_\infty \sqrt{u} J_{\frac{7-p}{2}}(u) \tag{3.27}$$

where $J(u)$ is the Bessel function and c_∞ is a constant. Now with this solution the graviton takes the form (see (3.24))

$$h(u) = \frac{g(u)}{\sqrt{u^{8-p} F(u) G(u)^{1-\frac{\delta}{2}}}} = \frac{c_\infty u^{-\frac{7-p}{2}} J_{\frac{7-p}{2}}(u)}{\sqrt{F(u) G(u)^{1-\frac{\delta}{2}}}} \tag{3.28}$$

Let us define another function by,

$$\tilde{g}(u) = \frac{g(u)}{\sqrt{u^{8-p}}}, \qquad \text{and so,} \quad h(u) = \frac{\tilde{g}(u)}{\sqrt{F(u) G(u)^{1-\frac{\delta}{2}}}} \tag{3.29}$$

then the solution in the far region is

$$\tilde{g}_\infty(u) = c_\infty u^{-\frac{7-p}{2}} J_{\frac{7-p}{2}}(u) \tag{3.30}$$

Now to find a solution in the near region we note that, unlike in the BPS case [1,37,38], here it is difficult to get an analytic solution of the equation (3.25) in general for all u in the near region. So, in the following we go over to certain region of space (within the near region) where we can solve (3.25) exactly and also have a matching in the overlapping region of the solution (3.30) in the far region and that in the near region. This is possible as we will see if the parameters of the solution are chosen appropriately. Now in the near region, we first make a coordinate transformation

$$z = \frac{2}{5-p} \frac{(\omega\rho_p)^{\frac{7-p}{2}}}{u^{\frac{5-p}{2}}}, \qquad \text{for,} \quad p < 5 \tag{3.31}$$

and note that as long as $z \gg (\omega\rho_p)^{\frac{7-p}{2}}$, $u \ll 1$, i.e., we are in the near region where we will find a solution of Eq.(3.25). The solution in the far region ($u \gg \omega\rho_p$) and that in the near region ($u \ll 1$) must be matched in the overlapping region to find the arbitrary constant c_∞ and so, $\omega\rho_p \ll 1$ and this gives a restriction on the parameter ρ_p of the non-susy Dp brane solutions. We further impose the condition that $z \ll \omega\rho_p$. Note that when $\omega\rho_p \ll 1$, there is no contradiction of this with the previous condition $z \gg (\omega\rho_p)^{\frac{7-p}{2}}$. So, z is in the range $(\omega\rho_p)^{\frac{7-p}{2}} \ll z \ll \omega\rho_p$. The scattering equation (3.25) then can be simplified as,

$$\left[\partial_z^2 + \left(\frac{1 - 4\frac{(7-p)^2}{(5-p)^2}}{4z^2}\right) + \left(\frac{\frac{2}{5-p}(\omega\rho_p)}{z}\right)^{\frac{2(7-p)}{5-p}} \times \left\{1 + \left(b^2 - \frac{6-p}{7-p}\right)\left(\frac{z}{\frac{2}{5-p}(\omega\rho_p)}\right)^{\frac{2(7-p)}{5-p}}\right\}\right] \hat{g}(z) = 0 \tag{3.32}$$

where we have defined,

$$b = \sqrt{\frac{\alpha}{2}\cosh^2\theta + \frac{\beta}{2}\sinh^2\theta}, \qquad \text{and} \qquad \hat{g}(z) = z^{\frac{7-p}{2(5-p)}} g(z) \tag{3.33}$$

Note from (3.32) that since $z \ll \omega\rho_p$, the first term in the curly bracket will dominate (assuming $b \sim \mathcal{O}(1)$ or $\ll 1$) and that will make the differential equation difficult to solve. On the other hand, if we further impose the condition on the parameter b such that

$z \gg b^{-\frac{5-p}{7-p}}(\omega\rho_p)$, then it is the b^2 term in the curly bracket which will dominate. We point out that this is possible if $b \gg 1$, implying that the parameter θ is large[6] and since θ is an independent parameter of the solution both the conditions $z \gg (\omega\rho_p)^{\frac{7-p}{2}}$ and the previous one can be simultaneously satisfied. Now with these conditions (3.32) can be rewritten in the following form,

$$\left[\partial_{\hat{z}}^2 + \left(1 + \frac{1 - 4\frac{(7-p)^2}{(5-p)^2}}{4\hat{z}^2}\right)\right]\hat{g}(\hat{z}) = 0 \tag{3.34}$$

Here the radial coordinate $\hat{z}$ is defined as $\hat{z} = bz$. Again we recognize (3.34) as the Bessel equation of order $\nu = \frac{7-p}{5-p}$ and since we are interested in incoming wave for the near region, the relevant solution is,

$$\hat{g}(z) = i\sqrt{b}z^{\frac{1}{2}}\left(J_{\frac{7-p}{5-p}}(bz) + iN_{\frac{7-p}{5-p}}(bz)\right) \tag{3.35}$$

where in the above J denotes the Bessel function, N denotes the Neumann function. The near solution $\tilde{g}_0(z)$ therefore takes the form,

$$\tilde{g}_0(z) = iz^{\frac{7-p}{5-p}}\left(J_{\frac{7-p}{5-p}}(bz) + iN_{\frac{7-p}{5-p}}(bz)\right) \tag{3.36}$$

Now the two solutions (3.30) and (3.36) can be matched in the common region when $\omega\rho_p \ll 1$ and this determines the constant c_∞ in terms of known constants as,

$$c_\infty = \frac{1}{\pi}2^{\frac{(7-p)^2}{2(5-p)}}b^{-\frac{7-p}{5-p}}\Gamma\left(\frac{7-p}{5-p}\right)\Gamma\left(\frac{9-p}{2}\right) \tag{3.37}$$

[6]Note that when we plotted the potential in Fig:3.1, Fig:3.2, Fig:3.3 to show the decoupling we have taken $\theta = 0$ for simplicity, but, we have checked that potential has very similar behavior even for large θ. This shows that decoupling actually occurs for generic values of θ, however, a closed form solution of the scattering equation is possible only for large value of θ satisfying the condition given above.

Therefore, we can write the solutions both in the near region ($u \ll 1$ or $(\omega\rho_p)^{\frac{7-p}{2}} \ll z \ll \omega\rho_p$ and $z \gg b^{-\frac{5-p}{7-p}}(\omega\rho_p)$) and in the far region ($u \gg \omega\rho_p$) as follows,

$$\tilde{g}_0(z) = iz^{\frac{7-p}{5-p}}\left(J_{\frac{7-p}{5-p}}(bz) + iN_{\frac{7-p}{5-p}}(bz)\right)$$
$$\tilde{g}_\infty(u) = \frac{1}{\pi}2^{\frac{(7-p)^2}{2(5-p)}}b^{-\frac{7-p}{5-p}}\Gamma\left(\frac{7-p}{5-p}\right)\Gamma\left(\frac{9-p}{2}\right)u^{-\frac{7-p}{2}}J_{\frac{7-p}{2}}(u) \tag{3.38}$$

Once we have the function $\tilde{g}(u)$, we can obtain the form of the graviton by using (3.29) and (3.22) as,

$$h_{\mu\nu} = \epsilon_{\mu\nu}\frac{\tilde{g}(\omega\rho)}{\sqrt{F(\omega\rho)G(\omega\rho)^{1-\frac{\delta}{2}}}}e^{i\omega t} \tag{3.39}$$

The graviton flux can then be calculated from (3.39) using the standard definition as,

$$F = i\int_{\rho=\rho_S}\sqrt{-\bar{g}}e^{-2\bar{\phi}}\bar{g}^{\rho\rho}\left((\partial_\rho h^*_{\mu\nu})h^{\mu\nu} - h^*_{\mu\nu}\partial_\rho h^{\mu\nu}\right)d^{p+1}xd\Omega_{8-p} \tag{3.40}$$

The integral is over a constant surface of radius $\rho = \rho_S$. We use (3.39) with the solutions (3.38) for the incoming waves of the graviton in (3.40) to obtain the absorption cross-section as,

$$\sigma_p(b,\omega,\rho_p) = \frac{(2\pi)^{8-p}}{\omega^{8-p}\Omega_{8-p}}\left|\frac{F_0^{\rm in}}{F_\infty^{\rm in}}\right|$$
$$= \frac{\pi^{\frac{11-p}{2}}\left(\frac{2}{5-p}\right)^{\frac{9-p}{5-p}}}{2^{\frac{4}{5-p}}\left[\Gamma\left(\frac{7-p}{5-p}\right)\right]^2\Gamma\left(\frac{9-p}{2}\right)}b^{\frac{2(7-p)}{5-p}}\omega^{\frac{9-p}{5-p}}\rho_p^{\frac{(7-p)^2}{5-p}} \tag{3.41}$$

where b is a particular combination of parameters of non-susy Dp brane solutions defined in (3.33) and $\Omega_d = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})}$ is the volume of a d-dimensional unit sphre. $F_0^{\rm in}$ and $F_\infty^{\rm in}$ are the graviton fluxes for the incoming waves in the near and the far regions respectively. We thus see that the absorption cross-sections (3.41) for the non-susy Dp branes depend on all the three parameters of the solutions, namely, ρ_p, θ and δ (through b) as expected.

Now since $\rho_p \sim \ell_s$, the absorption cross-sections in string units take the form,

$$\frac{\sigma_p(b,\omega,\rho_p)}{\ell_s^{8-p}} \sim (\omega\rho_p)^{\frac{9-p}{5-p}}, \tag{3.42}$$

which indeed vanish in the decoupling limit $\omega\rho_p \to 0$ as long as $p \leq 4$, showing that graviton does not reach the branes or it decouples for non-susy Dp branes for $p \leq 4$. We will discuss $p = 5,\ 6$ cases separately. The formula of the absorption cross-sections for the BPS Dp branes can be recovered from (3.41) by the BPS limit we discussed before. The BPS limit is given as $\rho_p \to 0$ and $\theta \to \infty$ (or $b \to \infty$) such that the product $b^2\rho_p^{7-p} = \bar{\rho}_p^{7-p} =$ fixed. We note that in this limit $\sigma_p(b,\omega,\rho_p) \to \sigma_p^{\rm BPS}(\omega,\bar{\rho}_p)$, where $\sigma_p^{\rm BPS}$ is the absorption cross-section on the BPS Dp branes obtained before in [1].

TABLE 3.1: We compare the BPS results we obtain with those given in the Table 1 of [1]. We find that there are some mismatch in the expressions of σ_p for $p = 1, 2$. The numerical factors for $p = 1$ we get is $\frac{\pi^4}{12}$ instead of $\frac{2\pi^4}{3}$ and for $p = 2$ the factor we get is $\frac{\pi^2(\Gamma(1/3))^2}{\sqrt[3]{3}\,5}$ instead of $\frac{\pi^3(\Gamma(1/3))^2}{\sqrt[3]{3}\,2^2\,5}$. For $p = 3, 4$ the numerical factors match with our results.

Dp-brane	$\frac{\sigma_p^{non-susy}}{\sigma_p^{BPS}}$	$\sigma_p^{BPS}/\bar{\rho}_p^{8-p}$
D1	$\left(\frac{b^2\rho_1^6}{\bar{\rho}_1^6}\right)^{\frac{3}{2}}$	$\frac{\pi^4}{12}(\omega\bar{\rho}_1)^2$
D2	$\left(\frac{b^2\rho_2^5}{\bar{\rho}_2^5}\right)^{\frac{5}{3}}$	$\frac{\pi^2(\Gamma(1/3))^2}{5\sqrt[3]{3}}(\omega\bar{\rho}_2)^{\frac{7}{3}}$
D3	$\left(\frac{b^2\rho_3^4}{\bar{\rho}_3^4}\right)^2$	$\frac{\pi^4}{8}(\omega\bar{\rho}_3)^3$
D4	$\left(\frac{b^2\rho_4^3}{\bar{\rho}_4^3}\right)^{\frac{3}{2}}$	$\frac{2\pi^3}{3}(\omega\bar{\rho}_4)^5$

For $p = 5$, the coordinate transformation in the near region (3.31) does not work and so the same method of obtaining the absorption cross-section would be problematic. Anyway, we will look at the potential (3.25) and argue how the decoupling occurs in this case.

The potential (3.25) for $p = 5$ takes the form,

$$V(u) = \frac{3}{4u^2} - \frac{1}{4}\left(\frac{\partial_u G(u)}{G(u)}\right)^2 - F(u)G(u)^{-\frac{1}{2}} \tag{3.43}$$

We simplify the potential in the near region $\omega\rho_5 \ll u \ll 1$ (this is possible when $\omega\rho_5 \ll 1$) as we have done for other Dp brane cases before. Then the potential (3.43) takes the form,

$$V(u) = \frac{3}{4u^2} - \left[1 + (b^2 - \frac{1}{2})\frac{(\omega\rho_5)^2}{u^2}\right] \tag{3.44}$$

where b is as defined in (3.33). If b is $\mathcal{O}(1)$ or $\ll 1$, then the second term in the square bracket of (3.44) can be neglected as $u \gg \omega\rho_5$ and so, $V(u) = 3/(4u^2) - 1$. Also since $u \ll 1$, the first term dominates and eventually becomes infinite in the near region. So, the graviton won't be able to reach the brane surmounting this infinite barrier and there is decoupling of gravity. On the other hand when θ is such that $b\omega\rho_5 \gg u$, then the potential becomes,

$$V(u) = \frac{3}{4u^2} - b^2\frac{(\omega\rho_5)^2}{u^2} \tag{3.45}$$

In this case, the potential acts as an infinite barrier as long as $\omega < \frac{\sqrt{3}}{2}\frac{1}{b\rho_5}$ and there is a decoupling and if $\omega > \frac{\sqrt{3}}{2}\frac{1}{b\rho_5}$, gravity couples to D5 brane. So, this gives a restriction on the energy of the incident graviton for the decoupling to occur. This situation is very similar to the BPS D5 brane discussed in [1].

For $p = 6$, the scattering potential (3.25) takes the form

$$V(u) = -\frac{1}{4}\left(\frac{\partial_u G(u)}{G(u)}\right)^2 - F(u) \tag{3.46}$$

Now since both the terms here are negative, there is no maximum anywhere and it is a monotonically decreasing function as u varies from ∞ to 0 without any barrier. Therefore, gravity will couple to non-susy D6 branes similar to BPS D6 branes.

We would like to remark that in obtaining the closed form expressions for the absorption cross-sections for the graviton we solved the scattering equation (3.25) analytically. However, in solving (3.25) we had to assume two conditions (i) $\omega\rho_p \ll 1$ (needed to have an overlapping region for the far and the near brane solutions) and also (ii) b (or θ) $\gg 1$ (needed to have a closed form solution of (3.32)). Note that these two conditions[7] together imply that the non-susy Dp brane solutions are near-BPS or near-extremal. So, our result for the absorption cross-sections (3.41) can be trusted only in this near-extremal case. When we are far away from the extremality point there can be large corrections to the absorption cross-sections. But since our numerical results suggest that the decoupling must occur even when we are far away from the extremality point, the corrections must vanish in the decoupling limit $\omega\rho_p \to 0$, which implies that the corrections must involve a positive power of $\omega\rho_p$.

Conclusion

To conclude, in this chapter we have studied graviton scattering on the non-susy Dp branes of type II string theories. We obtained the linearized equation of motion of the graviton in the background of non-susy Dp branes. We have shown that both the minimally coupled scalar and the graviton essentially satisfy the same Schrödinger-like scattering equation and from there we identify the potential the minimally coupled scalar or the graviton experiences while moving in this background. For all non-susy Dp brane backgrounds

[7]One might think that these two conditions suggest that non-susy branes are becoming BPS branes in the decoupling limit and what we are obtaining is basically the decoupling of gravity for these BPS branes and not the non-susy branes. However, the reason why this is not so is because these two conditions are not correlated. Note that in order to get BPS branes we must take $b \to \infty$ and $\omega\rho_p \to 0$ simultaneously such that the product $b^2(\omega\rho_p)^{7-p} = (\omega\bar{\rho}_p)^{7-p}$ = finite. Here in our calculation, there is no relation between the two conditions and they are taken independently. So, when we take the decoupling limit $\omega\rho_p \to 0$, b remains large but finite and therefore, the non-susy branes remain non-susy. But if we take b also to infinity in the correlated way we just mentioned, then we recover graviton scattering cross-section result for the BPS brane as expected.

we found that far away from the branes the potentials go over to -1, while near the branes they take the value $-\infty$ as in the case of BPS Dp branes. However, because of the complicated form of the potentials it is difficult to understand its behavior in between. So, we have studied them numerically and plotted the potentials $V(u)$ versus u in Fig:3.1, Fig:3.2, Fig:3.3 for various values of p in each Figure. As the non-susy branes have three independent parameters ρ_p, θ and δ, the potentials also depend on them. For simplicity we have set $\theta = 0$ and $\delta = \delta_c/2$, where δ_c is the maximum allowed value of δ described in section 2 and then plotted $V(u)$ for three different values of $\omega\rho_p$ (where ω is the energy of the graviton), namely, 1.0, 0.1 and 0.01 in the three Figures mentioned above to show the behavior in the intermediate region. We found that there are maxima in the potentials near the origin for each value of $p \leq 5$, but the maximum is absent for $p = 6$. When we lower the value of $\omega\rho_p$, the same feature remains, but the height of the maxima rise sharply and they shift more towards the origin. We thus concluded that when we take the decoupling limit $\omega\rho_p \to 0$, there will be infinite barriers close to the origin for all $p \leq 5$, but for $p = 6$, there will be no barrier and the potential will decrease monotonically to $-\infty$. Thus the graviton will not be able to reach the brane for $p \leq 5$ and there will be decoupling of bulk gravity from the brane world volume. On the other hand, for $p = 6$, since there is no barrier, graviton will couple to the brane.

To give further support to our claim, we tried to solve the graviton scattering equation for the non-susy Dp branes. We found that we can write a closed form solution in the far region away from the branes, but it is difficult to solve the differential equation in the near region for all u. We found that a closed form solution in the near region is possible if we go to a certain range of space within the near region. To make this possible we found that the parameters of the solutions must satisfy the conditions (i) $\omega\rho_p \ll 1$ and (ii) $b(\text{or } \theta) \gg 1$. We have emphasized that this does not mean that decoupling occurs only for large values of b. Decoupling occurs even for small values of b, but the closed form solutions is possible only if we use this condition. From these solutions we have computed the graviton absorption cross sections for the non-susy Dp branes and found that they depend on the parameter b as well as some positive powers of $\omega\rho_p$ for $p \leq 4$. Thus we found that in the decoupling limit $\omega\rho_p \to 0$, the graviton absorption cross-sections vanish for $p \leq 4$ and therefore gravity decouples. We have also compared our results with the BPS results obtained before in [1, 37, 38]. For $p = 5$, we were not able to solve the equation even with the conditions mentioned above, but by analyzing the scattering potential we concluded that in this case if the parameter b is $\mathcal{O}(1)$ or $\ll 1$, then gravity decouples for all energies of the graviton (unlike the BPS D5 brane), but if $b \gg 1$, then gravity decouples only if the energy of the graviton is below certain critical value, otherwise it couples, similar to BPS D5 brane. Even for $p = 6$, we argued by analyzing the scattering potential that the gravity in this case couples to D6 brane as there is no barrier in the potential for the graviton. Thus, contrary to what is known in the literature [47, 48], we have shown here that there is a decoupling of gravity on the non-susy Dp branes very similar to the BPS branes, modulo some differences for the case of D5 brane. We remarked that though the decoupling of gravity occurs in general for the non-susy Dp branes the closed form expressions for the absorption cross-sections can only be obtained for the near-BPS or near-extremal point. The expressions derived in (3.41) can not be trusted far away from the extremality and there can be large corrections proportional to some positive power of $\omega\rho_p$.

So far we have not discussed anything about the boundary theory or the theory on the brane. It would be very interesting to understand this aspect in detail. However, we would like to mention that since we are dealing with non-susy branes, there may be open string tachyon [54, 55] living on the brane. When $\theta = 0$, the charge of the non-susy branes vanish (see (3.3)) and the world volume theory in this case would be purely tachyonic field theory. But when $\theta \neq 0$, there are gauge fields on the brane and so, the theory in this case would be pure Yang-Mills theory coupled with tachyon. It can be checked that when $\alpha + \beta = 2$ and θ is large, the gravity theories reduce to some deformations of the near horizon geometry of BPS Dp branes. Thus the world volume theories would be some perturbations of pure Yang-Mills theories in various dimensions without any tachyon field. We hope to come back on these issues along with others in future.

In low energy limit, the decoupling of the graviton (a closed string mode) has given two decoupled geometries: the ten dimensional non-susy bulk and a new non-susy space near the brane. In the next chapter we will explore that 'near horizon geometry' of the non-susy brane.

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Chapter 4

Decoupling Limit And Gravity Theory

Previously mentioned AdS/CFT duality is very useful to understand the strong coupling behavior of field theory by studying the weakly coupled string theory or supergravity. However, the theories on both sides of this duality are supersymmetric and therefore not very realistic. For example, on the gravity side, the AdS geometry which is maximally supersymmetric arise from the near horizon limit of BPS D3 brane solution of type IIB string theory and is dual to a field theory which has superconformal symmetry, unlike the realistic QCD theory which is neither conformal nor supersymmetric. AdS/CFT correspondence has been extended for the less supersymmetric [43], non-conformal cases [44, 45] and even in other dimensions (other than three) [46] generally known as gauge/gravity duality (see [5] for a review). AdS/CFT type correspondence has also been studied for the non-supersymmetric (type 0) string theory solutions in [56–58].

There is no doubt that AdS/CFT type correspondence will be more useful if it can be understood for the non-supersymmetric case, where the associated field theory would be more like QCD and various strong coupling behavior of QCD can be understood by studying the dual gravity theory. However, the exact dual gravity theory which would correspond to QCD on the boundary is not known. But, it is clear that the relevant gravity solution must be non-supersymmetric. So, one could either start with a BPS brane-like solution and break the supersymmetry by compactification [59] or start directly with the non-supersymmetric brane-like solution of type II string theory [17,18,47]. Now, in order to see that gauge/gravity duality works for a brane-like gravitational background, there must exist a low energy or a decoupling limit for which a bulk graviton thrown towards the brane should not be able to reach it or in other words, the gravitational potential experienced by the bulk graviton due to the background must have an infinite barrier. One can also calculate the graviton absorption cross-section in the brane background and show that it vanishes in the decoupling limit. This exercise confirms that bulk gravity indeed gets decoupled from the brane in the decoupling limit and we have two equivalent decoupled theories, i.e., the theory on the brane which is non-gravitational and the theory in the bulk which is a gravitational theory. This is precisely what happens for the BPS Dp branes of type II string theory [1,37–42] and in chapter 3, we have shown that exactly the same phenomenon occurs for the non-supersymmetric Dp brane solutions of the same theory as well. Usually it is assumed that gauge/gravity duality should work even for the non-supersymmetric case and the results in previous chapter clearly indicate that this is indeed true.

In this chapter we will consider a specific case, namely, the non-supersymmetric D3 brane solution[1] of type IIB string theory and work out the decoupling limit more clearly.

[1]An anisotropic non-susy D3 brane solution has been shown by zooming into a particular space-time region, similar to the decoupling limit discussed in this chapter, to interpolate between AdS_5 black hole, AdS_5 soliton and a soft-wall gravity solution in [60].

We also obtain the throat geometry which would be the gravity dual of QCD like theory. Eventhough our results in chapter 3, show that decoupling should occur even when the non-susy D3 brane is chargeless, we haven't been able to figure out the concrete decoupling limit for this case. However, we can work out the decoupling limit only when the charge associated with the non-susy D3 brane is finite but large. In obtaining the decoupling limit for the non-susy case we will draw analogy from the BPS case and make sure that the decoupling limit goes over to the BPS D3 brane decoupling limit, when susy is restored. We will also show that the low energy excitations in the throat region and in the bulk get decoupled in the decoupling limit from the energy considerations. We then give the throat geometry which keeps the effective string action finite. Finally, by making an appropriate coordinate transformation, we show that the geometry is actually identical with the two parameter solution obtained previously by Constable and Myers [61]. They have given a gauge theory interpretation of this geometry using holography and have shown that indeed the gauge theory has certain QCD-like properties such as infrared confinement and running coupling constant in certain range of parameters. Our result actually justifies this interpretation since the decoupling of gravity on the brane suggests that there should be a dual non-gravitational field theory associated with this geometry in the decoupling limit. We further show that when we fix one of the parameters and make another coordinate transformation the geometry reduces precisely to the one studied by Csaki and Reece [62]. They interpreted the geometry as representing gauge theory with gluon condensate and thereby giving a natural IR cut-off in the theory. Again our result justifies this gauge theory interpretation due to decoupling of gravity on the brane.

This chapter is organized as follows. In the first section the non-supersymmetric D3-brane background is reviewed. We also discuss how to recover BPS D3 brane from this solution. In section 2, the decoupling limit is discussed. In section 3, the scaling assumed in the section 2 is applied to the non-susy background metric to obtain the throat geometry.

Using appropriate coordinate transformation, we have shown how background geometry reduce to the known geometry [61,62] claimed to be dual to QCD-like theory. Finally we conclude in section 4.

