SYNCHRONIZATION STUDIES BETWEEN TWO COUPLED GLOW DISCHARGE PLASMA SOURCES

By

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and the work has not been submitted earlier as a whole or in part for a degree/diploma at this or any other Institution or University.

Neeraj Chaubey

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To my family

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Contents

SYNOPSIS

Plasma is a complex medium, composed of charged and neutral particles, which inherently supports different types of nonlinear oscillations and instabilities. Depending upon the plasma conditions, these oscillations are classified as periodic, quasi-periodic, chaotic, turbulent etc. Controlling and understanding these oscillations are the biggest challenge in the plasma community. Synchronization is a universal phenomenon in which coupled oscillators adjust their rhythms to come into a unison motion. Synchronization helps in both controlling as well as understanding these plasma oscillations. Although synchronization phenomena looks very simple, however, the unison motion between the coupled oscillators goes through several dynamical processes. These processes are generally known as different synchronization types. Complete synchronization [1–3], lag synchronization [1–3], phase synchronization [1–3], imperfect synchronization [1–3], intermittent synchronization [1–3], oscillation death [1–3], chimera state [4], and phase-flip transition [5] are treated as various type of synchronization. Due to charge particle interactions, electrons, ions, and neutrals exhibit oscillatory motion in plasma. Landau damping of wave and particle interactions in plasma has been shown to have an analogous nature to the synchronization phenomenon among coupled oscillators by S. Strogatz [6]. Also, a type of synchronization method in which oscillation of a system is fedback to itself, known as feedback synchronization has been used in many plasma experiments for suppressing different types of nonlinear instabilities [7–11].

Although synchronization between plasma systems have already demonstrated its implication for controlling various types of plasma instabilities, there are only a few reported experimental works where such studies have been done. Most of these experimental work are concentrated around forced synchronization phenomenon also known as unidirectional synchronization, in which oscillations of a self-sustained oscillator are forced with the oscillations of a stable frequency source. By using this type of forced synchronization method, many different types of synchronization have been studied for various types of plasma instabilities like mode-mode coupling [12, 13], resonance effect [14], harmonic synchronization [15], and chaotic synchronization [16], etc. However, this method has some limitations. Oscillation death, chimera state, and phase-flip transition, etc. can not be studied by this way of synchronization method. The possible way to analyze is via mutual synchronization method. In plasma systems, there are a few reported works where mutual synchronization has been studied [17, 18]. One of the works is done by Stan et al. [17], in which they have reported mutual synchronization as a result of two coupled electrical discharges. Moreover in this experiment, both the discharge systems are kept inside a big chamber, where plasmas of two discharges directly interact with each other. In 2006, Fukuyama et al. have studied mutual synchronization between two isolated plasma systems [18]. In this work they have reported spatiotemporal synchronization between two mutually coupled chaotic oscillations.

Therefore, being a complex plasma medium and their self-consistent interaction among the plasma systems can give enormous opportunities for finding new interesting results via mutual synchronization. There are open areas to study mutual synchronization properties; such as (1) *frequency entrainment and frequency pulling states* (2) *Phase-flip transition* (3) *Harmonic synchronization between mutually coupled plasma sources*, which has been studied in this thesis. Both experimental and numerical results of mutual synchronization between two coupled glow discharge plasma systems are presented in this thesis.

Frequency entrainment is a synchronized state in which all the mutually coupled oscillators, oscillate with the same synchronized frequency. These states arise between the coupled oscillators when coupling strength is strong. If coupling strength is weak between the oscillators, then frequency pulling like states occur. In this state, side band or beat frequencies are formed around the fundamental frequencies of the oscillator.

Phase-flip is a type of synchronization process in which coupled oscillators abruptly change their phases either from in phase to out of phase or vice versa with only a slight change in the coupling strength. This is a unique type of synchronization process which has attracted the attention of many researchers in different areas of science [5, 19–25]. Prasad et al. in 2008 [5], have shown that this phase-flip transition can be universally observed in all the coupled systems with a time delay. Soon after them, the occurrence of this type of transition is observed among many different coupled oscillators [5, 19–23]. Although the theoretical study has been carried out on such phase-flip synchronization, there are only a few reported experimental works available in this direction [5, 19, 21]. One such experiment is reported by Cruz et al. in which phase-flip transition is observed between coupled chemical oscillators with the introduction of a time delay. Using the same time delay technique, in coupled electronic oscillators also such transitions [5, 19, 24] has been produced. Recently, Dana et al. have shown that phase-flip transition can also be observed without any time delay [24, 25]. However, their phase-flip synchronization has not been studied in two coupled plasma system. In this thesis phase-flip transition between two coupled plasma systems has been studied thoroughly.

Harmonic synchronization, is a type of synchronization in which oscillations of a self-sustained oscillator are forced at their harmonic values. In plasma systems, such studies are mainly done by unidirectional synchronization method, in which self-sustained plasma oscillations (driven system) are forced at harmonic values from the oscillations of a stable frequency source (driver system). In such studies, changes are only observed in driven system, not in the driver system. Therefore, it is interesting to look whether driver and the driven system both have any influence on their individual oscillations i.e., mutual synchronization process. Study of harmonic synchronization via mutual synchronization process by using both driver and driven systems are also another major objective of this thesis. In coupled plasma systems, where two plasma systems are synchronized at their harmonic values and also show generation of different type of nonlinear states beyond their harmonic region is indeed very unique observation.

A brief summary of each chapter with a comprehensive understanding of the main physical observations is described in the following:

Chapter-1 : *Introduction.* Brief introduction of synchronization phenomenon is presented with its manifestation in various fields. Detailed literature survey of the experimental works related to synchronization phenomenon in plasma systems is presented. Works related to mutual synchronization phenomenon in plasma physics are discussed in detail. To discuss the oscillation dynamics theoretically, Van der Pol model is introduced, which is one of the most fundamental equation in nonlinear dynamics and has wide range of applications in different field of science such as electronic circuits, neuron model, heart dynamics, modeling interaction of geological fault etc. The relevant works of this equation for the study of plasma oscillations are presented. Phase-flip transition and hysteresis dynamics are discussed and followed by open problems and motivation behind this thesis.

Chapter-2 : *Experimental setup and plasma characterization.* In this chapter, the experimental setup used for the study of mutual synchronization phenomenon is presented. A detailed description of vacuum chambers, power supplies, pumping system and shielding mechanism is provided. Langmuir probe diagnostic used for the measurement of floating potential fluctuations is discussed. A detailed description of electrode dimensions on the anode glow oscillation dynamics is given. We have discussed the possible ways to get anode glow oscillation by reducing the ratio of anode to the cathode surface area. The electrode size is made to fulfill the condition $S_a \leq a \times S_c\sqrt{m/M}$, where S_a and S_c represents effective anode and cathode surface areas respectively while factors m, M and a represent electron mass, ion mass, and proportionality constant respectively [26]. The plasma characterization methods like Paschen's curve and discharge voltage vs current analysis are also presented. The data analysis techniques like Lissajous plots, power spectrum, frequency bifurcation and Devil's stair case are discussed in detail.

Chapter-3 : *Van der Pol equation to study the dynamics of coupled plasma* *systems.* In this chapter, a brief introduction of Van der Pol equation is presented followed by its derivation from the basic plasma fluid equations. The physical interpretation of all the coefficients on the oscillation dynamics is provided, and the role of the additional DC term on the oscillation dynamics of the Van der Pol equation is discussed. Coupled oscillation dynamics between two coupled Van der Pol equations are discussed in detail for coupling schemes like reactive and dissipative couplings. Bath type coupling is introduced, and its relevance with experimental inductive (indirect) coupling is discussed.

Chapter-4 : *Mutual synchronization studies via direct coupling.* This chapter contains experimental and numerical results of synchronization dynamics between two directly coupled glow discharge plasma sources. The experimental system consists of two glow discharge tubes which are kept completely separate and coupling is established between them when anodes of the two systems are connected through a variable resistance bank. This type of coupling is also called as direct coupling because current oscillations of two systems are directly interacting with each oscillation. After this, oscillation dynamics of each of the individual discharge system is presented with increasing discharge voltage, for uncoupled configuration. In this case, it is observed that with increasing discharge voltage, oscillation frequencies increase linearly in each system. For synchronization experiment, oscillation frequencies of the two systems are chosen very close as 255 kHz and 262 kHz. Direct coupling is provided between the two systems, by connecting the anode of the two chambers through a variable resistance bank. When the value of the resistance is small, i.e., coupling strength is high, then frequency entrainment or frequency synchronized state is observed. In this state, the value of synchronized frequency is found to lie in between the oscillation frequencies of two systems in uncoupled configuration. With increasing resistance, i.e., decreasing coupling strength, this frequency entrainment state or frequency synchronized state changes into frequency pulling state. In frequency pulling state, side band frequencies are observed around the fundamental frequencies of two systems. Numerical modeling results of synchronization dynamics are presented using two coupled asymmetric Van der Pol type equations. These results are closely analogous to the experimental results. The introduction of additional added DC term in the equation give numerical results which closely match with experiment. With the increasing value of this DC parameter, oscillation frequencies are found to be increasing linearly, as observed in the experiments. After this, coupled dynamics are presented using dissipative type coupling between the two Van der Pol type equations. For synchronization results, uncoupled oscillation frequencies of two equations are chosen to be in the same ratio as observed in the experiments. When coupling is established and dissipative coupling strength is varied then all the different type of synchronized states like frequency entrainment and frequency pulling are observed which are found to be same as observed in the experiments.

Chapter-5 : *Phase-flip transition via indirect coupling.* In this chapter, both experimental and numerical simulation results of phase-flip transition associated with hysteresis dynamics between two inductively coupled glow discharge plasma sources are presented. A schematic indicating the inductive coupling between the two glow discharge plasma sources is shown. Inductive (indirect) coupling mechanism is discussed in detail. This inductive (indirect) coupling is provided by winding 10 turns of copper wires with thickness 2 mm, outside and on top of each of the two glass chambers. It is also observed that with increasing the discharge voltage, oscillation frequencies of each discharge systems are increasing linearly in uncoupled configuration. For synchronization, the discharge voltage of one system is kept fixed such that oscillation frequencies of this chamber do not change and inductive or non-intrusive coupling is established between the two systems by connecting outside wounded wires between two chambers. After this fundamental oscillation frequency of the other source is progressively increased with raising discharge voltage. A frequency pulling regime is observed followed by a synchronized regime that shows a frequency jump phenomenon. The jump is associated with a phase-flip transition that takes the synchronized state from in-phase to anti-phase state. When the process is reversed the transition takes place at a different frequency thereby exhibiting a hysteresis effect. Numerical simulation results are provided using two Van der Pol oscillators that are coupled to each other through a dynamic common medium also known as a bath coupling, which eminently captures all the essential features of our experimental observations.

Chapter-6 : *Mutual harmonic synchronization via direct coupling.* This chapter contains experimental results of harmonic synchronization between two coupled glow discharge plasma sources via a direct coupling. In our previous experiment, it is shown that with increasing discharge voltage, oscillation frequencies of anode glow oscillations are increasing linearly. In this work, we are using these oscillation frequencies for harmonic synchronization experiment. In this chapter, experimental results of oscillation dynamics with increasing discharge voltage is presented. It is observed that oscillation frequencies of each system are increasing linearly from 90 kHz to about 700 kHz. For harmonic synchronization experiment, the fundamental oscillation frequency of one of the system is chosen around 200 kHz, and kept fixed at that value by fixing its discharge voltage. After that, direct coupling is provided between the oscillations of two systems and fundamental oscillation frequencies of other system is increased from 90 kHz to 700 kHz with increasing its discharge voltage. Coupled dynamics of both the systems are measured using a Langmuir probe, inserted in each of the systems. It is observed that when the fundamental oscillation frequencies of both the systems lie very close to their respective harmonics then oscillations of both the systems goes to a frequency entrainment state and outside this regime goes to frequency pulling or chaotic type states. Also, chaotic states are prominent for higher harmonic synchronization region. A Devil's staircase like picture and frequency bifurcation diagram showing different region of frequency entrainment, frequency pulling, and chaotic type states are presented.

Chapter-7 : *Summary and Future Scope* The main results of this thesis are summarized as follow:-

(1) Our experiments provide a clear demonstration of the phenomena of frequency synchronization and frequency pulling of anode glow oscillations in plasma glow discharge devices. Also, model equations developed in support of our experiments can help in identifying parametric domains for such explorations.

(2) Experimental results of synchronization via inductive or non-intrusive coupling between two coupled glow discharge plasma sources are very unique and observed first time to the best of our knowledge.

(3) Experimental results of phase-flip transition between two coupled glow discharge plasma sources are reported first time in any plasma systems and also this transition is seen without introducing any time delay between the oscillators, which is a novel result. Since earlier investigation indicate that the occurrence of this transition can only happen with time delay. For the onset of the phase-flip transition, hysteresis is observed while scanning the synchronization regime in the increasing and decreasing directions. It is to be noted that this hysteresis is not due to the existence of a particular regime in the voltage-current characteristics of the source but is solely associated with the frequency synchronization mechanism which is also quite novel result. Numerical model matched quite well with the experimental results.

(4) Harmonic synchronization results between two mutually coupled plasma oscillators are presented. These results have significant implications for stabilizing harmonic oscillations between coupled plasma oscillators.

Future scope, This thesis work can be extended for studying synchronization states like chaotic synchronization, oscillation death, chimera states and explosive synchronization etc. in plasma systems. Inductive coupling mechanism which has been demonstrated for synchronization studies in this thesis, can be taken into account for controlling plasma oscillations remotely.

List of Figures

1 Introduction

Synchronization phenomenon is fundamental in nature and has been seen in a variety of natural as well as man-made systems. In this phenomenon two or more coupled oscillators adjust their rhythms to oscillate in a unison manner. This doctoral thesis reports on experimental and numerical results of synchronization dynamics between two coupled glow discharge plasma sources. The primary emphasis is given to experimental studies related to frequency entrainment, frequency pulling, phase flip transition, hysteresis dynamics and harmonic synchronization. In order to understand the experimental observations, numerical simulations are also done using a model of two coupled Van der Pol equations.

This introductory chapter is organized as follows. In the next section, Sec. (1.1) we provide a brief introduction on synchronization and how one can understand the dynamics of coupled plasma systems by using this phenomenon. Sec. (1.2) gives details of the various models and tools adopted for the depiction of coupled plasma systems. A review of earlier work in this area is provided in Sec. (1.3) to put the present work in a proper perspective.. Finally Sec. (1.4) and Sec. (1.5) discuss the motivation of this work and present an outline of the organization of the dissertation, respectively.

1.1 Synchronization

The first scientific observation of synchronization was reported way back in 1665 by Christian Huygen's [28], when he observed that the phases of the pendulum motion of two clocks hung side by side on a wall got synchronized after a while. However, for nearly two centuries after that observation, the phenomenon of synchronization did not receive much scientific attention. However, over the last fifty years or so, a large number of studies related to synchronization have been carried out started appearing in many branches of science and engineering. Some of the examples where this phenomenon has been observed and studied are: coherent flashes of light by a group of male fireflies [29–32], synchronized chirping of a group of crickets [33, 34], synchronous prolonged clapping by an audience [35], animal gaits [36,37] and biological clocks [31,34,38,39], circadian rhythm [31,34,40], pacemaker cells in the heart [2, 34, 41–43], the synchronization of human female menstrual cycles [34,44], synchronization between coupled metronomes [45], phase locking of relativistic magnetrons [46], synchronization of coupled organ pipes [2,34,47], synchronization in coupled lasers [48–53], synchronization of oscillations of coupled self-oscillatory electronic circuits [1–3, 54], chemical oscillators [55–58], synchronization between coupled mercury system [59–61] and Belousov-Zhabotinsky (BZ) reaction [34, 62–64], Josephson junctions SQUIDs [2, 34, 65, 66], Landau damping in plasma [6] etc.

The phenomenon of synchronization has also been exploited in many different types of applications like:

1. Enhancing the power output of coupled lasers: The power output from an array of coupled lasers does not necessarily get amplified even if all the lasers have the same frequency. This is because the asynchrony in their phases leads to a random distribution of their phases at any instant of time and consequently a sum of the individual amplitudes does not lead to a sum of the individual powers of the lasers. But if the lasers in the array are synchronized with one another then the total output power of the laser beam can be substantially increased [2, 67].

2. Controlling arrythmia in heart beats through externally induced synchrony of pacemaker cells: The invention of the pacemaker device, an electronic gadget that produces electrical oscillatory signals that can externally couple and regulate the synchronization of pacemaker cells of the heart has led to much amelioration of medical issues arising from an irregularly beating heart [2, 34].

3. Synchronization induced effects have also been used in the treatment of diseases like acute Leukemia [2,68] and have helped in the prevention of diseases like Down's syndrome and cancer [2, 69].

Synchronization does not always have beneficial effects. In many instances the onset of synchronization can lead to adverse consequences. Some well known examples of such adverse effects are:

1. Synchronization of neuronal cells in certain regions of the brain are suspected to be responsible for maladies like epileptic fits or Parkinson's disease whose effects can often be life threatening [2, 34, 70].

2. Danger to the structural integrity of mechanical structures as seen on the opening day of the Millennium bridge when synchronization between the lateral movement of the people's footsteps caused the bridge to sway more than 7 cms and required subsequent modifications in its structural design [34, 71].

As is evident, synchronization can take place only when there is coupling between the oscillators and when the coupling is sufficiently strong and sustained over a time long enough for the interactions to induce a common rhythm or force a locking of phases. The study of synchronization is meaningful only among oscillators that are self-sustained i.e. those that can individually maintain stable oscillations that are internally driven. Such is the case for most natural oscillators such as the pacemaker cells in the heart, or the pendulum clock of Huygens or neuronal cells in the brain. Self-sustained oscillators are characterized by the following features:

1. They oscillate with their self-sustained internal energies and do not damp out with the passage of time.

2. Oscillation dynamics of these oscillators are defined by the oscillator itself and not by any external source of energy.

3. With some applied external perturbation, the time period of these oscillators can be changed but after removal of the external perturbation, they regain their unperturbed oscillatory behavior.

For studying synchronous behavior between self-sustained oscillators, mainly two types of coupling mechanisms are used (1) synchronization by external forcing (2) mutual synchronization.

(1) Synchronization by external forcing:- In this type of synchronization, oscillations of a self-sustained oscillator are forced by the oscillations of a stable frequency source. This is also known as unidirectional synchronization where oscillations of former are forced by latter but not vice-versa. In terms of frequency detuning of stable frequency source with constant amplitude, three types of synchronization regions can be defined when forcing periodic oscillations of a self-oscillatory source:-

1. *Frequency locking*: - If detuning frequency is small, i.e., coupling strength is large between the externally applied frequency and the oscillations of the self-sustained oscillator, then the oscillations of the latter are completely pulled by the oscillations of the former and this state is known as frequency

locked state. In this state, self-sustained oscillator oscillates with the same frequency as of the externally applied source.

2. *Frequency entrainment*: - If detuning frequency is large such that externally applied frequency is not able to pull completely the oscillation frequency of the self-oscillatory source but strong enough that it changes the natural oscillation frequency of self-oscillatory source then this state is known as frequency entrainment state. In this state, self-oscillatory source oscillates with a frequency which is in between the natural oscillation frequency of the self-oscillatory source and the frequency of externally applied source.

3. *Frequency pulling*:- If detuning frequency is increased furthermore, i.e., coupling strength is decreased further such that externally applied frequency is not able to change the period of oscillations of self-sustained oscillator, but it makes the oscillator to oscillate in and around their natural frequency with the formation of sideband frequencies then this state is known as frequency pulling state.

(2) Mutual synchronization:- This type of synchronization is established between two or more than two coupled self-sustained oscillators where all the oscillators adjust their rhythms and come to a synchronized state. This kind of synchronization can be characterized in different ways with the change in the coupling strength between them. For two self-sustained oscillators, they are mainly defined as follows:-

1. *Complete synchronization*: - Both amplitude and phase of the oscillators are synchronized, and they are completely locked with each other.

2. *Lag synchronization*: - Amplitudes of both the oscillators are synchronized, but there is some time delay between their phases.

3. *Phase synchronization*: - Amplitudes of two oscillators are different, but phases are locked with each other.

4. *Generalized synchronization*: - There is some functional relationship between the output of the two oscillators. Like if one of the oscillators is defined as X and other as Y then after coupling their output should be like Y = f(X), where f is a functional parameter.

5. *Intermittent lag synchronization*: - Oscillators most of the times are lag synchronized but with some intermittent bursts.

6. *Imperfect phase synchronization*: - This is a kind of phase synchronization, in which there are some intermediate phase slips that occur between the oscillators.

Depending on the kind and strength of coupling some other types of synchronized behavior have also been observed in groups of coupled oscillators. These include *amplitude death* [2, 3, 72–74], *explosive synchronization* [60, 75], *Chimera states* [4, 76, 77] and *phase-flip transition* [5, 19–25, 78], etc.

1. *Amplitude death*: - In this type of synchronization state, the amplitudes of all the coupled oscillators become zero and the system collapses to a fixed point at the origin of the phase space [2, 3, 47, 72–74].

2. *Explosive synchronization*: - This is a type of synchronization state, in which a population of coupled oscillators transits from a disordered state to an ordered (synchronized) state abruptly at a critical value of the coupling parameter, resembling a first order phase transition [60, 75]. The system also displays hysteresis behavior such that the reverse transition does not happen at the same point when coupling parameter is decreased.

3. *Chimera state*: - This collective state is characterized by the co-existence of synchronous and asynchronous behavior in the system whereby a sub-population of the oscillators become synchronized while the rest behave asynchronously even though all the oscillators are identical and are identically coupled [4, 76, 77, 79].

4. *Phase-flip transition*: - Phase flip is a type of synchronization process in which coupled oscillators abruptly change their phases either from an in phase state to an out of phase state or vice versa with only a slight change in the coupling strength [5, 19–25, 78]. This is a unique type of synchronization process which has recently attracted some attention in different fields of science.

1.2 A Plasma Medium as an Oscillator

A plasma is an ionized medium made up of electrons, ions, and neutral particles. The constituents interact with each other through electrodynamics forces and also through collisions with the neutrals. The dynamics of the charged particles are also strongly influenced by the presence of any external electric or magnetic fields. A plasma can exhibit a host of collective behavior in the form of wave motion of the particles at different characteristic frequencies and wave lengths. These waves constitute various forms of synchronized motion of its components resulting from their mutual interactions (coupling) and interaction with external fields. In this sense the collective dynamics of a plasma can often be modeled in terms of the synchronized states of a system of coupled oscillators. An effective demonstration of such an analogy was brought out in [6] where it was shown that the decay of a perturbation imposed on a system of coupled oscillators closely resembles that of Landau damping of waves in a plasma. The mathematical equivalence of the perturbation analysis is uncanny and reveals a deep connection between the collective dynamics of a plasma to that of a system of coupled oscillators. The analogy has been further exploited in many experimental studies of plasma systems driven by external periodic sources where the synchronous behavior of the plasma system has been modeled by a single nonlinear oscillator model. We discuss these works in the following section.

1.3 Synchronization Studies in Plasma Systems

As mentioned above, the collective motion in a plasma constituting a wave motion can often be simply modeled as a synchronized state of a group of coupled oscillators. In a further simplification the dynamical behavior of the entire system is represented by a the solution states of single nonlinear oscillator such as a Van der Pol oscillator. Much of the past work related to synchronization studies in a plasma has adopted such an approach and we will summarize those experimental and theoretical studies in the next two subsections.

1.3.1 Experimental Work

A majority of the past works on synchronization in a plasma have relied on a forced synchronization method in which oscillations of the plasma are induced and synchronized to an external stable frequency source (e.g. a function generator). In this type of synchronization, changes are only observed in the plasma behavior and not in the stable frequency source. By using this sort of forcing mechanism, different types of plasma instabilities like ion sound [14, 80, 81], ionization waves [82–84], ion-ion instability [85, 86], relaxation oscillations [15, 87–90], drift waves [91–96], flute modes instabilities [97], dust acoustic waves [98, 99] and dust density waves [100, 101] etc. have been studied in terms of synchronization phenomenon.

In the early 1970s Keen and Fletcher have studied the suppression and enhancement of an ion-sound instability by nonlinear resonance effects [14], the mode-mode coupling of Van der Pol plasma instability [12], and subharmonic and harmonic forcing on ion- sound instability [80] etc. Similarly, Klinger et al. in 1990's and in early 2000, have studied various synchronization properties for different types of plasma instabilities like mode-locking and frequency pulling phenomenon for driven ionization waves [83], quasi-periodicity, mode-locking, and period doubling sequences towards chaos for driven plasma oscillations [13], frequency entrainment and frequency pulling process for driven self-oscillating thermionic discharges [87], frequency entrainment, quasi-periodicity, frequency pulling and period doubling etc. for periodically driven current oscillations. Similarly, various other synchronization properties for drift wave turbulence have been studied by applying external forcing in [91–94]. In 1990's Gyergyek et. al. have studied plasma relaxation oscillations and mode-suppression behavior for forced plasma relaxation oscillations [89] and also the nonlinear dynamics of a harmonically forced double layer [15]. Recently, in dusty plasma also, some works have been done where forced synchronization phenomenon is studied. Piel et. al. have studied synchronized dynamics of dust acoustic waves [98] and synchronization of dust density waves in anodic plasma [100]; Similarly, Goree et. al. have studied harmonic synchronization mechanism for dust density waves [99]. The chaotic synchronization between self-oscillatory chaotic oscillations of a plasma system and stable chaotic oscillations of a Chua circuit signal have been investigated in [16].

All the above mentioned studies are based on forced synchronization between a stable oscillatory source and a self-oscillatory source. Another class of studies has looked at synchronization phenomenon under different mechanisms such as delayed feedback and mutual synchronization between two evolving systems.

In delay feedback synchronization, oscillations of a plasma system are fed back to itself with some time delay or phase delay. This mechanism is very useful in suppressing various plasma instabilities [7–11,102–108] and has been implemented in some of the tokamak experiments for suppressing plasma fluctuations.

Mutual synchronization (as explained in section 1.1) is a type of synchronization mechanism in which, two or more than two oscillators are mutually coupled with each other and all the oscillators adjust their rhythms to come into a synchronized state. Mutual synchronization is a most common type of synchronization phenomenon which is usually observed in natural world oscillators like the synchronous flashing of lights by fireflies and unison chirping of crickets etc. This kind of synchronization phenomenon is also realized in many laboratory scale experiments like chemical oscillators, lasers, electronic oscillators, mechanical oscillators, etc. But in plasma systems, there are very few reported works, where this type of synchronization phenomenon is seen. One of the works is by Fukuyama et. al. where they have reported spatio-temporal synchronization between instabilities of two different plasma systems through mutual coupling [18]. In this work, they have produced chaotic waves in two separate plasma systems and coupled them through a coupler and observed that with the increase in the value of the coupling strength, oscillations of both the systems go from an unsynchronized state to a synchronized state. Mutual synchronization between two electrical discharges coupled through inter-anode currents has been reported by Stan et. al. [17] .

In the context of a tokamak plasma, Guo and Diamond have noted the transition from a phase locked to a phase slip state during the evolution of the edge-localized (ELMy) H-mode to the quiescent (Q) H-mode [109]. More recently Zhao et. al. have reported experimental observations of the synchronization of geodesic acoustic modes (GAMs) and magnetic fluctuations in the edge plasma of the HL-2A tokamak [110]

1.3.2 Numerical Modeling Work

The Van der Pol equation or a modified form of this equation has been widely used as a basic equation to describe the dynamics of self-oscillations in diverse physical systems. This equation was first introduced by Balthazar van der Pol (1889-1959) in 1927 to model triode oscillations in electrical circuits [111–114] and is given by,

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x = A * sin(t) \tag{1.1}$$

where the term on the right hand side represents an external driver. The solution of the basic Van der Pol equation, i.e. in the absence of the external forcing, and for $\epsilon > 1$ consists of a limit cycle oscillation - a self sustained oscillatory state. If external forcing is also included then, this equation is known as a forced oscillator equation, where self-oscillatory oscillations are forced by oscillations of a stable oscillator. The forced oscillator equation is a widely used equation for explaining many complicated forced synchronization processes like synchronization between heart beats and pacemaker [2, 42], forced electronic circuits [2, 3], etc. Coupled Van der Pol equations have been successfully used to model experimental results of mutual synchronization such as among coupled neurons [115, 116], the evolution of a geological fault between two coupled tectonic plates [117], etc.

Keen and Fletcher have used forced Van der Pol equation for describing experimental results of resonance effect [14] and harmonic synchronization effect [80] for ion sound instability. Similarly Klinger et. al. have used this equation for modeling experimental results of ionization wave [83] and relaxation type oscillations [14]. Recently Goree et. al. have also used forced Van der Pol equation for explaining experimental results of forced dust acoustic wave [99] etc. Stan et. al. have used coupled Van der Pol equations to explain experimental results of mutual synchronization between two coupled electrical discharges [17].

