Some aspects of Grand Unified Theory: gauge coupling unification with dimension-5 operators and neutrino masses in an SO(10) model

Joydeep Chakrabortty

Harish-Chandra Research Institute, Allahabad

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Certificate

This is to certify that the Ph. D. thesis titled "Some aspects of Grand Unified Theory: gauge coupling unification with dimension-5 operators and neutrino masses in an SO(10) model" submitted by Joydeep Chakrabortty is a record of bonafide research work done under my supervision. It is further certified that the thesis represents independent work by the candidate and collaboration was necessitated by the nature and scope of the problems dealt with.

Date:

Amitava Raychaudhuri
Thesis Adviser

Declaration

This thesis is a presentation of my original research work. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature and acknowledgment of collaborative research and discussions.

The work is original and has not been submitted earlier as a whole or in part for a degree or diploma at this or any other Institution or University.

This work was done under guidance of Amitava Raychaudhuri, at Harish-Chandra Research Institute, Allahabad.

Date:

Joydeep Chakrabortty
Ph. D. Candidate

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List of Publications and Preprints

List of papers & preprints that form the thesis of the candidate

- A Note on dimension-5 operators in GUTs and their impact.
  Joydeep Chakrabortty, Amitava Raychaudhuri.
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- Gaugino mass non-universality in an SO(10) supersymmetric Grand Unified Theory: Low-energy spectra and collider signals.
  Subhaditya Bhattacharya, **Joydeep Chakrabortty**.
  Published in Phys.Rev.D 81:015007,2010.

- GUTs with dim-5 interactions: Gauge Unification and Intermediate Scales.
  Joydeep Chakrabortty, Amitava Raychaudhuri.
  Published in Phys.Rev.D 81:055004,2010.

- Dimension-5 operators and the unification condition in SO(10) and E(6).
  Joydeep Chakrabortty, Amitava Raychaudhuri.
  e-Print: arXiv:1006.1252 [hep-ph]

- An SO(10) model with adjoint fermions for double seesaw neutrino masses.
  Joydeep Chakrabortty, Srubabati Goswami, Amitava Raychaudhuri.
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List of papers & preprints of the candidate that are not included in the thesis

- Renormalisation group evolution of neutrino masses and mixing in the Type-III seesaw mechanism.
  Joydeep Chakrabortty, Amol Dighe, Srubabati Goswami, Shamayita Ray.
  Published in Nucl.Phys.B 820:116-147,2009.

- Maximal mixing as a 'sum' of small mixings.
  Joydeep Chakrabortty, Anjan S. Joshipura, Poonam Mehta, Sudhir K. Vempati.
  e-Print: arXiv:0909.3116 [hep-ph].

- TeV scale double seesaw in left-right symmetric theories.
  Joydeep Chakrabortty.
  e-Print: arXiv:1003.3154 [hep-ph].

- Type I and *new* seesaw in left-right symmetric theories.
  Joydeep Chakrabortty.
  Published in Phys.Lett.B 690:382-385,2010.

- Multi-photon signal in supersymmetry comprising non-pointing photon(s) at the LHC.
  Sanjoy Biswas, **Joydeep Chakrabortty**, Sourov Roy.
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- Resonant Leptogenesis with nonholomorphic R-Parity violation and LHC Phenomenology.
  Joydeep Chakrabortty, Sourov Roy.
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Synopsis

- **Name: Joydeep Chakrabortty**
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The Standard Model (SM) based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ has three independent gauge couplings g_3, g_2, and g_1. Grand Unified Theory (GUT) aims for the unification of these couplings and also ensures the presence of quarks and leptons in a common multiplet of a single gauge group. The SM gauge couplings evolve logarithmically with energy leading to an unified coupling, g_{GUT}, if unification is achieved at some high scale (M_X). The experimental constraints on proton decay life-time set the lower bound of the GUT scale, $M_X \geq 10^{16}$ GeV.

A full theory at the Planck scale (M_{Pl}) is not yet known. In its absence it has been found useful to introduce higher dimensional effective operators at the GUT scale itself which will capture some of the higher scale physics implications. These operators might have significant impact on the predictions of the grand unified theory.

We focus on the corrections to the gauge kinetic term, $-\frac{1}{4c}Tr(F_{\mu\nu}F^{\mu\nu})$, through the operator, $-\frac{\eta}{M_{Pl}}[\frac{1}{4c}Tr(F_{\mu\nu}\Phi_D F^{\mu\nu})]$, where $F^{\mu\nu} = \Sigma_i \lambda_i . F_i^{\mu\nu}$ is the gauge field strength tensor with λ_i being the matrix representations of the generators normalised to $Tr(\lambda_i \lambda_j) = c\ \delta_{ij}$. Conventionally, for $SU(n)$ groups the λ_i are chosen in the fundamental representation with $c = 1/2$. η is a dimensionless parameter that determines the strength of the operator. Obviously the representations of Φ_D must appear among the representations in the symmetric product of two adjoint representations of the group.

When Φ_D develops a vacuum expectation value (*vev*) v_D, which sets the scale of grand unification M_X, an effective gauge kinetic term is generated and the modified gauge coupling unification condition reads as: $g_1^2(M_X)(1+\epsilon\delta_1) = g_2^2(M_X)(1+\epsilon\delta_2) = g_3^2(M_X)(1+\epsilon\delta_3)$, wherein the δ_i, $i = 1,2,3$ are the group factors, and $\epsilon = \eta v_D / 2M_{Pl} \sim \mathcal{O}(M_X/M_{Pl})$.

We work out the consequences of these dimension-5 operators for the unified theories based on $SU(5)$, $SO(10)$, and $E(6)$. We consider all the possible choices for Φ_D, namely, the representations **24**, **75**, **200** for $SU(5)$, **54**, **210**, **770** for $SO(10)$ and **650**, **2430** for $E(6)$. We propose a prescription to calculate the orientations of the *vev*s of these Higgs fields. The orientations depend on the pattern of symmetry breaking. $SU(5)$ directly breaks to the SM. But $SO(10)$ and $E(6)$ can be broken to the SM through different intermediate gauge groups. We calculate the corrections (δ_i's) which arise due to the dimension-5 operator for all possible breakings with all possible choices of Φ_D's.

We then calculate the β-coefficients and construct the renormalisation group equations (RGEs) to study the evolutions of the gauge couplings upto two-loop level to check whether unification is achieved or not.

We find these dimension-5 operators cannot help to achieve unification beyond 10^{16} GeV for $SU(5)$ in non-supersymmetric (non-SUSY) scenario. But in SUSY $SU(5)$ models unification is achieved with a high enough GUT scale. These dimension-5 operators also have impact on the prediction of $\sin^2\theta_W$ at low scale ($\sim M_Z$). This constrains the strength of these effective operators.

We also consider $SO(10)$ and $E(6)$ GUT gauge groups. We discuss no-, one-, and two-step breakings of these gauge groups to the SM. For each case we construct the RGEs and include the proper matching of the gauge couplings at the intermediate scales as well as at the GUT scale. In presence of one intermediate symmetry group, $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ for $SO(10)$ and $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ for $E(6)$, we explore D-parity (symmetry between left-right sector) conserving and broken cases. We extend our study through the inclusion of the second intermediate group, $SU(3)_C \otimes U(1)_{B-L} \otimes SU(2)_L \otimes U(1)_R$ and $SU(3)_C \otimes SU(2)_L \otimes U(1)_L \otimes SU(2)_R \otimes U(1)_R$ for $SO(10)$ and $E(6)$ respectively. In all the above cases we determine the ranges of the intermediate scales consistent with the viable unified scenario.

It has been noted that the operator that generates the gaugino masses in a supergravity (SUGRA) model is the same as the dimension-5 operator we consider. The gaugino mass non-universality is achieved through the *vev*s of the non-singlet Φ_D's and the ratios of the gaugino masses are same as the ratio of the δ_i's. We exhaustively explore all possibilities.

We also study neutrino mass generation in the context of a grand unified theory. We consider an $SO(10)$ based model with (10+120) Higgs fields. It has been noted that in such a model neutrino masses cannot be generated in tree level. One can generate neutrino masses at two-loop level. We aim for an alternate mechanism where all the couplings are at the tree level. To do so we extend this model by adding adjoint fermions (which transform as 45 of $SO(10)$) and $\overline{16}$ Higgs. Thus the neutrino mass matrix is extended and a double seesaw is achieved. Imposing $\mu - \tau$ symmetry we consider explicit form of the Dirac-Yukawa matrices that help to generate correct light neutrino masses and tri-bimaximal (TBM) mixing angles. We also check the gauge coupling unification in this model.

Contents

Chapter 1

Introduction

Our understanding of the fundamental particles and the interactions between them has evolved into a clear picture today. The basic constituents are the quarks and leptons, which appear to have no further structure. They interact among each other through four types of interactions: the strong, weak, and electromagnetic interactions besides the gravitational force. The first three of these can be elegantly expressed in a mathematical formulation known as gauge field theory which relies on group symmetries. Furthermore, it is possible to formulate the entire theory, i.e., the basic constituents and the three forces above, into an economical structure which is termed the standard model.

The standard model gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ contains three independent gauge couplings g_3, g_2, and g_1. Grand Unified Theory (GUT) aims for the unification of these couplings and also ensures the presence of quarks and leptons in a common multiplet of a single gauge group. The SM gauge couplings evolve logarithmically with energy leading to an unified coupling, g_{GUT}, if unification is achieved at some high scale.

In this thesis we have discussed the impact of dimension-5 operators, which may arise from quantum gravity, on gauge coupling unification and on non-universal gaugino masses. We also examine neutrino masses in an $SO(10)$ grand unified theory. In the second chapter we introduce the basic structure of the standard model and note its failures to explain some experimentally observed issues. Then in the next chapter we discuss the motivations for a grand unified theory and the possible GUT groups. In the fourth chapter we propose the prescription to calculate the vacuum orientations of the different symmetry breaking

Higgs fields. For $SU(5)$, $SO(10)$, and $E(6)$ GUT we calculate the β-coefficients for gauge coupling running and check the unification in chapter 5. In the subsequent chapters the group theoretic structures of the non-universal gaugino masses are calculated and the neutrino mass generation in an $SO(10)$ model is discussed respectively.

Chapter 2

The Standard Model and beyond

2.1 The standard model of particle physics

In 1960 Sheldon Glashow proposed a theory combining electromagnetism and weak interactions – known now as electroweak theory [1]. Later Weinberg and Salam in 1967 incorporated the Higgs mechanism in this electroweak theory [2,3] – that completes the basic structure of one part of the standard model (SM). Further to this, when strong interaction was incorporated the present form of the SM was achieved. The standard model was proposed on the basis of quantum field theory as a basic and fundamental theory of particle physics. In the SM all the participating entities are considered to be elementary. They can be categorised in three sectors – scalars, fermions, and gauge bosons – depending on their spins. As we commented before, SM encapsulates the features of the electromagnetic, weak, and strong forces. In the language of gauge theory the symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is introduced that describes the SM. The $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ depict the colour, weak isospin, and weak hypercharge symmetry groups respectively [1–6], named according to the quantum numbers. The only scalar particle in the SM is the Higgs multiplet which is colour singlet but doublet under $SU(2)_L$ and also has non-zero hypercharge. All the fermions are charged under $U(1)_Y$. Left-handed fermions transform as doublet while the right-handed fields are singlet under $SU(2)_L$. The fermions that do not have any colour quantum number are known as leptons and rest of them which form colour triplets are quarks. The details of these field contents are organised below as:

- **Scalars:**
  An $SU(2)$ doublet, with Y = 1 (Higgs field (H))
- **Fermions:**
  The left-handed fermionic fields are:
  $SU(3)_c$ triplet, $SU(2)_L$ doublet, with Y = 1/3 (left-handed quarks (Q_L))
  $SU(3)_c$ triplet, $SU(2)_L$ singlet, with Y = 2/3 (left-handed down-type antiquark (d_R))
  $SU(3)_c$ triplet, $SU(2)_L$ singlet, with Y = -4/3 (left-handed up-type antiquark (u_R))
  $SU(3)_c$ singlet, $SU(2)_L$ doublet, with Y = -1 (left-handed lepton (l_L))
  $SU(3)_c$ singlet, $SU(2)_L$ singlet, with Y = 2 (left-handed antilepton (l_R))
- **Gauge fields:**
  $SU(3)_c$ gauge field G_μ, with coupling constant g_3
  $SU(2)_L$ gauge field W_μ, with coupling constant g_2
  $U(1)_Y$ gauge field B_μ, with coupling constant g_1

In the SM, fermions carry the quantum numbers in such a way that anomaly gets canceled. When the SM gauge symmetry is unbroken all the particles are massless. The chiral symmetry and the gauge symmetry protect the masses of the fermions and the gauge bosons respectively. But the mass of the scalar particle, i.e., Higgs is not protected by any symmetry. Still the Higgs mass is expected to be of order 100 GeV to be consistent with experimental data. This fine tuning is still very uncanny. The fermion and gauge boson masses are generated in the SM through the Higgs mechanism [7] via spontaneous symmetry breaking (SSB). In the SM the scalar potential is constructed by writing all renormalisable Higgs self couplings. The minimum of this potential has a $O(2)$ symmetry and possesses an infinite set of vacua. When one of the directions is chosen among the infinite possibilities, the symmetry is spontaneously broken and the expectation value of this Higgs field for which the potential attains the minimum is called the vacuum expectation value (*vev*). As the vacuum must respect $U(1)_{em}$ symmetry only the neutral component of the Higgs field can acquire a *vev*. Thus after SSB the $SU(2)_L \otimes U(1)_Y$ symmetry group is broken to $U(1)_{em}$. But the colour symmetry remains as it is, and thus the gluons are massless. But among the four electroweak gauge bosons three eat Goldstone modes and become massive, and one remains massless – the photon. It has been noted that this *vev*, v, should be $\sim$ 246 GeV to agree with the experimentally measured masses and couplings of the gauge bosons. The fermion masses are generated once the Higgs gets *vev*, and these masses are proportional to the Yukawa couplings. In the SM all the Yukawa couplings are free parameters. In general the Yukawa couplings are non-diagonal and lead to mixings between different quark flavours and similarly for leptons. In the lepton and quark sectors mixing matrices are known as PMNS-, and CKM-matrix respectively. These are in general 3$\times$3 unitary matrices. The down-type fermions get masses from H, but that for up-type fermions are achieved from $\overline{H}$ ($\equiv i\sigma_2 H^*$), and we know that 2 and $\overline{2}$ transform in a same way under $SU(2)$. In the SM the neutrinos are massless due to the absence of the right-handed neutrino (ν_R).

The radiative corrections to the scalar masses are unprotected and that impact can be large in the presence of a very high scale. To address this problem one of the most attractive solutions is supersymmetry.

Supersymmetry (SUSY) is a symmetry between fermions and bosons [8], and is a unique extension of the Poincare group. SUSY is implemented by introducing the superpartners of the standard model – such as for fermions, sfermions which transform as scalars, and for gauge bosons and Higgs, gauginos and higgsinos which are fermions are introduced respectively. The scalar sector includes the sfermions and two Higgs doublets. The SM particles and their superpartners transform identically under the standard model gauge group. The requirement of two Higgs doublets can be argued as follows:
(a) From the construction of the SUSY theory it is well known that the superpotential must be holomorphic. It tells that in the superpotential a field and its conjugate cannot coexist. Thus we need to introduce two Higgs doublets, H_u and H_d, of opposite hypercharges to give masses to the up- and down-type fermions.
(b) Anomaly cancellation demands that there must be a pair of Higgsinos for the minimal model.
In supersymmetry the radiative correction to the Higgs mass gets contribution from new diagrams – the superpartner loops, and this helps to cancel the divergence in the Higgs mass [9].

Though the SM was thought to be a complete theory of particle physics, some experimental observations compel us to think beyond it. One of them is neutrino oscillation that tells that neutrinos are not massless but have tiny masses. In the next section we give a brief review on neutrino masses.

2.2 Neutrino mass

A number of experiments with solar, atmospheric, reactor and accelerator neutrinos have now unambiguously established that these elusive particles are massive. In addition, the data imply one small and two large mixing angles in complete contrast with the quark sector where all three mixing angles are small. In Table 2.1 we present the best-fit values and 3σ ranges of neutrino oscillation parameters as obtained from the global oscillation analysis [10]. These values are close to the so called tri-bimaximal mixing pattern [11] which implies $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\sin^2\theta_{13} = 0$.

There are several proposals to explain the origin of neutrino masses. Seesaw is the most promising one among them. In the seesaw mechanism a very heavy particle that couples to the lepton and Higgs doublet is exchanged. Now, at the low energy when this heavy particle is decoupled, i.e., integrated out, an effective tiny Majorana neutrino mass is generated. In this mechanism lepton number is violated by two units. The usual seesaw mass matrix where light neutrino masses are generated by integrating out heavy right handed neutrinos of mass (M_R) takes the form:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}; \tag{2.1}$$

where m_D is the Dirac coupling between ν_L and ν_R. M_R is a 3×3 matrix for three generations of right handed neutrinos. In general, seesaw scales are determined by the mass of an exchanged heavy particle. It has been noted that these scales need to be very high – 10^{12} GeV – to generate neutrino mass of $\mathcal{O}(eV)$ without fine tuning the neutrino Yukawa couplings. There are different types of seesaw models depending on the nature of the heavy particle. In type-I, -II, -III seesaw models a singlet fermion, a triplet scalar, and a triplet fermion are exchanged [12–14] respectively. But as the seesaw scales for these models are very high it is difficult to test these theories in current experiments. Besides these standard mechanisms some other variants of seesaw models are recently being studied – among them inverse and double seesaw are very popular. In general if the model con-

	best fit	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.59	7.03 - 8.27
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	2.40	2.07 - 2.75
$\sin^2\theta_{12}$	0.318	0.27 - 0.38
$\sin^2\theta_{23}$	0.50	0.36 - 0.67
$\sin^2\theta_{13}$	0.013	≤ 0.053

Table 2.1: The best-fit values and the 3σ ranges of neutrino mass and mixing parameters as obtained from a global analysis of oscillation data [10]. $\Delta m^2_{ij} = m^2_i - m^2_j$.

tains large fermion representations there is always scope to have more couplings among the SM neutrino and the extra neutral fermions. Thus the neutrino mass matrix in these cases are extended than the usual ones. The double seesaw, and the inverse seesaw mechanisms are gaining popularity because of the presence of TeV scale particles, i.e., these new models are testable at the colliders. The generic structure of the neutrino mass matrix for double seesaw is:

$$M_\nu = \begin{pmatrix} 0 & m_{D1} & m_{D2} & . \\ m^T_{D1} & 0 & M_{R1} & . \\ m_{D2} & M^T_{R1} & M_{N1} & . \\ . & . & . & . \end{pmatrix}. \tag{2.2}$$

When the usual seesaw model is extended by a singlet (consider only the displayed 3×3 block in eq. 2.2) then the double- and inverse seesaw mass matrix looks same. It is noted that if we set M_{N1}=0, then light neutrino masses cannot be generated.

Since in the standard model neutrinos are massless one is compelled to transcend beyond the realms of the SM. There are also several theoretical motivations for going beyond the SM, one of which is that the SM is a product of three gauge groups and so involves three independent couplings. A Grand Unified Theory (GUT), which is a theory of strong and electroweak interactions based on a single gauge group [15,16], aims to unify the three forces with a single coupling constant [17]. It also unifies the matter fields by placing the quarks and leptons in the same irreducible representation of the underlying gauge group [15]. Since GUTs aim to unify quarks and leptons it is a challenge to reconcile the large mixings in the lepton sector with the small mixings in the quark sector. The issue of fermion masses and mixing in the context of GUTs has received much attention from this perspective.

2.3 Evolution of gauge couplings

In a large class of quantum field theories higher order corrections in perturbation theory lead to divergent quantities. This is addressed by the theory of renormalisation that discriminates the bare parameters from the renormalised ones. It is well known that there should not be any ultraviolet divergence in a theory if it has to yield reasonable physical predictions. Because of higher order effects arising in an interacting quantum field theory parameters like coupling constants develop a dependence on the energy scale. Their values can change with distance, i.e., scale. For an example, in quantum electrodynamics the charge of an electron depends on at which energy we are measuring it, i.e., how deeply we are probing that electron. Similarly the gauge couplings, Yukawa couplings etc. also depend on the energy scale. Using the knowledge of quantum field theory a prescription is suggested that tracks this variation of the couplings with scales which are known as renormalisation group equations (RGEs). In quantum field theory the Lagrangian does not carry any signature of any scale parameter. Thus the Green functions are expected to be invariant under any scaling behaviour in the theory. But this is not true in reality. When we do the perturbative analysis, a hidden scale parameter enters in the theory. It is true that we do not have this parameter to start with and one must expect that physical observables should be independent of this. In principle if we include all orders of the perturbation there will not be any scale dependency in the theory. But most analyses consider only finite orders of the perturbation series thus scale dependency remains there. The renormalisation group equations describe the dependence of theory on this scale parameter.

Renormalisation group evolutions involve a parameter, μ, that carries the signature of the scale of the theory. The RGE is expressed as:

$$\mu \frac{\partial g}{\partial \mu} = \beta(g), \tag{2.3}$$

where g is the coupling and the exact form of the function β depends on the nature of the coupling. In chapter 5 we discuss the exact form of $\beta(g)$ for one-, and two-loop cases for the running of the gauge couplings.

References

[1] S. L. Glashow, Nucl. Phys. **22** (1961) 579.

[2] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.

[3] A. Salam, *In the Proceedings of 8th Nobel Symposium, Lerum, Sweden, 19-25 May 1968, pp 367-377*.

[4] For introduction, see, for example, F. Halzen and A.D. Martin, *Quarks and Leptons: An introductory course in modern particle physics*, (Ed. John Wiley, 1984); C. Quigg, *Gauge Theories of the Strong, Weak and Electromagnetic Interactions*, (Westview Press 1997); T.P. Cheng and L.F. Li, *Gauge theory of elementary particle physics*, (Oxford Univ. Press, 1991).

[5] Y. Ne'eman, Nucl. Phys. **26** (1961) 222; M. Gell-Mann, Phys. Lett. **8** (1964) 214; O. W. Greenberg, Phys. Rev. Lett. **13** (1964) 598; H. Fritzsch, M. Gell-Mann, H. Leutwyler, Phys. Lett. B **47** (1973) 365; D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343; H. D. Politzer, Phys. Rev. Lett. **30** (1973) 1346;

[6] G. 't Hooft, Nucl. Phys. B **35** (1971) 167; G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **44** (1972) 189.

[7] P. W. Higgs, Phys. Rev. Lett. **13** (1964) 508; F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321; G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13** (1964) 585; P. W. Higgs, Phys. Rev. **145** (1966) 1156; T. W. B. Kibble, Phys. Rev. **155** (1967) 1554.

[8] S. R. Coleman and J. Mandula, Phys. Rev. **159** (1967) 1251; R. Haag, J. Lopuszanski, and M. Sohnius, Nucl. Phys. B **88** (1975) 257.

[9] S. Dimopoulos and S. Raby, Nucl. Phys. B **192** (1981) 353; E. Witten, Nucl. Phys. B **188** (1981) 513; M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B **189** (1981) 575; S. Dimopoulos and H. Georgi, Nucl. Phys. B **193** (1981) 150; N. Sakai, Z. Phys. C **11** (1981) 153; R.K. Kaul and P. Majumdar, Nucl. Phys. B **199** (1982) 36.

[10] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10** (2008) 113011

[arXiv:0808.2016 [hep-ph]].

[11] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530** (2002) 167 [arXiv:hep-ph/0202074].

[12] See, for example, P. Minkowski, Phys. Lett. B **67** (1977) 421; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, O. Sawada and A. Sugamoto *(eds.)*, KEK, Tsukuba, Japan, 1979, p. 95; M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity*, P. van Nieuwenhuizen *et al. (eds.)*, North Holland, Amsterdam, 1980, p. 315; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, (1980) 912.

[13] See, for example, R.N. Mohapatra, A. Perez-Lorenzana and C.A. de Sousa Pires, Phys. Lett. B **474** (2000) 355; G. Altarelli and F. Feruglio, Springer Tracts Mod. Phys. **190** (2003) 169; G. Altarelli and F. Feruglio, New J. Phys. **6** (2004) 106; S. Bertolini, M. Frigerio and M. Malinsky, Phys. Rev. D **70** (2004) 095002; S.F. King, Rept. Prog. Phys. **67** (2004) 107.

[14] J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, (1980) 2227; J. Schechter and J.W.F. Valle, Phys. Rev. D **25**, (1982) 774; A. Mendez and F.I. Olness, Int. J. Mod. Phys. **A2**, (1987) 1085; J.F. Gunion, J. Grifols, A. Mendez, B. Kayser and F.I. Olness, Phys. Rev. D **40** (1989) 1546; N.G. Deshpande, J.F. Gunion, B. Kayser and F.I. Olness, Phys. Rev. D **44** (1991) 837; E. Ma, Phys. Rev. Lett. **81** (1998) 1171; P.Q. Hung, Phys. Lett. B **649** (2007) 275; A. Hektor, M. Kadastik, M. Muntel, M. Raidal and L. Rebane, Nucl. Phys. B **787** (2007) 198; T. Han, B. Mukhopadhyaya, Z. Si and K. Wang, Phys. Rev. D **76** (2007) 075013.

[15] J. C. Pati and A. Salam, Phys. Rev. Lett. **31** (1973) 66; Phys. Rev. D **10** (1974) 275.

[16] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.

[17] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33** (1974) 451.

Chapter 3

Grand Unified Theory

3.1 Motivations

Grand Unified Theory [1–3] is argued to be a complete and unique theory of all fundamental forces except gravity. First the unified picture of electric and magnetic forces, i.e., electromagnetism, and then electro-weak unification enlighten the hope of grand unification. GUT is described by a simple group (G) or a direct product of identical simple groups – related by some discrete symmetry. Thus theory can only have a single gauge coupling – so called unified coupling.