Non-supersymmetric D3 brane and its BPS limit

The forms of the non-supersymmetric Dp brane solutions of type II string theories are given in chapter 3. By putting $p = 3$ in (3.1) – (3.3) of that reference we write below the form of non-susy D3 brane solution of type IIB string theory as,

$$\begin{aligned} ds^2 &= F(r)^{-\frac{1}{2}} \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta}{2}} \left(-dt^2 + \sum_{i=1}^{3}(dx^i)^2\right) \\ &\qquad + F(r)^{\frac{1}{2}} \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta}{2}} \left(H(r)\tilde{H}(r)\right)^{\frac{1}{2}} \left(dr^2 + r^2 d\Omega_5^2\right) \\ e^{2\phi} &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{2\delta}, \qquad F_{[5]} = \frac{1}{\sqrt{2}}(1+*)Q\mathrm{Vol}(\Omega_5) \end{aligned} \tag{4.1}$$

The various functions appearing in the solution are defined as,

$$\begin{aligned} H(r) &= 1 + \frac{r_0^4}{r^4}, \qquad \tilde{H}(r) = 1 - \frac{r_0^4}{r^4} \\ F(r) &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\alpha} \cosh^2\theta - \left(\frac{\tilde{H}(r)}{H(r)}\right)^{\beta} \sinh^2\theta \end{aligned} \tag{4.2}$$

In the above the metric is given in the string frame and we have suppressed the string coupling constant g_s which is assumed to be small. $F_{[5]}$ is the self-dual RR 5-form and Q is the charge of the non-susy D3 brane. Note from (4.2) that because of the form of $\tilde{H}(r)$, the solution has a naked singularity at $r = r_0$ and the physical region is given by $r > r_0$. Further note that the solution is characterized by six parameters, namely, α, β, δ, θ, Q and r_0 of which r_0 has the dimension of length and others are dimensionless. The parameters appear as integration constants while solving the equations of motion. However, all these parameters are not independent as they must satisfy certain constraints for the consistency of the equations of motion. The constraints are,

$$\begin{aligned}
&\alpha = \beta \\
&\alpha^2 + \delta^2 = \frac{5}{2} \quad \Rightarrow \quad |\delta| \leq \sqrt{\frac{5}{2}} \\
&Q = 8\alpha r_0^4 \sinh 2\theta
\end{aligned} \tag{4.3}$$

Using (4.3) we have three independent parameters in the solution, namely, r_0, θ and δ. Now we rewrite the solution (4.1) in a form convenient for our study and to do that we make a coordinate transformation given by

$$r = \rho \left(\frac{1 + \sqrt{G(\rho)}}{2} \right)^{\frac{1}{2}}, \qquad \text{where,} \qquad G(\rho) = 1 + \frac{4r_0^4}{\rho^4} \equiv 1 + \frac{\rho_0^4}{\rho^4}. \tag{4.4}$$

The solution (4.1) then takes the form,

$$\begin{aligned}
ds^2 &= F(\rho)^{-\frac{1}{2}} G(\rho)^{\frac{\delta}{4}} \left(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + F(\rho)^{\frac{1}{2}} G(\rho)^{\frac{1+\delta}{4}} \left(\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2 \right) \\
e^{2\phi} &= G(\rho)^{\delta} \\
F_{[5]} &= \frac{1}{\sqrt{2}} (1 + *) Q \mathrm{Vol}(\Omega_5)
\end{aligned} \tag{4.5}$$

where, the function $F(\rho)$ is now given as,

$$F(\rho) = G(\rho)^{\frac{\alpha}{2}} \cosh^2 \theta - G(\rho)^{-\frac{\alpha}{2}} \sinh^2 \theta, \tag{4.6}$$

with $G(\rho)$ as given in (4.4). Note that the above non-supersymmetric D3-brane is asymptotically flat and the solution has a singularity at $\rho = 0$ which is the location of the brane. The constraint relations (4.3) do not change with the coordinate transformation except the charge relation where the old parameter r_0 is now replaced by the new parameter ρ_0 as, $Q = 2\alpha\rho_0^4 \sinh 2\theta$. We can compare the non-susy D3 brane solution (4.5) with the BPS D3 brane solution. First of all, note that the non-susy D3 brane solution (4.5) contains three parameters (r_0, θ, δ), whereas BPS D3 brane contains only one parameter (even the black D3 brane contains two parameters). Also BPS D3 brane is always charged under RR form-field, but the non-susy D3 brane can be chargeless by either putting θ or α (which is related to δ by (4.3)) or both to zero (see (4.3)). Finally, we note that for non-susy D3 brane, the dilaton is in general not constant, however, it can be made constant by setting δ to zero. But since α and δ are related by (4.3), they can not be simultaneously put to zero.

We will now see how one can recover BPS D3 brane solution from the non-susy D3 brane solution given in (4.5). For this we take the following double scaling limit

$$\begin{aligned} &\rho_0 \to 0 \\ &\theta \to \infty \\ \text{such that,}\quad &\frac{\alpha}{2}\rho_0^4(\cosh^2\theta + \sinh^2\theta) \to R^4 \;=\; \text{fixed} \end{aligned} \tag{4.7}$$

With this double scaling limit we easily check that $G(\rho) \to 1$ and $F(\rho)$ in (4.6) goes over to $\bar{H}(\rho) = 1 + R^4/\rho^4$ and $Q \to 4R^4$. Therefore the non-susy D3 brane solution reduces precisely to a BPS D3 brane solution. We will recollect this limit while considering the decoupling limit of non-susy D3 brane in the next section.

We would like to remark that as the solution given in (4.5) is not supersymmetric and has a naked singularity at $\rho = 0$, it is quite natural to ask whether the solution is stable under small classical perturbations. Unfortunately, the answer to this question with its full generality is not known. The study of stability under linear perturbations of non-supersymmetric space-time such as the Schwarzschild black holes both in four and higher dimensions has a long history and are given in [63–68]. These studies have been extended even for the globally naked singular solution in four and higher dimensions in [69, 70] and for the black p-brane solutions in higher dimensions in [71, 72]. Keeping in mind the cosmic censorship hypothesis one might think that globally naked singular solution, such as the one discussed in this Letter, must be unstable under linear perturbations, but careful analysis given in [69], suggests that this apprehension is not always correct and there are stable nakedly singular solutions for certain physical boundary conditions. This has also been corroborated in the study of [73].

In previous chapter, we have studied the dynamics of small classical graviton perturbations of scalar type (i.e., the perturbations are along the brane) and obtained a Schrödinger type equation satisfied by it. The analysis of the potential in this case suggests that at least for the scalar perturbations the background is stable. However, to claim that the space-time (4.5) is stable under linear perturbations we must also study the vector as well as the tensor perturbations with the proper boundary condition at the singularity [70].

Decoupling limit

In this section, we will discuss the decoupling limit of the non-susy D3 brane in analogy with BPS brane. As we know the decoupling limit is a low energy limit by which the fundamental string length $\ell_s = \sqrt{\alpha'} \to 0$. In this limit not only the interactions between the bulk theory and the theory living on the brane vanish, but also all the higher derivative terms in both the theories go to zero. Also as we have seen, in this limit, the classical scattering cross-section of a graviton moving in the brane background vanishes indicating that the bulk gravity possibly gets decoupled from the brane. This phenomenon is quite similar to the BPS case [1]. Now in order to find the decoupled geometry we make the following change of variables in analogy with BPS D3 brane [2, 5],

$$\begin{aligned}\rho &= \alpha' u\\ \rho_0 &= \alpha' u_0\\ \alpha \cosh^2\theta &= \tfrac{2g_{\rm YM}^2 N}{u_0^4 \alpha'^2} \quad = \frac{L^4}{u_0^4\alpha'^2}\end{aligned} \tag{4.8}$$

along with $\alpha' \to 0$. Note that in the above u and u_0 have the dimensions of energy i.e. $[L^{-1}]$ and are kept fixed as we take $\alpha' \to 0$, also $L^4 = 2g_{\rm YM}^2 N = \frac{R^4}{\alpha'^2}$ is a dimensionless parameter and remains fixed, where, $g_{\rm YM}^2 N$ is the 't Hooft coupling of the boundary theory. We would like to point out that in the limit, as $\alpha' \to 0$, $\rho_0 \to 0$ and $\theta \to \infty$, but, that does not imply that we have the BPS limit. This is because here ρ and ρ_0 go to zero with the same scale and therefore, $G(\rho)$ does not go to 1 as in the BPS limit. Furthermore, note from the last relation in (4.8) that in the limit $\alpha' \to 0$, $\alpha \cosh^2\theta \to \infty$. This implies from the charge relation in (4.3) that the charge of the non-susy D3 brane becomes very large in the decoupling limit. In chapter 3 we found that the decoupling must occur also for small or even zero charge of the non-susy D3 brane, but we have not been able to find the explicit decoupling limit for these cases.

Now to justify the decoupling limit (4.8), we will see how in this limit we can keep the energy of a particle in the throat region in string units as well as that measured by an observer at infinity fixed [5]. Since g_{tt} as given in (4.5) is not constant these two energies will not be the same. So, if E_p denotes the energy of a particle as measured by an observer at a finite distance ρ from the brane and E denotes that of the same particle as measured by an observer at infinity, then they are related by a red-shift factor given by,

$$E = \sqrt{g_{tt}} E_p = F(\rho)^{-\frac{1}{4}} G(\rho)^{\frac{\delta}{8}} E_p \tag{4.9}$$

Under the decoupling limit (4.8) the functions $G(\rho)$ and $F(\rho)$ become

$$\begin{aligned} G(\rho) &\to G(u) = 1 + \frac{u_0^4}{u^4} = \text{fixed} \\ F(\rho) &\to F(u) = \left(G(u)^{\frac{\alpha}{2}} - G(u)^{-\frac{\alpha}{2}}\right) \frac{L^4}{\alpha u_0^4 \alpha'^2} = \tilde{F}(u) \frac{L^4}{\alpha u_0^4 \alpha'^2} \end{aligned} \tag{4.10}$$

Then (4.9) takes the form

$$E = \tilde{F}(u)^{-\frac{1}{4}} \frac{\alpha^{\frac{1}{4}} u_0}{L} G(u)^{\frac{\delta}{8}} (\sqrt{\alpha'} E_p) \tag{4.11}$$

Therefore, if we keep $\sqrt{\alpha'}E_p$ fixed, then E will remain fixed since the other quantities on the rhs of (4.11) are fixed in the decoupling limit. This gives a consistency check of the decoupling limit with the energy of an arbitrary excited string state. We can recover the results for the BPS D3 brane from here by putting $u_0 \to 0$. Now, $G(u) \to 1$ and $F(u) = \tilde{F}(u)\frac{L^4}{\alpha u_0^4 \alpha'^2} \to \frac{L^4}{\alpha'^2 u^4}$ and the energy relation (4.11) reduces to $E = \frac{u}{L}(\sqrt{\alpha'}E_p)$, precisely that of a BPS D3 brane [5].

Throat geometry of non-susy D3 brane:

Here we will discuss the spacetime geometry for the non-susy D3 brane in the decoupling limit (4.8) we discussed in the previous section. In case of BPS D3 brane, the background becomes $AdS_5 \times S^5$ in the corresponding decoupling limit. We have seen in the last section that in the decoupling limit

$$\begin{aligned} G(\rho) &\to G(u) = 1 + \frac{u_0^4}{u^4} \\ F(\rho) &\to \tilde{F}(u) \frac{L^4}{\alpha u_0^4 \alpha'^2} \end{aligned} \tag{4.12}$$

where, $\tilde{F}(u) = G^{\frac{\alpha}{2}}(u) - G^{-\frac{\alpha}{2}}(u)$. The non-susy D3 brane solution (4.5) in the string frame becomes

$$ds^2 = \alpha' \frac{L^2}{u_0^2} \left[\tilde{F}(u)^{-\frac{1}{2}} G(u)^{\frac{\delta}{4}} \left(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + \tilde{F}(u)^{\frac{1}{2}} G(u)^{\frac{1+\delta}{4}} \left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \right]$$
$$e^{2\phi} = g_s^2 G(u)^{\delta} \tag{4.13}$$

Here we have restored the string coupling constant g_s. The Yang-Mills coupling constant is related to g_s by $g_{YM}^2 = 2\pi g_s$ and is independent of α'. Also in the above we have redefined the coordinates $(t,\, x^i) \to \frac{L^2}{\sqrt{\alpha} u_0^2}(t,\, x^i)$, for $i = 1,\, 2,\, 3$ and rescaled $L^2 \to \sqrt{\alpha} L^2$. The effective string coupling constant $e^{\phi} = \frac{g_{\rm eff}^2}{N} = g_s G(u)^{\frac{\delta}{2}} = \frac{g_{\rm YM}^2}{2\pi} G(u)^{\frac{\delta}{2}}$ is also independent of α' [46]. We, therefore, claim (4.13) to be the throat geometry of non-susy D3 brane. It can be easily checked that in the BPS limit $u_0 \to 0$ the above geometry reduces to $\mathrm{AdS}_5 \times \mathrm{S}^5$. The same geometry can also be obtained in the asymptotic limit, i.e., for $u \to \infty$. Now since there is decoupling of gravity on the non-susy D3 brane, this geometry must be dual to a QCD-like theory. To see that this is indeed true we first map the decoupled geometry (4.13) to the previously known geometry given by Constable and Myers [61] quite a while ago. In order to do that we redefine the function $F(u)$ as follows,

$$\tilde{F}(u) = \hat{F}(u) G(u)^{-\frac{\alpha}{2}}, \qquad \text{where,} \qquad \hat{F}(u) = G(u)^{\alpha} - 1 \tag{4.14}$$

the metric in the Einstein frame and the dilaton then take the forms,

$$ds^2 = \alpha' \frac{L^2}{u_0^2} \left[\hat{F}(u)^{-\frac{1}{2}} G(u)^{\frac{\alpha}{4}} \left(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + \hat{F}(u)^{\frac{1}{2}} G(u)^{\frac{1-\alpha}{4}} \left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \right]$$
$$e^{2\phi} = g_s^2 G(u)^{\delta} \tag{4.15}$$

Then we make a coordinate transformation

$$u = \bar{r}\left(1 + \frac{u_0^4}{4\bar{r}^4}\right)^{-\frac{1}{4}} \equiv \bar{r}\left(1 + \frac{\omega^4}{\bar{r}^4}\right)^{-\frac{1}{4}}. \qquad (4.16)$$

So the old harmonic function is reduced to the form,

$$G(u) \to \left(1 + \frac{2\omega^4}{\bar{r}^4}\right)^2, \qquad \text{and} \qquad \hat{F}(u) \to \left(1 + \frac{2\omega^4}{\bar{r}^4}\right)^{2\alpha} - 1 \equiv \hat{H}(\bar{r}) \qquad (4.17)$$

Therefore the metric and the dilaton in this new coordinate will be given as,

$$\begin{aligned} ds^2 &= \alpha' \frac{L^2}{u_0^2}\left[\hat{H}^{-\frac{1}{2}}\left(1 + \frac{2\omega^4}{\bar{r}^4}\right)^{\frac{\alpha}{2}}\left(-dt^2 + \sum_{i=1}^{3}(dx^i)^2\right)\right. \\ &\quad \left. + \hat{H}^{\frac{1}{2}}\left(1 + \frac{2\omega^4}{\bar{r}^4}\right)^{\frac{1-\alpha}{2}}\left(\frac{d\bar{r}^2}{\left(1 + \frac{\omega^4}{\bar{r}^4}\right)^{\frac{5}{2}}} + \frac{\bar{r}^2}{\left(1 + \frac{\omega^4}{\bar{r}^4}\right)^{\frac{1}{2}}} d\Omega_5^2\right)\right] \\ e^{2\phi} &= g_s^2\left(1 + \frac{2\omega^4}{\bar{r}^4}\right)^{2\delta} \end{aligned} \qquad (4.18)$$

Comparing this solution (4.18) with the Constable-Myers solution we find that they match exactly if we identify the parameters as $\alpha = \delta_{\rm CM}/2$ and $\delta = \Delta_{\rm CM}/2$, where we have denoted the Constable-Myers parameters with a subscript 'CM'. The parameter relation $\alpha^2 + \delta^2 = 5/2$ given in (4.3) then becomes $\delta_{\rm CM}^2 + \Delta_{\rm CM}^2 = 10$ and is precisely the parameter relation given in Constable-Myers solution. We thus claim that the throat geometry in the decoupling limit of the non-susy D3 brane solution is nothing but the Constable-Myers two parameter solution. Since we already found that the bulk gravity gets decoupled for the non-susy D3-brane in the decoupling limit, so, the throat geometry must be the gravity dual of some QCD-like theory as discussed by Constable and Myers and our calculation justifies that.

We remark that the Constable-Myers two parameter solution have also been shown in [61] to arise from a suitable scaling limit of a non-susy D3 brane solution very similar to the decoupling limit we have discussed. But how that scaling (β) is related to the physical low energy limit ($\alpha' \to 0$) is not clear there. Identifying $\beta = \cosh\theta$ in our solution we find from (4.8) that $\beta = L^2/(\sqrt{\alpha} u_0^2 \alpha')$ which indeed goes to infinity in the decoupling limit $\alpha' \to 0$. This clarifies why their scaled solution decouples gravity and represents a gravity dual of Yang-Mills type theory, actually a deformation of $D = 4$, $\mathcal{N} = 4$ super Yang-Mills theory, by breaking susy and conformal symmetry. This theory also contains massive fermions and scalars in the adjoint representation and has been shown to exhibit various QCD-like properties such as running coupling, confinement and mass gap in the glueball spectrum in certain range of parameters. Asymptotic bahavior of the dilaton and the volume scalar determine the expectation values of the gauge invariant dimension four Tr(F^2) and dimension eight Tr$(F^4 - (F^2)^2)$) operators in the gauge theory [61] in terms of the parameters of the gravity theory.

We will now show that the decoupled gravity background of non-susy D3 brane (4.15) describing QCD-like theory also studied by Constable and Myers can be reduced, in a special case, to another gravity background, supposed to describe infrared QCD-like theory which includes non-perturbative gluon condensate providing a natural IR cut-off for confinement, studied by Csaki and Reece [62]. For this we put $\alpha = 1$. So, by the constraint relation (4.3) we have $\delta = \pm\sqrt{\frac{3}{2}}$. Also, $\hat{F}(u)$ now takes the form

$$\hat{F}(u) = G(u) - 1 = \frac{u_0^4}{u^4} \tag{4.19}$$

Then the metric in the Einstein frame and the dilaton (4.15) take the forms in a new coordinate $z = \frac{L^2}{u}$,

$$ds^2 = \alpha' \left[\frac{L^2}{z^2} \left(G(z)^{\frac{1}{4}} \big(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 \big) + \frac{dz^2}{G(z)} \right) + L^2 d\Omega_5^2 \right]$$

$$e^{2\phi} = g_s^2 G(z)^{-\sqrt{\frac{3}{2}}} \tag{4.20}$$

where $G(z) = 1 + \frac{z^4 u_0^4}{L^8} \equiv 1 + \frac{z^4}{z_0^4}$. We have taken the negative sign for δ because as $z \to \infty$, we want to keep the dilaton small. Note also that the metric in (4.20) asymptotically ($z \to 0$) has the form AdS$_5 \times$ S^5. To cast the metric and the dilaton into the form of Csaki and Reece we need to go to another coordinate given by,

$$\hat{z} = z \left(\frac{1 + \sqrt{G(z)}}{2} \right)^{-\frac{1}{2}} \tag{4.21}$$

where $G(z)$ is as given above. From this coordinate relation it is clear that $\hat{z}$ is actually related to our original radial coordinate r (see (4.1)) by $\hat{z} = \frac{L^2}{r}$. Therefore, te original harmonic functions can now be expressed as,

$$\begin{aligned} H(r) &= 1 + \frac{r_0^4}{r^4} = 1 + \frac{\hat{z}^4}{\hat{z}_0^4} = H(\hat{z}) = \frac{2\sqrt{G(z)}}{1 + \sqrt{G(z)}} \\ \tilde{H}(r) &= 1 - \frac{r_0^4}{r^4} = 1 - \frac{\hat{z}^4}{\hat{z}_0^4} = \tilde{H}(\hat{z}) = \frac{2}{1 + \sqrt{G(z)}} \end{aligned} \tag{4.22}$$

where in the above we have defined $\hat{z}_0 = \frac{L^2}{r_0} = \sqrt{2} z_0$. Now using (4.22), we rewrite the metric and the dilaton given in (4.20) as,

$$\begin{aligned} ds^2 &= \alpha' \left[\frac{L^2}{\hat{z}^2} \left\{ \left(H(\hat{z}) \tilde{H}(\hat{z}) \right)^{\frac{1}{2}} \left(- dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + d\hat{z}^2 \right\} + L^2 d\Omega_5^2 \right] \\ &= \alpha' \left[\frac{L^2}{\hat{z}^2} \left\{ \sqrt{1 - \frac{\hat{z}^8}{\hat{z}_0^8}} \left(- dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) + d\hat{z}^2 \right\} + L^2 d\Omega_5^2 \right] \\ e^{2\phi} &= g_s^2 \left(\frac{1 + \frac{\hat{z}^4}{\hat{z_0}^4}}{1 - \frac{\hat{z}^4}{\hat{z_0}^4}} \right)^{-2\sqrt{\frac{3}{2}}} \end{aligned} \tag{4.23}$$

This is precisely the background studied by Csaki and Reece [62] as a gravity dual of a QCD-like theory. We have shown that this background is nothing but a special case of the throat limit of non-susy D3 brane. As we have seen in chapter 3 that the bulk gravity indeed gets decoupled from the non-susy D3 brane in the decoupling limit, so our calculation justifies the gauge theory (or QCD-like theory) interpretation of this gravity background.

Conclusion

To conclude, in this chapter we have obtained the decoupling limit and the throat geometry of the well-known non-susy D3 brane solution of type IIB string theory. In chapter 3 we have shown that there is a decoupling of gravity on non-susy Dp branes and so, there must exist a decoupling limit for which the corresponding geometry would represent the gravity dual of a gauge theory (non-gravitational) by gauge/gravity duality. Here we have explicitly shown that this expectation is indeed true and we have found the decoupling limit of the non-susy D3 brane in analogy with the BPS brane. The geometry under this decoupling limit gives the throat geometry of the non-susy D3 brane. Since D3 brane we considered is non-supersymmetric and non-conformal (it has non-zero dilaton) so, the decoupled geometry in this case must be the gravity dual of a non-supersymmetric, non-conformal gauge theory like QCD. To show that the throat geometry we found already has an interpretation as a gauge theory with QCD-like properties, we have performed a coordinate transformation and mapped the decoupled non-susy D3 brane geometry to the two parameter Constable-Myers solution which has been shown to have a running coupling and confinement in certain range of parameters. Further we have shown that if we restrict one of the parameters of the theory to a special value then the resulting geometry after another coordinate transformation can be mapped exactly with the gravity background studied by Csaki and Reece. This geometry has been argued to describe the gravity dual of a QCD-like gauge theory with non-perturbative gluon condensate providing a natural

IR cut-off in the theory. Our results in this chapter supports the gauge theory interpretation of this geometry as it is obtained from the non-susy D3 brane in the decoupling limit.

We would like to point out that the decoupling limit we have obtained in this chapter is only for the non-susy D3 brane with large RR charge. However, as we have seen in our previous work that decoupling occurs also for the zero charge non-susy D3 brane. But we have not been able to obtain the decoupling limit for the zero charge case. Since BPS branes are always charged, we can take a guidance from it to obtain the decoupling limit for the charged non-susy D3 branes, as we have done in this chapter, but there is no such analog for the chargeless case. However, it will certainly be interesting to understand the decoupling limit for the chargeless case and it remains an open problem.

It would be interesting to use gauge/gravity duality further to explore various properties of the QCD-like theory [74–77] that can be obtained from the throat geometry of the non-susy D3 brane background. For example, it may be possible to calculate the Wilson loop, static as well as velocity-dependent quark-antiquark potential, screening length, jet quenching parameter and compare with the results already known for the susy $\mathcal{N} = 4$ gauge theory. We will discuss some of these in the following chapter.

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Chapter 5

Study Of Some Properties Of QGP

According to the AdS/CFT duality, when the field theory is strongly coupled, the dual string theory is weakly coupled, i.e., given by supergravity and *vice-versa*. The correspondence has since been generalized to encompass a wider variety of gauge theories with different gravity duals and is now more aptly called the gauge-gravity duality [5]. The duality has proved to be an extremely powerful tool that allows us to gain valuable insights into the behavior of strongly coupled field theories (by mapping the system to a suitable holographic dual gravity theory) which are otherwise not accessible via the standard perturbative formalism. In the original proposal, both sides of the duality are required to respect supersymmetry and conformal symmetry. In particular, on the string theory side, one takes a large number (N) of coincident BPS D3 branes of type IIB string theory and looks at the decoupled geometry (throat of the D3 brane) near the branes which is a maximally symmetric AdS_5 ($\times$ S^5) space and relates it with $(3+1)$-dimensional $\mathcal{N}$=4, SU(N) super Yang-Mills (SYM) theory at large N, living on the boundary of AdS space.