In our thesis work, we have used coupled Van der Pol equations with different sorts of coupling mechanisms for explaining experimental results of mutual synchronization between two coupled plasma systems.

1.4 Motivation of Thesis

The primary motivation for the research reported in this thesis is to understand how two isolated plasma systems can be synchronized and what the effect of different coupling schemes are on this process and what are the different synchronized states that one can achieve. Hence the study focuses on the following investigations:

- mutual synchronization between two coupled glow discharge plasma sources with direct coupling.
- mutual synchronization between two coupled plasma sources showing phase-flip transition using inductive coupling.
- mutual harmonic synchronization between two coupled plasma sources, in which one source acts as the driver for the other.

Wherever possible the experimental results have been explained using the Van der Pol equation, modified as per the requirements.

1.5 Organization of Thesis

This doctoral thesis reports the experimental and numerical results of synchronization dynamics between two coupled glow discharge plasma sources. The primary emphasis is given to experimental studies like frequency entrainment, frequency pulling, phase flip transition, hysteresis dynamics and harmonic synchronization, etc. Experimental results are explained on the basis of a model consisting of two coupled Van der Pol equations.

Chapter 2 describes the experimental setup, the diagnostics, the plasma production method, plasma characterization results, and the plasma oscillation dynamics. A description of the vacuum chamber, pumping unit, power supplies and shielding mechanism is also provided. The effect of the size of the electrodes on the anode glow oscillation dynamics is discussed and also results of change in oscillation dynamics with change in different plasma parameters are presented.

Chapter 3 is devoted to a theoretical/numerical study of the oscillation dynamics of two mutually coupled Van der Pol equations in order to provide a framework for the interpretation of experimental results. Firstly, a brief history of Van der Pol equations and its derivation from the basic plasma fluid equations are provided. The physical meaning of all the coefficients is discussed and their relation with plasma parameters is delineated. The role of the additional term in the equation on the oscillation dynamics is provided, and its importance with DC discharge voltage is discussed. The model oscillation dynamics of two mutually coupled Van der Pol equations is also discussed with different types of coupling mechanisms like direct and indirect couplings. For both direct and indirect couplings, effects of reactive and dissipative type couplings are discussed. Also, bath type coupling is introduced and its relevance to inductive coupling in an experimental situation is discussed.

In Chapter 4, the experimental results on the nonlinear dynamics of two coupled glow discharge plasma sources are presented. Firstly, the details of the anode glow oscillation dynamics are shown for each each glow discharge system with the variation in the discharge voltage. Then the dynamics of the coupled oscillation dynamics as a function of the variation in the coupling strength between the two glow discharge plasma sources is presented. The coupled dynamics displays a variety of nonlinear phenomena like frequency synchronization and frequency pulling. Numerical simulation results obtained using two coupled asymmetric Van der Pol type equations are compared to the experimental results and shown to be in good agreement with each other.

In Chapter 5, the experimental observation of a phase-flip transition in the frequency synchronization of two DC glow discharge plasma sources that are coupled in a non-invasive fashion is reported. When the fundamental oscillation frequency of the potential fluctuations of one of the sources is progressively increased, by raising its discharge voltage, a frequency pulling regime is observed followed by a synchronized regime that shows a frequency jump phenomenon. The jump is associated with a phase-flip transition that takes the synchronized state from an in-phase to an anti-phase state. When the process is reversed the transition takes place at a different frequency thereby exhibiting a hysteresis effect. A theoretical model consisting of two Van der Pol oscillators that are coupled to each other through a dynamic common medium eminently captures the essential features of our experimental observations.

In Chapter 6, synchronization dynamics is studied between two glow discharge plasma systems by coupling the anode glow oscillation frequencies of these discharges. In both the discharge systems anode glow oscillation frequencies were found to increase linearly with the increase in the discharge voltage and we have coupled these oscillations by keeping one at a fixed discharge voltage and varying the other. The nonlinear phenomenon like frequency entrained states were observed when oscillation frequencies of the varied discharge voltage system were close to the harmonic frequencies of fixed discharge voltage system and beyond these regions frequency pulling and chaos type states were observed. In this chapter, we also show a Devil's staircase like behavior of the two coupled plasma systems. Some region of the experimental results were modeled by numerical simulation of two coupled asymmetric Van der Pol type equations and these results are found to be in good agreement.

In Chapter 7, a detailed summary and possible future extensions of this thesis work are presented. In summary, results of unidirectional and mutual synchronization studies are discussed in detail. For unidirectional synchronization, the asymmetric nature of the frequency synchronized states and the merging of higher harmonic states with higher strength of external forcing are discussed. In mutual synchronization, results of both direct and indirect couplings are discussed and the main results are elaborated in detail. In direct coupling between fundamental oscillation frequencies of two sources, results of different synchronized states like frequency entrainment and frequency pulling with the change in coupling strength are discussed. Also, results of mutual harmonic synchronization are presented with the change in harmonic multiple of one of the source frequency. In indirect mutual coupling, results like phase-flip transition and hysteresis are presented. Finally a short discussion on how the results of this thesis work can be extended in the future is given. The possible directions to take are observations chaotic synchronization, chimera states and amplitude death phenomenon in coupled plasma systems. We also discuss some potential studies relevant for practical applications like plasma based surface coating technology using physical vapor deposition(PVD) mechanism. Synchronization effects could be useful in controlling defects and in increasing the thickness of such coatings. Also, the experimental setup used in this thesis work could be used as a table top experiment for understanding and teaching synchronization behavior in a class room environment.

2

Experimental setup and plasma characterization

In this chapter we provide a description of the experimental setup used for plasma formation and the different types of coupling employed to study the synchronization phenomena between waves in two coupled plasma devices. We also discuss various data analysis techniques that are used for plasma characterization.

2.1 Introduction

One of the earliest experimental studies related to mutual synchronization between plasma systems was carried out by Fukuyama et al. [18]. In their experiment plasma was produced in two separate Gisler tubes which were then coupled through a variable resistance. In the pressure regime in which they were operating, they had to deal with ionization instabilities that are typically very difficult to control. This difficulty limited their explorations to a very narrow regime of experimental parameters and synchronization phenomena. Our present objective is similar to that of Fukuyama et al [18] but our approach and experimental setup is considerably more flexible allowing us to carry out a host of different synchronization experiments. Our experimental system consists of two individual glass chambers in which glow discharge plasmas are created. In this system, anode glow oscillations are formed by making the size of the anode surface area to be very much smaller than the cathode surface area. The oscillations produced in this way can be controlled in a much more efficient manner as a function of the background pressure and discharge voltages. One can in fact obtain a wide range of oscillations that can be periodic, quasi-periodic or chaotic in nature. This broadens the range of synchronization experiments that can be done. Before carrying out the synchronization experiments we have done a detailed characterization of the plasma properties and the nature of oscillations in each device as a function of the background pressures and discharge voltages. Typical Paschen curves and discharge voltage vs current characteristics have been obtained in each case. The self-sustained periodic type oscillations observed near the minimum of the Paschen curve are used for our synchronization studies. For our regime of operation the discharge voltage vs current curve shows a linear relationship. The data consisting of floating potential fluctuation signals are visualized using a variety of techniques such as Lissajous plots, power spectra etc. and their variations noted as a function of the coupling strengths. In our analysis we also look for distinct signatures like frequency bifurcation, Devil's stair case behavior etc. to identify different synchronization regimes.

This chapter has been organized as follows. In Section 2.2, the experimental setup for synchronization studies is discussed in detail. Section 2.3 and Section 2.5 presents a description of the tools and techniques used to characterize the plasma medium and type/regime of synchronization. The requirement of plasma conditions for doing synchronization studies are explained in Section 2.4. Finally, Section 2.6 contains some discussion and a summary of the conclusions.

2.2 Experimental Setup

Our experimental setup, consisting of two coupled glow discharge plasma sources, is schematically shown in Fig.2.1.

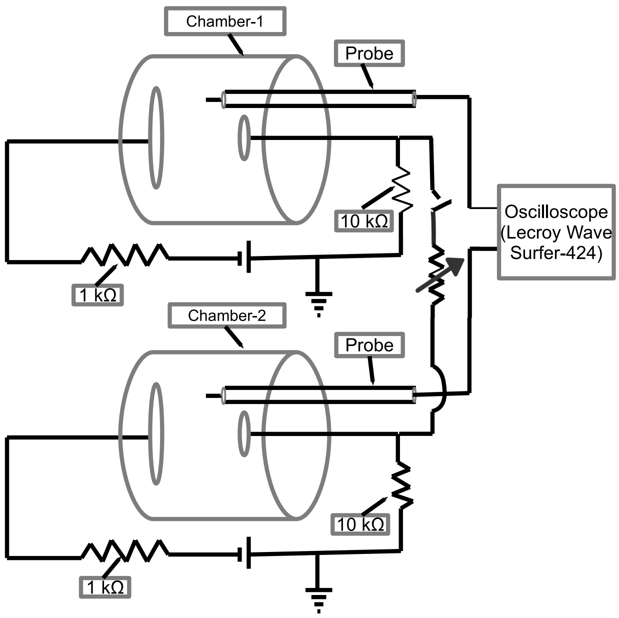

Figure 2.1: *Schematic of experimental setup for two coupled glow discharge plasma sources.*

The pictures of experimental setup corresponding to above schematically diagram is shown in Fig.2.2.

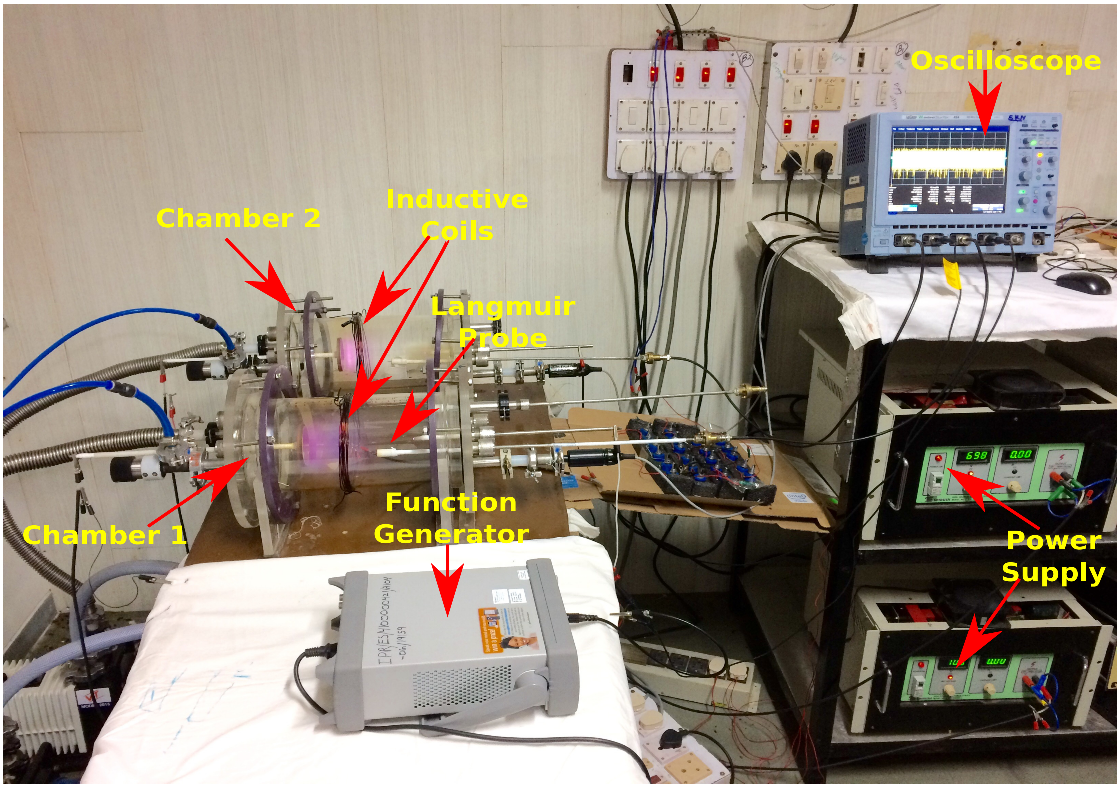

Figure 2.2: *Synchronization experimental setup picture.*

The setup consists of two identical DC glow discharge plasma systems with almost same configurations. Each individual device primarily consists of a cylindrical glass chamber, a rotary pump, a gas feeding mechanism, shields from external noise, electrodes, power supply, etc., as can be clearly seen in Fig. 2.2

In the following subsections, the function and role of each component of our experimental setup is explained in detail.

2.2.1 Chambers and Pumping System

The entire set of synchronization experiments have been carried out in two separate cylindrical glass chambers with diameters of 15 cm and lengths of 35 cm, shown in Fig.2.3.

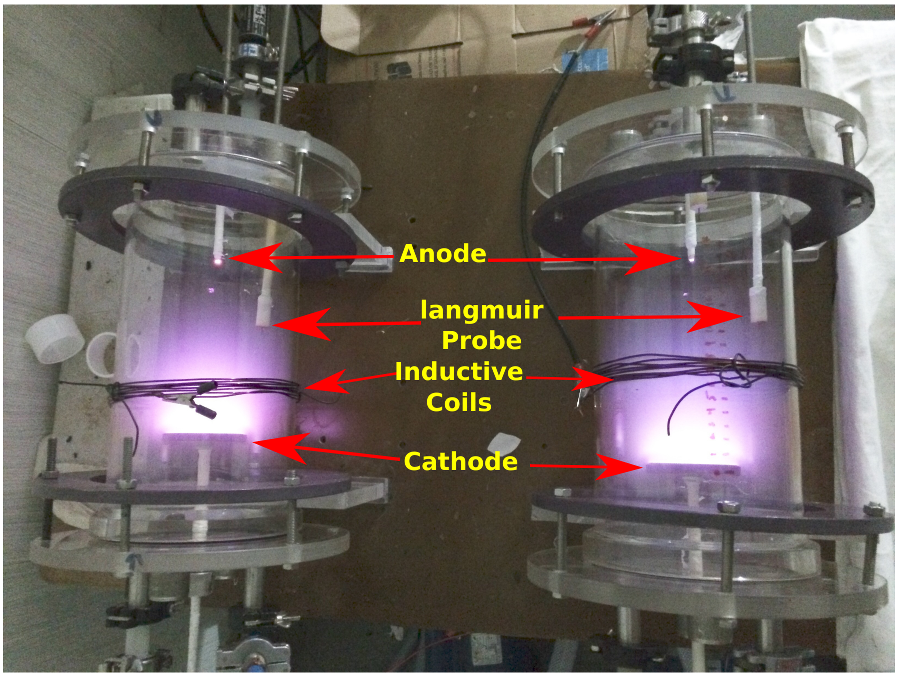

Figure 2.3: *Picture of cylindrical glass chambers.*

Both the chambers are pumped down, using two different rotary pumps with pumping speeds of 250 lit/sec, to a base pressure of 0.001 mbar. After this, each chamber is filled with Argon gas (99.999% pure) through a precision gas dosing valve to reach working pressures in the range of (0.1 - 0.2) mbar. The choice of working pressure depends upon the experimental requirements of stable oscillations. The pressures in the chambers are measured by digital Pirani gauges which are mounted in each of the chambers.

2.2.2 Electrodes

For producing a plasma, planar electrodes are used with the diameters of the cathode and anode taken to be 7 cm and 0.2 cm respectively and the separation distance between them fixed at 20 cm. The size of the anode is chosen deliberately small as compared to the cathode in order to promote the formation of an anode glow around the anode surface area. This anode glow is the main source of the periodic oscillations in our system which are then used for the synchronization experiments. Nikulin [26] has provided a general prescription for producing anode glows or anode double layers in DC glow discharge plasma sources in which if the anode area is made smaller than $A_c\sqrt{m/M}$ (where A_c, m and M represent cathode area, electron mass and ion mass respectively) then the drop in potential around the anode becomes positive as compared to the plasma potential and that results in the formation of an anode glow or anode double layer around the anode. For a zero drop in potential, the anode area formula can be written as:

$$A_0 = a \times A_c \sqrt{m/M} \tag{2.1}$$

where a represents the proportionality constant. If the anode size is made smaller than A_0, then the electron collection area of the anode does not change but remains A_0, which is a necessary requirement for maintaining current due to ions on cathode equal to current due to electrons on anode. For maintaining this condition both anode fall and anode sheath area increases, which results in a positive potential around the anode. In this positive potential electrons gain energy, which creates extra ionization in this region. The electrons produced by this extra ionization are captured by the anode and result in an accumulation of extra ions. These ions are repelled by the positive anode and that results in an increment of the anode sheath area. Due to this increment, electron flux entering in the anode sheath area increases and for maintaining the current equality, again this region shrinks. This process is repetitive in nature and produces an unstable double layer. This unstable double layer is the cause of oscillations in the system.

In our experiments, we have taken the cathode radius as 3.5 cm. Putting ($\sqrt{m/M} = 1/272$) for Argon gas and taking the value of proportionality constant as $a = 1$ (for no loss of ions from anode) in equation (2.1), we get the anode radius equal to 0.21 cm, below which anode glow or anode double layer could be formed. We have chosen the anode radius as 0.1 cm which is less than the calculated radius 0.21 cm.

2.2.3 DC Power Supply

Power supplies are one of the vital components of a discharge system for producing a plasma and for controlling the plasma properties. For synchronization experiments, the role of power supplies is very crucial for controlling the discharge properties and the oscillation characteristics in a precise manner. Keeping in mind this requirement, we have used a separate regulated DC power supply for each of the chambers with a variable voltage rating ranging from 30 V to 1000 V. These power supplies have a least count of 1 Volt, i.e., voltages can be increased in steps of 1 volt which gives us precise control over the oscillation dynamics.

2.2.4 Shielding From External Noise

Each chamber is separately covered with SS mesh grids with proper grounding mechanism, such that oscillations of one chamber plasma do not influence the oscillations of other chamber plasma by leaking out from this SS enclosure. Also, these SS enclosures help in reducing noise by shielding frequencies present in the environment which can influence the plasma oscillations.

2.2.5 Diagnostic

For measuring plasma properties like the electron temperature, the electron density, the floating potential and the plasma potential we use a standard Langmuir probe diagnostic. It consists of a cylindrical tungsten rod with length 20 mm and diameter 1 mm which is fitted inside a teflon tube. The Tungsten rod is brazed with stainless steel wire and the whole arrangement is fitted inside a teflon coated 6 mm hollow SS (stainless steel) pipe. The exposed part of the tungsten wire in the plasma was 10 mm. Two probes are made with the same configurations and placed in each of the chambers through ports as shown in the above figures 2.1 and 2.2.

All signals are recorded in an oscilloscope (Lecroy wave Surfer 424) and the floating potential oscillations are used to study the oscillation dynamics.

2.3 Plasma Characterization

In direct current glow discharge plasma systems, plasma exhibits different characteristics depending upon the applied discharge voltage between the electrodes, size of the electrodes, distance between the electrodes and pressure in the system etc. Also, for doing synchronization studies in this system some special type of self sustained oscillations has to generated. For this, systems have to be optimized for different working pressure and discharge voltages regimes, in which such type of oscillations can be generated. In this regard, we have done Paschen's curve and discharge voltage vs current analysis for these systems. The results are presented in the following subsections:

2.3.1 Paschen Curve

In 1889, F. Paschen formulated an experimental relation between the breakdown voltage of a gas the operating gas pressure (p) and the electrode spacing or gap (d). The curve describing this relation is known as the Paschen curve or Paschen law. The breakdown voltage is a function of the product of the pressure (p) and the inter electrode distance (d): $V_{breakdown} = pd$. For characterization of the plasma, the Paschen curve is drawn for each of the plasma systems by varying the neutral pressure while keeping the electrode spacing to be constant. A typical Paschen curve for the chamber-1 plasma system is shown in Fig. 2.4.

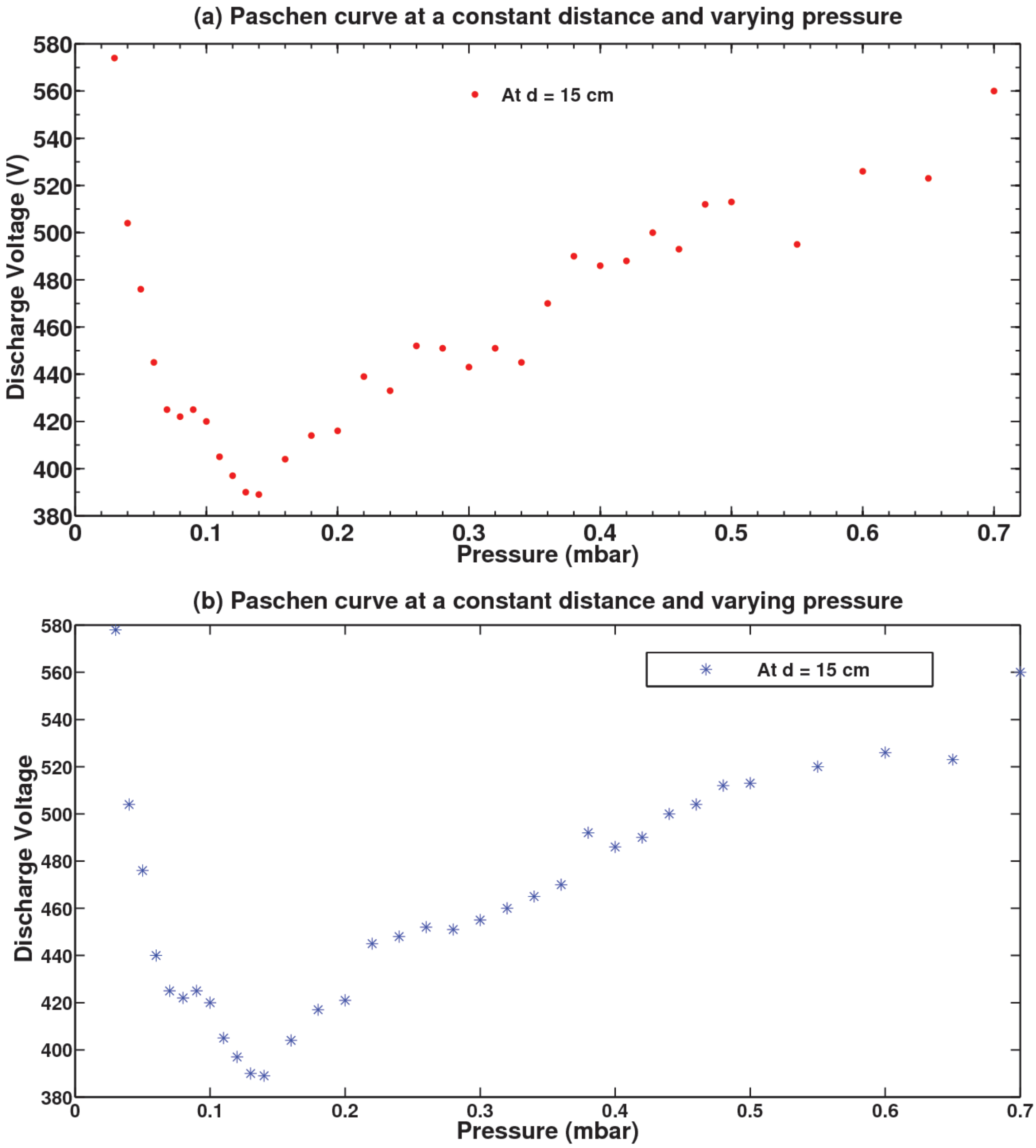

Figure 2.4: *Paschen curve for (a) chamber-1 and (b) chamber-2 plasma systems with the variation in the neutral pressure for a constant distance of 15 cm between the electrodes. Paschen minimum is found to be at 0.12 mbar for both the systems.*

It is observed that the Paschen minimum for each of the plasma system is around 0.12 mbar for a constant distance of 15 cm between the electrodes.

2.3.2 Discharge Voltage vs Current Characteristics

The Voltage-Current (VI) characteristics are strongly tied to the plasma characteristics and are used for determining the different regimes of plasma discharges. Klinger et al. have used this method for quantifying the different stable discharge modes for thermionic discharge systems [87, 118]. They have shown that at low discharge current a regime of anode glow mode (AGM) is produced while at a high discharge currents a temperature limited mode (TLM) occurs. Also, this transition from AGM to TLM mode occurs with a sudden jump in the value of the discharge current. We have also done such an analysis for our experimental system. In Fig. 2.5 (a, b, c and d), these characteristics are shown for chamber-1 plasma system at different pressures of 0.1, 0.2, 0.3 and 0.4 mbar respectively.

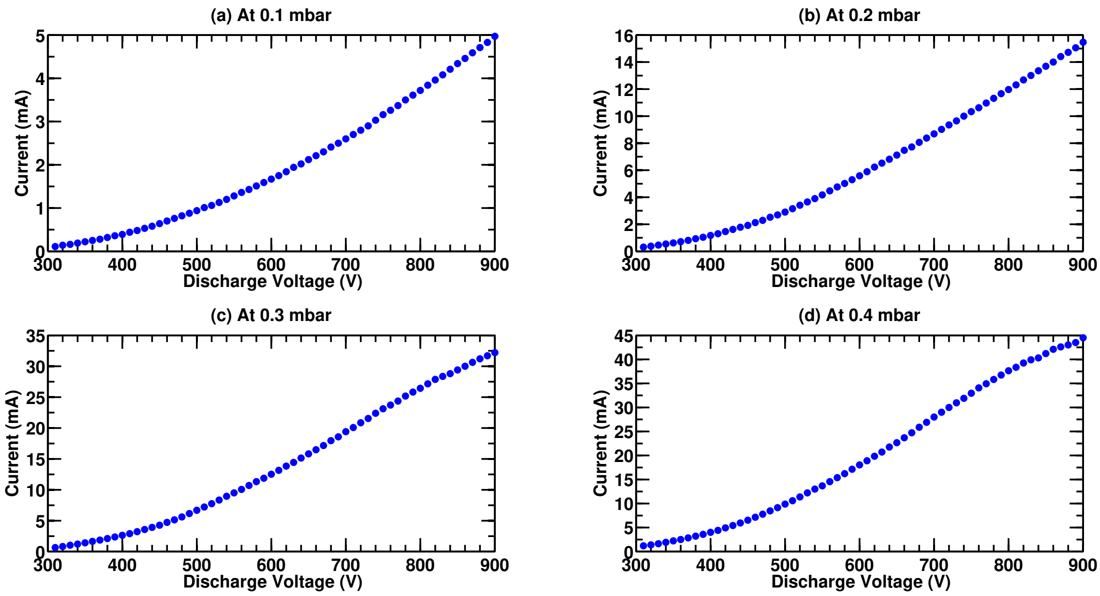

Figure 2.5: *Discharge voltage vs current plot. At four different pressures of chamber-1 plasma system.(a) 0.1 mbar (b) 0.2 mbar (c) 0.3 mbar (d) 0.4 mbar respectively*

From these plots, it is seen that with the increase in the discharge voltage, the discharge current is increasing and this increase is following an almost linear trend. It is also seen from these plots that at a given discharge voltage, the discharge current value is higher at high-pressure discharges. From these results, it is observed that there is no sharp transition occurring in the current value of the discharge current. This establishes the fact that our system remains in the anode glow regime and does not transit into the temperature limited mode in the range of pressures and discharge voltages considered in our experiment.

2.4 Plasma Conditions Required For Synchronization

In the Subsection 2.2.2, we have mentioned that the main source of oscillations in our experimental system is the fluctuations in the anode glow which are produced due to an asymmetry in the size of the electrodes. The oscillations produced in this way also depend upon various other factors like the discharge voltage, the pressure and the distance between the electrodes etc. By changing the values of these factors, different types of oscillations can be produced in the system. For our experimental results, oscillation dynamics of these oscillations are studied by changing the value of the discharge voltages and pressures with keeping the distance between the electrodes constant. For synchronization studies, one necessary requirement for the oscillation source is that it should be a self-oscillatory type. So before doing synchronization studies, it is checked that plasma source (i.e. anode glow oscillations) is of self-oscillatory type. To check this a perturbation method has been used, in which plasma oscillations are forced with some external source of oscillation like function generator. If plasma oscillations are changed with forcing by these external oscillations and after removal of this they regain their initial oscillations then such oscillation source is known as self-oscillatory type. For all the experimental results which we have report in this thesis, this test has been done and it is found that oscillations are of self-oscillatory type.

For a better understanding of the synchronization dynamics between the two glow discharge sources, oscillations in each of the plasma chamber are kept in the periodic regime. These oscillations are characterized with changing pressure as well as discharge voltages with constant anode to cathode surface area and distance between these electrodes. It is found that plasma oscillations have periodic characteristics at low discharge voltages for the pressure range of 0.07 mbar to 0.25 mbar and chaotic characteristics at higher pressures and discharge voltages. For synchronization experiments, pressures in each chambers are kept around 0.1 mbar (Paschen minima) such that periodic oscillations could be produced in each chamber. After this by playing with discharge voltages, oscillations of the two systems are brought close to each other.