The failure of the SM to explain some experimental issues and the large number of free parameters present in it forces one to think of a theory beyond it. For example:

- Neutrinos are massless in the SM. But now it is well established through different experiments that neutrinos do have very tiny masses.
- There is no reason why there are such differences in the strengths of the strong, weak, and electromagnetic couplings – $\alpha_3, \alpha_2, \alpha$ respectively. The strong coupling is completely unrelated to the electro-weak couplings.
- In the SM electric charge is given as: $Q = T_3 + Y/2$, where the hypercharge, Y, can be assigned independently for each representation. We know that in a doublet the electric charge of the two fields differ by one unit – this is a group theoretic constraint. But there is no underlying symmetry which tells that these charges of leptons, quarks and Higgs fields should be related.
- The Yukawa couplings and the couplings in the scalar potential are all free parameters, thus the fermion masses, mixings, phases and gauge boson masses are quite arbitrary.

- In the SM the colour gauge group is decoupled by its nature from other sectors of the Lagrangian.

A number of these unanswered questions can be addressed within the framework of grand unified theory.

The standard model being a successful theory at low scale ($\sim M_z$), G must contains $G_s \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as a subgroup. Grand Unified Theory dictates that at some high scale, say M_X, these SM gauge couplings unify. GUT does not mean only unification of couplings but also implies the family unification. Like the unified coupling, all the fermions are contained in the multiplets of the GUT gauge group, G. Thus all the quarks, anti-quarks, leptons, anti-leptons are accommodated in a single (or two) representation(s) of G and their charges are no more arbitrary but now related.

The GUT gauge group has a larger symmetry than the SM thus there must be other extra gauge bosons. These gauge bosons achieve masses once the GUT symmetry is broken at very high energy which sets the unification scale. The presence of these extra particles cause proton decay. The main decay modes of the proton in the simplest form of the grand unified theory are: $p \to e^+(\mu^+)\pi^0$, $p \to \bar{\nu}\pi^+$.

In non-supersymmetric theory proton decay occurs via mass dimension-6 operators suppressed by M_{GUT}^{-2}. But in supersymmetric models with conserved matter parity, $(-1)^{3(B-L)}$, through sfermion exchange, dimension-5 operators can cause proton decay. Through the dimension-4 operators, the matter parity violating couplings, e.g., $\lambda''_{ijk} U_i^c D_j^c D_k^c$ and $\lambda'_{ijk} L_i Q_j D_k^c$ in terms of the superfields, accelerate proton decay. These couplings need to be very small to satisfy the present limit on proton decay life-time, $\tau_p \geq 10^{33}$ yrs. The approximated proton decay life-time, via dimension-6 operators, is given as:

$$\tau_p \sim \frac{1}{\alpha_X^2} \frac{M_X^4}{m_p^5}; \tag{3.1}$$

where $\alpha_X = g_X^2/4\pi$ is the unified gauge coupling. This unification scale (M_X) is also restricted and M_X must be greater than $10^{15.5}$ GeV. As these gauge bosons are very heavy, beyond the reach of recent colliders, it is very hard to find their footprints to justify the theory.

As we discuss in the earlier section the idea of GUT is to have a single gauge coupling and put quarks, and leptons in one multiplet. There are several candidates that can serve as grand unified gauge group. Among them $SU(5), SO(10)$, and $E(6)$ are very popular.

3.2 SU(5)

In 1974 Georgi and Glashow proposed an $SU(5)$ grand unified theory [2]. The rank of this group is 4, and the dimensionality of the adjoint representation is 24. Thus it has 24 generators, i.e., 24 gauge bosons. These generators can be written as 5×5 complex unitary matrices with determinant unity. The complex unitary matrices can be expressed as:

$$U = exp(-i \sum_{j=1}^{24} \theta^i T^i), \tag{3.2}$$

where these T^i are the hermitian and traceless generators, and θ^i are real parameters. The generators are normalised as: $Tr(T_i T_j) = 2\delta_{ij}$ when they are in fundamental representation. These generators can be written explicitly as:

$$T^a = \left[\begin{array}{ccc|cc} & & & 0 & 0 \\ & \lambda^a & & 0 & 0 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]; \tag{3.3}$$

where λ^a (a=1,...,8) are the $SU(3)_c$ generators, i.e., the Gell-Mann Zweig matrices;

$$T^{9,10} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \sigma_{1,2} & \\ 0 & 0 & 0 & & \end{array}\right]; \tag{3.4}$$

where $\sigma_{1,2}$ are the $SU(2)$ generators (non-diagonal Pauli spin matrices);

$$T^{11} = diagonal(0, 0, 0, 1, -1); \tag{3.5}$$

$$T^{12} = \frac{1}{\sqrt{15}} diagonal(-2, -2, -2, 3, 3); \tag{3.6}$$

are proportional to the third component of weak isospin and weak hypercharge generators respectively.

The other 12 generators ($T^{13,...,18}$ and $T^{19,...,24}$) of $SU(5)$ that do not belong to the SM are written as:

$$T^{13} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]; \tag{3.7}$$

$$T^{14} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]; \tag{3.8}$$

$$T^{19} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right]; \tag{3.9}$$

$$T^{20} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \end{array}\right]. \tag{3.10}$$

The other generators can be obtained by putting 1, i, and -i in the same pattern in the entries of the off-diagonal blocks. There are 24 gauge bosons, one associated with each generator.

Among the 24 , 12 are SM ones (under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$):

$$24 = (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-\frac{5}{3}) + (\overline{3},2,\frac{5}{3}). \tag{3.11}$$

The 12 gauge bosons associated with $T^{13,...,24}$ are known as X and Y bosons. These lepto-quark type gauge bosons can cause proton decay. To satisfy the present bound on proton decay life-time, mass of these gauge bosons ($M_{x,y}$) must be $\geq 10^{15.5}$ GeV. The proton decay, mediated by the X and Y bosons, life-time is given as: $\tau_p \sim 10^{30}(\frac{M_{x,y}}{10^{14}GeV})^4$ yrs.

The weak mixing angle $\sin^2\theta_W = g'^2/(g^2+g'^2)$ is expressed in terms of the SM gauge couplings g' and $g \equiv g_2$. Of these, the $U(1)_Y$ coupling g' is related to the coupling g_1 arising in a unified theory through $g_1^2 = c^2 g'^2$ where $c^2 = \frac{5}{3}$. In the limit of unification of all couplings at a GUT-scale, M_X, this leads to the prediction $\sin^2\theta_W(M_X) = 3/8$. After considering the RG evolution of the gauge couplings, one can find that at the low scale ($M_Z \sim 90$ GeV) $\sin^2\theta_W(M_Z)$ is given as:

$$\sin^2\theta_W(M_Z) = \frac{1}{6} + \frac{5}{9}\frac{\alpha(M_Z^2)}{\alpha_s(M_Z^2)}; \tag{3.12}$$

where $\alpha_i = g_i^2/4\pi$, which is not inconsistent with the measured low scale value of θ_W.

SM contains 15 fermions for each generation. In $SU(5)$, these fermions belong to $\overline{5}$- and 10- dimensional representations:

$$\overline{5} \equiv \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L ; \tag{3.13}$$

$$10 \equiv \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}. \tag{3.14}$$

In the above the 10-dimensional fermions are written as a 5×5 antisymmetric matrix. Minimal $SU(5)$ contains 24- and $\overline{5}$-dimensional Higgs fields. The 24-dimensional Higgs breaks $SU(5)$ to the SM directly once it acquires vacuum expectation value along the direction singlet under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. This *vev* sets the scale of the unification and because of this spontaneous symmetry breaking X and Y gauge bosons are massive. Subsequently the *vev* of $\overline{5}$ breaks the $SU(2)_L \otimes U(1)_Y$ symmetry and generates the masses for the fermions and $W^\pm, Z$ gauge bosons. Actually this $\overline{5}$ contains the SM Higgs doublet.

Though $SU(5)$ was found to be the simplest GUT group but the detailed studies of this model failed to address a few basic issues. As we have discussed in the earlier chapter, seesaw mechanism is a popular and viable candidate to

generate the light neutrino masses. And for that a very high scale (mass of the heavy particle) is associated with the theory. But in minimal $SU(5)$ there is no room for the right-handed neutrino, thus neutrino masses cannot be generated through seesaw mechanisms. In this grand unified model left-right symmetry is not realised thus the origin of parity violation at electroweak scale remains unexplained. The prime aim of a viable grand unified theory, i.e., the unification of gauge couplings is achieved at a scale that is too low in comparison with the one suggested by the experimental bounds on proton decay.
These problems provide encouragement to look for other viable gauge groups like $SO(10)$, $E(6)$ etc.

3.3 SO(10)

$SO(10)$ is a simple group of rank 5 whose generators are orthogonal matrices with determinant unity [4]. The fermions are in a 16-dimensional representation and dimensionality of the adjoint representation is 45. 16 can be decomposed in terms of the $SU(5)$ representations as:

$$16 \equiv 1 + 10 + \overline{5}. \tag{3.15}$$

Thus along with the 15 SM fermions the right-handed neutrino (ν_R) is accommodated. Because of this ν_R, the type-I seesaw is very much there to generate light neutrino masses.

As the rank of $SO(10)$ is one unit more than that of the SM, there can be several intermediate scales on the way to descend down to the SM. The Pati-Salam $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ is one of them. Through this intermediate scale left-right symmetry is realised and the parity violation at low energy is understood as an artifact of the breaking of the left-right symmetry [5]. Inclusion of the intermediate scales in the theory helps to achieve the gauge coupling unification at an appropriate high scale.

In minimal $SO(10)$ only 10-dimensional Higgs along with either 54-, 770- or 210-dimensional Higgs fields are there. When $SO(10)$ is broken via the *vev*s of either 54 or 770 then D-parity [6] [1] is kept intact, while the *vev* of 210 Higgs breaks this discrete symmetry. In the next chapter we discuss the impact of these Higgs fields on the unification boundary condition.

[1] This is a discrete symmetry that connects $SU(2)_L$ and $SU(2)_R$ multiplets.

The 10-dimensional Higgs contain a Higgs bi-doublet. The *vev* of 10 Higgs breaks the SM gauge symmetry spontaneously and generates the masses for the fermions. As all the leptons and quarks belong to a single multiplet there is a single Yukawa coupling and thus it is not sufficient to satisfy the mass relations for all. It has been noted that instead of using one Higgs field, it is useful to consider 10-, 120-, and 126-dimensional Higgs fields to generate correct fermion masses and mixings. In the last chapter we present a possible scenario to generate neutrino masses in a specific $SO(10)$ model.

3.4 E(6)

$E(6)$ is the exceptional group of rank 6 [7]. The fermions are in 27-dimensional representation, and the dimensionality of the adjoint representation is 78. $E(6)$ contains $SO(10)$ as a subgroup thus all the features in $SO(10)$ can be captured. 27 can be decomposed in terms of the $SO(10)$ representations as:

$$27 \equiv 1 + 10 + 16. \tag{3.16}$$

Thus it contains other exotic fermions that transform as 1 and 10 of $SO(10)$. These are very heavy and usually do not couple to the SM fermions. There can be several intermediate breaking patterns depending on the choices of the orientations of the symmetry breaking Higgs fields. The possible choices for these symmetry breaking Higgs fields are 650-, 2430-dimensional. The left-right symmetry can be realised through the intermediate gauge group $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$. 650-dimensional Higgs contains two directions – one is D-parity even and other one is odd, but 2430-dimensional Higgs has a single direction that respects D-parity. We calculate the effects of these different choices of Higgs fields in the next chapter.

3.5 SUSY GUTs

In the earlier chapter we have discussed SUSY as one of the most promising theories beyond the standard model. It has been noted that in SUSY theories unification improves as now the superparticles are contributing in the RGEs of the gauge couplings. The unification condition also constrains the SUSY breaking scale $\sim$ TeV. In the next chapter we discuss the ranges of the intermediate scales

for $SO(10)$ and $E(6)$, and we find that SUSY always pushes the intermediate scales towards the unification point. Thus in SUSY GUT [8] theories intermediate scales are in general not widely separated from the unification scale.
The supersymmetric $SU(5)$ is constructed by extending $SU(5)$ multiplets with SUSY multiplets. The chiral supermultiplets transform as ($\overline{5}+10$) for each generation, and vector multiplets are 24-dimensional. But unlike the non-supersymmetric case, one needs to consider both 5 and $\overline{5}$ that serve the job of H_u and H_d respectively.

In non-supersymmetric GUT the stability of the electroweak scale (M_W) is questioned because of the hierarchy between M_W and M_X. This problem is resolved by introducing supersymmetry.

In SUSY a discrete symmetry, R-parity $\equiv (-1)^{3(B-L)+2S}$, is introduced to protect proton decay, where B, L, S are the baryon, lepton, and spin quantum numbers respectively. This symmetry forbids dimension-4 proton decay operators. As either the baryon number or the lepton number conserving operator is sufficient to protect the proton to decay through the renormalisable operators, R-parity is an overconstraint. Thus R-parity violating terms are allowed in the SUSY Lagrangian, though these new couplings are highly constrained from many physics results.

References

[1] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275 [Erratum-ibid. D **11** (1975) 703].

[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.

[3] G. G. Ross, *Grand Unified Theories* (Benjamin/Cummings, Reading, USA, 1984); R. N. Mohapatra, *Unification and Supersymmetry. The frontiers of quark - lepton physics* (Springer, Berlin, Germany, 1986).

[4] H. Georgi, in *Particles and Fields – 1974*, ed. C. A. Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93** (1975) 193; T. Clark, T. Kuo and N. Nakagawa, Phys. Lett. B **115** (1982) 26; C. S. Aulakh and R. N. Mohapatra, Phys. Rev. D **28** (1983) 217.

[5] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275; R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 566; R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 2558; G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12** (1975) 1502.

[6] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072; Phys. Rev. D **30** (1984) 1052.

[7] F. Gürsey, P. Ramond and P. Sikivie, Phys. Lett. B **60** (1976) 177.

[8] U. Amaldi, W. de Boer, H. Furstenau, Phys. Lett. B **260** (1991) 447.

Chapter 4

Gauge coupling unification boundary conditions

4.1 Dimension-5 operators in gauge kinetic sector

A full quantum-theoretic treatment of gravity is not available currently. Nonetheless, it has been found useful to attempt to mimic some of its implications on grand unification through higher dimension effective contributions, suppressed by powers of the Planck mass, M_{Pl}. In a string theory setting, similar effective operators may also originate from string compactification, M_{Pl} being then replaced by the compactification scale M_c.
In this chapter we focus on the corrections to the gauge kinetic term:

$$\mathcal{L}_{kin} = -\frac{1}{4c} Tr(F_{\mu\nu} F^{\mu\nu}). \tag{4.1}$$

where $F^{\mu\nu} = \Sigma_i \lambda_i . F_i^{\mu\nu}$ is the gauge field strength tensor with λ_i being the matrix representations of the generators normalised to $Tr(\lambda_i \lambda_j) = c\ \delta_{ij}$. Conventionally, for $SU(n)$ groups the λ_i are chosen in the fundamental representation with $c = 1/2$. In the following, we will often find it convenient to utilise other representations.

The lowest order contribution from quantum gravitational (or string compactification) effects, which is what we wish to consider here, is of dimension five and has the form:

$$\mathcal{L}_{dimension-5} = -\frac{\eta}{M_{Pl}} \left[\frac{1}{4c} Tr(F_{\mu\nu} \Phi_D F^{\mu\nu}) \right], \tag{4.2}$$

where Φ_D denotes the D-component Higgs multiplet which breaks the GUT symmetry and η parametrises the strength of this interaction. In order for it to be possible to form a gauge invariant of the form in eq. 4.2, Φ_D can be in any representation included in the symmetric product of two adjoint representations of the group.

When Φ_D develops a vacuum expectation value v_D, which sets the scale of grand unification M_X and drives the symmetry breaking[1] $\mathcal{G}_{GUT} \to \mathcal{G}_1 \otimes \mathcal{G}_2 \otimes \ldots \mathcal{G}_n$, an effective gauge kinetic term is generated from eq. (4.2). Depending on the structure of the *vev*, this additional contribution, in general, will be different for the kinetic terms for the subgroups $\mathcal{G}_1, \ldots \mathcal{G}_n$. After an appropriate scaling of the gauge fields this results in a modification of the gauge coupling unification condition to:

$$g_1^2(M_X)(1+\epsilon\delta_1) = g_2^2(M_X)(1+\epsilon\delta_2) = \ldots = g_n^2(M_X)(1+\epsilon\delta_n), \qquad (4.3)$$

wherein the δ_i, $i = 1, 2, \ldots n$, are group theoretic factors which arise from eq. 4.2 and $\epsilon = \eta v_D / 2M_{Pl} \sim \mathcal{O}(M_X/M_{Pl})$. Thus, the presence of the dimension-5 terms in the Lagrangian modify the usual boundary conditions on gauge couplings, namely, that they are expected to unify at M_X.

4.2 Prescription to calculate orientations of VEVs

4.2.1 SU(5)

For $SU(5)$ the dimensionality of adjoint representation is 24. Thus Φ_D can be in the 24, 75, and 200 representations as $(24 \otimes 24)_{symm} = 1 \oplus 24 \oplus 75 \oplus 200$.

The prototype example of the vacuum expectation values found useful in the calculations is the case of $SU(5)$ with a Φ_{24} scalar. The *vev* of this field can be represented as a traceless 5×5 diagonal matrix ($Tr(\lambda_i \lambda_j) = 1/2\, \delta_{ij}$):

$$< \Phi_{24} >= v_{24} \frac{1}{\sqrt{60}}\, diag(3, 3, -2, -2, -2) \equiv v_{24} < 24 >_5 . \qquad (4.4)$$

Obviously, $< \Phi_{24} >$ can be expressed in matrix form using the representations of the generators of any other dimensionality. The group theoretic factors, δ_i,

[1]Since Φ_D arises from the symmetric product of two adjoint representations the symmetry breaking is rank preserving.

obtained therefrom, should be the same in all cases. In particular 10- and 15-dimensional forms of the *vev*, identified through the property that the resulting δ_i are the same as from (4.4), are also found useful. Under $\mathcal{G}_{SM}$ the $SU(5)$ 10 = $(1,1)_2$ + $(\overline{3},1)_{-\frac{4}{3}}$ + $(3,2)_{\frac{1}{3}}$. Noting that for the 10-dimensional representation $Tr(\lambda_i\lambda_j) = 3/2\ \delta_{ij}$, one finds:

$$< \Phi_{24} >= v'_{24}\frac{1}{\sqrt{60}}\ diag(6,-4,-4,-4,\underbrace{1,\ldots,1}_{6\ entries}) \equiv v'_{24} < 24 >_{10}\ . \qquad (4.5)$$

Under $\mathcal{G}_{SM}$ the 15 of $SU(5)$ is $(6,1)_{-\frac{4}{3}}$ + $(3,2)_{\frac{1}{3}}$ + $(1,3)_2$ and one has ($Tr(\lambda_i\lambda_j) = 7/2\ \delta_{ij}$):

$$< \Phi_{24} >= v''_{24}\frac{1}{\sqrt{60}}\ diag(\underbrace{-4,\ldots,-4}_{6\ entries},\underbrace{1,\ldots,1}_{6\ entries},6,6,6) \equiv v''_{24} < 24 >_{15}\ . \qquad (4.6)$$

(4.4), (4.5), and (4.6) yield the same δ_i if $v_{24} = v'_{24} = 9v''_{24}$.

It comes of use for the discussions of $SO(10)$ to also list the 24×24 forms of the different $SU(5)$ *vev*s. In this case $Tr(\lambda_i\lambda_j) = 5\ \delta_{ij}$.
The 24 of $SU(5)$ is $(1,1)_0$ + $(1,3)_0$ + $(8,1)_0$ + $(3,2)_{-\frac{5}{3}}$ + $(\overline{3},2)_{\frac{5}{3}}$. Thus

$$\begin{aligned} < \Phi_{24} >= &\quad v'''_{24}\sqrt{\frac{5}{252}}\ diag(2,6,6,6,\underbrace{-4,\ldots,-4}_{8\ entries},\underbrace{1,\ldots,1}_{6\ entries},\underbrace{1,\ldots,1}_{6\ entries}) \\ \equiv &\quad v'''_{24} < 24 >_{24}\ . \end{aligned} \qquad (4.7)$$

For the *vev* of the 75-dimensional representation one uses the $SU(5)$ relation: $10 \otimes \overline{10} = 1 \oplus 24 \oplus 75$. This allows the *vev* to be expressed as a 10×10 traceless diagonal matrix. Taking into consideration that $< \Phi_{75} >$ must be orthogonal to $< \Phi_{24} >$, i.e., (4.5), it can be expressed as:

$$< \Phi_{75} >= v_{75}\frac{1}{\sqrt{12}}\ diag(3,1,1,1,\underbrace{-1,\ldots,-1}_{6\ entries}) \equiv v_{75} < 75 >_{10}\ . \qquad (4.8)$$

The 24×24 form of $< \Phi_{75} >$ which yields the same δ_i as (4.8) is:

$$\begin{aligned} < \Phi_{75} >= &\quad v'_{75}\sqrt{\frac{5}{72}}\ diag(5,-3,-3,-3,\underbrace{-1,\ldots,-1}_{8\ entries},\underbrace{1,\ldots,1}_{6\ entries},\underbrace{1,\ldots,1}_{6\ entries}) \\ \equiv &\quad v'_{75} < 75 >_{24}\ . \end{aligned} \qquad (4.9)$$

Similarly, the relation $15 \otimes \overline{15} = 1 \oplus 24 \oplus 200$ permits the *vev* for Φ_{200} to be written as a 15×15 traceless diagonal matrix. Ensuring orthogonality with

$< \Phi_{24} >$, i.e., (4.6), one has:

$$< \Phi_{200} >= v_{200} \frac{1}{\sqrt{12}} \, diag(\underbrace{1, \ldots, 1}_{6 \; entries}, \underbrace{-2, \ldots, -2}_{6 \; entries}, 2, 2, 2) \equiv v_{200} < 200 >_{15} . \quad (4.10)$$

$< \Phi_{200} >$ can be also cast in a 24×24 form. Keeping (4.7), (4.9), and (4.10) in mind, it is found to be:

$$\begin{aligned} < \Phi_{200} >= & \quad v'_{200} \sqrt{\frac{5}{168}} \, diag(10, 2, 2, 2, \underbrace{1, \ldots, 1}_{8 \; entries}, \underbrace{-2, \ldots, -2}_{6 \; entries}, \underbrace{-2, \ldots, -2}_{6 \; entries}) \\ \equiv & \quad v'_{200} < 200 >_{24} . \end{aligned} \quad (4.11)$$

4.2.2 SO(10)

The dimensionality of the adjoint representation is 45 for $SO(10)$. The possible choices for Φ_D are 54, 210, and 770 as $(45 \otimes 45)_{symm} = 1 \oplus 54 \oplus 210 \oplus 770$,

The $SO(10)$ relation $10 \otimes 10 = 1 \oplus 45 \oplus 54$ ensures that $< \Phi_{54} >$ can be expressed as a 10×10 traceless diagonal matrix. It is readily checked that the normalisation condition is $Tr(\lambda_i \lambda_j) = \delta_{ij}$.

Similarly, $\overline{16} \otimes 16 = 1 \oplus 45 \oplus 210$ permits $< \Phi_{210} >$ to be represented in a 16×16 traceless diagonal form. For the 16×16 matrices $Tr(\lambda_i \lambda_j) = 2 \, \delta_{ij}$.

Finally, $< \Phi_{770} >$ can be written as a 45 × 45 matrix which is traceless and diagonal since $(45 \otimes 45)_{symm} = 1 \oplus 54 \oplus 210 \oplus 770$. Note that $< \Phi_{54} >$ and also $< \Phi_{210} >$ can be written in a similar form and orthogonality with them has to be ensured when obtaining $< \Phi_{770} >$. For these matrices $Tr(\lambda_i \lambda_j) = 8 \, \delta_{ij}$.

The above observations for $SO(10)$ are valid no matter which chain of symmetry breaking is under consideration.

SO(10) → SU(5)⊗U(1)

For Φ_{54} there is no $SU(5) \otimes U(1)_X$ invariant direction.

Under $SU(5) \otimes U(1)_X$, 16 = (1,-5) + ($\overline{5}$,3) + (10,1). Further, the diagonal matrix $< \Phi_{210} >$ must be orthogonal to the one corresponding to $U(1)_X$, i.e., $\frac{1}{2\sqrt{10}}$ *diag*(-5,3,3,3,3,3,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1). Satisfying these, we find:

$$< \Phi_{210} >= v_{210} \frac{1}{\sqrt{20}} \, diag(5, 1, 1, 1, 1, 1, \underbrace{-1, \ldots, -1}_{10 \; entries},) \equiv v_{210} < 210 >_{16} . \quad (4.12)$$

Under $SU(5) \otimes U(1)_X$ 45 = (1, 0) + (10, 4) + ($\overline{10}$, -4) + (24, 0). Asking the results from (4.12) be reproduced one arrives at:

$$\begin{aligned} < \Phi_{210} >= & \quad v'_{210}\sqrt{\frac{2}{15}}\, diag(-4, \underbrace{-1,\ldots,-1}_{10\ entries}, \underbrace{-1,\ldots,-1}_{10\ entries}, \underbrace{1,\ldots,1}_{24\ entries}) \\ \equiv & \quad v_{210} < 210 >_{45} . \end{aligned} \tag{4.13}$$

$< \Phi_{770} >$ is chosen to be singlet under $SU(5) \otimes U(1)_X$ and orthogonal to $< \Phi_{210} >$ in (4.13). It is:

$$\begin{aligned} < \Phi_{770} >= & \quad v_{770}\frac{1}{3\sqrt{5}}\, diag(16, \underbrace{-2,\ldots,-2}_{10\ entries}, \underbrace{-2,\ldots,-2}_{10\ entries}, \underbrace{1,\ldots,1}_{24\ entries}) \\ \equiv & \quad v_{210} < 770 >_{45} . \end{aligned} \tag{4.14}$$

SO(10) $\rightarrow$ SU(3)$_c\otimes$ SU(2)$_L\otimes$ U(1)$_Y\otimes$ U(1)$_X$

The case we consider in this subsection is a typical example of several symmetry breaking chains (see the $E(6)$ cases below) where the vacuum expectation values can be easily written down using the *vev*s for GUT groups which are themselves subgroups of the one under consideration. Here we exploit the findings of sec. 4.2.1 on $SU(5)$ symmetry breaking to obtain the required results providing enough details. In subsequent subsections we simple write down the results since the method is the same.

