LINKING INFLATION WITH DARK MATTER, BARYOGENESIS AND NEUTRINO MASSES

By

ABHASS KUMAR

Enrolment No: PHYS08201304003

Harish-Chandra Research Institute, Allahabad

A thesis submitted to the

Board of Studies in Physical Sciences

In partial fulfillment of requirements

for the Degree of

DOCTOR OF PHILOSOPHY

of

HOMI BHABHA NATIONAL INSTITUTE

July, 2019

Homi Bhabha National Institute[1]

Recommendations of the Viva Voce Committee

As members of the Viva Voce Committee, we certify that we have read the dissertation prepared by Abhass Kumar entitled "Linking Inflation with Dark Matter, Baryogenesis and Neutrino Masses" and recommend that it may be accepted as fulfilling the thesis requirement for the award of Degree of Doctor of Philosophy.

Chairman - Prof. S. Naik S Naik	Date: 24/10/19
Guide /Convener - Prof. Sandhya Choubey Sandhya Choubey	Date: 24/10/19
Examiner - Prof. Subhendra Mohanty Subhendra Mohanty	Date: 24. 10. 19.
Member 1- Prof. B. Mukhopadhyaya B Mukhopadhyaya	Date: 24/10/19
Member 2 - Prof. Santosh Kumar Rai [signature]	Date: 24/10/19
Member 3 - Prof. Ujjwal Sen Ujjwal Sen	Date: 24/10/19

Final approval and acceptance of this thesis is contingent upon the candidate's submission of the final copies of the thesis to HBNI.

I/We hereby certify that I/we have read this thesis prepared under my/our direction and recommend that it may be accepted as fulfilling the thesis requirement.

Date: 24.10.2019

Place: Prayagraj

Sandhya Choubey
Prof. Sandhya Choubey
Guide

[1] This page is to be included only for final submission after successful completion of viva voce.

STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfilment of requirements for an advanced degree at Homi Bhabha National Institute (HBNI) and is deposited in the Library to be made available to borrowers under rules of the HBNI.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgement of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the Competent Authority of HBNI when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

Abhass Kumar

Abhass Kumar

Name & Signature

of the student

DECLARATION

I, Abhass Kumar, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree or diploma at this or any other Institution or University.

Abhass Kumar

Abhass Kumar

Name & Signature

of the student

List of Publications arising from the thesis

Journal

1. "TeV Scale Leptogenesis, Inflaton Dark Matter and Neutrino Masses in a Scotogenic Model", Debasish Borah, P.S. Bhupal Dev, Abhass Kumar, *Phys. Rev. D*, **2019**, *99*, 055012

2. "Inflation and Dark Matter in the Inert Doublet Model" Sandhya Choubey, Abhass Kumar, *JHEP,* **2017**, *11,* 080

Preprints

1. "Inflation and Reheating with a Fermionic Field", Abhass Kumar, *arXiv:1811.12237[gr-qc],* **2018**

Conferences

1. "Linking Inflation with Dark Matter, Baryogenesis & Neutrino Masses in the Scotogenic Model", Abhass Kumar, *Neutrino & Dark Matter Activity*, **2019**, *HRI Allahabad,* India.
2. "TeV Scale Leptogenesis, Inflaton Dark Matter & Neutrino Mass in the Scotogenic Model", Abhass Kumar, *XXIII DAE-BRNS High Energy Physics Symposium,* **2018**, *IIT Chennai,* India.
3. "Inflation and Dark Matter in the Inert Doublet Model", Abhass Kumar, *Nu HoRIzons VII,* **2017**, *HRI Allahabad*, India.

Abhass Kumar

Abhass Kumar

Name & Signature

of the student

DEDICATIONS

To my sister, Aastha, the strongest person I know

Kāyen vācā mansendriyairvā, buddhyātmanā ca prakṛte svabhāvāt,

karomi yadyatsakalam parasmai, nārāyaṇāyeti samarpayāmī

ACKNOWLEDGEMENTS

I have spent quite some time in HRI and during my stay, I have had opportunity to thank a lot of people. The first of them is my supervisor Prof. Sandhya Choubey. I absolutely liked the independence she gave me to pursue my own ideas. The weekly arXiv discussion sessions that we had were of immense help and motivated me to keep up with the recent research topics. Her insight into physics topics and the round of questioning on my ideas that followed helped me become comfortable with my work.

I would also like to thank my teachers and thesis committee members, in no particular order, Prof. Jayant Bhattacharya, Prof. Ashoke Sen, Prof, Dileep Jatkar, Prof. Sandhya Coubey, Prof. Santosh Rai, Prof. Arun Pati, Prof. Ujjwal Sen, Prof. Sumathi Rao, Prof. Anirban Basu and Prof. Rajesh Gopakumar, Prof. S. Naik, Prof. B Mukhopadhyaya, Prof. Santosh Rai, Prof. Ujjwal Sen.

I made some of my best friendships at HRI. Thanks a lot Gautam, Sandeep, Samiran, Sarif, Shouvik, Dipyaman, Ritabrata, Chiranjib, Bhuwanesh, Samrat, Deepak, Sitender, Mrityunjay, Ashutosh, Juhi, Ruchi, Kasinath, Siddharth, Abhishek, KMT, Waleed and more

I also acknowledge the help I received from the administration staff from time to time.

A special thanks to my parents and sister for being being there whenever I looked.

Abhass Kumar

Abhass Kumar

Contents

Summary

In this thesis we explore scenarios where inflation can be linked with dark matter, baryogenesis and neutrino masses in particle physics models such that the same field is responsible for all them. We construct a simple and minimal model that provides a common framework for inflation, dark matter, baryogenesis and neutrino masses. Dark matter and inflaton are the same field while they participate with another set of fields for neutrino masses and baryogenesis.

In our work, we extend the standard model by a $SU(2)$ scalar doublet and three massive SM singlet fermions which can act as right handed neutrinos. Both these sets of particles are odd under an extra $\mathbb{Z}_2$ symmetry in which the SM particles are even. The non-minimal gravity coupling forces us to use a conformal transformation of the action. The conformally transformed inert doublet acts as the inflaton. The spectral index - n_s and the tensor to scalar ratio - r are calculated. We find $n_s = 0.9678$ and $r =$ 0.0029 which are quite consistent with Planck 2018 results [1] which give $n_s = 0.9649 \pm 0.0042$ and $r = 0.0029$ at 95% C.L. The scalar power spectrum gives

the relation between the inert doublet quartic self-coupling λ_2 and the non-minimal gravity coupling ξ_2 to be $\xi_2 = 5.33\times10^4\sqrt{\lambda_2}$. The reheating dynamics give us a lower bound on $\lambda_2 \gtrsim 1/60$. Later, the lightest SM singlet fermion decays to an SM lepton and the inert doublet particles. The interference of the tree-level and the loop level decay terms produces a CP asymmetric decay that generates a lepton asymmetry. The sphaleron processes of the standard model convert it to the baryon asymmetry of the Universe. The results after matching with observations give bounds on the CP violating coupling between the inert doublet and the Higgs doublet λ_5. We find that it should be of $\mathcal{O}(10^{-4} - 10^{-5})$ for successful baryogenesis if the SM singlet fermion masses are of $\mathcal{O}(10 - 100)$ TeV. The lightest neutral particle of the inert doublet freezes-out near the electroweak scales to give the dark matter relic with a mass of around 1.5 TeV. Neutrino masses are generated radiatively at the one loop level by interaction between the SM singlet fermions and the inert doublet in the loop. The neutrino masses thus obtained are matched with the neutrino oscillation data. For baryogenesis to occur at the 10 TeV scale, it is necessary that the lightest SM neutrino have a mass of $\mathcal{O}(10^{-11} - 10^{-12})$ eV.

We also explore driving inflation using a fermion condensate. A right handed neutrino (RHN) condensate drives inflation. The RHN can later generate neutrino masses and baryon asymmetry. If the dynamics of inflation in such a scenario are successfully calculated, such a model can be the most minimal model for combining inflation with baryogenesis, neutrino masses and dark matter. Some of the problems with such a model are the justification of the formation of condensates and the calculation of the power spectrum and its spectral index.

List of Figures

1 Introduction

The Universe has always evoked a strong sense of awe and curiosity in human minds. The urge to understand it has given birth to some very active research topics in physics. Perhaps the subjects most directly related to the Universe are cosmology and particle physics. ***Cosmology*** is the science of the birth, nature and development of the Universe while ***particle physics*** deals with the nature of the elementary particles constituting everything in the known world. Over the years, we have come a long way in understanding the nature of many of the particles and the larger and macroscopic phenomena occurring in the cosmos to develop a standard model for particle physics and for cosmology.

The standard model of particle physics is a gauge theory that describes three of the fundamental forces of nature – the strong force, electromagnetism and the weak force. It also classifies all the known elementary particles and the interactions among them. It also describes the interactions between various particles and the predictions from the standard model of particles have held firm through various collider based tests.

The standard model for cosmology, better known as the ΛCDM model is the most widely accepted model for the origin of the Universe via the Big Bang and its three major components – the cosmological constant Λ associated with dark energy making up around 70% of the known Universe, dark matter making up about 27%, and ordinary matter making up the rest. The Big Bang theory states that a singular point exploded around 14 billion years ago into the Universe and has been expanding ever since to end up in the present Universe. Indeed evidence of such a case has been observed through various experiments including the Hubble Space Telescope [3, 4].

Both of these have been incredible in explaining why the Universe is as it is. More and more observations have also shown that the microscopic particle physics and the macroscopic cosmology are linked. Inflation, baryogenesis, dark matter and the neutrino sector are some of the topics where there is an inexplicable link between cosmology and particle physics. The ΛCDM model describes the structure of the Universe and tells us the ratios between the three components making up the known Universe – ordinary matter, dark matter and dark energy. It can be extended by adding cosmological inflation which is the era of exponential expansion shortly after the Big Bang. The nature of the particle driving inflation, the *inflaton*, is not needed for the ΛCDM model and is part of the particle physics approach to the Universe. Similarly, while the dark matter abundance and its distribution go into the details of the ΛCDM model, its nature and interactions are the purview of particle physics. Baryons and anti-baryons are both a part of the matter in the Universe and do not affect the macroscopic structure which is governed by gravity. However, the excess of baryons over anti-baryons is explained by particle physics mechanisms.

Experiments like the WMAP [5] and Planck which observe the cosmic microwave background have produced a map of the Universe. Some of the most obvious features of the map are that our Universe is:

- isotropic at large scales,
- homogeneous at large scales (Mega-parsecs) with small inhomogeneities of the order of 10^{-5} at small scales,
- there exists a horizon problem – regions in the Universe which are causally separated are still correlated in-spite of the information about the correlation functions moving at only the speed of light,
- the Universe is flat,
- all the known matter comprises less than 5% of the Universe,
- 27% of the Universe is composed of a mysterious non-baryonic [6] dark matter that seems to interact only gravitationally,
- the remaining $68 - 69\%$ is made by dark energy, something characterised by a negative pressure,
- the baryon to photon ratio is measured very precisely.

Apart from the above, another area where the standard model has no answer is the mass of neutrinos [6]. In the SM, neutrinos are massless elementary particles while observations of oscillations between neutrino flavours suggest that neutrinos must have a non-zero small mass.