2.5 Data Analysis Techniques

For analysis of the synchronization phenomenon between floating potential fluctuations of two systems, various analysis techniques are used. In this time series, power spectrum and Lissajous plots are drawn for oscillations of two systems. Frequency bifurcation and Devil's stair case plots are also drawn for defining different synchronization regimes. In the following subsections basics of this analysis are elaborated.

2.5.1 Time Series Plot

For describing oscillations in the two systems, floating potential fluctuations recorded at equal interval of spaced time is plotted. These oscillations are recorded with a sampling frequency of 25 MS/s and a frequency resolution of 100 Hz.

2.5.2 Power Spectrum

This is a type of plot which provides the information of distribution of energy in different frequency components of a waveform. For a periodic time series, power spectrum can be expressed as a linear combination of oscillations whose frequencies are integer multiples of the basic frequency which is the Fourier series. For finding the characteristic modes present in the floating potential fluctuations, power spectral method has been used. Power spectrum has been calculated using the fast Fourier transform(FFT) method. The FFT is a mathematical method in which a function of time is transformed into a function of frequency.

2.5.3 Lissajous Plot

These plots were investigated by the French mathematician Jules-Antoine Lissajous in (1857-58) [27]. The shape of Lissajous figures depends on the ratio of the periods (frequencies), phases, and amplitudes of the two oscillations. In the simplest case, if the amplitude and phases of the two signals are equal then a straight line at an angle 45° is produced while if one of the oscillations is 180° out of phase then a straight line at an angle 135° is produced. If the phase difference between the two signals is 90° then a circle is formed. Similarly, if the periods of the two oscillations do not coincide exactly, then the shape of the formed ellipse change continuously. Lissajous pattern will not be observed if the periods differ substantially. However, if the periods are related by an integer ratio , the moving point will return to its original position in a period of time equal to the least multiple of the two periods, and a Lissajous figure with a more complex shape is produced. Some of the common shapes formed in Lissajous plots are shown in Fig. 2.6.

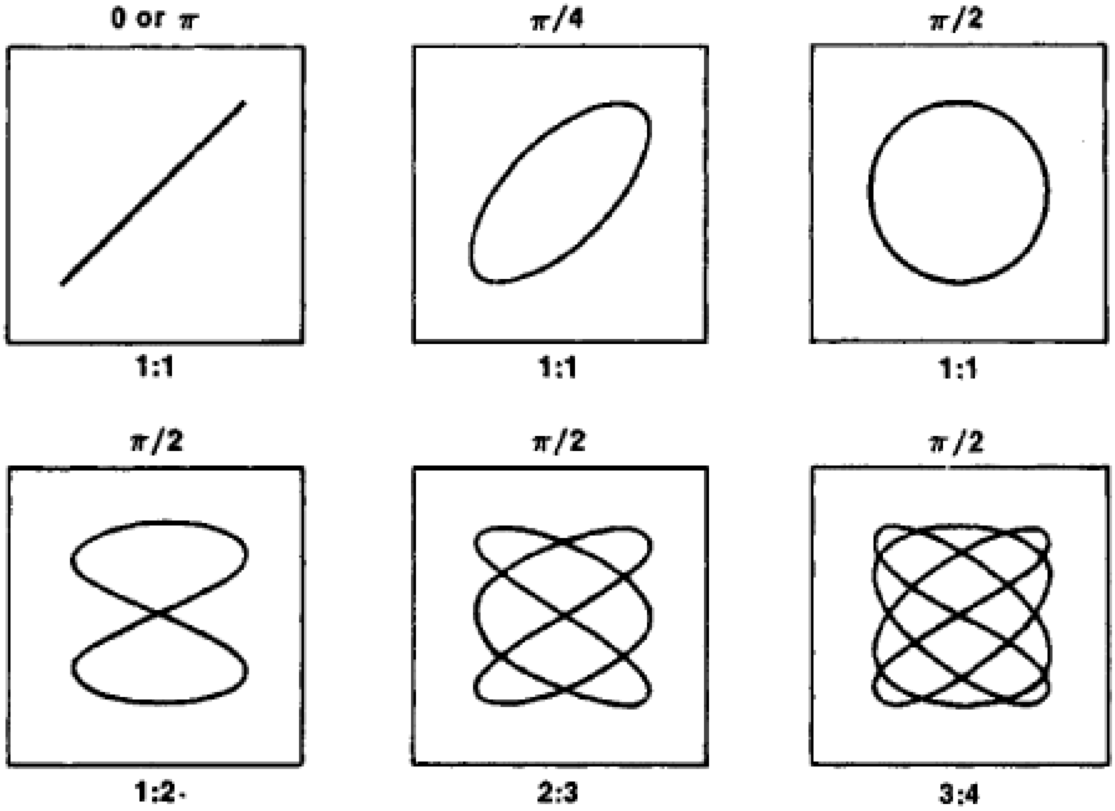

Figure 2.6: *Upper three figures are showing Lissajous plots for two signal having same frequencies with varying phase differences while bottom three figures are showing Lissajous plots for phase difference of* 90° *with changing frequency [27]*

For describing synchronization behaviour between coupled oscillations of two glow discharge plasmas and coupled oscillations of Van der Pol equations the Lissajous plot technique has been used.

2.5.4 Frequency Bifurcation Plot

This is a type of plot which is used for describing different types of nonlinear modes present in a system. In this plot, frequencies present in a system are plotted on Y axis while variation in the controlled parameter is plotted on the X axis. Debajyoti et al. [119] have in the past plotted the oscillation frequencies of a DC glow discharge plasma system against the variation in the discharge voltage. Such plots are also used in describing different types of synchronization phenomenon. For forced synchronization process such plots are used in describing frequency synchronized and frequency pulling states as shown in Fig. 2.7, where the applied frequency vs forced plasma oscillation frequencies are plotted. In this figure, non-frequency locked region have three regimes which signify the formation of harmonics and their side bands. The upper two bands signify the first and second harmonics bands while lower band is formed due to the interaction between the side band frequencies of the first harmonic frequency. In frequency locked region(in middle part of the figure), the lower band frequencies are not formed because plasma oscillations are completely locked with the externally applied frequency. Due to this, side band frequencies of first harmonic frequency is not formed whose interaction creates lower bands of frequencies.

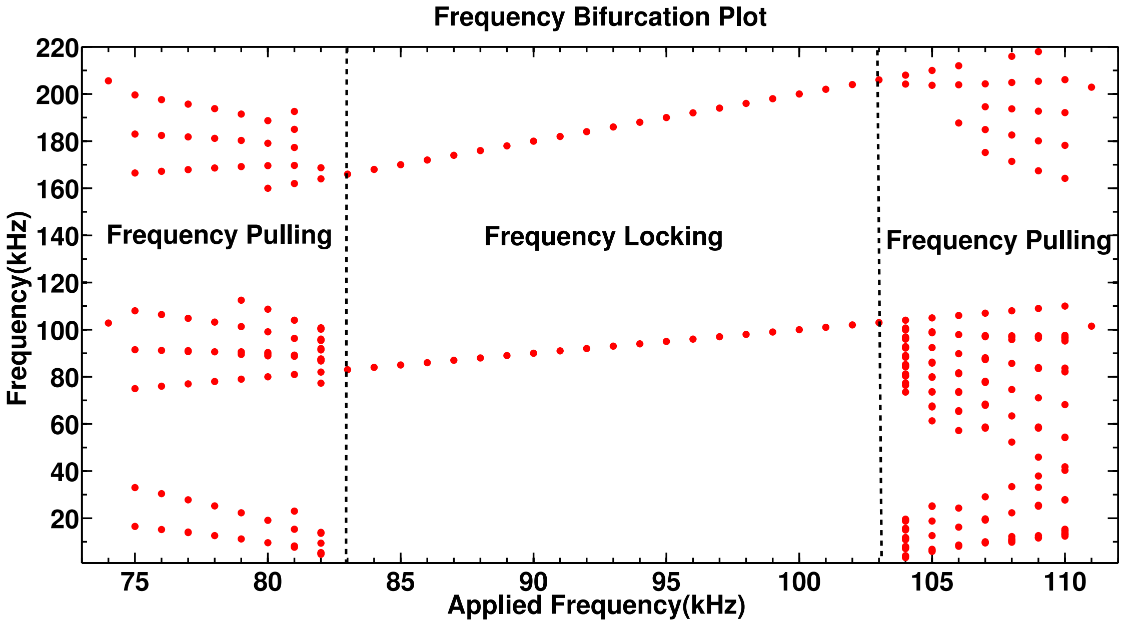

Figure 2.7: *Frequency bifurcation plot with the variation in the applied forced frequency.*

We have used these plots for describing different regimes of unidirectional and mutual synchronization phenomenon.

2.5.5 Devil's Stair Case Plot

This is a type of plot which is used for describing different harmonic synchronization regimes for a forced oscillatory system. In this plot winding number(W) is plotted against the driver frequency(f2) where winding number is a ratio of the driver frequency(f2) and the peak frequency(f1) of the forced self oscillatory system [99]. Flat regions of this plot indicate the existence of synchronization while the rising part of the curve indicates lack of synchronization. A typical Devil's staircase plot is shown in Fig. 2.8, for two coupled glow discharge plasma sources, where the oscillations of one of the systems are used as a driver for the other.

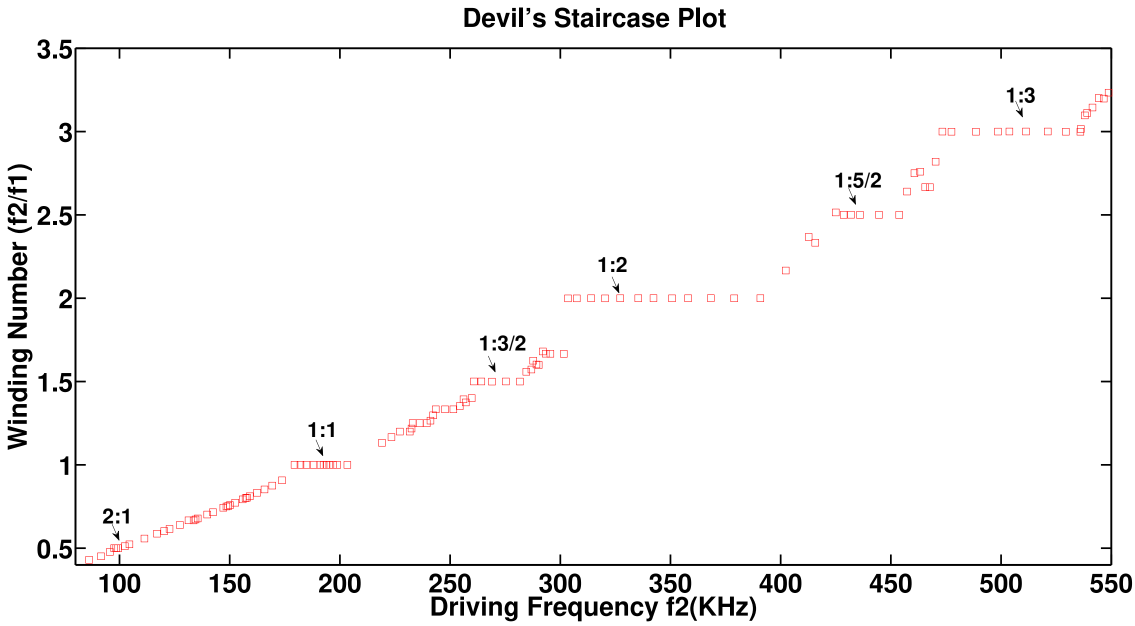

Figure 2.8: *Lissajous plot, winding number(ratio of driver frequency(f2) and driven frequency(f1)) vs driver frequency(f2).*

2.6 Summary and Conclusions

We have presented a detailed description of the experimental setup which has been used for the studies of different types of synchronization. The results of plasma characterizations are shown, which are used for optimizing the system for working pressure and discharge voltage regimes. Different data analysis techniques are introduced, which are used for analyzing the time series data and finding different synchronized states.

In later chapters (4, 5 and 6), mutual synchronization have been studied in detail. In the next chapter, the Van der Pol equation is introduced and a model based on it is used for investigating the mutual synchronization phenomenon numerically.

3

Van der Pol equation to study the dynamics of coupled plasma systems

In this chapter we give a brief overview of the Van der Pol equation, which is widely used as a model equation to represent the nonlinear dynamics of a variety of oscillators and to study the synchronization phenomenon between two or more coupled oscillators. We then discuss how this equation can also serve as a useful model to represent nonlinear oscillations in a plasma by providing a derivation of this equation from first principles by using a fluid model of the plasma. Next, the dynamics of two such coupled equations under various coupling conditions are presented to provide a framework for comparison with experimental data on synchronization that are reported in subsequent chapters.

3.1 Introduction

The Van der Pol equation was proposed by B. Van der Pol in 1920 [112] to model current oscillations observed in triode vacuum tubes. Since then this equation or modified form of this equation has been used in several branches of science for studying a variety of nonlinear dynamic phenomena [2,3]. This equation possesses oscillatory solutions which are self-sustained and of the relaxation type which are quite appropriate for representing many real world oscillations [2,3]. Van der Pol had used this equation for simulating and controlling the irregular beating of a heart (cardiac "arrhythmias") [120]. There are numerous other examples where such type of oscillations are observed such as in electronic devices for generating relaxation oscillations [112–114, 121], oscillations observed in biological systems [42, 120, 122], neuronal excitations in the brain [115, 116], pacemaker neurons [41, 123], circadian rhythms in the chemistry of the eyes [124], power line radiation in the magnetosphere [125], dynamics of a geological fault between two plates [117], relativistic magnetrons [126], semiconductor laser [127] etc.
In its classical form the Van der Pol equation can be written as:

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x = 0 \tag{3.1}$$

Here, dots over the variables represent derivatives over time t and ϵ is a parameter that controls the magnitude of the damping term. Note that the term proportional to ϵ can change its sign. Taking ϵ to be positive the sign of this term depends upon whether x^2 is greater or less than unity. For $x^2 > 1$ the term $(1 - x^2)$ is negative which implies that the damping force term will be positive and the value of x will decrease. If the value of x^2 is below unity then the damping force term has a negative value and serves to boost the value of x. Thus the solution settles on a limiting curve known as a limit cycle. The amplitude and frequency of the limit cycle solution can depend on the value of ϵ. If ϵ is small and positive i.e., $0 < \epsilon << 1$ then the solution of this equation is close to that of a simple harmonic oscillator(S.H.O) with a frequency close to that of S.H.O. (which is unity in this case). However, if ϵ is not small then the oscillations still have periodic characteristics but the frequency will be lower than that of the simple harmonic oscillator. Also, with increase in the value of ϵ relaxation properties of the oscillations increases. These results can be seen from Fig. 3.1(a and b) where with the increase in the value of ϵ, the period of the oscillations is seen to decrease with a concomitant increase in the relaxation properties.

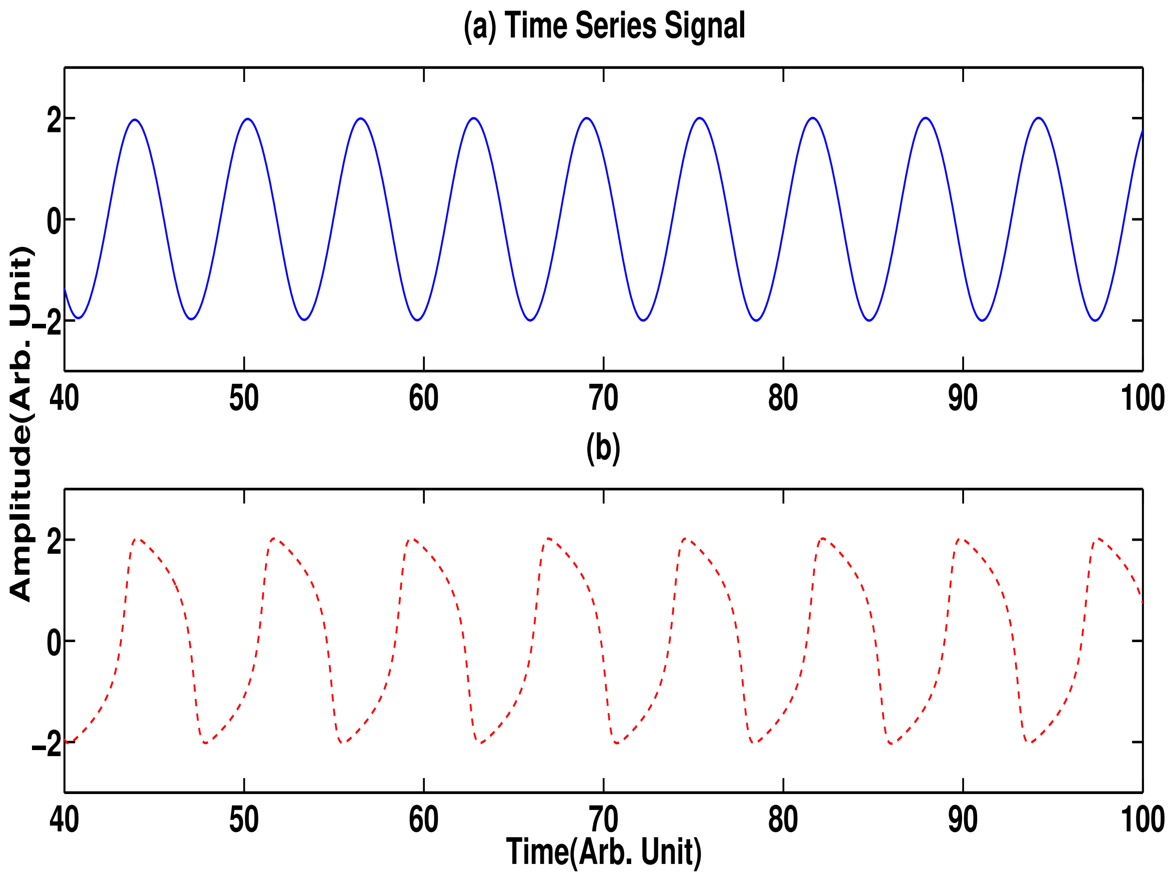

Figure 3.1: *In figures (a) and (b) values of nonlinear parameter ϵ is taken as 0.1 and 2 respectively.*

This chapter has been organized as follows. Section 3.2 presents a brief overview of past studies of the Van der Pol (VPD) equations in the context of plasma systems followed by a derivation of the VPD equation from a fluid model of the plasma. A physical discussion of the various terms of the VPD equation, their relation to plasma parameters and the role of a constant DC source term is also provided. In section 3.3 numerical solutions of two coupled VPD equations are given and discussed for various scenarios relevant for our experiments. This includes various forms of coupling between the oscillators e.g. direct coupling and indirect or bath coupling. Section 3.4 contains a brief discussion and concluding remarks.

3.2 Van der Pol Model in Plasma Systems

In plasma systems, the Van der Pol equation has been used for studying various nonlinear instabilities occuring in ion acoustic waves [12,14,80,128,129], relaxation type oscillations [87, 89], ionization waves [82, 84], beam driven waves [85, 86, 130], drift instability [91, 131, 132], anode oscillations [133], dust acoustic waves [98–101], etc. This equation has also been used for understanding various plasma nonlinear phenomena like plasma resonance [134], feedback stabilization [128], mode suppression [14, 89, 135], mode-mode coupling [12], frequency entrainment [132], amplitude collapse [136], periodic pulling [15, 137, 138], wave-wave interaction [139], spectral broadening [140] etc. In 1969, Keen and Fletcher used this equation for explaining mode-mode coupling phenomenon in a plasma system for the ion acoustic instability and gave a heuristic derivation of this equation from the basic plasma hydrodynamic equations [12, 14, 80]. After that, Shut'ko et al. [141], klinger et al. [83], etc. have also used this equation for studying various plasma instabilities and provided its derivation from the basic plasma fluid equations.

Kadji et al. [142] and and Debajyoti et al. [119] have used Van der Pol model for explaining results of an un-magnetized plasma and given a heuristic derivation using a two-fluid model. Our work also involves an un-magnetized DC plasma with all the essential features that are similar to their setup [119, 142]. We therefore outline a derivation of the Van der Pol equation from the basic plasma two-fluid model along similar lines to that of [119, 142].

The electron equation of motion is written as:

$$m_e \frac{dv_e}{dt} = e\nabla\phi + (T_e/n)\nabla n, \tag{3.2}$$

where v_e is the electron velocity, T_e is the electron temperature, n is the plasma density, ϕ is the potential. Plasma is considered to be un-magnetized, therefore magnetic field part is taken to be zero in above Eq. (3.2). As instabilities present in our system are in the ion acoustic region, we can neglect the electron inertia and take $m_e = 0$.

The total density and potential can be written as

$$n = n_0 + n_1, \tag{3.3}$$

$$\phi = \phi_0 + \phi_1, \tag{3.4}$$

where n_0 and ϕ_0 are unperturbed density and potential respectively while n_1 and ϕ_1 are perturbed density and potential respectively. Using these values in Eq. (3.2) and retaining only linear terms we obtain

$$\phi_1 = \frac{n_1 T_e}{n_0 e}, \tag{3.5}$$

The ion equation of motion is written as

$$m_i \frac{dv_i}{dt} = -e\nabla\phi - m_i v_i \nu_i \ , \tag{3.6}$$

where m_i, ν_i and v_i are ion mass, ion neutral collision frequency and ion velocity respectively.

The equation of continuity for ions is

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i) = S_i \ , \tag{3.7}$$

Here, n_i is ion density and S_i is the ion source term. Using thermodynamic argument [12, 14, 80, 142], ion source term S_i can be given as

$$S_i = \alpha n_1 - \beta n_1^2 - \gamma n_1^3 - .... \tag{3.8}$$

where the constants α, β, and γ represent the strength of the ionization, two body recombination and three body recombination processes respectively [119].

Next, linearizing Eq. (3.7) by putting $n_i = n_0 + n_1$ and $v_i = v_0 + v_1$ and excluding higher order terms, we get

$$\frac{\partial n_1}{\partial t} + n_0(\nabla.v_1) + (v_1.\nabla)n_0 = S_i \tag{3.9}$$

After differentiating w.r.t t

$$\frac{\partial^2 n_1}{\partial t^2} + \frac{\partial[n_0(\nabla.v_1)]}{\partial t} + \frac{\partial[(v_1.\nabla)n_0]}{\partial t} = \frac{dS_i}{dt} \tag{3.10}$$

Combining the first and third terms on left hand side we obtain,

$$\frac{d^2 n_1}{dt^2} + \frac{\partial[n_0(\nabla.v_1)]}{\partial t} = \frac{dS_i}{dt} \tag{3.11}$$

Using Eq. (3.6) in Eq.(3.11), we have

$$\frac{d^2 n_1}{dt^2} - \frac{dS_i}{dt} + n_0 \frac{d[-(e\nabla\phi)/m_i - v_i\nu_i]}{dx} = 0 \tag{3.12}$$

Putting Eq. (3.5) in Eq. (3.12), Eq. (3.12) becomes

$$\frac{d^2 n_1}{dt^2} - \frac{dS_i}{dt} - \frac{n_0 e}{m_i} \frac{\partial[\nabla(n_1 T_e/n_0 e)]}{\partial x} - n_0 \nu_i \frac{\partial v_1}{\partial x} = 0 \tag{3.13}$$

Solving Eqs. (3.9) and (3.13), we get

$$\frac{d^2 n_1}{dt^2} - \frac{T_e}{m_i}\nabla^2 n_1 - \nu_i[S_i - \frac{\partial n_1}{\partial t}] = \frac{dS_i}{dt} \tag{3.14}$$

Now putting the value of S_i from Eq. (3.8) in above Eq. (3.14), we get

$$\frac{d^2 n_1}{dt^2} - \frac{T_e}{m_i}\nabla^2 n_1 + [\nu_i - \alpha + 2\beta n_1 + 3\gamma n_1^2]\frac{dn_1}{dt} - \nu_i[\alpha n_1 - \beta n_1^2 - \gamma n_1^3] = 0 \tag{3.15}$$

Taking spatial variation of the perturbed quantity n_1 of the form e^{ikz} and using expressions of ion sound velocity $c_s = \sqrt{(T_e/m_i)}$ and $\omega_0 = kc_s$ in Eq. (3.15), we get

$$\frac{d^2 n_1}{dt^2} + [\nu_i - \alpha + 2\beta n_1 + 3cn_1^2]\frac{dn_1}{dt} - \nu_i[\alpha n_1 - \beta n_1^2 - cn_1^3] + \omega_0^2 n_1 = 0 \tag{3.16}$$

For ν_i (ion neutral collision frequency) $\rightarrow 0$, this equation can be reduced to

$$\frac{d^2 n_1}{dt^2} - [\alpha - 2\beta n_1 - 3\gamma n_1^2]\frac{dn_1}{dt} + w_0^2 n_1 = 0 \tag{3.17}$$

Equation (3.17) is of the same form as that of the classical van der Pol type equation.

3.2.1 Physical Interpretation of Coefficients

In Eq. (3.17), the ionization coefficient α defines the linear growth rate of the self-sustained oscillations. The oscillations remain harmonic for small and positive values of $0 < \alpha << 1$, and oscillation frequency will be close to ω_0. For large α values the oscillations will still remain periodic but with a oscillation frequency slower than ω_0 and a higher amplitude [3].

The two body recombination term β defines the degree of the asymmetry in Eq. (3.17). This term is not present in the ideal classical Van der Pol equation but inherently comes in the derivation of the Van der Pol equation from the two-fluid model. In two fluid model too, this term can be neglected because the probability of occurrence of two body recombination is very less. Although this term is less significant, but in some works in which harmonic of the oscillations are studied and have importance of both odd as well as even harmonics then, this asymmetry factor β plays a very important role and cannot be ignored. This asymmetry factor β inherently produces even harmonics in the oscillations of the Van der Pol equation [143, 144].

The three-body recombination term γ constitutes the main losses in a plasma. For a Van der Pol Eq. (3.17), this term provides a nonlinear saturation or damping effects in the oscillations of this equation. The self-sustained oscillations produced by this equation are due to interplay between the linear growth rate/ionization coefficient α and the nonlinear saturation term γ.

3.2.2 Role of Additional DC Constant Term A in Eq. (3.17)

The general form of Van der Pol Eq. (3.17) with additional DC constant term A on the right hand side is given as

$$\frac{d^2n_1}{dt^2} - [\alpha - \beta n_1 - \gamma n_1^2]\frac{dn_1}{dt} + \omega_0^2 n_1 = A \tag{3.18}$$

Debajyoti et al. in their paper [119], have shown that an additional added DC constant term in the Van der Pol equation can change the ionization rate or linear growth term α of Eq. (3.17). Similar to [119], additional term influences the oscillations of our numerical and experimental systems. We found a good agreement between our numerical/experimental results and observation in [119]. Numerically, in Eq. (3.18) we have observed that with an increase in the additional term A, oscillation frequency of this equation is increasing linearly under certain limits. This can be seen in Figs. (3.2(a, b, and c)) where, three time series plots are shown with three increasing values of A term.