To accomplish the desired symmetry breaking the *vev* has to be assigned to a component of Φ_D which is not only a non-singlet under $SO(10)$ but also under its subgroup $SU(5)$. In fact, from the discussions in sec. 4.2.1 it must transform as a 24, 75, or 200 of $SU(5)$.

The 54-dimensional $SO(10)$ representation contains an $SU(5) \otimes U(1)_X$ (24,0) which is appropriate for the symmetry breaking under consideration. Under $SU(5) \otimes U(1)_X$ 10 = (5,2) + ($\overline{5}$,-2). Using (4.4) one finds

$$\begin{aligned} < \Phi_{54,24} > &= v'_{54}\frac{1}{\sqrt{60}}\, diag(3,3,-2,-2,-2,3,3,-2,-2,-2) \\ &= v'_{54}\, diag(< 24 >_5, < 24 >_5) \equiv v'_{54} < 54, 24 >_{10} . \end{aligned} \tag{4.15}$$

Under $SU(5) \otimes U(1)_X$ 210 $\supset$ (24,0) + (75,0).
Bearing in mind 16 = (1,-5) + ($\overline{5}$,3) + (10,-1) and employing (4.4) and (4.5)

$$\begin{aligned} < \Phi_{210,24} > &= v'_{210}\frac{1}{\sqrt{60}}\, diag(0,3,3,-2,-2,-2,6,-4,-4,-4, \underbrace{1,\ldots,1}_{6\ entries}) \\ &= v'_{210}(0, < 24 >_5, < 24 >_{10}) \equiv v'_{210} < 210, 24 >_{16} . \end{aligned} \tag{4.16}$$

Ensuring orthogonality and using (4.8) one has:

$$
\begin{aligned}
< \Phi_{210,75} > &= v''_{210}\frac{1}{3}\, diag(0,\underbrace{0,\ldots,0}_{5\ entries},3,1,1,1,\underbrace{-1,\ldots,-1}_{6\ entries}) \\
&= v'_{210}\frac{2}{\sqrt{3}}(0,\underbrace{0,\ldots,0}_{5\ entries},<75>_{10}) \equiv v'_{210} < 210,75 >_{16} . \quad (4.17)
\end{aligned}
$$

The 770 representation of $SO(10)$ contains within it (24,0), (75,0), and (200,0) submultiplets under $SU(5)\otimes U(1)_X$. As already discussed, $<\Phi_{770}>$ can be expressed as a traceless, diagonal 45×45 matrix.
Further 45 = (1,0) + (10,4) + ($\overline{10}$, -4) + (24,0). Using (4.11) one gets:

$$
\begin{aligned}
< \Phi_{770,200} > = \quad & v'''_{770}\sqrt{\frac{8}{5}}(0,\underbrace{0,\ldots,0}_{10\ entries},\underbrace{0,\ldots,0}_{10\ entries},<200>_{24}) \\
& \equiv v'''_{770} < 770,200 >_{45} . \quad (4.18)
\end{aligned}
$$

SO(10) $\rightarrow$ SU(4)$_c\otimes$SU(2)$_L\otimes$SU(2)$_R$

Under $SU(4)_c\otimes SU(2)_L\otimes SU(2)_R \equiv \mathcal{G}_{422}$, 10 ≡ (1,2,2) + (6,1,1). From the tracelessness condition one can immediately obtain

$$
< \Phi_{54} >= v_{54}\frac{1}{\sqrt{60}}\, diag(3,3,3,3,\underbrace{-2,\ldots,-2}_{6\ entries}). \quad (4.19)
$$

As noted earlier, $<\Phi_{210}>$ can be represented as a traceless and diagonal 16×16 matrix. Since 16 ≡ (4,2,1) + ($\overline{4}$,1,2) one can readily identify

$$
< \Phi_{210} >= v_{210}\frac{1}{\sqrt{8}}\, diag(\underbrace{1,\ldots,1}_{8\ entries},\underbrace{-1,\ldots,-1}_{8\ entries}), \quad (4.20)
$$

Similarly, noting 45 ≡ (15,1,1) + (1,3,1) + (1,1,3)+ (6,2,2) under $\mathcal{G}_{422}$, one can write $<\Phi_{770}>$ as:

$$
< \Phi_{770} >= v_{770}\frac{1}{\sqrt{180}}\, diag(\underbrace{-4,\ldots,-4}_{15\ entries},\underbrace{-10,\ldots,-10}_{3+3\ entries},\underbrace{5,\ldots,5}_{24\ entries}). \quad (4.21)
$$

The $<\Phi_{54}>$ and $<\Phi_{210}>$ can also be written in a similar 45×45 form and care must be taken to ensure that $<\Phi_{770}>$ is orthogonal to them.

4.2.3 E(6)

In $E(6)$, $(78 \otimes 78)_{symm} = 1 \oplus 650 \oplus 2430$, where 78 is the dimensionality of the adjoint representation and Φ_D are 650- and 2430 -dimensional.

In $E(6)$ $\overline{27} \otimes 27 = 1 \oplus 78 \oplus 650$. So, Φ_{650} can be expressed as a 27×27 traceless diagonal matrix. For this case $Tr(\lambda_i \lambda_j) = 3\ \delta_{ij}$.

Also, as already noted $(78 \otimes 78)_{symm} = 1 \oplus 650 \oplus 2430$. Hence both $< \Phi_{650} >$ and $< \Phi_{2430} >$ can be represented as 78×78 diagonal traceless matrices. For them $Tr(\lambda_i \lambda_j) = 12\ \delta_{ij}$.

E(6) → SU(2)⊗SU(6)

Both 650 and 2430 have directions which are singlets under $SU(2) \otimes SU(6)$. Under $SU(2) \otimes SU(6)$ 27 = (2,$\overline{6}$) + (1,$\overline{15}$). Therefore one can readily write $< \Phi_{650} >$ for this channel of symmetry breaking as the 27×27 diagonal traceless matrix:

$$< \Phi_{650} >= v_{650} \frac{1}{\sqrt{180}}\, diag(\underbrace{5, \ldots, 5}_{12\ entries}, \underbrace{-4, \ldots, -4}_{15\ entries}). \tag{4.22}$$

The 2430 *vev* can be written down using 78 = (3,1) + (1, 35) + (2, 20) and maintaining orthogonality with $< \Phi_{650} >$ one can write

$$< \Phi_{2430} >= v_{2430} \frac{1}{\sqrt{3640}}\, diag(70, 70, 70, \underbrace{18, \ldots, 18}_{35\ entries}, \underbrace{-21, \ldots, -21}_{40\ entries}). \tag{4.23}$$

E(6) → SO(10)⊗U(1)$_\eta$

The 650 representation has a singlet under $SO(10) \otimes U(1)$ which as before can be expressed as a 27×27 matrix. Under $SO(10) \otimes U(1)$ 27 = (1, 4) + (16, 1) + (10, -2). Using this one finds

$$< \Phi_{650} >= v_{650} \frac{1}{12\sqrt{5}}\, diag(40, \underbrace{-5, \ldots, -5}_{16\ entries}, \underbrace{4, \ldots, 4}_{10\ entries}). \tag{4.24}$$

To write down Φ_{2430} we note that the decomposition under $SO(10) \otimes U(1)$ is 78 = (1, 0) + (45, 0) + (16, -3) + ($\overline{16}$, 3). Ensuring the requirements of orthogonality to $< \Phi_{650} >$ and tracelessness we have

$$< \Phi_{2430} >= v_{2430} \frac{1}{4\sqrt{78}}\, diag(-108, \underbrace{-4, \ldots, -4}_{45\ entries}, \underbrace{9, \ldots, 9}_{16\ entries}, \underbrace{9, \ldots, 9}_{16\ entries}). \tag{4.25}$$

E(6) $\rightarrow$ SU(5)$\otimes$U(1)$_\xi\otimes$U(1)$_\eta$

The results for this option of symmetry breaking can be obtained by referring to those in sec. 4.2.2 for $SO(10) \rightarrow SU(5) \otimes U(1)_X$. Here the *vev* must be assigned to a direction which is a singlet under $SU(5) \otimes U(1)_\xi \otimes U(1)_\eta$ but not under $SO(10) \otimes U(1)_\eta$. Such possibilities are the following: 650 of $E(6)$ contains the submultiplets (54,0) and (210,0) under the latter group and the 2430 of $E(6)$ includes (210,0) and (770,0). As already noted the $SO(10)$ 54 does not have a singlet direction of $SU(5) \otimes U(1)_X$. So, we need to consider only the other possibilities.

It is useful to recall the decomposition 27 = (1,4) + (10,-2) + (16,1) under $SO(10) \otimes U(1)_\eta$. Then from (4.12) we have

$$< \Phi_{650,210} >= v_{650}\sqrt{\frac{3}{2}}\, diag(0, \underbrace{0, \ldots, 0}_{10\ entries}, < 210 >_{16}). \tag{4.26}$$

$< \Phi_{650} >$ can also be expressed as a 78$\times$78 traceless diagonal matrix. Here one uses 78 = (1,0) + (45,0) + (16, -3) + ($\overline{16}$,3) under $SO(10) \otimes U(1)_\eta$. Then using (4.12) and (4.13):

$$< \Phi_{2430,210} >= v'_{650}\, diag(0, < 210 >_{45}, < 210 >_{16}, < 210 >_{16}). \tag{4.27}$$

The remaining *vev* is $< \Phi_{2430} >$ which can be written down using (4.14)

$$< \Phi_{2430,770} >= v_{2430}\sqrt{\frac{3}{2}}\, diag(0, < 770 >_{45}, \underbrace{0, \ldots, 0}_{16\ entries}, \underbrace{0, \ldots, 0}_{16\ entries}). \tag{4.28}$$

E(6) $\rightarrow$ SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y\otimes$ U(1)$_\xi\otimes$U(1)$_\eta$

For this symmetry breaking we can utilise the results in sec. 4.2.2 for $SO(10) \rightarrow SU(3) \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. The relevant submultiplets are the following: 650 of $E(6)$ contains (54,0) and (210,0) under $SO(10) \otimes U(1)_\eta$ and the 2430 of $E(6)$ includes (54,0), (210,0) and (770,0). The desired symmetry breaking can occur through the further $SU(5)$ 24, 75, or 200 content of the $SO(10)$ multiplets, viz.,
54 $\supset$ 24; 210 $\supset$ 24 and 75; and 770 $\supset$ 24, 75 and 200.

The explicit forms of the *vev*s are not of much use since ultimately it is the $SU(5)$ representation which will fix the δ_i.

E(6) $\to$ SU(3)$_c\otimes$SU(3)$_L\otimes$SU(3)$_R$

As before, $< \Phi_{650} >$ can be written as a 27×27 matrix. It turns out that the 650 representation has two directions which are singlet under $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \equiv \mathcal{G}_{333}$. One of them is even under D-parity while the orthogonal direction is odd. These are of interest from the physics standpoint. Of course, a *vev* in any one of these directions or linear combinations thereof may be chosen to break the symmetry.

In this option of $E(6)$ symmetry breaking to $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ one has 27 = (1,$\overline{3}$,3) + (3,1,$\overline{3}$) + ($\overline{3}$,3,1). The D-even case is:

$$< \Phi_{650} >= v_{650} \frac{1}{\sqrt{18}}\, diag(\underbrace{-2,\ldots,-2}_{9\ entries},\underbrace{1,\ldots,1}_{9\ entries},\underbrace{1,\ldots,1}_{9\ entries}). \tag{4.29}$$

while the D-odd *vev* is

$$< \Phi'_{650} >= v'_{650} \frac{1}{\sqrt{6}}\, diag(\underbrace{0,\ldots,0}_{9\ entries},\underbrace{1,\ldots,1}_{9\ entries},\underbrace{-1,\ldots,-1}_{9\ entries}). \tag{4.30}$$

As in the other cases, $< \Phi_{2430} >$ can be written as a 78×78 traceless diagonal matrix. Noting that 78 = (8,1,1) + (1,8,1) + (1,1,8) + (3,3,$\overline{3}$) + ($\overline{3}$,$\overline{3}$,3) and maintaining orthogonality with $< \Phi_{650} >$ and $< \Phi'_{650} >$ one can write

$$< \Phi_{2430} >= v_{2430} \frac{1}{\sqrt{234}}\, diag(\underbrace{9,\ldots,9}_{8\ entries},\underbrace{9,\ldots,9}_{8\ entries},\underbrace{9,\ldots,9}_{8\ entries},\underbrace{-4,\ldots,-4}_{27\ entries},\underbrace{-4,\ldots,-4}_{27\ entries}). \tag{4.31}$$

4.3 Corrections from dimension-5 operators to the unification conditions

In this section we calculate the corrections, δ_i, which arise due to the dimension-5 operators for different orientations of the Higgs fields discussed in the earlier section.

4.3.1 SU(5) GUT

Here, we summarise the results for $SU(5)$ [1,2]. Φ_D can be in the 24-, 75- or 200-dimensional representation of $SU(5)$ and the symmetry is broken to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$.

$SU(5)$ Representations	δ_1	δ_2	δ_3	N
24	1	3	-2	$2/\sqrt{15}$
75	5	-3	-1	$8/15\sqrt{3}$
200	10	2	1	$1/35\sqrt{21}$

Table 4.1: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. N is an overall normalisation which has been factored out from the δ_i.

The procedure to obtain these results [2] is to express $< \Phi_D >$ as a diagonal matrix of dimensionality of some $SU(5)$ irreducible representation as has been done in the earlier section. From the $\mathcal{G}_{SM}$ structure of this representation, the δ_i can be read off.

The δ_i arising in the different cases are listed in Table 4.1.

4.3.2 SO(10) GUT

$SO(10)$ [3] offers the option of descending to $\mathcal{G}_{SM}$ through a left-right symmetric route [4] – the intermediate Pati-Salam $\mathcal{G}_{PS} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ – or *via* $SU(5) \otimes U(1)_X$ or in one step to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. The effects of dimension-5 interactions for different breaking patterns of $SO(10)$ are discussed in the next sections .

SO(10) $\rightarrow$ SU(5)$\otimes$U(1)

Under $SU(5) \otimes U(1)_X$ the $SO(10)$ spinorial representation decomposes[2] as follows: 16 $\equiv$ (1,-5) + ($\overline{5}$,3) + (10,-1). The SM families belong to this representation. The particle assignments within the 16-plet can be chosen in two distinct ways with different physics consequences: (a) **conventional SU(5)**: $U(1)_X$ commutes with the SM, so the low scale hypercharge ($U(1)_Y$) is the same as the $U(1)_{Y'}$ in

[2]The correctly normalised ($Tr(\lambda_i \lambda_j) = 2\ \delta_{ij}$) $U(1)_X$ charges are obtained by multiplying the displayed quantum numbers by a factor of $\frac{1}{2\sqrt{10}}$.

$SU(5)$; e.g., for the ($\overline{5}$,3) multiplet $T_{Y'} \equiv \sqrt{\frac{3}{5}}\, diag(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$. The SM generators are entirely within the $SU(5)$ and a singlet under it is uncharged. Therefore, the (1,-5) submultiplet has to be identified with the neutral member in the 16-plet, the ν_i^c (i = 1,2,3). The other option is (b) **flipped SU(5)**: Here $U(1)_{Y'}$ and $U(1)_X$ combine to give $U(1)_Y$: $T_Y = -(2\sqrt{6}\, T_X + T_{Y'})/5$ [6]. The difference can be illustrated using the ($\overline{5}$,3) multiplet. For it the $U(1)_{Y'}$ assignment is, as before, $T_{Y'} \equiv \sqrt{\frac{3}{5}}\, diag(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$ while the *normalised* $U(1)_X$ is $\frac{3}{2\sqrt{10}}$ so that $T_Y \equiv \sqrt{\frac{3}{5}}\, diag(-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, -\frac{1}{2}, -\frac{1}{2})$. Thus, this submultiplet now contains (u_i^c, L_i) rather than the usual (d_i^c, L_i). The (1,-5) state is $SU(3)_c \otimes SU(2)_L$ singlet but carries a non-zero hypercharge, $Y = 1$. The only particle that satisfies this requirement is l_i^c.

The complete particle assignments for the first generation in the two options are:

(a) For conventional $SU(5)$:

$$(\mathbf{1}, -\mathbf{5}) \;=\; \nu_1^c\,, \;\; (\overline{\mathbf{5}}, \mathbf{3}) \;=\; (d_1^c,\, l_1)\,, \;\; (\mathbf{10}, -\mathbf{1}) \;=\; (q_1,\, u_1^c,\, e_1^c)\,, \tag{4.32}$$

and (b) for flipped $SU(5)$:

$$(\mathbf{1}, -\mathbf{5}) \;=\; e_1^c, \;\; (\overline{\mathbf{5}}, \mathbf{3}) \;=\; (u_1^c,\, l_1)\,, \;\; (\mathbf{10}, -\mathbf{1}) \;=\; (q_1,\, d_1^c,\, \nu_1^c)\,, \tag{4.33}$$

where q and l are respectively the left-handed quark and lepton doublets, u^c, d^c, e^c, and ν^c are the CP conjugated states corresponding to the right-handed up-type quark, down-type quark, lepton, and neutrino, respectively.

In $SO(10)$ GUT, at the unification scale one has $g_5 = g_1$. The presence of any dim-5 effective interactions of the form of eq. 4.2 will affect this relation generating corrections as shown in eq. 4.3 which in this case will involve two parameters δ_5 and δ_1.

As noted earlier, Φ_D can be chosen only in the 54, 210, and 770-dimensional representations. Of these, the 54 does not have an $SU(5) \otimes U(1)$ singlet. So, only the 210- and 770-dimensional cases need examination.

Using $(\overline{16} \otimes 16) = 1 \oplus 45 \oplus 210$, $< \Phi_{210} >$ can be expressed as a 16-dimensional traceless diagonal matrix. The form of this *vev* for this symmetry breaking is given in (4.12). It yields $\delta_5 = -\frac{1}{4\sqrt{5}}$ and $\delta_1 = \frac{1}{\sqrt{5}}$.

In a similar fashion the *vev* of Φ_{770} can be written as the 45×45 diagonal traceless matrix in (4.14). This results in $\delta_5 = -\frac{1}{24\sqrt{5}}$, $\delta_1 = -\frac{2}{3\sqrt{5}}$.

$SO(10)$ Representations	δ_5	δ_1	N
210	-1	4	$1/4\sqrt{5}$
770	1	16	$-1/24\sqrt{5}$

Table 4.2: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $SO(10) \to SU(5) \otimes U(1)$. N is an overall normalisation which has been factored out from the δ_i.

$SO(10)$ Representations	δ_1	δ_2	δ_3	δ_{1X}	N
54 (24)	1	3	-2	0	$1/2\sqrt{15}$
210 (24)	1	3	-2	0	$1/4\sqrt{15}$
210 (75)	5	-3	-1	0	$1/12$
770 (24)	1	3	-2	0	$2/\sqrt{15}$
770 (75)	5	-3	-1	0	$8/15\sqrt{3}$
770 (200)	10	2	1	0	$-1/8\sqrt{21}$

Table 4.3: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $SO(10) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. $SU(5)$ subrepresentations are indicated within parentheses. N is an overall normalisation which has been factored out from the δ_i.

The results for this chain of symmetry breaking are summarised in Table 4.2. The δ_i are completely group theoretic in nature and obviously do not depend on whether the particle assignments follow the conventional or flipped $SU(5)$.

SO(10) $\to$ SU(3)$_c\otimes$ SU(2)$_L\otimes$ U(1)$_Y\otimes$ U(1)$_X$

The unification condition in the presence of dimension-5 effective interactions of the form of eq. 4.2 will now involve the parameters δ_i (i = 1,2,3) as for $SU(5)$ and an additional one δ_{1X}.

In order to break $SO(10)$ directly to $\mathcal{G}_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$

the *vev* must be a non-singlet not just under $SO(10)$ but also under $SU(5)$. The decompositions of $SO(10)$ representations under $SU(5) \otimes U(1)$ are useful for identifying these *vev*s. The calculation can be considerably simplified by using the $SU(5)$ symmetry breaking patterns at our disposal from sec. 4.3.1. One simply has to look for 24, 75, and 200 submultiplets within the 54, 210, and 770 $SO(10)$ multiplets.

The 54 representation of $SO(10)$ has a singlet under $\mathcal{G}_{3211}$ which is contained in a 24 of $SU(5)$. The *vev* for this case is shown in (4.15) and the contributions to the δ_i can be immediately read off from the $SU(5)$ result in Table 4.1. These δ_i are listed in Table 4.3.

Notice, that in this case the effect of dimension-5 terms cannot distinguish between an $SU(5)$ theory with $< \Phi_{24} >$ driving the symmetry breaking and an $SO(10)$ one with $< \Phi_{54} >$. For Φ_{210} or Φ_{770} the situation is different as they have multiple $\mathcal{G}_{3211}$ singlet directions.

Φ_{210} has three directions which are all singlets under $\mathcal{G}_{3211}$. Of these one is also an $SU(5)$ singlet. In the subspace defined by them, three convenient orthogonal directions can be identified, all singlets under $U(1)_X$, and corresponding to 1-, 24- and 75-directions of the $SU(5)$ subgroup. If the *vev* is along one of these directions[3] it can be simply read off from the results of sec. 4.3.1. The *vev*s corresponding to the 24 and 75 directions are given in (4.16) and (4.17). The δ_i derived therefrom are shown in Table 4.3. In the most generic situation one can write $< \Phi_{210} > = \alpha_1 < \Phi_{210,1} > + \alpha_{24} < \Phi_{210,24} > + \alpha_{75} < \Phi_{210,75} >$, where the α_i are complex numbers and the concomitant δ_i will be appropriately weighted combinations of the above results.

$< \Phi_{770} >$ has four $\mathcal{G}_{3211}$ invariant directions which can be classified under the $SU(5)$ representations 1, 24, 75, and 200. The results for these are also shown in Table 4.3. Here again, in general, the *vev* may lie in an arbitrary direction in the space spanned by the four $SU(5)$-identified ones and the resultant δ_i can be readily obtained from the above.

Unlike the case of $< \Phi_{54} >$ where the singlet direction is unique, the other possible *vev*s, $< \Phi_{210} >$ and $< \Phi_{770} >$, provide a more general option and therefore the predictions for the δ_i are not unique but cover a range. In this sense the

[3]For the $SU(5)$ singlet direction the δ_i are all equal. A *vev* in this direction alone will not break $SO(10)$ to $\mathcal{G}_{3211}$.

$SO(10)$ Representations	δ_1	δ_2	δ_3	N
54 (24)	1	3	-2	$1/2\sqrt{15}$
210 (1)	-19/5	1	1	$-1/4\sqrt{5}$
210 (24)	-7/5	3	-2	$1/4\sqrt{15}$
210 (75)	1/5	-3	-1	1/12
770 (1)	77/5	1	1	$-1/24\sqrt{5}$
770 (24)	29/5	3	-2	$2/\sqrt{15}$
770 (75)	1/5	-3	-1	$8/15\sqrt{3}$
770 (200)	2/5	2	1	$-1/8\sqrt{21}$

Table 4.4: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $SO(10) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ (flipped $SU(5)$). $SU(5)$ subrepresentations are indicated within parentheses. N is an overall normalisation which has been factored out from the δ_i.

model becomes less predictive[4].

This route of symmetry breaking of $SO(10)$ *does not* admit the flipped $SU(5)$ option by itself since in that case the $U(1)_X$ combines with a $U(1)$ subgroup of $SU(5)$ to produce $U(1)_Y$ and thus $SO(10)$ is broken to $\mathcal{G}_{SM}$, which is of rank 4, not 5. So this symmetry breaking will have to be through some other $SO(10)$ scalar multiplet. Nonetheless, assuming that such a symmetry breaking is operational, we may ask what would be the impact of the *vev*s of Φ_{54}, Φ_{210}, and Φ_{770} of this subsection on the unification parameters δ_i, $i = 1, 2, 3$. Using the *vev*s used before and noting that $U(1)_Y$: $T_Y = -(2\sqrt{6}\, T_X + T_{Y'})/5$ one finds the results presented in Table 4.4.

SO(10) $\to$ SU(4)$_c \otimes$SU(2)$_L \otimes$SU(2)$_R$

Left-right symmetry is realised through this intermediate breaking pattern of $SO(10)$. Φ_D can be chosen only in the 54-, 210-, and 770-dimensional representa-

[4]This also applies to the non-universality options for gaugino masses in supersymmetric theories.

$SO(10)$ Representations	δ_{4c}	δ_{2L}	δ_{2R}	N
54	-2	3	3	$1/2\sqrt{15}$
210	0	1	-1	$1/2\sqrt{2}$
770	2	5	5	$1/24\sqrt{5}$

Table 4.5: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $SO(10) \to SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. N is an overall normalisation which has been factored out from the δ_i.

tions, as in the earlier section, ensuring that $< \Phi_D >$ leaves $\mathcal{G}_{422}$ unbroken.