1.1 Inflation

Inflation [7, 8] was invented as a solution to the flatness and the horizon problem. It can also explain the homogeneity and isotropy of the Universe. The WMAP experiment in 2010 [5] and the Planck experiment results of 2018 [1] have constrained inflationary parameters like the scalar power spectrum and the spectral index and the tensor to scalar perturbations ratio. The inflationary paradigm states that shortly after the Big Bang, the nascent Universe underwent a rapid exponential expansion phase during which it grew by atleast 60 e-folds (e^{60} times). Before the expansion, all the points in the Universe were causally connected, solving the horizon problem. Different regions were correlated to each other explaining homogeneity and isotropy. After inflation, as the different regions became causally disconnected, each region evolved independently but due to similar initial conditions, the Universe still remains homogeneous and isotropic at large scales. However, small scale inhomogeneities arise because of random quantum fluctuations resulting in the formation of structure in the Universe.

A consequence of inflation is the disappearance of any information before the inflationary phase from the Universe. Any fields or particle densities present in the Universe before inflation get diluted to negligible amounts. In essence, the end of inflation marks a cold, empty Universe which needs to be heated up again and repopulated by all the particles of the standard model. This is usually achieved by the decay or annihilation of the inflaton – the particle responsible for driving inflation – into other relativistic standard model particles and the process is called *reheating* [8, 9].

1.2 Baryogenesis

Matter and anti-matter cannot exist together. This tells us that there is no anti-matter around us. In fact, there is very less anti-matter compared to matter in the known Universe and the CMB data gives us a very precise value of the over-density of matter particles over anti-matter particles. Matter anti-matter asymmetry must be created sometime during the evolution of the Universe. Even if the Universe started with an asymmetric phase, there must be a mechanism to generate it again after inflation as inflation dilutes all kinds of information including the asymmetry between particles and anti-particles. ***Baryogenesis*** is the study of generation of excess baryons over anti-baryons. Sakharov had in 1967, given the three necessary conditions for generating the baryon asymmetry in the Universe [10] which are:

1. B (baryon number) violation,
2. C (charge conjugation) and CP (charge conjugation and parity) violation,
3. out-of-equilibrium interactions.

The first condition is quite obvious otherwise equal number of baryons and anti-baryons will be created in any reaction. If CP is conserved, for every interaction producing a baryon, an analogous reaction exists that produces anti-baryons:

$$X + Y \rightarrow B + others \tag{1.1}$$

$$\bar{X} + \bar{Y} \rightarrow \bar{B} + others \tag{1.2}$$

To see the necessity of out-of-equilibrium interactions consider where both B and C and CP are violated adequately but the processes occur in equilibrium. The thermal average of the generated baryon number $\langle B \rangle$ can be calculated using the trace of density matrix, $\rho = e^{\beta H}$, where H is the Hamiltonian and $\beta = \frac{1}{k_B T}$ ($k_B =$ Boltzmann constant, $T =$ temperature) [11].

$$\begin{aligned} \langle B \rangle &= \mathrm{Tr}\left(e^{\beta H} B\right) \\ &= \mathrm{Tr}\left(CPT\,\rho (CPT)^{-1} CPT\, B\, (CPT)^{-1}\right) \\ &= \mathrm{Tr}\,(-\rho B) = -\,\mathrm{Tr}\,(\rho B) = -\langle B \rangle \\ \Rightarrow \langle B \rangle &= 0 \end{aligned} \tag{1.3}$$

Out of the above three conditions, B violation occurs in the standard model of particles and the electroweak phase transition offers a natural out-of-equilibrium condition but there is not enough C and CP violation. Therefore we require some beyond the standard model mechanism with enough C and CP violating features. The out-of-equilibrium condition in such models can be easily satisfied by the accelerated expansion of the Universe. The interaction rate for any process must remain greater than the expansion rate of the Universe (the *Hubble rate parameter*) for it to be in equilibrium. As the Universe keeps expanding, eventually all interactions fall out of equilibrium.

1.3 Dark Matter

Dark matter is a type of non-luminous, non-baryonic matter whose only known interactions are gravitational. It is present in the Universe as a relic with negligible interactions with any other known particle through the other three forces. This makes it extremely difficult to detect the dark matter particle. It constitutes over 27% of the energy density in the Universe [2]. Dark matter was discovered due to the anomaly obtained in the calculation of the mass of the Milky Way galaxy using the visible stars and using the velocity dispersion curves of the stars rotating around the galaxy, specially of those near the periphery of the galaxy. There are many observational evidences of dark matter including galaxy rotation curves, the velocity dispersion relations of stars, gravitational lensing, structure formation in the Universe and the cosmic microwave background.

Dark matter could be cold, warm or hot depending on its velocities. The ΛCDM model of cosmology uses cold dark matter and reproduces the cosmological data quite well. However, it should be noted that the ΛCDM model still has some problems like the missing satellite problem (also called the dwarf galaxy problem) [12–16] and the "too big to fail" problem [17].

Some of the candidates for the dark matter particle are weakly interacting massive particle (WIMP), axions, strongly interacting massive particle (SIMP), feebly interacting massive particle (FIMP), gravitationally interacting massive particle (GIMP), the lightest supersymmetric particle (LSP) or primordial black holes. WIMPs are the most common candidates for dark matter. They are massive particles that undergo weak interactions with other standard model particles, giving rise to something called the ***WIMP miracle***: particles in the mass range of $\mathcal{O}(10^2-10^3)$ GeV with couplings of the weak scale order produce the observed relic abundance of dark matter $\Omega_{DM}h^2 \approx 0.1$. In recent years, however, with the non-observation of WIMPs in colliders or other dark matter detection experiments, interest in other candidates like axions, FIMPs and SIMPs has increased.

There are two main types of dark matter detection experiments: direct and indirect. In direct detection experiments, the dark matter particle scatters off a nucleus which produces nuclear recoil. The nuclear recoil energies are measured to get constraints on the dark matter scattering cross sections and their masses. In indirect detection experiments, one detects secondary particles like neutrinos that are produced as a result of dark matter decays and annihilations inside the sun or in places with a high dark matter density like the galaxy center.

1.4 Unification

1.4.1 Inflation and dark matter

One of the first attempts at combining inflaton and dark matter was done by Liddle and Ureña-López [18] in the string theory landscape. They consider the conditions required for merging inflaton, dark matter and dark energy as a single field and find that it is possible for a single field to become a dark matter candidate while still being the inflaton only if the inflaton does not decay completely during reheating. They also consider the possibility of the same field being dark energy and dark matter but not inflaton and find that such a case is not possible.

Once it was shown that it was possible to combine inflaton and dark matter in a single field, people studied it in the context of particle physics without invoking string theory. Such attempts considered a gauge singlet scalar which acts as the inflaton particle and later freezes-out to give the dark matter relic [19–21] while a feebly interacting massive particle (FIMP) dark matter, also being a light inflaton field was considered in [22]. A unification of inflaton and dark matter was also sought in two Higgs doublet model [23] while [24] studies gravity waves in the context of inflaton and dark matter being combined. All such models constructed have a common feature of the inflaton field being coupled to gravity non-minimally *i.e.* apart from the usual Einstein-Hilbert term in the action, there is another term $\xi\phi^2 R$ term where ϕ is the inflaton field, R is the Ricci scalar and ξ is the coupling strength.

1.4.2 Baryogenesis, dark matter and neutrino masses

One of the most common means to generate the baryon asymmetry of the Universe is through ***leptogenesis***. Individually baryon number, B, and lepton number, L, are not conserved in the standard model and neither is $B + L$. However, $B - L$ is conserved and is free from chiral or gravitational anomalies. $B - L$ is also conserved by sphaleron processes[1] that occur in the Universe until the electroweak phase transition occurs. In leptogenesis [25], initially a lepton asymmetry is created which is then converted by the sphalerons to a baryon asymmetry because of the $B-L$ conservation. Usual case of leptogenesis involves a right handed neutrino decaying into a scalar and a lepton through a complex CP violating Yukawa interaction. The asymmetry is obtained due to the interference of tree-level and one loop decay diagrams of the right handed neutrino. Addition of right handed neutrinos which are singlets under the gauge groups of the standard model also gives mass to the neutrinos. If the coupling strengths of the various Yukawa couplings are small enough such that these extra right handed neutrinos can fall out of equilibrium, they can also be dark matter candidates. In this manner, all three *viz.* baryogenesis, dark matter and neutrino masses can be unified.

1.4.3 Combining all four

Since inflation can be combined with dark matter and dark matter can be combined with baryogenesis and neutrino masses, it is only natural that we consider a case where all four can be unified in a single framework. Previously, supersymmetry scenarios have been studied to combine inflation with dark matter, baryogenesis and neutrino masses *e.g.* in [26, 27]. Non-supersymmetric models have also been studied including a minimal νMSM one with three right handed neutrinos in addition to the standard model [28] or a simple type-I see-saw model with a complex scalar singlet

[1] A sphaleron is a classical static solution for the field configuration. It is calculated as the saddle point of the energy functional

whose real part is the inflaton and the imaginary part is the Nambu-Goldstone dark matter particle [29]. Axion based models that consider unifying all the four phenomena are given in [30–32]

1.4.4 Our approach

This thesis is a result of our approach to unifying inflation, dark matter, baryogenesis and neutrino masses in one non-supersymmetric beyond the standard model framework. We will expand the standard model of particles with two sets of particles – an inert $SU(2)$ scalar doublet and three SM singlet fermions which act as right handed neutrinos. We also introduce a discrete $\mathbb{Z}_2$ symmetry in the model which keeps the standard model particles unchanged but changes the sign of the inert doublet and the SM singlet fermions. The inert doublet is so called due to the $\mathbb{Z}_2$ symmetry which prevents it from having a Yukawa interaction with a pair of SM fermions. The inert doublet is the inflaton and its lightest neutral component is the dark matter. Apart from the interactions of the inert doublet with Higgs and the gauge bosons, the only other possible interactions are the Yukawa interaction between the inert doublet, the SM singlet fermions and the SM leptons. Neutrino masses are generated radiatively and baryogenesis proceeds through leptogenesis – the SM singlet fermion decays to the neutral components of the inert doublet and neutrinos or the charged scalar and a charged lepton to create a lepton asymmetry.

2 Unifying inflation with dark matter

This chapter is based on the work done in the paper: "Inflation and dark matter in the inert doublet model" [33]. In this work we showed that the same field could be the inflaton in the early Universe and become a dark matter candidate much later in the Universe. During reheating, there is an incomplete decay of the inflaton such that it can become a part of the equilibrium plasma and later its lightest neutral component can freeze-out as dark matter. We use the inert doublet model with both the scalar doublets – the Higgs and inert doublet – coupled to gravity non-minimally to combine inflation with dark matter. Because of the non-minimal coupling, a conformal transformation is needed. The transformed field acts as the inflaton. The transformation essentially becomes identity at the end of the inflationary and the reheating era to bring us back to the usual physical frame and the lightest scalar particle of the doublet becomes the dark matter candidate.

2.1 The inert doublet model

The inert doublet model is a very simple model with a rich phenomenology. It is a special case of the general two Higgs doublet models due to an extra discrete $\mathbb{Z}_2$ symmetry (over the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)$) under which the inert doublet changes its sign while the SM particles are unchanged. The SM particles are even under $\mathbb{Z}_2$ while the extra scalar doublet is odd. It interacts with the Higgs doublet via the potential terms of the model and with the gauge bosons through the covariant derivative in the kinetic term. It was first introduced in the year 1978 by Deshpande and Ma [34]. In the 2000s, it was studied as a means to solve the *LEP paradox* [35–37]. Later, the inert doublet also started being studied as a candidate for dark matter [36, 38–44] The most general quartic renormalizable potential for this model is:

$$\begin{aligned} V(\Phi_1, \Phi_2) \; = \; & m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 \\ & + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right], \end{aligned} \tag{2.1}$$

where Φ_1 is the Higgs doublet, Φ_2 is the inert doublet containing the following field configuration:

$$\Phi_1 \; = \; \frac{1}{\sqrt{2}}\begin{pmatrix} \chi \\ h \end{pmatrix}, \qquad \Phi_2 \; = \; \begin{pmatrix} H^\pm \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix}. \tag{2.2}$$

Other terms in the potential which could possibly contain a single or three copies of Φ_2 are prohibited by the $\mathbb{Z}_2$ symmetry under which Φ_2 is odd. χ, h and $H^\pm$

are comlpex fields with χ being a charged scalar while h is neutral and made up of a scalar and a pseudoscalar. H_0 and A_0 are neutral scalar and pseudoscalar respectively.