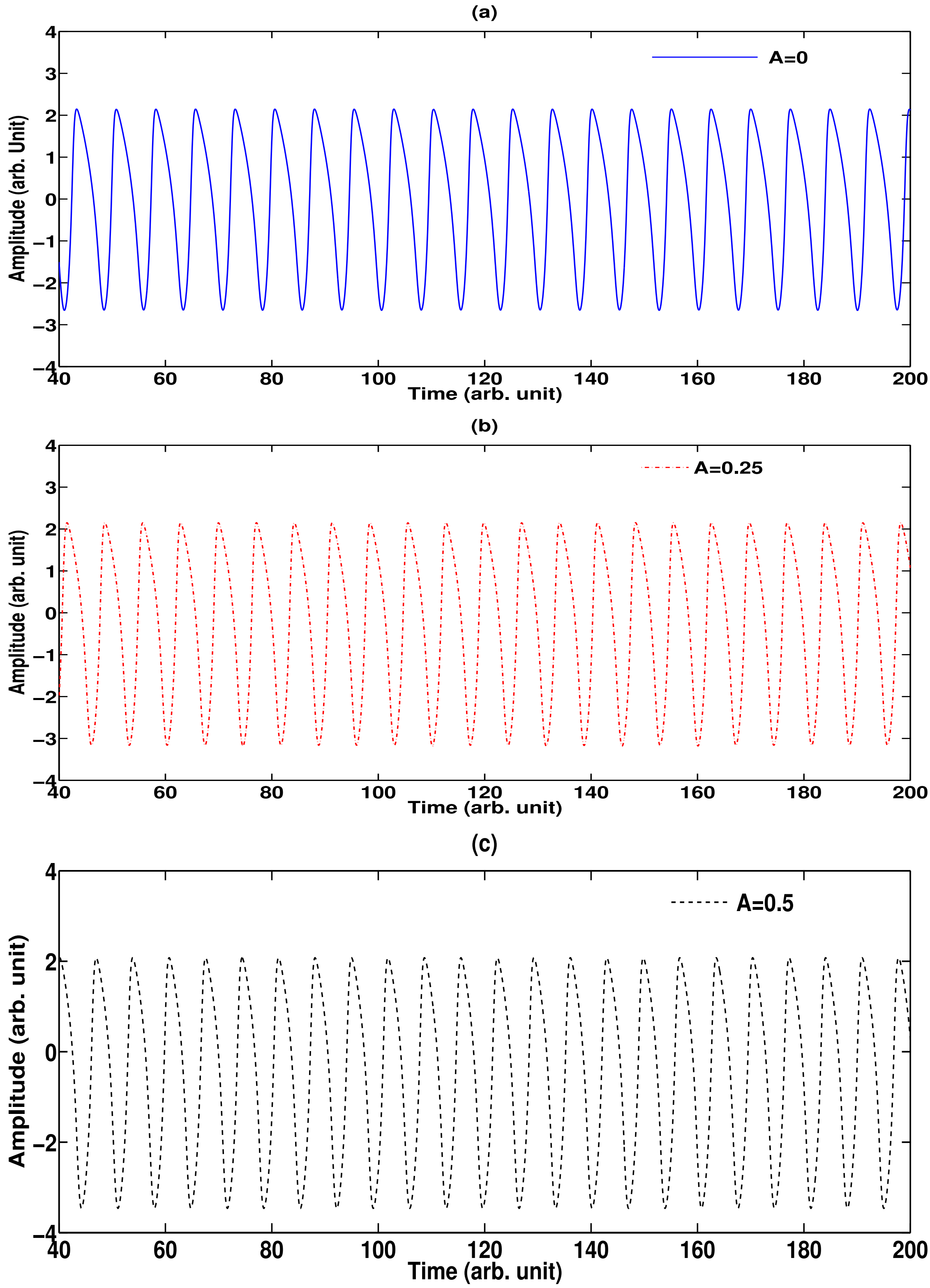

Figure 3.2: *In figures (a), (b) and (c) values of* a, b *and* c *of equation (3.18) are taken as 0.5, 1 and 0.5 respectively and values of* A *are increased as 0, 0.25 and 0.5 respectively.*

From these plots it can be seen that oscillations are increasing with an increase in the value of constant term A. A clearer picture of this dynamics can be seen from the power spectrum plot of these time series, shown in Fig. (3.3).

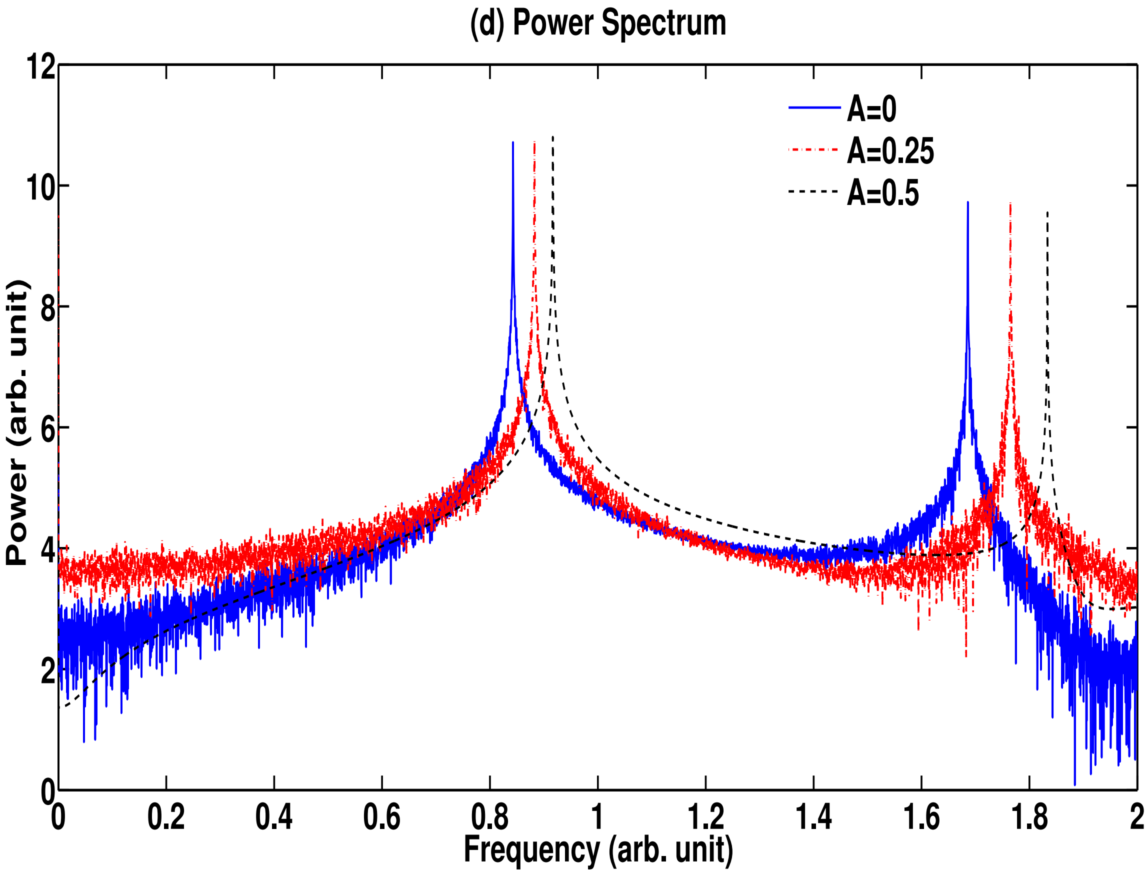

Figure 3.3: *The power spectrum is shown for $A = 0$, 0.25 and 0.5*

The DC term can be related to the discharge voltage in our experiments and as we will subsequently show in later chapters it helps in modeling our synchronization experimental results.

Thus far we have considered an individual system with and without a DC source term. In the next section, we will consider the coupled dynamics of two VDP equations and examine their mutual synchronization phenomena.

3.3 Coupled Oscillation Dynamics

The main motivation of this thesis work is to study experimentally, mutual synchronization phenomenon between two coupled glow discharge plasma sources. For getting more insight into the experimental results, numerical modeling is also done by using two mutually coupled van der Pol type equations. The main reasons for choosing this equation for such studies are: (1) This equation inherently produces self-oscillatory oscillations, which is a necessary requirement for doing synchronization studies. (2) The oscillations of this equation have relaxation properties, which is of the same type as in our experimental signals. (3) This equation can be derived from the basic plasma fluid equations and contains all the essential features for describing different types of plasma oscillations. By considering these characteristics of the Van der Pol equation, two Van der Pol type equations with different sorts of coupling schemes namely direct and indirect couplings are used for explaining experimental results of mutual synchronization between two coupled glow discharge plasma sources. In next two sections, direct coupling and indirect coupling schemes are described in detail.

3.3.1 Direct Coupling

In this type of coupling mechanism, all the oscillators are directly connected with each other. This is a well-studied coupling mechanism which has been observed in a variety of systems [1–3,39,145,146]. In the experimental results of chapters 5 and 7, of this thesis, this type of coupling mechanism is used for synchronizing oscillations of two glow discharge plasma sources [147]. For explaining our experimental results, we do numerical modeling using same type of direct coupling mechanism between two Van der Pol type equations. In Eqs. (3.19) and (3.20) [147–149], such coupling is shown between two Van der Pol type equations:-

$$\ddot{x} - (a_1 - b_1 x - c_1 x^2)\dot{x} + x = M(y - x) + N(\dot{y} - \dot{x}), \tag{3.19}$$

$$\ddot{y} - (a_2 - b_2 y - c_2 y^2)\dot{y} + y = M(x - y) + N(\dot{x} - \dot{y}), \tag{3.20}$$

where (a_1, b_1, c_1) and (a_2, b_2, c_2) in (3.19) and (3.20) are constant quantities that are a measure of the linear growth rate, the degree of asymmetry and the amount of nonlinear damping respectively while x and y represent the dependent variables. Here, in Eqs. (3.19) and (3.20), two types of couplings are shown in the right hand side of these equations. First term is known as reactive coupling or displacement coupling which is proportional to the difference of the two dependent variables while second term is known as dissipative or velocity coupling which is proportional to the difference in the first derivative quantities of the two dependent variables. The constant parameters M and N quantify the strength of reactive and dissipative couplings respectively. If M term in Eqs. (3.19) and (3.20) are taken as zero then only dissipative coupling is provided with a strong coupling strength N. Then in mutual coupling, frequencies of both the oscillators move towards each other and have same frequencies which lies in between their original frequencies [3]. If N term is taken as zero and only reactive coupling is provided then results will not be the same as in the case of dissipative coupling. In this case, with mutual coupling there is a competition between oscillations of two oscillators and the final frequency does not settle in between the original natural frequencies of the two individual oscillators [3].

3.3.2 Indirect Coupling or Bath Coupling

In the previous section, the direct coupling mechanism was discussed in which, oscillators were directly coupled with each other. But in nature there are numerous examples where oscillators get synchronized, even though there is not any direct interaction between the oscillators. For instance; cellular populations communicating via a common medium [150, 151], pendulum clocks mounted on the same wooden beam [152], chemical oscillators interacting via a common solution [153], global oscillation of circadian oscillator [40], cold atoms interacting with electromagnetic field [154], lasers connected via a common amplifying medium [155], and indirectly coupled systems [156]. In this type of coupling mechanism, the coupling can be established between the oscillators by various ways like, via same common medium in which oscillators are placed or via some dynamic environment etc. In this thesis work, we have also presented experimental results of synchronization between two indirectly coupled plasma sources [157]. These results are verified by numerical modeling results which are done by using two bath coupled van der Pol equations. Here, the bath is acting as a third common medium for maintaining indirect coupling between the two equations [124]. Below in Eqs. (3.21), (3.22) and (3.23), we present this form of bath coupling between two VPD oscillators:

$$\ddot{x} - \epsilon_1(1 - x^2)\dot{x} + x = M(z - y) \tag{3.21}$$

$$\ddot{y} - \epsilon_2(1 - y^2)\dot{y} + y = M(z - x) \tag{3.22}$$

$$\dot{z} = M(z - x) + M(z - y) \tag{3.23}$$

where x and y represent the dependent variables of oscillators (3.21) and (3.22) respectively. The variable z represents the dependent variable in the bath and its dynamics is governed by the inputs from both the oscillators. Here, M is a measure of the coupling strength and ϵ_1 and ϵ_2 are the nonlinear coefficients governing the dynamics of the two systems. In Eq. (3.23), bath coupling is shown which represents a third medium where oscillations of two oscillators (3.21) and

(3.22) are linked with each other. This is an indirect coupling mechanism, where oscillators are not directly connected with each other but connected via a third medium.

3.4 Summary and Conclusions

In this chapter, a derivation of the Van der Pol equation from the basic plasma fluid equations has been provided and the physical meaning of all the constants and variables have been discussed. Also, the role of a DC constant source term on the oscillation dynamics of Van der Pol equation is presented. Coupled dynamics between two Van der Pol equations with both direct and indirect couplings is introduced in order to use them as models for interpreting our experimental results of synchronization between two coupled plasma sources.

In chapter 4 and 6, mutually coupled Van der Pol equations are used for simulating the experimental results of two directly coupled plasma sources. Similarly, in chapter 5, bath coupled Van der Pol equations are used for explaining the results of two indirectly coupled plasma sources. In the next chapter, we will present experimental/numerical results of mutual synchronization between two coupled plasma sources.

4

Mutual synchronization studies via direct coupling

In this chapter, we discuss the mutual synchronization between two directly coupled plasma systems. We also present numerical simulations to understand the experimental observations with the help of two coupled asymmetric Van der Pol type equations which are directly coupled with each other.

4.1 Introduction

In the past, very few studies have been devoted to the investigation of mutual synchronization between two plasma systems except [18] where this phenomenon was studied between two double plasma devices and [17] where it was investigated in two electrical discharges. In [18] the emphasis was on investigating the interaction between chaotic oscillations excited in two coupled glow discharge plasmas and the observation of spatio-temporal synchronization between them.

In this chapter, we present experimental studies on interaction between coherent oscillations in two glow discharge plasma devices. For this, a direct coupling mechanism is chosen for connecting the anodes of the two systems with a variable resistance in between them. The basic oscillations in each device are due to anode glow instabilities [15,83,88,158,159] and we demonstrate the occurrence of various nonlinear collective states as a function of the coupling strength. These include frequency pulling as well as frequency entrainment (synchronization) with increasing strengths of the coupling. We have also carried out a numerical simulation to understand the experimental observations with the help of two coupled asymmetric Van der Pol type equations [143, 144] which are directly coupled with each other.

The chapter is organized as follows. In Section 4.2 we have provided details of the experimental setup. Section 4.3 contains results of the characterization studies of each device followed by the results from their mutual coupling. In section 4.4 we have discussed the numerical simulation results obtained from the model equations of the asymmetric Van der Pol type. Brief conclusion is given in section 4.5.

4.2 Experimental Setup

The experiments were carried out using two separate glass chambers with diameters of 15 cm and lengths of 35 cm each. A schematic of the experimental setup is shown in Fig. 1. In both the chambers the diameters of the anode and the cathode were taken as 0.2 cm and 7 cm respectively and the distance between these electrodes was set at about 20 cms. The diameter of the anode was chosen to be smaller than the cathode diameter to facilitate the formation of an anode glow which has been attributed to the presence double layers [26,160,161] by concentrating the space charges on to a small surface area. Both the chambers were independently covered with a steel mesh and a proper grounding was done so that the oscillations generated during the experiments would not leak out of a chamber and interact with the plasma in the other chamber. In each chamber a cylindrical Langmuir probe of length 10 mm and diameter 1 mm was placed for measuring the plasma floating potential fluctuations and other plasma parameters. Both the chambers were evacuated to a base pressure of 0.001 mbar and after that were filled with Argon gas to a pressure of 0.09 mbar and 0.1 mbar respectively. A DC glow discharge plasma was produced in chamber-1 by grounding the anode through a 10 $k\Omega$ resistance. When a discharge was struck in the plasma chamber an anode glow was observed around the anode accompanied by floating potential fluctuations which settled to a stable periodic type of oscillations with further increase of the discharge voltage.

A similar procedure was followed in chamber-2 and when the floating potential fluctuations of this chamber also displayed a periodic type of oscillations then the anodes of the two chambers were connected through a variable resistance as shown in Fig. 6.1. The floating potential fluctuations of the two chambers were recorded in a Lecroy Wave Surfer-424 oscilloscope. The typical data length of the recorded signals consisted of about 250000 points with a sampling frequency of 25 MS/s and a frequency resolution of 100 Hz. From Langmuir probe measurements the typical plasma density and electron temperatures in these chambers were determined to be in the range of $(10^8 - 10^9)$ cm^{-3} and (1-2) eV respectively.

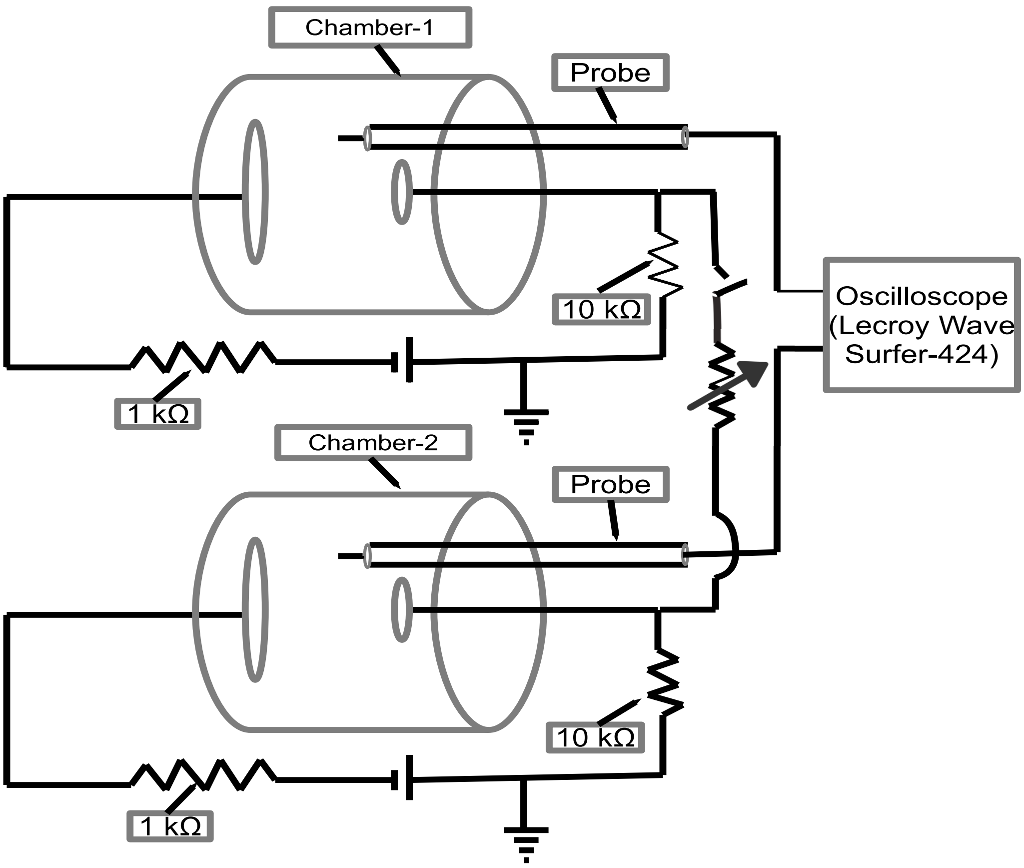

Figure 4.1: *Experimental setup for two coupled plasma sources. The pressures in chamber-1 and chamber-2 are 0.09 mbar and 0.1 mbar respectively.*

4.3 Experimental Results

4.3.1 Frequency Dynamics of the Uncoupled System

To understand the combined dynamics of the two coupled plasma devices we first characterized the dynamical behaviour of each device in their uncoupled state. In this subsection we describe the experimental findings from each individual plasma device - in particular the characteristics of their anode glow oscillations [15,160,161] as a function of the discharge voltage. Data from chamber-1 is shown in Fig. 4.2(a, b and c) as time series signals for three different discharge voltages and the power spectra of these signals are shown in Fig. 4.2(d). Similarly Figs. 4.3(a, b, c and d) show the time series signals and their power spectra for three different discharge voltages of chamber-2. From these figures it can be seen that, within the range of the applied voltages, the oscillation frequencies of the fluctuations in both the systems increase monotonically with the discharge voltage. This is seen even more clearly in Figs. 4.4(a) and 4.4(b) where the fundamental frequencies vs the discharge voltage plots for chamber-1 and chamber-2 respectively, are shown. These frequencies have a near linear dependence on the discharge voltages for both the systems.

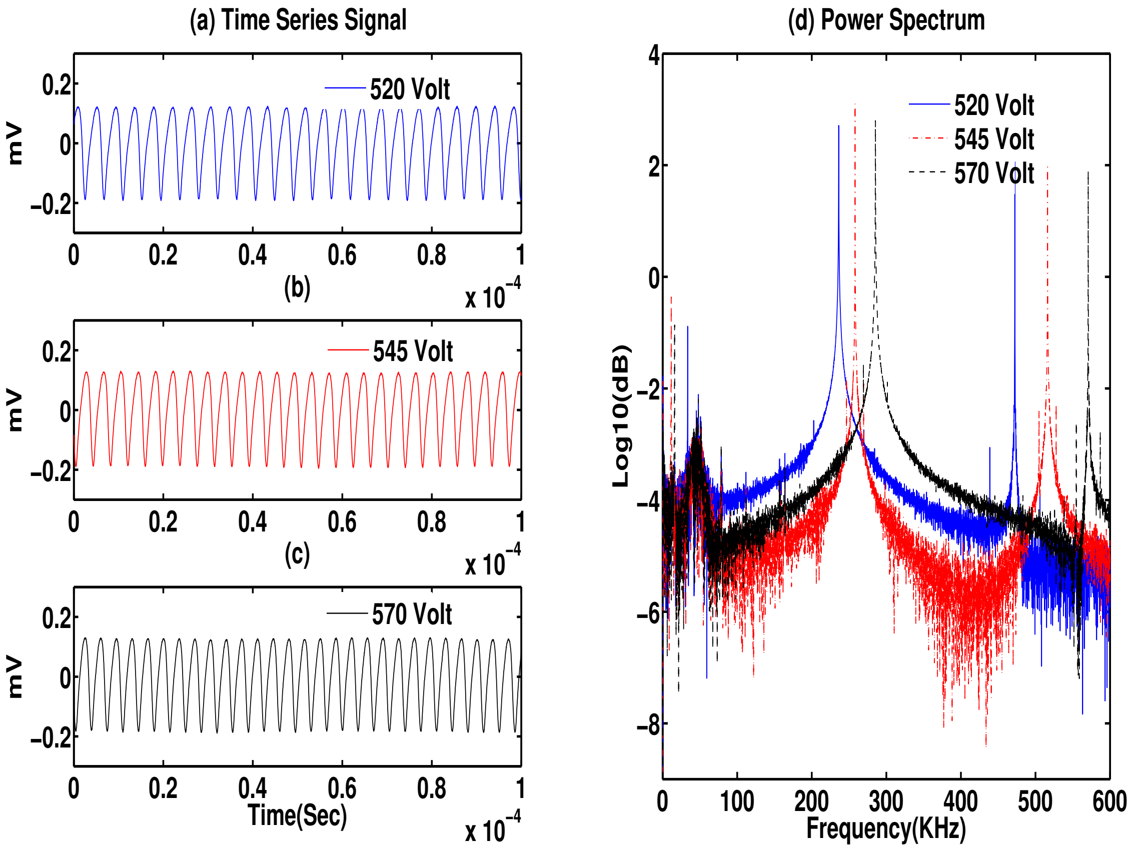

Figure 4.2: *Floating potential fluctuation signals for three different discharge voltages of Chamber-1 (a, b, c) are shown with corresponding power spectrum in (d).*

4.3.2 Synchronization Between Two Coupled DC Glow Discharge Plasmas

We now discuss the combined dynamics of the two coupled plasma devices. In Fig. 4.5 the floating potential fluctuation signals, Lissajous plots and power

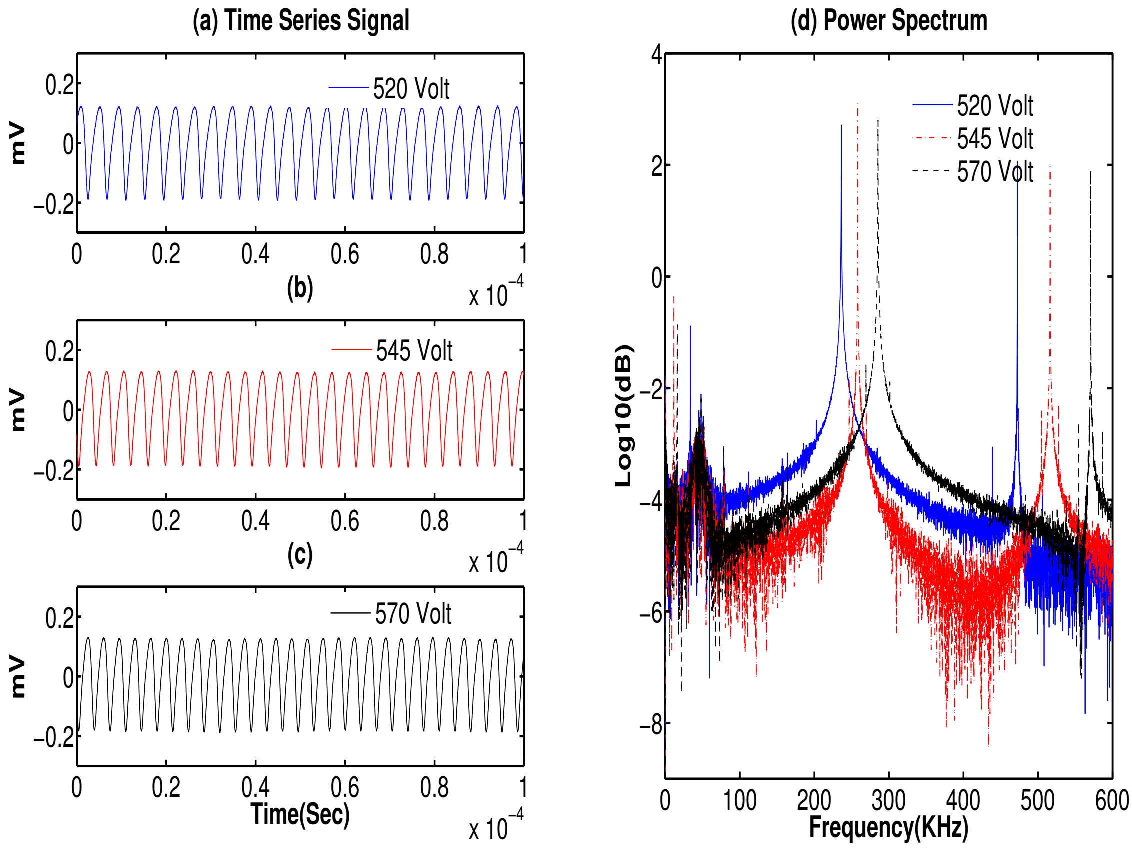

Figure 4.3: *Floating potential fluctuation signals for three different discharge voltages of Chamber-2 (a, b, c) are shown with corresponding power spectrum in (d).*

spectrum graphs are shown for chamber-1 and chamber-2 plasmas with different coupling strengths between their anodes. For the synchronization experiment the discharge voltages in the two chambers were chosen to be such that the floating potential fluctuation frequencies of both the systems were close to each other. Thus, in the absence of coupling, the individual fundamental frequencies of the two chambers were around 255.2 kHz and 262.5 kHz with the corresponding DC currents being 0.93 mA and 1.21 mA respectively. The two devices were then coupled through a variable resistor as shown in Fig. 1 which allowed us to vary the coupling strength by varying the value of the resistance. Zero resistance corresponds to maximum coupling strength.

Fig. 4.5(d)-4.5(f) show the results of the maximum coupling case where we find the floating potential fluctuation frequencies of both the systems to be locked to

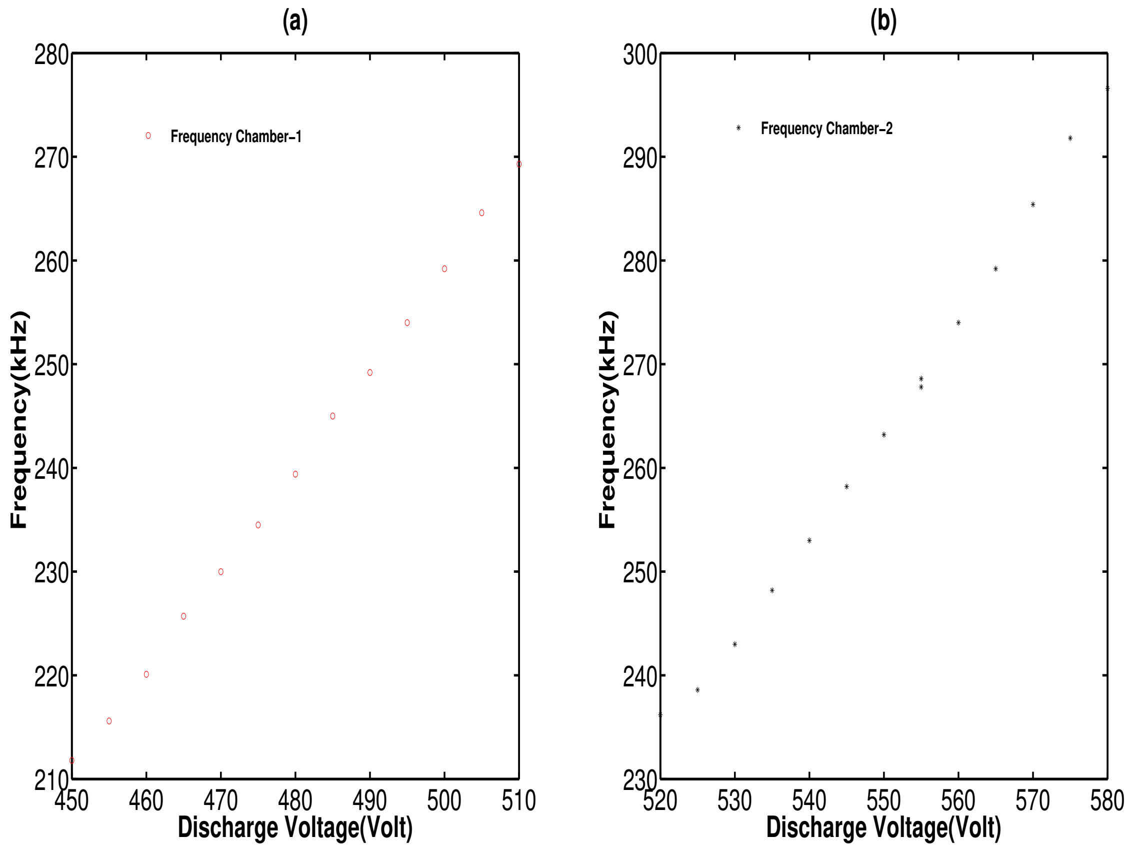

Figure 4.4: *(a), (b) fundamental frequency (error bar ±5%) vs discharge voltage plots of chamber-1 and chamber-2 respectively.*

a common frequency that is intermediate in value between the frequencies of the uncoupled system. This entrained state is maintained with decreasing coupling strengths upto a coupling resistance value of 4.5 $k\Omega$ as can be seen in Fig. 4.5(g)-4.5(i). With a further increase in the value of the resistance the oscillations in the two systems go over to a frequency pulling state as is evidenced by the emergence of sideband frequencies shown in Fig. 4.5(j)-4.5(l). These states can also be easily observed in Fig. 4.6 in which plots of the coupling strength vs fundamental frequencies of the two systems are shown. This figure shows the nature of the oscillation frequencies in the two systems as a transition from the frequency entrained state to the frequency pulling state takes place.