For Φ_{54} the appropriate *vev* is given in (4.19) and this results in $\delta_{4c} = -\frac{1}{\sqrt{15}}$ and $\delta_{2L} = \delta_{2R} = \frac{3}{2\sqrt{15}}$. Notice that this correction to unification ensures that $g_{2L}(M_X) = g_{2R}(M_X)$, i.e., D-parity [11] is preserved.

A 16×16 form of $< \Phi_{210} >$ is given in (4.20) from which one can calculate δ_{4c} =0 and $\delta_{2L} = -\delta_{2R} = \frac{1}{2\sqrt{2}}$. D-parity is broken through $< \Phi_{210} >$ and thus $g_{2L}(M_X) \neq g_{2R}(M_X)$ though $SU(2)_L \otimes SU(2)_R$ remains unbroken at M_X.

The final option is Φ_{770}. One can write the *vev* in terms of a 45-dimensional diagonal traceless matrix and this is given in (4.21). From this one finds $\delta_{4c} = \frac{1}{12\sqrt{5}}$ and the D-parity conserving $\delta_{2L} = \delta_{2R} = \frac{5}{24\sqrt{5}}$. The results for this chain of $SO(10)$ breaking are collected together in Table 4.5.

4.3.3 E(6) GUT

$E(6)$ can have different subgroups of same rank: $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$, $SU(3)_c \otimes SU(2)_L \otimes U(1)_L \otimes SU(2)_R \otimes U(1)_R$, $SU(2) \otimes SU(6)$, $SO(10) \otimes U(1)_\eta$, $SU(5) \otimes U(1)_\xi \otimes U(1)_\eta$, and $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_\xi \otimes U(1)_\eta$. All of these intermediate gauge groups accommodate $\mathcal{G}_{SM}$ as a subgroup and lead to different low scale phenomenology. For $E(6)$ the adjoint representation is 78-dimensional. We note that Φ_D can be either 650- or 2430-dimensional as $(78 \otimes 78)_{symm} = 1 \oplus 650 \oplus 2430$. Both of them contain singlets under the above mentioned intermediate gauge groups we are interested in.

$E(6)$ Representations	δ_2	δ_6	N
650	5	-1	$1/6\sqrt{5}$
2430	-35	-9	$1/12\sqrt{910}$

Table 4.6: Effective contributions (see eq. 4.3) to gauge kinetic terms from different Higgs representations in eq. 4.2 for $E(6) \to SU(2) \otimes SU(6)$. N is an overall normalisation which has been factored out from the δ_i.

E(6) $\to$ SU(2)$\otimes$SU(6)

The inconsistency of the Georgi-Glashow $SU(5)$ model with the proton decay and gauge unification requirements has been a motivation to seek alternative GUT models. $SU(6)$ is one of them. It can naturally guarantee strong-CP invariance and a supersymmetrised version implements doublet-triplet splitting by the missing *vev* mechanism [9].

The subgroups from the breaking $E(6) \to SU(2) \otimes SU(6)$ have been identified in several physically distinct manners: $SU(2)_R \otimes SU(6)'$, $SU(2)_L \otimes SU(6)''$, and $SU(2)_X \otimes SU(6)$. The results that we discuss are valid irrespective of these alternative interpretations.

The contributions from the 650-dimensional representation for this symmetry breaking chain can be obtained from eq. 4.22. One finds $\delta_2 = \frac{5}{6\sqrt{5}}$ and $\delta_6 = -\frac{1}{6\sqrt{5}}$,

For the 2430-dimensional $E(6)$ representation the *vev* is given in (4.23). From it we get $\delta_2 = -\frac{35}{12\sqrt{910}}$, $\delta_6 = -\frac{9}{6\sqrt{910}}$. The results for this symmetry breaking chain can be found in Table 4.6.

E(6) $\to$ SO(10)$\otimes$U(1)$_\eta$

$E(6)$ contains $SO(10) \otimes U(1)$ as a maximal subgroup. Breaking patterns based on this chain are well discussed in the literature [8]. Here, we consider the effect of dimension-5 operators on the gauge unification condition.

$< \Phi_{650} >$ is given in (4.24). From it one obtains $\delta_{10} = -\frac{1}{6\sqrt{5}}$, $\delta_1 = \frac{5}{6\sqrt{5}}$. Using (4.25) for $< \Phi_{2430} >$ one can similarly get $\delta_{10} = \frac{1}{72\sqrt{26}}$, $\delta_1 = \frac{3}{8\sqrt{26}}$. These results are listed in Table 4.7.

$E(6)$ Representations	δ_{10}	δ_1	N
650	-1	5	$1/6\sqrt{5}$
2430	1	27	$1/72\sqrt{26}$

Table 4.7: Effective contributions (see eq. (4.3)) to gauge kinetic terms from different Higgs representations in eq. (4.2) for $E(6) \rightarrow SO(10) \otimes U(1)$. N is an overall normalisation which has been factored out from the δ_i.

$\mathbf{E(6)} \rightarrow \mathbf{SU(5)}\otimes\mathbf{U(1)}_\xi\otimes\mathbf{U(1)}_\eta$

The results in this case are very similar to that for sec. 4.3.2. There it was noted that the $SO(10)$ 210 and 770 representations contain singlets under $SU(5) \otimes U(1)_X$ and the δ_5 and δ_1 in the two cases were presented in Table 4.2. These results can be immediately taken over for the current case with the change that the $U(1)_X$ is here termed $U(1)_\xi$ and that $\delta_\eta = 0$ in all cases.

The two relevant representations of $E(6)$ are 650 and 2430. Of these, 650 contains a (210,0) submultiplet under $SO(10) \otimes U(1)_\eta$. So for the 650 the δ_i will be exactly as for the 210 in Table 4.2.

The $E(6)$ 2430 representation contains both the (210,0) as well as the (770,0) within it. If the *vev* is assigned to any one of these directions the resultant δ_i will be as in the respective case in Table 4.2. In general, the *vev* will be a superposition of these two and so the δ_i will be the appropriately weighted value.

$\mathbf{E(6)} \rightarrow \mathbf{SU(3)}_c\otimes\mathbf{SU(2)}_L\otimes\mathbf{U(1)}_Y\otimes\ \mathbf{U(1)}_\xi\otimes\mathbf{U(1)}_\eta$

As for the previous subsection, this alternative can be disposed off straightforwardly using the results of sec. 4.3.2. This time there is one extra step. In sec. 4.3.2 results are presented for the $SO(10)$ representations 54, 210, and 770. They can be immediately taken over by noting that the $E(6)$ 650 contains (54,0) and (210,0) submultiplets while the 2430 contains (54,0), (210,0), and (770,0).

The main changes compared to sec. 4.3.2 are that the $U(1)_X$ there is called $U(1)_\xi$ here and for all cases $\delta_\xi = \delta_\eta = 0$. For the 650 representation if the *vev* is chosen along either the (54,0) or the (210,0) directions then the results of Table 4.3 apply directly. In general, of course, the δ_i will be a weighted combination of

$E(6)$ Representations	δ_{3c}	δ_{3L}	δ_{3R}	N
650	-2	1	1	$-1/6\sqrt{2}$
650′	0	1	-1	$1/2\sqrt{6}$
2430	1	1	1	$-1/4\sqrt{26}$

Table 4.8: Effective contributions (see eq. (4.3)) to gauge kinetic terms from different Higgs representations in eq. (4.2) for $E(6) \to SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$. Note that there are two $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ singlet directions in 650. N is an overall normalisation which has been factored out from the δ_i.

these. Similar conclusions can be drawn about the $< \Phi_{2430} >$ except that here, in general, the δ_i will be a linear combination of the ones in Table 4.3.

E(6) $\to$ SU(3)$_c\otimes$SU(3)$_L\otimes$SU(3)$_R$

This breaking chain possesses the left-right symmetry [4]. A Z_2 symmetry – D-Parity – is assumed to be active between $SU(3)_L$ and $SU(3)_R$. The *vev* $< \Phi >$ can be classified by its D-Parity behaviour. $< \Phi_{650} >$ has two directions which are singlets under $\mathcal{G}_{333}$ and are even, and odd under D-Parity.

The form of $< \Phi_{650} >$ is given in (4.29) for the D-Parity even case while (4.30) is for the D-parity odd *vev*. This results in $\delta_{3c} = -\frac{1}{3\sqrt{2}}$ and $\delta_{3L} = \delta_{3R} = \frac{1}{6\sqrt{2}}$ for the former and δ_{3c} =0, $\delta_{3L} = -\delta_{3R} = \frac{1}{2\sqrt{6}}$ for the latter. $< \Phi_{2430} >$ is listed in (4.31). From it one can readily read off $\delta_{3c} = \delta_{3L} = \delta_{3R} = -\frac{1}{4\sqrt{26}}$. In Table 4.8 we collect the findings for the different representations of $E(6)$. It is worth remarking that the three $SU(3)$ subgroups in this chain are on an equal footing. It is possible to relate *any* two of them through a Z_2-type discrete symmetry. For the purpose of illustration and for phenomenological interest we have identified it with D-Parity. Obviously, one could just as well choose the Z_2-type symmetry to be between $SU(3)_c$ and $SU(3)_L$ (or $SU(3)_R$). The symmetry breaking *vev*s of Φ_{650}, either even or odd under this changed parity-like symmetry, are simply linear combinations between the *vev*s which are odd and even under D-Parity discussed above.

References

[1] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52** (1984) 875; C. T. Hill, Phys. Lett. B **135** (1984) 47; L. J. Hall and U. Sarid, Phys. Rev. Lett. **70** (1993) 2673.

[2] J. Chakrabortty and A. Raychaudhuri, Phys. Lett. B **673** (2009) 57.

[3] H. Georgi, in *Particles and Fields – 1974*, ed. C. A. Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93** (1975) 193; T. Clark, T. Kuo and N. Nakagawa, Phys. Lett. B **115** (1982) 26; C. S. Aulakh and R. N. Mohapatra, Phys. Rev. D **28** (1983) 217.

[4] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275; R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 566; R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11** (1975) 2558; G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12** (1975) 1502.

[5] J. Chakrabortty and A. Raychaudhuri, Phys. Rev. D **81** (2010) 055004.

[6] S.M. Barr, Phys. Lett. B **112** (1982) 219; I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231.

[7] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072; Phys. Rev. D **30** (1984) 1052.

[8] F. Gürsey, P. Ramond and P. Sikivie, Phys. Lett. B **60** (1976) 177.

[9] J. E. Kim, Phys. Lett. B **107** (1981) 69; A. Sen, Phys. Rev. D **31** (1985) 900; Z. Chacko and R. N. Mohapatra, Phys. Lett. B **442** (1998) 199.

Chapter 5

Impact of dimension-5 operators on gauge coupling unification

5.1 Renormalisation group equations

The generic RG equations governing gauge coupling evolution are:

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i, g_j), \quad (i, j = 1, \ldots, n), \tag{5.1}$$

where n is the number of couplings in the theory. At one-loop order β_i depends on g_i only and for a gauge theory involving fermions and scalars is given by:

$$\beta_i(g_i) = \frac{g_i^3}{16\pi^2}[\frac{2}{3}T(R_i)d(R_j) + \frac{1}{3}T(S_i)d(S_j) - \frac{11}{3}C_2(G_i)]. \tag{5.2}$$

The fermions and scalars transform according to the representations R_i and S_i with respect to G_i respectively. T(R) is expressed as: $C_2(R)d(R) = T(R)r$, where $C_2(R)$ is the quadratic Casimir operator for the representation R, d(R) is the dimension of the representation and r is the number of generators of the group. $C_2(G)$ is the quadratic Casimir for the adjoint representation. For the SM particle content β_i were used for the first time to estimate M_X in [1].

In general the RGEs of the gauge couplings also include contributions from the Yukawa couplings. The Yukawa couplings being mostly small result in negligible contributions. Among them only the top quark Yukawa coupling is significant and its contribution is comparable to the two-loop contributions coming from the gauge couplings. But in our study we have checked and noticed that for all the gauge coupling running equations the numerical contributions for the two-loop case do not change significantly the one-loop results. Thus in the numerical analysis we ignore the Yukawa coupling contributions.

$$\beta_i(g_i, g_j) = (16\pi^2)^{-1} b_i g_i^3 + (16\pi^2)^{-2} \sum_{j=1}^{n} b_{ij} g_j^2 g_i^3. \tag{5.3}$$

The two-loop coefficients b_{ij} for non-SUSY and SUSY theories can be found in [2]. When using this two-loop formula, the matching of the coupling constant $\alpha_k = g_k^2/4\pi$ below an intermediate scale M_I which goes over to α_l thereafter follows the relation [3,4]:

$$\frac{1}{\alpha_k(M_I)} - \frac{C_k}{12\pi} = \frac{1}{\alpha_l(M_I)} - \frac{C_l}{12\pi}, \tag{5.4}$$

where C_k is the quadratic Casimir for the k-th subgroup. At the unification scale, M_X, the above condition has to be supplemented with the contributions from the dimension-5 operators in eq. 4.3. We consider both non-supersymmetric (non-SUSY) as well as supersymmetric (SUSY) versions of the theory. In the latter case the contributions of the superpartners to the beta functions are included (We assume that the SUSY scale is at M_{SUSY} = 1 TeV.). As is well-known [5], unification of coupling constants is compatible with TeV-scale supersymmetry. We find that addition of the dimension-5 contributions does not spoil this. It is also observed that the requirement of unification pushes any intermediate scale towards high values forbidding observable $n - \overline{n}$ oscillations[1].

5.2 Intermediate scales

In this chapter we explore the possibilities of different unification scenarios with and without intermediate scales. We consider the cases for $SU(5)$, $SO(10)$, and $E(6)$.

5.2.1 No intermediate scale

This is the case when the GUT gauge group is directly broken to the SM.

[1]For exceptional cases see the discussion on one and two intermediate scales for $SO(10)$ in the non-SUSY scenarios in the later sections.

Results for $SU(5)$

As the ranks of $SU(5)$ and the SM gauge group are the same, there cannot be any intermediate gauge group. We incorporate the impact of the dimension-5 operator to find whether unification is achieved or not. The shift in the gauge couplings as dictated by eq. 4.3 leaves its mark at low energies through the Renormalisation Group equations. Moreover, besides the low energy value of the weak mixing angle $\sin^2\theta_W$, even its GUT-level prediction is affected. In chapter 2 we discuss the GUT prediction of $\sin^2\theta_W(M_X) = 3/8$. Now, due to the modified GUT relationship of eq. 4.3 one has for the weak mixing angle $\hat{\theta}_W$:

$$\sin^2\hat{\theta}_W(M_X) = \frac{3}{8} + \frac{15}{64}\epsilon(\delta_2 - \delta_1). \tag{5.5}$$

The experimentally determined value of $\sin^2\theta_W$ at low energies receives further RG-dependent corrections to which we now turn.

In this case the β-coefficients b_i and b_{ij} are:

$$b_1 = \frac{1}{10}n_H + \frac{4}{3}n_G; \quad b_2 = \frac{1}{6}n_H + \frac{4}{3}n_G - 22/3; \quad b_3 = \frac{4}{3}n_G - 11, \tag{5.6}$$

and

$$\begin{aligned} b_{ij} = \quad & n_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} + n_G \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix} \\ & + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix}. \end{aligned} \tag{5.7}$$

and n_H and n_G are respectively the number of Higgs doublets and the number of fermion generations in the theory. The RG equations must satisfy the boundary conditions set by eq. 4.3 on the $g_i^2(M_X)$.

In our numerical analyses below we show the full two-loop RG equation results. For ease of discussion if only the lowest order contributions are retained, then in the absence of dimension-5 operators ($\alpha_i = g_i^2/4\pi$):

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_X)} + \frac{2b_i}{2\pi}\ln\left[\frac{M_X}{\mu}\right], \quad (i = 1, 2, 3). \tag{5.8}$$

These equations can be combined to yield:

$$\frac{\alpha}{2\pi}\ln\frac{M_X}{M_Z} = \left[\frac{3}{5b_1 + 3b_2 - 8b_3}\right]\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right\}, \tag{5.9}$$

$SU(5)$ Representations	ϵ (from eq. 5.13)	ϵ (using eq. 5.1)	M_X (GeV)
24	0.087	0.088	5.01×10^{13}
75	-0.048	-0.045	4.79×10^{15}
200	-1.92	-1.40	1.05×10^{18}

Table 5.1: $SU(5)$ dimension-5 interaction strength ϵ and the gauge unification scale, M_X, for different Φ representations for non-SUSY one-loop case.

and therefrom

$$\sin^2\theta_W(M_Z) = \frac{3}{8} - \frac{15}{8}\left[\frac{b_1 - b_2}{5b_1 + 3b_2 - 8b_3}\right]\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right\}, \quad (5.10)$$

where α – the fine structure constant – and α_3 are the couplings at the scale M_Z.

Inclusion of the boundary condition, eq. 4.3, dictated by the dimension-5 interactions, alters eqs. 5.9 and 5.10 to:

$$\begin{aligned}\frac{\alpha}{2\pi}\ln\frac{\hat{M}_X}{M_Z} &= \left[\frac{3}{5b_1 + 3b_2 - 8b_3}\right]\left\{1 - \frac{8}{3}\frac{\alpha}{\alpha_3}\right\} \\ &+ \left(\frac{\epsilon}{5b_1 + 3b_2 - 8b_3}\right)\left[\frac{-3(8\delta_3 - 3\delta_2 - 5\delta_1)b_3}{5b_1 + 3b_2 - 8b_3} - (5\delta_1 + 3\delta_2)\frac{\alpha}{\alpha_3}\right] \\ &+\mathcal{O}(\epsilon^2),\end{aligned} \quad (5.11)$$

and

$$\begin{aligned}\sin^2\hat{\theta}_W(M_Z) &= \frac{3(1+\epsilon\delta_2)}{8+\epsilon(3\delta_2+5\delta_1)} \\ &- \left(\frac{5(1+\epsilon\delta_1)(1+\epsilon\delta_2)}{8+\epsilon(3\delta_2+5\delta_1)}\right)\left[\frac{b_1}{1+\epsilon\delta_1} - \frac{b_2}{1+\epsilon\delta_2}\right] \\ &\frac{3(1+\epsilon\delta_3) - [8+\epsilon(3\delta_2+5\delta_1)][\alpha/\alpha_3]}{(5b_1+3b_2)(1+\epsilon\delta_3) - [8+\epsilon(3\delta_2+5\delta_1)]b_3},\end{aligned} \quad (5.12)$$

which reduces to eq. 5.10 in the appropriate limit. In fact,

$$\begin{aligned}\sin^2\hat{\theta}_W(M_Z) &= \sin^2\theta_W(M_Z) \\ &- \epsilon\left[\frac{5[\delta_1(b_3-b_2)+\delta_2(b_1-b_3)+\delta_3(b_2-b_1)]}{(5b_1+3b_2-8b_3)^2}\right] \\ &\left\{3b_3 - (5b_1+3b_2)\frac{\alpha}{\alpha_3}\right\} + \mathcal{O}(\epsilon^2).\end{aligned} \quad (5.13)$$

The first term on the r.h.s. of eq. 5.13 is fixed from eq. 5.10. From the measured value of $\sin^2\theta_W$ [8] one can extract the value of ϵ. These are presented for the different Φ representations in Table 5.1.

These $\mathcal{O}(\epsilon)$ one-loop analytic results can be cross-checked using the full RG equations with $n_G = 3$ and $n_H = 1$. Using the low energy ($\sim M_Z$) measured values [8], $\sin^2\theta_W$ = 0.231 19(14) and α_3 = 0.11 76(20), the RG equations can be numerically integrated. The scale M_X is fixed through the requirement that the modified unification condition, eq. 4.3, is satisfied there. From this analysis one can determine ϵ and M_X. The conclusions from one-loop RG running are shown in Table 5.1 and Fig. 5.1.

The two-loop results, shown as insets in Fig. 5.1, incorporate the proper matching conditions eq. 5.4 as well as eq. 4.3 at M_X, namely,

$$\frac{1}{\alpha_i(M_X)(1+\epsilon\delta_i)} - \frac{C_i}{12\pi} = \text{constant, independent of } i, \quad \text{for } i = 1,2,3. \quad (5.14)$$

It is noteworthy that the results are not significantly affected and the coupling constants still unify. The unification scales (obtained using two-loop evolutions), M_X, 5.01 $\times 10^{13}$, 2.09 $\times 10^{15}$, and 3.02 $\times 10^{17}$ GeV respectively for Φ_{24}, Φ_{75}, and Φ_{200}, are consistent with one-loop results given in Table 5.1. Though unification is achieved within the Planck scale for all three choices, for Φ_{24} and Φ_{75} the results are not consistent with the existing limits from proton decay. Thus only a 5-dimensional operator with Φ_{200} yields a viable solution.

In [9], noting that the dimension-5 operator in eq. 4.2 with Φ_{24} cannot by itself provide satisfactory gauge unification, it has been proposed that including gravitational contributions in the beta functions can help ameliorate this problem. Alternatively, within SUSY $SU(5)$ it has been argued in [10] that one-loop (as well as two-loop) RG evolution with Φ_{24}-driven boundary conditions in eq. 4.3 can yield satisfactory unification solutions provided the possible modification of the Planck scale itself due to the large number of GUT fields is given consideration.

It is well known that gauge coupling unification is possible if SUSY is manifested at the TeV scale [5]. If dimension-5 interactions are also present then that will further affect this unification. In fact, it was shown within SUSY SU(5) that if the δ_i $(i = 1,2,3)$ in eq. 4.3 are fixed as determined (see Table 4.1) by the

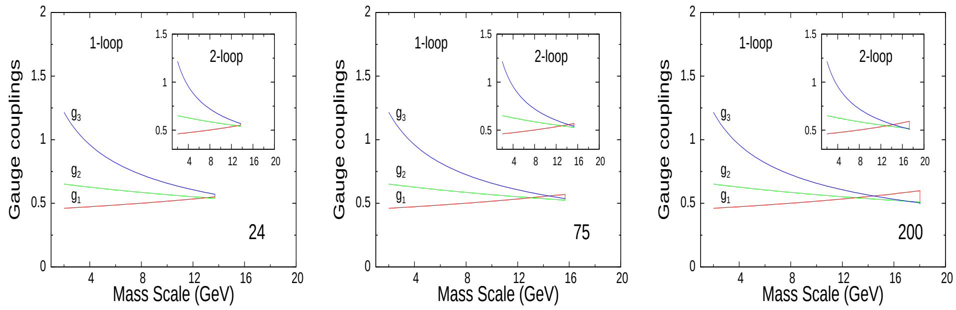

Figure 5.1: The evolution of the coupling constants for different choices of Φ for non-SUSY $SU(5)$ GUTs: Φ_{24} (left), Φ_{75} (center), Φ_{200} (right). In the inset the results for two-loop evolution are shown.

24-dimensional representation [6] or permitted to vary arbitrarily [7] then unification, at the one-loop level, is always possible.

Here we perform a one-loop as well as a two-loop analysis. Above the SUSY scale (chosen as 1 TeV) this entails the replacement of eqs. 5.6 and 5.7 (with $n_G = 3, n_H = 2$) by

$$b_1 = \frac{33}{5}; \quad b_2 = 1; \quad b_3 = -3, \tag{5.15}$$

and

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}. \tag{5.16}$$

We find that unification is allowed for all three choices of Φ – namely, 24, 75, and 200 – when the δ_i $(i = 1, 2, 3)$ are appropriately identified. The results are presented in Table 5.2. Unlike the non-SUSY alternative in Table 5.1, now for every case one gets $M_X \sim 10^{16}$ GeV which is consistent with the proton decay limit. In line with expectation, the size of ϵ is reduced in this SUSY case as the couplings tend to unify even without these interactions. The trend of agreement between the one-loop and two-loop results is gratifying.

Results for $SO(10)$

No intermediate scale is the most straight-forward symmetry breaking for $SO(10)$ and is much like the $SU(5)$ case discussed in the previous section.

$$SO(10) \xrightarrow{M_X} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y. \tag{5.17}$$

SU(5) Representations	1-loop		2-loop	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
24	0.017	1.10×10^{16}	-0.003	1.38×10^{16}
75	-0.007	1.92×10^{16}	0.001	1.24×10^{16}
200	-0.204	3.16×10^{16}	0.071	1.10×10^{16}

Table 5.2: SU(5) dimension-5 interaction strength ϵ and the gauge unification scale, M_X, for different Φ representations in a supersymmetric theory.

When there are no intermediate scales the gauge coupling evolutions are governed by eqs. 5.6 and 5.7 for the non-supersymmetric case and eqs. 5.15 and 5.16 for the SUSY version.

$SO(10)$ representations	non-SUSY		SUSY	
	ϵ	M_X (GeV)	ϵ	M_X (GeV)
54	0.170	3.99×10^{13}	-0.013	1.54×10^{16}
210	0.088	4.39×10^{14}	-0.008	1.35×10^{16}
770	0.274	4.10×10^{13}	-0.018	1.54×10^{16}

Table 5.3: Dimension-5 interaction strength, ϵ, and the gauge unification scale, M_X, for different Φ_D representations using two-loop RG equations when $SO(10)$ descends directly to the SM.

The results are shown in Table 5.3. As for $SU(5)$, we find that the non-supersymmetric solutions are untenable. For all three choices of Φ_D the unification scale is $\mathcal{O}(10^{13} - 10^{14})$ GeV, which is excluded by the current observational bounds on the proton decay lifetime.

Results for $E(6)$

This corresponds to the situation when $E(6)$ is directly broken to the SM and the symmetry breaking chain is simply:

$$E(6) \xrightarrow{M_X} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y, \tag{5.18}$$

The gauge coupling evolution is determined by the eqs. 5.6, 5.7 (non-SUSY) and eqs. 5.15, 5.16 (SUSY) in the entire range. The results obtained including the dimension-5 operators in eq. 4.2 are shown in Table 5.4.