After electroweak (EW) symmetry breaking, the Φ_1 takes a vacuum expectation value (vev), v and its doublet can be written as:

$$\Phi_1 = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} . \tag{2.3}$$

In the broken symmetry phase, the masses of the various scalars become:

$$\begin{aligned} m_h^2 &= \lambda_1 v^2, \\ m_{H^\pm}^2 &= \mu_2^2 + \frac{1}{2}\lambda_3 v^2, \\ m_{H^0}^2 &= \mu_2^2 + \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)v^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4+\lambda_5)\, v^2, \\ m_{A^0}^2 &= \mu_2^2 + \frac{1}{2}(\lambda_3+\lambda_4-\lambda_5)v^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4-\lambda_5)\, v^2 . \end{aligned} \tag{2.4}$$

Later, without losing any generality, we will consider $\lambda_5 < 0$ and $\lambda_4 + \lambda_5 < 0$ so that the CP-even scalar H_0 is the lightest Z_2 odd particle and hence a stable DM candidate.

For ease of calculations during inflation, we will use the polar coordinate representation for the Φ_2 in which it can be written as:

$$\Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ x e^{i\theta} \end{pmatrix} . \tag{2.5}$$

In the curved space-time of the FLRW metric $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$, the full action of the inert doublet model coupled non-minimally to gravity is given as follows:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_{Pl}^2 R - D_\mu\Phi_1 D^\mu\Phi_1^\dagger - D_\mu\Phi_2 D^\mu\Phi_2^\dagger - V(\Phi_1,\Phi_2) - \xi_1|\Phi_1|^2 R - \xi_2|\Phi_2|^2 R\right], \tag{2.6}$$

where D stands for the gauge covariant derivative including the $SU(2)$ gauge bosons of the SM and the affine connection due to gravity. Since Φ_1 and Φ_2 are scalars, the affine term doesn't occur. Also, with no fields other than the inflaton present during inflation, the gauge covariant terms can also be neglected. This allows us to reduce to the normal derivative $D_\mu \to \partial_\mu$. The metric convention of $(-,+,+,+)$ is followed which explains the $-$ sign in front of the derivative terms. M_{Pl} is the reduced Planck mass, R is the Ricci scalar and ξ_1 and ξ_2 are dimensionless couplings of the doublets to gravity. Such terms involving non-minimal gravitational couplings arise naturally in quantum gravity theories at the Planck scales [45].

2.2 Inflation

Inflation occurs along the Φ_2 direction if $\frac{\lambda_2}{\xi_2^2} \ll \frac{\lambda_1}{\xi_1^2}$. Once this choice is made, we can neglect the terms containing the Higgs field in the potential for high field inflation scenarios. This assumption will ease up later analytical calculations. Because of the non-minimal gravitational coupling term in the model, the action is non-canonical. It cannot be handled by usual quantum field theory methods. Here we make use of a tool called the ***conformal transformation*** to the *Einstein frame*. The physical frame where the action is originally written is called the *Jordan frame* while in the Einstein frame, there are no explicit terms with curvature couplings which means that regular field theory methods become applicable.

2.2.1 Conformal transformation to the Einstein Frame

We take help from [46] where general equations for conformal tranformations involving multiple scalars is given. To begin with, we transform the metric:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \tag{2.7}$$

$$\text{where } \Omega^2 = 1 + \frac{\xi_1}{M_{Pl}^2}(|\chi|^2 + |h|^2) + \frac{\xi_2}{M_{Pl}^2}(|q|^2 + |x|^2). \tag{2.8}$$

If we define $\phi = \{\chi, h, q, x, \theta\}$, we end up with the following transformed action.

$$S = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{2}M_{Pl}^2\tilde{R} - \frac{1}{2}G_{ij}\tilde{g}^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_j - \tilde{V}(h, q, x, \theta)\right], \tag{2.9}$$

where

$$G_{ij} = \frac{1}{\Omega^2}\delta_{ij} + \frac{3}{2}\frac{M_{Pl}^2}{\Omega^4}\frac{\partial\,\Omega^2}{\partial\,|\phi_i|}\frac{\partial\,\Omega^2}{\partial\,|\phi_j|}, \tag{2.10}$$

$$\tilde{V} = \frac{V}{\Omega^4} \text{ for the first 4 fields of } \phi \text{ namely } \chi, h, q, x. \tag{2.11}$$

The goal of the transformation was to get a canonical Lagrangian. However, the prefix 2.10 of the kinetic term contains non-diagonal elements as can be seen from its full matrix form, dropping the modulus symbols | from the equations for brevity:

$$G = \begin{bmatrix} \frac{\Omega^2+6\xi_1^2\chi^2/M_{Pl}^2}{\Omega^4} & 6\frac{\xi_1^2}{M_{Pl}^2\Omega^4}\chi\, h & \frac{6\xi_1\xi_2}{M^2\Omega^4}\chi\, q & \frac{6\xi_1\xi_2}{M^2\Omega^4}\chi\, x & 0 \\ 6\frac{\xi_1^2}{M_{Pl}^2\Omega^4}\chi\, h & \frac{\Omega^2+6\xi_1^2h^2/M_{Pl}^2}{\Omega^4} & \frac{6\xi_1\xi_2}{M^2\Omega^4}hq & \frac{6\xi_1\xi_2}{M^2\Omega^4}hx & 0 \\ \frac{6\xi_1\xi_2}{M^2\Omega^4}\chi\, q & \frac{6\xi_1\xi_2}{M^2\Omega^4}hq & \frac{\Omega^2+6\xi_2^2q^2/M_{Pl}^2}{\Omega^4} & \frac{6\xi_2^2}{M^2\Omega^4}qx & 0 \\ \frac{6\xi_1\xi_2}{M^2\Omega^4}\chi\, x & \frac{6\xi_1\xi_2}{M^2\Omega^4}hx & \frac{6\xi_2^2}{M^2\Omega^4}qx & \frac{\Omega^2+6\xi_2^2x^2/M_{Pl}^2}{\Omega^4} & 0 \\ 0 & 0 & 0 & 0 & \frac{x^2}{\Omega^2} \end{bmatrix}. \tag{2.12}$$

The assumption that fields other than inflaton do not contribute means that we can take the Higgs fields χ and h to be zero. This simplifies 2.10 into:

$$G = \begin{bmatrix} \frac{1}{\Omega^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\Omega^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\Omega^2+6\xi_2^2q^2/M_{Pl}^2}{\Omega^4} & \frac{6\xi_2^2}{M^2\Omega^4}qx & 0 \\ 0 & 0 & \frac{6\xi_2^2}{M^2\Omega^4}qx & \frac{\Omega^2+6\xi_2^2x^2/M_{Pl}^2}{\Omega^4} & 0 \\ 0 & 0 & 0 & 0 & \frac{x^2}{\Omega^2} \end{bmatrix}. \tag{2.13}$$

Now we are left with a 2×2 part of the matrix which is non-diagonal. Here, we define new fields A and B in terms of the old ones q and x:

$$A = \sqrt{\frac{3}{2}} M_{Pl} \log\left(\Omega^2\right), \tag{2.14}$$

$$B = M_{Pl} \, \frac{x}{q}. \tag{2.15}$$

The prefix in terms A and B becomes diagonal and we get the following kinetic term in the Einstein frame action:

$$\frac{1}{2\Omega^2}\left((\partial_\mu \chi)^2 + (\partial_\mu h)^2\right) + \left[\frac{1}{2} + \frac{1}{12\xi_2 F(A)}\right](\partial_\mu A)^2 + \left[\frac{F(A)}{2\xi_2(1+B^2/M_{Pl}^2)^2}\right](\partial_\mu B)^2 + \left[\frac{F(A)\, B^2}{2\xi_2(1+B^2/M_{Pl}^2)}\right](\partial_\mu \theta)^2, \tag{2.16}$$

where $F(A) = 1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{A}{M}\right)$.

Eq. (2.16) is diagonal but still far from being canonical as the coefficients of each derivative term are not $1/2$. Note however that the form of $F(A)$ is such that for high valued fields ($A \gtrsim M_{Pl}$) it is almost 1 as can be seen from Fig. 2.1.

Also, inflation along Φ_2 needs large ξ_2. We can neglect $\frac{1}{12\xi_2 F(A)}$ in comparison to $1/2$ and in the remaining terms, with $B \approx$ const. $\sim \mathcal{O}(M_{Pl})$, we can perform a rescaling of the fields to get the prefix $1/2$ for kinetic terms of B and θ. The fields χ and h can also be rescaled appropriately to end up with an approximate, effectively canonical action for the relevant region of field configuration during inflation.

Having settled with the kinetic part of the action, we move to the potential part which will determine all the aspects of inflation and its observable parameters.

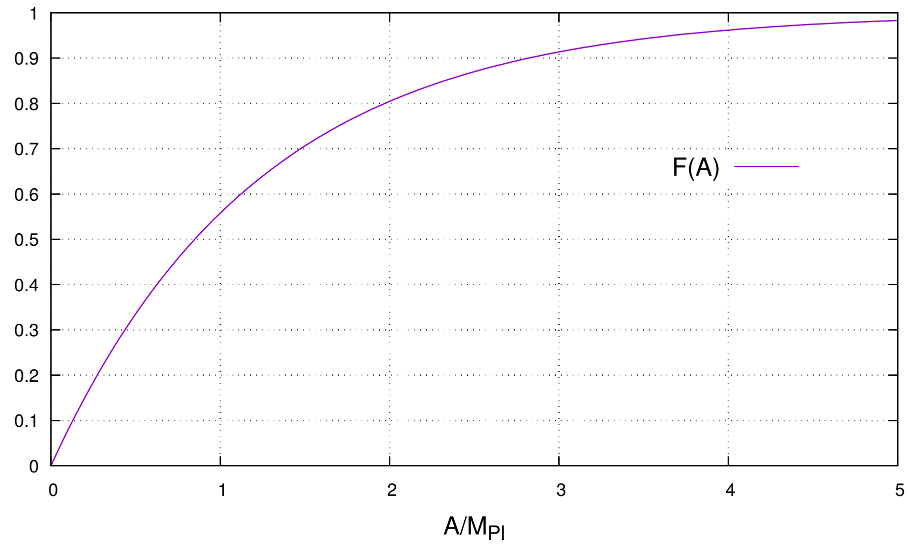

Figure 2.1: *The graph of $F(A)$*

The only relevant term in the potential for inflation is the self-quartic term of Φ_2, $\frac{1}{4}\lambda_2(|q|^2+|x|^2)^2$ as all others contain some powers of the Higgs term while reducing the power of Φ_2. The mass term of Φ_2 can also be neglected in the large field limit. In terms of the new fields A and B, we get the following potential:

$$V_e \approx \frac{\lambda_2 M_{Pl}^4}{4\xi_2^2}\left[1-\exp\left(-\sqrt{\frac{2}{3}}\frac{A}{M_{Pl}}\right)\right]^2. \tag{2.17}$$

This form of the potential belongs to the Starobinsky class (see [47, 48]). This class of potentials sits in the sweet spot of all inflationary observations from various experiments; see Fig. 2.2. The behaviour of the potential itself can be seen in Fig. 2.3.