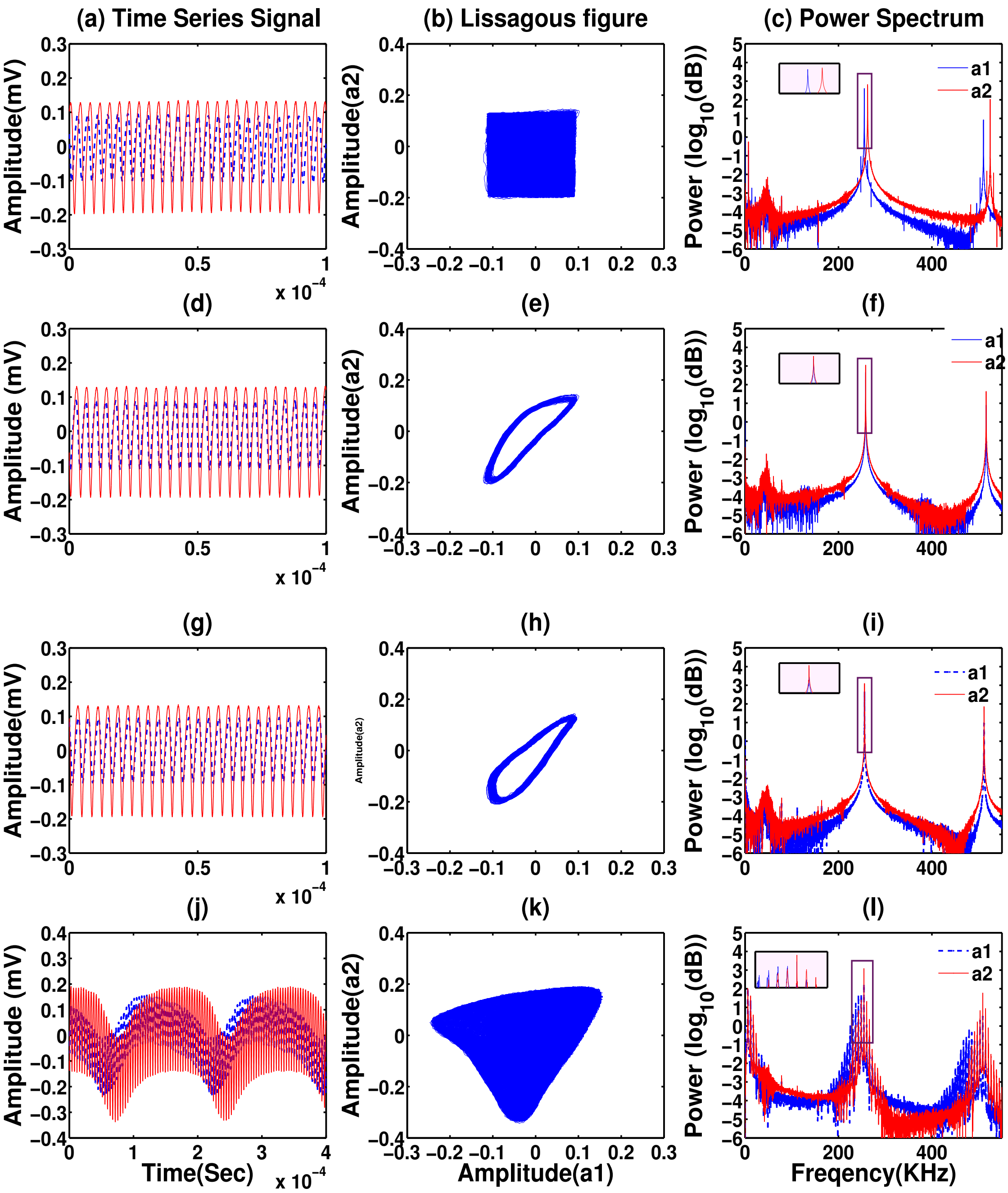

Figure 4.5: *Time series, Lissajous and power spectrum plots with changing coupling strength between chamber-1 and chamber-2:- (a, b, c) uncoupled, (d, e, f) Max. Coupling(0 kΩ), (g, h, i) at 4.5 kΩ, (j, k, l) at 15 kΩ.*

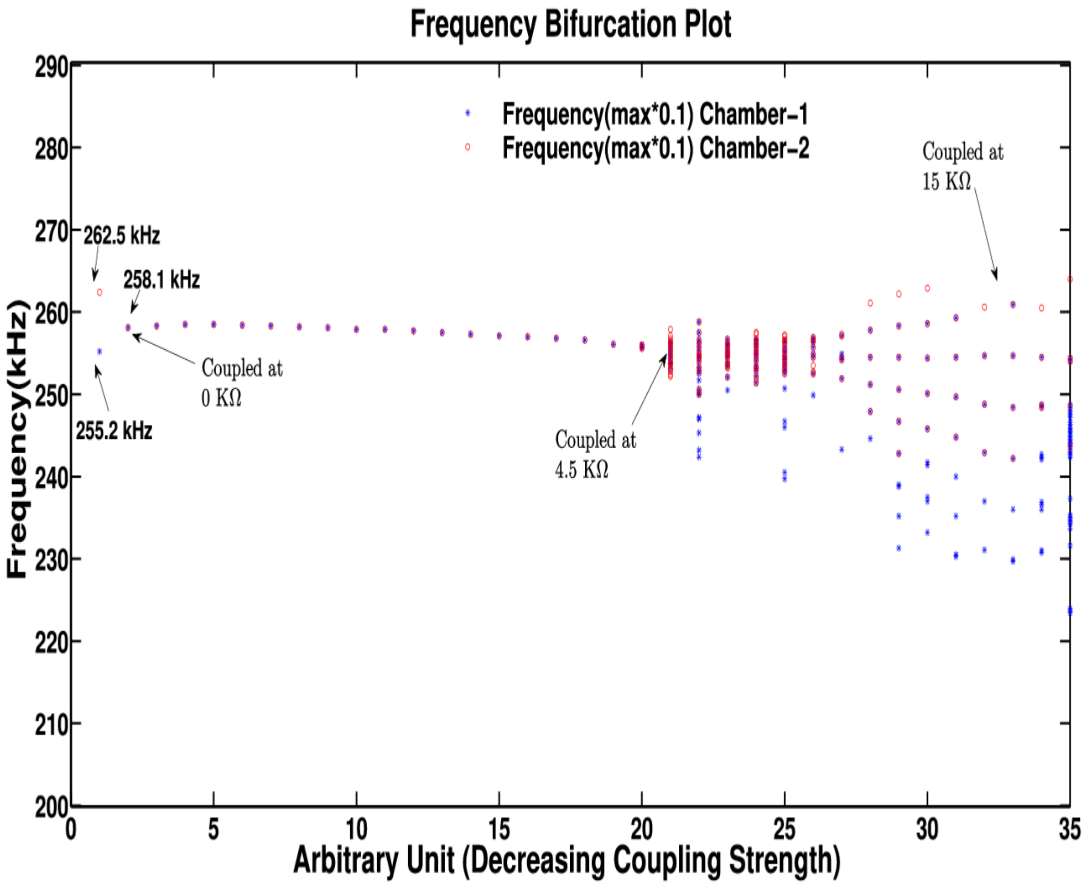

Figure 4.6: *Frequency bifurcation plot of time series signals of chamber-1 and chamber-2 with decreasing value of coupling strength.*

4.4 Numerical Modeling Results

To get some insight into the individual as well as the coupled state behaviour of the two plasma devices we have modeled their individual dynamics with the help of an asymmetric Van der Pol type equation. Variants of such a model have been used in the past to understand the nonlinear behaviour of glow discharge plasmas [14,80,83,119,142] and a heuristic derivation [14] has been given in Chapter 3. Our model coupled equations are of the form,

$$\ddot{x} - (a_1 - b_1 * x - c_1 * x^2)\dot{x} + x = N * (\dot{y} - \dot{x}) - A_1 \tag{4.1}$$

$$\ddot{y} - (a_2 - b_2 * y - c_2 * y^2)\dot{y} + y = N * (\dot{x} - \dot{y}) - A_2 \tag{4.2}$$

where (a_1, b_1, c_1, A_1) and (a_2, b_2, c_2, A_2) in equations (1) and (2) are constant quantities that are a measure of the linear growth rate, the degree of asymmetry, the amount of nonlinear damping and the magnitude of DC bias respectively. The dependent variables x and y represent amplitudes of voltage oscillations and the parameter N quantifies the strength of the coupling parameter. The form of the coupling is chosen to be of the dissipative type and is proportional to the difference in the first derivative quantities of the two dependent variables.

4.4.1 Frequency Dynamics of a Single Asymmetric Van Der Pol Type Model Equation

A numerical solution of eqn.(4.1) with constant parameter values of $a_1 = 1, b_1 = 1, c_1 = 0.5, N = 0$ for various values of the DC parameter A_1 is found to qualitatively display a behaviour similar to that observed for anode glow oscillations in a single glow discharge plasma device. This can be seen from the plots of the signal time series, and power spectra obtained from the numerical solution of eqn.(4.1) and plotted in Fig. 4.7(a-d) that show a near linear increase in the oscillation frequencies with increasing values of A_1. Similarly in Fig. 4.8(a-d) the time series signals and the corresponding power spectra obtained from numerical solutions of (4.2) are shown for a set of parameter values of A_2 with $a_2 = 0.5, b_2 = 1, c_2 = 0.5$ and N=0.

In Figs. 4.9(a) and 4.9(b) plots of the DC parameter values vs the fundamental frequency of the time series signals generated for different parameters of A_1 and A_2 are shown. From Figs. 4.7,4.8 and 4.9 it can be seen that the natural frequencies of the signals of the two uncoupled asymmetric Van der Pol oscillators keep increasing with the increase in the DC parameter values (A_1 and A_2) over the range plotted in the figures. These equations were solved using a fourth order Runge-Kutta method with a time step of δt=0.06 and the total number of data points were taken to be around 250000.

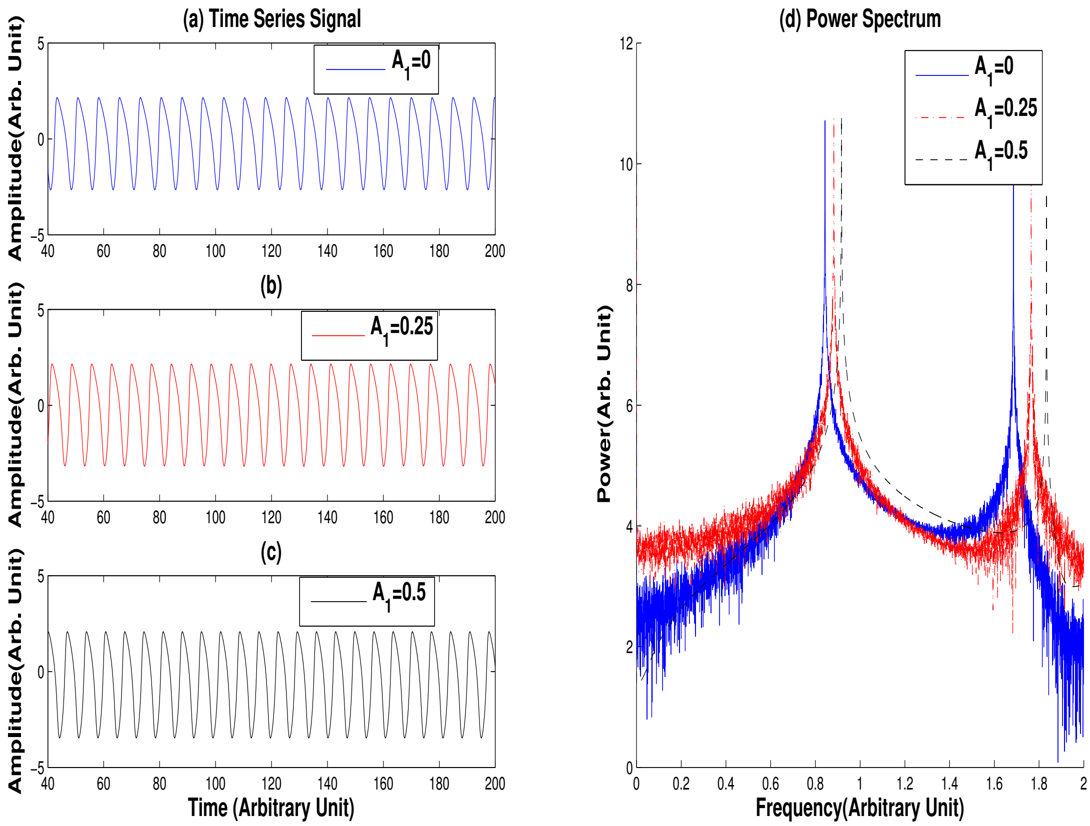

Figure 4.7: *In figures (a), (b) and (c) values of a_1, b_1, c_1 and N of equation (1) were taken as 1, 1, 0.5 and N=0 respectively and values of A_1 were increased as 0, 0.25 and 0.5 respectively. In figure (d) corresponding power spectrum is shown for A_1 = 0, 0.25 and 0.5*

4.4.2 Dynamics of Two Coupled Asymmetric Van Der Pol Type Equations

We now discuss the nature of the dynamics when the two model equations are coupled to each other in the manner shown in (4.1) and (4.2). We note that in the absence of coupling the individual Van der Pol oscillator model that we have chosen has 4 parameters (a, b, c, A) which govern the nonlinear dynamics of the oscillation. The parameter $a > 0$ (termed as the linear growth rate term) is physically identified with the ionization rate in the plasma and is the source of the linear self-excited instability in the system. In the nonlinear regime it influences both the amplitude and frequency of the saturated state. The amplitude is governed by the balance between the growth rate and the nonlinear damping (represented by the $c > 0$ term). The frequency of the nonlinear oscillation (as discussed in [3]) is

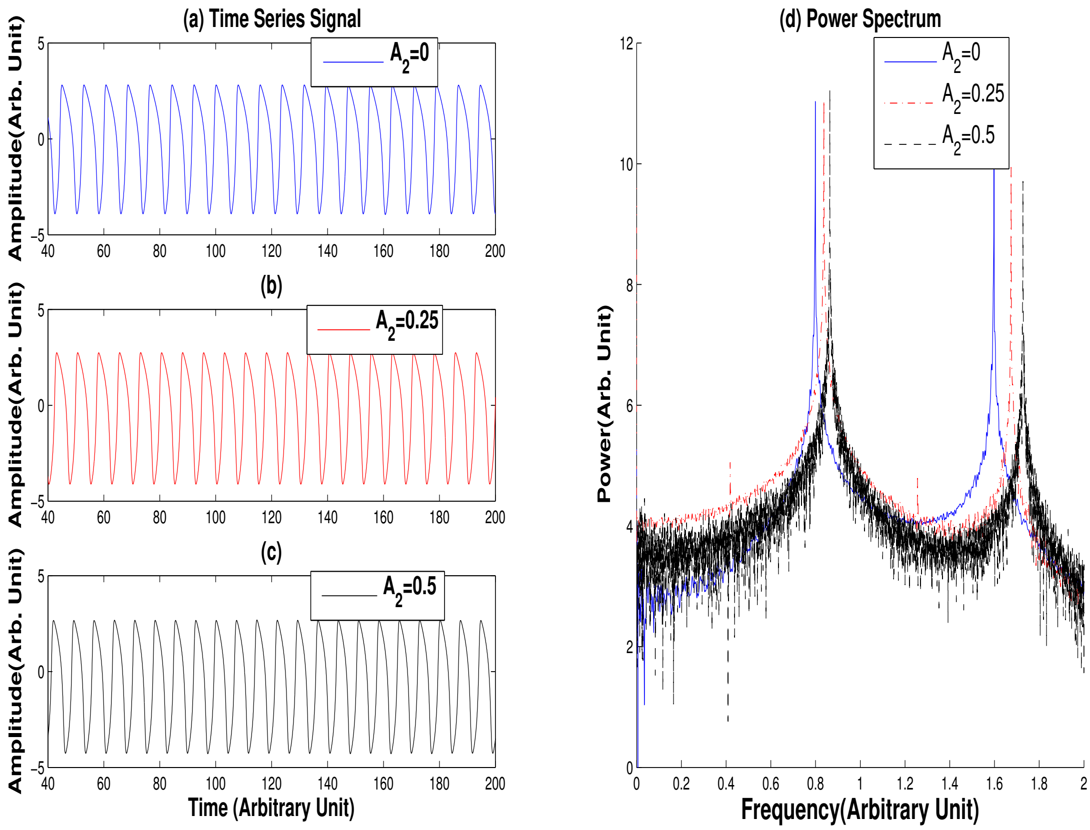

Figure 4.8: *In figures (a), (b) and (c) values of a_2, b_2, c_2 and N=0 of equation (2) were taken as 0.5, 1, 0.5 and N=0 respectively and values of A_2 were increased as 0, 0.25 and 0.5 respectively. In figure (d) corresponding power spectrum is shown for A_2 = 0, 0.25 and 0.5*

inversely proportional to a i.e. the frequency decreases as a increases. The driving term A (associated with the discharge voltage) also contributes to the ionization and in turn influences both the growth of the instability (and hence the saturated amplitude) as well as the frequency of the oscillation. The dependence of the frequency on A is however opposite to that of a i.e. the frequency increases as A is increased as shown in Fig. 4.9. To mimic the experimental results, where the two chambers have different pressures and different discharge voltages we have primarily adjusted the parameters $(a_1 A_1)$ and (a_2, A_2) while holding all other parameters constant, so that the ratio of the fundamental frequencies of the two oscillators are in the same ratio as in the experimentally uncoupled situation. Our chosen parameter values are $(a_1 = 1, b_1 = 1, c_1 = 0.5, A_1 = 0.3)$ in equation (1) and $(a_2 = 0.5, b_= 1, c_2 = 0.5, A_2 = 0.6)$ in equation (2). In Fig. 4.10 the time series

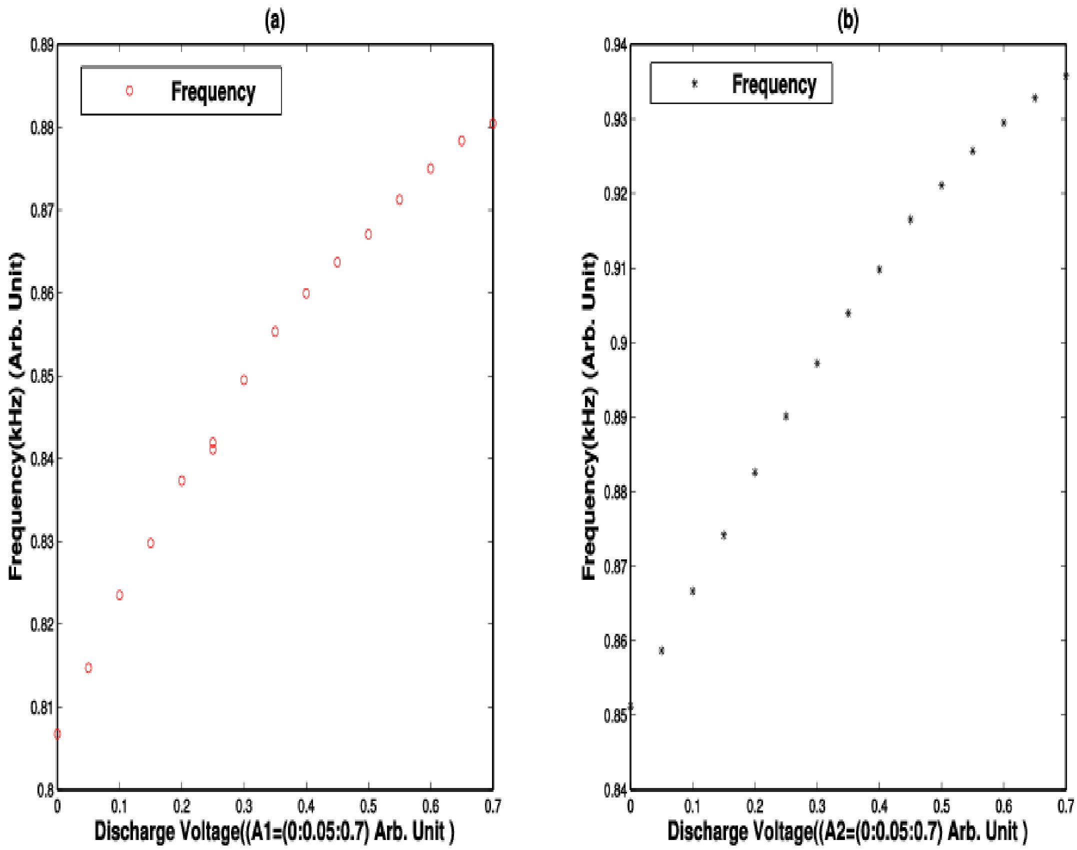

Figure 4.9: *Discharge voltage vs fundamental frequency plot:(a) In equation (1) parameter values of a_1, b_1, c_1 and N is fixed at 1, 1, 0.5 and 0 respectively and A_1 is varied as (0:0.05:0.7), (b) In equation (2) parameter values of a_2, b_2, c_2 and N is fixed at 0.5, 1, 0.5 and 0 respectively and A_2 is varied as (0:0.05:0.7)*

signals, Lissajous plots and corresponding power spectra are shown for different values of the coupling parameter N in equation (1) and (2). From this figure it can be seen that when the two equations are uncoupled (N=0) then the corresponding frequencies are different and there exists no correlation between their time series (see Fig. 10(a-c)). When they are coupled with a coupling strength of N=0.05, the frequencies of the two systems become the same i.e their frequencies are entrained to a value which is in between that of the frequencies of the uncoupled systems (see Fig. 10 (d-i)). As the value of the coupling strength is decreased to N=0.015, we see the formation of beat frequencies (see Fig. 4.10 (j-l)) indicating the onset of the frequency pulling state. In Fig. 4.11 the frequency bifurcation plot is shown with decreasing values of the coupling parameter N. These results closely resemble the experimental dynamical behaviour of the two coupled plasma discharges and lend credence to the utility of the model equations in understanding the dynamics of the coupled anode glow oscillations.

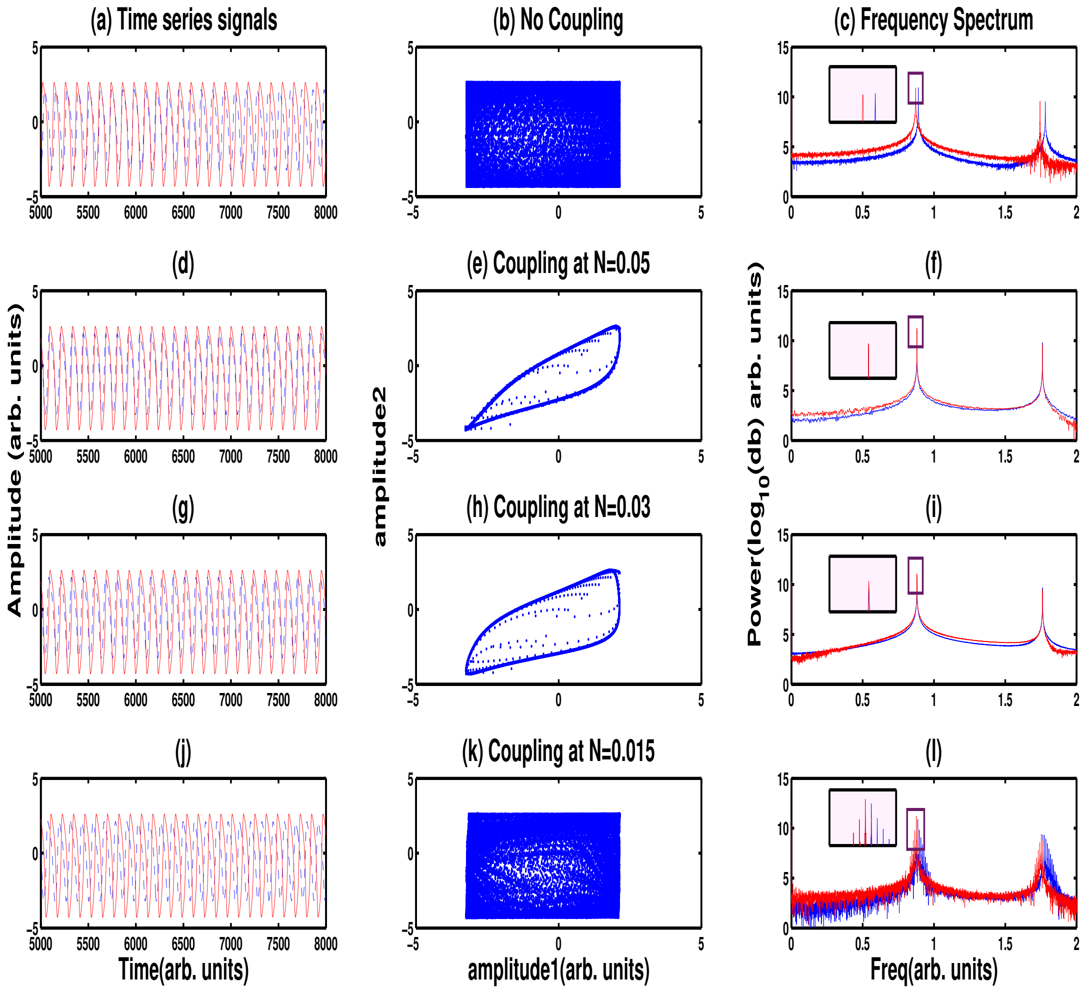

Figure 4.10: *Time series, Lissajous and power spectrum plots with changing coupling strength(N) between equation-1 and equation-2:- fig.(a, b, c) No Coupling(N=0), fig.(d, e, f) N=0.05, fig.(g, h, i) N=0.03, fig.(j, k, l) N=0.015.*

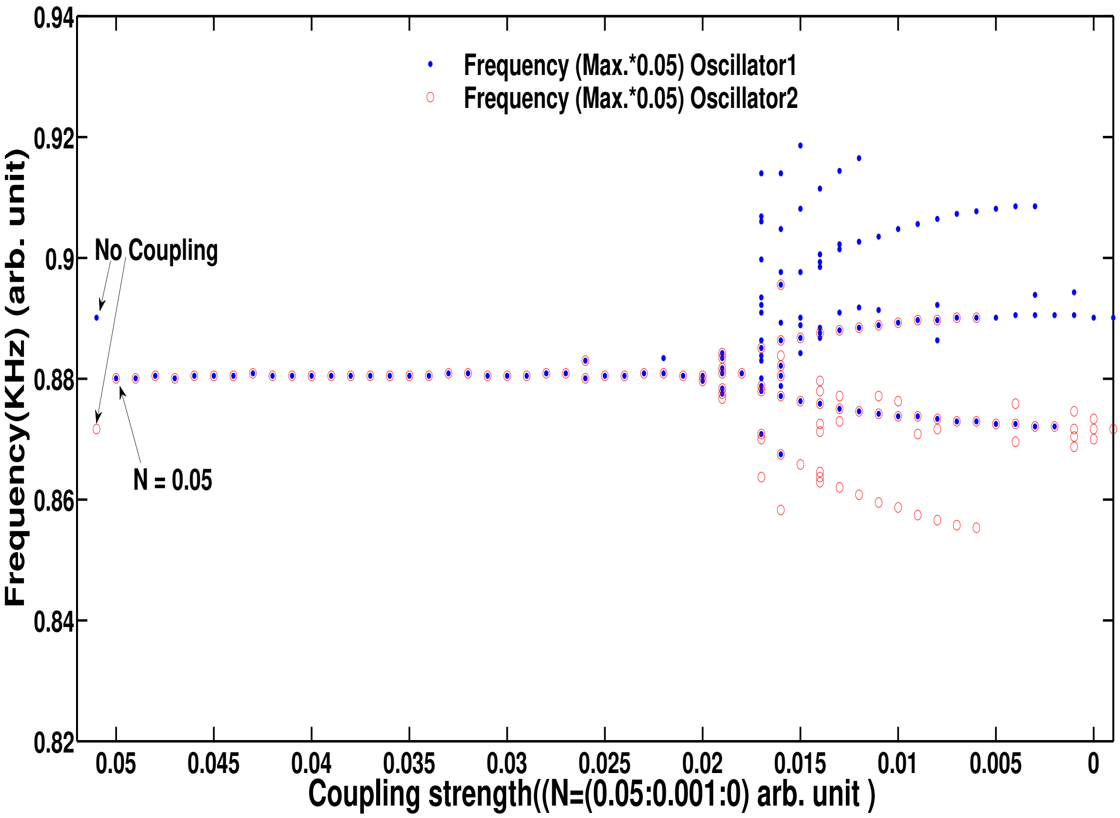

Figure 4.11: *Frequency bifurcation plot of equation (1) and equation (2) with decreasing value of coupling parameter (N=0.05:0.001:0).*

4.5 Discussion and Conclusions

In this chapter we have investigated the coupled dynamics of two glow discharge plasma devices with a particular focus on synchronization phenomenon. The basic oscillation frequencies we have studied are those corresponding to anode glow oscillations. Characterization studies of the individual plasma devices show that the frequency of these oscillations increases with increase in the discharge voltage. These oscillations are probably due to ion acoustic instabilities or ionization instabilities, and since we satisfy the conditions for triggering these instabilities, the increase in the frequencies also may be due to a combination of both temperature and density increase with discharge voltage. The primary cause for this increase can be attributed to an increase in the electron temperature resulting from a higher voltage and the direct impact this higher temperature has on the ionization rate contributing to the anode glow oscillation [15, 160, 161]. To model this behaviour of the anode glow oscillations, we have adopted an asymmetric Van der Pol type equation with a DC source term that represents the bias voltage. Numerical solutions of this equation for different values of the DC bias term display a similar rise in the oscillation frequency as a function of the DC parameter. The presence of the asymmetry term was found to be quite important. If the asymmetry term is absent in the model equations (4.1) and (4.2) then the frequency spectrum of the numerical solutions have only odd multiples of frequencies of the fundamental and therefore do not correctly model the experimental situation where both odd and even multiples of the basic frequencies are seen to exist.