$E(6)$	non-SUSY		SUSY	
representations	ϵ	M_X (GeV)	ϵ	M_X (GeV)
650	0.126	8.04×10^{12}	-0.012	1.72×10^{16}
650′	0.101	4.15×10^{14}	-0.011	1.30×10^{16}
2430	0.000	3.76×10^{12}	0.000	1.25×10^{15}

Table 5.4: Dimension-5 interaction strength, ϵ, and the gauge unification scale, M_X, for different Φ_D representations using two-loop RG equations when $E(6)$ descends directly to the SM.

As for the other GUT groups, though gauge unification is possible in the non-SUSY case, the scale of unification is too low and is ruled out by the proton decay limits. The SUSY solutions are acceptable for Φ_{650}. For Φ_{2430} the scale M_X is too low (Note that all the δ_i are equal! see Table 3.8.) but this can be addressed easily by changing the SUSY scale, M_{SUSY}.

5.2.2 One intermediate scale

Only $SO(10)$ and $E(6)$ can accommodate this feature.

Results for $SO(10)$

Here we consider the following breaking chain of $SO(10)$

$$SO(10) \xrightarrow{M_X} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \xrightarrow{M_C} SM. \quad (5.19)$$

The $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \equiv \mathcal{G}_{422}$ intermediate group offers a new discrete symmetry – D-parity [4, 11]. This symmetry relates the gauged $SU(2)_L$ and $SU(2)_R$ subgroups of $SO(10)$ much the same way that ordinary Parity relates the $SU(2)_L$ and $SU(2)_R$ subgroups of the Lorentz group $SO(3,1)$. Alternative routes of $SO(10)$ symmetry breaking are admissible which either preserve or violate D-parity at the intermediate stages. We will consider both in the following. The first step of symmetry breaking from $SO(10)$ to $\mathcal{G}_{422}$ is accomplished by assigning an appropriate *vev* to a 54, 210, or 770-dimensional Higgs. $\langle\Phi_{54}\rangle$ or $\langle\Phi_{770}\rangle$ ensure that D-parity is conserved (i.e., $\delta_{2L} = \delta_{2R}$) but $\langle\Phi_{210}\rangle$ breaks D-parity (i.e., $\delta_{2L} = -\delta_{2R}$, see Table 4.5).

The next step breaking of $\mathcal{G}_{422}$ to the SM is achieved through the *vev* of a 126-dimensional Higgs. The submultiplet of 126_H that develops a *vev* at the scale M_C for this purpose transforms as ($\overline{10}$,1,3) under $\mathcal{G}_{422}$. Here we use the Extended Survival Hypothesis (ESH) [12] which posits that at any energy scale only those scalars which are required for symmetry breaking at that or lower energies remain light. Since scalar fields have no mass protection mechanism they would normally tend to have masses at the highest energies involved. Maintaining a light scalar involves a fine tuning. The Extended Survival Hypothesis limits the fine tuning to that which is essential for the symmetry breaking. Reflecting this sense it is also termed the principle of minimum fine tuning in the scalar potential. In supersymmetric theories the symmetry protects a tree-level choice of masses from higher order corrections due to what is known as the set and forget theorem. According to the ESH the entire submultiplet, ($\overline{10}$,1,3), acquires a mass $\mathcal{O}(M_C)$ while the other members of 126_H are at M_X. This is true if D-parity is not conserved. When D-parity remains unbroken then it relates the ($\overline{10}$,1,3) submultiplet to the (10,3,1) $\subset$ 126_H and it too has a mass of $\mathcal{O}(M_C)$.

One must also consider the Higgs scalars ϕ_{SM} responsible for the breaking of SM at $\sim M_Z$. They transform under $\mathcal{G}_{SM}$, $\mathcal{G}_{422}$, and $SO(10)$ as {(1,2,1) + (1,2,-1)}, (1,2,2) and 10 respectively. Notice that the Extended Survival Hypothesis mandates that the (6,1,1) under $\mathcal{G}_{422}$ contained in the $SO(10)$ 10-dimensional representation has a mass at M_X while the (1,2,2) is at M_Z.

The scalars contributing to the RG evolution in different stages are summarised in Table 5.5.

When the couplings are evolved from their low energy inputs the key matching formula at M_C is[2]:

$$\frac{1}{\alpha_{1Y}(M_C)} = \frac{3}{5}\left[\frac{1}{\alpha_{2R}(M_C)} - \frac{1}{6\pi}\right] + \frac{2}{5}\left[\frac{1}{\alpha_{4c}(M_C)} - \frac{1}{3\pi}\right] \quad . \tag{5.20}$$

This is a consequence of the relation $Y/2 = T_{3R} + (B-L)/2$. On the r.h.s. T_3 resides within the $SU(2)_R$ while $(B-L)$ is included in $SU(4)_c$ and eq. 5.4 has

[2] α_{1Y} is the GUT-normalised $U(1)_Y$ coupling.

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG	
		$M_Z \to M_C$ Under $\mathcal{G}_{SM}$	$M_C \to M_X$ Under $\mathcal{G}_{422}$
10	$\mathcal{G}_{SM} \to EM$	(1,2,±1)	(1,2,2)
126	$\mathcal{G}_{422} \to \mathcal{G}_{SM}$	-	($\overline{10}$,1,3) {(10,3,1)}

Table 5.5: Higgs scalars for the symmetry breaking of $SO(10)$ with one intermediate stage and the submultiplets contributing to the RG evolution according to the ESH. The submultiplet in the braces also contributes if D-parity is conserved.

been used. Similarly, $\alpha_{4c}(M_C) = \alpha_{3c}(M_C) + 1/12\pi$ and is fixed from the RG evolution of α_{3c} from M_Z. The two cases that we discuss here are:

(a) If D-parity is conserved at M_C then in eq. 5.20 we must further impose $\alpha_{2R}(M_C) = \alpha_{2L}(M_C)$, with the latter fixed by the RG evolution of α_{2L} from its low energy value. This identifies a unique M_C. M_X can then be determined in terms of ϵ.

(b) If D-parity is not conserved then for every choice of M_C, eq. 5.20 determines $\alpha_{2R}(M_C)$. The three couplings have to be further evolved to determine M_X and ϵ.

We discuss these options in detail below.

From M_Z to M_C: For the RG running of the coupling constants in this range eqs. 5.6, 5.7 (for non-SUSY) and 5.15, 5.16 (for SUSY) are applicable irrespective of whether D-parity is conserved or not.

D-parity not conserved

This is the case when Φ_{210} is responsible for the $SO(10)$ GUT symmetry breaking.

From M_C to M_X (D-parity not conserved):

The β-function coefficients receive contributions from $(\overline{10}, 1, 3) \subset 126_H$ along with the (1,2,2)$\subset$ 10_H scalars and the three generations of fermions: $(4,2,1) + (\overline{4},1,2) = 16_F$. These are:

$$\text{non-SUSY:} \qquad b_{2L} = -3; \;\; b_{2R} = 11/3; \;\; b_{4c} = -23/3;$$
$$b_{ij} = \begin{pmatrix} 8 & 3 & 45/2 \\ 3 & 584/3 & 765/2 \\ 9/2 & 153/2 & 643/6 \end{pmatrix}. \tag{5.21}$$

$$\text{SUSY:} \; b_{2L} = 1; \;\; b_{2R} = 21; \;\; b_{4c} = 3; \quad b_{ij} = \begin{pmatrix} 25 & 3 & 45 \\ 3 & 265 & 405 \\ 9 & 81 & 231 \end{pmatrix}. \tag{5.22}$$

The one- and two-loop β-function coefficients we have calculated are in agreement[3] with those obtained in [4].

For this chain, the low energy measured gauge couplings allow a range of values for M_C. The results for this case are shown in the left (non-SUSY) and middle (SUSY) panels of Fig. 5.2. As shown in the Figure, for every allowed M_C one can determine M_X (red dark solid curve) and ϵ (green pale broken curve) from the unification of coupling constants satisfying eq. 4.3. As a general observation, lower values of M_C correspond to increased M_X and larger ϵ. Notice that in the non-SUSY case, M_C can be as low as 10^3 GeV and therefore within the range of detectability for the Large Hadron Collider. Further, the ($\overline{10}$,1,3) scalars which have mass $\sim M_C$ can mediate $n - \overline{n}$ oscillations[4] and it is known that current experimental limits place a lower bound on M_C around 10 TeV depending on hadronic factors not precisely known [14]. The mass of the ν_R is also $\mathcal{O}(M_C)$. While a low M_C is desirable for detectability of $n - \overline{n}$ oscillations it is not the preferred choice for a seesaw mechanism for generating light neutrino masses. In the SUSY case M_X and M_C are restricted to a very limited range, a reflection of the large beta functions beyond M_C. Here M_C ($10^{14} - 10^{16}$ GeV) is too high for observable $n - \overline{n}$ oscillations but quite appropriate for light neutrino seesaw masses.

[3]There are minor differences in b_{2L2R} and b_{2L4c} between our results and that in [4].
[4]The oscillation period $\tau_{n-\overline{n}} \sim (M_{(\overline{10},1,3)})^5$.

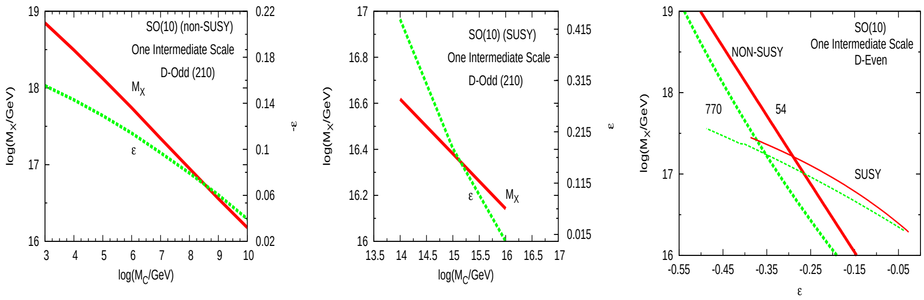

Figure 5.2: $SO(10)$ one intermediate scale results: The unification scale, M_X, (red dark solid lines) and the strength of the dim-5 interaction, ϵ, (green pale broken lines) as a function of the intermediate scale M_C for the D-parity nonconserving (Φ_{210}) case for (left) non-SUSY and (centre) SUSY. M_X as a function of ϵ for the D-parity conserving case (right). Thick (thin) lines correspond to non-SUSY (SUSY). The results for both Φ_{54} (red dark solid) and Φ_{770} (green pale broken) are shown.

D-parity conserved

This is the situation which arises when either Φ_{54} or Φ_{770} is responsible for the $SO(10)$ breaking.

From M_C to M_X (D-parity conserved):

According to the Extended Survival Hypothesis the only change from the previous subsection is that one must include contributions from both ($\overline{10}$,1,3) and (10,3,1) within the 126_H. This gives:

$$\text{non-SUSY:} \qquad b_{2L} = b_{2R} = 11/3; \quad b_{4c} = -14/3; \\ b_{ij} = \begin{pmatrix} 584/3 & 3 & 765/2 \\ 3 & 584/3 & 765/2 \\ 153/2 & 153/2 & 1759/6 \end{pmatrix}. \tag{5.23}$$

$$\text{SUSY: } b_{2L} = b_{2R} = 21; \quad b_{4c} = 12; \quad b_{ij} = \begin{pmatrix} 265 & 3 & 405 \\ 3 & 265 & 405 \\ 81 & 81 & 465 \end{pmatrix}. \tag{5.24}$$

The β-function coefficients for the non-SUSY case agree with those in [13]. In this case, the relationship between the $SU(2)_L$ and $SU(2)_R$ couplings uniquely fixes the intermediate scale M_C.

We find that for the non-SUSY case $M_C = 5.37 \times 10^{13}$ GeV while in the SUSY case it is higher and is around 1.9 $\times 10^{16}$ GeV. This fixed intermediate scale, M_C, is the same for Φ_{54} and Φ_{770}. The $(\overline{10}$,1,3) and (10,3,1) scalars at $\sim M_C$ are thus too heavy for observable $n - \overline{n}$ oscillations. Depending on whether the non-SUSY or the SUSY theory is under consideration, a range of allowed M_X can be obtained as a function of ϵ for either choice of Φ_D. The results for the non-SUSY (thick lines) and SUSY (thin lines) cases are shown in the right panel of Fig. 5.2. The dark solid (red) lines correspond to Φ_{54} while the pale broken (green) lines are for Φ_{770}.

Results for $E(6)$

Here we consider only the breaking pattern:

$$E(6) \xrightarrow{M_X} SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \xrightarrow{M_R} SM. \tag{5.25}$$

For this case, the symmetry breaking at M_R and subsequently the one at M_Z are through the *vev*s to components within the ($\overline{3}$,3,1) submultiplet under $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \equiv \mathcal{G}_{333}$ which is present in a 27 of $E(6)$. According to the Extended Survival Hypothesis this entire (1,$\overline{3}$,3) submultiplet, but for the ϕ_{SM} fields which are at M_Z, has a mass M_R. Since it is symmetric under $SU(3)_L \leftrightarrow SU(3)_R$, the evolution of the couplings from M_R to M_X are controlled by the same RG-equations for both the D-parity violating and D-parity conserving cases. The β-function coefficients in this case are:

From M_R to M_X:

$$\text{non-SUSY: } b_{3L} = b_{3R} = -9/2; \;\; b_{3c} = -5; \quad b_{ij} = \begin{pmatrix} 23 & 20 & 12 \\ 20 & 23 & 12 \\ 12 & 12 & 12 \end{pmatrix}. \tag{5.26}$$

$$\text{SUSY: } b_{3L} = b_{3R} = 3/2; \;\; b_{3c} = 0; \quad b_{ij} = \begin{pmatrix} 65 & 32 & 24 \\ 32 & 65 & 24 \\ 24 & 24 & 48 \end{pmatrix}. \tag{5.27}$$

From M_Z to M_R: For the RG running of the coupling constants below M_R eqs. 5.6, 5.7 (non-SUSY) and eqs. 5.15, 5.16 (SUSY) are applicable irrespective of whether D-parity is conserved or not.

The chain of $E(6)$ breaking considered in this subsection is rather constrained. The matching formula at M_R is now:

$$\frac{1}{\alpha_{1Y}(M_R)} = \frac{4}{5}\left[\frac{1}{\alpha_{3R}(M_R)} - \frac{1}{4\pi}\right] + \frac{1}{5}\left[\frac{1}{\alpha_{3L}(M_R)} - \frac{1}{4\pi}\right] \quad . \tag{5.28}$$

This is a consequence of the relation $Y/2 = T_{3R} + (Y'_L + Y'_R)/2$. On the r.h.s. T_{3R} and Y'_R reside within the $SU(3)_R$ while Y'_L is included in $SU(3)_L$. The two cases are:

(a) If D-parity is conserved at M_R then in eq. 5.28 $\alpha_{3R}(M_R) = \alpha_{3L}(M_R)$, with the latter fixed by the RG evolution of α_{2L} from its low energy value. This identifies a unique M_R, effectively the scale at which $\alpha_{1Y}(M_R) = \alpha_{2L}(M_R)$. M_X can then be determined in terms of ϵ.

(b) If D-parity is not conserved then for any chosen M_R, through eq. 5.28 $\alpha_{3L}(M_R)$ is fixed since $\alpha_{2L}(M_R)$ is determined from its low energy value through RG evolution. The three couplings have to be further evolved to determine M_X and ϵ.

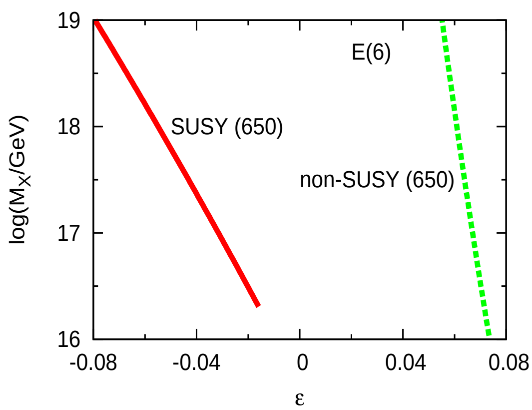

Figure 5.3: $E(6)$ Results: M_X as a function of $E(6)$ for one-step breaking of $E(6)$ in the D-parity conserving case for 650. The green pale broken (red dark solid) line corresponds to non-SUSY (SUSY).

We discuss these options in detail below.

When D-parity is violated, i.e., for $\Phi_{650'}$, we find that the intermediate scale at M_R is rather tightly restricted from the twin requirements that M_X satisfies the

$E(6)$ representation	Symmetry breaking	Scalars contributing to RG	
		$M_Z \to M_R$ Under $\mathcal{G}_{SM}$	$M_R \to M_X$ Under $\mathcal{G}_{333}$
27	$\mathcal{G}_{SM} \to EM$	(1,2,$\pm$1)	(1,$\overline{3}$,3)
27	$\mathcal{G}_{333} \to \mathcal{G}_{SM}$	-	(1,$\overline{3}$,3)

Table 5.6: Higgs scalars for the symmetry breaking of $E(6)$ with one intermediate stage and the submultiplets contributing to the RG evolution according to the ESH.

proton decay bound and is within the upper limit set by the Planck mass as well as all couplings remain perturbative. It is in the ballpark of 10^{14} (10^{16}) GeV for the non-SUSY (SUSY) case. The unification scale is 7.0×10^{18} (3.5×10^{16}) GeV for the respective cases with ϵ almost fixed at = -0.04 (0.02).

When D-parity is conserved, which corresponds to Φ_{650} and Φ_{2430}, the intermediate scale M_R is uniquely fixed in both cases at the value 1.5×10^{13} (1.7×10^{16}) GeV for non-SUSY (SUSY). A plot of the unification scale M_X vs. ϵ is shown in the Fig. 5.3 for Φ_{650}. For Φ_{2430} we have $\delta_{3L} = \delta_{3R} = \delta_{3c}$ and so the dim-5 operator does not affect the unification. We find that for non-SUSY as well as SUSY with M_{SUSY} = 1 TeV the couplings unify at an energy beyond the Planck scale. For both Φ_{650} and $\Phi_{650'}$ the scale M_R is in the right range for the mass of the right-handed neutrinos to drive a type-I seesaw.

5.2.3 Two intermediate scales

The ranks of the GUT groups $SO(10)$ and $E(6)$ are larger than that of the SM. This ensures the possibility to have more than one intermediate scale. A subtle feature [15, 16], considered most recently within the context of $SO(10)$ in [17], has to do with the dynamical mixing of two $U(1)$ subgroups of an intermediate gauge symmetry even at the one-loop level. The $U(1)$ gauge currents and the $U(1)$ gauge boson fields are by themselves gauge invariant and so cross couplings between them are not forbidden by gauge symmetry. Even if the mixing is set to zero at some scale it emerges again through the RG flow. The origin of this mixing in the RG equations lies in the following fact: while the trace of the product of two different U(1) generators vanishes over an entire gauge multiplet, when only a submultiplet is light (e.g., some scalars of a multiplet remaining light due to the Extended Survival Hypothesis in $SO(10)$ or $E(6)$, or incomplete light fermion multiplets in $E(6)$) this is no longer so. This requires a more sophisticated analysis leading to a coupling of g_{1m} and g_{1n} in the one and two-loop RG equations where m and n identify two $U(1)$ groups. These terms arise in the two-step breaking options for both $SO(10)$ and $E(6)$ and are detailed in the discussions in the respective sections.

Results for $SO(10)$

Here we consider the breaking of $SO(10)$ to SM *via* two intermediate steps:

$$\begin{aligned} SO(10) &\xrightarrow{M_X} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \\ &\xrightarrow{M_C} SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)} \\ &\xrightarrow{M_R} SM. \end{aligned} \tag{5.29}$$

The symmetry breaking at different stages is arranged as follows. The breaking of the Pati-Salam $\mathcal{G}_{422}$ to $\mathcal{G}_{2131}$ is through the *vev* of a (15,1,3) component of 210_H. The subsequent descent to the SM is through the *vev* to a (1,3,1,-2) $\subset (\overline{10},1,3) \subset 126_H$. The Higgs scalars responsible for the SM symmetry breaking, ϕ_{SM}, transform as (1,2,$\pm$1) under the SM group and as $(1,0,2,\pm\frac{1}{2}) \subset (1,2,2) \subset 10$ under $\mathcal{G}_{2131}$, $\mathcal{G}_{422}$, and $SO(10)$ respectively. The contributing scalars at different stages of RG evolution, as determined by the ESH, are summarised in Table 5.7.

If D-parity is conserved, and it can be conserved only till M_C in this chain, then one must include the contribution from a (10,3,1) and a (15,3,1) in the final stage of evolution (see Table 5.7).

In the energy range M_R to M_C there are two $U(1)$ gauge groups. As observed in [15, 16] and stressed most recently in [17], due to incomplete scalar multiplets remaining light due to the Extended Survival Hypothesis there is a dynamical mixing between these two $U(1)$ subgroups which is manifested in the RG evolution equations. In particular, below the M_R threshold there is one $U(1)$ coupling corresponding to hypercharge, Y, while above one must consider the

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG		
		$M_Z \to M_R$ Under $\mathcal{G}_{SM}$	$M_R \to M_C$ Under $\mathcal{G}_{2131}$	$M_C \to M_X$ Under $\mathcal{G}_{422}$
10	$\mathcal{G}_{SM} \to EM$	(1,2,$\pm$1)	(2,$\pm\frac{1}{2}$,1,0)	(1,2,2)
126	$\mathcal{G}_{2131} \to \mathcal{G}_{SM}$	-	(1,3,1,-2)	($\overline{10}$,1,3) {(10,3,1)}
210	$\mathcal{G}_{422} \to \mathcal{G}_{2131}$	-	-	(15,1,3) {(15,3,1)}

Table 5.7: Higgs scalars for the symmetry breaking of $SO(10)$ with two intermediate stages and the submultiplets contributing to the RG evolution according to the ESH. The submultiplets in the braces also contribute if D-parity is conserved.

possibility of a 2×2 matrix of $U(1)$ couplings, G:

$$G = \begin{pmatrix} g_{RR} & g_{RX} \\ g_{XR} & g_{XX} \end{pmatrix}, \tag{5.30}$$

where $X \equiv (B - L)$. This is the most general form permitted for the coupling of the gauge currents to gauge bosons which for the $U(1)$ groups are both by themselves gauge invariant. Here, g_{ij} is the strength of the coupling of the ith current to the jth gauge boson. In the range M_R to M_C the evolution of all elements of G will occur[5]. The RG equations for g_{RX} and g_{XR} at the one-loop level involve one additional β-function coefficient, $\tilde{b}_{XR} = \tilde{b}_{RX} \propto \sum_i Q^i_R Q^i_X$. At the two-loop level, besides the usual ones, one requires the following independent coefficients:

1. $\tilde{b}_{RX,RR}$, $\tilde{b}_{XR,XX}$
2. $\tilde{b}_{RX,p}$, $\tilde{b}_{XR,p}$
3. $\tilde{b}_{p,RX}$.

[5]Because of the mixing of the two $U(1)$ groups, the RG equations will be somewhat more involved [16, 17]. Here we list all the β-function coefficients which are non-vanishing.

The first β-coefficient appears in, among others, the evolution equation of g_{RX} as the coefficient of $g_{RR}^4 g_{XX}$ while the second is readily obtainable from the above through $R \leftrightarrow X$. For 2 and 3 above, p represents a non-abelian subgroup of the gauge symmetry. The coefficient of $g_{RX}^3 g_p^2$ ($g_{XR}^3 g_p^2$) in the RG equation of g_{RX} (g_{XR}) is listed under 2 above. Similarly, in 3, $\tilde{b}_{p,RX}$ is the coefficient of $g_p^3(g_{RR}g_{XR} + g_{XX}g_{RX})$. For the $SO(10)$ model we are considering, the entries in 2 and 3 turn out to be zero.

At the boundary M_R there is freedom to choose G to be upper triangular. On RG evolution all elements will, however, become non-zero. The matching of the elements of G with the coupling below M_R and those above M_C is made through projection operators which relate the basis of evolution with the $U(1)$ gauge basis defining the groups at the boundary.