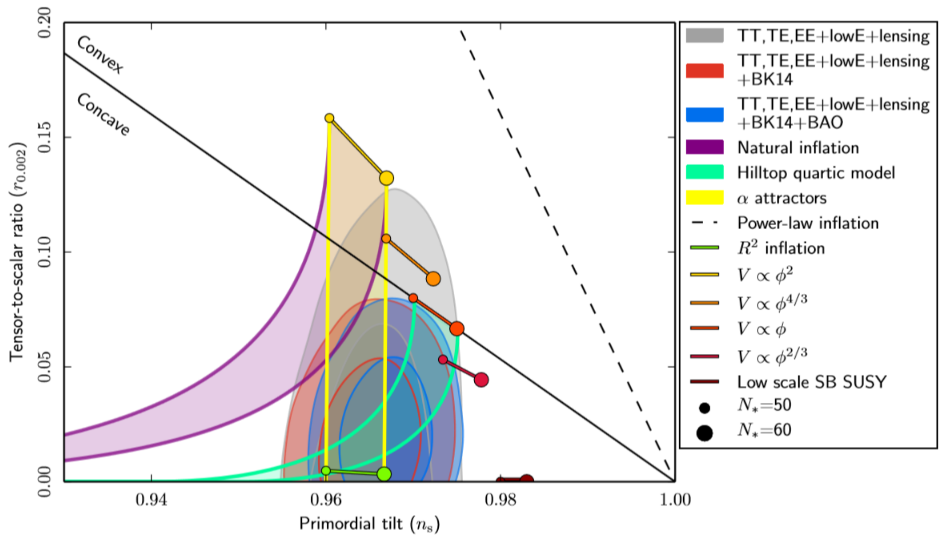

Figure 2.2: *A comparison of the various inflation models stacked against the experimental observations [1]*

.

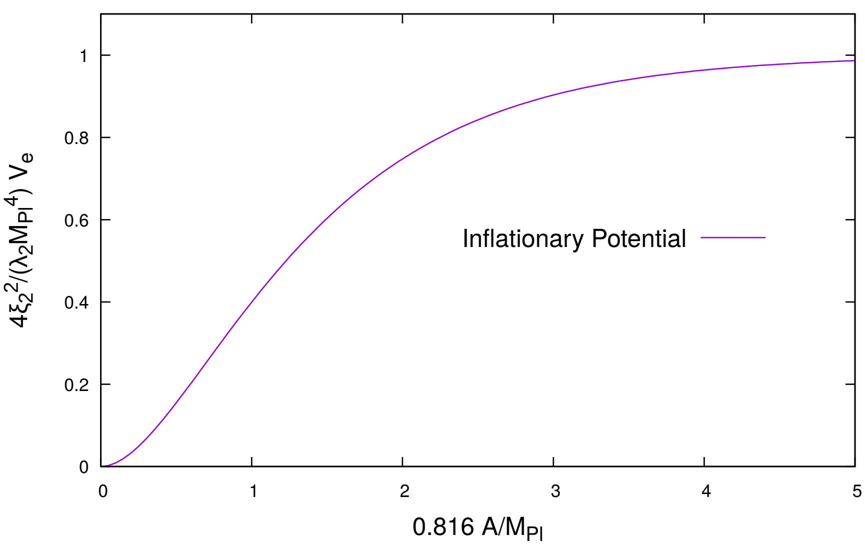

Figure 2.3: *The slow-rolling inflationary potential. (The number* 0.816 *is used instead of* $\sqrt{2/3}$ *in the figure)*

We use this potential to calculate the slow roll-parameters as follows:

$$\epsilon = \frac{1}{2}M_{Pl}^2\left(\frac{1}{V_e}\frac{dV_e}{dA}\right)^2 = \frac{4}{3}\left[-1+\exp\left(\sqrt{\frac{2}{3}}\frac{A}{M_{Pl}}\right)\right]^{-2}, \tag{2.18}$$

$$\eta = M_{Pl}^2\frac{1}{V_e}\frac{d^2V_e}{dA^2} = \frac{4}{3}\frac{\left[2-\exp\left(\sqrt{\frac{2}{3}}\frac{A}{M_{Pl}}\right)\right]}{\left[-1+\exp\left(\sqrt{\frac{2}{3}}\frac{A}{M_{Pl}}\right)\right]^2}. \tag{2.19}$$

The form of the slow-roll potentials given above holds only for a canonical action. Extra terms get added to them for non-canonical actions. The conformal transformed action for our case is only approximately canonical in the relevant large field limits. It is important to note that the values for slow-roll parameters thus obtained are approximate. However, it is not a cause for worry because the corrections they would get are negligible in the region of interest.

Exponentially expanding phase results when the equation of state for the dominant fluid of the Universe, in this case the inflaton, satisfies the following constraint between the energy density $\rho = T + V_e$ and the pressure density $p = T - V_e$ where T is the kinetic density term in the Lagrangian.

$$p = -\rho. \tag{2.20}$$

This will hold if $T \ll V_e$ such that T can be neglected and ρ is just V_e. From the Friedmann equation, $H^2 = \frac{\rho}{3M_{Pl}^2}$, we learn that $H^2 = \frac{V_e}{3M_{Pl}^2}$ which is constant in the slow roll region of the potential occurring in the large field limit.

2.2.2 Obtaining the inflationary parameters

There are three most important observables to be calculated in any model for inflation: the spectral tilt (also called the spectral index) n_s, the ratio of the tensor perturbations to the scalar perturbations r and the curvature power spectrum P_s.

The slow roll-parameters are approximately constant during inflation,

$$\frac{d\epsilon}{dN} = 2\epsilon(\epsilon - 2\eta) \approx \mathcal{O}(\epsilon^2), \tag{2.21}$$

where N is the number of e-folds by which the Universe expands during inflation. Therefore, we can write the inflationary parameters like the tensor perturbations to the scalar perturbations ratio (tensor-to-scalar ratio for short), r, and the spectral index (also called spectral tilt), n_s as follows:

$$n_s = 1 - 6\epsilon + 2\eta, \tag{2.22}$$

$$r = 16\epsilon. \tag{2.23}$$

The scalar power spectrum is given by:

$$P_s = \frac{1}{12\,\pi^2}\frac{V_e^3}{M_{Pl}^6\,V_e'^2}. \tag{2.24}$$

These parameters are calculated at inflaton field value where at least N e-folds expansion can take place *i.e.* if the Universe expands by N e-folds, the appropriate inflaton field value should be its value at the beginning of the first e-fold expansion,

A_{ini}. The end of inflation is marked by the slow roll parameter ϵ becoming ≈ 1 at the inflaton field value A_{end}. To get numbers for the spectral index and tensor to scalar ratio, we need the values of A_{ini} which can be obtained from the number of e-folds, N.

The Hubble parameter $H = \frac{\dot{a}}{a}$ is constant during inflation. Solving for $a(t)$ gives us $a(t) = a(t_i)\, e^{t-t_i}$ where t_i is the initial time. Thus the total number of e-folds during inflation is given by

$$N = -\int_{t_e}^{t_i} H\, dt, \tag{2.25}$$

where t_e marks the end of inflation. The minus sign takes care of the convention that starting from the end of inflation, we count upto N number of e-folds. During slow roll inflation, the Friedman equation $\ddot{A} + 3H\dot{A} + V_e' = 0$ (prime denoting a derivative with respect to A) reduces to:

$$3H\dot{A} + \frac{dV_e}{dA} = 0 \tag{2.26}$$

Using this, we can rewrite N as a field space integral instead of a time integral as follows:

$$\begin{aligned} dt &= -\frac{3H}{V_e'} dA \\ \Rightarrow N &= \int_{A_{end}}^{A_{ini}} H\, \frac{3H}{V_e'}\, dA = \frac{1}{M_{Pl}^2} \int_{A_{end}}^{A_{ini}} \frac{V_e}{V_e'}\, dA \end{aligned} \tag{2.27}$$

After integration, this gives:

$$N = \frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}}\right) - \exp\left(\sqrt{\frac{2}{3}}\frac{A_{end}}{M_{Pl}}\right) - \sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}} + \sqrt{\frac{2}{3}}\frac{A_{end}}{M_{Pl}}\right] \quad (2.28)$$

We obtain A_{end} by the $\epsilon = 1$ condition in Eq. (2.18) to get:

$$\exp\left(\sqrt{\frac{2}{3}}\frac{A_{end}}{M_{Pl}}\right) \simeq 2.15, \quad (2.29)$$

$$\sqrt{\frac{2}{3}}\frac{A_{end}}{M_{Pl}} \simeq 0.77. \quad (2.30)$$

We can substitute Eq. (2.29) into Eq. (2.28) to obtain the value for A_{ini} if we know the number of e-folds expansion the Universe undergoes during the inflationary period. Any number $N \geq 50$ can solve the flatness and horizon problems. Inflationary scales of $\mathcal{O}(10^{16})$ GeV require 60 e-folds with lower scales requiring higher e-folds to satisfy the baryon asymmetry conditions of the Universe [49]. We stick to using $N = 60$ in all our analysis. With this we calculate, for A_{ini},

$$\frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}}\right) - \sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}} - 1.387\right] = 60, \quad (2.31)$$

$$\Rightarrow \exp\left(\sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}}\right) - \sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}} = 81.387$$

$$\Rightarrow \sqrt{\frac{2}{3}}\frac{A_{ini}}{M_{Pl}} \approx 4.45. \quad (2.32)$$

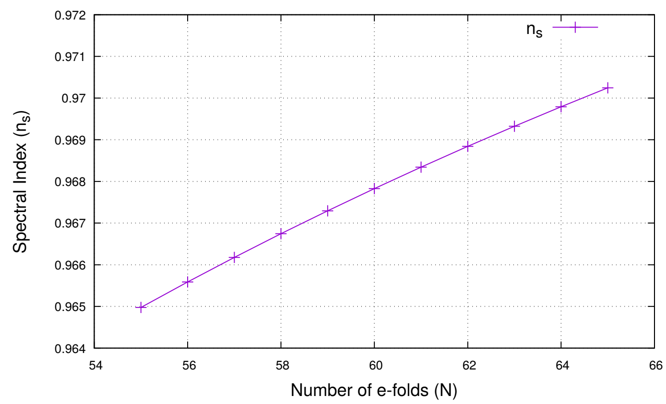

Figure 2.4: *Variation of n_s as N is changed.*

Now, we can calculate the inflationary parameters:

$$r = 0.0029, \tag{2.33}$$

$$n_s = 0.9678, \tag{2.34}$$

which are well within range of the values observed by the Planck experiment (Planck, TT, TE, EE + low E + lensing) [1]:

$$r < 0.11 \qquad \text{(at 95\% CL)}\,, \tag{2.35}$$

$$n_s = 0.9649 \pm 0.0042 \qquad \text{(at 68\% CL)}\,. \tag{2.36}$$

These values can be checked for other possible and relevant values for N from 55 to 65. The results are shown in Figs. 2.4 and 2.5.