It is well-known that when heavy ions like gold or lead, moving in opposite directions, collide head-on at ultra-relativistic energies, they produce a fluid-like state of matter made up of strongly interacting quarks and gluons better known as quark-gluon plasma (QGP) (see [78] for a recent review). Many of the properties of QGP like thermalization, chiral symmetry breaking, deconfinement, etc. have been studied in the past in RHIC and is also being currently studied extensively in LHC (For example, see this comprehensive review [79] and the references therein.). The theoretical frameworks at our disposal to explore this strongly coupled plasma are primarily lattice QCD and the AdS/CFT duality. Unlike lattice field theory, the AdS/CFT correspondence is tailor-made for studying the real-time dynamics that is of interest in many cases. In particular, the AdS/CFT formalism has been extensively used to study, among other properties, the screening length of a heavy quark-antiquark ($Q\bar{Q}$) pair as well as its potential [80–82]. In the holographic picture, one takes a stack of BPS D3 branes in the decoupling limit which is the gravity dual of $D = 4$, $\mathcal{N}$=4 SYM theory at large N and introduces a probe fundamental string in this background. To examine the effect of velocity on the $Q\bar{Q}$ pair, one boosts the gravity solution along a brane direction and to incorporate the effect of a non-zero temperature, one considers a 'black' brane. However, QCD being neither supersymmetric nor conformal, one may question whether BPS D3 brane is the appropriate framework to study the properties of QGP from a holographic perspective. This motivates us to consider non-supersymmetric gravity solutions that can potentially be used to model real-world QCD more faithfully. While the literature abounds with works addressing various observables related to supersymmetric gauge theories using the holographic correspondence [79], our purpose here is to examine whether the lack of supersymmetry has any significant effect on the qualitative features of heavy quark observables. Non-supersymmetric backgrounds representing the holographic dual of QCD-like theories have been constructed earlier from string theory [56, 59, 83, 84] as well as from phenomenological point of view [85–88] to study various aspects of QCD using the gauge/gravity duality. Type II string theories are known to admit BPS as well as non-supersymmetric (non-susy) Dp brane solutions [17, 18, 47]. If, like BPS branes, the non-susy branes also have a decoupling limit, they will naturally represent the gravity dual of a non-supersymmetric gauge theory like QCD. Indeed, in our recent works, we have shown, by studying graviton scattering in the background of non-susy Dp branes, that bulk gravity gets decoupled on the brane very similar to the case of BPS branes discussed in chapter 3. We also worked out the details of the decoupling limit for non-susy D3 branes and obtained the throat geometry discussed in chapter 4. This geometry, under a suitable coordinate transformation, has been shown to match with the Constable-Myers solution having many interesting properties like confinement and running coupling constant similar to QCD [61]. In this chapter, we consider a 'black' version of this solution which corresponds to the holographic dual of a non-susy gauge theory at finite temperature [24, 26]. We intend to study the screening length and the potential of a heavy $Q\bar{Q}$ pair (equivalently, called a dipole) moving through a hot plasma whose gravity dual is the non-susy background just mentioned. Our computation is similar in spirit to the one performed by Liu, Rajagopal and Wiedemann (LRW) [80, 82] for the $D = 4$, $\mathcal{N}$ = 4 SYM plasma. We introduce a fundamental string as a probe whose end points (representing the $Q\bar{Q}$ pair), separated by a distance ℓ, lie on the boundary in the x^1-x^3 plane and makes an angle θ with the x^3 direction, along which we have given the background a boost. Following LRW and the holographic dictionary, we compute the thermal expectation value of the time-like Wilson loop[1] $\langle W^F(\mathcal{C})\rangle$ in the fundamental representation by calculating the minimal world-sheet area swept out by the

[1] See [89–93] for some early computations of Wilson loop in AdS/CFT.

open string with a boundary which coincides with the loop $\mathcal{C}$. The precise relation between them is $\langle W^F(\mathcal{C})\rangle = \exp[i\mathcal{S}(\mathcal{C})] = \exp[iE(\mathcal{C})\mathcal{T}]$, where $\mathcal{S}(\mathcal{C})$ is the action (finite) for the extremal world-sheet and for time-like Wilson loop, $\mathcal{S}(\mathcal{C})$ is proportional to time $\mathcal{T}$, and, therefore, $E(\mathcal{C})$ represents the $Q\bar{Q}$ potential. Typically, this potential suffers from a divergence which can be cured by subtracting out the self-energy of the free quark and antiquark. In the process, we also compute the separation length of the $Q\bar{Q}$ pair or the dipole length whose maximum value gives the screening length. In the absence of analytical expressions, we obtain numerically the variations of the screening length as well as the potential with the velocity, orientation of the dipole with respect to the direction of the moving plasma and other parameters of the theory. Comparison of the obtained results with the supersymmetric LRW [82] counterparts leads us to the interesting observation that all the results exhibit qualitatively similar pattern irrespective of the presence of supersymmetry. That supersymmetry need not be an essential ingredient for these features to exist, is an important step towards a better understanding of QGP since the plasma generated in the collider experiments is not itself supersymmetric. The other parameters will be shown to be related to the temperature and the coupling in the boundary theory.

This chapter is organized as follows. In section 1, we review the non-susy D3 brane solution and its decoupling limit. Section 2 is devoted to the calculation of the thermal expectation value of the time-like Wilson loop from which we extract the formal expressions for the $Q\bar{Q}$ separation length and the $Q\bar{Q}$ potential. This is followed by the numerical results and their discussion in section 3. Finally, we conclude in section 4.

The ‘black’ non-susy D3 brane and the decoupling limit

The non-supersymmetric D3 brane solution and its decoupling limit has been discussed in chapter 4. Here, we consider a ‘black’ version of this solution. We take the non-susy Dp brane solution, anisotropic in t as well as one of the brane directions x^1, given in eqs.(4) and (5) of ref. [26]. For D3 brane, we put $p = 3$ and make it anisotropic only in t direction by setting $\delta_2 = \delta_0$ and $\bar{\delta} = (3/4)\delta_2$. We further put $\delta_1 + 2\delta_2 = \delta$ which enables us to eliminate δ_2 as an independent parameter. The resulting solution takes the form[2]

$$\begin{aligned}
ds^2 &= F(r)^{-\frac{1}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{-\frac{\delta}{4}-\frac{3\delta_1}{8}}\left[\left(\frac{H(r)}{\tilde{H}(r)}\right)^{\delta}(-dt^2)+\sum_{i=1}^{3}(dx^i)^2\right]\\
&\quad +F(r)^{\frac{1}{2}}(H(r)\tilde{H}(r))^{\frac{1}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{3\delta_1}{8}}\left(dr^2+r^2d\Omega_5^2\right)\\
e^{2\phi} &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{-3\delta+\frac{7\delta_1}{2}}, \qquad F_{[5]}=\frac{1}{\sqrt{2}}\left[1+*\right]Q\mathrm{Vol}(\Omega_5).
\end{aligned} \tag{5.1}$$

The metric is given in the Einstein frame and we have suppressed the string coupling constant g_s, in the above, which is assumed to be small. The ‘$*$’ stands for the Hodge dual. The various functions introduced above are defined as

$$\begin{aligned}
H(r) &= 1+\frac{r_0^4}{r^4}\\
\tilde{H}(r) &= 1-\frac{r_0^4}{r^4}\\
F(r) &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\alpha}\cosh^2\theta-\left(\frac{\tilde{H}(r)}{H(r)}\right)^{\beta}\sinh^2\theta.
\end{aligned} \tag{5.2}$$

[2]Note that the metric in the solution does not have the full Poincare symmetry ISO(1, 3) in the brane world-volume directions, rather it is broken to R $\times$ ISO(3) and that is the reason we call it ‘black’ non-susy D3 brane solution. However, we put black in inverted comma because this solution does not have a regular horizon like true black brane rather it has a singular horizon. But still one can define a temperature for this solution as we mention later.

The solution is characterized by seven parameters α, β, δ, δ_1, θ, r_0, and Q. However, not all of them are independent - rather they are constrained by the three relations

$$\begin{aligned}
&\alpha-\beta=-\frac{3}{2}\delta_1\\
&\alpha+\beta=\sqrt{10-\frac{21}{2}\delta^2-\frac{49}{4}\delta_1^2+21\delta\delta_1}\equiv\gamma(\delta,\delta_1)\\
&Q=4\gamma r_0^4\sinh 2\theta.
\end{aligned}\tag{5.3}$$

The constraints allow us to eliminate three of the parameters and the 'black' non-susy D3 brane solution is actually a four-parameter solution depending upon (δ, δ_1, r_0, θ) . To cast the solution in a simpler form, we make a coordinate transformation from r to ρ defined by

$$r=\rho\left(\frac{1+\sqrt{G(\rho)}}{2}\right)^{\frac{1}{2}},\qquad \text{where,}\qquad G(\rho)=1+\frac{4r_0^4}{\rho^4}\equiv 1+\frac{\rho_0^4}{\rho^4}.\tag{5.4}$$

Under this coordinate transformation, the various functions introduced above look like,

$$\begin{aligned}
&H(r)=1+\frac{r_0^4}{r(\rho)^4}=\frac{2\sqrt{G(\rho)}}{\sqrt{G(\rho)}+1}\quad \tilde{H}(r)=1-\frac{r_0^4}{r(\rho)^4}=\frac{2}{\sqrt{G(\rho)}+1}\\
&\frac{H(r)}{\tilde{H}(r)}=\sqrt{G(\rho)},\quad \left(H(r)\tilde{H}(r)\right)^{\frac{1}{2}}dr^2=G(\rho)^{-\frac{3}{4}}d\rho^2,\\
&\left(H(r)\tilde{H}(r)\right)^{\frac{1}{2}}r^2=G(\rho)^{\frac{1}{4}}\rho^2.
\end{aligned}\tag{5.5}$$

Plugging in (5.5) into the solution (5.1) and expressing the metric in the string frame by the relation $ds^2_{\text{str}}=e^{\phi/2}ds^2$, we obtain the 'black' non-susy D3 brane solution,

$$\begin{aligned}
ds^2_{\text{str}} &= F(\rho)^{-\frac{1}{2}}G(\rho)^{-\frac{\delta}{2}+\frac{\delta_1}{4}}\left[-G(\rho)^{\frac{\delta}{2}}dt^2+\sum_{i=1}^{3}(dx^i)^2\right]\\
&\quad +F(\rho)^{\frac{1}{2}}G(\rho)^{\frac{1}{4}-\frac{3\delta}{8}+\frac{5\delta_1}{8}}\left[\frac{d\rho^2}{G(\rho)}+\rho^2 d\Omega_5^2\right]
\end{aligned}$$

$$e^{2\phi} = G(\rho)^{-\frac{3\delta}{2}+\frac{7\delta_1}{4}}, \qquad F_{[5]} = \frac{1}{\sqrt{2}}(1+*)Q\mathrm{Vol}(\Omega_5). \tag{5.6}$$

The function $F(\rho)$ is given as,

$$F(\rho) = G(\rho)^{\alpha/2}\cosh^2\theta - G(\rho)^{-\beta/2}\sinh^2\theta. \tag{5.7}$$

The metric in the original solution (5.1) has a singularity at $r = r_0$ arising from $\tilde{H}(r)$ in (5.2), but the coordinate change shifts the singularity to $\rho = 0$ in the resultant metric (5.6). The parameter relations (5.3), however, are unaffected by the change of coordinate. It can be easily checked that for the following values of the parameters

$$\alpha+\beta = 2, \qquad \delta_1 = -\frac{12}{7}, \qquad \delta = -2, \quad \text{which imply,} \quad \alpha = \frac{16}{7}, \qquad \beta = -\frac{2}{7}, \tag{5.8}$$

the solution (5.6) reduces exactly to the standard black D3 brane solution [94] which, in the present coordinate, takes the form,

$$\begin{aligned} ds^2_{\mathrm{blackD3}} &= \tilde{F}(\rho)^{-\frac{1}{2}}G(\rho)^{\frac{1}{2}}\left[-G(\rho)^{-1}dt^2 + \sum_{i=1}^{3}(dx^i)^2\right] + \tilde{F}(\rho)^{\frac{1}{2}}\left(\frac{d\rho^2}{G(\rho)} + \rho^2 d\Omega_5^2\right) \\ e^{2\phi} &= 1, \qquad F_{[5]} = \frac{1}{\sqrt{2}}(1+*)Q\mathrm{Vol}(\Omega_5) \end{aligned} \tag{5.9}$$

where $\tilde{F}(\rho)$ is defined as,

$$\tilde{F}(\rho) = 1 + \frac{\rho_0^4\cosh^2\theta}{\rho^4} = F(\rho)G(\rho)^{1-\frac{\alpha}{2}}. \tag{5.10}$$

In order to recover the BPS D3 brane solution, as usual, we have to take a double scaling limit $\rho_0 \to 0$, $\theta \to \infty$ such that $\rho_0^4\cosh^2\theta \approx \rho_0^4\sinh^2\theta \to R^4$ (fixed). In that case, $G(\rho) \to 1$ and $\tilde{F}(\rho) \to 1 + R^4/\rho^4$ and the solution (5.9) reduces to the BPS D3 brane solution. The black D3 brane given in (5.9) has a horizon at $\rho = 0$. The temperature of the black D3 brane can be shown to have the value

$$T = \frac{1}{\pi\rho_0 \cosh\theta} \longrightarrow \frac{1}{\pi\rho_0 \sinh\theta}, \quad \text{(near extremality)}. \tag{5.11}$$

It has been argued in [95], that even though 'black' non-susy D3 brane (5.6) has an essential singularity at $\rho = 0$, still it is possible to define a temperature. By comparing the solution, given in eq.(3.7) of [95], with the solution (5.6) in the present chapter, supplemented with certain coordinate transformation, we find the temperature of the 'black' non-susy D3 brane solution to have the form near extremality[3],

$$T_{\text{nonsusy}} = \frac{(-2\delta)^{\frac{1}{4}}}{\sqrt{\gamma}} \frac{1}{\pi\rho_0 \sinh\theta}. \tag{5.12}$$

For the temperature to have a real value, we must demand that δ be negative. Further, when $\delta = -2$ and $\gamma = (\alpha + \beta) = 2$, the 'black' non-susy D3 brane reduces to ordinary black D3 brane and its temperature (5.12) reduces to that of the black D3 brane given in (5.11). On the other hand, when $\delta = 0$, the temperature vanishes - this is consistent with the fact that in this limit the metric becomes isotropic with ISO(1,3) symmetry and reduces to zero temperature non-susy D3 brane solution.

Next, we discuss the decoupling limit of the 'black' non-susy D3 brane (5.6). Decoupling limit is a low-energy limit in which the fundamental string length $\ell_s = \sqrt{\alpha'} \to 0$ and in analogy with black D3 brane, we make the following change of variables:

$$\rho = \alpha' u, \qquad \rho_0 \to \alpha' u_0, \qquad \cosh^2\theta = \frac{2L^4}{\gamma u_0^4 \alpha'^2}. \tag{5.13}$$

[3] It is not difficult to compute the ADM mass and also the charge of the 'black' non-susy D3 brane from the metric and the form-field given in (5.6). We get $M = \frac{\Omega_5 \rho_0^4}{2\kappa^2}\left[2(\alpha\cosh^2\theta + \beta\sinh^2\theta) + \left(5 - \frac{27\delta}{2} + \frac{31\delta_1}{2}\right)\right]$ and $|e| = \frac{\Omega_5\rho_0^4}{\sqrt{2}\kappa} 2(\alpha + \beta)\cosh\theta\sinh\theta$ and taking the ratio we get, $\frac{\sqrt{2}\kappa M}{|e|} \geq 1$ for finite θ. However, for large θ, which is assumed in this expression we find $\sqrt{2}\kappa M \to |e|$, indicating that in this limit the solution is near extremal. This also happens in the decoupling limit discussed below.

As we take $\alpha' \to 0$, the variable u and the parameter u_0 which have dimensions of energy, are kept fixed. Also in the above $L^4 = 2Ng_{\rm YM}^2 = R^4/\alpha'^2$, similar to the BPS D3 brane [2], is kept fixed. Now substituting (5.13), the decoupled geometry of 'black' non-susy D3 brane given in (5.6) assumes the form[4]

$$\begin{aligned} ds^2_{\rm str} &= \alpha' \left[\left(\frac{\sqrt{\gamma/2} u_0^2}{L^2} \right) F(u)^{-\frac{1}{2}} G(u)^{-\frac{\delta}{2} + \frac{\delta_1}{4}} \left(-G(u)^{\frac{\delta}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right) \right. \\ &\qquad \left. + \left(\frac{L^2}{\sqrt{\gamma/2} u_0^2} \right) F(u)^{\frac{1}{2}} G(u)^{\frac{1}{4} - \frac{3\delta}{8} + \frac{5\delta_1}{8}} \left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \right] \\ e^{2\phi} &= g_s^2 G(u)^{-\frac{3\delta}{2} + \frac{7\delta_1}{4}}, \qquad (5.14) \\ \text{with} &\qquad F(u) = G(u)^{\frac{\alpha}{2}} - G(u)^{-\frac{\beta}{2}}, \quad G(u) = 1 + \frac{u_0^4}{u^4}. \end{aligned}$$

where we have restored the string coupling constant g_s in the dilaton expression. To check the correctness of the decoupling limit and the geometry (5.14), we notice that for $\delta = -2$, $\gamma = 2$, $\beta = -2/7$, the metric reduces to the Schwarzschild black hole solution and for $u_0 \to 0$, the decoupling limit (5.13) reduces to that of BPS D3 brane and the metric reduces to $\text{AdS}_5 \times \text{S}^5$ form.

In the next section, we use this geometry to compute the screening length and $Q\bar{Q}$ potential in a hot, windy non-supersymmetric plasma. We note here that the parameters δ and δ_1 (notice that α, β are given in terms of δ and δ_1) can not take arbitrary values. First of all, since α, β are real, therefore, we get a restriction on δ and δ_1 from the second relation in (5.3). Also, for the supergravity description to remain valid $e^{2\phi}$ and the curvature of the string metric in (5.6) in units of α' must remain small. These two conditions will put further restrictions on the parameters δ and δ_1. We have taken these restrictions into account in our calculations in the following two sections.

[4]Note that since in the decoupling limit θ becomes very large as we take $\alpha' \to 0$ (see (5.13)), the solution becomes near-extremal by the discussion given in previous footnote. This also makes the solution stable as in the case of near extremal black D3 brane. We would like to thank Juan Maldacena for an e-mail correspondence on this issue.

Screening length and $Q\bar{Q}$ potential

Here we introduce a fundamental string as a probe in the gravity background we just described whose end points lie on the boundary ($\rho \to \infty$). The end points describing a $Q\bar{Q}$ pair is introduced in this way in the boundary non-supersymmetric gauge theory. These are heavy quarks and suppose the $Q\bar{Q}$ pair is moving with a velocity v along the x^3 direction of the boundary. We can pass on to the rest frame of the dipole in which the plasma is seen to move with a velocity $-v$ along the x^3 direction by inflicting the Lorentz boost

$$\begin{aligned} dt &\to dt \cosh\eta - dx_3 \sinh\eta \\ dx_3 &\to -dt \sinh\eta + dx_3 \cosh\eta \end{aligned} \tag{5.15}$$

where $\tanh\eta = v$ is the boost velocity. In terms of the boosted coordinates, the background reads

$$\begin{aligned} ds^2 &= \alpha' \Bigg[\left(\frac{\sqrt{\gamma/2} u_0^2}{L^2} \right) F(u)^{-\frac{1}{2}} G(u)^{-\frac{\delta}{2}+\frac{\delta_1}{4}} \Big(- \left(G(u)^{\frac{\delta}{2}} \cosh^2\eta - \sinh^2\eta \right) dt^2 \\ &\quad + \left(\cosh^2\eta - G(u)^{\frac{\delta}{2}} \sinh^2\eta \right) (dx^3)^2 - \left(1 - G(u)^{\frac{\delta}{2}} \right) \sinh 2\eta dt dx^3 + (dx^1)^2 + (dx^2)^2 \Big) \\ &\quad + \left(\frac{L^2}{\sqrt{\gamma/2} u_0^2} \right) F(u)^{\frac{1}{2}} G(u)^{\frac{1}{4}-\frac{3\delta}{8}+\frac{5\delta_1}{8}} \left(\frac{du^2}{G(u)} + u^2 d\Omega_5^2 \right) \Bigg] \qquad (5.16) \\ &\equiv \alpha' g_{\mu\nu} dx^\mu dx^\nu \\ e^{2\phi} &= g_s^2 G(u)^{-\frac{3\delta}{2}+\frac{7\delta_1}{4}}. \qquad (5.17) \end{aligned}$$

Note from the g_{tt} component of the metric in (5.16) that it has a singularity at a finite distance $u_c = u_0 \left(\tanh^{\frac{4}{\delta}} \eta - 1 \right)^{-\frac{1}{4}}$.

In order to study the dynamics of a probe string in this gravity background, we need to compute the Nambu-Goto string world-sheet action

$$S = -\frac{1}{2\pi}\int d\sigma d\tau \sqrt{-\det\left[h_{\alpha\beta}\right]}. \tag{5.18}$$

Here $h_{\alpha\beta}$ is the induced metric on the string world-sheet, i.e.,

$$h_{\alpha\beta} = g_{\mu\nu}\frac{\partial x^\mu}{\partial \xi^\alpha}\frac{\partial x^\nu}{\partial \xi^\beta} \tag{5.19}$$

and $\xi^{\alpha,\beta}$ are the world-sheet coordinates, $\xi^0 = \tau$ and $\xi^1 = \sigma$. For evaluating the Nambu-Goto action, we need to fix the parametrization of the string world-sheet. We choose our coordinates along the brane directions in such a way that the dipole lies in the x^1-x^3 plane and makes an angle θ with the x^3 direction while the Lorentz boost is in the t-x^3 plane. The parameterization of the coordinates are $\tau = t$, $x_1 = \sigma$, $x_2 =$ constant, $x_3 = x_3(\sigma)$ and $u = u(\sigma)$. If ℓ be the separation between the quark and the antiquark in the bound state, the projection of the dipole on the x_1 and x_3 directions are $\ell\sin\theta$ and $\ell\cos\theta$ respectively. At this stage, it is useful to introduce the following dimensionless coordinates

$$y = \frac{u}{u_0}, \quad x = u_0 x^1, \quad z = u_0 x^3. \tag{5.20}$$

In terms of the scaled coordinates, the boundary condition reads

$$\begin{aligned} y\left(x = \pm\frac{u_0\ell}{2}\sin\theta\right) &= \Lambda \\ z\left(x = \pm\frac{u_0\ell}{2}\sin\theta\right) &= \pm\frac{u_0\ell}{2}\cos\theta \end{aligned}$$

where $x \in [-\frac{u_0\ell}{2}\sin\theta, \frac{u_0\ell}{2}\sin\theta]$ and we assume that the gauge theory lives at $y = \Lambda$ (we will take $\Lambda \to \infty$ in the end). With the parametrization fixed, we can now evaluate the relevant components of the ten-dimensional metric $g_{\mu\nu}$ and hence, the components of the world-sheet metric $h_{\alpha\beta}$:

$$
\begin{aligned}
h_{\tau\tau} &= g_{tt} = -\left(\frac{\sqrt{\gamma/2}}{L^2}\right)\frac{\left(G(y)^{\frac{\delta}{2}}\cosh^2\eta - \sinh^2\eta\right)}{F(y)^{\frac{1}{2}}G(y)^{\frac{\delta}{2}-\frac{\delta_1}{4}}} \\
h_{\sigma\sigma} &= g_{11} + g_{33}\left(\frac{dx_3}{d\sigma}\right)^2 + g_{uu}\left(\frac{du}{d\sigma}\right)^2 \\
&= g_{xx} + g_{zz}z'^2 + g_{yy}y'^2 \\
&= \frac{\sqrt{\gamma/2}}{L^2}\frac{G(y)^{-\frac{\delta}{2}+\frac{\delta_1}{4}}}{\sqrt{F(y)}}\Big[1 + (\cosh^2\eta - G(y)^{\frac{\delta}{2}}\sinh^2\eta)z'^2 \\
&\qquad + \frac{2L^4}{\gamma}F(y)G(y)^{-\frac{3}{4}+\frac{\delta}{8}+\frac{3\delta_1}{8}}y'^2\Big] \qquad (5.21) \\
h_{\tau\sigma} &= 0
\end{aligned}
$$

Here we have also scaled t by $u_0 t$ to make it dimensionless. The functions are now given as

$$
G(y) = 1 + y^{-4}, \qquad \text{and} \qquad F(y) = G(y)^{\frac{\alpha}{2}}\cosh^2\theta - G(y)^{-\frac{\beta}{2}}\sinh^2\theta \qquad (5.22)
$$