Taking all this into account, the gauge couplings evolve as follows:

i-a) From M_C to M_X (D-parity not conserved):

$$\text{non-SUSY:} \quad b_{2L} = -3; \ \ b_{2R} = 41/3; \ \ b_{4c} = -11/3;$$
$$b_{ij} = \begin{pmatrix} 8 & 3 & 45/2 \\ 3 & 1424/3 & 1725/2 \\ 9/2 & 345/2 & 1987/6 \end{pmatrix}. \tag{5.31}$$

$$\text{SUSY:} \ b_{2L} = 1; \ \ b_{2R} = 51; \ \ b_{4c} = 15; \quad b_{ij} = \begin{pmatrix} 25 & 3 & 45 \\ 3 & 625 & 885 \\ 9 & 177 & 519 \end{pmatrix}. \tag{5.32}$$

i-b) From M_C to M_X (D-parity conserved):

$$\text{non-SUSY:} \quad b_{2L} = b_{2R} = 41/3; \ \ b_{4c} = 10/3;$$
$$b_{ij} = \begin{pmatrix} 1424/3 & 3 & 1725/2 \\ 3 & 1424/3 & 1725/2 \\ 345/2 & 345/2 & 4447/6 \end{pmatrix}. \tag{5.33}$$

$$\text{SUSY:} \ b_{2L} = b_{2R} = 51; \ \ b_{4c} = 36; \quad b_{ij} = \begin{pmatrix} 625 & 3 & 885 \\ 3 & 625 & 885 \\ 177 & 177 & 1041 \end{pmatrix}. \tag{5.34}$$

ii) From M_R to M_C:

Below M_C, where the gauge group is $SU(2)_L \otimes U(1)_R \otimes SU(3)_c \otimes U(1)_{(B-L)}$, there is no $L \leftrightarrow R$ symmetry and hence there can be no D-parity. Thus for the two cases just discussed the evolution will be identical. Here we are giving the decompositions of the contributing fields under the gauge symmetry at this level:

$$\begin{aligned} 16_F &= [2,0,3,-1/3] + [2,0,1,1] + [1,1/2,\overline{3},1/3] + \\ &\quad +[1,1/2,1,-1] + [1,-1/2,\overline{3},1/3] + [1,-1/2,1,-1], \\ 10_H &\supset [2,1/2,1,0] + [2,-1/2,1,0], \quad 126_H \supset [1,-1,1,2] \ . \end{aligned} \tag{5.35}$$

whence[6]

$$\begin{aligned} \text{non-SUSY:} \quad & b_{2L} = -3;\ b_{RR} = 14/3;\ b_{3c} = -7; \\ & b_{(B-L)(B-L)} = 9/2;\ \tilde{b}_{R(B-L)} = \tilde{b}_{(B-L)R} = -1/\sqrt{6}, \end{aligned} \tag{5.36}$$

$$\begin{aligned} b_{ij} &= \begin{pmatrix} 8 & 1 & 12 & 3/2 \\ 3 & 8 & 12 & 15/2 \\ 9/2 & 3/2 & -26 & 1/2 \\ 9/2 & 15/2 & 4 & 25/2 \end{pmatrix}; \\ \tilde{b}_{(B-L)R,RR} &= -2\sqrt{6};\ \tilde{b}_{R(B-L),(B-L)(B-L)} = -3\sqrt{6}. \end{aligned} \tag{5.37}$$

$$\begin{aligned} \text{SUSY:} \quad & b_{2L} = 1;\ b_{RR} = 8;\ b_{3c} = -3; \\ & b_{(B-L)(B-L)} = 15/2;\ \tilde{b}_{R(B-L)} = \tilde{b}_{(B-L)R} = -\sqrt{6}/2, \end{aligned} \tag{5.38}$$

$$\begin{aligned} b_{ij} &= \begin{pmatrix} 25 & 1 & 24 & 3 \\ 3 & 11 & 24 & 9 \\ 9 & 3 & 14 & 1 \\ 9 & 9 & 8 & 16 \end{pmatrix}; \\ \tilde{b}_{(B-L)R,RR} &= -2\sqrt{6};\ \tilde{b}_{R(B-L),(B-L)(B-L)} = -3\sqrt{6}. \end{aligned} \tag{5.39}$$

iii) From M_Z to M_R:

In this range eqs. 5.6, 5.7 (for non-SUSY) and 5.15, 5.16 (for SUSY) are applicable.

The one- and two-loop β-function coefficients in the D-parity conserving case agree with those obtained in [13] and [17] with the proviso that in [13] only one Higgs doublet is assumed to contribute in the range M_Z to M_R. In addition, the $U(1)$ mixing contribution at the one-loop level has been included only in [17].

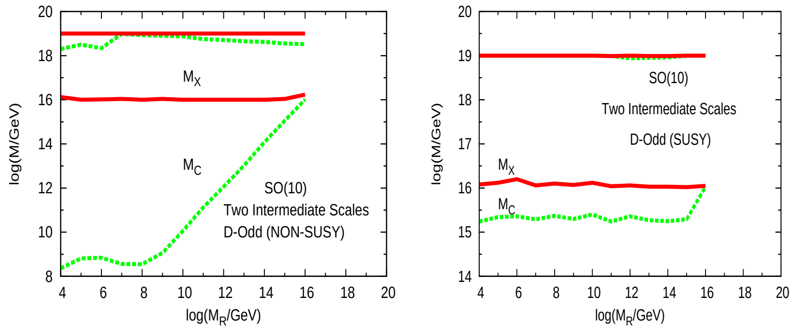

Figure 5.4: Results for $SO(10)$ with two intermediate scales when D-parity is not conserved. The allowed ranges of M_X and M_C vs. M_R for the non-SUSY (SUSY) case is in the left (right) panel. Note that the upper limits for M_X and M_C are almost identical here.

At M_R one must now use the matching relation:

$$\frac{1}{\alpha_{1Y}(M_R)} = 4\pi\; P\;(G\;G^T)^{-1} P^T\;. \tag{5.40}$$

where $P = (\sqrt{\frac{3}{5}}\;\;\sqrt{\frac{2}{5}})$. At the M_C boundary, the $U(1)_R$ and $U(1)_{B-L}$ couplings are obtained from the RG evolved G using a similar formula while choosing P = (1 0) and (0 1), respectively.

When D-parity is conserved, M_R must be such that the α_{1R} and $\alpha_{1(B-L)}$ matches with α_{2L} and α_{3c} (as per eq. 5.4) at precisely the same energy scale M_C. This is severely constraining. We find that in the non-SUSY case, for both Φ_{54} and Φ_{770} the solution is pushed to $M_R \simeq M_C = M_X = 1.02 \times 10^{16}$ GeV with $\epsilon \simeq 0.005$. For the SUSY case one has $M_R = 10^{13}$ - 10^{16} GeV while $M_C = M_X \sim 1.51 \times 10^{16}$ GeV. The high values of M_C preclude the possibility of detectable $n - \overline{n}$ oscillations. On the other hand, such a high M_{ν_R} will be able to accommodate the light neutrino masses through a type-I seesaw.

When D-parity is not conserved, eq. 5.40 fixes the couplings at M_R. The meeting of the $U(1)_{(B-L)}$ and $SU(3)_c$ couplings determines M_C and at that scale α_{1R} goes over to α_{2R}. At M_R, the ratios $g_{RR}/g_{(B-L)(B-L)}$ and $g_{R(B-L)}/g_{(B-L)(B-L)}$ can be varied to first determine M_C *via* eq. 5.40 and subsequently M_X. The range

[6]The coefficients superscribed with a *tilde* arise due to $U(1)$ mixing.

of values for these ratios and M_R are tightly constrained by the requirements of perturbativity and consistency with proton decay limits. In the left and right panels of Fig. 5.4 are shown the ranges of M_C and M_X consistent with the above choice for the non-SUSY and SUSY cases. For the non-SUSY case, M_C is at $10^{8.5}$ GeV or above which is probably a bit high for the detectability of $n - \overline{n}$ oscillations. For SUSY M_C is above 10^{15} GeV which is much too high. Here, the non-SUSY range of M_C is not high enough for the light neutrino seesaw mechanism but the SUSY solutions are quite suitable from this angle.

Results for $E(6)$

The symmetry breaking steps are:

$$\begin{aligned} E(6) \xrightarrow{M_X} \quad & SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \\ \xrightarrow{M_I} \quad & SU(2)_L \otimes U(1)_{Y'_L} \otimes SU(2)_R \otimes U(1)_{Y'_R} \otimes SU(3)_c \\ \xrightarrow{M_R} \quad & SM. \end{aligned} \tag{5.41}$$

Here, $\langle \Phi_{650} \rangle$ or $\langle \Phi_{2430} \rangle$ breaks $E(6)$ to $\mathcal{G}_{333}$ which reduces to $SU(2)_L \otimes U(1)_{Y'_L} \otimes SU(2)_R \otimes U(1)_{Y'_R} \otimes SU(3)_c \equiv \mathcal{G}_{21213}$ when the (1,8,8) submultiplet of a 650_H acquires a *vev*. The SM is reached by assigning a *vev* to the (1,$\overline{3}$,3) component of 27_H. The final step of SM symmetry breaking is accomplished through a different component of (1,$\overline{3}$,3) (see Table 5.8). It is seen that there is room for D-parity to be conserved or broken during the running in the M_R to M_I range. But the Higgs submultiplets which acquire masses at M_I according to the Extended Survival Hypothesis, namely, (1,$\overline{3}$,3) and (1,8,8), are $SU(3)_L \leftrightarrow SU(3)_R$ symmetric and so the running from M_I to M_X will be identical in both cases.

Below we list the one- and two-loop β-function coefficients for gauge coupling evolution in the different stages. Notice that in the range M_R to M_I there are two $U(1)$ components and the RG evolution here has to take into account mixing and follows the same procedure as discussed in detail for $SO(10)$ in the previous section.

i) From M_I to M_X:

The fermion and scalar fields which contribute in the RG equations are:

$$27_F = [1,\overline{3},3] + [3,3,1] + [\overline{3},1,\overline{3}], \quad 650_H \supset [1,8,8], \quad 27_H \supset [1,\overline{3},3]. \tag{5.42}$$

$E(6)$ representation	Symmetry breaking	Scalars contributing to RG		
		$M_Z \to M_R$ Under $\mathcal{G}_{SM}$	$M_R \to M_I$ Under $\mathcal{G}_{21213}$	$M_I \to M_X$ Under $\mathcal{G}_{333}$
27	$\mathcal{G}_{SM} \to EM$	(1,2,$\pm$1)	(2, -$\frac{1}{2\sqrt{3}}$, 2, $\frac{1}{2\sqrt{3}}$,1)	(1,$\overline{3}$,3)
27	$\mathcal{G}_{21213} \to \mathcal{G}_{SM}$	- -	(1, $\frac{1}{\sqrt{3}}$, 2, $\frac{1}{2\sqrt{3}}$,1) {(2, $\frac{1}{2\sqrt{3}}$, 1, $\frac{1}{\sqrt{3}}$,1)}	(1,$\overline{3}$,3)
650	$\mathcal{G}_{333} \to \mathcal{G}_{21213}$	-	-	(1,8,8)

Table 5.8: Higgs scalars for the symmetry breaking of $E(6)$ with two intermediate stages and the submultiplets contributing to the RG evolution according to the ESH. The submultiplet in the braces also contributes if D-parity is conserved.

Thus:

$$\text{non-SUSY:} \quad b_{3L} = 7/2; \;\; b_{3R} = 7/2; \;\; b_{3c} = -5;$$
$$b_{ij} = \begin{pmatrix} 359 & 308 & 12 \\ 308 & 359 & 12 \\ 12 & 12 & 12 \end{pmatrix}. \tag{5.43}$$

$$\text{SUSY:} \; b_{3L} = 51/2; \;\; b_{3R} = 51/2; \;\; b_{3c} = 0; \quad b_{ij} = \begin{pmatrix} 497 & 320 & 24 \\ 320 & 497 & 24 \\ 24 & 24 & 48 \end{pmatrix}. \tag{5.44}$$

iia) From M_R to M_I (D-parity not conserved):

At this stage the non-SM fermions have acquired mass and decoupled. Taking the Extended Survival Hypothesis into consideration, the fields that contribute in the RG equations are:

$$\begin{aligned} 27_F \;\; &\supset \;\; [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 0, 3] + \\ &\quad\;\; [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [1, 0, 2, -1/2\sqrt{3}, \overline{3}], \\ 27_H \;\; &\supset \;\; [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [2, -1/2\sqrt{3}, 2, 1/2\sqrt{3}, 1]. \end{aligned} \tag{5.45}$$

This gives[7]:

$$\text{non-SUSY:} \quad b_{2L} = -3;\ b_{LL} = 3;\ b_{2R} = -17/6;\ b_{RR} = 17/6;$$
$$b_{3c} = -7;\ \tilde{b}_{LR} = \tilde{b}_{RL} = 4/3, \tag{5.46}$$

$$\begin{aligned}
b_{ij} &= \begin{pmatrix} 8 & 4/3 & 3 & 4/3 & 12 \\ 4 & 8/3 & 6 & 1 & 4 \\ 3 & 2 & 61/6 & 3/2 & 12 \\ 4 & 1 & 9/2 & 11/6 & 4 \\ 9/2 & 1/2 & 9/2 & 1/2 & -26 \end{pmatrix}; \\
\tilde{b}_{LR,RR} &= 5/6;\ \tilde{b}_{RL,LL} = 7/6;\ \tilde{b}_{2R,RL} = 1/2;\ \tilde{b}_{2L,LR} = 1/6; \\
\tilde{b}_{RL,2R} &= \tilde{b}_{LR,2R} = 3/2;\ \tilde{b}_{RL,2L} = \tilde{b}_{LR,2L} = 1/2.
\end{aligned} \tag{5.47}$$

$$\text{SUSY:} \quad b_{2L} = 1;\ b_{LL} = 5;\ b_{2R} = 3/2;\ b_{RR} = 9/2;$$
$$b_{3c} = -3;\ \tilde{b}_{LR} = \tilde{b}_{RL} = 2, \tag{5.48}$$

$$\begin{aligned}
b_{ij} &= \begin{pmatrix} 25 & 7/3 & 3 & 7/3 & 24 \\ 7 & 13/3 & 9 & 5/3 & 8 \\ 3 & 3 & 57/2 & 5/2 & 24 \\ 7 & 5/3 & 15/2 & 7/2 & 8 \\ 9 & 1 & 9 & 1 & 14 \end{pmatrix}; \\
\tilde{b}_{LR,RR} &= 5/3;\ \tilde{b}_{RL,LL} = 2;\ \tilde{b}_{2R,RL} = 1;\ \tilde{b}_{2L,LR} = 2/3; \\
\tilde{b}_{RL,2R} &= \tilde{b}_{LR,2R} = 3;\ \tilde{b}_{RL,2L} = \tilde{b}_{LR,2L} = 2.
\end{aligned} \tag{5.49}$$

iib) From M_R to M_I (D-parity conserved):

Due to D-Parity conservation the scalar sector is slightly enlarged and the fields contributing to the RG equations are:

$$\begin{aligned}
27_F &\supset [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 0, 3] + \\
&\quad [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [1, 0, 2, -1/2\sqrt{3}, \overline{3}], \qquad (5.50) \\
27_H &\supset [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 1/\sqrt{3}, 1] + [2, -1/2\sqrt{3}, 2, 1/2\sqrt{3}, 1].
\end{aligned}$$

We find:

$$\text{non-SUSY:} \quad b_{2L} = -17/6;\ b_{LL} = 55/18;\ b_{2R} = -17/6;\ b_{RR} = 55/18;$$
$$b_{3c} = -7;\ \tilde{b}_{LR} = \tilde{b}_{RL} = 13/9, \tag{5.51}$$

[7]The coefficients superscribed with a *tilde* arise due to $U(1)$ mixing.

$$
\begin{aligned}
b_{ij} &= \begin{pmatrix} 61/6 & 3/2 & 3 & 2 & 12 \\ 9/2 & 49/18 & 6 & 11/9 & 4 \\ 3 & 2 & 61/6 & 3/2 & 12 \\ 6 & 11/9 & 9/2 & 49/18 & 4 \\ 9/2 & 1/2 & 9/2 & 1/2 & -26 \end{pmatrix} ; \\
\tilde{b}_{LR,RR} &= 23/18; \;\; \tilde{b}_{RL,LL} = 23/18; \;\; \tilde{b}_{2R,RL} = 1/2; \;\; \tilde{b}_{2L,LR} = 1/2; \\
\tilde{b}_{RL,2R} &= \tilde{b}_{LR,2R} = 3/2; \;\; \tilde{b}_{RL,2L} = \tilde{b}_{LR,2L} = 3/2. \qquad (5.52)
\end{aligned}
$$

$$
\begin{aligned}
\text{SUSY:} \qquad & b_{2L} = 3/2; \quad b_{LL} = 31/6; \; b_{2R} = 3/2; \; b_{RR} = 31/6; \\
& b_{3c} = -3; \; \tilde{b}_{LR} = \tilde{b}_{RL} = 7/3, \qquad (5.53)
\end{aligned}
$$

$$
\begin{aligned}
b_{ij} &= \begin{pmatrix} 57/2 & 5/2 & 3 & 3 & 24 \\ 15/2 & 79/18 & 9 & 17/9 & 8 \\ 3 & 3 & 57/2 & 5/2 & 24 \\ 9 & 17/9 & 15/2 & 79/18 & 8 \\ 9 & 1 & 9 & 1 & 14 \end{pmatrix} ; \\
\tilde{b}_{LR,RR} &= 19/9; \;\; \tilde{b}_{RL,LL} = 19/9; \;\; \tilde{b}_{2R,RL} = 1; \;\; \tilde{b}_{2L,LR} = 1; \\
\tilde{b}_{RL,2R} &= \tilde{b}_{LR,2R} = 3; \;\; \tilde{b}_{RL,2L} = \tilde{b}_{LR,2L} = 3. \qquad (5.54)
\end{aligned}
$$

From M_Z to M_R: For the RG running of the coupling constants below M_R eqs. 5.6, 5.7 (non-SUSY) and eqs. 5.15, 5.16 (SUSY) are applicable irrespective of whether D-parity is conserved or not.

When $E(6)$ breaks to the SM through two intermediate steps, at M_R one must set:

$$
\frac{1}{\alpha_{1Y}(M_R)} = \frac{3}{5}\left[\frac{1}{\alpha_{2R}(M_R)} - \frac{1}{6\pi}\right] + 4\pi \, P \, (G \, G^T)^{-1} P^T \, . \qquad (5.55)
$$

where $P = (\sqrt{\frac{1}{5}} \;\; \sqrt{\frac{1}{5}})$, which follows from $Y/2 = T_{3R} + (Y'_L + Y'_R)/2$.

When the first stage of symmetry breaking is driven through the Φ_{650}, D-parity is preserved. This implies that $\alpha_{2R}(M_R) = \alpha_{2L}(M_R)$ and is fixed by the RG evolution of g_{2L} from M_Z. Also at M_R, $g_{Y'_L Y'_L} = g_{Y'_R Y'_R}$ and one can choose

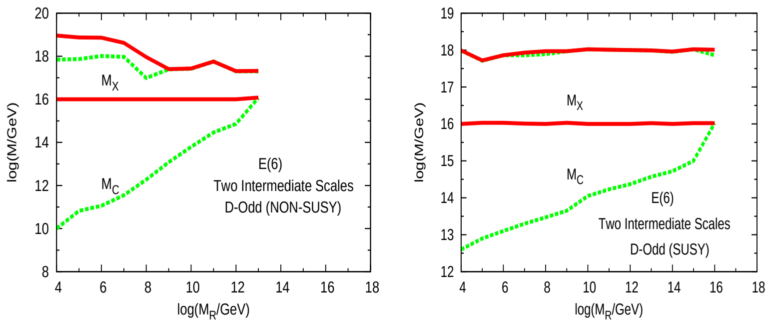

Figure 5.5: $E(6)$ Results: The allowed ranges of M_X and M_I vs. M_R for the non-SUSY (left) and SUSY (right) cases for $E(6)$ breaking through two intermediate steps when D-parity is not conserved. Note that the upper limits for M_X and M_I are almost identical.

$g_{Y'_L Y'_R} = g_{Y'_R Y'_L} = 0$, so all couplings are determined once M_R is chosen. Requiring that the constraints on M_X be satisfied along with perturbativity, we find that M_R is in the range 3.9×10^8 - 2.5×10^{10} (2.5×10^{15} - 6.3×10^{15}) GeV for the non-SUSY (SUSY) case. M_I is above 10^{13} GeV in all cases with M_X between 10^{16} and 10^{19} GeV.

The case of Φ_{2430} is not distinguishable from the situation of no dimension-5 operators at all since here $\delta_1 = \delta_2 = \delta_3$.

When the initial symmetry breaking of $E(6)$ is through the $\Phi_{650'}$, D-parity is not conserved. It might seem that there is more flexibility here and at M_R one can choose $g_{Y'_R Y'_R}$, $g_{Y'_R Y'_L}$, and g_{2R} independently, determining $g_{Y'_L Y'_L}$ from eq. 5.55. In fact, there is a rather severe constraint that $\alpha_{Y'_R}$ and α_{2R} must meet at M_I and at precisely the same scale $\alpha_{Y'_L}$ must equal α_{2L}. In the left and right panels of Fig. 5.5 we show the allowed range of the intermediate scale M_I and the unification scale M_X as a function of M_R. Note that for both cases these scales are on the high side. The scale of the second stage of symmetry breaking, M_R, is permitted to be as low as 10^4 GeV for the non-SUSY as well as the SUSY case which may offer room for experimental probing.

References

[1] H. Georgi, H. R. Quinn and S, Weinberg, Phys. Rev. Lett. **33** (1974) 451.

[2] D. R. T. Jones, Phys. Rev. D **25** (1982) 581.

[3] L. J. Hall, Nucl. Phys. B **178** (1981) 75.

[4] D. Chang, R. N. Mohapatra, J. M. Gipson, R. E. Marshak and M. K. Parida, Phys. Rev. D **31** (1985) 1718.

[5] U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B **260** (1991) 447.

[6] J. Ellis, K. Enqvist, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155** (1985) 381.

[7] M. Drees, Phys. Lett. B **158** (1985) 409.

[8] C. Amsler et al., (The Particle Data Group), Phys. Lett. B **667** (2008) 1.

[9] J. R. Bhatt, S. Patra, U. Sarkar, Mod. Phys. Lett. **A25** (2010) 283-293.

[10] X. Calmet, S. D. H. Hsu and D. Reeb, Phys. Rev. Lett. **101** (2008) 171802 [arXiv:0805.0145 [hep-ph]].

[11] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072; Phys. Rev. D **30** (1984) 1052.

[12] F. del Aguila and L. E. Ibanez, Nucl. Phys. B **177** (1981) 60; S. Dimopoulos and H. M. Georgi, Phys. Lett. B **140** (1984) 67.

[13] N. G. Deshpande, E. Keith and P. B. Pal, Phys. Rev. D **46** (1992) 2261.

[14] R. N. Mohapatra, J. Phys. G: Nucl. Part. Phys. **36** (2009) 104006.

[15] B. Holdom, Phys. Lett. B **166** (1986) 196.

[16] F. del Aguila, G. D. Coughlan and M. Quiros, Nucl. Phys. B **307** (1988) 633 [Erratum *ibid.* B **312** (1989) 751.

[17] S. Bertolini, L. Di Luzio and M. Malinsky, Phys. Rev. D **80** (2009) 015013 [arXiv:0903.4049 [hep-ph]].

Chapter 6

Non-universal gaugino masses

Minimal supergravity (mSUGRA) is the most popular framework of supersymmetry breaking, where SUSY is broken in the 'hidden sector' and is connected to the visible sector via gravity mediation and, as a result, one can parametrise all the SUSY breaking terms by a universal gaugino mass ($M_{1/2}$), a universal scalar mass (m_0), a universal trilinear coupling parameter A_0, the ratio of the vacuum expectation values of the two Higgs fields ($\tan\beta$) and the sign of the SUSY-conserving Higgs mass parameter, ($sgn(\mu)$) [1,2].

However, within the ambit of a SUGRA-inspired GUT scenario itself, one might find some deviations from the simplified and idealised situations mentioned above. For instance, the gaugino mass parameter ($M_{1/2}$) or the common scalar mass parameter (m_0) can become *non-universal* at the GUT scale. In this chapter, we explore a situation with non-universal gaugino masses in a supersymmetric scenario embedded in the $SO(10)$ GUT group.

Gaugino masses, arising after GUT-breaking and SUSY-breaking at a high scale, crucially depend on the gauge kinetic function, as discussed in the next section. One achieves universal gaugino masses if the hidden sector fields (Higgs scalars, in particular), involved in GUT-breaking, are singlets under the underlying GUT group. However if we include the higher dimensional terms (dimension five, in particular) in the non-trivial expansion of the gauge-kinetic function, the Higgs fields belonging to the symmetric products of the adjoint representation of the underlying GUT group can be non-singlets. If these non-singlet Higgs scalars are responsible for GUT breaking, the gaugino masses M_1, M_2 and M_3 become non-universal at the high scale itself. It is also possible to have more than one non-singlet representations involved in GUT breaking, in which case the non-universality arises from a linear combination of the effects mentioned above.

Although this issue has been explored in earlier papers, particularly in the context of $SU(5)$ [3–6], there had been one known effort [7] to study $SO(10)$. In this chapter, we calculate the non-universal gaugino mass ratios for the non-singlet representations **54** and **770**, based on the results obtained in [8], for the intermediate gauge group, namely, Pati-Salam $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ ($\mathcal{G}_{422}$) with conserved D-parity [9].

We adhere to a situation where all soft SUSY breaking effects arise via hidden sector interactions in an underlying supergravity (SUGRA) framework, specifically, in $SO(10)$ gauge theories with an arbitrary chiral matter superfield content coupled to N=1 supergravity.

All gauge and matter terms including gaugino masses in the N=1 supergravity Lagrangian depend crucially on two fundamental functions of chiral superfields [10]: (i) gauge kinetic function $f_{\alpha\beta}(\Phi)$, which is an analytic function of the left-chiral superfields Φ_i and transforms as a symmetric product of the adjoint representation of the underlying gauge group (α, β being the gauge indices, run from 1 to 45 for $SO(10)$); and (ii) $G(\Phi_i, \Phi_i^*)$, a real function of Φ_i and gauge singlet, with $G = K + ln|W|$ (K is the K*ä*hler potential and W is the superpotential).