The scalar power spectrum P_s given in Eq. (2.24) can also be calculated. We

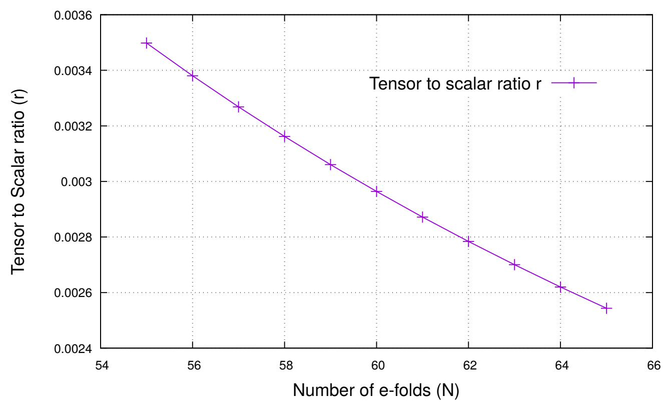

Figure 2.5: *Variation of r as N is changed.*

get:

$$P_s = 5.57\frac{\lambda_2}{\xi_2^2}. \tag{2.37}$$

We get a relation between the inert doublet quartic self-coupling, λ_2, and its coupling to gravity ξ_2 if we use the results for the power spectrum given by the Planck 2018 results [1]: $\log(10^{10}P_s) = 3.047 \pm 0.014$ at 68% C.L.

$$\xi_2 \;=\; 5.33 \times 10^4\sqrt{\lambda_2} \tag{2.38}$$

Later, our analysis of reheating will put a lower bound on λ_2 which in turn by virtue of Eq. (2.38) puts a lower bound on ξ_2. An $\mathcal{O}(0.1)$ λ_2 would require $\xi_2 \sim \mathcal{O}(10^3)$ which is quite large. Such large values of the non-minimal coupling can give rise to unitarity issues about which more can be found in the appendix.

2.3 Reheating

Inflation dilutes any previous particle content or information by at-least a factor of e^{60} if the Universe expands by 60 e-folds. The resulting Universe at the end of inflation is completely devoid of any known particles or any temperature at this point – a cold and dark Universe. However, we know that such a fate of the Universe is not true. There must be something that creates all the known and unknown particles and the temperature in the Universe. Indeed the process of repopulating the Universe with SM particles and giving it a non-zero finite temperature is called ***reheating*** [8]. During this phase of the early evolution of the Universe, the inflaton field decays or annihilates to create relativistic SM particles which are often clubbed together as *radiation*. At the beginning of the reheating era, the Universe is dominated by matter as the inflaton after having climbed down the potential now oscillates in an approximate harmonic oscillator well. These oscillations produce a classical coherent state which is non-relativistic. As the inflaton evolves and oscillates in this potential, its interactions with other SM particles cause it to decay or annihilate according to the nature of its interactions. The inflaton not just oscillates but because of its decay, its field amplitude decreases as well implying that the oscillations are not pure simple harmonic but damped. If the inflaton decays into fermions directly, because of the Pauli exclusion principle and the resulting Fermi-Dirac statistics, there is no runaway production of radiation. Rather, a much controlled radiation production occurs. If the decay rate of the inflaton is larger than the Hubble rate, the reheating process completes and a hot Universe is obtained which can then evolve to its present state. If, however, the Hubble parameter is larger than the decay rate, the Universe gets stuck in a perennial reheating phase without it ever attaining any temperature – it remains cold, dark and empty throughout its history. Instead of producing fermions, if the inflaton decays or annihilates into bosons, non-perturbative parametric resonance production of bosons almost always ensures efficient reheating [50, 51] (see also [52, 53]).

2.3.1 The inflaton field dynamics

Two distinct regions in the conformal field frame can be identified [54]:

$$A \approx \begin{cases} (q^2 + x^2)^{1/2} & \text{for } A < A_{cr}, \\ \sqrt{\frac{3}{2}}\, M_{Pl} \, \log\left(\Omega^2\right) & \text{for } A > A_{cr}. \end{cases} \tag{2.39}$$

where $A_{cr} = \sqrt{\frac{2}{3}\frac{M_{Pl}}{\xi_2}}$. Below A_{cr}, the conformal transformation is almost 1 and we can go back to using the physical Jordan frame. Inflation occurs as long as the potential remains sufficiently flat. It ends at around A_{end} which is still much larger than A_{cr}. The inflaton field keeps rolling down the potential until much below M_{Pl}, it ends up in the valley of the potential in Eq. (2.17) (see Fig. 2.3). This valley can be well approximated by a harmonic oscillator potential as can be seen below:

$$\begin{aligned} V_e &= \frac{\lambda_2 \, M_{Pl}^4}{4 \, \xi_2^2} \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{A}{M_{Pl}} \right) \right]^2, \\ &\simeq \frac{\lambda_2 M_{Pl}^2}{6 \xi_2^2} A^2, \end{aligned} \tag{2.40}$$

which can be written as

$$V_e \;=\; \frac{1}{2}\omega^2\, A^2, \;\; \text{where } \omega^2 = \frac{\lambda_2\, M_{Pl}^2}{3\xi_2^2}, \tag{2.41}$$

The average energy density in the inflaton during this oscillatory phase is $\bar{\rho}_A = \langle \dot{A}^2 \rangle$. The energy densiy satisfies the equation

$$\bar{\rho}_A + 3H\dot{\bar{\rho}}_A = 0 \Rightarrow \bar{\rho}_A \propto \frac{1}{a^3}\,, \tag{2.42}$$

which means that the Universe is matter dominated at this time and the inflaton now behaves as classical matter fluid. The scale factor $a(t)$ goes as $t^{2/3}$ during the matter dominated era which means that the Hubble rate is

$$H = \frac{2}{3t}\,. \tag{2.43}$$

Using the equation of motion for A during this time,

$$\ddot{A} + 3H\dot{A} + \frac{dV_e}{dA} = 0, \tag{2.44}$$

we get, for $\omega \gg H$,

$$A(t) = A_0(t)\cos(\omega t), \tag{2.45}$$

where,

$$A_0 = 2\sqrt{2}\frac{\xi_2}{\sqrt{\lambda_2}}\frac{1}{t}. \tag{2.46}$$

When the amplitude of the field A_0 falls below A_{cr}, this approximation breaks down. Since reheating proceeds when the inflaton is oscillating in a harmonic potential while decaying into other particles, breaking of the quadratic approximation also means the end of the reheating phase. The other terms in the potential which include quadratic terms in the inert doublet also start affecting the dynamics. This is also when reheating will have produced enough SM particles which will also start affecting the dynamics. Looking at the time dependence of the inflaton amplitude, we can know the time when this happens. We call it t_{cr} which marks the end of reheating:

$$t_{cr} = \frac{2\xi_2}{\omega} \tag{2.47}$$

2.3.2 Transfer of energy density from inflaton to SM particles

The odd behaviour of the extra doublet under the $\mathbb{Z}_2$ symmetry in this model prevents it from coupling directly to SM fermions. The only way for producing the SM particles is through the Higgs-inert doublet couplings and via the knietic couplings to gauge bosons through the covariant derivative terms. As this happens much above the electroweak symmetry breaking scale, the SM particles don't have an intrinsic mass. However, the amplitude of the inflaton gives these particles an effective mass. The interaction between the inert doublet fields and the gauge bosons is $\frac{g^2}{4}(|q|^2 + |x|^2)W^2$ where W represents $W^{\pm}, Z$ and B fields depending on charge conservation, g represents the weak or the hypercharge coupling strengths. The

Higgs interaction occurs through the $\lambda_{3,4,5}$ terms.

While $A_0 \ll \sqrt{\frac{3}{2}} M_{Pl}$ but above A_{cr}, we can re-write $(|q|^2 + |x|^2)$ which in turn is much smaller than $\frac{M_{Pl}^2}{\xi_2}$ as:

$$|q|^2 + |x|^2 \approx \sqrt{\frac{2}{3}} \frac{M_{Pl}}{\xi_2} |A|, \tag{2.48}$$

assuming that the oscillation frequency ω is much larger than H.

Substituting Eq. (2.48) for $q^2 + x^2$ terms in the various interaction terms, we get effective masses for the gauge bosons and the Higgs which are as shown below:

$$m_W^2 = \frac{g^2}{2\sqrt{6}} \frac{M_{Pl}}{\xi_2} |A| \qquad \text{gauge boson effective mass}\,, \tag{2.49}$$

$$m_h^2 = \frac{1}{\sqrt{6}} \left(\lambda_3 + \frac{\lambda_4}{2} \right) \frac{M_{Pl}}{\xi_2} |A| \qquad \text{Higgs effective mass}\,. \tag{2.50}$$

The Higgs effective mass in Eq. (2.50) assumes equal contribution from q and x for all the three coupling terms.

It is worth mentioning that Eq. (2.48) is not a vacuum expectation value arising from some spontaneous breaking of symmetry which is calculated at the minimum of the potential. It is a classical value and describes just the transformation law between the fields in the Jordan frame and the Einstein frame. Thus the masses in Eqs. (2.49) and (2.50) are effective masses only when averaged over the oscillation period of A. They are not masses that arise out of a spontaneous symmetry breaking.

Later when we will see how the inert doublet gives a dark matter candidate, we will find that the couplings $\lambda_3 + \lambda_4 + \lambda_5$ should be of order 1. At the same time,

λ_5 must be small [55]. The gauge coupling g similarly is of order $0.1 - 1$. Thus the effective masses of the gauge and the Higgs bosons are quite large for much of the duration of the oscillation period except when they are zero around the zeros of the oscillations of the inflaton field A. The inflaton can decay or annihilate into these SM bosons only near the zeros of their effective masses. At the same time large couplings also mean that these SM bosons are non-relativistic. The Universe will be reheated only with production of relativistic particles or radiation. This is achieved through decays or annihilation of the SM bosons into SM fermions. Since these fermions don't have any coupling to the inert doublet, they don't get any effective mass due to the inflaton oscillations, remaining relativistic in nature.

The production of the gauge and the Higgs bosons proceeds by two methods

- Linear production when the number density of these boson is small
- Parametric resonance production when their number density is large.

The rate of production of the SM bosons for both of these methods is given in Eqs. (2.51) and (2.52)

$$\frac{d(n_W a^3)}{dt} = \begin{cases} \frac{P}{2\pi^3}\omega k_1^3 a^3, & \text{(linear)}, \\ \\ 2\, a^3\, \omega Q\, n_W, & \text{(resonance)}, \end{cases} \qquad \text{gauge boson production rates}\,, \tag{2.51}$$

$$\frac{d(n_h a^3)}{dt} = \begin{cases} \frac{P}{2\pi^3}\omega k_2^3 a^3, & \text{(linear)}, \\ \\ 2a^3\omega Q n_h. & \text{(resonance)}. \end{cases} \quad \text{Higgs production rate}\,, \tag{2.52}$$

where $P \approx 0.0455$ and $Q \approx 0.045$ and $\alpha_W = \frac{g^2}{4\pi}$ is the weak coupling constant. The details for obtaining the numbers P and Q and the factors K_1 and K_2 can be found in the appendix of [54] (see also, [56, 57]). k_1 and k_2 which have dimensions of energy are given as:

$$k_1 = \left[\frac{g^2 M_{Pl}^2}{6\xi_2^2}\sqrt{\frac{\lambda_2}{2}}A_0(t_{i,0})\right]^{1/3}, \tag{2.53}$$

$$k_2 = \left[\frac{\lambda_3\, M_{Pl}^2}{3\xi_2^2}\sqrt{\frac{\lambda_2}{2}}A_0(t_{i,0})\right]^{1/3}, \tag{2.54}$$

where $t_{i,0}$ are instants when the inflaton $A = 0$ near which the inflaton actually produces the SM bosons.