Our synchronization experiments trace the changes in the oscillation frequencies of the two plasma devices as a function of the coupling strength. When the coupling strength is high then a frequency entrained state is observed with the frequencies of the two devices synchronized to a common frequency that is intermediate in value to the individual frequencies of the uncoupled devices. As the coupling strength is decreased a bifurcation behaviour is found to occur below a threshold value resulting in a frequency pulling state that is characterized by the formation of beat frequencies. This behaviour is also well simulated by the model equations (4.1) and (4.2).

To conclude, our experiments provide a clear demonstration of the phenomena of frequency synchronization and frequency pulling of anode glow oscillations in plasma glow discharge devices that can form the basis of future dynamical system experiments in plasma systems such as the exploration of chaos or amplitude death phenomenon. The model equations developed in support of our present experiments can help in identifying parametric domains for such explorations.

5

Phase-flip transition via indirect coupling

In this chapter, a detailed experimental study of two glow discharge plasma sources that are indirectly coupled is presented. Novel results pertaining to phase-flip transitions and hysteresis are highlighted.

5.1 Introduction

In nature, apart from direct coupling, there are numerous examples of oscillators which show synchronization without any direct interaction [28]. In such synchronization, oscillators can establish an indirect form of coupling between them by interacting through a common medium in which they are placed or via some dynamic environment etc. Such a type of coupling mechanism is also known as bath coupling.

In synchronization, an interesting collective phenomenon that has attracted some recent attention is that of a phase-flip (PF) transition. In this collective phenomenon a fully synchronized state of oscillators abruptly changes its relative phase e.g. a system of two coupled oscillators switches from an in-phase state to a π out-of-phase state [5, 19–25]. A striking example of a PF transition is the spontaneous change of gait in quadruped mammals e.g. when they switch from a trot to a galloping gait [162,163]. PF transitions have been induced in some simple experimental systems such as coupled electro-chemical cells [21] or coupled non-linear electronic circuits [5] by introducing time delay in the coupling mechanism. More recently two theoretical studies have demonstrated that a PF transition can also be induced by a relay coupling [24] or a form of environmental coupling [25].

To the best of our knowledge, a PF transition has never been observed in a more complex medium like a plasma system although a number of studies have observed frequency synchronization in coupled or externally driven plasma sources [14–16,18,80,83,84,87,88,90,91,95,99,138,147,164,165]. In this chapter we report the first experimental observation of a PF transition in the frequency synchronized state of two coupled DC glow discharge plasma sources. These sources are coupled in a non-invasive fashion that relies on an inductive mechanism of communication between the two systems and which is responsible for the PF transition without the need for any time delay in the coupling. We also observe a hysteresis in the onset of the transition as we scan the synchronization region along the increasing and decreasing directions of the frequency of one of the sources. The essential features of our observations are well reproduced by a mathematical model of the system consisting of two Van der Pol oscillators coupled through a common dynamical medium. Our results can be useful in understanding the collective behavior of plasmas subjected to external radio frequency waves and can also provide a convenient means of controlling fluctuations of such plasmas by using the non-invasive technique demonstrated in our experiment.

The present chapter has been organized as follows. In Section 5.2, we have described the experimental setup which is used to study the indirect coupling phenomena and investigated the existence of a phase-flip transition in the frequency synchronized regime of two indirect coupled glow discharge plasma sources. Numerical results are described in Section 5.3. Finally, Section 5.4 contains some discussion and concluding remarks.

5.2 Experimental Setup

Our experimental setup, consisting of two coupled glow discharge plasma sources, is schematically shown in Fig. 5.1. Each discharge system, built from a glass chamber of diameter 15 cm and length 35 cms, contains an anode of diameter 0.2 cms and a cathode of diameter 7 cms. The distance between the cathode and anode is set at 15 cm and a glow discharge Argon plasma is created by applying a DC voltage between the anode and cathode.

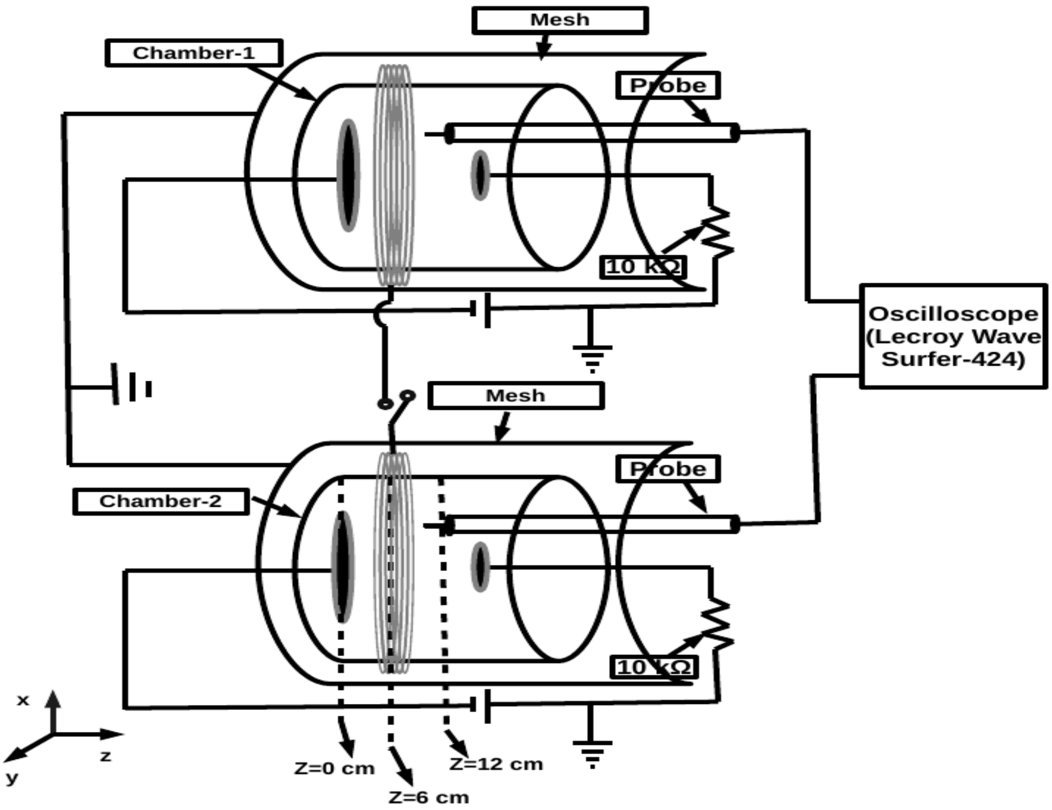

Figure 5.1: *Experimental setup for two coupled plasma sources.*

The two systems are operated independently and are isolated from each other by enclosing them individually with a steel mesh and giving a proper grounding. The systems are then coupled in an inductive manner by a length of copper wire (of diameter 2 mm) which is closely wound ten times around each glass chamber and whose two ends are shorted. The coupling can be broken by a mechanical on/off switch that breaks the wire continuity. The position of the closely wound bunch of wires can be moved by sliding it along the surface of the glass tube. This form of coupling is totally non-invasive as it does not involve the use of any physical probes or density sources within the plasma and acts solely through the external wire inductively picking up electromagnetic signals of the plasma and in turn influencing the signals by its own electromagnetic activity.

In each of the chamber a DC glow discharge plasma is typically formed at an operating pressure of 0.1 mbar and at a discharge voltage above 320 V. The fluctuations of the plasma are characterized through direct measurements of the floating potential oscillations of the two Langmuir probes using a channel-isolated digital storage oscilloscope and then analyzed using a MATLAB data analysis software package. As a precaution against stray electrostatic pickups in the probe we have checked that when there is no plasma in the second chamber then the Langmuir probe in that chamber does not show any oscillations. As has been previously reported in many studies [15,26,147,160,161] these potential fluctuations are due to anode glow oscillations that arise because of dissimilar anode-cathode areas. The frequency of these fluctuations are a function of the discharge voltage and can be changed by changing the discharge voltage.

For our synchronization experiment we keep the discharge voltage of one of the chambers fixed and vary the voltage of the other in a systematic fashion and at every step record the floating potential oscillations of the two Langmuir probes. Typically, the discharge voltage of chamber-1 is fixed at 385 Volts such that the fundamental frequency of the floating potential oscillation of this chamber is around 130 kHz and the discharge voltage of chamber-2 is increased from 325 Volt to 350 Volt in steps of 1 Volt. Consequently the fundamental oscillation frequency of this chamber rises from 120 kHz to about 137 kHz. After this the process is reversed by decreasing the voltage of chamber-2 in steps of 1 volt. We observe several interesting features in our experiment. Initially due to different discharge voltages the frequencies of the two sources are quite disparate and gives rise to side band creations - a regime that can be described as a frequency pulling region. As the discharge voltage of chamber-2 (DV-2) is increased its fundamental frequency keeps increasing and approaching that of chamber-1. When the frequency difference between the two sources falls below a threshold value the two sources synchronize to a common frequency and the side band production dramatically ceases. The oscillations of the two sources are then found to be in an in-phase synchronized state. They continue to remain synchronized in this state till at a certain value of the voltage there occurs an abrupt or discontinuous change in the synchronized frequency taking it to a lower value. The synchronized state of this lower frequency is found to be a π out-of-phase state. This discontinuous change marks a phase-flip transition. The out-of-phase synchronization continues till a certain value of the voltage beyond which one again enters a frequency pulling regime. When one reverses the process from this point by decreasing the DV-2 one first observes the onset of an out-of-phase state followed by an in-phase state. However this transition now takes place at different values of the voltage indicating a hysteresis effect.

We display our experimental results in Figs. 5.2 and 5.3 for observations carried out for two different positions of the bound copper coil on chamber-2, namely, Fig. 5.2 right on top of the cathode region and Fig. 5.3 when it is 6 cms away from the cathode region. In Fig. 5.2 we have only plotted the fundamental frequencies(FF) of the two sources to provide an uncluttered picture of the synchronization process.

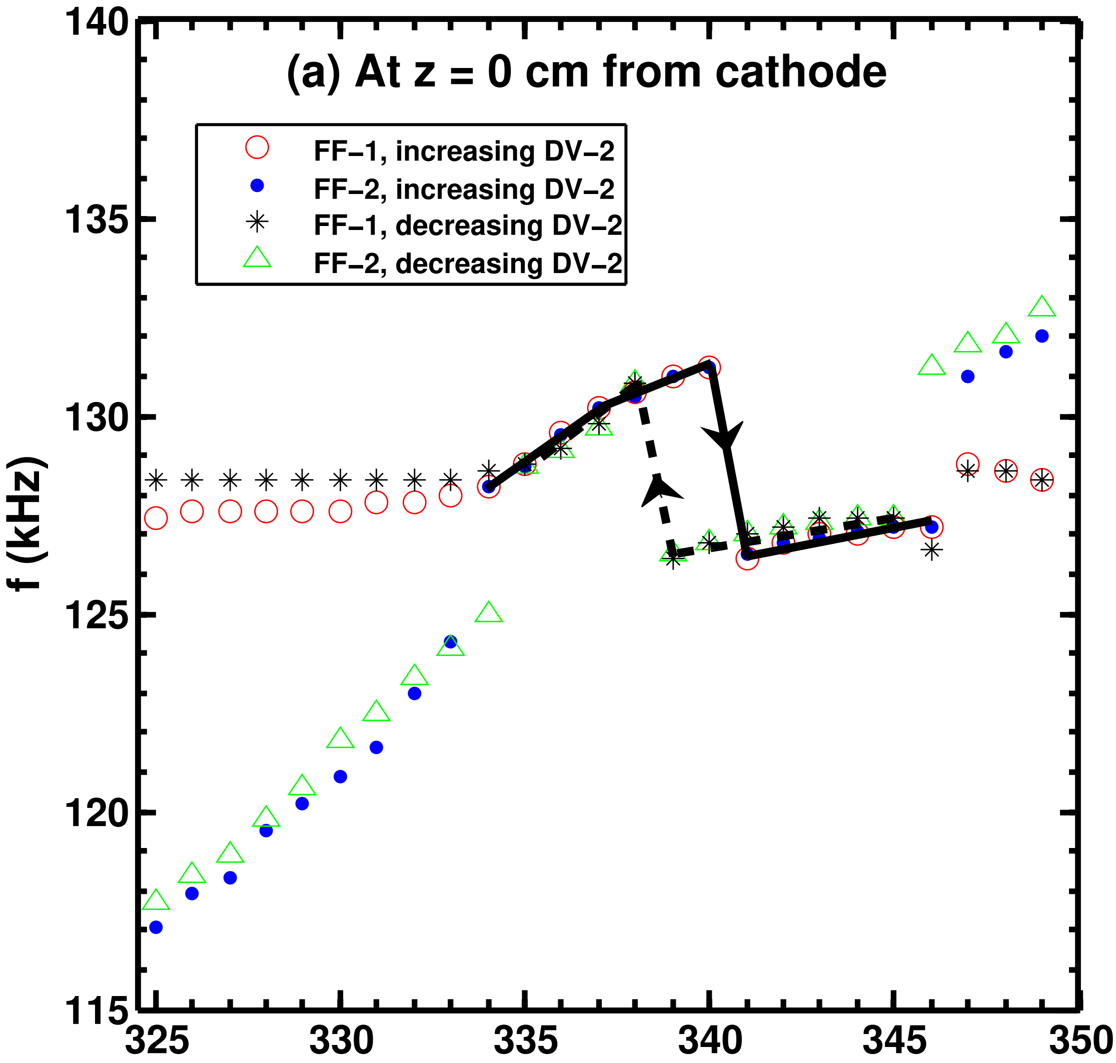

Figure 5.2: *Experimental observations of frequency pulling, frequency synchronization, phase flip bifurcations and hysteresis phenomenon between two inductively coupled plasma sources. Here, the fundamental frequencies of the two sources have been plotted.*

The frequency pulling region in both the figures resembles that shown in Fig. 5.3 where all the sideband frequencies(f) have been plotted to the left and right of the synchronization region.

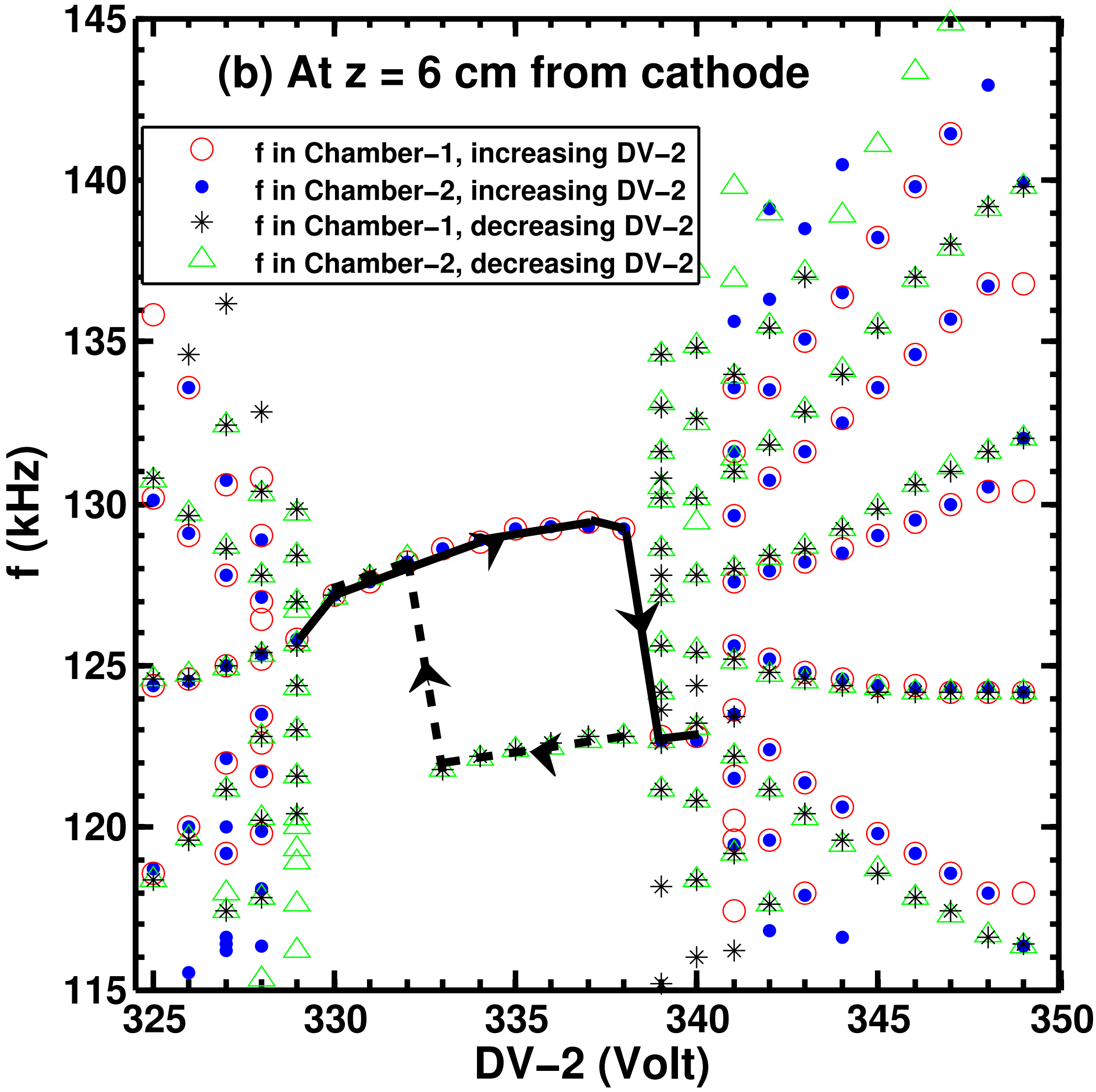

Figure 5.3: *Here, the synchronization region is enlarged due to stronger coupling between the sources when the inductive coil is placed at 6 cms from the cathode and is hence closer to the plasma column.*

When the coupling coil bunch on chamber-2 is placed 6 cms away from the cathode the coil is close to the plasma column and hence senses the fluctuations more intensely leading to a stronger coupling between the two sources.

This results in a broadening of the synchronization region and correspondingly the hysteresis region as can be clearly seen in Fig. 5.3 when compared to Fig. 5.2. It should be mentioned that the origin of this hysteresis is not related to the hysteresis seen in the voltage-current (V-I)characteristics of glow discharge plasmas that is typically observed for certain operating regimes [166–169]. We have ascertained experimentally that our sources do not operate in that regime and we do not see any hysteresis in the V-I characteristics. The hysteresis in our experiment is solely associated with the frequency synchronization phenomenon, a novel effect that has never been observed before. The abrupt nature of the phase-flip transition is illustrated in Figs. 5.4 and 5.5 where we display the oscillation time series, the power spectra and the Lissajous plots of the signals from chamber-1 plotted against that of chamber-2 for the increasing and decreasing discharge voltages of chamber-2 plasma system(DV-2) corresponding to Fig. 5.2.

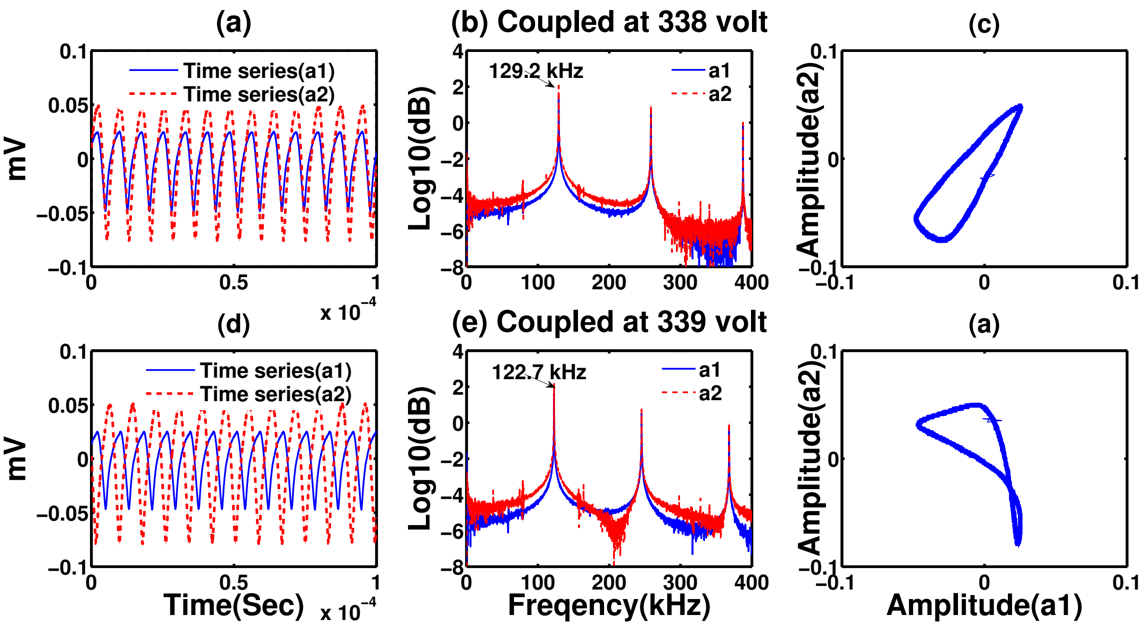

Figure 5.4: *Phase flip transition from an in-phase state to an anti-phase state occurring within one volt increase in the voltage of chamber-2.*

As can be seen the phase-flip transition takes place with just one step increase or decrease in the DV-2.

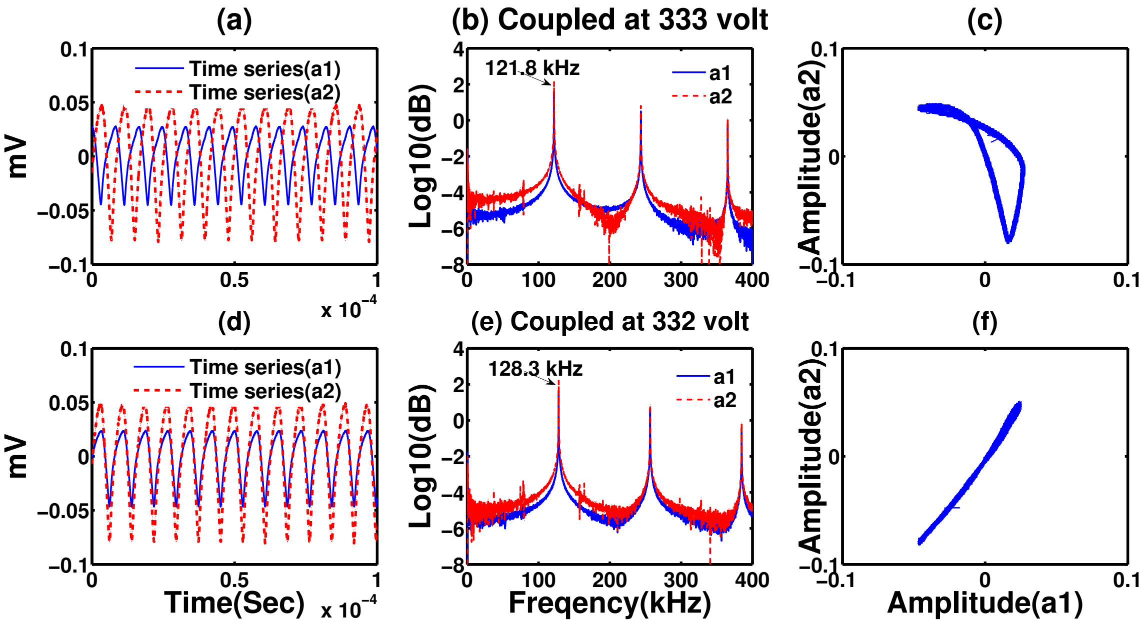

Figure 5.5: *Phase flip transition from an anti-phase state to an in-phase state occurring within one volt decrease in the voltage of chamber-2.*

5.3 Numerical Modeling Results

We next discuss the dynamical origin of the phase-flip transition observed in our experiment. As stated earlier, past investigations of such transitions have identified two principal mechanisms for such transitions, namely, due to the presence of time delayed coupling or under the presence of an 'environmental' coupling in which the two systems are not directly coupled but influence each other by interacting with a common interactive medium. We believe that a form of the latter mechanism is at work in our case and is due to the nature of the non-invasive coupling that we have adopted. The induced fluctuating currents in the copper coil arise from the fluctuating fields in the two plasma columns that it senses and in turn these currents act back on the fluctuating fields of the plasma sources. Thus the coil acts as a dynamical medium that forms a bath through which the two sources interact with each other.

To understand the nonlinear dynamics of this coupled system we consider a mathematical model consisting of two Van der Pol equations coupled via a bath

[124]. A single Van der Pol model has been frequently used in the past to model the oscillatory behaviour of glow discharge plasmas and is capable of providing a realistic description of synchronization phenomena occurring when it is driven by an external oscillating source [99]. Following Ref. [124] we choose the model equations in the form,

$$\ddot{x} - \epsilon_1(1 - x^2)\dot{x} + x = M(z - y) \tag{5.1}$$

$$\ddot{y} - \epsilon_2(1 - y^2)\dot{y} + (0.7 + S)y = M(z - x) \tag{5.2}$$

$$\dot{z} = M(z - x) + M(z - y) \tag{5.3}$$

where x and y represent the amplitudes of the fluctuations in systems 1 and 2 respectively. The variable z represents the amplitude of the fluctuations in the bath (representing the oscillating signals in the copper wire in our case) and its dynamics is governed by the inputs from both the systems. M is a measure of the coupling strength and ϵ_1 and ϵ_2 are the nonlinear coefficients governing the dynamics of the two systems. To depict the experimental situation, we have taken the intrinsic frequencies of the two systems to be different (namely 1 and 0.7 respectively) and introduced the parameter S to vary the frequency of the second system. We have solved equations (1-3) numerically by setting $\epsilon_1 = 0.2$, $\epsilon_2 = 0.1$ and increasing or decreasing the parameter S from 0 to 0.6 in steps of 0.01 for two different values of M, namely, 0.06 and 0.1. The results are shown in Fig. 5.6 where we clearly see the occurrence of phase flip transitions as well as the existence of hysteresis.