The part of the N=1 supergravity Lagrangian containing kinetic energy and mass terms for gauginos and gauge bosons (including only terms containing the real part of $f(\Phi)$) reads (using the natural units in which $M_{Pl}/\sqrt{8\pi}$=1)

$$e^{-1}\mathcal{L} = -\frac{1}{4}Ref_{\alpha\beta}(\phi)(-1/2\overline{\lambda}^{\alpha}\not{D}\lambda^{\beta}) - \frac{1}{4}Ref_{\alpha\beta}(\phi)F^{\alpha}_{\mu\nu}F^{\beta\mu\nu} + \frac{1}{4}e^{-G/2}G^{i}((G^{-1})^{j}_{i})[\partial f^{*}_{\alpha\beta}(\phi^{*})/\partial\phi^{*j}]\lambda^{\alpha}\lambda^{\beta} + h.c, \tag{6.1}$$

where $G^i = \partial G/\partial\phi_i$ and $(G^{-1})^i_j$ is the inverse matrix of $G^j{}_i \equiv \partial G/\partial\phi^{*i}\partial\phi_j$, λ^α is the gaugino field, and ϕ is the scalar component of the chiral superfield Φ, and $F^{\alpha}_{\mu\nu}$ is defined in unbroken $SO(10)$, and $M_{Pl} = 10^{19}$ GeV is the Planck scale. The F-component of Φ enters the last term to generate gaugino masses with a consistent SUSY breaking with non-zero *vev* of the chosen $\tilde{F}$, where

$$\tilde{F}^{j} = \frac{1}{2}e^{-G/2}[G^{i}((G^{-1})^{j}_{i})]. \tag{6.2}$$

The Φ^j can be a set of GUT singlet supermultiplets Φ^S, which are part of the hidden sector, or a set of non-singlet ones Φ^N, fields associated with the spontaneous breakdown of the GUT group to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The non-trivial gauge kinetic function $f_{\alpha\beta}(\Phi^j)$ can be expanded in terms of the non-singlet components of the chiral superfields in the following way

$$f_{\alpha\beta}(\Phi^j) = f_0(\Phi^S)\delta_{\alpha\beta} + \sum_N \eta_N(\Phi^s)\frac{{\Phi^N}_{\alpha\beta}}{M_{Pl}} + \mathcal{O}(\frac{\Phi^N}{M_{Pl}})^2, \tag{6.3}$$

where f_0 and η^N are functions of chiral singlet superfields, essentially determining the strength of the interaction.

In eq. 6.3, the contribution to the gauge kinetic function from Φ^N has to come through symmetric products of the adjoint representation of the associated GUT group, since $f_{\alpha\beta}$ on the left side of eq. 6.3 has such transformation property. For $SO(10)$, one can have contributions to $f_{\alpha\beta}$ from all possible non-singlet irreducible representations to which Φ^N can belong :

$$(45 \otimes 45)_{symm} = 1 \oplus 54 \oplus 210 \oplus 770. \tag{6.4}$$

As an artifact of the expansion of the gauge kinetic function $f_{\alpha\beta}$ mentioned in eq. 6.3, corrections from Φ^N to the gauge kinetic term (2nd term) in the Lagrangian (eq. 6.1) can be recast in the following form

$$Ref_{\alpha\beta}(\phi)F^{\alpha}_{\mu\nu}F^{\beta\mu\nu} \supset \frac{\eta_N(\Phi^s)}{M_{Pl}}Tr(F_{\mu\nu}\Phi^N F^{\mu\nu}), \tag{6.5}$$

where $F_{\mu\nu}$, under unbroken $SO(10)$, contains $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge fields. It has been noted that the operator structure of the above eq. 6.5 is same as the dimension-5 operators we considered in the chapter 4 (eq. 4.2).

Next, the kinetic energy terms are restored to the canonical form by rescaling the gauge superfields, by defining

$$ {F^{\alpha}}_{\mu\nu} \rightarrow \hat{F}^{\alpha}_{\mu\nu} = \langle Ref_{\alpha\beta}\rangle^{\frac{1}{2}} {F^{\beta}}_{\mu\nu}, \tag{6.6}$$

and

$$\lambda^{\alpha} \rightarrow \hat{\lambda}^{\alpha} = \langle Ref_{\alpha\beta}\rangle^{\frac{1}{2}}\lambda^{\beta}. \tag{6.7}$$

Simultaneously, the gauge couplings are also rescaled (as a result of eq. 6.3):

$$g_{\alpha}(M_X)\langle Ref_{\alpha\beta}\rangle^{\frac{1}{2}}\delta_{\alpha\beta} = g_c(M_X), \tag{6.8}$$

where g_c is the universal coupling constant at the GUT scale (M_X). This shows clearly that the first consequence of a non-trivial gauge kinetic function is non-universality of the gauge couplings g_α at the GUT scale [3–5, 11, 12].

Once SUSY is broken by non-zero *vev*'s of the $\tilde{F}$ components of hidden sector chiral superfields, the coefficient of the last term in eq. 6.1 is replaced by [3–5]

$$\langle \tilde{F}^i_{\alpha\beta} \rangle = \mathcal{O}(m_{\frac{3}{2}} M), \tag{6.9}$$

where $m_{\frac{3}{2}} = exp(-\frac{\langle G \rangle}{2})$ is the gravitino mass. Taking into account the rescaling of the gaugino fields (as stated earlier in eqs. 6.7) in eq. 6.1, the gaugino mass matrix can be written down as [3, 4, 6]

$$M_\alpha(M_X)\delta_{\alpha\beta} = \sum_j \frac{\langle F^j_{\acute{\alpha}\acute{\beta}} \rangle}{2} \frac{\langle \partial f_{\alpha\beta}(\phi^*)/\partial \phi^{*j}{}_{\acute{\alpha}\acute{\beta}} \rangle}{\langle Re f_{\alpha\beta} \rangle}, \tag{6.10}$$

which demonstrates that the gaugino masses are non-universal at the GUT scale. In [4] the gaugino mass matrix was written as a sum of two contributions, coming from the singlet and the adjoint scalars. When the contribution from the latter one dominates the gaugino masses are proportional to the group theoretic factors same as δ_i's calculated in chapter 4. In this chapter we consider the dominance of the last term and calculate the non-universal gaugino mass ratios for the Pati-Salam breaking pattern of $SO(10)$.

The underlying reason for this is the fact that $\langle f_{\alpha\beta} \rangle$ can be shown to acquire the form $f_\alpha \delta_{\alpha\beta}$, where the f_α 's are purely group theoretic factors, as we will see. On the contrary, if symmetry breaking occurs via gauge singlet fields only, one has $f_{\alpha\beta} = f_0 \delta_{\alpha\beta}$ from eq. 6.3 and as a result, $\langle f_{\alpha\beta} \rangle = f_0$. Thus both gaugino masses and the gauge couplings are unified at the GUT scale (as can be seen from eqs. 6.8 and 6.10).

As mentioned earlier, we would like to calculate here, the f_α 's for Higgs (Φ^N) belonging to the representations **54** and **770** which break $SO(10)$ to the intermediate gauge group $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ with unbroken D-parity (usually denoted as $\mathcal{G}_{422P}$) [1]. We associate the non-universal contributions to the gaugino mass ratios with the group theoretic coefficients f_α 's that arise here. It in turn, indicates that we consider the non-universality in the gaugino masses of $O(1)$. This, however, is not a generic situation in such models. This can be rather achieved under some special conditions like dynamical generation of

[1]Higgs fields that break $\mathcal{G}_{422P}$ to SM, do not contribute to gaugino masses.

SUSY-breaking scale from the electroweak scale, no soft-breaking terms for the GUT or Planck scale particles and with the simplified assumption $M_{GUT} = M_{Pl}$ which is also reflected in the RGE specifications.

In this chapter we derive non-universal gaugino mass ratios for the representations **54** and **770** for the breaking chain $\mathcal{G}_{422}$ in an $SO(10)$ SUSY GUT scenario. We have assumed that the breaking of $SO(10)$ to the intermediate gauge group and the latter in turn to the SM gauge group takes place at the GUT scale itself.

The representations of $SO(10)$ [13], decomposed into that of the Pati-Salam gauge group are

$$\begin{aligned} SO(10) &\rightarrow \mathcal{G}_{422} = SU(4)_C \otimes SU(2)_L \otimes SU(2)_R \\ \mathbf{45} &= (\mathbf{15},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{3}) + (\mathbf{6},\mathbf{2},\mathbf{2}) \\ \mathbf{10} &= (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}) \\ \mathbf{16} &= (\mathbf{4},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}). \end{aligned} \tag{6.11}$$

Using the $SO(10)$ relation, $(10 \otimes 10) = 1 \oplus 45 \oplus 54$, one can see that *vev* of 54-dimensional Higgs ($\langle 54 \rangle$) can be expressed as a 10×10 diagonal traceless matrix. Thus the non-zero *vev* of **54**-dimensional Higgs can be written as [12]

$$< 54 >= \frac{v_{54}}{2\sqrt{15}}\, diag(3,3,3,3,-2,-2,-2,-2,-2,-2). \tag{6.12}$$

Since $(45 \otimes 45)_{symm} = 1 \oplus 54 \oplus 210 \oplus 770$, one can write the non-zero *vev* [8] of **770**-dimensional Higgs as 45×45 diagonal matrix:

$$\langle 770 \rangle = \frac{v_{770}}{\sqrt{180}}\, diag(\underbrace{-4, ..., -4}_{15}, \underbrace{-10, ..., -10}_{3+3}, \underbrace{5, ..., 5}_{24}). \tag{6.13}$$

In the intermediate scale (M_C), G_{422} is broken to the SM group. Here $SU(4)_C$ is broken down to $SU(3)_C \otimes U(1)_{B-L}$ and at the same time, $SU(2)_R$ is broken to $U(1)_{T_{3R}}$. It is noted that $SU(2)_R \otimes SU(4)_C$ is broken to $SU(3)_C \otimes U(1)_Y$ and hence the hypercharge is given as $\frac{Y}{2} = T_{3R} + \frac{1}{2}(B - L)$. Below we note the branchings of $SU(4)_C$ representations:

$$\begin{aligned} SU(4)_C &= SU(3)_C \otimes U(1)_{B-L} \\ \mathbf{4} &= (\mathbf{3}, 1/3) + (\mathbf{1}, -1) \\ \mathbf{15} &= (\mathbf{8}, 0) + (\mathbf{3}, 4/3) + (\overline{\mathbf{3}}, -4/3) + (\mathbf{1}, 0) \\ \mathbf{10} &= (\mathbf{6}, 2/3) + (\mathbf{3}, -2/3) + (\mathbf{1}, -2) \\ \mathbf{6} &= (\mathbf{3}, -2/3) + (\overline{\mathbf{3}}, 2/3). \end{aligned} \tag{6.14}$$

Combining these together, we achieve the branchings of $SO(10)$ representations in terms of the SM gauge group.

$$
\begin{aligned}
SO(10) \quad &: SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
\mathbf{45} \quad &: \\
(\mathbf{15},\mathbf{1},\mathbf{1}) \quad &= (\mathbf{8},\mathbf{1},0) + (\mathbf{3},\mathbf{1},4/3) + (\overline{\mathbf{3}},\mathbf{1},-4/3) + (\mathbf{1},\mathbf{1},0) \\
(\mathbf{1},\mathbf{3},\mathbf{1}) \quad &= (\mathbf{1},\mathbf{3},0) \\
(\mathbf{1},\mathbf{1},\mathbf{3}) \quad &= (\mathbf{1},\mathbf{1},2) + (\mathbf{1},\mathbf{1},0) + (\mathbf{1},\mathbf{1},-2) \\
(\mathbf{6},\mathbf{2},\mathbf{2}) \quad &= (\mathbf{3},\mathbf{2},1/3) + (\mathbf{3},\mathbf{2},-5/3) + (\overline{\mathbf{3}},\mathbf{2},5/3) + (\overline{\mathbf{3}},\mathbf{2},1/3). \qquad (6.15) \\
\mathbf{10} \quad &: \\
(\mathbf{6},\mathbf{1},\mathbf{1}) \quad &= (\mathbf{3},\mathbf{1},-2/3) + (\overline{\mathbf{3}},\mathbf{1},2/3) \\
(\mathbf{1},\mathbf{2},\mathbf{2}) \quad &= (\mathbf{1},\mathbf{2},1) + (\mathbf{1},\mathbf{2},-1). \qquad (6.16) \\
\mathbf{16} \quad &: \\
(\mathbf{4},\mathbf{2},\mathbf{1}) \quad &= (\mathbf{3},\mathbf{2},1/3) + (\mathbf{1},\mathbf{2},-1) \\
(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) \quad &= (\overline{\mathbf{3}},\mathbf{1},2/3) + (\overline{\mathbf{3}},\mathbf{1},-4/3) + (\mathbf{1},\mathbf{1},2) + (\mathbf{1},\mathbf{1},0). \qquad (6.17)
\end{aligned}
$$

We have $U(1)_{T_{3R}}$ and $U(1)_{B-L}$ from $SU(2)_R$ and $SU(4)_C$ respectively. Thus the weak hypercharge generator (T_Y) can be expressed as a linear combination of the generators of $SU(2)_R$ (T_{3R}) and $SU(4)_C$ (T_{B-L}) sharing the same quantum numbers. In 10-dimensional representation T_{3R}, T_{B-L} and T_Y are written as:

$$T_{3R} = diag(0,0,0,0,0,0,\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2}); \tag{6.18}$$

$$T_{B-L} = \sqrt{\frac{3}{2}}diag(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0,0,0); \tag{6.19}$$

$$T_Y = \sqrt{\frac{3}{5}}diag(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2}); \tag{6.20}$$

Using these explicit forms of the generators we find the following relation

$$T_Y = \sqrt{\frac{3}{5}}T_{3R} + \sqrt{\frac{2}{5}}T_{B-L}; \tag{6.21}$$

and this leads to the following mass relation,

$$M_1 = \frac{3}{5}\,M_{2R} + \frac{2}{5}\,M_{4C}; \tag{6.22}$$

which is same for all representations.

• For **54**-dimensional Higgs:

Using **54**-dimensional Higgs we have [12], for D-parity even scenario, $M_{4C} = 1$ and $M_{2R} = M_{2L} = -\frac{3}{2}$. We have identified $M_3 = M_{4C}$ and $M_2 = M_{2R}$. Hence, using the above mass relation we obtain $M_1 = -\frac{1}{2}$. Therefore the gaugino mass ratio is given as:

$$M_1 : M_2 : M_3 = (-\frac{1}{2}) : (-\frac{3}{2}) : 1. \tag{6.23}$$

We have already mentioned that the *vev* [8] of **770**-dimensional Higgs can be expressed as a 45×45 diagonal matrix. So to calculate the gaugino masses, using **770**-dimensional Higgs, we repeat our previous task in 45-dimensional representation.

• **770**-dimensional Higgs:

Using **770**-dimensional Higgs we find [8] for D-parity even case $M_{4C} = 2$ and $M_{2R} = M_{2L} = 5$. Hence, using the above mass relation we obtain $M_1 = 3.8$. Therefore the gaugino mass ratio is given as:

$$M_1 : M_2 : M_3 = 1.9 : 2.5 : 1. \tag{6.24}$$

We tabulate the gaugino mass ratios, obtained above, in Table 1.

Representation	$M_3 : M_2 : M_1$ at M_{GUT}
1	1:1:1
54: $H \to SU(4) \otimes SU(2) \otimes SU(2)$	1:(-3/2):(-1/2)
770: $H \to SU(4) \otimes SU(2) \otimes SU(2)$	1:(2.5):(1.9)

Table 1: *High scale gaugino mass ratios for the representations* **54** *and* **770**.

References

[1] S. P. Martin, arXiv:hep-ph/9709356 and in G. Kane (ed), Perspectives On Supersymmetry, *World Scientific* (1998); M. E. Peskin, arXiv:0801.1928 [hep-ph].

[2] L. Alvarez-Gaume, J. Polchinski and M. B. Wise, Nucl. Phys. B **221** (1983) 495; A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. **49** (1982) 970.

[3] J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247** (1984) 373.

[4] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155** (1985) 381.

[5] M. Drees, Phys. Lett. B **158** (1985) 409.

[6] A. Corsetti and P. Nath, Phys. Rev. D **64** (2001) 125010.

[7] N. Chamoun, C. S. Huang, C. Liu and X. H. Wu, Nucl. Phys. B **624** (2002) 81.

[8] J. Chakrabortty and A. Raychaudhuri, Phys. Lett. B **673** (2009) 57.

[9] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072.

[10] E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara and L. Girardello, Phys. Lett. B **79** (1978) 231; E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B **147** (1979) 105; E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. B **116** (1982) 231; E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B **212** (1983) 413.

[11] C. T. Hill, Phys. Lett. B **135** (1984) 47.

[12] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52** (1984) 875.

[13] R. Slansky, Phys. Rept. **79** (1981) 1.

Chapter 7

Neutrino mass in SO(10) GUT

In a number of papers it has been shown that renormalisable $SO(10)$ – with and without supersymmetry (SUSY) – is quite predictive and powerful in constraining fermion mass patterns because of the underlying $SU(4)_c$ symmetry which relates the quark and lepton Yukawa couplings. In $SO(10)$, $16 \otimes 16 = 10 \oplus 120 \oplus 126$ and so Higgs fields giving mass to the 16_F can reside in the 10_H, 120_H and $\overline{126}_H$ representations. Obtaining correct masses for the quarks and the charged leptons requires at least two Higgs multiplets. It has been noted, for example in [1], that any one of the combinations $(10_H, 120_H)$, $(10_H, \overline{126}_H)$, or $(120_H, \overline{126}_H)$ can, in principle, be utilised. Among these the model with 10_H and $\overline{126}_H$ has been extensively considered as the most successful candidate for the minimal $SO(10)$ GUT [2]. $\overline{126}_H$ contains colour singlet submultiplets which transform as a triplet under $SU(2)_L$ and a singlet under $SU(2)_R$ or *vice versa*; these are the cornerstones of the seesaw mechanism [3]. Both type-I (mediated through singlets [3]) and type-II (mediated through scalar triplets [4]) seesaw have been examined for both supersymmetric [5] and non-supersymmetric [6] cases. The $\overline{126}_H$ relates the Majorana mass of the neutrinos to the Dirac mass as well as other charged fermion masses making the model predictive. It is also possible and in some cases advantageous to include all the three Higgs representations [7, 8]. The model with $10_H + 120_H$ [9, 10], on the other hand, does not have the requisite scalars to lead to neutrino masses through the seesaw mechanism. Here, neutrino mass can be obtained at two-loop through the radiative seesaw mechanism due to Witten [11] by adding $16_H + \overline{16}_H$ multiplets. This model has been studied in [12] and it was shown that under plausible assumptions it predicts $b - \tau$ unification, natural occurrence of large leptonic and small quark mixing and large value for the atmospheric mixing angle. However, the radiative seesaw runs into difficulty with low-energy SUSY although it works well in the context of split SUSY [13]. Moreover, as has been shown in [10] the SUSY $SO(10)$ model containing 10_H and 120_H cannot reproduce the charged fermion masses correctly. On the other hand in non-SUSY $SO(10)$ the two-loop neutrino mass is very small.

In this chapter we consider the generation of neutrino masses in the $10_H + 120_H$ model embellished with a $\overline{16}_H$ by adding fermions belonging to the adjoint representation (45_F) of $SO(10)$. Such fermions couple to the usual sixteen-plet of quarks and leptons *via* the $\overline{16}_H$ and can give rise to neutrino masses through the 'double seesaw' mechanism. In models with $10_H + 120_H$ this can serve as an alternative option for generating small neutrino masses[1]. Fermions in the triplet adjoint representation of $SU(2)_L$ are also considered in the so called type-III [15] seesaw mechanism. Such models have become quite popular in the context of $SU(5)$ GUTs [16]. $SU(2)_L$ triplet fermions fit naturally into the 24-dimensional representation of $SU(5)$ and can cure two main problems of these theories, *viz.* generation of neutrino masses and unification of gauge couplings. The latter requires the mass of the fermionic triplets to be $\sim \mathcal{O}(1\ \text{TeV})$ making the model testable at the LHC [17]. Presence of adjoint fermions in the context of left-right symmetric models has been considered in [18], and generation of neutrino masses and possible collider signatures were discussed. From this point of view our model can also be considered as a generalization of type-III seesaw for $SO(10)$. However as in LR symmetric models the mechanism of mass generation here is actually the 'double seesaw' mechanism.

We discuss the conditions which the Yukawa coupling matrices should satisfy for the model to have predictive power. This requires ascribing some additional flavour symmetry to the model which we choose to be the generalized $\mu - \tau$ symmetry that has been considered widely for explaining the neutrino mixing angles [19]. It predicts θ_{23} to be $\pi/4$ which is the best-fit value of this angle from global fits. In addition it implies $\theta_{13} = 0$ which is also consistent with the data. Small deviation from these exact values may be generated by breaking the $\mu - \tau$ symmetry by a small amount. Combining $\mu - \tau$ flavour symmetry with GUTs has been considered in the case of $SU(5)$ in [20] and also for $SO(10)$ [8]. Here we impose $\mu - \tau$ symmetry on the Yukawa matrix for the 10_H and $\overline{16}_H$ whereas the one for 120_H is taken to be antisymmetric. We also impose a parity symmetry leading to Hermitian Yukawa matrices. Thus we consider the model

[1]It is also possible to get a double seesaw type mass matrix using singlet fields [14].

$SO(10) \otimes Z_2^{(\mu-\tau)} \otimes Z_2^{\mathcal{P}}$ [8]. Imposition of these two symmetries help in reducing the number of unknown parameters in the Yukawa sector. In addition, we make an ansatz relating the effective ν_R mass matrix arising due to the inclusion of adjoint fermions with the Yukawa matrix for 10_H. As a result the light neutrino mass matrix after seesaw mechanism obtains a simple form and can be written as a sum of two contributions. It turns out that with the above choice the neutrino mass matrix is $\mu - \tau$ symmetric so that one immediately gets $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. It is straight-forward to get the consequence for the neutrino masses and θ_{12} and obtain the conditions on the parameters such that tri-bimaximal mixing is obtained.

7.1 The Model

We explore an $SO(10)$ model where the three fermion families acquire mass through the 10_H and/or 120_H. The model also includes additional fermion multiplets in the $SO(10)$ adjoint representation, 45_F, and a $\overline{16}_H$.

In this model the Yukawa terms for the fermions can be expressed as:

$$\mathcal{L} = Y_{10} 16_F 16_F 10_H + Y_{120} 16_F 16_F 120_H. \tag{7.1}$$

In general, Y_{10} is a complex symmetric matrix while Y_{120} is complex antisymmetric. When the 10_H and 120_H scalars obtain their vacuum expectation values (*vevs*) quarks and leptons obtain masses which can be represented as:

$$\begin{aligned} m_d &= M_0 + iM_2, \;\; m_u = c_0 M_0 + ic_2 M_2, \\ m_l &= M_0 + ic_3 M_2, \;\; m_D = c_0 M_0 + ic_4 M_2. \end{aligned} \tag{7.2}$$

Above, m_d (m_u) denotes the mass matrix for the down-type (up-type) quarks, m_l is the charged lepton mass matrix, whereas m_D is the Dirac mass matrix of the neutrinos. The matrices M_0 and M_2 are proportional to Y_{10} and Y_{120} respectively.

$$M_0 = M_0^T, \;\; M_2 = -M_2^T. \tag{7.3}$$

c_0, c_2, c_3, and c_4 are constants fixed by Clebsch-Gordan (CG) coefficients and *vev* ratios which are taken to be real. We impose a generalized parity symmetry and make appropriate choices of the *vev*s [21] which make M_0 and M_2 real thereby reducing the number of free parameters and ensuring the hermiticity of the mass matrices in eq. 7.2.

For neutrinos the above implies the presence of only the Dirac mass term which cannot reproduce the correct neutrino mass pattern [12]. Since the $\overline{126}_H$ field is not present the type-I and type-II seesaw mass terms are absent in this model. One can of course generate the neutrino mass through the Witten mechanism of radiative seesaw [11] but then for non-SUSY $SO(10)$ such contributions are too small [12].

In this chapter we propose a new mechanism to generate a neutrino mass in a non-SUSY $SO(10)$ with 10_H and 120_H. We introduce additional matter multiplets (45_F) which belong to the adjoint representation of $SO(10)$. Note that this is similar to the so called type-III seesaw mechanism where one adds additional matter fields in the adjoint representation. However, as we will see, the neutrino mass is generated here through the 'double seesaw' mechanism. $SO(10)$ breaks to the SM through two intermediate steps:

$$\begin{aligned} SO(10) &\xrightarrow{M_X} \quad SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \\ &\xrightarrow{M_C} \quad SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{(B-L)} \\ &\xrightarrow{M_R} \quad \mathcal{G}_{SM}. \end{aligned} \tag{7.4}$$

The Pati-Salam ($\mathcal{G}_{422} \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$) decomposition gives:

$$45 = (\Sigma_{3L}, \Sigma_{3R}, \Sigma_{4C}, \Sigma_{LRC}) = (1,3,1) \oplus (1,1,3) \oplus (15,1,1) \oplus (6,2,2). \tag{7.5}$$

It is useful to note the $SU(3)_c \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ decompositions

$$\begin{aligned} (15,1,1) &\equiv (1,1,0,0) + (3,1,0,-4/3) + (\overline{3},1,0,4/3) + (8,1,0,0)\ , \quad (7.6) \\ (4,1,2) &\equiv (1,1,\pm\frac{1}{2},1) + (3,1,\pm\frac{1}{2},-1/3)\ . \end{aligned}$$

The colour, $U(1)_R$, and $U(1)_{(B-L)}$ singlet members of Σ_{3R} and Σ_{4c} couple to ν_R when $\overline{16}_H$ gets a *vev* along $(1,1,-\frac{1}{2},1)$ $\subset$(4,1,2) that breaks $U(1)_R \otimes U(1)_{B-L}$. The relevant Yukawa coupling is:

$$\begin{aligned} Y_{16}16_F45_F\overline{16}_H \supset \quad & Y_{16}\left[a_1(1,1,\frac{1}{2},-1)_F(1,1,0,0)_F^{\Sigma_{3R}} + a_2(1,1,\frac{1}{2},-1)_F(1,1,0,0)_F^{\Sigma_{4c}}\right] \\ & (1,1,-\frac{1}{2},1)_H\ . \end{aligned} \tag{7.7}$$

$a_{1,2}$ are CG coefficients. The *vev* $v_R \equiv <(1,1,-\frac{1}{2},1)_H>$ sets the scale M_R.