The world-sheet action, therefore, can be written as

$$
S = -\frac{\mathcal{T}}{2\pi}\int_{-\frac{u_0\ell}{2}\sin\theta}^{+\frac{u_0\ell}{2}\sin\theta} dx\mathcal{L}(y(x)) \qquad (5.23)
$$

where $\mathcal{T}$ is the temporal span and the Lagrangian $\mathcal{L}$ is

$$
\begin{aligned}
\mathcal{L} &= \left[-g_{tt}\left\{g_{xx} + g_{zz}z'^2 + g_{yy}y'^2\right\}\right]^{\frac{1}{2}} \\
&= \frac{\sqrt{\gamma/2}}{L^2}\frac{\sqrt{G(y)^{\frac{\delta}{2}}\cosh^2\eta - \sinh^2\eta}}{F(y)^{\frac{1}{2}}G(y)^{\frac{\delta}{2}-\frac{\delta_1}{4}}}\Big[1 + (\cosh^2\eta - G(y)^{\frac{\delta}{2}}\sinh^2\eta)z'^2 \\
&\qquad + \frac{2L^4}{\gamma}F(y)G(y)^{-\frac{3}{4}+\frac{\delta}{8}+\frac{3\delta_1}{8}}y'^2\Big]^{\frac{1}{2}}. \qquad (5.24)
\end{aligned}
$$

A mere inspection of the Lagrangian tells us about the existence of two integrals of motion. Firstly, since z does not appear explicitly in the Lagrangian, we have a constant of motion

$$\frac{-g_{tt}g_{zz}}{\mathcal{L}}z' = p. \tag{5.25}$$

A second constant of motion arises from the fact that the Lagrangian does not depend explicitly upon x either, implying

$$\mathcal{L} - z'\frac{\partial\mathcal{L}}{\partial z'} - y'\frac{\partial\mathcal{L}}{\partial y'} = \frac{-g_{tt}g_{xx}}{\mathcal{L}} = k = \text{constant}. \tag{5.26}$$

To find the scaled radial distance where the string profile turns around, we impose the condition $\frac{dy}{dx} = 0$ which is tantamount to the constraint

$$k^2 + g_{xx}g^{zz}p^2 + g_{tt}g_{xx}\Big|_{y_t} = 0. \tag{5.27}$$

From the two constants of motion, we obtain

$$\frac{dz}{dx} = \frac{p}{k}g_{xx}g^{zz} \tag{5.28}$$

$$\frac{dy}{dx} = \frac{1}{k}\sqrt{g_{xx}g^{yy}}\left[-g_{tt}g_{xx} - p^2 g_{xx}g^{zz} - k^2\right]^{\frac{1}{2}} \tag{5.29}$$

which, upon integration, results in

$$\frac{u_0\ell}{2}\sin\theta = k\int_{y_t}^{\infty}\frac{1}{\sqrt{g_{xx}g^{yy}}\left[-g_{tt}g_{xx} - p^2 g_{xx}g^{zz} - k^2\right]^{\frac{1}{2}}}dy \tag{5.30}$$

$$\frac{u_0\ell}{2}\cos\theta = p\int_{y_t}^{\infty}\frac{g_{xx}g^{zz}}{\sqrt{g_{xx}g^{yy}}\left[-g_{tt}g_{xx} - p^2 g_{xx}g^{zz} - k^2\right]^{\frac{1}{2}}}dy. \tag{5.31}$$

The gravity solution has a IR cut-off at $y_c = \left(\tanh^{\frac{4}{\delta}}\eta - 1\right)^{-\frac{1}{4}}$ as we mentioned after (5.17) and the turning point y_t must satisfy $y_t \geq y_c$. y_t, in turn, is found by demanding that the terms in the denominator vanish separately and accepting the greater among the two possibilities. y_t is found out numerically and it is indeed found to satisfy the lower bound $y_t \geq y_c$. At the same time, changing the integral variable from x to y, the Nambu-Goto action in(5.23) is rewritten as

$$S = \frac{\mathcal{T}}{\pi} \int_{y_t}^{\infty} \frac{g_{tt} g_{xx}}{\sqrt{g_{xx} g^{yy}} \left[-g_{tt} g_{xx} - p^2 g_{xx} g^{zz} - k^2 \right]^{\frac{1}{2}}} dy. \tag{5.32}$$

Inserting the appropriate expressions for the metric components, it is easy to figure out that the action is afflicted by a divergence. This is, in fact, not surprising and is typical of calculations of this sort. The reason for this divergence is not far too seek. The action, in this form, actually receives contribution from the interaction energy of the $Q\bar{Q}$ pair (which we seek to find out) and also the self-energies of the quark and the antiquark. Since we are interested in the interaction energy, the next step is to get rid of the contribution coming from the self energies of the quark and the antiquark. To calculate the self energy, we use an open string hanging downwards from the boundary and whose end point on the boundary contains a quark/antiquark in fundamental representation. The relevant parametrization is $\tau = t$, $\sigma = u$, $x_3 = x_3(u)$, and $x_1 = x_2 =$ constant, which furnishes the following relations

$$h_{\tau\tau} = g_{tt}, \qquad h_{\sigma\sigma} = g_{yy} + g_{zz} \left(\frac{dz}{dy} \right)^2, \qquad h_{\tau\sigma} = g_{tz} \left(\frac{dz}{dy} \right). \tag{5.33}$$

The Nambu-Goto action now takes following form

$$S_{\text{free}} = -\frac{\mathcal{T}}{\pi} \int_{y_c}^{\infty} \mathcal{L}_0 dy$$

where

$$\mathcal{L}_0 = \sqrt{-g_{tt}g_{yy} + (g_{tz}^2 - g_{tt}g_{zz})\left(\frac{dz}{dy}\right)^2} = a(y)\sqrt{1 + b(y)\left(\frac{dz}{dy}\right)^2}$$

with the following definitions for a and b:

$$\begin{aligned} a(y) &= \sqrt{-g_{tt}g_{yy}} = G(y)^{-\frac{3}{8}-\frac{7}{16}\delta+\frac{7}{16}\delta_1}\sqrt{G(y)^{\frac{\delta}{2}}\cosh^2\eta - \sinh^2\eta} \\ b(y) &= \left(\frac{\gamma}{2L^4}\right)F(y)^{-1}G(y)^{\frac{3}{4}-\frac{\delta}{8}-\frac{3}{8}\delta_1}\frac{G(y)^{\frac{\delta}{2}} + \frac{3}{4}\left(1 - g^{\frac{\delta}{2}}\right)^2\sinh^2 2\eta}{G(y)^{\frac{\delta}{2}}\cosh^2\eta - \sinh^2\eta}. \end{aligned} \tag{5.34}$$

Further, we have multiplied the action of a free quark by 2 to take into account the fact there is contribution to the diverging part from both the quark and the antiquark. z being a cyclic coordinate, $\frac{\partial\mathcal{L}_0}{\partial z'}$ is a constant, say k_0 which yields

$$\left(\frac{dz}{dy}\right)^2 = \frac{2L^4k_0^2}{\gamma}\frac{F(y)^2G(y)^{-\frac{3}{4}+\frac{9}{8}\delta-\frac{\delta_1}{8}}}{\left[G(y)^{\frac{\delta}{2}} + \frac{3}{4}\left(1 - G^{\frac{\delta}{2}}\right)^2\sinh^2 2\eta\right]} \times \frac{\left(G(y)^{\frac{\delta}{2}}\cosh^2\eta - \sinh^2\eta\right)}{\left[G(y)^{\frac{\delta}{2}} + \frac{3}{4}\left(1 - G^{\frac{\delta}{2}}\right)^2\sinh^2 2\eta - k_0^2F(y)G(y)^{\delta-\frac{\delta_1}{2}}\right]}$$

We expect a free string to extend right up to y_c. In the present case, actually the denominator can vanish at some y greater than y_c depending on the value of k_0, thereby providing a potential turning point before the string hits y_c. This possibility is eliminated by constraining the value of the constant k_0 such that the numerator vanishes at the same point. In other words, we tune the value of k_0 in such a way that the zeros of the numerator and the denominator coincide and keeps z'^2 finite. This restriction enables us to extract the value of k_0 as

$$k_0 = \frac{2}{\sqrt{\tanh^{\frac{2\delta-2\delta_1+2\alpha}{\delta}}\eta - \tanh^{\frac{2\delta-2\delta_1-2\beta}{\delta}}\eta}}. \tag{5.35}$$

Finally, the action of two freely hanging strings is written as

$$S_{\text{free}} = -\frac{\mathcal{T}}{\pi}\int_{y_c}^{\infty} a(y)^2\sqrt{\frac{b(y)}{a(y)^2 b(y) - k_0^2}}dy$$

and the energy of the $Q\bar{Q}$ pair reads

$$E = \frac{\mathcal{S}(\ell)}{\mathcal{T}} = \frac{S - S_{\text{free}}}{\mathcal{T}}. \tag{5.36}$$

We have thus obtained the formal expressions for the $Q\bar{Q}$ separation length ℓ or the dipole length in (5.30) and (5.31). We can square and add these relations to obtain ℓ in terms of the metric components and the constants of motion k and p. Note that ℓ is not completely independent of θ, the angle, the dipole makes with the boost direction, but depends on it through k and p. We will numerically solve these equations in the next section and show the variation of ℓ with the various parameters in the theory. The maximum value of the dipole length is called the screening length, above which the dipole dissociates. The formal expression for the $Q\bar{Q}$ potential is obtained in (5.36). The numerical solution and the variations of the potential with the parameters of the theory will be shown and discussed in the next section.

Numerical results

In this section we provide the numerical results. First of all, it is important to realize that although we have found two constants of motion p and k, actually they are not independent but tied through a constraint equation. This will be evident once we take the ratio of (5.30) to (5.31). The L.H.S. yields $\tan\theta$, whereas, the R.H.S. results in some function of p and k. This allows us in principle to eliminate one of the constants - either k or p in

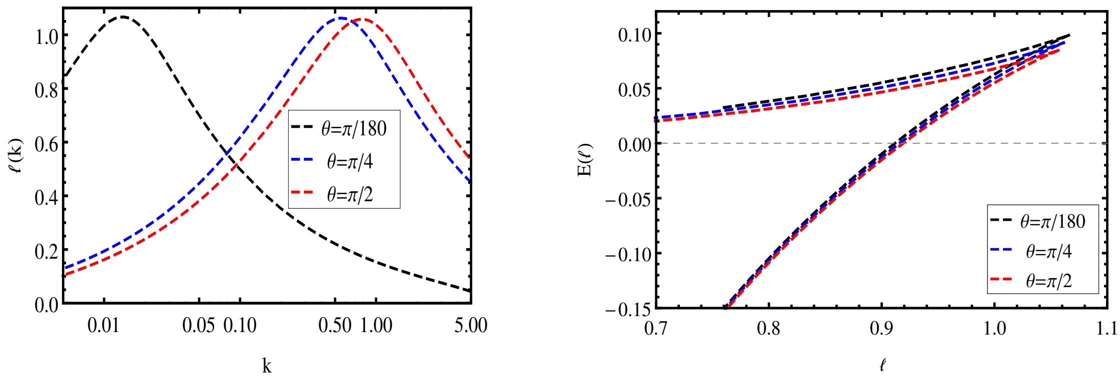

FIGURE 5.1: The plot in the left panel shows the variation of the $Q\bar{Q}$ separation length $\ell(k)$ (scaled) with k, a constant of motion, given in (5.26) and the plot in the right panel shows the variation of $Q\bar{Q}$ potential $E(\ell)$ with ℓ, when some parameters of the theory are fixed to the values $\delta = -0.1$, $\delta_1 = -0.1$ and $\eta = 1.0$. In both panels we have shown the variations for three different values of θ, the angle the dipole makes with the direction of the velocity.

terms of the other one and the angle θ. Here, for definiteness, we shall, henceforth, choose the constant k as the independent one. To evaluate the dipole separation (ℓ) one needs to have knowledge of the string turning point y_t that depends both upon the boost parameter η and the integration constant k. For a given background, we have then $\ell = \ell(\eta, \theta, k)$ and $\mathcal{S} = \mathcal{S}(\eta, k)$. We can invert the first relationship to extract $k = k(\eta, \theta, \ell)$ and plug in into the second one to finally obtain $\mathcal{S} = \mathcal{S}(\eta, \theta, \ell)$. Without any loss of generality, we have set $L = u_0 = 1$ so that in our figure ℓ actually represents a scaled $Q\bar{Q}$ separation. Fig.5.1 shows the variation of this scaled dipole length ℓ with the constant of motion of the string k and the variation of the dipole potential E with the dipole length ℓ for three different orientations of the dipole. We have set the boost parameter η to unity while the two remaining parameters of the gravity background are set to $\delta = \delta_1 = -0.1$. The $\ell(k) - k$ plots tells us that as the orientation angle θ increases, the screening length occurs for higher values of the integration constant k. The maximum possible value of the dipole length, i.e., the screening length changes only marginally with the orientation angle θ. The

$E - \ell$ plot shows that the screening length is maximum when the dipole is almost parallel to the direction of boost velocity, i.e., $\theta \sim 0$[5]. The screening length decreases with θ and takes minimum value when dipole is exactly perpendicular to the direction of boost, i.e., $\theta = \pi/2$. The $E(\ell) - \ell$ plot also shows that the dipole energy is practically insensitive to its orientation with respect to the boost direction. Fig.5.2 again shows the variation of

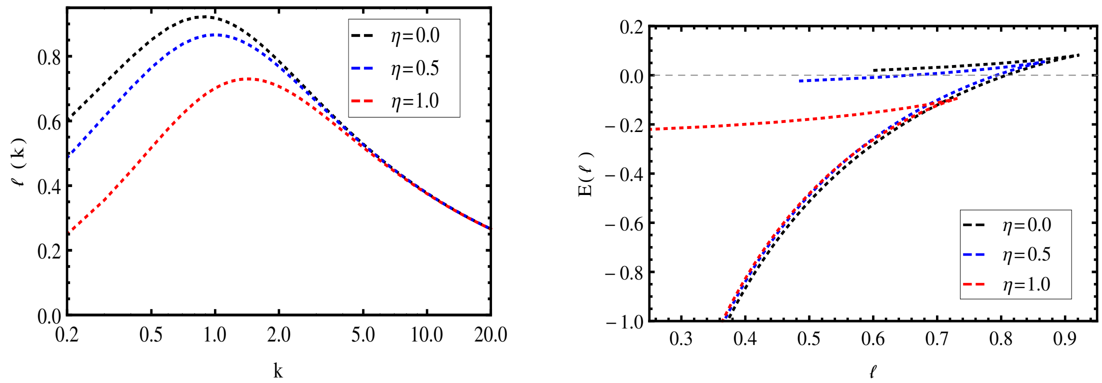

FIGURE 5.2: The plot in the left panel shows the variation of the $Q\bar{Q}$ separation length $\ell(k)$ (scaled) with k, a constant of motion, given in (5.26) and the plot in the right panel shows the variation of $Q\bar{Q}$ potential $E(\ell)$ with ℓ, when some parameters of the theory are fixed to the values $\delta = -1.0$, $\delta_1 = -1.0$ and $\theta = \pi/2$. In both panels we have shown the variations for three different values of η, where $\tanh \eta = v$.

ℓ with k and the variation of E with ℓ for the values of $\delta = \delta_1 = -1.0$, but this time we have fixed the value of θ at $\pi/2$. Instead, we have shown the variations for three different values of the boost parameter η. Again, the interaction energy is not much affected by the presence of a velocity. In this context, it is worth mentioning that we are only concerned with the lower branch of the $E - \ell$ curves since they represent the lower energy states and are more stable compared to the upper branches which have higher energy and does not represent stable configuration of the dipole. However, the $\ell(k) - k$ plot clearly shows that the screening length is quite sensitive to the boost parameter and is the maximum when

[5]We exclude $\theta = 0$ because it is not allowed for the parametrization we have used for the calculation of ℓ or the potential in the previous section and we need to use a different parametrization to include this case.

$\eta = 0$. This means the dipole is most stable (for this particular configuration) when it is sitting still in the plasma - any motion through the hot plasma leads to a decrease in the screening length and makes the dipole more vulnerable to dissociation. This is expected because as the velocity increases there is more chance of a collision of the dipole with the background making it more easily dissociable.

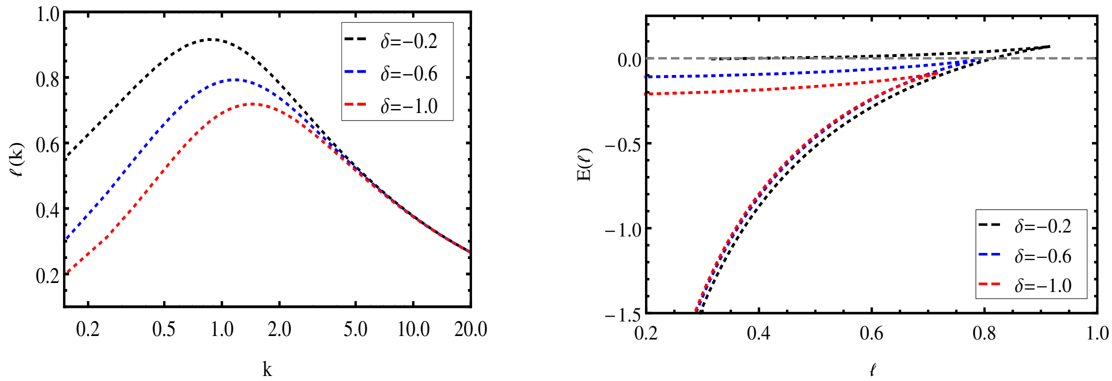

FIGURE 5.3: The plot in the left panel shows the variation of the $Q\bar{Q}$ separation length $\ell(k)$ (scaled) with k, a constant of motion, given in (5.26) and the plot in the right panel shows the variation of $Q\bar{Q}$ potential $E(\ell)$ with ℓ, when some parameters are fixed to the values $\eta = 1.0$, $\theta = \pi/2$ and $\theta = \pi/2$, $-(3/2)\delta + (7/4)\delta_1 = -0.5$. Keeping the the last combination fixed keeps the effective coupling of the theory fixed. In both panels we have shown the variations for three different values of δ which is directly related to the temperature.

The next two figures Fig.5.3 and Fig.5.4 show the variations ℓ with k and also the variations of $Q\bar{Q}$ potential E with ℓ when some parameters of the gravity theory are varied. The parameters which we denoted δ and δ_1 in the non-susy D3 brane configuration (5.6) do not appear in the supersymmetric theory and therefore the plots under their variations reveal new features not observed before (can not be compared with the supersymmetric theory). We also give some physical interpretations of the parameters. Fig.5.3 shows the behavior of the curves ℓ with k and also $E(\ell)$ with ℓ when the parameter δ is varied. Here we have kept θ, η and the combination $-\frac{3}{2}\delta + \frac{7}{4}\delta_1$ fixed to the values 1.0, $\pi/2$ and

-0.5 respectively. The reason for keeping the particular combination of δ and δ_1 fixed is that this combination appears in the expression for the dilaton given in (5.14) and so, keeping this combination fixed will keep the effective coupling, g_{eff} = fixed. We find that the screening length of the dipole have a strong dependence on the value of δ, while the interaction energy changes mildly. As $|\delta|$ increases, the screening length decreases. It is not difficult to understand why this is so, if we remember that non-susy D3 brane has a temperature near extremality proportional to δ (given in (5.12)). Expressed in terms of u_0 it has the form,

$$T = \frac{(-2\delta)^{1/4}}{\sqrt{2}\pi L^2} u_0 = \frac{(2|\delta|)^{1/4}}{\sqrt{2}\pi L^2} u_0, \qquad \text{as}, \quad \delta \leq 0 \tag{5.37}$$

Therefore as $|\delta|$ increases the temperature of the system increases which makes the dipole to dissociate more easily resulting in the decrease of the screening length. Also as the temperature increases the interaction energy of the $Q\bar{Q}$ decreases making the dipole less stable and that is seen in the figure as the slight increment of $E(\ell)$ with $|\delta|$. Next, Fig.5.4 explores how the $\ell - k$ plot and the $E - \ell$ plot depend upon δ_1 when all other parameters are kept fixed. Now the screening length changes mildly with variation in δ_1 while the corresponding change in the potential energy is more prominent. Let us now see how the parameter δ_1 affects the screening length and the potential. We have already seen that the temperature of the non-susy brane (which is also interpreted as the temperature of the gauge theory) depends upon δ for fixed value of u_0, whereas, the effective coupling g_{eff} (related to the dilaton field) depends upon both δ and δ_1. Now if we vary δ_1, keeping δ fixed, the temperature remains fixed but the effective coupling changes. From (5.17), we find the dimensionless gauge coupling to be

$$g_{\text{eff}}^2 \sim N e^{\phi} = N g_s G(u)^{\frac{3}{2}|\delta| - \frac{7}{4}|\delta_1|}, \qquad \text{as} \qquad \delta,\, \delta_1 \leq 0 \tag{5.38}$$

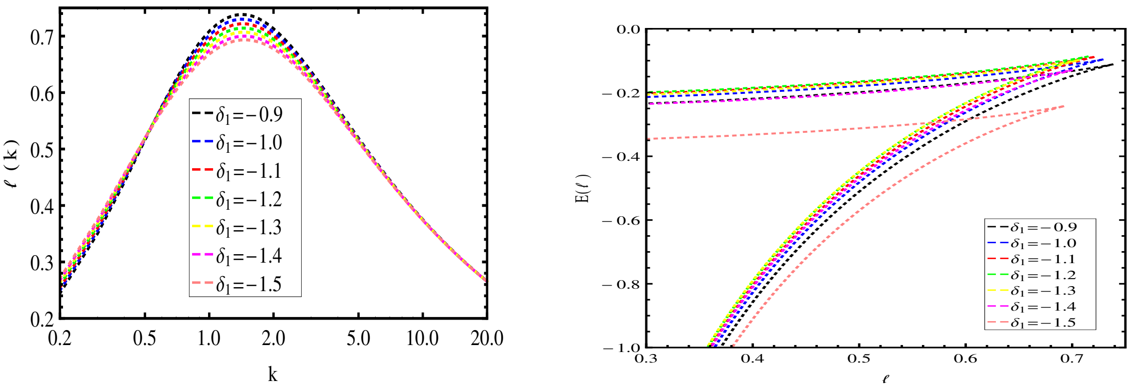

FIGURE 5.4: The plot in the left panel shows the variation of the $Q\bar{Q}$ separation length $\ell(k)$ (scaled) with k, a constant of motion, given in (5.26) and the plot in the right panel shows the variation of $Q\bar{Q}$ potential $E(\ell)$ with ℓ, when some parameters are fixed to the values $\eta = 1.0$, $\delta = -1.0$, and $\theta = \pi/2$. In both panels we have shown the variations for seven different values of δ_1 which is directly related to the effective coupling of the theory.

At a fixed energy scale u, the coupling g_{eff} decreases with the increasing $|\delta_1|$. As the coupling becomes weaker, the quark-antiquark pair is loosely bound and the maximum allowed length of the dipole decreases - this is clearly visible in Fig.5.4. On the other hand, as the coupling becomes weaker with increasing $|\delta_1|$, the dipole will be loosely bound and becomes less stable. This is actually seen in the $E(\ell)$ vs. ℓ plot in Fig.5.4. Indeed we see that the physically relevant lower portion of the curve actually goes up as we increase $|\delta_1|$. But this happens only upto certain value of $|\delta_1|$ between 1.3 and 1.4. But beyond that the effect gets reversed. As we increase $|\delta_1|$ further the potential goes down showing that the dipole becomes more stable. This latter stability of the dipole is quite counterintuitive and we do not have a satisfactory physical explanation for its occurrence.

Lastly, we mention that for the superconformal theory as in $D = 4$, $\mathcal{N} = 4$ SYM theory, it is known that the variation of the screening length with velocity is given as,

$$\ell_{\text{max}}(v) \sim \left(1 - v(\eta)^2\right)^{\nu} \ell_{\text{max}}(0) \tag{5.39}$$

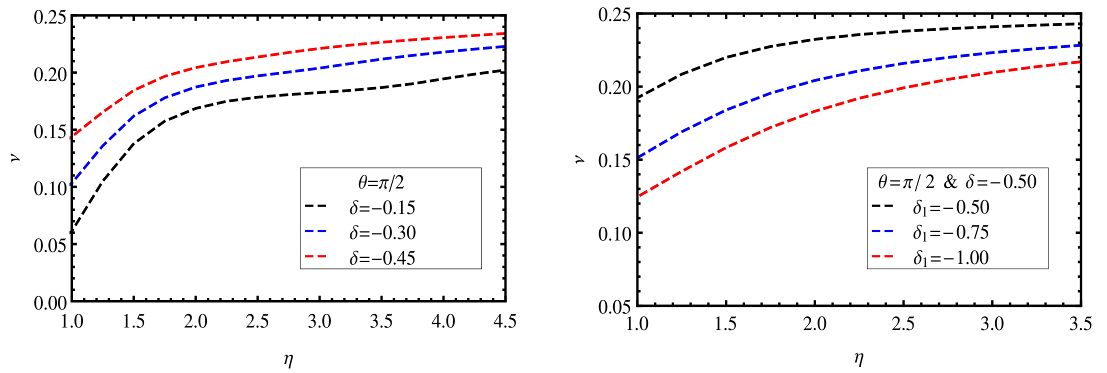

FIGURE 5.5: Here in both the panels we have shown the variations of the exponent ν (defined in eq.(5.39)) with η (i.e., the velocity). In the left panel the parameters are fixed to the values $\theta = \pi/2$ and $-(3/2)\delta + (7/4)\delta_1 = -0.5$ (this means the effective coupling is fixed) and in the right panel the parameters are fixed to the values $\theta = \pi/2$ and $\delta = -0.5$ (this means the temperature is fixed).

where $\ell_{\max}(0)$ is the screening length when the background is at rest and ν takes a value 0.25. However, it is also observed in [82], that for non-conformal theory the power ν in (5.39) actually drops from the value 0.25. Since in our background there is no conformal symmetry, we would like to see how the exponent changes with velocity. This can be obtained by studying the equations (5.30) and (5.31). In Fig.5.5 we have shown this variation. In the first figure we have shown the variation of ν with δ (i.e., the temperature) while keeping θ and $-\frac{3}{2}\delta + \frac{7}{4}\delta_1$ (i.e., effective coupling) fixed and in the second figure we have shown the variation of ν with δ_1 (i.e., the effective coupling) while keeping θ and δ (i.e., temperature) fixed. The curves can be trusted for high velocity and we see that in both cases, the curves saturate to some value less than 0.25, a characteristic of a non-conformal background.