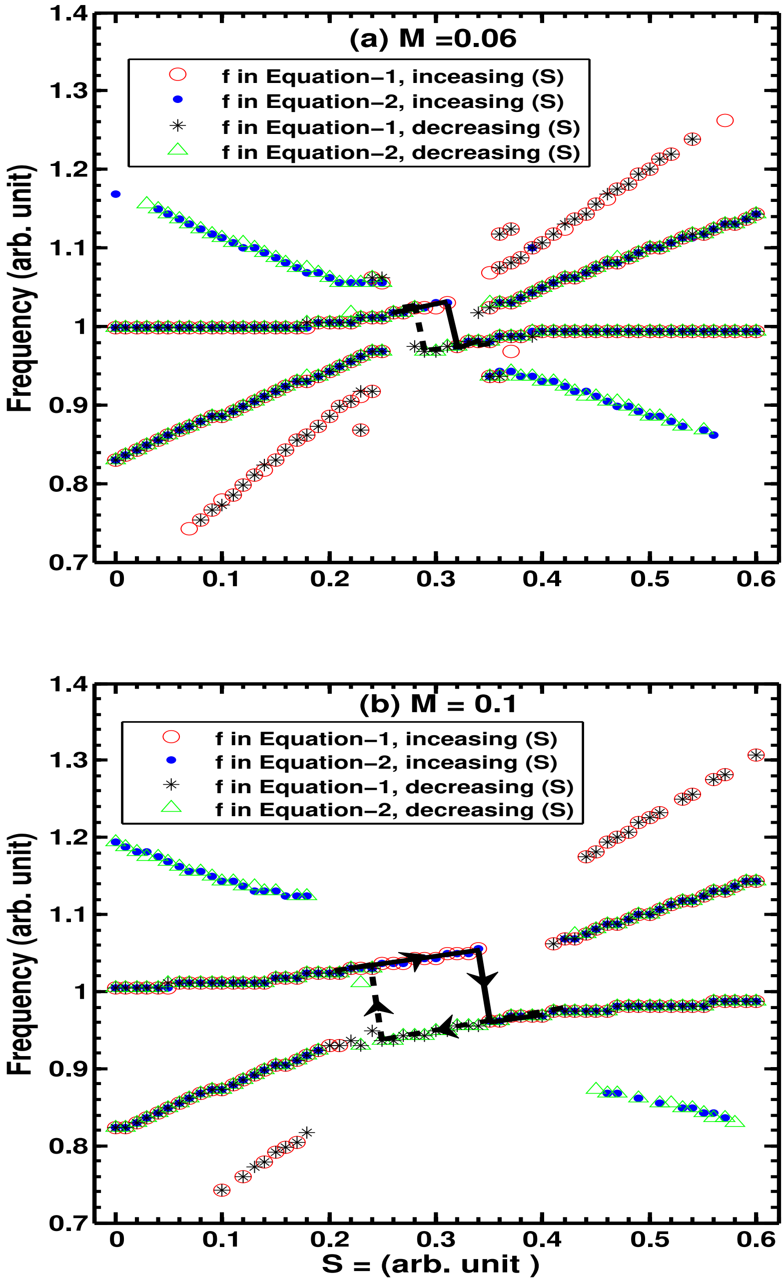

Figure 5.6: *Numerical observations of frequency pulling, frequency synchronization, phase flip bifurcations and hysteresis phenomenon between two bath coupled Van der Pol equations. Coupling parameters are : (a) M = 0.06 and (b) M = 0.1*

It should be mentioned that the hysteresis behaviour taking place in this model system is associated with changes in initial conditions as pointed out in [25] and our numerical results are obtained by initiating a random change in the initial condition as we reverse the change in S. Such a change in initial conditions would naturally occur in the plasma environment and hence the model system provides a realistic description of the synchronization process. We also note that the region of hysteresis is enlarged in the model results when the coupling strength is enhanced in agreement with experimental observations.

5.4 Summary and Conclusions

In conclusion, we have investigated the synchronization dynamics of two coupled DC glow discharge plasma sources and shown for the first time the existence of a phase-flip transition in the frequency synchronized regime of such a system. The PF transition is associated with a discontinuous jump in the synchronized frequency of the two sources and does not require the presence of any time delay in the coupling mechanism. The key feature responsible for the PF is the inductive nature of the coupling that acts as a sort of a common interactive medium through which the two sources communicate with each other. Another novel result observed in our experiments is the presence of hysteresis in the onset of the PF transition as we scan the synchronized regime in the increasing or decreasing directions of the dc voltage of one of the sources. Unlike past results, the observed hysteresis is not due to any hysteresis in the voltage-current characteristics of the source but is solely associated with the frequency synchronization mechanism.

Our experimental results are well supported by a theoretical model that represents the dynamics of the individual plasma sources by Van der Pol oscillators and mimics the inductive wire by a bath coupling. The numerical results from the model capture all the essential characteristics of the experiment including frequency pulling, frequency synchronization, phase-flip transitions and hysteresis. Finally we would like to point out that the successful implementation of an inductive coupling mechanism for effecting synchronization between two plasma sources opens up novel possibilities for non-invasive control of plasma fluctuations and instabilities that can have wider applications in industrial or larger scale laboratory setups.

6

Mutual harmonic synchronization via direct coupling

In chapters 4 and 5, we have established that mutual synchronization between the oscillations of two coupled plasma sources can be achieved via direct or indirect coupling between the sources. The synchronization happens through a interaction between the fundamental frequencies of the two systems. In this chapter, we report on mutual harmonic synchronization between two plasma sources. In other words the interaction of the oscillations takes place at higher harmonic frequencies.

6.1 Introduction

In a plasma, higher harmonic synchronization has been studied in various systems like arc discharges [14, 80], glow discharges [83, 90], magnetized plasma discharges [88], thermionic plasma discharges [87], double plasma devices [15], triple plasma devices [138], high beta plasmas [81] and dusty plasmas [99, 165]. All these studies were done by synchronizing the plasma oscillations to an oscillation from an external frequency source such as a function generator. However, to the best of our knowledge there is no reported work where plasma signals themselves were used for such studies (mutual harmonic synchronization). But previously such studies have been done in other fields like lasers [170] and electrochemical oscillators [171].

We present experimental investigations of mutual harmonic synchronization between plasma oscillations of two glow discharge plasma sources with a variation in the oscillation frequencies in one of the sources. The oscillations in each device are due to anode glow instabilities [15,83,88,158,159] and these are of a self-sustained type [2,3]. We show that when oscillations frequencies in both the systems are near their harmonic frequencies then the frequency entrained state is observed and beyond these regions other nonlinear states like frequency pulling, chaos etc. are observed. For a better understanding of the experimental results we have also done numerical simulation by using two mutually coupled asymmetric Van der Pol type equations [143,144,148,149].

The chapter is organized as follows. In Section 6.2 the experimental setup and procedures followed are given. Section 6.3 contains experimental results of harmonic synchronization between two coupled plasma sources. In Section 6.4 we have provided numerical simulation results and finally, in section 6.5 a brief discussion and conclusions are presented.

6.2 Experimental Setup and Procedure

The schematic of the experimental setup for two glow discharge plasma sources is shown in Fig. 6.1. This setup is similar to that presented in chapter-5, with the only difference being that the coupling is provided via a conducting wire in place of a variable resistance [147].

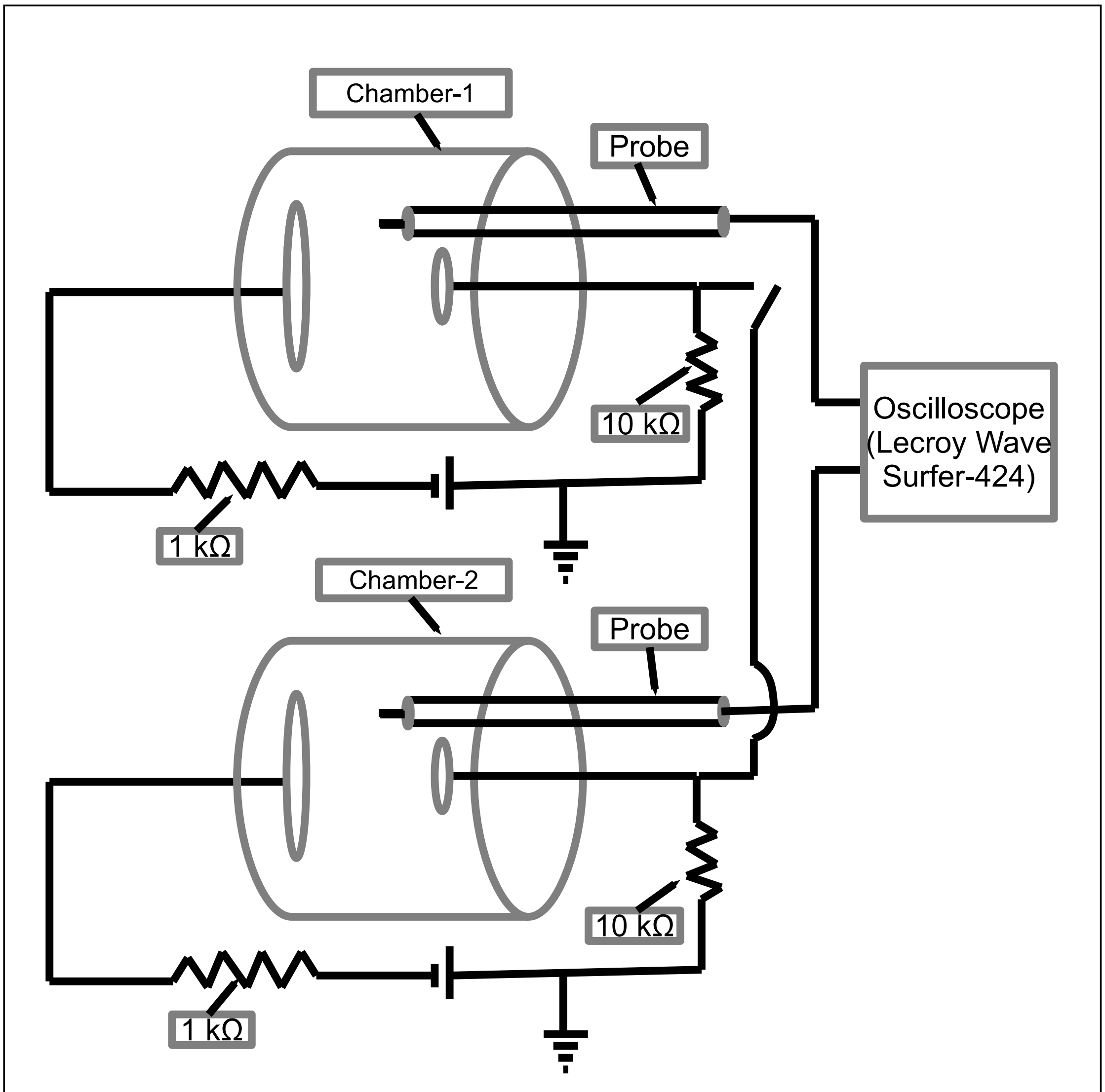

Figure 6.1: *Experimental setup for two coupled plasma sources.The pressures in chamber-1 and chamber-2 are 0.1 mbar and 0.1 mbar respectively.*

In both the discharge systems, a DC glow discharge plasma was formed by applying a potential difference to the electrodes and floating potential fluctuation frequencies were measured using cylindrical Langmuir probes. It was observed that with the increase in discharge voltage, floating potential fluctuation frequencies in the plasmas in both chambers increased linearly. As noted previously in [147]. In the present experiment, the discharge voltage in one of the plasma systems was kept fixed while the discharge voltage (i.e oscillation frequencies) of the other system was varied by connecting the anodes of the two chambers as shown in Fig. 6.1.

6.3 Experimental Results: Mutual Harmonic Synchronization

6.3.1 Frequency Dynamics of the Uncoupled Systems

In chapter-4 [147] and chapter-5 [157], it was shown that the anode glow oscillation [15,26,160,161] frequencies which arose due to the instabilities formed around the anode were increasing linearly with an increase in the discharge voltage. A similar kind of observations was seen in the present set of experiment but with over a much greater range of frequencies compared to chapter-5 [147] and chapter-6 [157]. These results are shown in Fig. 6.2 (a and b) for the chamber-1 and chamber-2 plasma systems respectively. For the synchronization experiment, chamber-2 plasma oscillations are chosen as a driver system while chamber-1 plasma oscillations as a driven system.

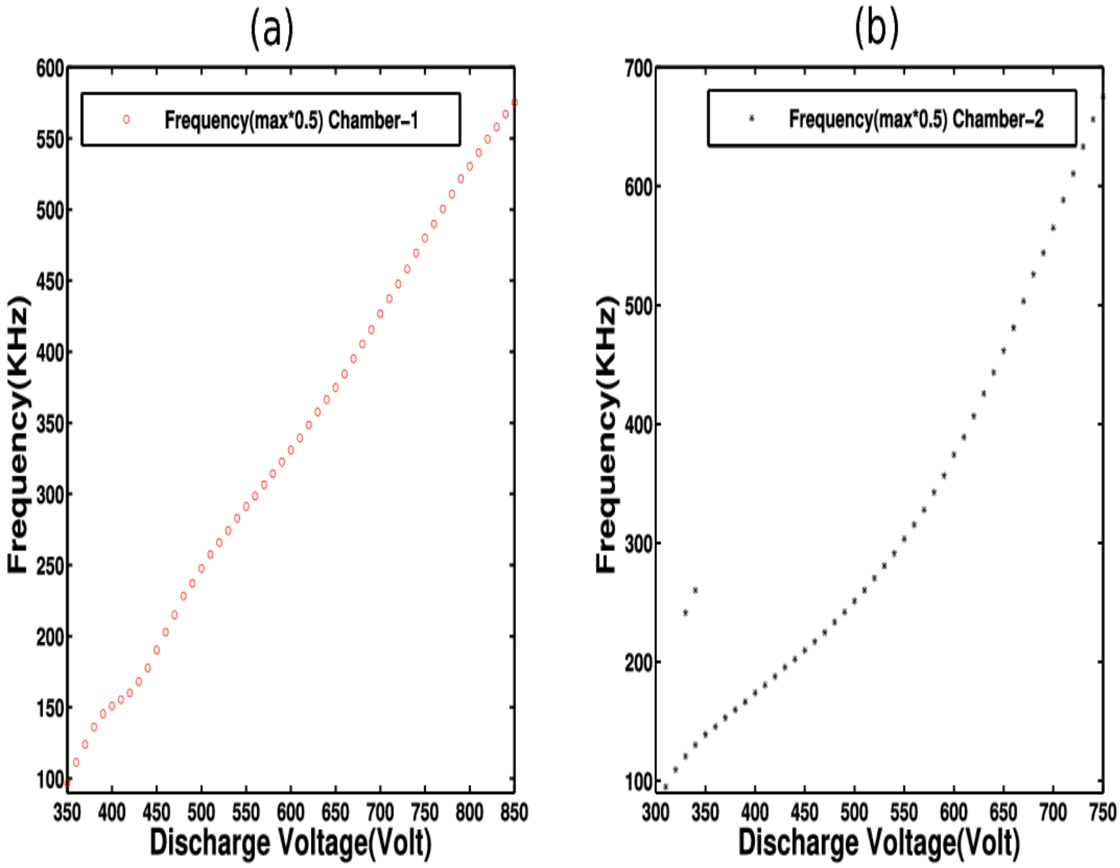

Figure 6.2: *Fundamental frequencies of chamber-1(left) and chamber-2(right) are shown with increasing discharge voltages.*

6.3.2 Dynamics of Mutual Harmonic Synchronization

We now discuss the dynamics of mutual harmonic synchronization between two coupled DC glow discharge plasma sources. For this experiment, the discharge voltage in chamber-1 is fixed at 470 volt such that oscillation frequencies in this chamber were around 203 kHz and discharge voltage of chamber-2 is varied from 310 volt to 800 volt such that oscillation frequencies in this chamber's plasma are increased from 90 kHz to 700 kHz. When the anodes of the two chambers are connected, different synchronized states were observed depending upon the ratio of the oscillation frequencies in chamber-1 and chamber-2 plasmas.

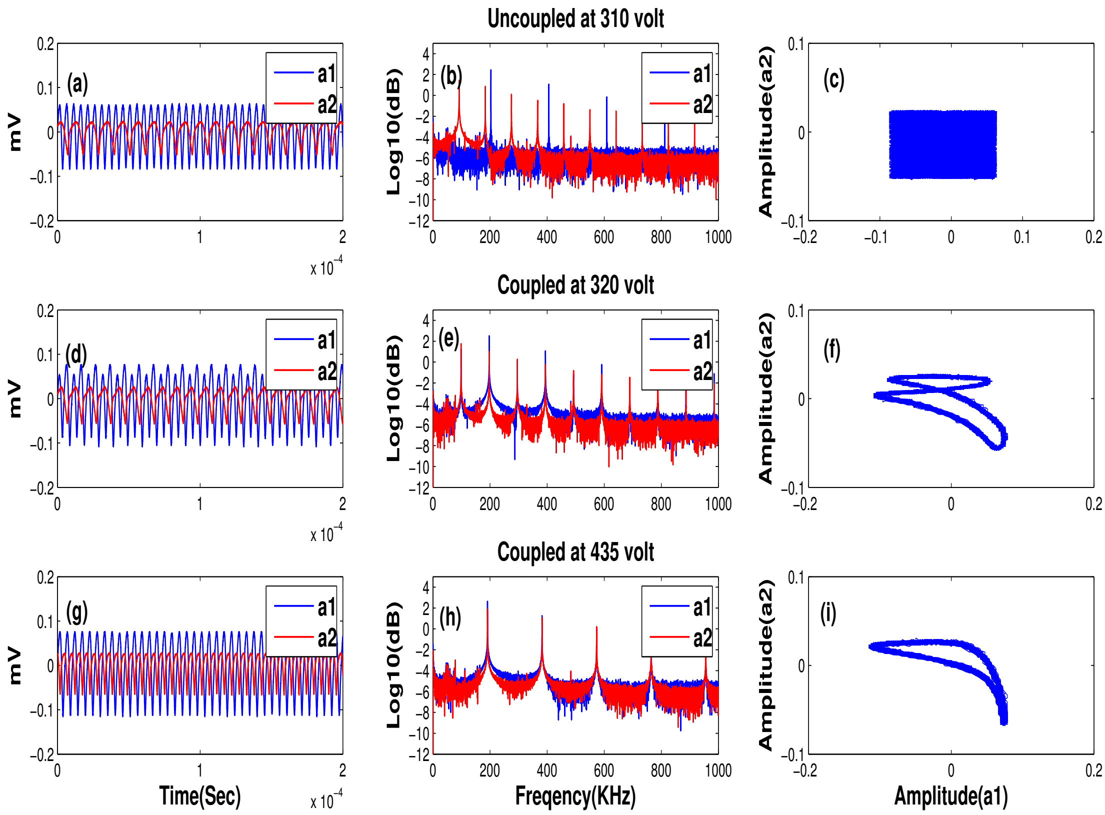

Figure 6.3: *In these figures a1 and a2 denotes the signals from chamber-1 and chamber-2 plasmas respectively. In figures (a,b,c) time series, power spectrum and Lissajous plots are shown for chamber-1(at 470 Volt) and chamber-2(at 310 volt) plasmas when anodes of two chambers were not connected. In figures (d,e,f) and (g,h,i) time series, power spectrum and Lissajous plots are shown when anodes of two chambers were connected and discharge voltage of chamber-2 plasma was at 320 Volt and 435 Volt respectively.*

In Figs. 6.3 and 6.4, time series, power spectrum and Lissajous plots are shown for chamber-1 and chamber-2 plasma systems. In these results, the discharge voltage of chamber-1 is fixed at 470 V while discharge voltages of chamber-2 are varied. In Fig. 6.3 (a,b,c), time series, power spectrum, and Lissajous plots are shown respectively for chamber-1 and chamber-2 plasma system in uncoupled configuration, with a discharge voltage of 310 V for chamber-2 plasma system. From Fig.6.3 (b), it can be seen that fundamental oscillation frequencies in chamber-1 and chamber-2 plasmas are different and its values are 203 kHz and 91.6 kHz respectively.

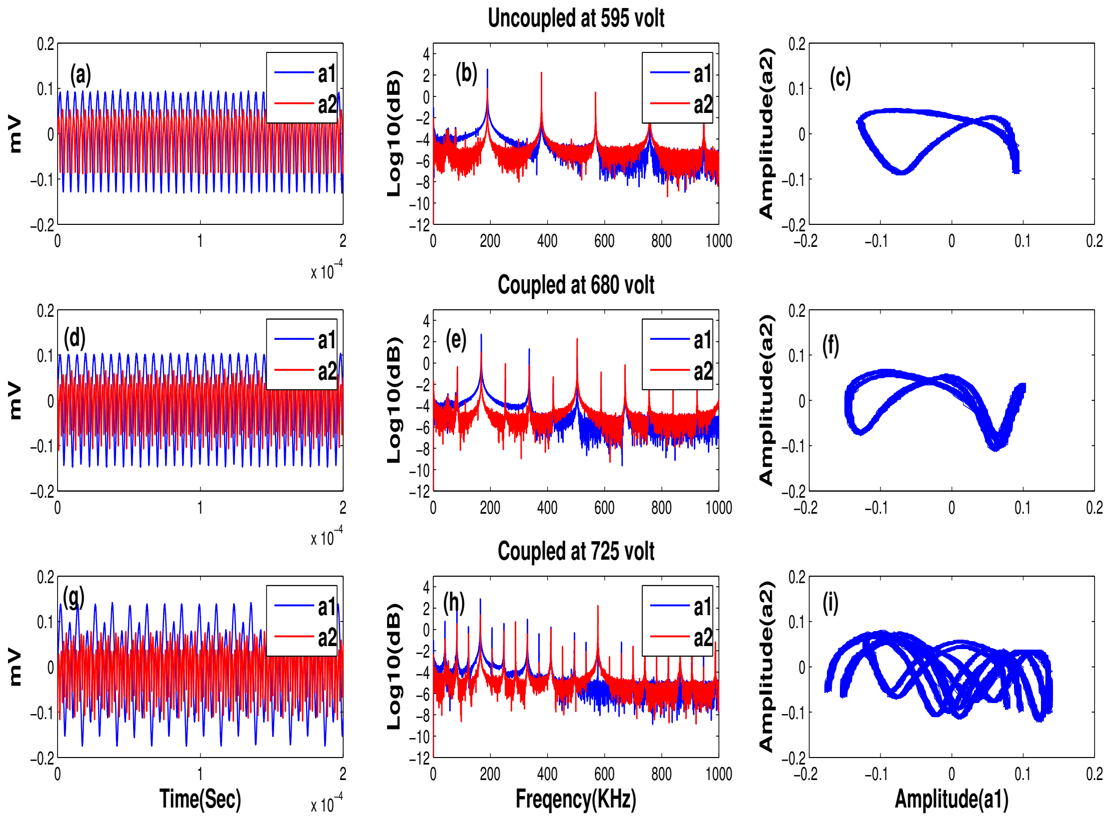

Figure 6.4: *In figures (a,b,c),(d,e,f) and (g,h,i) time series, power spectrum and Lissajous plots are shown for chamber-1 and chamber-2 plasmas when the anodes of the two chambers were connected and the discharge voltages of chamber-2 were at 595, 680 and 725 Volt respectively*

In Figs. 6.3 (d,e,f), 6.3 (g,h,i), 6.4 (a,b,c), 6.4 (d,e,f), and 6.4 (g,h,i), time series, power spectrum and Lissajous plots are shown for chamber-1 and chamber-2 plasmas, when anodes of the two chambers are connected and the discharge voltages in chamber-2 plasma are at 320, 435, 595, 680 and 725 Volts respectively. In Fig. 6.3 (d,e,f), subharmonic synchronization(2:1) results are shown between chamber-1 and chamber-2 plasma oscillations. Here, fundamental oscillation frequency in chamber-1 and chamber-2 plasmas are measured as 197 kHz and 98.5 kHz respectively which are in the ratio of 2:1. This can also be realized from the Lissajous plot in Fig. 6.3 (f), that an eight shape is formed which is a signature of two locked frequencies. In Fig. 6.3 (g,h,i), fundamental frequency synchronization(1:1) are shown between chamber-1 and chamber-2 plasma oscillations. For this case, the fundamental oscillation frequency in both the systems is measured as 191.1 kHz. Similarly, in Figs. 6.4 (a,b,c), 6.4 (d,e,f)and 6.4 (g,h,i), super harmonic synchronization results $1:2$, $1:3$ and $1:4$ are shown respectively. In Figs. 6.4 (b), 6.4 (e) and 6.4 (h) fundamental frequency of chamber-1 and chamber-2 plasma oscillations are measured as (167.7 kHz, 335.4 kHz), (167.9 kHz, 503.8 kHz) and (164.6 kHz, 576.2 kHz) respectively which are in the ratio of $1:2$, $1:3$, and $1:4$ respectively.

6.3.3 Devil's Staircase and Frequency Bifurcation Plots

The Devil's staircase [2, 3, 99] is a way of representation to show the harmonic synchronization phenomenon. The flat portion of such plots describes the synchronization region and type of harmonic synchronization while region of increase shows the unsynchronized part. In Fig. 6.5, this plot is shown for describing mutual harmonic synchronization phenomenon between two coupled plasma sources. This plot is drawn by plotting the ratio of maximum peak frequency of chamber-2 and chamber-1 plasma oscillations(known as the winding number) on the Y-axis and the maximum peak frequency of chamber-2 plasma oscillations on the horizontal X-axis. From this plot, it can be seen that with the increase in the driving frequency of chamber-2 plasma oscillations, different flat portions emerged, corresponding to different harmonic frequencies of chamber-1 plasma. This figure clearly gives a picture of different harmonic synchronization regions between two coupled plasma sources.

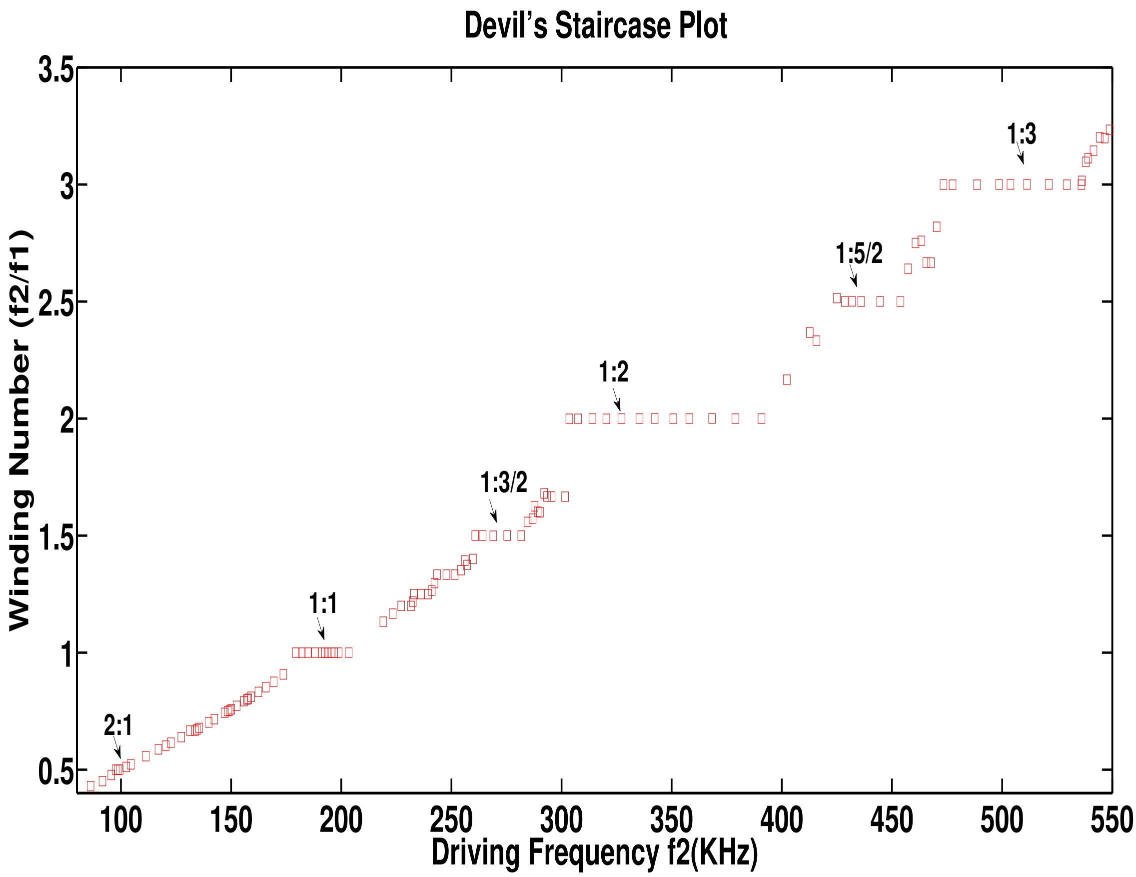

Figure 6.5: *In this figure, we have plotted the winding number (ratio of driver frequency (f2) of chamber-2 and driven frequency (f1) of chamber-1) vs driver frequency (f2) of chamber-2 plasma. This plot is known as Devil's staircase plot which is used for quantifying the range in which synchronization occur.*

For explaining the complete dynamics of the driven (chamber-1) and driver (chamber-2) systems, in Figs. 6.6 (a and b), frequency bifurcation plots are shown for chamber-1 and chamber-2 plasma oscillations with the change in the discharge voltages of chamber-2 plasma system. These figures clearly show the regions in which the oscillation frequencies of chamber-1 and chamber-2 plasmas are frequency entrained and frequency pulled with each other. When the oscillations of chamber-2 plasmas were at harmonic multiples of chamber-1 plasma then frequency entrained states are observed as mentioned by (2:1), (1:1), (1:3/2), (1:2), (1:5/2), and (1:3) numbers in Fig. 6.6 (a and b) and outside these regions frequency pulling and chaos type states are observed. Also, these chaos type states are more prominent for higher harmonic synchronization which is a very interesting observation.