The decay rate of the gauge bosons and the Higgs into fermions is given by:

$$\Gamma_W = 0.75\frac{g^2}{4\pi}\langle m_W\rangle \qquad \text{gauge boson to fermions decay}, \tag{2.55}$$

$$\Gamma_h = \frac{y^2}{16\pi}\langle m_h\rangle \qquad \text{Higgs to fermions decay}. \tag{2.56}$$

In addition, two gauge bosons can also annihilate together to produce two fermions with the scattering cross-section given as:

$$\sigma_{WW} \approx \frac{g^4}{16}\frac{2N_l + 2N_q N_c}{8\pi\langle m_W^2\rangle} \approx 10\pi\frac{g^4}{16\pi^2\langle m_W^2\rangle}. \tag{2.57}$$

When the number density of the gauge bosons is less they mostly undergo decay with each boson producing a pair of fermions. As their number density increases with more and more gauge bosons being produced by the inflaton, their annihilation can start occurring and dominate over their decay as the source for producing relativistic fermions. Parametric resonance production of the SM bosons can start only when their resonance production rates given in Eqs. (2.51) and (2.52) exceeds their decay rates given in Eqs. (2.55) and (2.56) respectively.

This means that for the gauge bosons, parametric resonance production can start only if:

$$A_0 < \frac{2}{0.5625\,\pi}\frac{Q^2\,\lambda_2}{\alpha_W^3}A_{cr} \approx 61.88\,\lambda_2\,A_{cr}. \tag{2.58}$$

while the corresponding condition for Higgs production by parametric resonance is:

$$A_0 < \frac{64\pi\,Q^2\lambda_2}{\lambda_3\,y^4}A_{cr} \approx 0.41\frac{\lambda_2}{\lambda_3 + \frac{\lambda_4}{2}}\,A_{cr}. \tag{2.59}$$

For the gauge bosons, the production of radiation is much faster during their annihilation than during their decay. In fact the Universe would keep reheating long after the resonance production of bosons had started and ended if decay was the only possibility [54]. Annihilation will occur only if there are a large number of bosons. This in turn requires that parametric resonance production of gauge bosons does take place. Using this fact in Eq. (2.58), we obtain a lower bound on the inert doublet quartic self-coupling λ_2:

$$\lambda_2 \gtrsim \frac{1}{60}. \tag{2.60}$$

Unlike the gauge bosons, only the decay channel is available for the Higgs. For simplicity, let's assume $\lambda_4 \ll \lambda_3$ such that $\lambda_3 + \frac{\lambda_4}{2} \approx \lambda_3$. Comparing Eq. (2.59) with Eq. (2.58), we see that the Higgs production will enter the resonance regime around the same time as the gauge bosons only when $\lambda_3 \lesssim 0.006$ which is unlikely considering the phenomenology of dark matter (which we will see later). If the Higgs production has to enter the resonance production regime at all, λ_3 must be less than $\sim 0.41\lambda_2$. This means that in all realistic cases, Higgs production can enter the resonance phase only after the end of the oscillatory phase of the inflaton. Therefore the production rate of Higgs and thus the amount of radiation produced by it remains small compared to the gauge bosons during the entire reheating period.

We can now calculate the amount of radiation energy density produced during the reheating process by the annihilation of the gauge bosons using Eq. (2.57) as follows [54]

$$\frac{d(\rho_\gamma a^4)}{dt} = 2a^4\sqrt{\langle m_W^2\rangle}\frac{4Q^2\omega^2}{\sigma_{WW}}\,. \tag{2.61}$$

After integration we get

$$\rho_\gamma = \frac{8Q^2\omega^2}{10\pi\alpha_W^2}\frac{6}{13}\left(\frac{4\pi\alpha_W M_{Pl}}{\sqrt{3\lambda_2}}\right)^{3/2}\left[\frac{t_{cr}^{13/6} - t_p^{13/6}}{t_{cr}^{8/3}}\right], \tag{2.62}$$

where t_p is the time when the parametric resonance production of gauge bosons starts and t_{cr} as mentioned previously is the time at the end of reheating. Almost all of the gauge bosons get converted into radiation during reheating such that at the end, only radiation and the inert doublet particles remain. The distribution of energy density in radiation and the inflaton at the end of reheating, after putting in the numbers (using Planck 2018 [1] values for the scalar power spectrum to determine λ_2/ξ_2^2) for all the various factors is:

$$\rho_\gamma \approx \frac{1.06 \times 10^{57}}{\lambda_2}\ \mathrm{GeV}^4\,. \tag{2.63}$$

At this time, energy density in A is:

$$\rho_A = \frac{\omega^2 A_{cr}^2}{2} \approx \frac{4.84 \times 10^{53}}{\lambda_2}\ \mathrm{GeV}^4\,. \tag{2.64}$$

The temperature of the Universe at the end of reheating is given in terms of the radiation energy density and taking $\lambda_2 \sim \mathcal{O}(1)$ in Eq. (2.63) as:

$$T_r = \left(\frac{30\,\rho_\gamma}{\pi^2\, g_*}\right)^{1/4}\ \mathrm{GeV}\,, \tag{2.65}$$

where $g_* = 107$ is the number of degrees of freedom in the relativistic plasma including the inert doublet. Putting the numbers for $\lambda_2 \simeq 1$, the reheating temperature is 7.1×10^{13} GeV.

At the end of reheating, as collisions and interaction keep occurring between the inert doublet particles and the other SM particles, the inert doublet also becomes a part of the equilibrium plasma through its interactions with the Higgs doublet and the gauge bosons. The conformal transformation Ω becomes 1 as the fields are much below M_{Pl} (see Eq. (2.8)) and we can switch back to using the physical Jordan frame. As the Universe expands, eventually the inert doublet particles freeze-out of equilibrium. This gives us a dark matter candidate, thus combining inflation with dark matter.

2.4 Dark matter in the inert doublet model

We will consider the neutral scalar particle of the inert doublet as the dark matter particle. Recall that in the polar coordinates, we had written the inert doublet as $\Phi_2 = \frac{1}{\sqrt{2}}\left(q, x\, e^{i\theta}\right)^T$ while the actual physical particle content was $\Phi_2 = \left(H^\pm, \frac{H_0+iA_0}{\sqrt{2}}\right)^T$. Thus the dark matter candidate is $x\cos\theta$ or the particle H_0. This is similar to the inert doublet model of dark matter discussed in literature [36, 38, 41–44, 58–64].

The evolution of the dark matter particle is given by the Boltzmann equation:

$$\frac{dn_{\rm DM}}{dt} + 3Hn_{\rm DM} \;=\; -\langle\sigma v\rangle\left[n_{\rm DM}^2 - (n_{\rm DM}^{\rm eq})^2\right], \tag{2.66}$$

where $n_{\rm DM}^{\rm eq}$ is the equilibrium number density of DM, n_{DM} is its number density at time t and $\langle\sigma v\rangle$ is the thermally averaged annihilation cross section, given by [65]

$$\langle\sigma v\rangle \;=\; \frac{1}{8m_{\rm DM}^4 T K_2^2\left(\frac{m_{\rm DM}}{T}\right)}\int\limits_{4m_{\rm DM}^2}^{\infty} \sigma(s-4m_{\rm DM}^2)\sqrt{s}\,K_1\left(\frac{\sqrt{s}}{T}\right)ds\,, \tag{2.67}$$

with K_i being the modified Bessel functions of order i. The freeze-out temperature T_f and the dark matter relic abundance $\Omega_{DM} = \frac{\rho_{DM}}{\rho_c}$ can be obtained by solving the Boltzmann equation given in Eq. (2.66). ρ_c is the critical energy density required for the flatness of the Universe and is given as $\frac{3H_0^2}{8\pi G_N}$ where G_N is the Newton's gravitational constant and $H_0 \approx 70\ \mathrm{Km\,s^{-1}\,Mpc^{-1}} = 100\,h\ \mathrm{Km\,s^{-1}\,Mpc^{-1}}$ is the present value of the Hubble parameter.

If we assume only s-wave annihilation and define $x_f = \frac{m_2}{T_f}$ where m_2 is the mass of the inert doublet (see Eq. (2.1)), we can get the following form for x_f and subsequently the relic abundance $\omega_{DM}h^2$ (see [66, 67]):

$$x_f \equiv \frac{m_{\rm DM}}{T_f} \; = \; \ln\left(0.038\frac{g}{g_*^{1/2}} M_{\rm Pl} m_{\rm DM} \langle\sigma v\rangle_f\right) , \tag{2.68}$$

$$\Omega_{\rm DM}h^2 \; = \; \left(1.07\times 10^9\ \mathrm{GeV}^{-1}\right)\frac{x_f g_*^{1/2}}{g_{*s} M_{\rm Pl}\langle\sigma v\rangle_f} , \tag{2.69}$$

where g is the number of internal degrees of freedom of the inert doublet while g_* and g_{*s} are the number of effective number of degrees of freedom contributing to the energy density and the entropy density respectively. The entropy density gets contribution only from relativistic particle species which means that g_{*s} counts only the relativistic degrees of freedom. g_* however counts all the degrees of freedom as it forms a part of the total energy density. For the most part of the history of the Universe and specially for our situation where we are still much above the EW phase transition, all the particle species are relativistic except for the dark matter candidate near freeze-out. Therefore, we can take $g_{*s} \approx g_*$.

These two equations are solved simultaneously with the relic abundance being constrained by the Planck 2018 results [2] which give:

$$\Omega_{DM} h^2 = 0.120 \pm 0.001 \text{ at } 68\% \text{ C.L.} \,. \tag{2.70}$$

The inert doublet particles are part of the equilibrium as long as their interaction rate is greater than the Hubble rate at that time. Freeze-out is the process of the falling out of equilibrium for any particle. This happens when its interaction rate becomes smaller than the Hubble rate. Thus the freeze-out condition is $\Gamma(t_f, T_f) = \langle \sigma v \rangle n_{DM} = H(t_f, T_f) = \sqrt{\frac{\pi^2, g_*}{90}} \frac{T_f^2}{M_{Pl}}$ where Γ is the interaction rate, t_f is the freeze-out time and T_f is the freeze-out temperature. Both x_f and $\Omega_{DM} h^2$ are calculated at freeze-out.

At the tree level, the interactions of H_0 keeping it in equilibrium before freeze-out are its interactions with the Higgs via the $\lambda_{3,4,5}$ terms and the interactions with the gauge bosons through the covariant derivative in the kinetic term. These are all 4-point interactions and shown in Fig. 2.6. Apart from the 4-point diagram, there are trilinear diagrams which include the momentum dependent gauge boson mediated diagrams shown in Fig. 2.7. Their contribution is small near freeze-out as the particles are non-relativistic, thus have almost zero momentum. The trilinear Higgs mediated diagram also shown in Fig. 2.7 also does not contribute as it does not exist above the electroweak symmetry breaking phase.