The masses of the adjoint matter fields are generated from

$$M\ Tr(45_F^2)\ +\ \lambda\ Tr(45_F^2 210_H)\ . \tag{7.8}$$

Once 210_H acquires a *vev* along the (1,1,1) direction, $SO(10)$ is broken to $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. In the mass term M_N of $(1,1,0,0)_F \subset (15,1,1)_F$ and $M_{\Sigma_{3R}}$ of $(1,1,0,0)_F \subset (1,1,3)_F$, an extra contribution (from the second term of eq. 7.8) is added, *i.e.*, $M_{\Sigma_{3R}} = M_N = M + \lambda < (1,1,0,0)_H >$. There is no symmetry that protects the masses of these adjoint fermions. So naturally these are very heavy ($\sim M_X$).

7.2 Constraints from gauge coupling unification

In this section, we discuss the Renormalisation Group (RG) evolution of the gauge couplings at the one-loop level, check for the scale of unification and determine the possible intermediate scales. The symmetry breaks in two stages following the steps given in (7.4). The contributions in the RG running from scalars at the different scales are included according to the 'extended survival hypothesis' (ESH) [22] which amounts to minimal fine tuning of the parameters of the potential. Our model contains extra adjoint fermions. But these fermions are very heavy $\sim \mathcal{O}\ (M_X)$, so they do not contribute in the renormalisation group evolution of the gauge couplings.

When the $SO(10)$ symmetry is broken to the Pati-Salam group [23] $\mathcal{G}_{422}$ by a 210_H multiplet through the *vev* in the $< (1,1,1) >$ direction, D-parity [24] is spontaneously broken at this scale (M_C).

The gauge coupling evolution is usually stated as [25]:

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i, g_j), \qquad (i, j = 1, \ldots, n), \tag{7.9}$$

where n is the number of couplings in the theory and at one-loop order

$$\beta_i(g_i, g_j) = (16\pi^2)^{-1} b_i g_i^3. \tag{7.10}$$

There is, however, a subtlety which must be taken into account since the gauge symmetry in the energy range M_R to M_C includes two $U(1)$ factors. According to the ESH the $SO(10)$ multiplets are split in mass with some submultiplets having mass above and some below this range. The incomplete scalar and fermion multiplets that contribute to the RG evolution at this stage lead to a mixing between these two $U(1)$ gauge groups. Thus even at the one-loop level one cannot treat the evolution of these $U(1)$ couplings in separation and in a generic scenario one

$SO(10)$ representation	Symmetry breaking	Scalars contributing to RG evolution		
		$M_Z \to M_R$ Under $\mathcal{G}_{SM}$	$M_R \to M_C$ Under $\mathcal{G}_{3211}$	$M_C \to M_X$ Under $\mathcal{G}_{422}$
10	$\mathcal{G}_{SM} \to EM$	(1,2,$\pm$1)	(1,2,$\pm\frac{1}{2}$,0)	(1,2,2)
120		...	...	(1,2,2), (15,2,2)
$\overline{\mathbf{16}}$	$\mathcal{G}_{3211} \to \mathcal{G}_{SM}$	...	(1,1,$-\frac{1}{2}$,1)	(4,1,2)
210	$\mathcal{G}_{422} \to \mathcal{G}_{3211}$	...	...	(15,1,3)

Table 7.1: Higgs submultiplets contributing to the RG evolution as per the extended survival hypothesis when symmetry breaking of $SO(10)$ takes place with two intermediate stages – see (7.4).

must include a 2×2 matrix of $U(1)$ couplings. The details of this $U(1)$ mixing has been discussed in the sec. 4.2.3 of chapter 4. We have computed the RG-coefficients following the proposals given in [26] at the one-loop level including the $U(1)$ mixings. The b_i are the ordinary β-coefficients and the $\tilde{b}_j$ are the additional ones which arise due to the mixings stated above.

Taking all this into account, the gauge couplings evolve as follows:

i) From M_C to M_X :

$$b_{2L} = 7/3; \;\; b_{2R} = 13; \;\; b_{4c} = -1. \tag{7.11}$$

ii) From M_R to M_C:

$$\begin{aligned} &b_{2L} = -3;\; b_{RR} = 53/12;\; b_{3c} = -7;\; b_{(B-L)(B-L)} = 33/8; \\ &\tilde{b}_{R(B-L)} = \tilde{b}_{(B-L)R} = -1/4\sqrt{6}. \end{aligned} \tag{7.12}$$

iii) From M_Z to M_R:

$$b_{1Y} = 21/5; \;\; b_{2L} = -3; \;\; b_{3c} = -7. \tag{7.13}$$

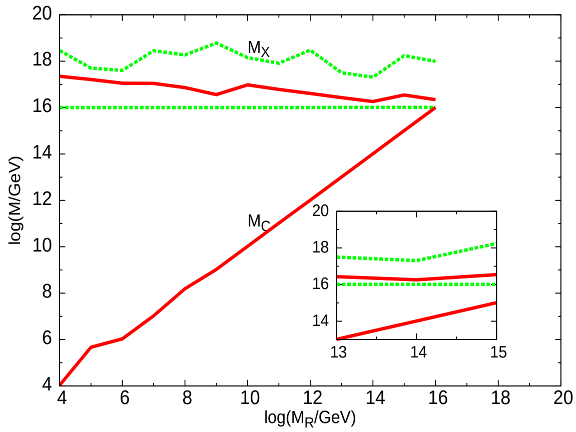

Figure 7.1: The allowed ranges of the unification (M_X, pale, green) and intermediate Pati-Salam (M_C, dark, red) scales as a function of the $U(1)_{(B-L)}$ breaking scale (M_R) for $SO(10)$ with two intermediate scales. The inset is a zoom of the region of interest for generating neutrino masses of the right magnitude.

The mixing of the two $U(1)$ groups adds flexibility to the model. With this, we find for every M_R a range of consistent solutions for M_C and M_X (see Fig. 7.1). In the plot we have exhibited the maximum and minimum values of both M_C and M_X consistent with unification. In a Grand Unified Theory low intermediate scales are always perceived with extra interest. These low intermediate scale scenarios keep alive the hope that signals of the GUT may be identified at accessible energies. In Fig. 7.1, we have shown that M_R and M_C can be quite low – ~ 10 TeV – which is within the reach of recent colliders, such as the LHC; this is an artifact of the inclusion of the $U(1)$ mixings. The *vev* v_R of the scalar $(1, 1, -\frac{1}{2}, 1) \subset \overline{16}$, sets the scale M_R. In the next section we have shown that v_R needs to be very high ($\sim 10^{14}$ GeV) to yield the correct neutrino mass with the Yukawa couplings $\sim \mathcal{O}(1)$. In the inset of Fig. 7.1 we magnify this range of M_R. It is to be noted that this establishes that the proposed model of 'double-seesaw' mechanism is compatible with gauge coupling unification at a scale which is not in conflict with the present bound on the proton lifetime.

7.3 Neutrino Mass

The neutrino mass matrix in the basis $((\nu_L)^c, \nu_R, \Sigma^0_R, N)$ is:

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & 0 & a_1 Y_{16} v_R & a_2 Y_{16} v_R \\ 0 & a_1 Y_{16}^T v_R & M_N & 0 \\ 0 & a_2 Y_{16}^T v_R & 0 & M_N \end{pmatrix} . \tag{7.14}$$

The left-handed fermionic triplets, Σ_{3L}, having a mass matrix identical to M_N, do not mix with other fermions since the left-handed analogue of v_R is chosen to be zero. From the mass matrix (7.14) it is seen that the masses of the light neutrinos are obtained by integrating out the heavy triplet and singlet fermions. Thus we can have type-III and type-I seesaw mechanism in succession. The right-handed neutrino mass term is generated once the heavy triplet fermion Σ^0_{3R} and N are integrated out – an effective type-I + type-III seesaw. Assuming $M_N \gg v_R Y_{16} \gg m_D$, the right-handed neutrino mass matrix is:

$$M_R = v_R^2 Y_{16} M_M^{-1} Y_{16}^T, \tag{7.15}$$

where,

$$M_M^{-1} = (a_1^2 + a_2^2) M_N^{-1} \ , \tag{7.16}$$

and the light neutrino mass matrix after an effective type-I seesaw becomes:

$$m_\nu = m_D M_R^{-1} m_D^T \ . \tag{7.17}$$

Substituting for m_D from eq. 7.2 one arrives at the general expression of m_ν as

$$m_\nu = c_0^2 M_0 M_R^{-1} M_0 - c_0 c_4 M_0 M_R^{-1} M_2 + c_4 c_0 M_2 M_R^{-1} M_0 + c_4^2 M_2 M_R^{-1} M_2 \ . \tag{7.18}$$

Typical values for the various parameters are $v_R \sim 10^{14}$ GeV, $M_N \sim 10^{15}$ GeV, and $c_i \sim \mathcal{O}(1)$, $Y_i \sim \mathcal{O}(1)$ which gives $M_R \sim 10^{12}$ GeV. Then with $m_D \sim 100$ GeV one gets $m_\nu \sim 1$ eV.

With three neutrino generations, the model has 6 real parameters in M_0 and 3 in M_2. In addition there are 5 *vev*s $(c_0, c_2, c_3, c_4, v_R)$. Besides, there are additional parameters in Y_{16} and M_N. However the low energy neutrino mass matrix is characterized by 9 parameters. Neutrino oscillation experiments have so far determined and/or bounded 5 of these. The general case is obviously not sufficiently constrained. One way to address this lacuna requires invoking some flavour symmetry. We consider this to be the $\mu - \tau$ symmetry.

7.4 $\mu - \tau$ symmetry and allowed textures

$\mu - \tau$ symmetry has been considered widely for explaining the large atmospheric mixing angle in the neutrino sector [19]. In addition it gives $\theta_{13} = 0$ which is also consistent with the current global fits[2]. We impose the condition of a generalized $\mu - \tau$ symmetry on the Yukawa matrices stemming from 10_H and $\overline{16}_H$. This implies that these matrices are invariant under the exchange of the second and third rows and columns. This reduces the number of unknown parameters in the Yukawa sector. However, this symmetry cannot be exact in the quark and lepton sector. This is accomplished by the term M_2 in the fermion mass matrices which originates from the 120_H which is taken to be antisymmetric under the exchange of $2 \leftrightarrow 3$ and breaks $\mu - \tau$ symmetry spontaneously.

In addition we had imposed a generalized parity symmetry [21] which makes the complex matrices M_0 and M_2 real thereby reducing the number of free parameters. Thus the model that we consider is $SO(10) \otimes {Z_2}^{\mu-\tau} \otimes Z_2^{\mathcal{P}}$ [8]. However it is to be mentioned that if we assume exact $\mu - \tau$ (anti)symmetry in (M_2) M_0 then a generalized CP-invariance holds [8] and the CKM matrix comes out as real. This can be rectified either by assuming some of the *vevs* to be complex or by allowing a small explicit breaking of $\mu - \tau$ symmetry in M_0. This induces CP-violation phases in both U_{CKM} and U_{PMNS} [8]. We work in the basis where the charged lepton mass matrix is diagonal and the PMNS matrix is solely determined by the mixing in the neutrino sector.

The structures for M_0 and M_2 under the above symmetries are given by

$$M_0 = \begin{pmatrix} a' & b' & b' \\ b' & c' & d' \\ b' & d' & c' \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & x' & -x' \\ -x' & 0 & y' \\ x' & -y' & 0 \end{pmatrix}. \tag{7.19}$$

We consider a model with three adjoint fermion multiplets, *i.e.,* the model consists of $(3\nu_L + 3\nu_R + 3N + 3\Sigma_R)$. Thus, Y_{16} and M_N are also 3×3 matrices which we take to be $\mu - \tau$ symmetric. It follows from eq. 7.15 that M_R also respects this symmetry. Thus we have both M_0 and M_R to be $\mu - \tau$ symmetric. In order to make the model predictive we make the further assumption that M_R and M_0 are proportional, *i.e.,*

$$KM_R = M_0. \tag{7.20}$$

[2]Recent global fits have found indication for non-zero θ_{13} although this is only a 1σ effect. A small non-zero value of θ_{13} can be induced by breaking the $\mu - \tau$ symmetry.

where K is a constant. m_ν in eq. 7.18 then takes the form

$$m_\nu = Kc_0^2 M_0 + Kc_4^2 M_2 M_0^{-1} M_2 = M_1 + M_1' \ . \tag{7.21}$$

The number of free real parameters in the theory are now 4 from M_0, 2 in M_2, and 4 real *vevs*. Because of eq. 7.20 M_R adds just one further parameter. Thus in total we have 11 real parameters. The *vev* ratios c_2 and c_3 do not affect eq. 7.21 and thus we have 9 parameters involved in the neutrino sector. Some of these appear only as overall scale factors.

We note that although M_2 is $\mu - \tau$ antisymmetric the product $M_2 M_0^{-1} M_2$ possesses $\mu - \tau$ symmetry. Thus m_ν is $\mu - \tau$ symmetric. This immediately implies $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Therefore the mixing matrix in the basis where the charged lepton mass matrix is diagonal is given as,

$$U_{\rm PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \tag{7.22}$$

which can be brought to the standard $U_{\rm PMNS}$ form by a suitable redefinition of fermion phases. We have

$$m_\nu = U_{\rm PMNS} M_{dia} U_{\rm PMNS}^T, \tag{7.23}$$

where $M_{dia} = Diag(m_1, m_2, m_3)$. m_1, m_2, m_3, the mass eigenvalues are real [3], and are given as

$$\begin{aligned} m_1 &= \frac{X - \sqrt{X^2 - 4(d-c)Y}}{2(d-c)}, \\ m_2 &= \frac{X + \sqrt{X^2 - 4(d-c)Y}}{2(d-c)}, \\ m_3 &= \frac{Y}{2b^2 - ac - ad}. \end{aligned} \tag{7.24}$$

Here

$$\begin{aligned} X &= -ac - c^2 + ad + d^2 + 2x^2 + y^2; \\ Y &= 2b^2c - ac^2 - 2b^2d + ad^2 + 2cx^2 + 2dx^2 + 4bxy + ay^2, \end{aligned} \tag{7.25}$$

[3]Since the mass matrices have real entries, complex roots can appear only in conjugate pairs leading to unacceptable degenerate neutrinos. We take the eigenvalues to be all non-negative.

and

$$a = Kc_0^2 a', \;\; b = Kc_0^2 b', \;\; c = Kc_0^2 c', \;\; d = Kc_0^2 d', \;\; x = Kc_4^2 x', \;\; y = Kc_4^2 y' \quad . \tag{7.26}$$

Note that the eigenstate m_3 is determined to be the one associated with the eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$. Whether this is the highest mass state or the lowest mass state, *i.e.,* whether the hierarchy is normal or inverted will depend on the values of the parameters. We further require $\Delta m_{21}^2 > 0$ from the solar data. This implies that for our choice of m_2 and m_1

$$\frac{X}{(d-c)^2}\sqrt{X^2 - 4(d-c)Y} \;\; > \;\; 0 \tag{7.27}$$

Using eqs. 7.22 and 7.23 we obtain,

$$\tan\theta_{12} = \frac{1}{\sqrt{2}}\frac{(a-m_1)(c-d) - 2x^2}{b(c-d) + xy} \quad . \tag{7.28}$$

The condition for tri-bimaximal mixing implies

$$(a - m_1 - b)(c-d) = 2x^2 + xy \quad . \tag{7.29}$$

7.4.1 10_H dominance

In this case, $a, b, c, d \gg x, y$. The light neutrino mass matrix m_ν is approximated as $Kc_0^2 M_0$ with M_0 defined in eq. 7.19. In this limit the mass eigenvalues are given as,

$$m_1 = \frac{1}{2}(f_1 - R), \;\; m_2 = \frac{1}{2}(f_1 + R), \;\; m_3 = c - d \quad , \tag{7.30}$$

with

$$R = +\sqrt{8b^2 + f_2^2} \quad , \tag{7.31}$$

where,

$$f_1 = a + c + d, \;\; f_2 = -a + c + d \quad . \tag{7.32}$$

Again, m_3 is identified as the eigenvalue for the state
eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$. Since the solar data has determined the ordering of the 1 and 2 mass states to Then the mass squared differences can be expressed as,

$$\Delta m_{21}^2 = f_1 R \qquad \Delta m_{31}^2 = (f_1 R - a^2 - 4b^2 + c^2 + d^2 - 6cd)/2 \quad . \tag{7.33}$$

Again, the mass ordering will depend on the values of the parameters. In general both normal and inverted hierarchy are possible. In addition, the solar neutrino data require $\Delta m^2_{21} > 0$ which implies $f_1 R > 0$ for the above selection of states.

The mixing angles are given as,

$$\theta_{13} = 0\,, \;\; \theta_{23} = \pi/4\,, \;\; \tan\theta_{12} = \frac{(R - f_2)}{2\sqrt{2}b}\,. \tag{7.34}$$

Tri-bimaximal mixing implies $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\tan^2\theta_{12} = 1/2$. We see that the requirements for θ_{13} and θ_{23} are already satisfied. If in addition we impose

$$f_2 = b \;\; \Longrightarrow \;\; R = 3b,\; f_1 = (2a + b), \tag{7.35}$$

tri-bimaximal mixing is obtained. In this limit

$$\Delta m^2_{21} = 3\,b\,(2a + b) \qquad \Delta m^2_{31} = (c - d)^2 - (a - b)^2\;\;. \tag{7.36}$$

7.4.2 120_H dominance

In this limit $a, b, c, d \ll x, y$ and the low energy neutrino mass matrix is given as

$$m_\nu = M_4 = K c_4^2 M_2\, M_0^{-1}\, M_2\;\;. \tag{7.37}$$

The U_{PMNS} continues to be given by eq. 7.22. The eigenvalues, in terms of the parameters defined in eq. 7.26, are given as,

$$m_1 = 0,\; m_2 = \frac{2x^2 + y^2}{d - c},\; m_3 = \frac{2cx^2 + 2dx^2 + 4bxy + ay^2}{2b^2 - ac - ad}\;\;. \tag{7.38}$$

Since the eigenvector $(0, 1/\sqrt{2}, -1/\sqrt{2})$ belongs to the eigenvalue m_3 so that the zero eigenvalue has to be associated with the eigenstate m_1. Therefore this case corresponds to the normal hierarchy. Since $m_1 = 0$, $\Delta m^2_{21} = m^2_2$ and $\Delta m^2_{31} = m^2_3$. Then, using eqs. 7.22 and 7.23 one obtains the 1-2 mixing angle as,

$$\tan\theta_{12} = -\frac{\sqrt{2}x}{y} \tag{7.39}$$

Thus, the mixing matrix in this case is completely determined by the parameters of M_2. The condition for obtaining exact tri-bimaximal mixing is $y = -2x$.

In summary in this chapter we have considered a non-SUSY $SO(10)$ model in which the fermion masses originate from Yukawa couplings to 10_H and 120_H.

In such a model the usual type-I and type-II seesaw mass terms which arise from $\overline{126}_H$ are not present. It is possible to generate the neutrino mass at two-loops by the radiative seesaw mechanism [11]. But for non-SUSY $SO(10)$ the contribution is very small.

Here we suggest a new possibility to generate neutrino masses in a non-SUSY $SO(10)$ model with $10_H + 120_H$ using fermions in the 45_F representation and an additional $\overline{16}_H$ scalar multiplet. Constraints from gauge coupling unification requires the *vev* $< \overline{16}_H >$ to be in the range $\sim 10^4 - 10^{16}$ GeV. However from the standpoint of generation of naturally small neutrinos masses the range $\sim 10^{13} - 10^{15}$ GeV is preferred. We show that in this case one can generate small neutrino masses through the 'double seesaw' mechanism. Predictions for mixing angles require further imposition of a flavour symmetry which we chose to be the $\mu - \tau$ symmetry for the Yukawa matrices due to 10_H and $\overline{16}_H$ whereas for the one originating from 120_H we take the matrix to be $\mu - \tau$ antisymmetric. We further assume the right-handed matrix (M_R) due to the heavy fields to be proportional to the one (M_0) originating from 10_H. With this the light neutrino mass matrix is given by the sum of two terms which are both $\mu - \tau$ symmetric. This automatically satisfies $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. We present the neutrino masses and θ_{12} obtained from this model and determine the condition for satisfying tri-bimaximality. We also discuss the limiting values when one of the terms dominate. For the 10_H-dominance case both hierarchies are possible whereas if the 120_H dominates the hierarchy can only be normal.

References

[1] G. Senjanović, arXiv:hep-ph/0612312.

[2] J. A. Harvey, D. B. Reiss and P. Ramond, Nucl. Phys. B **199** (1982) 223; G. Lazarides and Q. Shafi, Nucl. Phys. B **350** (1991) 179; K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70** (1993) 2845 [arXiv:hep-ph/9209215]; C. H. Albright and S. Nandi, Phys. Rev. Lett. **73** (1994) 930 [arXiv:hep-ph/9311227].

[3] P. Minkowski, Phys. Lett. B **67** (1977) 421; T. Yanagida, Proceedings of the *Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, 1979, eds. A. Sawada, A. Sugamoto; S. Glashow, in *Cargèse 1979, Proceedings, Quarks and Leptons* NATO Adv. Study Inst. Ser.B Phys. **59** (1979) 687; M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the *Supergravity Stony Brook Workshop*, New York, 1979, eds. P. Van Niewenhuizen, D. Freedman *et al.* (North-Holland, Amsterdam, 1980); R. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.

[4] M. Magg and C. Wetterich, Phys. Lett. B **94**, (1980) 61; G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, (1981) 287.

[5] K. y. Oda, E. Takasugi, M. Tanaka and M. Yoshimura, Phys. Rev. D **59** (1999) 055001 [arXiv:hep-ph/9808241]; H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Rev. D **68** (2003) 115008 [arXiv:hep-ph/0308197]; H. S. Goh, R. N. Mohapatra, S. Nasri and S. P. Ng, Phys. Lett. B **587** (2004) 105 [arXiv:hep-ph/0311330]; S. Bertolini, M. Frigerio and M. Malinsky, Phys. Rev. D **70** (2004) 095002 [arXiv:hep-ph/0406117]; T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C **42** (2005) 191 [arXiv:hep-ph/0401213]; B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Rev. D **70** (2004) 035007 [arXiv:hep-ph/0402122]; C. S. Aulakh and A. Girdhar, Nucl. Phys. B **711** (2005) 275 [arXiv:hep-ph/0405074].

[6] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Rev. D **73** (2006) 055001

[arXiv:hep-ph/0510139].

[7] W. Grimus and H. Kühböck, Phys. Lett. B **643** (2006) 182 [arXiv:hep-ph/0607197]; B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Lett. B **603** (2004) 35 [arXiv:hep-ph/0406262].

[8] A. S. Joshipura, B. P. Kodrani and K. M. Patel, Phys. Rev. D **79** (2009) 115017 [arXiv:0903.2161 [hep-ph]].

[9] K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. D **64** (2001) 053015 [arXiv:hep-ph/0010026].

[10] L. Lavoura, H. Kühböck and W. Grimus, Nucl. Phys. B **754** (2006) 1 [arXiv:hep-ph/0603259].

[11] E. Witten, Phys. Lett. B **91** (1980) 81.

[12] B. Bajc and G. Senjanović, Phys. Rev. Lett. **95** (2005) 261804 [arXiv:hep-ph/0507169].

[13] B. Bajc and G. Senjanović, Phys. Lett. B **610** (2005) 80 [arXiv:hep-ph/0411193].

[14] R. N. Mohapatra, Phys. Rev. Lett. **56** (1986) 561; S. M. Barr, Phys. Rev. Lett. **92** (2004) 101601 [arXiv:hep-ph/0309152]; S. M. Barr and I. Dorsner, Phys. Lett. B **632** (2006) 527 [arXiv:hep-ph/0507067].

[15] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C **44** (1989) 441.

[16] P. Fileviez Pérez, Phys. Rev. D **76** (2007) 071701 [arXiv:0705.3589 [hep-ph]]; I. Dorsner and P. Fileviez Pérez, JHEP **0706** (2007) 029.

[17] B. Bajc, M. Nemevsek and G. Senjanović, Phys. Rev. D **76** (2007) 055011 [arXiv:hep-ph/0703080]; A. Arhrib, B. Bajc, D. K. Ghosh, T. Han, G. Y. Huang, I. Puljak and G. Senjanović, Phys. Rev. D **82** (2010) 053004 [arXiv:0904.2390 [hep-ph]].

[18] P. Fileviez Pérez, JHEP **0903** (2009) 142 [arXiv:0809.1202 [hep-ph]].

[19] C. S. Lam, Phys. Lett. B **507** (2001) 214 [arXiv:hep-ph/0104116].

[20] R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **636** (2006) 114 [arXiv:hep-ph/0603020].

[21] W. Grimus and H. Kühböck, Eur. Phys. J. C **51** (2007) 721 [arXiv:hep-ph/0612132].

[22] F. del Aguila and L. E. Ibañez, Nucl. Phys. B **177** (1981) 60.

[23] J. C. Pati and A. Salam, Phys. Rev. Lett. **31** (1973) 66; Phys. Rev. D **10** (1974)

275.

[24] D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **52** (1984) 1072.

[25] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33** (1974) 451.

[26] B. Holdom, Phys. Lett. B **166** (1986) 196; F. del Aguila, G. D. Coughlan and M. Quiros, Nucl. Phys. B **307** (1988) 633 [Erratum-ibid. B **312** (1989) 751]; S. Bertolini, L. Di Luzio and M. Malinsky, Phys. Rev. D **80** (2009) 015013 [arXiv:0903.4049 [hep-ph]]; J. Chakraborty and A. Raychaudhuri, Phys. Rev. D **81** (2010) 055004 [arXiv:0909.3905 [hep-ph]].