Conclusion

To conclude, in this chapter we have shown an application of non-supersymmetric AdS/CFT in the calculation of Wilson loop in QGP. The non-supersymmetric AdS/CFT has been proposed earlier by two of us, by proposing a decoupling limit for the non-supersymmetric D3 brane solution of type IIB string theory. By this procedure we obtained the throat geometry of the non-susy D3 brane and that gave us the gravity dual of a non-supersymmetric gauge theory on the boundary. We have taken a 'black' version of this solution and the decoupled geometry of this solution has given a finite temperature, non-supersymmetric, non-conformal gauge theory on the boundary. We have taken this geometry and boosted it along one of the brane directions. We have also introduced a probe string in this background whose end points represent the quark-antiquark pair on the boundary theory. The $Q\bar{Q}$ pair has been made to lie at an angle θ from the direction of the boost. Then following LRW [82] we have computed the time-like Wilson loop in this background which in turn has given us the formal expression of $Q\bar{Q}$ separation length (5.30), (5.31) and the interaction potential (5.36). We have numerically solved those equations and plotted the $Q\bar{Q}$ separation length (ℓ) as a function of certain integral of motion (k) and also the interaction energy (E) of the diople as a function of ℓ. The maximum allowed $Q\bar{Q}$ separation is called the screening length since beyond this Q and $\bar{Q}$ in the dipole dissociates. As happens in real heavy ion collision experiment, we have shown the variation of screening length $\ell_{\rm max}$ as well as the interaction potential E with the angle θ (angle between the dipole and the direction of the velocity), $\eta = \tanh^{-1} v$ (v, the background velocity) and also with $|\delta|$ (related to the temperature of the gauge theory) and $|\delta_1|$ (related to the effective gauge coupling), keeping other parameters fixed. These variations with θ and the velocity have also been studied for supersymmetric theory by Liu, Rajagopal and Wiedemann and surprisingly, we found qualitative agreement with their results. This clearly shows that these results are quite robust as they do not depend on the presence of any supersymmetry (also the conformal symmetry) of the theory. We have also shown how the screening length and the potential change with the change of $|\delta|$ (i.e., temperature) and $|\delta_1|$ (i.e., the effective coupling) not seen in the supersymmetric theory. We observed some peculiar behavior of the potential when the coupling changes. Initially when $|\delta_1|$ increases, the potential slightly increases as expected, but beyond some value in between 1.3 and 1.4, the effect gets reversed, namely, as $|\delta_1|$ increases further the potential suddenly decreases, whose physical explanation is not clear to us.

One of the motivations for studying QGP behavior using AdS/CFT for LRW [80] was to see the observed quarkonium suppression when the background is not static but has a velocity. They calculated the screening length for the superconformal theory and found that the screening length gets reduced with velocity by a factor $(1-v^2)^{1/4}$ from its static value. This indeed supports the observed quarkonium suppression as the QGP produced in heavy ion collision moves with high relative velocity with the dipole. They also observed that for theories without conformal symmetry the exponent ν should have values less than 0.25. Since the non-supersymmetric theory we are dealing with also does not have a conformal symmetry we have plotted the exponent ν with velocity. We found that at high velocity, ν indeed saturates to values somewhat below 0.25, when both δ and δ_1 are varied confirming that for theories without conformal symmetry, the velocity dependence of screening length would be such that it would enhance the quarkonium suppression.

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Chapter 6

Accelerating Cosmologies And Conformally de Sitter Space-time

So far our discussion was entirely about the various space-dependent, time-like Dp brane solutions. But we have seen in chapter 2 these Dp branes have been transformed into time-dependent, space-like S-brane solutions under Wick's rotations. The S(pace)-like branes are topological defects localized on a space-like hypersurface which exist as time dependent solutions of many field theories as well as of string/M theory [96, 97]. In string theory just like Dp branes arise as space-like tachyonic kink solution of world volume field theory of non-BPS D$(p+1)$ brane or D$(p+2)$ – antiD$(p+2)$ brane [54], space-like Dp (or SDp) branes arise as the time-like tachyonic kink solution of the above unstable brane systems [96, 98]. SDp-branes have $(p+1)$ dimensional Euclidean world-volume and carry the same RR charges as their time-like cousins. The original motivation for studying SDp-branes was to understand holography in the temporal context. Just as Dp branes give rise to a space-like direction from a Lorentzian world-volume field theory, SDp branes give rise to a time-like direction from the Euclidean world-volume theory of SDp branes and this is a necessary ingredient for dS/CFT correspondence [99]. One of the reasons for the space-time construction of these SDp branes[1] was to understand the so-called dS/CFT correspondence.

In [101] the anisotropic (in one direction) SD3 brane solution of type IIB string theory has been constructed and compactified on a six dimensional product space of the form $H_5 \times S^1$, where H_5 is a five dimensional hyperbolic space[2] and S^1 is a circle. The resulting external space was then shown to be conformal to a four dimensional de Sitter space. This brought out the connection between SD3 brane and the four dimensional de Sitter space which may be helpful in understanding dS/CFT correspondence [99] in the same spirit as AdS/CFT correspondence [2]. It may be of interest to see if similar structure exists for other SDp branes for $p \neq 3$. Moreover, it is well-known that S-brane solutions of string/M theory give rise to four dimensional accelerating cosmologies (similar to the acceleration of our universe observed in the present epoch [105–107]) upon time dependent hyperbolic space compactification [30, 108–112] and we have seen this, in particular, for SD2-brane compactified on six dimensional hyperbolic space and expressing the resultant metric in Einstein frame [109, 113]. It would be of interest to see whether similar accelerating cosmologies can be obtained in other dimensions and under what conditions.

Motivated by this, we construct the isotropic SDp brane solutions having isometries ISO$(p+1) \times$ SO$(8-p, 1)$, from the double Wick rotation of the static, non-supersymmetric,

[1] The space-time constructions of S-branes were given in [16, 35, 36, 100].

[2] Hyperbolic space compactifications are discussed in [102–104].

charged Dp brane solutions [18] of type II string theories. The isotropic SDp brane solutions will be characterized by three independent parameters (τ_0, θ, δ_0). The parameter τ_0 sets a time scale in the sense that when $\tau \gg \tau_0$, the solutions become flat. On the other hand when $\tau \sim \tau_0$, the isotropic SDp brane metrics can be compactified on $(8-p)$ dimensional hyperbolic spaces of time dependent radii, to obtain a $(p+1)+1$ dimensional flat FLRW metrics in the Einstein frame. We show that these resultant metrics give rise to transient accelerating cosmologies for all p (where $1 \leq p \leq 6$) i.e., from $(2+1)$ to $(7+1)$ space-time dimensions. The amount of acceleration and the duration vary with the variations of the various parameters and we study them only in realistic $(3+1)$ space-time dimensions. When $\tau \ll \tau_0$, we will fix the parameter δ_0 for calculational simplicity (without loss of any generality) and find after a similar compactification on $(8-p)$ dimensional hyperbolic spaces that the resultant metrics can be cast into de Sitter forms in $(p+1)+1$ dimensions upto a conformal factor after a suitable coordinate transformation. This clarifies the relation between SDp branes and de Sitter spaces. The other two parameters θ and δ_0 in the solutions are related to the charge of SDp branes and the dilaton, respectively.

Here we briefly mention that the isotropic SDp brane solutions described in section 2 are not new and they are just rewriting the already known solutions [16, 18, 26] in a convenient form. The accelerating cosmologies were known [30, 109, 113] to follow from the SD2 brane solutions upon time dependent hyperbolic space compactification. Here we show that similar accelerating cosmologies also follow from all the SDp brane solutions (for $1 \leq p \leq 6$) upon similar time dependent hyperbolic space compactification and is described in section 3. The results of this section are new. Also, a four dimensional de Sitter space upto a conformal factor was obtained before in [101] from the near horizon limit of an anisotropic SD3 brane solution of type IIB string theory upon hyperbolic space compactification. In this chapter we show that de Sitter solutions upto a conformal factor in $(p+1)+1$ dimensions also follow from all the isotropic SDp brane solutions of type II string theory upon hyperbolic space compactification. This is described in section 4 and here also the results are new. So, the accelerating cosmologies and also the conformally de Sitter solutions are not specific to a particular SDp brane (that were known before), but they are quite generic, as we show in this chapter, for all the SDp branes for $1 \leq p \leq 6$.

The chapter is organized as follows. In the first section, we give the construction of isotropic SDp brane solutions of type II string theories and write them in a suitable coordinate. In section 2, we show how FLRW type cosmological solutions in various dimensions can be obtained from the isotropic SDp brane solutions by compatifications. We also discuss about the solutions in various dimensions. In section 3, we show how the same solutions give rise to $(p+1)+1$ dimensional de Sitter spaces upto conformal factors in early times. Finally, we conclude in section 4.

Isotropic SDp brane solutions

In this section we will give the construction of isotropic SDp brane solutions of type II string theories characterized by three independent parameters and write them in a suitable coordinate system for the ease of our discussion in the next two sections. These solutions can actually be obtained either from the static, non-supersymmetric, isotropic p-brane solutions in arbitrary space-time dimensions given in [18] and using a double Wick rotation, or from the isotropic S-brane solutions in arbitrary dimensions given in [16]. But for convenience we will use the solutions given in (2.44) of chapter 2, representing nonsupersymmetric intersecting brane solutions involving charged Dp branes. These solutions contain several parameters and to obtain isotropic nonsupersymmetric Dp brane solutions from here we will put the conditions $q = 0$, $\delta_2 = \delta_3 = \cdots = \delta_p = \delta_0$ and also $\delta_1 = -2\delta_0$.

The solutions (2.44), then take the form,

$$\begin{aligned}
ds^2 &= F(r)^{\frac{p+1}{8}}\left(H(r)\tilde{H}(r)\right)^{\frac{2}{7-p}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{-\frac{p(p+1)}{4(7-p)}\delta_0}\left(dr^2+r^2d\Omega^2_{8-p}\right)\\
&\quad +F(r)^{-\frac{7-p}{8}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{p}{4}\delta_0}\left(-dt^2+\sum_{i=1}^{p}(dx^i)^2\right)\\
e^{2(\phi-\phi_0)} &= F(r)^{\frac{3-p}{2}}\left(\frac{H(r)}{\tilde{H}(r)}\right)^{-(4+p)\delta_0}, \qquad F_{[8-p]} = Q\mathrm{Vol}(\Omega_{8-p}) \qquad (6.1)
\end{aligned}$$

We remark that the other two references mentioned above also give the same solutions, but the parameter relations are simpler here. Note that the metrics in (6.1) are given in the Einstein frame. The various functions appearing in the solutions are defined as,

$$\begin{aligned}
F(r) &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\alpha}\cosh^2\theta-\left(\frac{\tilde{H}(r)}{H(r)}\right)^{\beta}\sinh^2\theta\\
H(r) &= 1+\frac{\omega^{7-p}}{r^{7-p}}, \qquad \tilde{H}(r) = 1-\frac{\omega^{7-p}}{r^{7-p}} \qquad (6.2)
\end{aligned}$$

There are six parameters α, β, δ_0, θ, ω, and Q associated with the solutions. However, from the equations of motion, the parameters can be seen to satisfy the following three relations,

$$\begin{aligned}
\alpha-\beta &= 3\delta_0\\
\frac{14+5p}{7-p}\delta_0^2+\frac{1}{2}\alpha(\alpha-3\delta_0) &= \frac{8-p}{7-p}\\
Q &= (7-p)\omega^{7-p}(\alpha+\beta)\sinh 2\theta \qquad (6.3)
\end{aligned}$$

Using these relations we can eliminate three parameters out of the six we mentioned above and therefore, the solutions have three independent parameters, namely, ω, θ and δ_0. Note from the form of $\tilde{H}(r)$ in (6.2) that the solutions have curvature singularities at $r=\omega$ and therefore, the solutions are well defined only for $r>\omega$. Also in (6.1) ϕ_0 denotes the asymptotic value of the dilaton and $F_{[8-p]}$ is the $(8-p)$ form and Q is the charge associated with the Dp branes which in this case are magnetically charged. We point out that the singularities at $r=\omega$ are naked singularities where the dilaton becomes plus or minus infinity (depending on the parameters and the value of p) and can not be removed by coordinate transformations or going to a different (dual) frame. However, since these are string theory solutions it may be plausible that these singularities will go away when the higher order stringy effects are taken into account. As far as we know the status of these singularities in full string theory is still not clearly understood.

Now in order to get isotropic SDp brane solutions we apply the double Wick rotation [18] $r \to it$, $t \to -ix^{p+1}$ to the solutions (6.1) along with $\omega \to i\omega$, $\theta \to i\theta$ and $\theta_1 \to i\theta_1$, where θ_1 is one of the angles parameterizing the sphere Ω_{8-p} and then we obtain,

$$\begin{aligned} ds^2 &= F(t)^{\frac{p+1}{8}} \left(H(t)\tilde{H}(t)\right)^{\frac{2}{7-p}} \left(\frac{H(r)}{\tilde{H}(r)}\right)^{-\frac{p(p+1)}{4(7-p)}\delta_0} \left(-dt^2 + t^2 dH^2_{8-p}\right) \\ &\quad + F(t)^{-\frac{7-p}{8}} \left(\frac{H(t)}{\tilde{H}(t)}\right)^{\frac{p}{4}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2 \\ e^{2(\phi-\phi_0)} &= F(t)^{\frac{3-p}{2}} \left(\frac{H(t)}{\tilde{H}(t)}\right)^{-(4+p)\delta_0}, \qquad F_{[8-p]} = (-1)^{8-p} Q \mathrm{Vol}(H_{8-p}) \end{aligned} \tag{6.4}$$

where the various functions are now given as,

$$\begin{aligned} F(t) &= \left(\frac{H(t)}{\tilde{H}(t)}\right)^{\alpha} \cos^2\theta + \left(\frac{\tilde{H}(t)}{H(t)}\right)^{\beta} \sin^2\theta \\ H(t) &= 1 + \frac{\omega^{7-p}}{t^{7-p}}, \qquad \tilde{H}(t) = 1 - \frac{\omega^{7-p}}{t^{7-p}} \end{aligned} \tag{6.5}$$

Note that under the Wick rotation the solutions have become time dependent. The naked singularities at $r=\omega$ has now changed to the singularities at $t=\omega$. Also, the metric of the sphere $d\Omega^2_{8-p}$ has changed to negative of the metric of the hyperbolic space dH^2_{8-p}. The metrics now has the symmetry ISO$(p+1)\times$ SO$(8-p,1)$. The hyperbolic functions

$\sinh\theta$, $\cosh\theta$ have become trigonometric functions and the function F has relative plus sign in the two terms instead of minus. Most importantly the form field remains real and retains its form upto a sign which does not happen for the BPS Dp branes (Wick rotation actually makes the form field imaginary for BPS Dp branes and the solutions in that case do not remain solutions of type II theories, instead they become solutions of type II* theories [114]). The first two parameter relations in (6.3) remain the same under the Wick rotation, whereas the last relation changes to $Q = (7-p)\omega^{7-p}(\alpha+\beta)\sin 2\theta$ if we insist that Q should also change under the Wick rotation as $Q \to (i)^{8-p}Q$. Eq.(6.4) represents real isotropic SDp brane solutions of type II string theories characterized by three independent parameters ω, θ, δ_0.

Now for the discussion in the next two sections we will make a coordinate transformation from t to τ given by,

$$t \;=\; \tau\left(\frac{1+\sqrt{g(\tau)}}{2}\right)^{\frac{2}{7-p}}, \qquad \text{where,} \qquad g(\tau) \;=\; 1+\frac{4\omega^{7-p}}{\tau^{7-p}} \equiv 1+\frac{\tau_0^{7-p}}{\tau^{7-p}} \tag{6.6}$$

Under this coordinate change we get,

$$\begin{aligned}
&H(t) = 1+\frac{\omega^{7-p}}{t^{7-p}} = \frac{2\sqrt{g(\tau)}}{1+\sqrt{g(\tau)}}, \qquad \tilde{H}(t) = 1-\frac{\omega^{7-p}}{t^{7-p}} = \frac{2}{1+\sqrt{g(\tau)}},\\
&H(t)\tilde{H}(t) = \frac{4\sqrt{g(\tau)}}{(1+\sqrt{g(\tau)})^2}, \qquad \frac{H(t)}{\tilde{H}(t)} = \sqrt{g(\tau)},\\
&-dt^2+t^2dH_{8-p}^2 = g(\tau)^{\frac{1}{7-p}}\left(-\frac{d\tau^2}{g(\tau)}+\tau^2 dH_{8-p}^2\right)
\end{aligned} \tag{6.7}$$

Using these relations we can rewrite the isotropic SDp brane solutions given in (6.4) as follows,

$$ds^2 \;=\; F(\tau)^{\frac{p+1}{8}} g(\tau)^{\frac{1}{7-p}-\frac{p(p+1)}{8(7-p)}\delta_0}\left(-\frac{d\tau^2}{g(\tau)}+\tau^2 dH_{8-p}^2\right) + F(\tau)^{-\frac{7-p}{8}} g(\tau)^{\frac{p}{8}\delta_0}\sum_{i=1}^{p+1}(dx^i)^2$$

$$e^{2(\phi-\phi_0)} = F(\tau)^{\frac{3-p}{2}} g(\tau)^{-\frac{(4+p)}{2}\delta_0}, \qquad F_{[8-p]} = (-1)^{8-p} Q \mathrm{Vol}(H_{8-p}) \tag{6.8}$$

where $g(\tau)$ is as given in (6.6) and $F(\tau)$ is given by,

$$F(\tau) = g(\tau)^{\frac{\alpha}{2}} \cos^2\theta + g(\tau)^{-\frac{\beta}{2}} \sin^2\theta \tag{6.9}$$

The parameter relations remain the same as given in (6.3) with the factor $\sinh 2\theta$ in the last one replaced by $\sin 2\theta$. It should be noted from (6.8), that in the new coordinate, the original singularities at $t = \omega$ have been shifted to $\tau = 0$. Now the solutions have three independent parameters, namely, τ_0, θ, δ_0. Also note that as $\tau \gg \tau_0$, $g(\tau)$, $F(\tau) \to 1$ and therefore, the solutions reduce to flat space. In the next two sections we will use the solutions (6.8) to see how one can get cosmologies in various dimensions and also how to obtain de Sitter spaces upto a conformal factor.

SDp brane compactifications

FLRW cosmologies

In this section we will see how we can get flat FLRW cosmologies in various dimensions from the isotropic SDp brane solutions given in (6.8). We will assume that $\tau \sim \tau_0$, so that the two terms in the function $g(\tau) = 1 + \tau_0^{7-p}/\tau^{7-p}$ are comparable and we must keep both the terms. Keeping this in mind we can rewrite the metrics in (6.8) in the following form,

$$ds^2 = F(\tau)^{-\frac{(p+1)(8-p)}{8p}} g(\tau)^{-\frac{8-p}{p(7-p)}+\frac{(8-p)(p+1)}{8(7-p)}\delta_0} \tau^{-\frac{2(8-p)}{p}} ds_E^2 + F(\tau)^{\frac{p+1}{8}} g(\tau)^{\frac{1}{7-p}-\frac{p(p+1)}{8(7-p)}\delta_0} \tau^2 dH_{8-p}^2 \tag{6.10}$$

where

$$ds_E^2 = -F(\tau)^{\frac{p+1}{p}} g(\tau)^{\frac{8}{p(7-p)} - \frac{p+1}{7-p}\delta_0 - 1} \tau^{\frac{2(8-p)}{p}} d\tau^2 + F(\tau)^{\frac{1}{p}} g(\tau)^{\frac{8-p}{p(7-p)} - \frac{1}{7-p}\delta_0} \tau^{\frac{2(8-p)}{p}} \sum_{i=1}^{p+1} (dx^i)^2 \tag{6.11}$$

is the $(p+1)+1$ dimensional metrics in the Einstein frame. One can think of these metrics as coming from the compactification of the ten dimensional metrics (6.10) on $(8-p)$ dimensional hyperbolic space with time dependent radius given by

$$R(\tau) = F(\tau)^{\frac{p+1}{16}} g(\tau)^{\frac{1}{2(7-p)} - \frac{p(p+1)}{16(7-p)}\delta_0} \tau \tag{6.12}$$

and expressing the resulting metrics in the Einstein frame. Now defining a new time coordinate η by,

$$d\eta = F(\tau)^{\frac{p+1}{2p}} g(\tau)^{\frac{4}{p(7-p)} - \frac{p+1}{2(7-p)}\delta_0 - \frac{1}{2}} \tau^{\frac{8-p}{p}} d\tau \equiv C(\tau) d\tau \tag{6.13}$$

we can rewrite the Einstein frame metrics ds_E^2 in the standard flat FLRW form in $(p+1)+1$ dimensions as

$$ds_E^2 = -d\eta^2 + S^2(\eta) \sum_{i=1}^{p+1} (dx^i)^2 \tag{6.14}$$

where the scale factor $S(\eta)$ is given by,

$$S(\eta) \equiv A(\tau) = F(\tau)^{\frac{1}{2p}} g(\tau)^{\frac{8-p}{2p(7-p)} - \frac{1}{2(7-p)}\delta_0} \tau^{\frac{8-p}{p}} \tag{6.15}$$

Now we define another function

$$B(\tau) = \frac{A(\tau)}{C(\tau)} = F(\tau)^{-\frac{1}{2}} g(\tau)^{\frac{6-p}{2(7-p)} + \frac{p}{2(7-p)}\delta_0} \tag{6.16}$$

where $C(\tau)$ is defined in (6.13). Now the universe is expanding if the scale factor $S(\eta)$ satisfies $dS(\eta)/d\eta > 0$ and the expansion is accelerating if it further satisfies $d^2S(\eta)/d\eta^2 > 0$. Since $S(\eta)$ is a complicated function of η, we will translate [113] these two conditions in terms of the two known functions $A(\tau)$ and $B(\tau)$ given in (6.15) and (6.16). The conditions are,

$$\begin{aligned} m(\tau) &\equiv \frac{d\ln A(\tau)}{d\tau} > 0 \\ n(\tau) &\equiv \frac{d^2\ln A(\tau)}{d\tau^2} + \frac{d\ln A(\tau)}{d\tau}\frac{d\ln B(\tau)}{d\tau} > 0 \end{aligned} \tag{6.17}$$

where $m(\tau)$ is the expansion parameter and $n(\tau)$ is the rate of expansion parameter. The parameters α and β which appear in the definition of $F(\tau)$ given in (6.9) can be given in terms of δ_0 from the second relation in (6.3) as,

$$\begin{aligned} \alpha &= \frac{3}{2}\delta_0 \pm \sqrt{\frac{8(8-p) - 49(p+1)\delta_0^2}{4(7-p)}} \\ \beta &= -\frac{3}{2}\delta_0 \pm \sqrt{\frac{8(8-p) - 49(p+1)\delta_0^2}{4(7-p)}} \end{aligned} \tag{6.18}$$

In Fig:6.1, Fig:6.2, we have plotted the expansion parameter $m(\tau)$ and the rate of expansion parameter $n(\tau)$, respectively. We have used the $(p+1)+1$ dimensional metrics ds_E^2 given in (6.11) and the functions $A(\tau)$ and $B(\tau)$ given in (6.15) and (6.16). We have also used the value of α, β given in (6.18). The positive value of $m(\tau)$ in Fig:6.1 indicates the expansion of the universe. From the above plot, we see that the universe expands for all values of p (where $1 \le p \le 6$) and therefore, we get the expanding $2+1$, $3+1$ upto $7+1$ dimensional universes. In Fig:6.2, the values of the various parameters we have chosen are $\theta = 0$ (this means that the form field is zero and therefore the solution is chargeless and simpler), $\tau_0 = 1$ (this is a typical value we have chosen to show the cosmologies in various dimensions and if τ_0 is less than this value the acceleration is