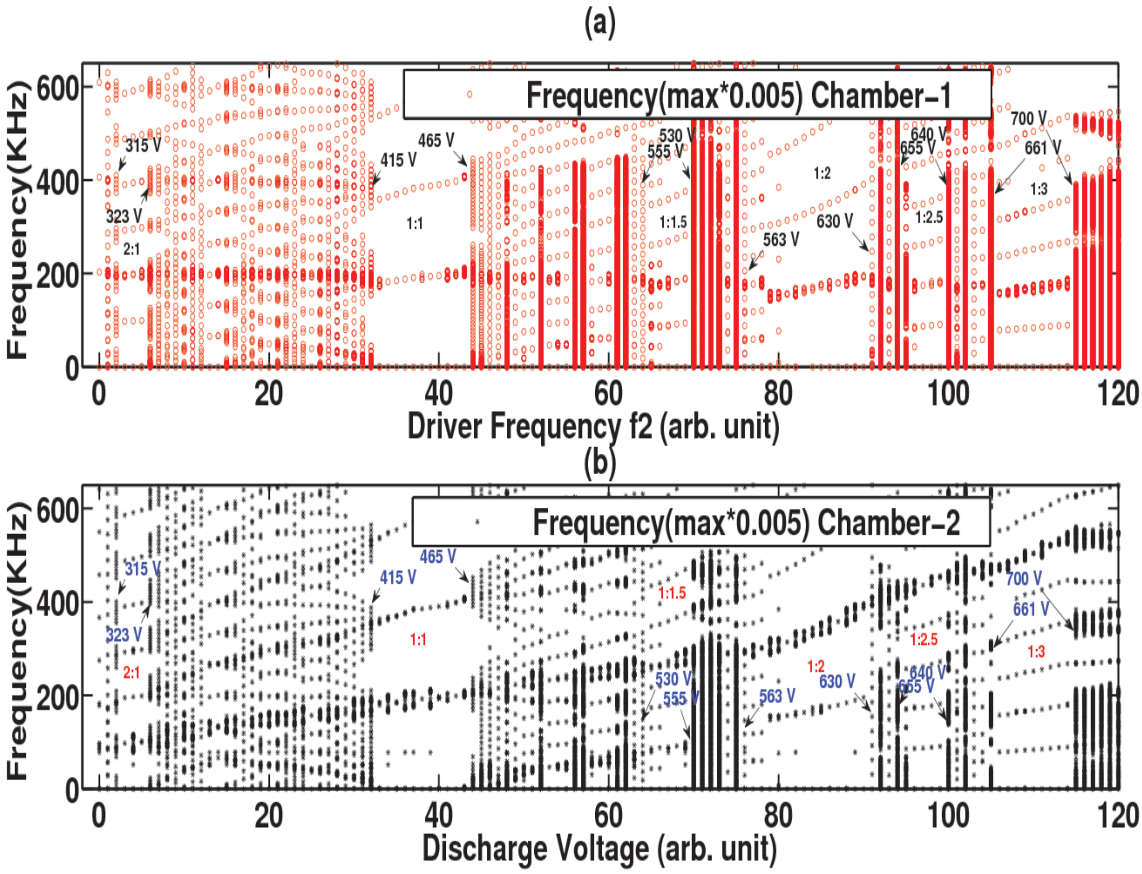

Figure 6.6: *In figures 6(a) and 6(b), we have shown frequency bifurcation plots of chamber-1 and chamber-2 plasma signals with the change in the discharge voltage of chamber-2 plasma respectively.*

6.4 Numerical Modeling

In chapter-5 [147], Van der Pol type equations [143, 144] are used for modelling the oscillation dynamics of two uncoupled as well as coupled glow discharge plasma sources. In this chapter, a similar type of equations are used for quantifying the experimental results of mutual harmonic synchronization between two coupled glow discharge plasma sources. These are shown in the equations (6.1) and (6.2) [147].

$$\ddot{x} - (a_1 - b_1 * x - c_1 * x^2)\dot{x} + x = N * (\dot{y} - \dot{x}) - A_1 \tag{6.1}$$

$$\ddot{y} - (a_2 - b_2 * y - c_2 * y^2)\dot{y} + y = N * (\dot{x} - \dot{y}) - A_2 \tag{6.2}$$

In chapter-5 [147], it was shown that with the increase in the DC parameter A_1 and A_2 of equations (6.1) and (6.2) respectively, the oscillation frequencies are increased linearly in certain limits. In this chapter too, the same concept is used for modeling the experimental results of mutual harmonic synchronization between two coupled plasma sources. Here, the parameter values are taken as $a_1 = 0.5, b_1 = 0.5, c_1 = 0.1, N = 0$ and $a_2 = 0.5, b_2 = 0.5, c_2 = 0.1, N = 0$ of equation (6.1) and (6.2) respectively. In Fig. 6.7, plot of the DC parameter values vs the fundamental frequency of the time series signals generated for the constant parameter $A_1 = 1$ and increasing parameter values of $A_2 = [0\colon\ 0.1\colon\ 2]$ is shown.

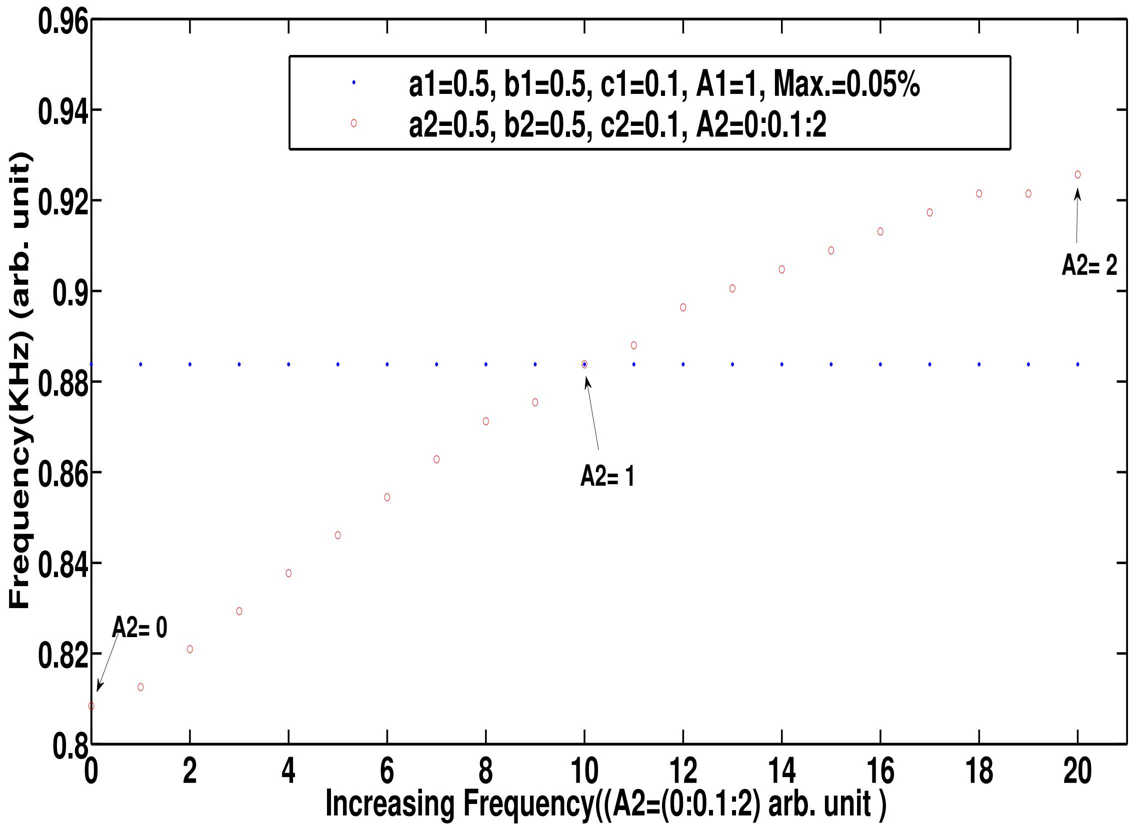

Figure 6.7: *Fundamental frequency plot for equation (6.1)(dot) and equation (6.2)(circle): In Eq.(1) parameter values of* a_1, b_1, c_1, N *and* A_1 *are fixed at 0.5, 0.5, 0.1, 0 and 1, respectively and in Eq.(2), parameter values of* a_2, b_2, c_2 *and* N *are fixed at 0.5, 0.5, 0.1 and 0, respectively and* A_2 *is varied as (0:0.1:2)*

From this figure, it can be seen that with the increase in the value of DC parameter A_2 the natural frequencies of the signals of the asymmetric Van der Pol oscillators is increasing while for a fixed value of A_1 =1 it remains constant. These equations are solved using a fourth order Runge-Kutta the method with a time step of δt=0.06 and the total number of data points are taken to be around 250000.

6.4.1 Mutual Harmonic Synchronization

In the experimental results, it is shown that when the harmonic frequencies of two coupled plasma systems are close to each other then frequency entrained states are observed and outside this region, frequency pulling and chaotic states are seen. Similar to these, in the numerical simulation results also, mutual harmonic synchronization is presented for first harmonic frequency regions for two coupled asymmetric Van der Pol type equations. For these, the parameters (a_1, b_1, c_1) of (6.1) and (a_2, b_2, c_2) of (6.2) are taken as $(0.5, 0.5, 0.1)$ and $(0.5, 0.5, 0.1)$ respectively and coupling parameter (N) is taken as 0.03 while coupling.

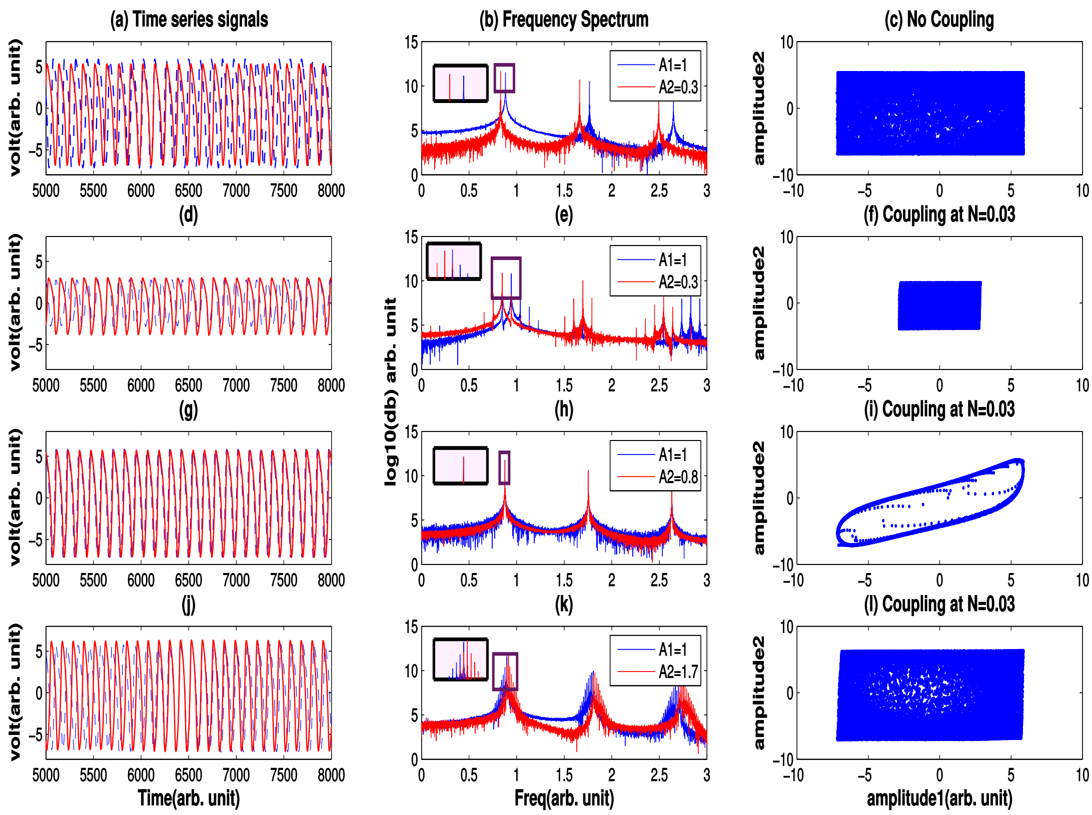

Figure 6.8: *(a-c) Time series, power spectrum and Lissajous plots for uncoupled $(N = 0)$ equations (6.1) and (6.2) when parameters $(A_1 = 1, A_2 = 0.3)$. (d-i) Time series, power spectrum and Lissajous plots for constant coupling $(N = 0.03)$ for equations (6.1) and (6.2): (d)-(f) parameters $(A_1 = 1, A_2 = 0.3)$, (g)-(i) parameters $(A_1 = 1, A_2 = 0.8)$, (j)-(k) parameters $(A_1 = 1, A_2 = 1.7)$*

In Fig. 6.8(a-l) time series, frequency spectrum and Lissajous plots are shown for equations (6.1) and (6.2) with a constant DC parameter ($A_1 = 1$) and a varied parameter ($A_2 = 0.3, 0.8, 1.7$). In Fig. 6.8 (a,b,c) plots are shown corresponding to coupling parameter ($N = 0$) and DC parameters ($A_1 = 1$) and ($A_2 = 0.3$) of equations (6.1) and (6.2). From these figures, it can be seen that when the two equations are uncoupled then oscillation frequencies corresponding to DC parameters ($A_1 = 1$) and ($A_2 = 0.3$) are different and there is no correlation between these oscillations. In Fig. 6.8 (d,e,f), plots for coupled equations (6.1) and (6.2) are shown corresponding to coupling factor ($N = 0.03$) and fixed DC parameters ($A_1 = 1$) and ($A_2 = 0.3$). From Fig. 6.8 (e), it can be seen that side band frequencies are forming around the harmonic frequencies of two coupled equations. In Fig. 6.8 (g,h,i), frequency entrainment results are shown corresponding to DC parameter values of ($A_1 = 1$) and ($A_2 = 0.8$) of equation (6.1) and (6.2) respectively and constant coupling parameter ($N = 0.03$) for both the equations. From Fig. 6.8 (h), it can be seen that though the DC parameter values in both the equations are different but for coupled case they are generating the same frequencies i.e oscillation frequencies of both the equations are entrained with each other.

Similarly in Fig. 6.8 (j,k,l), frequency pulling results are shown corresponding to ($A_1 = 1$) and ($A_2 = 1.7$) of equation (6.1) and (6.2) respectively and constant coupling parameter ($N = 0.03$). From Fig. 6.8 (k), side band frequencies can be clearly seen around the harmonic frequencies of both the equations.

6.4.2 Frequency Bifurcation Plot

For a better representation of the synchronization dynamics between two coupled asymmetric Van der Pol type equations, in Fig. 6.9 frequency bifurcation plot is shown for a fixed DC parameter $A_1 = 1$ of the equation (6.1) and a varying parameter $A_2 = 0 : 0.05 : 2$ of equation (6.2) while keeping coupling parameter ($N = 0.03$) constant for both the equations. From this plot, it can be clearly seen that when the difference of fundamental oscillation frequencies of two equations is large then frequency pulling states are coming Fig. 6.9 (left and right sides of the plot) and when this difference is small then frequency entrained states are observed as seen in Fig. 6.9 (middle part of the plot).

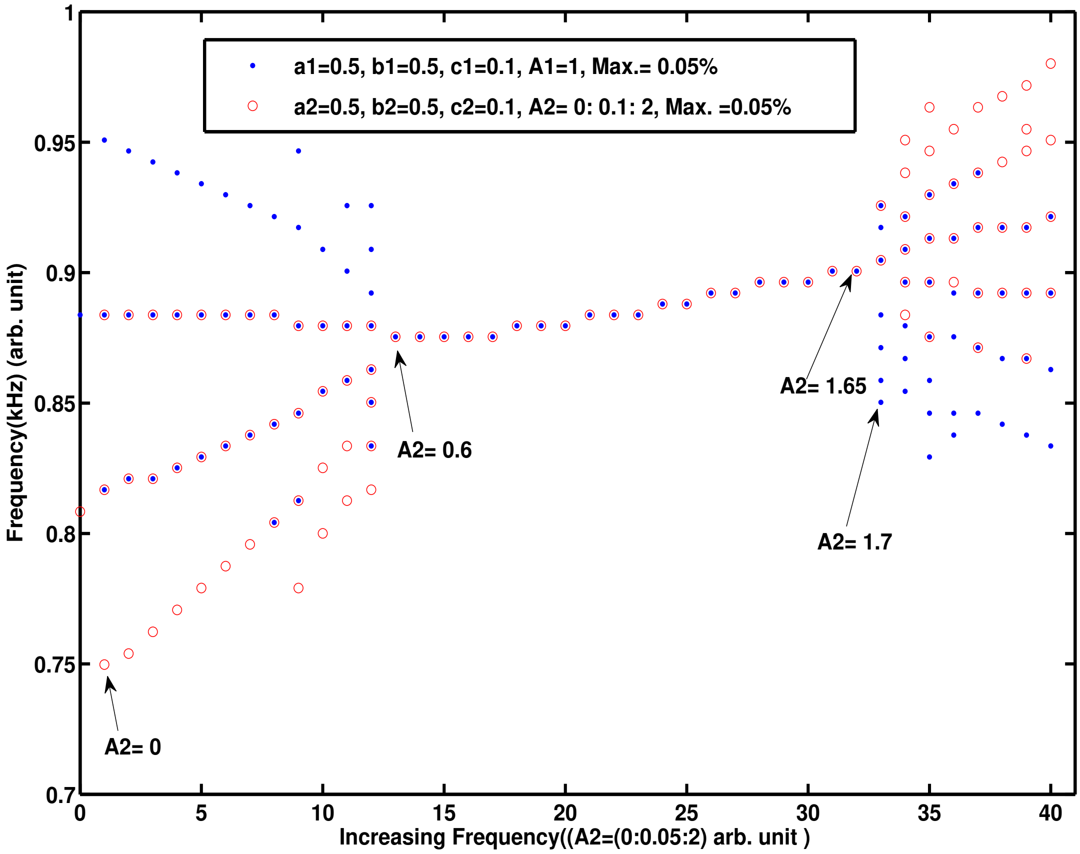

Figure 6.9: *Frequency bifurcation plot for fixed frequency ($A_1 = 1$) of equation (1) and increasing frequency ($A_2 = (0 : 0.05 : 2)$) of equation (6.2) by keeping coupling parameter ($N = 0.03$) fixed.*

6.5 Summary and Conclusions

In this chapter, we have focused on an investigation of the mutual harmonic synchronization between two coupled plasma sources. In our experimental observations, it is found that with the increase in discharge voltage oscillation frequencies increases linearly. For mutual harmonic synchronization experiment the oscillation frequencies in one of the plasma chamber are kept fixed by keeping its discharge voltage fixed and oscillation frequencies of the other plasma chamber are increased by increasing its discharge voltage. A nonlinear phenomenon like frequency entrainment is observed when oscillation frequencies of two chamber plasmas are harmonic multiples of each other and states like frequency pulling and chaotic oscillations are observed beyond these frequency entrained states. Frequency bifurcation plots of two coupled glow discharge plasma sources provide a clear picture of different synchronized states between the oscillations of the two discharges.

In the numerical simulation, the effect of the DC factor on the oscillation frequencies of the asymmetric Van der Pol type equation is discussed, which is found to be increasing linearly with increase in the DC factor. The coupled dynamics of the system is discussed by making the oscillation frequency in one of the equation constant and increasing the oscillation frequency of other equation. It is found that when oscillation frequencies of the two equations are close then frequency entrainment results are obtained and beyond this region frequency pulling results are seen which is similar to what is seen in the experimental results.

7

Summary and future scope

This chapter provides a summary of the experimental details and major results of our investigations into the mutual synchronization phenomenon between two coupled plasma sources that are reported in earlier chapters and also briefly discusses potential applications of our research and possible future extensions of our research work.

7.1 Summary

The main focus of this dissertation research has been the study of mutual synchronization phenomenon between two coupled DC glow discharge plasma sources. In particular, investigations have centered on finding different frequency synchronized states via direct and indirect coupling of the two sources. In the direct coupling mechanism, states displaying frequency entrainment and frequency pulling are observed as the coupling strength is changed. In the indirect coupling mechanism, a phase-flip transition associated with a frequency jump phenomenon is observed. Numerical simulation results based on a model of two coupled Van der Pol type equations are found to be in good agreement with the experimental results.

The synchronization experiments have been carried out in two DC glow discharge plasma systems which are completely isolated from each other. For performing the experiments, self-sustained anode glow oscillations are produced in each of the chambers. These oscillations are produced by choosing the anode surface area to be such that $S_a \leq a \times S_c\sqrt{m/M}$, where S_a and S_c represent effective anode and cathode surface areas respectively while m, M and a are the electron mass, ion mass, and proportionality constant respectively [26]. If the chosen anode surface area follows this criterion, then for maintaining current equality at cathode and anode by ions and electrons respectively, additional ionization takes place near the anode, which is highly unstable and causes the anode glow oscillations. For performing synchronization experiments, these oscillations are brought to a periodic regime by appropriate choice of the discharge voltage and pressure. The following subsections summarize our results for different types of synchronization studies.

7.1.1 Mutual Synchronization Studies via Direct Coupling

In this investigation, mutual synchronization studies are explored between two directly coupled glow discharge plasma system. The anode glow oscillations are produced in each of the systems. In individual characterization studies of these oscillations it is found that these oscillations increase with the increase in the discharge voltage. These oscillations are probably produced due to ion acoustic instabilities or ionization instabilities [26]. The increase in the frequencies may be due to a combination of both temperature and density increase with the discharge voltage. The primary cause for this increase can be attributed to an increase in the electron temperature resulting from a higher voltage and the direct impact this higher temperature has on the ionization rate contributing to the anode glow oscillation [15, 160, 161]. To model this behaviour of the anode glow oscillations, we have adopted an asymmetric Van der Pol type equation with a DC source term that represents the bias voltage. Numerical solutions of this equation for different values of the DC bias term display a similar rise in the oscillation frequency as a function of the DC parameter. The presence of the asymmetry term was found to be quite important. If the asymmetry term is absent in the model equations, i.e Van der Pol type equation, then the frequency spectrum of the numerical solutions have only odd multiples of frequencies of the fundamental and therefore do not correctly model the experimental situation where both odd and even multiples of the basic frequencies are seen to exist.

Our synchronization experiments trace the changes in the oscillation frequencies of the two plasma devices as a function of the coupling strength. When the coupling strength is high then a frequency entrained state is observed with the frequencies of the two devices synchronized to a common frequency that is intermediate in value to the individual frequencies of the uncoupled devices. As the coupling strength is decreased a bifurcation behaviour is found to occur below a threshold value resulting in a frequency pulling state that is characterized by the formation of beat frequencies. This behaviour is also well simulated by the model equations of the Van der Pol type equations.

In conclusion, our experiments provide a clear demonstration of the phenomena of frequency synchronization and frequency pulling of anode glow oscillations in plasma glow discharge devices that can form the basis of future dynamical system experiments in plasma systems such as the exploration of chaos or amplitude death phenomenon. The model equations developed in support of our present experiments can help in identifying parametric domains for such explorations.

7.1.2 Phase-flip Transition via Indirect Coupling

We have investigated the synchronization dynamics of two coupled DC glow discharge plasma sources and shown the existence of a phase-flip transition in the frequency synchronized regime of such a system. The PF transition is associated with a discontinuous jump in the synchronized frequency of the two sources and does not require the presence of any time delay in the coupling mechanism. The key feature responsible for the PF is the inductive nature of the coupling that acts as a sort of a common interactive medium through which the two sources communicate with each other. Another novel result observed in our experiments is the presence of hysteresis in the onset of the PF transition as we scan the synchronized regime in the increasing or decreasing directions of the dc voltage of one of the sources. Unlike past results, the observed hysteresis is not due to any hysteresis in the voltage-current characteristics of the source but is solely associated with the frequency synchronization mechanism. Our experimental results are well supported by a theoretical model that represents the dynamics of the individual plasma sources by Van der Pol oscillators and mimics the inductive wire by a bath coupling.

The numerical results from the model capture all the essential characteristics of the experiment including frequency pulling, frequency synchronization, phase-flip transitions and hysteresis. Here, it should also be pointed out that the successful implementation of an inductive coupling mechanism for effecting synchronization between two plasma sources opens up novel possibilities for non-invasive control of plasma fluctuations and instabilities that can have wider applications in industrial or larger scale laboratory setups.

7.1.3 Mutual Harmonic Synchronization via Direct Coupling

In this chapter, we have focused on the investigation of the mutual harmonic synchronization between two coupled plasma sources. For the harmonic synchronization experiment, the oscillation frequencies in one of the plasma chambers are kept fixed by keeping its discharge voltage fixed and the oscillation frequencies in the other chamber plasma are increased by increasing its discharge voltage. A nonlinear phenomenon like frequency entrainment is observed when the oscillation frequencies of the two plasma systems are at harmonic multiples of each other and states like frequency pulling and chaotic oscillations are observed beyond these frequency entrained states. A frequency bifurcation plot of the two coupled glow discharge plasma sources provides a clear picture of the different synchronized states between the oscillations of the two discharges.

In numerical simulations, the effect of the DC factor on the oscillation frequencies of the asymmetric Van der Pol type equation is discussed. It is found that the frequency increases linearly with an increase of this factor. Coupled dynamics is studied by making the oscillation frequency in one of the equation constants and increasing the oscillation frequency of the other equation. It is found that when the oscillation frequencies of the two equations are close then frequency entrainment is observed and beyond this region frequency pulling results are seen which is analogous to that seen in the experimental results.

7.2 Future Scope

In this thesis work, we have performed synchronization studies by producing self-sustained periodic type oscillations in two glow discharge plasma systems. However, oscillations in this system are not only limited to a periodic type of oscillations and a variety of different types of nonlinear oscillations could also be produced. The synchronization studies using these oscillations have a huge potential to provide new insights into different types of synchronized states. Some of the studies where the results of this thesis work could be used are as follows:

- ***Chaotic Synchronization***
  In studies of this type, chaotic systems adjust their rhythms to oscillate in a unified way, due to coupling between the oscillators [1]. This type of synchronization phenomenon has been studied in a variety of fields [1–3, 53, 54, 145, 155]. One of the most important immediate use of such type of synchronization is in the field of secure communication [54]. But due to various complexity involved with this method, it is still not fully explored. The results presented in this thesis work could be extended for carrying out such chaotic synchronization studies in plasma systems by operating in regimes where stable chaotic oscillations occur in the plasma instead of the periodic oscillations used in our experiments. Also, in the chapter 6, for mutual harmonic synchronization studies, it has been shown that at higher harmonic synchronization, chaotic states emerge. These results can also be extended for performing chaotic synchronization studies.

- ***Oscillation Death***
  This is a type of synchronization phenomenon in which coupled oscillators cease their oscillations [2, 3, 72–74]. Ramana et al. have shown that by the introduction of some time delay between the coupled oscillators, this state can be obtained [72,72]. Such steady state (amplitude death) has been seen in many different types of oscillators [2, 3, 74] but in plasma systems, such state has not been explored. The results of this thesis work can also be extended for performing such studies in plasma systems. In our experimental system, this state may be obtained by introducing a time delay between the oscillations of two systems or by introducing a mismatch between the oscillations of two

systems [74] etc.

- ***Chimera States***
  This is a special type of synchronization state, which is observed in many mutually coupled oscillators. In this state, some of the oscillators come to a synchronized state while others remain in an unsynchronized state [4, 76, 77] even though all the oscillators are identical in their characteristics and the coupling between all the oscillators are also identical. In plasma systems, such studies have not been done before, but our thesis work can also be extended to do such studies in plasma systems. For these studies, the number of plasma oscillation sources has to be increased and by using the mutual inductive coupling method tit might be possible to obtain such a state.

- ***Application in PVD Based Coatings***
  The physical vapor deposition(PVD) technique is widely used for coating different types of materials on substrates. There is however one disadvantage with this method, namely, that coatings formed by this way are not uniform or have some defects. This happens due to the presence of various types of instabilities in the plasma medium. Our synchronization results could help in controlling these instabilities and that in turn will help in forming defect less and uniform coatings. This can be done by forcing PVD oscillations by some external source of oscillation or by self feedback mechanism.

- ***Science Popularization***
  Experimental synchronization work that has been presented in this work could be used in general science popularization in the field of nonlinear dynamics and synchronization. Usually, students who wish to study nonlinear phenomena have very limited resources for doing hands on research in this field. They perform such experiments in electronic circuits which have lots of limitations due to the very limited range of its operation in nonlinear regimes. Our experimental setup which has a very wide range of operating regimes and supports for different types of nonlinear dynamic behaviour, could be used for such studies. Also, being a table top setup such a device is very cost effective for use in a teaching environment.

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