Solving the Eqs. (2.68) and (2.69) using the constraint in Eq. (2.70) gives us

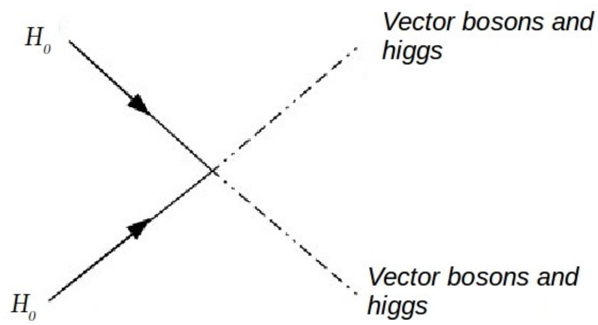

Figure 2.6: *The 4-point vertex interactions*

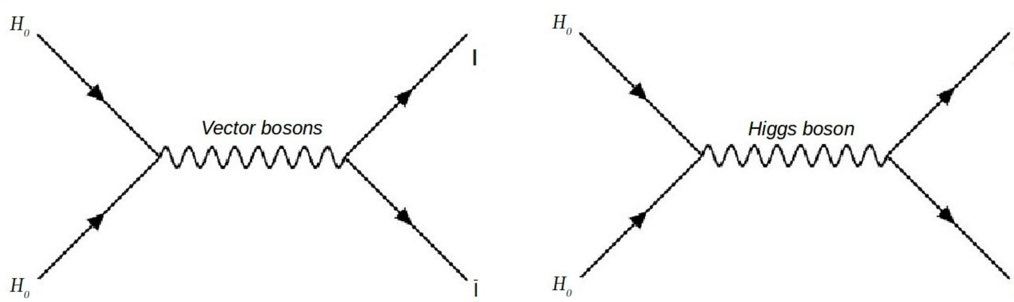

Figure 2.7: *The other tree level interaction diagrams.* H_0 *is the neutral scalar component of the inert doublet. These diagrams are not relevant for the relic density calculation*

the relic abundance values for different masses m_2 of the inert doublet and different values of $\lambda_s = \lambda_3 + \lambda_4 + \lambda_5$. The graph in Fig. 2.8 shows the relic abundance as a function of the mass of the inert doublet m_2 in GeV and the coupling strength. For simplicity, we choose λ_4 and λ_5 to be negligible compared to λ_3 so that $\lambda_s \approx \lambda_3$.

We find that the relic abundance is satisfied for TeV scale dark matter. The spin independent cross-section of the DM candidate is of $\simeq 10^{45} - 10^{46}$ cm^2 which makes

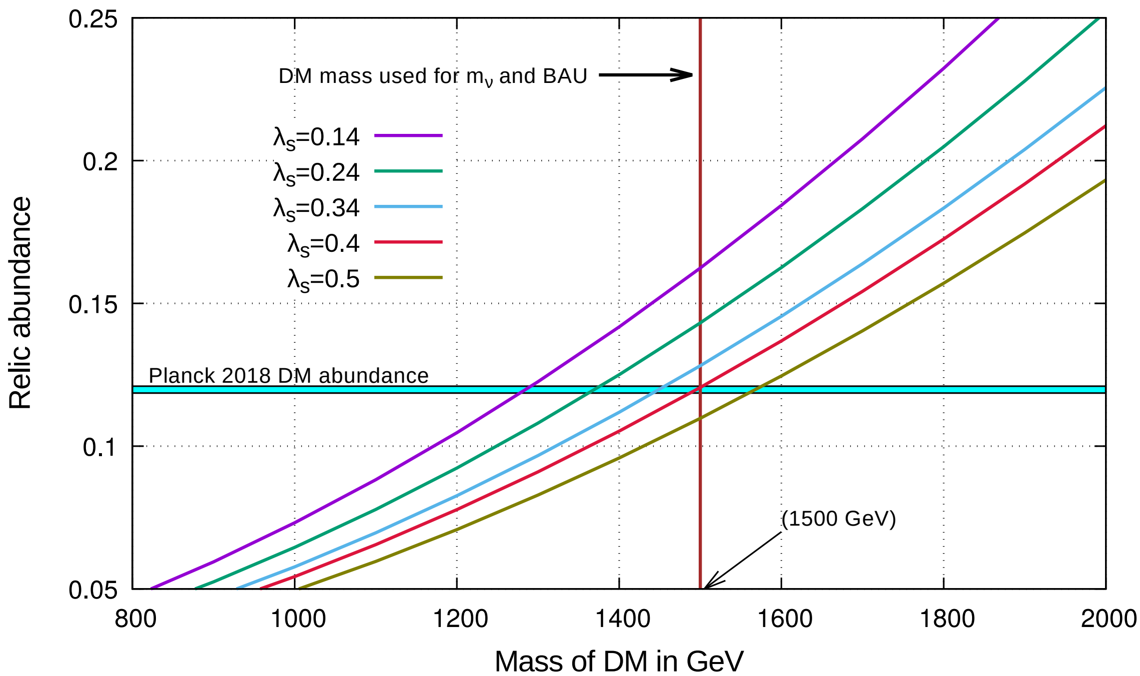

Figure 2.8: *The relic abundance of dark matter vs. the mass of the dark matter* m_2 *in GeV. The horizontal band is the Planck 2018 result. The vertical line at 1500 GeV is a benchmark value for calculating baryogenesis in Sec. 3.4.1*

it within reach of the near-future direct detection experiments like XENONnT [68], LZ [69], DARWIN [70] and PandaX-30T [71]

2.5 Conclusion

We have now showed that in the inert doublet model coupled non-minimally to gravity, inflation and dark matter can be unified into one particle. The non-minimal coupling forces us to perform a conformal transformation that leads to a Starobinsky class potential for inflation. This class of potentials sits in the sweet spot of all inflationary observations. The power spectrum, the tensor to scalar ratio and the spectral index were calculated and match well with experimental findings. After reheating, the inert doublet particles attain equilibrium and later the neutral scalar component freezes-out to become the cold dark matter relic of mass in the TeV range with a nuclear scattering cross section within reach of the ongoing and planned dark matter direct detection experiments.

In the next chapter, we will see how to combine these two with baryogenesis and neutrino masses.

3

Introducing baryogenesis and neutrino masses into the mix

This chapter is based on our paper titled "TeV scale leptogenesis, inflaton dark matter and neutrino mass in a scotogenic model" [72]. This work naturally follows up and builds on the previous work mentioned in the beginning of Chapter 2. Apart from a unified framework for inflaton and dark matter we show that baryogenesis via leptogenesis and neutrino masses can also be linked to the dark matter which is also the inflaton. In [73, 74] baryon asymmetry was generated by the decay of a heavy particle. This can be applied to the generation of the asymmetry by the decay of heavy right handed neutrinos in leptogenesis [25]. As mentioned in section 1.4.2, leptogenesis is one of the most popular ways to generate baryon asymmetry. Typically this involves a heavy right handed neutrino decaying into a scalar and a lepton through a complex Yukawa coupling. The asymmetry is generated due to the interference between the tree-level diagram and the one loop diagrams for the vertex correction and the self-energy correction as can be seen in Fig. 3.1.

The tree level decay is not CP asymmetric. Anti-leptons and leptons are created equally. However, once we take into account the one loop corrections, the imaginary parts of the complex Yukawa couplings come into play and the decay becomes CP asymmetric, enough to generate an asymmetry in the leptons. If this asymmetry is created before the EW symmetry breaking, the sphaleron processes [75, 76] convert it into a baryon asymmetry and we have a viable mechanics for baryogenesis.

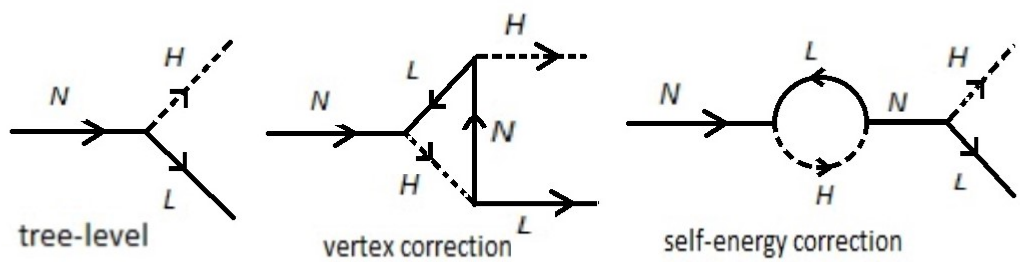

Figure 3.1: *The three diagrams for the decay of right handed neutrino N into a scalar H and a lepton L which interfere to produce a lepton asymmetry*

Addition of right handed neutrinos to the standard model also generates masses for the active neutrinos via the seesaw mechanism [77–82]. The right handed neutrinos being a singlet in the standard model have a Majorana mass matrix M which are not constrained by any gauge symmetry. The presence of the right handed neutrinos allows a Yukawa interaction term $Y_{ij}\bar{N}_i H \ell_j$ where N_i is the i-th flavour of the right handed neutrino, ℓ_j is the j-th flavour of the lepton doublet, H is the Higgs doublet and Y_{ij} is the Yukawa coupling. This induces Dirac mass terms such that we can now write a mass matrix for both the left and right handed neutrinos

$$M_{all} = \begin{bmatrix} 0 & D \\ D^T & M \end{bmatrix}, \tag{3.1}$$

where D and M are 3×3 matrices. Diagonalizing this gives the following mass matrix for the light left-handed neutrinos:

$$M_\nu = -D^T M^{-1} D\,. \tag{3.2}$$

A very large M then means a very small mass for the SM neutrinos.

In our work, we make use of these two concepts to add baryogenesis and neutrino masses to our mix of inflation and dark matter to get a combined minimal framework for explaining all four together. We add three SM singlet fermions which can be the right handed neutrinos to the model which already has an extra inert doublet. Both the inert doublet and the SM singlet fermions are odd under the extra $\mathbb{Z}_2$ symmetry. All the other SM particles are even under $\mathbb{Z}_2$. This model is called the ***scotogenic model*** [58].

3.1 The scotogenic model

The minimal scotogenic model involves extending the standard model by three SM-singlet $\mathbb{Z}_2$ odd fermions N_i, (with $i = 1, 2, 3$) and another scalar doublet Φ_2 which is also odd under $\mathbb{Z}_2$. In other words, the minimal scotogenic model is obtained by extending the inert doublet model by three $\mathbb{Z}_2$ odd SM-singlet fermions. These fermions can play the role of the heavy right handed neutrinos to generate masses for SM neutrinos at the loop level. The $\mathbb{Z}_2$ symmetry which is put by hand in an ad hoc manner in this setup could arise naturally as a subgroup of a continuous gauge group like $U(1)_{B-L}$ with some non-minimal field content [59, 83]. It was used as a neutrino mass model first by Ernest Ma in 2006 [58].

Due to the $\mathbb{Z}_2$ symmetry, the inert doublet does not develop a non-zero vev which forbids neutrino mass generation at the tree level through a conventional type-I see-saw mechanism [77–82].

The scalar part of the potential of this minimal scotogenic model is the same as for the inert doublet model given in Eq. (2.1). We repeat it here for completeness:

$$\begin{aligned} V(\Phi_1, \Phi_2) \;=\; & m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 \\ & + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right], \end{aligned} \tag{3.3}$$

where Φ_1 is the Higgs doublet, Φ_2 is the inert doublet containing the following field configuration:

$$\Phi_1 \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix}\chi \\ h\end{pmatrix}, \qquad \Phi_2 \;=\; \begin{pmatrix} H^\pm \\ \frac{H^0 + iA^0}{\sqrt{2}}\end{pmatrix}. \tag{3.4}$$

In addition to the scalar part, the $\mathbb{Z}_2$ odd SM-singlet fermions contribute the following terms:

$$\mathcal{L} \;\supset\; \frac{1}{2}(M_N)_{ij}N_iN_j + \left(Y_{ij}\,\bar{L}_i\tilde{\Phi}_2N_j + \text{H.c.}\right)\,, \tag{3.5}$$

where $(M_N)_{ij}$ are the Majorana mass matrix elements for the 3×3 mass matrix for these fermions, Y_{ij} are the Yukawa coupling strengths and $\tilde{\Phi}_2$ is given by

$$\tilde{\Phi}_2 = \sigma_2 \Phi_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \Phi_2$$

The scalar sector in this model is also coupled to gravity non-minimally so as to accommodate the inflationary physics as done in Chapter 2 with the action for the scalar sector given by Eq. (2.6) repeated here:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_{Pl}^2 R - D_\mu \Phi_1 D^\mu \Phi_1^\dagger - D_\mu \Phi_2 D^\mu \Phi_2^\dagger - V(\Phi_1, \Phi_2) - \xi_1 |\Phi_1|^2 R - \xi_2 |\Phi_2|^2 R \right]. \tag{3.6}$$

Therefore the complete model involves an extra $\mathbb{Z}_2$ odd, $SU(2)$ scalar doublet which is coupled non-minimally to gravity and three $\mathbb{Z}_2$ odd SM-singlet fermions. The inert doublet particles after the conformal transformation of the action and the fields become the inflaton and later reheat the Universe. The neutral scalar of the doublet becomes a freeze-out dark matter candidate later in the history of the Universe while the $\mathbb{Z}_2$ odd SM-singlet fermions are used to generate neutrino masses and the lepton asymmetry leading to the baryon asymmetry of the Universe via loops involving the inert doublet particles. In this manner, we achieve the goal of combining inflation, dark matter, baryogenesis and neutrino masses in a single minimal framework.