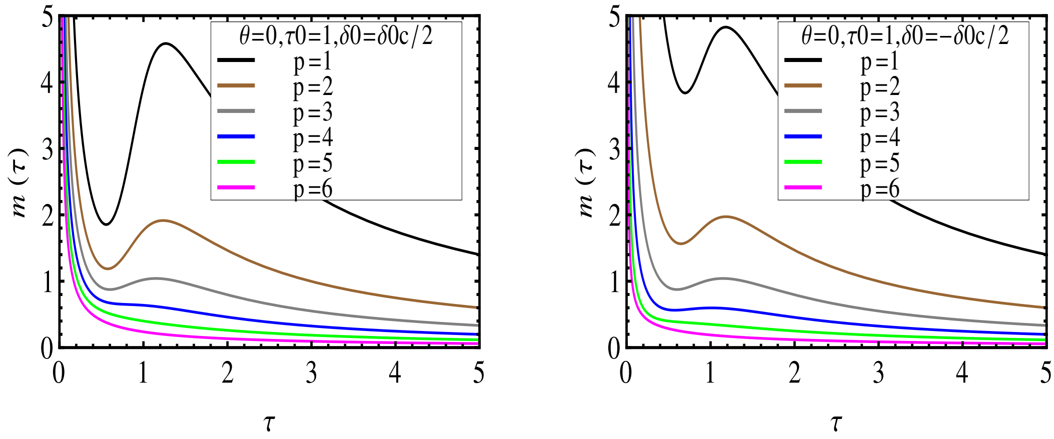

FIGURE 6.1: Plot of expansion paramater $m(\tau)$ for SDp brane compactifications on hyperbolic space H_{8-p} given in (6.17). The functions are plotted for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = \frac{\delta_{0c}}{2} = \frac{1}{14}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, \ldots, 6$ in the left panel and for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = -\frac{\delta_{0c}}{2} = -\frac{1}{14}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, \ldots, 6$ in the right panel.

more but the duration is less as seen in Fig:6.5) and $\delta_0 = \delta_{0c}/2$ (defined below) in the left panel and $\delta_0 = -\delta_{0c}/2$ in the right panel. Actually, the parameter δ_0 can not take any arbitrary value. From (6.18) we note that since the parameters α and β are real δ_0 must lie in between

$$-\frac{2}{7}\sqrt{\frac{2(8-p)}{p+1}} \leq \delta_0 \leq \frac{2}{7}\sqrt{\frac{2(8-p)}{p+1}} = \delta_{0c}, \qquad (6.19)$$

where we have called the maximum value of δ_0 as δ_{0c}. In Fig:6.1, we have chosen the value of δ_0 as $\pm 1/2$ of its maximum value in the left panel and in the right panel respectively and get expanding universes in all dimensions. The reason for choosing these particular values is that we get accelerating expansion for these values for different p as shown in Fig:6.2. Fig:6.2 also contains two panels. Here again the positivity of $n(\tau)$ gives an accelerating phase of expansion. On the left panel of Fig:6.2, we show that $n(\tau)$ remains positive for certain interval of time for $p = 1, 2, 3$ and on the right panel we show the positivity of $n(\tau)$ for certain interval of time for $p = 4, 5, 6$. Therefore, we get accelerating expansions for all values of p from 1 to 6. Note that the magnitude of

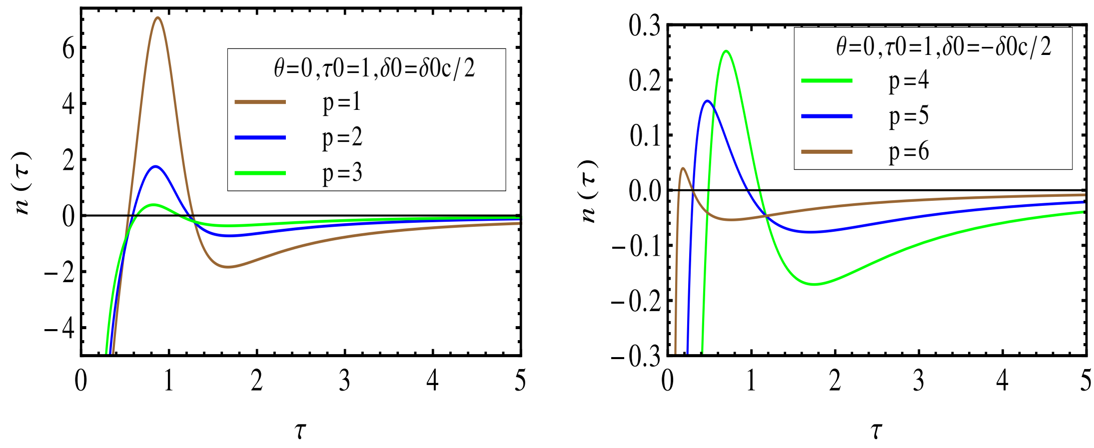

FIGURE 6.2: Plot of rate of expansion parameter $n(\tau)$ for SDp brane compactifications on hyperbolic space H_{8-p} given in (6.17). Here the functions are plotted for $\theta = 0$, $\tau_0 = 1$, $\delta_0 = \frac{\delta_{0c}}{2} = \frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 1, 2, 3$ in the left panel and $\theta = 0$, $\tau_0 = 1$, $\delta_0 = -\frac{\delta_{0c}}{2} = -\frac{1}{7}\sqrt{\frac{2(8-p)}{p+1}}$ and $p = 4, 5, 6$ in the right panel.

acceleration and the duration depend crucially on the parameters θ, τ_0 and particularly δ_0. If the parameters are not chosen judiciously, we do not get accelerations. For $p = 1,\ 2,\ 3,$ we have chosen $\theta = 0$, $\tau_0 = 1$ and $\delta_0 = \delta_{0c}/2$ in the left panel of Fig:6.2 to get accelerations. If we keep the same values of the parameters we get acceleration for $p = 4$ but no accelerations for $p = 5,\ 6$. This is the reason, for $p = 4,\ 5,\ 6$ we have chosen $\theta = 0$, $\tau_0 = 1$ and $\delta_0 = -\delta_{0c}/2$ in the right panel of Fig:6.2 and get accelerations in all the cases. This shows that we can get accelerating cosmologies for all values of p by the appropriate choice of the various parameters characterizing the SDp solutions. The expansion, however, becomes decelerating in the remote past, i.e., for $\tau \ll \tau_0$ and also in the far future $\tau \gg \tau_0$ irrespective of spacetime dimensions and other parameters and all the curves tend to merge in those two regions. We will discuss those cases later. We have tabulated the values of δ_0 for which the rate of expansion parameter $n(\tau)$ is maximum for different values of p in Table 1. We have also given those maximum values and the values of τ where these maxima occurs. We have chosen $\theta = 0$ and $\tau_0 = 1$. In all the cases the

$(p+1)+1$	δ_0	τ	$n(\delta_0,\tau)$
2+1	$\frac{\delta_{0c}}{2}$	0.87197	7.05882
3+1	$\frac{\delta_{0c}}{4}$	0.84025	1.93012
4+1	0	0.78463	0.72178
5+1	$-\frac{\delta_{0c}}{4}$	0.68025	0.30926
6+1	$-\frac{\delta_{0c}}{2}$	0.47336	0.16178
7+1	$-\frac{3\delta_{0c}}{4}$	0.11068	0.32242

TABLE 6.1: Here $n(\delta_0,\tau)$ is treated as a function of two variables δ_0 and τ. We have found the particular value of δ_0 which makes the rate of expansion maximum for $\theta=0$, $\tau_0=1$ and $p=1,\ldots,6$.

maximum values are found to be positive and so there are accelerations for all p.

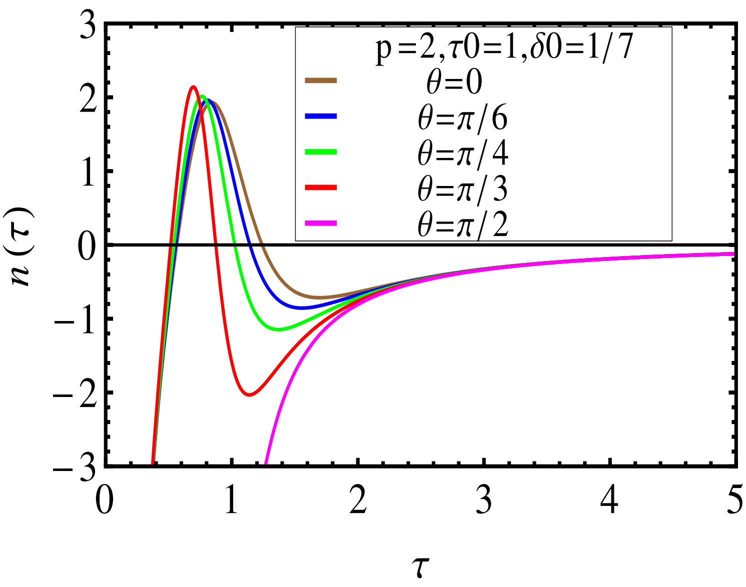

FIGURE 6.3: The plot of $n(\tau)$ for different values of θ where $p=2$, $\tau_0=1$ and $\delta_0=\frac{\delta_{0c}}{4}=\frac{1}{7}$.

In Fig:6.3, Fig:6.4, Fig:6.5 below, we have plotted the rate of expansion parameter $n(\tau)$ for various values of θ, the charge parameter, δ_0, the dilaton parameter and τ_0, the time scale, respectively. We have taken $p=2$, so that the space-time is $3+1$ dimensional. In Fig:6.3, we have taken $\tau_0=1$ and $\delta_0=\delta_{0c}/4=1/7$. We find that there is acceleration for all values of θ in the range $0\leq\theta<\pi/2$. The acceleration is minimum for $\theta=0$

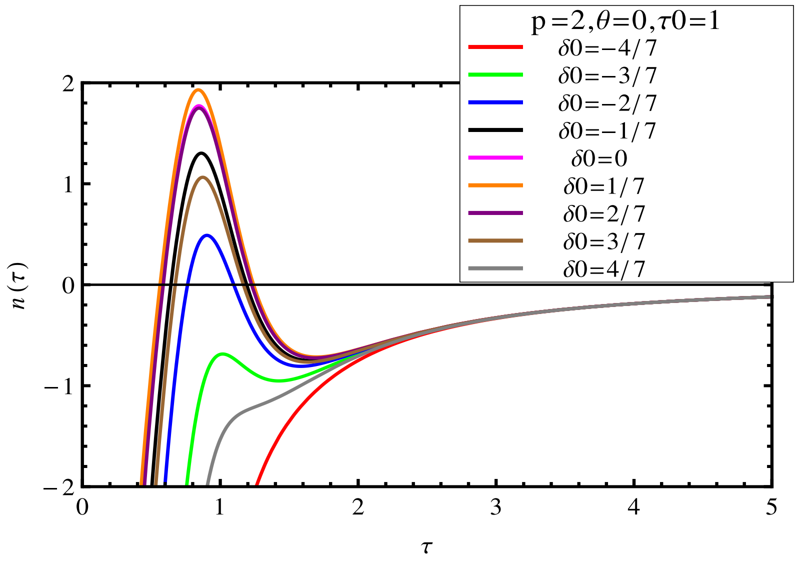

FIGURE 6.4: The plot of $n(\tau)$ for $\delta_0 = -\delta_{0c}(= -\frac{4}{7})$, $-\frac{3\delta_{0c}}{4}(= -\frac{3}{7})$, $-\frac{\delta_{0c}}{2}(= -\frac{2}{7})$, $-\frac{\delta_{0c}}{4}(= -\frac{1}{7})$, 0, $\frac{\delta_{0c}}{4}(= \frac{1}{7})$, $\frac{\delta_{0c}}{2}(= \frac{2}{7})$, $\frac{3\delta_{0c}}{4}(= \frac{3}{7})$, $\delta_{0c}(= \frac{4}{7})$, $p = 2$ and $\tau_0 = 1$.

and it gradually increases as we increase the value of θ, except at $\theta = \pi/2$. The duration of the accelerating phase gradually decreases with increasing θ and becomes zero exactly at $\frac{\pi}{2}$. This happens for every dimension where there is an accelerating phase. Note that here we have used the upper sign of α given in (6.18). If we use the lower sign we get exactly the same behavior with the interchange of $\theta = 0$ and $\theta = \pi/2$. Similar results can also be obtained for other values of p. In Fig:6.4, we have plotted the rate of expansion parameter $n(\tau)$ for different values of δ_0, with the other parameters kept fixed at $\theta = 0$ and $\tau_0 = 1$. We have again chosen $p = 2$, corresponding to $3 + 1$ dimensional universe. The accelerating phase depends on the value of δ_0. We varied δ_0 from δ_{0c} to $-\delta_{0c}$ in an interval of $(1/4)\delta_{0c}$. We find from the figure that the acceleration is maximum for $\delta_0 = \frac{\delta_{0c}}{4}$. Acceleration decreases for other absolute values of δ_0. For $\delta_0 = 0$ and $\delta_{0c}/2$ we get reasonably large acceleration although plus value gives more acceleration than minus value. On the other hand, for $\delta_0 = \pm\delta_{0c}$ we always get deceleration. Again this happens

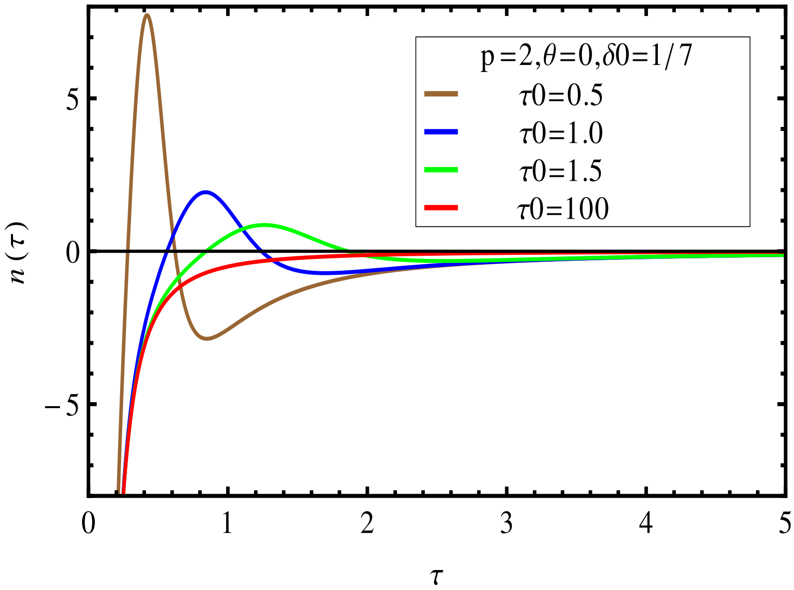

FIGURE 6.5: Plot of $n(\tau)$ for different values of τ_0, with the other parameters kept fixed at $\theta = 0$, $\delta_0 = \frac{\delta_{0c}}{4} = \frac{1}{7}$ and $p = 2$

for each dimension where there is an accelerating phase. As before there exists a critical absolute value of δ_0 for each p above which there is an accelerating phase and below which there will always be deceleration. Similar conclusion can be drawn for other values of p. In Fig:6.5, we have plotted $n(\tau)$ for different values of τ_0, with the other parameters kept fixed at $\theta = 0$ and $\delta_0 = \delta_{0c}/4$. We have chosen $p = 2$ as before such that we have $3 + 1$ dimensional space-time. Here we find that the acceleration is more but of shorter duration as we decrease the value of τ_0. However, as τ_0 is increased beyond a certain value we always have deceleration. This also happens for each dimension where there is an accelerating phase. There is a critical value of τ_0 at each dimension $p = 1, \ldots, 6$, below which there is an accelerating phase and above which there is always a deceleration.

We remark that even though it is difficult to obtain an exact relation between η and τ from the relation (6.13), but it can be integrated in the far future $\tau \gg \tau_0$ and also in the remote past $\tau \ll \tau_0$. The case of $\tau \ll \tau_0$ will be discussed in the next section. Here we mention that for $\tau \gg \tau_0$, τ is related to η by the relation $\tau \sim (\eta - \eta_0)^{\frac{p}{8}}$. Therefore, we have the scale factor (6.15) to take the form $S(\eta) \sim (\eta - \eta_0)^{1-\frac{p}{8}}$. It is clear from here that in the far future the universe will expand with deceleration for all p as we have seen in Fig:6.1, Fig:6.2.

Conformally de Sitter spaces in various dimensions

In this section we will see how at early times $\tau \ll \tau_0$, we can get de Sitter solutions upto a conformal transformation in various dimensions from the SDp solutions given in (6.8). Note that in this case the function $g(\tau)$ can be approximated as,

$$g(\tau) = 1 + \frac{\tau_0^{7-p}}{\tau^{7-p}} \approx \frac{\tau_0^{7-p}}{\tau^{7-p}} \tag{6.20}$$

and so the function $F(\tau)$ given in (6.9) can be approximated as,

$$F(\tau) \approx \left(\frac{\tau_0}{\tau}\right)^{\frac{7-p}{2}\alpha} \cos^2\theta \tag{6.21}$$

Here we have assumed $\theta \neq \pi/2$, otherwise, it is arbitrary. If $\theta = \pi/2$, then $F(\tau)$ takes the form $F(\tau) = (\frac{\tau_0}{\tau})^{-(7-p)\beta/2}$. But, as we will see that since the final answer will be independent of the parameters α, β, we can take the form of $F(\tau)$ as given in (6.21) without any loss of generality. We will further choose $\alpha + \beta = 2$ for calculational simplicity and again without losing any generality. Now since from the parameter relations given in (6.3) we have $\alpha - \beta = 3\delta_0$, combining these two we get $\alpha = 1 + (3/2)\delta_0$. For more simplification we will set $\tau_0 = 1$ and $\theta = 0$. With all these the metric and the dilaton in (6.8) take the forms,

$$ds^2 = -\tau^{\frac{(7-p)(15-p)-16}{16} - \frac{7(3-p)(p+1)}{32}\delta_0} d\tau^2 + \tau^{\frac{(7-p)^2}{16} + \frac{7(7-p)(3-p)}{32}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2$$

$$+\tau^{\frac{16-(7-p)(p+1)}{16}-\frac{7(3-p)(p+1)}{32}\delta_0}dH_{8-p}^2$$

$$e^{2(\phi-\phi_0)} = \tau^{-\frac{(3-p)(7-p)}{4}+\frac{7(7-p)(p+1)}{8}\delta_0} \tag{6.22}$$

It should be mentioned that here δ_0 is not a free parameter, unlike in the previous section where we did not use $\alpha+\beta=2$. In fact, since $\alpha = 1+(3/2)\delta_0$, we can use the second parameter relation in (6.3) to obtain the value of δ_0 as,

$$\delta_0 = \pm\frac{2}{7}\sqrt{\frac{9-p}{p+1}} \tag{6.23}$$

Now the metrics in (6.22) can also be written as,

$$ds^2 = -\left(\tau^{\frac{16-(7-p)(p+1)}{16}-\frac{7(3-p)(p+1)}{32}\delta_0}\right)^{\frac{p-8}{p}} ds_E^2 + \tau^{\frac{16-(7-p)(p+1)}{16}-\frac{7(3-p)(p+1)}{32}\delta_0}dH_{8-p}^2 \tag{6.24}$$

where ds_E^2 is a $(p+1)+1$ dimensional metrics in the Einstein frame and have the forms,

$$ds_E^2 = \tau^{\frac{(9-p)(p+1)}{2p}-\frac{7(3-p)(p+1)}{4p}\delta_0}\left[-\frac{d\tau^2}{\tau^2}+\tau^{-\frac{(9-p)}{2}+\frac{7(3-p)}{4}\delta_0}\sum_{i=1}^{p+1}(dx^i)^2\right] \tag{6.25}$$

Actually the metrics in (6.25) can be seen to arise from a $(8-p)$ dimensional hyperbolic space compactifications with time dependent radius $R(\tau) = \tau^{\frac{16-(7-p)(p+1)}{8}-\frac{7(3-p)(p+1)}{16}\delta_0}$ and then expressing the resulting $(p+1)+1$ dimensional metrics in the Einstein frame. We notice that for $p=3$, $R(\tau)=1$ and for this case the transverse hyperbolic space H_5 gets decoupled from the rest of the space-time, similar to what happens for D3 brane where the transverse S^5 gets decoupled. This simplification for $p=3$ case occurs because of our particular choice of parameters, namely, $\alpha+\beta=2$. Defining a canonical time by the relation,

$$\eta^2 = \tau^{\frac{(9-p)}{2}-\frac{7(3-p)}{4}\delta_0} \tag{6.26}$$

we can rewrite the metrics in (6.25) as,

$$ds_E^2 = \eta^{\frac{2(p+1)}{p}} \left[-\frac{d\eta^2}{\eta^2} + \frac{\sum_{i=1}^{p+1}(dx^i)^2}{\eta^2} \right] \tag{6.27}$$

Note that in writing the above metrics we have scaled η and x^i's as follows,

$$\begin{aligned} \eta &\rightarrow \left(\frac{9-p}{4} - \frac{7(3-p)}{16}\delta_0 \right)^{\frac{p}{p+1}} \eta \\ x^i &\rightarrow \left(\frac{9-p}{4} - \frac{7(3-p)}{16}\delta_0 \right)^{\frac{p}{p+1}} x^i, \qquad \text{for} \quad i = 1, \ldots, (p+1) \end{aligned} \tag{6.28}$$

The dilaton given in (6.22) can also be written in terms of canonical time using (6.26). We recognize the metrics in (6.27) to be the de Sitter metrics in $(p+1)+1$ dimensions upto the conformal factor $\eta^{2(p+1)/p}$. For $p = 2$, i.e., for the four dimensional case the conformal factor becomes η^3 precisely the form we obtained in [101].

We can compare the situation here with the time-like or static BPS Dp-brane cases. For the usual Dp branes when $p = 3$, the near horizon limit gives AdS$_5$ $\times$ S^5 solution. In other words, compactifying on S^5, we get AdS$_5$ solution. So, here the boundary theory is conformally invariant. But for other Dp branes (except for $p = 5$), compactifying on S^{8-p}, we do not get AdS$_{p+2}$ spaces directly, but we get them upto a conformal factor. So, the boundary theories for these cases do not have conformal symmetry (as the bulk spaces are not AdS spaces – conformal factors make the bulk spaces to be different from AdS spaces), but still the connection with the AdS spaces upto a conformal transformation proves to be useful for calculational purposes. For, space-like Dp branes compactifying on hyperbolic spaces H$_{8-p}$, we never get (for any p) de Sitter spaces, but we get them (dS$_{p+2}$) upto a conformal factor exactly like static Dp branes with $p \neq 3, 5$. Here also the boundary theories do not have conformal symmetry since the bulk solution is not a de Sitter solution. However, this connection with de Sitter solution with the space-like Dp

branes might prove to be useful for calculational purposes (for example, calculation of correlation function) as in the static Dp brane cases.

To see that the space-times given in (6.27) describe decelerating expansions we rewrite them in flat FLRW forms by defining a new canonical coordinate by $d\tilde{\eta} = \eta^{\frac{1}{p}} d\eta$. The metrics in (6.27) then takes the forms,

$$ds_E^2 = -d\tilde{\eta}^2 + S^2(\tilde{\eta}) \sum_{i=1}^{p+1} (dx^i)^2 \tag{6.29}$$

where the scale factor is given by $S(\tilde{\eta}) \sim (\tilde{\eta} - \tilde{\eta}_0)^{1-\frac{p}{p+1}}$. This clearly shows that the universes expand with deceleration for all p. For $p = 2$, we get $S(\tilde{\eta}) \sim (\tilde{\eta} - \tilde{\eta}_0)^{\frac{1}{3}}$, the result that was obtained in [108].

Thus we have seen how starting from isotropic SDp brane solutions of type II string theories, we get $(p + 1) + 1$ dimensional de Sitter spaces upto a conformal factor by compactifying on $(8 - p)$ dimensional hyperbolic spaces. This brings out the connection between space-like branes and the de Sitter space which might be helpful in understanding dS/CFT correspondence in the same spirit as AdS/CFT correspondence. From the metrics in this case we find that the space-times undergo decelerating expansion for all p, but only in particular conformal frames we get de Sitter spaces, i.e., eternal accelerations.

Conclusion

To conclude, in this chapter we have studied the various cosmological scenarios that are obtained from the isotropic space-like Dp brane solutions of type II string theories by compactifications on $(8-p)$ dimensional hyperbolic spaces and also found the connection between SDp branes and $(p+1)+1$ dimensional de Sitter spaces. The SDp brane solutions are characterized by three independent parameters τ_0, θ and δ_0. τ_0 sets a time scale in the theory, θ is related to the RR charge associated with SDp branes and δ_0 is associated with the dilaton in the sense that when $p = 3$, the dilaton is trivial for $\delta_0 = 0$ much like time-like D3 branes. τ_0 gives a time scale because when $\tau \gg \tau_0$, the SDp brane solutions reduce to flat spaces and in that sense these solutions are asymptotically ($\tau \to \infty$) flat. At large time or in the far future we found that the external space-times undergo decelerating expansions where the scale factors behave like $S(\eta) \sim (\eta - \eta_0)^{1-\frac{p}{8}}$, for all values of p from 1 to 6. On the other hand, when $\tau \sim \tau_0$, the SDp branes upon compactifications by hyperbolic spaces give external space-times which in suitable coordinate can be recast into flat FLRW forms. Here we kept the parameter δ_0 to be arbitrary and found that $(p+1)+1$ dimensional external spaces undergo accelerating expansions for all p. We have studied various cases numerically; because of the complicated nature of the solutions, it is not possible to study them analytically. We have plotted the expansion parameter $m(\tau)$ and the rate of expansion parameter $n(\tau)$ defined in the text, for various values of p to show the cosmologies in various dimensions. We found that for all p lying between 1 to 6, there is a region where $n(\tau)$ becomes positive for certain finite interval of time indicating that universes undergo a transient phase of accelerating expansion. We have also plotted $n(\tau)$ when we vary the three parameters θ, δ_0 and τ_0 while keeping the other parameters fixed in Fig:6.3, Fig:6.4, and Fig:6.5 respectively. These show how the acceleration changes as we vary the parameters. Finally, we have shown that at early time, i.e., for $\tau \ll \tau_0$, the $(p + 1) + 1$ dimensional external spaces can be cast into the form of de Sitter metrics upto a conformal transformation for all values of p. Here we have fixed the parameter δ_0 for calculational simplicity. This brings out the connection between the SDp branes and the de Sitter space which was the original motivation for constructing the space-like branes, and might be useful in understanding dS/CFT correspondence in the same spirit as AdS/CFT correspondence. We mentioned that the cosmologies here again are decelerating, but they give eternal accelerations only in a special conformal frame.

We know our current universe is not completely isotropic. However we have obtained the isotropic cosmology in this chapter, to get a picture closed to the realistic universe one can start with the anisotropic S-brane. It will be discussed in next chapter.

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Chapter 7

Kasner-like Cosmologies

Having seen the isotropic picture, we are now in position to derive the anisotropic cosmology from S-brane solutions. It is well-known [108] that cosmological solution of higher dimensional vacuum Einstein equation can give rise to interesting four dimensional cosmology (with a period of accelerated expansion) upon time dependent hyperbolic space compactifications [102]. This process, therefore, evades a no-go theorem [115,116] of obtaining such accelerated expansion in standard time-independent compactifications. Similar cosmologies also follow if one includes fluxes [30] and/or a dilaton field [109] in the higher dimensional theories such as M/string theory.

The previous S2 brane solutions considered in the last chapter and also in the literature [16, 30, 35, 109] were isotropic in the brane directions and so the four dimensional accelerating cosmologies obtained from these solutions were isotropic. In this chapter we will construct an anisotropic SD2 brane solutions of type IIA string theory and try to see whether similar four dimensional accelerating cosmologies can be obtained in all three spatial directions upon compactification. Another motivation to look at the anisotropic SD2 brane solution is to see whether one can get a four dimensional Kasner-like [117] solution from it upon compactification where one can get expansions in all three spatial directions which is not possible in conventional Kasner solution from four dimensional vacuum Einstein equation. The construction of anisotropic SD2 brane solution follows from the standard double Wick rotation [18] of the known anisotropic non-susy D2 brane solution [26] of type IIA string theory. This solution is characterized by five independent parameters. We then cast the solution in a suitable time-like coordinate and is given in terms of a single harmonic function containing a characteristic time t_0. Next, we compactify the space-time on a six dimensional hyperbolic space with time dependent volume. The resultant metric when expressed in Einstein frame gives us a four dimensional FLRW type space-time with three different scale factors in three spatial directions. We find that when $t \sim t_0$, we can get accelerating cosmologies in all three directions when other parameters of the solution take some specific values. Although the expansions and the accelerations in all three directions are not the same but they do not differ drastically and the accelerations are all transient. However, when $t \ll t_0$, the resultant four dimensional metric takes a Kasner-like form when the parameters characterizing the solution satisfy certain conditions. But because of the presence of the dilaton as well as the volume scalar of the six dimensional hyperbolic space, all the Kasner exponents could be positive definite, leading to expansions in all three spatial directions. However, the expansions in this case are decelerating. This can be contrasted with the standard four dimensional Kasner space-time [117] (obtained from the solution of vacuum Einstein equation) where expansions in all three directions are not possible.

This chapter is organized as follows. In the first section we give the construction of anisotropic SD2 brane solution from its time-like counterpart and cast the solution in a coordinate system suitable for our purpose. In section 2, we obtain the anisotropic accelerating cosmologies from this solution upon compactification on six dimensional hyperbolic space of time dependent volume. In section 5, we show how a four dimesional Kasner-like geometry arises from this string theory solution, where all the Kasner exponents could be positive definite leading to expansions in all three spatial directions unlike the standard Kasner solution in four dimensions. Finally, we conclude in section 4.

Anisotropic SD2 brane solutions

In this section we construct the anisotropic SD2 brane solution from the known anisotropic non-susy D2 brane solution of type IIA string theory. In chapter 2, we have constructed an anisotropic non-susy Dp brane solution and interpreted it as the intersecting solution of various non-susy branes. Here we make use of that solution and write the anisotropic non-susy D2 brane solution in the following by putting $p = 2$ and $q = 0$ in (2.44) of the chapter 2,

$$\begin{aligned} ds^2 &= F(r)^{\frac{3}{8}} \left(H(r)\tilde{H}(r)\right)^{\frac{2}{5}} \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{10}+\frac{\delta_3}{10}} \left(dr^2 + r^2 d\Omega_6^2\right) \\ &\quad + F(r)^{-\frac{5}{8}} \Big\{ - \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}+\frac{\delta_3}{2}} dt^2 + \left(\frac{H(r)}{\tilde{H}(r)}\right)^{-\frac{3\delta_1}{4}-\frac{3\delta_2}{2}+\frac{\delta_3}{2}} (dx^1)^2 \\ &\quad + \left(\frac{H(r)}{\tilde{H}(r)}\right)^{-\frac{3\delta_1}{4}+\frac{\delta_2}{2}-\frac{3\delta_3}{2}} (dx^2)^2 \Big\} \end{aligned} \tag{7.1}$$

$$e^{2(\phi-\phi_0)} = F(r)^{\frac{1}{2}} \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\delta_1-2\delta_2-2\delta_3}, \qquad F_{[6]} = \hat{Q}\mathrm{Vol}(\Omega_6) \tag{7.2}$$

The metric in the above is given in the Einstein frame. The various functions appearing in the solution are defined as,

$$
\begin{aligned}
F(r) &= \left(\frac{H(r)}{\tilde{H}(r)}\right)^{\alpha}\cosh^2\theta - \left(\frac{\tilde{H}(r)}{H(r)}\right)^{\beta}\sinh^2\theta \\
H(r) &= 1+\frac{\omega^5}{r^5}, \qquad \tilde{H}(r) = 1-\frac{\omega^5}{r^5}
\end{aligned}
\tag{7.3}
$$

Note that the solution has eight parameters α, β, δ_1, δ_2, δ_3, θ, ω, and $\hat{Q}$. ϕ_0 is the asymptotic value of the dilaton and $F_{[6]}$ is a six form and $\hat{Q}$ is the magnetic charge associated with the D2 brane. The solution becomes isotropic in the brane directions when $\delta_1 = -2\delta_2 = -2\delta_3$. So, in that sense these parameters can be called anisotropy parameters. Now for the consistency of the field equations the eight parameters of the solution must satisfy the following relations [26],

$$
\begin{aligned}
\alpha-\beta &= -\frac{3}{2}\delta_1 \\
\frac{1}{2}\delta_1^2+\frac{1}{2}\alpha(\alpha+\frac{3}{2}\delta_1)+\frac{2}{5}\delta_2\delta_3 &= \frac{6}{5}\left(1-\delta_2^2-\delta_3^2\right) \\
\hat{Q} &= 5\omega^5(\alpha+\beta)\sinh 2\theta
\end{aligned}
\tag{7.4}
$$

These three relations reduce the number of independent parameters from eight to five, which are ω, θ, and the anisotropy parameters δ_1, δ_2 and δ_3. Using the second and the first relations in (7.4), we can express α and β in terms of the other parameters as,

$$
\begin{aligned}
\alpha &= -\frac{3}{4}\delta_1 \pm \frac{1}{2}\sqrt{\frac{48}{5}(1-\delta_2^2-\delta_3^2)-\frac{7}{4}\delta_1^2-\frac{16}{5}\delta_2\delta_3} \\
\beta &= \frac{3}{4}\delta_1 \pm \frac{1}{2}\sqrt{\frac{48}{5}(1-\delta_2^2-\delta_3^2)-\frac{7}{4}\delta_1^2-\frac{16}{5}\delta_2\delta_3}
\end{aligned}
\tag{7.5}
$$

The form of the harmonic function $\tilde{H}(r)$ in (7.3) indicates that there is a naked singularity of the solution at $r=\omega$ and therefore, the solution is well defined only for $r>\omega$. Now we apply the double Wick rotation [18] $r \to i\tau$, $t \to -ix^3$ to the solution (7.1) along with $\omega \to i\omega$, $\theta \to i\theta$ and $\theta_1 \to i\theta_1$, where θ_1 is one of the angular coordinates of the sphere Ω_6 of the transverse space. This operation gives us anisotropic space-like D2 brane from the anisotropic static non-susy D2 brane and the change in the angular coordinate converts spherical Ω_6 to hyperbolic H_6. Thus the transformed solution is,

$$\begin{aligned} ds^2 &= F(\tau)^{\frac{3}{8}} \left(H(\tau)\tilde{H}(\tau)\right)^{\frac{2}{5}} \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{10}+\frac{\delta_3}{10}} \left(-d\tau^2 + \tau^2 dH_6^2\right) \\ &\quad + F(\tau)^{-\frac{5}{8}} \Bigg\{ \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{\frac{\delta_1}{4}+\frac{\delta_2}{2}+\frac{\delta_3}{2}} (dx^3)^2 + \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{-\frac{3\delta_1}{4}-\frac{3\delta_2}{2}+\frac{\delta_3}{2}} (dx^1)^2 \\ &\quad + \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{-\frac{3\delta_1}{4}+\frac{\delta_2}{2}-\frac{3\delta_3}{2}} (dx^2)^2 \Bigg\} \end{aligned} \tag{7.6}$$

$$e^{2(\phi-\phi_0)} = F(\tau)^{\frac{1}{2}} \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{\delta_1-2\delta_2-2\delta_3}, \qquad F_{[6]} = \hat{Q}\mathrm{Vol}(H_6) \tag{7.7}$$

The various functions associated with the solution are also changed under the above rotation and are given below,

$$\begin{aligned} F(\tau) &= \left(\frac{H(\tau)}{\tilde{H}(\tau)}\right)^{\alpha} \cos^2\theta + \left(\frac{\tilde{H}(\tau)}{H(\tau)}\right)^{\beta} \sin^2\theta \\ H(\tau) &= 1 + \frac{\omega^5}{\tau^5}, \qquad \tilde{H}(\tau) = 1 - \frac{\omega^5}{\tau^5} \end{aligned} \tag{7.8}$$

Thus we see that the anisotropic static non-susy D2 brane has been converted to anisotropic time dependent or space-like D2 brane. For the former solution the radial coordinate r was transverse to the D2 brane's world-volume, whereas, for the latter the timelike coordinate τ is transverse to the SD2 brane's world-volume. The metric of the transverse sphere $d\Omega_6^2$ has been converted to negative of the metric of the hyperbolic space dH_6^2. The hyperbolic functions $\sinh^2\theta$ and $\cosh^2\theta$ become $-\sin^2\theta$ and $\cos^2\theta$ respectively, therefore, the relative sign of the two terms of the function $F(\tau)$ has been flipped. But the form field remains unchanged with $\hat{Q} \to -\hat{Q}$. Thus the first two parameter relations in (7.4) remain the same, while the last relation has changed to $\hat{Q} = 5\omega^5(\alpha+\beta)\sin 2\theta$. Now for our purpose we will make a coordinate transformation from τ to t given by,

$$\tau = t\left(\frac{1+\sqrt{g(t)}}{2}\right)^{\frac{2}{5}}, \qquad \text{where,} \qquad g(t) = 1+\frac{4\omega^5}{t^5} \equiv 1+\frac{t_0^5}{t^5} \tag{7.9}$$

Under this coordinate change we have,

$$\begin{aligned}
&H(\tau) = 1+\frac{\omega^5}{\tau^5} = \frac{2\sqrt{g(t)}}{1+\sqrt{g(t)}}, \qquad \tilde{H}(\tau) = 1-\frac{\omega^5}{\tau^5} = \frac{2}{1+\sqrt{g(t)}},\\
&H(\tau)\tilde{H}(\tau) = \frac{4\sqrt{g(t)}}{(1+\sqrt{g(t)})^2}, \qquad \frac{H(\tau)}{\tilde{H}(\tau)} = \sqrt{g(t)},\\
&-d\tau^2+\tau^2 dH_6^2 = g(t)^{\frac{1}{5}}\left(-\frac{dt^2}{g(t)}+t^2 dH_6^2\right)
\end{aligned} \tag{7.10}$$

Using (7.10) we can rewrite the anisotropic SD2 brane solution given in (7.6) as follows,

$$\begin{aligned}
ds^2 &= F(t)^{\frac{3}{8}} g(t)^{\frac{\delta_1}{8}+\frac{\delta_2}{20}+\frac{\delta_3}{20}+\frac{1}{5}}\left(-\frac{dt^2}{g(t)}+t^2 dH_6^2\right) + F(t)^{-\frac{5}{8}}\Big[g(t)^{-\frac{3\delta_1}{8}-\frac{3\delta_2}{4}+\frac{\delta_3}{4}}(dx^1)^2\\
&\quad + g(t)^{-\frac{3\delta_1}{8}+\frac{\delta_2}{4}-\frac{3\delta_3}{4}}(dx^2)^2 + g(t)^{\frac{\delta_1}{8}+\frac{\delta_2}{4}+\frac{\delta_3}{4}}(dx^3)^2\Big]\\
e^{2(\phi-\phi_0)} &= F(t)^{\frac{1}{2}} g(t)^{\frac{\delta_1}{2}-\delta_2-\delta_3}, \qquad F_{[6]} = \hat{Q}\mathrm{Vol}(H_6)
\end{aligned} \tag{7.11}$$

where $g(t)$ is as given in (7.9) and $F(t)$ is given by,

$$F(t) = g(t)^{\frac{\alpha}{2}}\cos^2\theta + g(t)^{-\frac{\beta}{2}}\sin^2\theta \tag{7.12}$$

It is important to note that in the new coordinate, the original singularity at $\tau = \omega$ has been shifted to $t=0$. Also note that as $t \gg t_0$, $g(t),\ F(t) \to 1$ and therefore, the solution reduces to flat space. In the next two sections we will impose the assumption $t \sim t_0$ and also $t \ll t_0$ into the solution (7.11) to see how one can get accelerating cosmology in the first case and a Kasner-like cosmology in the second case in (3+1) dimensions upon compactification.

Compactification

Accelerating cosmology

In this section we will compactify the anisotropic SD2 brane solution given in (7.11) on a six dimensional hyperbolic space of time dependent volume and write the resultant four dimensional metric in the Einstein frame[1]. This four dimensional metric will have the standard FLRW form whose cosmology we want to study. We rewrite the metric in (7.11) in a four dimensional part and the transverse six dimensional part as,

$$ds^2 = ds_4^2 + e^{2\psi} dH_6^2 \tag{7.13}$$

where ψ is the radion field and $e^{2\psi} = F(t)^{\frac{3}{8}} g(t)^{\frac{\delta_1}{8}+\frac{\delta_2}{20}+\frac{\delta_3}{20}+\frac{1}{5}} t^2$. The four dimensional metric ds_4^2 is given as,

$$\begin{aligned} ds_4^2 &= -F(t)^{\frac{3}{8}} g(t)^{\frac{\delta_1}{8}+\frac{\delta_2}{20}+\frac{\delta_3}{20}-\frac{4}{5}} dt^2 + F(t)^{-\frac{5}{8}} \Big[g(t)^{-\frac{3\delta_1}{8}-\frac{3\delta_2}{4}+\frac{\delta_3}{4}} (dx^1)^2 \\ &\quad + g(t)^{-\frac{3\delta_1}{8}+\frac{\delta_2}{4}-\frac{3\delta_3}{4}} (dx^2)^2 + g(t)^{\frac{\delta_1}{8}+\frac{\delta_2}{4}+\frac{\delta_3}{4}} (dx^3)^2 \Big] \end{aligned} \tag{7.14}$$

[1]Here one might ask that since hyperbolic spaces are in general non-compact in what sense are we compactifying the ten dimensional space on six dimensional hyperbolic space and studying the four dimensional cosmology? To address this question we remark that it is quite well-known how to construct compact hyperbolic manifolds (CHM) from hyperbolic spaces and there is a vast mathematical literature some of which are given in [102]. In short, the CHM's are obtained from H_d (with $d \geq 2$), the universal covering space of d dimensional hyperbolic manifold by modding out by an appropriate freely acting discrete subgroup of the isometry group SO$(1, d)$ of H_d. CHM's have many interesting properties and we refer the reader to some of the original literature given in [102] for details.

The compactified four dimensional metric (7.14) when expressed in Einstein frame takes the form [109],

$$\begin{aligned} ds^2_{4E} &= e^{6\psi} ds^2_4 \\ &= -F(t)^{\frac{3}{2}} g(t)^{-\frac{1}{5}+\frac{\delta_1}{2}+\frac{\delta_2}{5}+\frac{\delta_3}{5}} t^6 dt^2 + F(t)^{\frac{1}{2}} g(t)^{\frac{3}{5}-\frac{3\delta_2}{5}+\frac{2\delta_3}{5}} t^6 dx_1^2 \\ &\quad + F(t)^{\frac{1}{2}} g(t)^{\frac{3}{5}+\frac{2\delta_2}{5}-\frac{3\delta_3}{5}} t^6 dx_2^2 + F(t)^{\frac{1}{2}} g(t)^{\frac{3}{5}+\frac{\delta_1}{2}+\frac{2\delta_2}{5}+\frac{2\delta_3}{5}} t^6 dx_3^2 \\ &= -A(t)^2 dt^2 + \sum_{i=1}^{3} S_i(t)^2 dx_i^2 \end{aligned} \tag{7.15}$$

where the various time-dependent coefficients are

$$\begin{aligned} A(t) &= F(t)^{\frac{3}{4}} g(t)^{-\frac{1}{10}+\frac{\delta_1}{4}+\frac{\delta_2}{10}+\frac{\delta_3}{10}} t^3, \qquad S_1(t) = F(t)^{\frac{1}{4}} g(t)^{\frac{3}{10}-\frac{3\delta_2}{10}+\frac{\delta_3}{5}} t^3 \\ S_2(t) &= F(t)^{\frac{1}{4}} g(t)^{\frac{3}{10}+\frac{\delta_2}{5}-\frac{3\delta_3}{10}} t^3, \qquad S_3(t) = F(t)^{\frac{1}{4}} g(t)^{\frac{3}{10}+\frac{\delta_1}{4}+\frac{\delta_2}{5}+\frac{\delta_3}{5}} t^3 \end{aligned} \tag{7.16}$$

Note that in the compactified four dimensional space there are three fields, namely $g_{\mu\nu},\ \phi,\ \psi$. Now we perform another coordinate transformation

$$d\eta^2 = F(t)^{\frac{3}{2}} g(t)^{-\frac{1}{5}+\frac{\delta_1}{2}+\frac{\delta_2}{5}+\frac{\delta_3}{5}} t^6 dt^2 \;\Rightarrow\; \eta = \int F(t)^{\frac{3}{4}} g(t)^{-\frac{1}{10}+\frac{\delta_1}{4}+\frac{\delta_2}{10}+\frac{\delta_3}{10}} t^3 dt \tag{7.17}$$

and rewrite the Einstein frame metric ds^2_{4E} in the standard flat FLRW form as

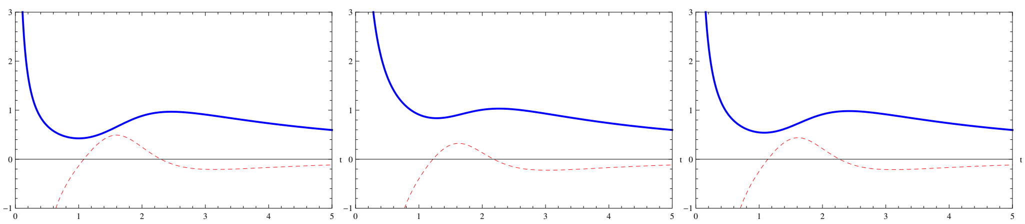
$$ds^2_{4E} = -d\eta^2 + s_i^2(\eta) \sum_{i=1}^{3} (dx^i)^2 \tag{7.18}$$

with η being the canonical time and the scale factor $s_i(\eta) \equiv S_i(t)$. Note that since $s_i(\eta)$ are different for each i, the cosmology here will be anisotropic. Now because of the complicated relation between t and η let us define (6.17)

$$m_i(t) \equiv \frac{d \ln S_i(t)}{dt}$$

$$n_i(t) \equiv \left[\frac{d^2}{dt^2}\ln(S_i(t)) + \frac{d}{dt}\ln(S_i(t))\frac{d}{dt}\ln\left(\frac{S_i(t)}{A(t)}\right)\right] \tag{7.19}$$

and with these one can easily see that $m_i(t) > 0$ implies that $ds_i(\eta)/d\eta > 0$, amounting to expansion of our universe, and similarly, $n_i(t) > 0$ implies that $d^2s_i(\eta)/d\eta^2 > 0$, amounting to acceleration of our universe. Therefore, from (7.19) it is clear that in the four-dimensional spacetime (7.15) we get an accelerated expansion in the i-th coordinate direction only if the parameters $m_i(t)$ and $n_i(t)$ are simultaneously positive in that direction. It can be checked that for $t \ll t_0$, accelerating expansion is not possible at all in any direction. However, it is possible only if $t \sim t_0$. In this case the first term in the harmonic function $g(t)$ given in (7.9) is of the same order as the second. The other parameters of the solution, namely, δ_1, δ_2 and δ_3 can not be totally arbitrary. From Eq.(7.5), we see that the reality of α and β imposes some restriction on the value of these three anisotropy parameters. Also it can be checked that by changing the value of θ does not change the cosmological behavior of the solution very much. Thus we have chosen some typical values of these parameters (as given in the Figure) and plotted the functions $m_i(t)$ and $n_i(t)$ in Fig:7.1, to show that it is indeed possible to have accelerating expansions in all three directions. We notice as shown in (a), (b) and (c) in Fig:7.1, we always get



(A) along x_1 direction

(B) along x_2 direction

(C) along x_3 direction

FIGURE 7.1: The plot of $m(t)$(solid blue line) and $n(t)$(dashed red line) in different spatial coordinate directions at $\theta = \pi/6$, $\delta_1 = -0.5$, $\delta_2 = 0.2$, $\delta_3 = 0.4$ and $t_0 = 2.0$.

expanding universe (given by the solid blue line) in all three directions, but the expansion is accelerating only for a short period of time, i.e., the acceleration is transient (given by the dotted red line). Also note that since $m_i(t)$ and $n_i(t)$ are different for different i, the cosmology is anisotropic, however, the anisotropy is not too much.

To understand the accelerating expansion, we can write down the four dimensional compactified action from the original ten dimensional one and obtain the form of the potential of the dilaton and the radion field [109]. The ten dimensional action has the form,

$$S = \int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\cdot 6!}e^{-\phi/2}F_{[6]}^2\right] \tag{7.20}$$

Reducing the action on a six dimensional hyperbolic space H_6, the four dimensional action we get[2] [**?**, 118]

$$S_4 = \int d^4x\sqrt{-g_{4E}}\left[R_{4E} - \frac{1}{2}(\partial\phi)^2 - 24(\partial\psi)^2 - V(\phi,\psi)\right] \tag{7.21}$$

where,

$$V(\phi,\psi) = \frac{\hat{Q}^2}{2}e^{-\frac{\phi}{2}-18\psi} + 30e^{-8\psi}. \tag{7.22}$$

Here $\hat{Q}$ is the magnetic charge of the D2 brane given in (7.1). Note that because of the hyperbolic space compactification the potential is always positive irrespective of the charge and therefore there is always a possibility that the system will be driven to an accelerating phase [110, 119].

[2]Here reduction on the hyperbolic space H_6 to obtain the four dimensional action is done in the sense decribed in footnote 3. This has also been done in the references [**?**, 118].

Kasner-like solution

In this section we will show how a four dimensional Kasner-like cosmological solution follows from the anisotropic SD2 brane solution upon six dimensional hyperbolic space compactification discussed in the previous section. The compactified action expressed in Einstein frame is given in (7.15). We take this four dimensional metric and express it at early times, $t \ll t_0$. In this case the function $g(t)$ can be approximated as,

$$g(t) = 1 + \frac{t_0^5}{t^5} \approx \frac{t_0^5}{t^5} \sim t^{-5}, \tag{7.23}$$

Also since we want to express the metric components in (7.15) as some powers of t, we note from the form of $F(t)$ in (7.12) that this can be done (assuming $\alpha > 0$ without any loss of generality) in three ways as follows. (a) Put $\theta = 0$, with α, β as given in (7.5), (b) put $\alpha = -\beta = -(3/4)\delta_1$, with θ arbitrary and (c) both $\alpha > 0$, $\beta > 0$, with θ arbitrary. There is another possibility with $\theta = \pi/2$ and $\beta < 0$, but this case can be seen to be equivalent to case (a). Note that for case (a) and (b) we have $\hat{Q} = 0$ (since $\hat{Q} = 5\omega^5(\alpha + \beta)\sin 2\theta$), however, for case (c) $\hat{Q}$ is non-zero and the non-susy brane is magnetically charged. In either case (a) or (b) we have

$$F(t) \sim t^{-\frac{5\alpha}{2}} \tag{7.24}$$

In the above we have absorbed t_0 in t. But for case (c) $F(t)$ has an additional $\cos^2\theta$ factor which can be absorbed in t as well as in $x^{1,2,3}$. Thus in all cases $F(t)$ has the form as given in (7.24). So, in this near region, the space-time metric (7.15), the dilaton and the radion fields take the forms,

$$ds^2 = -t^{2\left(\frac{7}{2} - \frac{15\alpha}{8} - \frac{5\delta_1}{4} - \frac{\delta_2}{2} - \frac{\delta_3}{2}\right)} dt^2 + t^{2\left(\frac{3}{2} - \frac{5\alpha}{8} + \frac{3\delta_2}{2} - \delta_3\right)} (dx^1)^2$$

$$
\begin{aligned}
& + t^{2\left(\frac{3}{2}-\frac{5\alpha}{8}-\delta_2+\frac{3\delta_3}{2}\right)}(dx^2)^2 + t^{2\left(\frac{3}{2}-\frac{5\alpha}{8}-\frac{5\delta_1}{4}-\delta_2-\delta_3\right)}(dx^3)^2 \\
e^{2(\phi-\phi_0)} &= t^{2\left(-\frac{5\alpha}{8}-\frac{5\delta_1}{4}+\frac{5\delta_2}{2}+\frac{5\delta_3}{2}\right)}, \qquad e^{2\psi} = t^{2\left(\frac{1}{2}-\frac{15}{32}\alpha-\frac{5}{16}\delta_1-\frac{\delta_2}{8}-\frac{\delta_3}{8}\right)} \qquad (7.25)
\end{aligned}
$$

Now since we are taking $t \ll 1$ here, we have to be careful about the validity of the gravity solution. The gravity solution will be valid as long as the dilaton remains small and the curvature of the transverse space in string units also remains small. These two conditions impose certain restrictions on the parameters of the solution and they are given as,

$$
\begin{aligned}
& 5\alpha + 5\delta_1 - 4\delta_2 - 4\delta_3 > 4 \\
& \alpha + 2\delta_1 - 4\delta_2 - 4\delta_3 \leq 0 \qquad (7.26)
\end{aligned}
$$

where α is as given in (7.5). Furthermore, the reality of α also restricts the parameters as

$$
\frac{35}{4}\delta_1^2 + 48\delta_2^2 + 48\delta_3^2 + 16\delta_2\delta_3 \leq 48 \qquad (7.27)
$$

We have checked numerically that all these three conditions can be satisfied simultaneously for certain range of values of the parameters δ_1, δ_2 and δ_3 and only for those values we have a valid gravity solution (7.25). We would like to remark here that the validity of the supergravity solution also requires that we cannot take t arbitrarily close to zero as we are considering $t \ll 1$. In fact t has to be much larger than the string scale if the supergravity solution remains valid. This can be seen if we calculate $\dot{\phi}^2$, $\dot{\psi}^2$ and also the scalar curvature with the solution given in (7.25). All of these terms come out to be proportional to $1/t^2$ and so, when $t \ll 1$, they can become very large invalidating the supergravity solution and stringy corrections must be included. To avoid this we require $\sqrt{\alpha'} \ll t \ll t_0$ or in terms of scaled t, we must have $\sqrt{\alpha'}/t_0 \ll t \ll 1$. Now, keeping those restrictions in mind, we can rewrite the solution in terms of canonical time $\eta \equiv \frac{8t^{\frac{9}{2}-\frac{15\alpha}{8}-\frac{5\delta_1}{4}-\frac{\delta_2}{2}-\frac{\delta_3}{2}}}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3}$

as,

$$
\begin{aligned}
ds^2 &= -d\eta^2 + \eta^{2p_1}(dx^1)^2 + \eta^{2p_2}(dx^2)^2 + \eta^{2p_3}(dx^3)^2 \\
e^{2(\phi-\phi_0)} &= C(\delta_1,\delta_2,\delta_3)\,\eta^{2\gamma_\phi} \qquad\qquad e^{2\psi} = D(\delta_1,\delta_2,\delta_3)\eta^{2\gamma_\psi}
\end{aligned} \tag{7.28}
$$

Note that in writing the metric in (7.28) we have rescaled the coordinates x^1, x^2 and x^3 by some constant factors involving the parameters δ_1, δ_2, δ_3. Also in the dilaton and the radion field C and D are constants involving these paramaters whose explicit form will not be important. It can be easily checked in the defining relation of η, that the coefficient in front of t is always positive definite and that also ensures that as $t \to 0$, $\eta \to 0$. The Kasner exponents p_1, p_2 and p_3 in the metric and γ_ϕ, γ_ψ are defined as,

$$
\begin{aligned}
p_1 &= \frac{12-5\alpha+12\delta_2-8\delta_3}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3} \\
p_2 &= \frac{12-5\alpha-8\delta_2+12\delta_3}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3} \\
p_3 &= \frac{12-5\alpha-10\delta_1-8\delta_2-8\delta_3}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3} \\
\gamma_\phi &= \frac{-5\alpha-10\delta_1+20\delta_2+20\delta_3}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3} \\
\gamma_\psi &= \frac{1}{4}\frac{16-15\alpha-10\delta_1-4\delta_2-4\delta_3}{36-15\alpha-10\delta_1-4\delta_2-4\delta_3}
\end{aligned} \tag{7.29}
$$

Now since this a solution to the compactified four dimensional action given in (7.21), it must satisfy the equations of motion. The Einstein equation, the dilaton and radion equations following from (7.21) have the forms,

$$
\begin{aligned}
&R_{\mu\nu,E} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - 24\partial_\mu\psi\partial_\nu\psi = 0 \\
&\frac{1}{\sqrt{-g_E}}\partial_\mu\left(\sqrt{-g_E}g_E^{\mu\nu}\partial_\nu\phi\right) = 0, \qquad \frac{1}{\sqrt{-g_E}}\partial_\mu\left(\sqrt{-g_E}g_E^{\mu\nu}\partial_\nu\psi\right) = 0
\end{aligned} \tag{7.30}
$$

here μ, ν run over $(1+3)$-dimensional space-time. Note that since we have $t \ll 1$, the potential in (7.22) is trivial (the first term is zero even when $\hat{Q} \neq 0$ because the exponential factor effectively goes to zero due to the relations given in (7.26) and similarly the exponential in the second term also effectively goes to zero because of (7.26)). Substituting the above solution (7.28) in (7.30), we get two conditions

$$p_1 + p_2 + p_3 = 1, \qquad \text{and} \qquad p_1^2 + p_2^2 + p_3^2 = 1 - \frac{1}{2}\gamma_\phi^2 - 24\gamma_\psi^2 \tag{7.31}$$

The first condition of (7.31) can be seen to be satisfied trivially from (7.29). On the other hand when we substitute the parameter values from (7.29) to the second condition of (7.31), we find that it gives the same parametric relation as the second relation of (7.4) verifying the consistency of the solution. This therefore shows how one can get a four dimensional Kasner-like solution from the ten dimensional anisotropic SD2 brane solution by six dimensional hyperbolic space compactification. It is well-known that the standard Kasner solution [117] obtained as the solution of vacuum Einstein equation, does not lead to expansions in all spatial directions. The reason is that in standard Kasner cosmology the Kasner exponents satisfy $p_1 + p_2 + p_3 = 1$ and $p_1^2 + p_2^2 + p_3^2 = 1$. Since these two conditions can not be satisfied together when p_i's are all positive, the expansions can not occur in all the directions. However, for the four-dimensional Kasner cosmology we obtained from string theory solutions, the parameters p_i's can all be positive definite because the second condition here (7.31) is different. This is the essentially the reason that we can have expansions in all the directions, but, it can be easily checked that the expansions are decelerating.

Conclusion

To summarize, in this chapter we have constructed an anisotropic SD2 brane solution starting from an anisotropic non-susy D2 brane solution of type IIA string theory by the standard trick of double Wick rotation. We wanted to see whether it is possible to generate accelerating cosmologies in all the directions which is known for the isotropic SD2 brane solution upon compactification on six dimensional hyperbolic space of time dependent volume. Indeed we found that when the resultant four dimensional metric is expressed in Einstein frame there are some windows of the parameters of the solution where one can get accelerating cosmologies in all the directions and is discussed in section 3. Here both the expansions and the accelerations we found are anisotropic. But, in order to get accelerating expansions we noted that the anisotropy can not be too drastic in three different directions. We also noted that accelerations are possible only for $t \sim t_0$, where t_0 is some characteristic time given as one of the parameters of the solution. Next, we looked at the four dimensional metric at early times, i.e., for $t \ll t_0$ and found that in a suitable coordinate and under certain conditions on the parameters of the solution, it can be expressed in a standard four dimensional Kasner-like form. But unlike in the standard Kasner cosmology, where expansions in all three directions are not possible, here we can get expansions in all the three directions. The reason is that in this case the relations among the Kasner exponents get modified due to the presence of the dilaton and the radion field. It would be interesting to see what effect (such modification to Kasner solution at early time) does it have on the cosmological singularities [120, 121].

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Chapter 8

Summary and Conclusion

An important sector of type II supergravity solutions has been discussed in this thesis. The term 'type II' indicates that this ten dimensional supergravity theory has $N = 2$ supersymmetry. However all of the brane solutions of this theory are not supersymmetric. Here we have discussed the non-supersymmetric (in short, non-susy) branes, their various properties and applications.

In chapter 2 we have started with the supergravity action of type II superstring theory. This gravity background contains a scalar field ϕ known as dilaton and a magnetic form-field $A_{[7-p]}$. Variations of these background fields have given the Einstein's field equation along with the scalar field equation and the form-field equation. Assuming a particular condition on the background metric a special kind of Dp brane solutions have been found which has broken supersymmetry of the background. These solutions have been called non-supersymmetric (in short, non-susy) Dp branes. The brane solutions obtained here are static, isotropic and have isometries $ISO(p,1) \times SO(9-p)$. There are three parameters which characterize these solutions. Among them r_p is the mass parameter. Unlike the BPS branes, these non-susy branes have two more parameters; θ is known as the charge parameter and δ is identified as the dilaton parameter. In BPS limit, these brane solutions have been seen to transform into the BPS brane solutions. In (2.42) we mentioned the ratio of ADM mass and the charge of a non-susy Dp-brane and from there we find that if we put $\theta = \tanh^{-1}(\alpha/\beta)$ or $\theta \to \infty$, the total charge of the brane becomes $\sqrt{2}\kappa_{10}$ times the ADM mass where $\kappa_{10} = \sqrt{8\pi G_{10}} = g_s \ell_s^4$ is the square root of the ten dimensional Newton's constant. So, for these two values of θ, the non-susy Dp branes become extremal. We just like to point out that although the mass and charge satisfy $\sqrt{2}\kappa_{10}\mathcal{M}_p \geq \mathcal{Q}_p$, the non-susy D$p$ branes have naked singularity. We have also introduced the anisotropic non-susy brane solutions. We have found the Dp brane solution in which $p-q$ number of brane directions are anisotropic. One can find this solution from the non-susy, static, Dq brane which is delocalised in $(p-q)$ transverse directions. Then applying the T-dualities on those $(p-q)$ delocalised coordinates the above anisotropic solution has been obtained. This has more free parameters like δ_2, δ_3, $\cdots$, δ_{p-q+1}. Anisotropy of each direction is controlled by a particular δ parameter. Setting these parameters to certain values one can easily get different solutions. For example, $\delta_2 = \delta_3 = \cdots = \delta_{p-q+1}$ and $q = 0$ give 'black' Dp brane. Again, for $p = q$ we get isotropic Dp brane. The various metric components and the background fields in this non-susy brane solution depend on the transverse radial coordinate r. Here time t is the longitudinal coordinate on the brane. We have also discussed the Wick rotation in this background. Here Wick rotation rotates the brane in $t-r$ plane with an angle $\pi/2$. It changes the signatures of these two coordinates, so, the metric and the various background fields become time dependent. The Wick rotation on the static non-susy Dp brane, has changed the space-time isometries as

$ISO(p,1) \times SO(9-p) \to ISO(p+1) \times SO(8-p,1)$. The brane world-volume is now a $(p+1)$ dimensional Euclidean space. These space-like Dp brane solutions have been named as S-brane or SDp brane.

We have seen the transformation of the non-susy branes into BPS branes under BPS limit. And it can also be shown that various properties of these non-susy solutions are very similar to the BPS branes. So the concept of the gauge/gravity duality is also expected to be analogous in these two classes of solutions, i.e., susy and non-susy. We have discussed about it in chapter 3. We have studied the graviton scattering on these non-susy Dp brane solutions to check the possibility of the non-susy version of AdS/CFT correspondence. Here we have considered the isotropic non-susy brane solution. The graviton perturbation in this background has given the Schrödinger-like wave equation. It has been shown that the same equation can also be obtained by perturbing a minimally coupled scalar in the bulk. Now from this scattering equation we have identified the scattering potential. For D1, D2, D3 and D4 branes the potentials have been found to have the maxima near the brane which has acted as a barrier in this scattering. The Numerical calculations Fig:3.1, Fig:3.2 and Fig:3.3 have shown that these barriers become higher and higher as $\omega\rho_p$ becomes lower and lower. As $\rho_p \sim \ell_s^2$, in low energy limit $\ell_s \to 0$, the barrier becomes infinitely high. So no graviton can cross this barrier to enter into the near brane region. On the other hand, for non-susy D6 brane no barrier have been encountered by the graviton to reach the near brane region. In fact for D6 brane, the potential monotonically decreases to $-\infty$ near the brane which means it attracts all types of gravitons. From the potential of D5 brane we have found a cut-off on the energy of the incoming graviton, $\omega_c = \sqrt{3}/(2b\rho_5)$. Here the potential barrier is allowed only for the graviton having energy $\omega < \omega_c$. Thus, in low energy limit, the non-susy Dp branes (for $1 \leq p \leq 4$) decoupled from the bulk gravity. To be more precise, about this decoupling, we have calculated the absorption cross-section of the graviton. Because of its complicated functional form we have solved the scattering equation particularly in two regions: the region very closed to the brane ($\rho \ll \rho_p$) and the asymptotic region ($\rho \gg \rho_p$). Using these two solutions we have found that the graviton absorption cross-section σ_p for non-susy Dp brane is proportional to $\ell_s^{\frac{2(7-p)^2}{5-p}}$. So in the low energy limit, absorption cross-section vanished for $1 \leq p \leq 4$ which means no graviton can be absorbed by the near brane region. It is the same conclusion we have found from the scattering potential. In short, these studies have proved that the bulk gravity theory decouples from the theory on the non-susy Dp ($p \leq 4$) brane in the low energy limit.

In fact, these non-susy branes and the BPS Dp branes have responded to the incoming graviton exactly in same way. Thus like the near brane decoupled geometry of BPS brane, a similar type of decoupled geometry is naturally expected for these non-susy Dp branes and we have found it in chapter 4. Here we have focused only on static non-susy D3 brane. We have seen that the BPS limit and the low energy limit are totally different and independent. So these two operations must commute. Here we have proposed the decoupling scaling in (4.8) for non-susy D3 brane which relates the length scale ρ to the fixed energy scale u with an α' factor. The decoupling scaling has been taken in such a way that it has given the finite value of red-shifted energy (4.11) in string unit, and under the BPS limit this has been reduced to $\rho = \alpha' u$, $R^4 = \alpha'^2 L^4$, the popular decoupling scaling given by Maldacena for BPS D3 brane. Using this scaling we have found the near brane decoupled geometry for the static non-susy D3 brane. The geometry has been characterized by three parameters u_0, δ and L^4. Here, the decoupled geometry is $(4 + 1)$ dimensional asymptotically AdS and has been reduced to some known asymptotically AdS background (described by Constable and Myers) under some coordinate transformations. Unlike BPS D3 brane the dilaton field depends on energy parameter. So, g_{eff} changes with the energy parameter which is analogous to the running coupling of QCD. The existence of such decoupled geometry ensures that the AdS/CFT like gauge/gravity duality is also applicable for the non-susy brane solutions of type II supergravity. Running coupling and asymptotically AdS structure of this gravity theory suggests that its corresponding gauge theory is fully non-conformal, Yang-Mills type theory. The parameters of this gravity theory are related to $g_{\rm YM}^2$, N and the gluon condensate in the corresponding gauge theory. In reality, QCD is a non-supersymmetric, non-conformal gauge theory. So, one can use this decoupled geometry to study various properties of QCD.

Conversely, the successful application of this duality in QCD also ensures the existence of the above decoupling limit and throat geometry. In chapter 5 we have used this non-susy gauge/gravity duality to study the Wilson loop in quark gluon plasma (QGP). Here we have considered a 'black' version of the non-supersymmetric D3 brane. Here the brane is anisotropic in the time direction where the anisotropy is tuned with a parameter δ. In spite of having a singularity in this type of solution one can define a temperature (given in (5.12)) of the background to have a finite value proportional to δ. Even at zero temperature the background is non-supersymmetric and this non-supersymmetry is driven by another parameter δ_1. Finally the solution in (5.6) has been characterized by four parameters – δ, δ_1, ρ_0 and θ. Then, in the decoupling limit, we have found a four parameter decoupled gravity theory. In fact, these parameters on gravity side are related to the temperature (T), t'Hooft coupling ($g_{\rm YM}^2 N$) and gluon condensate. This gravity background has been used to holographically compute the expectation value of a time-like Wilson loop which, in turn, is related to the potential of a heavy quark-antiquark pair. By boosting the gravity solution along one of the brane directions and placing the pair at an arbitrary orientation with this direction, we have numerically obtained the variation of the screening length ($\ell_{\rm max}$) as well as the potential ($E(\ell)$) with velocity parameter ($\eta = \tanh^{-1} v$), its orientation (θ) with respect to the direction of motion and other parameters (δ, δ_1) of the theory. Remarkably enough, our results given in Fig:5.1 and Fig:5.2 are in qualitative agreement with those obtained holographically in supersymmetric gauge theories (derived by LRW) indicating that these features are quite robust and universal as they are insensitive to the presence of any supersymmetry in the theory. The physical interpretations of the variations with respect to the other parameters of the theory, not observed in supersymmetric theory, have also been given. The variations of the screening length and Q-$\bar{Q}$ potential with δ have been explained as the variations with respect to the temperature ($T \sim \sqrt[4]{-2\delta}$) of the plasma. It has been found in Fig:5.3 that the Q-$\bar{Q}$ dipole has become more unstable at higher temperature. The variations with δ_1 have been interpreted as the variations with the effective coupling (g^2_{eff}). It has been found that $\ell_{\max}$ has been decreased for smaller g^2_{eff}. But the Q-$\bar{Q}$ potential has been found in Fig:5.4 to depend on the effective coupling g^2_{eff} in an unexpected way. The reason for this peculiar behavior of $E(\ell)$ is unclear to us till the date. After all we remark that this work is probably the first attempt to study QGP properties using non-supersymmetric AdS/CFT following from the decoupling limit of non-susy D3 brane. There are various other properties of QGP, like jet quenching parameter, thermalization, phase transition, chiral symmetry breaking, photon and dilepton production etc. which can also be studied using the background described in this chapter.

Unlike susy branes, the various accessible parameters of the non-susy branes give us the advantage in Wick rotations to get the consistent real time dependent brane solutions. These time dependent branes are the well-known space-like Dp brane or SDp brane. These SDp branes have been used here to study some cosmological scenarios. The static Dp brane solutions having isometries $ISO(p, 1) \times SO(9-p)$ have been considered in chapter 6. By the Wick rotations $r \to i\tau$ and $t \to -ix_{p+1}$ we have found the SDp branes with isometries $ISO(p+1) \times SO(8-p, 1)$. Here the time-like coordinate τ is the real time in this background. The brane is a $(p+1)$ dimensional Euclidean space whereas the transverse space-time is hyperbolic. It has three free parameters τ_0, δ_0 and θ. τ_0 has been identified as the time scale of the theory whereas δ_0 and θ are related to the dilaton and RR charge respectively. These solutions are asymptotically ($\tau \gg \tau_0$) flat. The compactification of the transverse hyperbolic space H_{8-p} has given the $(p+1)+1$ dimensional time dependent geometry which can be mapped to the cosmological space-time. Now re-defining the time coordinate $\tau \to \eta = \eta(\tau)$ we have found the FLRW metric in $(p+1)+1$ dimension. Unlike the standard cosmological solutions, the background has two fields like dilaton and volume scalar (due to compactification, it is also known as radion field). The numerical analysis of the scale factor $S(\eta)$ has shown that this background has an everlasting expansion for all values of p ($1 \leq p \leq 6$). This has been shown from $m(\tau)$ vs τ plots. Again from $n(\tau)$ vs τ plots it is clear that, at a certain window of time $\tau \sim \tau_0$, the expansions are accelerated. For $p = 2$ we get a $(3 + 1)$ dimensional cosmology. The nature of accelerations of this solution have been shown in various numerical plots. We have seen that with increasing τ_0 the acceleration period has been increased Fig:6.5 and it has been decreased with increasing θ Fig:6.3. For $\delta_0 = 0$ the acceleration period is maximum and it decreases with increasing $|\delta_0|$ Fig:6.4. Now we have studied the solution in near singularity regime i.e. $\tau \ll \tau_0$ which can be thought as the early time cosmology. In this approximation the above FLRW metrics have been reduced to the de Sitter metrics upto a conformal factor $\eta^{2(p+1)/p}$. These de Sitter metrics have also the flat FLRW form which gives a decelerating expansion with the scale factor $(\tilde{\eta}(\eta) - \tilde{\eta}_0)^{\frac{1}{p+1}}$.

In chapter 7 we have discussed about anisotropic (in all directions including the time direction of the brane) non-susy SD2 brane solution. It has been constructed from an anisotropic D2 brane solution by the standard trick of double Wick rotation. This solution is characterized by five independent parameters – t_0, θ, δ_1, δ_2 and δ_3. Here the brane is a three dimensional Euclidean space, time is one of its transverse directions and other transverse direction has made a six dimensional hyperbola H_6. The compactification on this hyperbolic space (H_6) of time dependent volume of this SD2 brane solution has led to the accelerating cosmologies. It is anisotropic in the sense that each of the three spatial directions is multiplied by different time-dependent factors. In fact, this anisotropy can be controlled by δ_1, δ_2 and δ_3. Here the cosmologies are also expanding, it has been shown in Fig7.1. Due to anisotropies the nature of expansions are different in different directions. Now in the plot, $n(t)$ has positive value over some period of time, which means that during this period the expansions are accelerated expansions. It has been found that the accelerated expansions are possible in all of the spatial directions in same instant. At early times, $t \ll t_0$, these cosmologies reduces to a Kasner-like solutions with background dilaton $\phi(t)$ and radion fields $\psi(t)$. These Kasner solutions have been found to be very important in many aspects. One of them is that, unlike standard Kasner solution of vacuum Einstein equation, all of its exponents p_1, p_2, p_3 can be positive. So one can have expanding solutions in all of its spatial directions.

The non-susy solutions discussed throughout this thesis are asymptotically flat and have the naked singularities at their locations. From the point of view of gravity, this singularity is an essential problem in the sense that it gives a lower bound in radial coordinate. In other word, one can not access the full space from $\rho = 0$ to infinity. The accessible region is $\epsilon \leq \rho < \infty$, where ϵ is very small but non-zero. Again the standard definition of Hawking temperature is not applicable here due to this open singularity. The non-susy brane also has Gregory-Laflamme instability. However, in the decoupling limit, this instability goes away. Hence, as a gravity solution, these non-susy branes need more studies to be sure about these issues or to resolve these issues. It is known that the two or more number of parallel non-BPS branes do not have 'no-force' condition, i.e. they apply net force on one another. So when they they become coincident they may become unstable. This instability should be studied in detail. Here we have also discussed about the finite temperature non-susy D brane solutions. It will be interesting to study the thermodynamics [122] in this non-susy regime. In decoupling limit we have found the decoupled near brane geometry for near extremal non-susy D3 brane, i.e., the non-susy branes with large charge. It is till unclear to us whether there is any decoupling limit for these branes with zero charge. In other words, is it possible to have a general decoupling limit for non-susy branes with an arbitrary charge? One can try to explore the non-susy gauge/gravity correspondence in more detail. However, various QCD properties have been shown to exist in some certain non-susy brane solutions [61]. Thus, it is expected that our non-susy branes also have those QCD-like properties e.g. the chiral symmetry breaking, gluon condensate, jet quenching parameter [123, 124], confinement [125–127], glue-ball mass spectrum [128, 129], etc. In second part of this thesis we have studied just some accelerating cosmological solutions. But more details about inflation [119, 130] like the slow-roll model, e-foldings [131] can also be studied from these solutions.

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