Before we move on to show the baryon asymmetry and neutrino masses generation in this model, we must make sure that addition of the three extra $\mathbb{Z}_2$ odd fermions do not disturb the setup for inflationary physics and dark matter if we are to attempt a unification of all the four aspects of the Universe.

3.2 Changes to inflation, reheating and dark matter

Since the conformally transformed inert doublet fields continue to be the fields responsible for inflation, and we can safely assume negligible abundance of all other fields including the new $\mathbb{Z}_2$ odd fermions, the inflationary dynamics remain the same as discussed in Chapter 2. However, there can occur some changes to the reheating process because of the Yukawa coupling between the inert doublet, the new fermions and the SM leptons. In addition to the gauge bosons and the Higgs channel for producing relativistic standard model degree of freedom, another channel now opens up that involves direct decays of the inert doublet to relativistic SM leptons along with relativistic $\mathbb{Z}_2$ odd fermions.

From the Eq. (3.5), we know the form of the interaction between the SM leptons, the $\mathbb{Z}_2$ odd fermions and the inert doublet. During reheating, we still work with the conformally transformed inflaton field $A = A_0(t)\cos(\omega t)$ which oscillates in its damped harmonic oscillator potential while annihilating or decaying into relativistic degrees of freedom and the other scalar field B (see Eqs. (2.14) and 2.15 for their definitions). Denoting the lepton doublets as $L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$, with i being the flavour index, we have the following coupling term:

$$-Y_{ij}\sqrt{\frac{M_{Pl}A}{\sqrt{6}\xi_2(1+B^2/M_{Pl}^2)}}(\bar{e}_i N_j+\bar{N}_j e_i)+Y_{ij}\frac{B}{M_{Pl}}\sqrt{\frac{M_{Pl}A}{\sqrt{6}\xi_2(1+B^2/M_{Pl}^2)}}(\bar{\nu}_i N_j+\bar{N}_j \nu_i)\,, \tag{3.7}$$

the first of which is the decay of the charged scalar in the inert doublet while the second is from the decay of the neutral scalar. During reheating the inflaton A oscillates quite rapidly with frequency ω which is much larger than the Hubble rate. Because of this, we can replace A by its time average over each period, $\frac{A_0}{2}$ in the interaction term. With $B \approx$ constant $\sim \mathcal{O}(M_{Pl})$ (see Section 2.2.1), we can reduce the above expression to

$$\begin{aligned} &-Y_{ij}\sqrt{\frac{M_{\rm Pl}A_0}{\sqrt{24}\xi_2}}(\bar{e}_i N_j+\bar{N}_j e_i)+Y_{ij}\sqrt{\frac{M_{\rm Pl}A_0}{\sqrt{24}\xi_2}}(\bar{\nu}_i N_j+\bar{N}_j \nu_i) \\ =&-Y_{ij}\sqrt{\frac{M_{\rm Pl}}{\sqrt{24}\xi_2 A_0}}A_0(\bar{e}_i N_j+\bar{N}_j e_i)+Y_{ij}\sqrt{\frac{M_{\rm Pl}}{\sqrt{24}\xi_2 A_0}}A_0(\bar{\nu}_i N_j+\bar{N}_j \nu_i)\,, \end{aligned} \tag{3.8}$$

where i, j are the lepton flavor indices.

The decay rate of the scalar A of mass ω to two fermions (say N_1 and ν_1) with a coupling strength of $Y\sqrt{\frac{M_{Pl}}{\sqrt{24}\xi_2 A_0}}$ is:

$$\Gamma = \omega Y^2 \frac{M_{Pl}}{8\pi\sqrt{24}\xi_2 A_0}. \tag{3.9}$$

The decay of the energy density of the inflaton due to this decay is given by:

$$\begin{aligned}\frac{d(\rho_A a^3)}{dt} &= -\Gamma \rho_A\, a^3\,, \\ &= -\omega Y^2 \frac{M_{Pl}}{8\pi\sqrt{24}\xi_2 A_0}\,\frac{\omega^2 A_0^2}{2} a^3\,, \\ &= -\frac{a^3\omega^3 Y^2 M_{Pl} A_0}{8\pi\sqrt{24}\xi_2}\,. \end{aligned} \tag{3.10}$$

From the Friedman equation, it follows that the total energy density ρ_t and the total pressure density p_t obey

$$\frac{d(\rho_t a^3)}{dt} = -3p_t a^2 \dot{a}\,. \tag{3.11}$$

With $\rho_t = \rho_A + \rho_\gamma$ and $p_t = p_\gamma = \frac{\rho_\gamma}{3}$ where p_γ is the pressure density of radiation (matter has zero pressure density implying pressure density of A during reheating, $p_A = 0$), Eq. (3.11) gives us the relation between the energy density of radiation and that of matter:

$$\frac{d(\rho_\gamma a^4)}{dt} = -a\,\frac{d(\rho_A a^3)}{dt} = \frac{a^4\omega^3 Y^2 M_{Pl} A_0}{8\pi\sqrt{24}\xi_2} \quad \text{(follows from Eq. (3.10))} \tag{3.12}$$

For a single generation of leptons a simple integration gives us:

$$\rho_\gamma = 2\times\sqrt{\frac{3}{\lambda_2}}\frac{Y^2 M_{Pl}\omega^3}{8\pi} = \frac{6.16\times 10^{49}\ \text{GeV}^4}{\sqrt{\lambda_2}}\,, \tag{3.13}$$

for a typical Yukawa value of $Y \approx 10^{-4}$. The factor 2 stands for two different decays occuring in one generation of leptons for a given generation of the $\mathbb{Z}_2$ odd fermion (which can be interpreted as a right handed neutrino in the context of neutrino mass generation) – one into a charged lepton and another into the neutrino. The energy density produced in other generations of the leptons and $\mathbb{Z}_2$ odd fermions can similarly be obtained by varying the Yukawa values. For $\lambda_2 \sim \mathcal{O}(1)$, this is much smaller compared to the relativistic energy density produced by the gauge boson channel.

As long as the Yukawa strengths remain significantly smaller than 1, the relativistic energy density produced by direct decay of the inflaton into $\mathbb{Z}_2$ odd fermions and SM leptons will be subdominant to the gauge boson channel. This is in general true for scotogenic models which also explain the very neutrino masses.

Since the neutral scalar component of the inert doublet is the dark matter, the Yukawa coupling to the SM-singlet $\mathbb{Z}_2$ odd fermions can in principle affect the relic abundance calculation. However, with typical small Yukawa values of the $\mathcal{O}(10^{-4}$, the contribution of these fermions to the dark matter interaction cross section is negligible compared to the Higgs and the gauge boson interactions. Therefore, addition of the extra fermions to our existing model of the inert doublet non-minimally coupled to gravity does not affect inflation and reheating physics or dark matter relic abundance and allows us to include baryogenesis and neutrino masses with inflation and dark matter without any worries.

3.3 Generation of neutrino masses

Because of the $\mathbb{Z}_2$ symmetry with both the extra SM singlet fermions and the inert doublet being odd in it while the SM particles are even, the usual Dirac mass term arising from the Yukawa coupling of the Higgs to the $\mathbb{Z}_2$ odd fermion and the SM neutrinos, $\bar{L}_i\tilde{\Phi}_1 N_j$, is prohibited. Therefore the usual type I seesaw mechanism for generating neutrino masses doesn't work in this model. Instead, this model utilizes the Yukawa interaction involving the inert doublet and the $\mathbb{Z}_2$ odd fermions, see Eq. (3.5). At the loop level, two such Yukawa terms can come together where the inert doublet particles' line is coupled to two copies of the Higgs vev, with another $\mathbb{Z}_2$ odd fermion crossing as they are Majorana particles. The relevant Feynman diagram is shown in Fig. 3.2

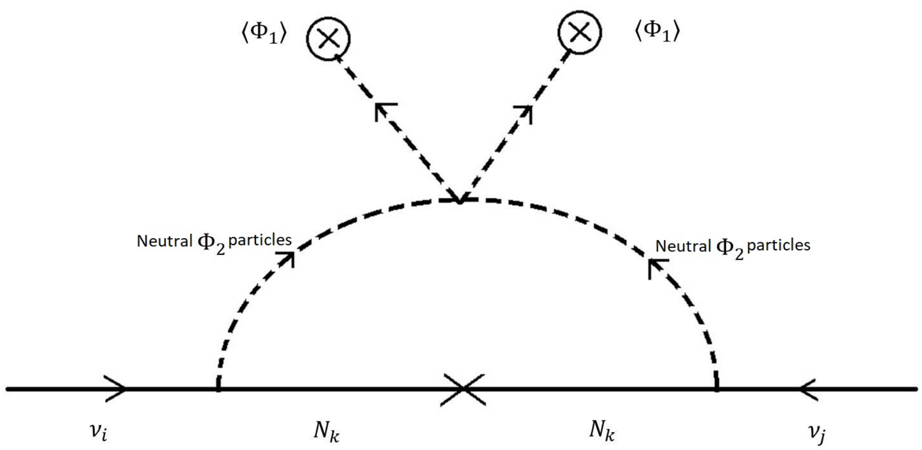

Figure 3.2: *Generation of small neutrino masses at the one loop level in the scotogenic model*

If we choose a basis where the mass matrix, M_{ij} of the $\mathbb{Z}_2$ odd fermions is diagonal i.e. we work in the $\mathbb{Z}_2$ odd fermions mass basis, we can write the SM neutrino mass matrix as follows [58, 84]

$$(M_\nu)_{ij} = \sum_k \frac{Y_{ik}Y_{jk}M_k}{32\pi^2}\left(\frac{m_{H^0}^2}{m_{H^0}^2 - M_k^2}\ln\frac{m_{H^0}^2}{M_k^2} - \frac{m_{A^0}^2}{m_{A^0}^2 - M_k^2}\ln\frac{m_{A^0}^2}{M_k^2}\right)$$
$$\equiv \sum_k \frac{Y_{ik}Y_{jk}M_k}{32\pi^2}\left[L_k(m_{H^0}^2) - L_k(m_{A^0}^2)\right] , \quad (3.14)$$

where M_k is the mass eigenvalue of the $\mathbb{Z}_2$ odd fermion state N_k in the internal line while the indices $i, j = 1, 2, 3$ run over the three neutrino generations and the three copies of N_i. The function $L_k(m^2)$ is defined as:

$$L_k(m^2) = \frac{m^2}{m^2 - M_k^2}\ln\frac{m^2}{M_k^2} . \quad (3.15)$$

It is interesting to note that from Eqs. (2.4), $m_{H_0}^2 - m_{A_0}^2 = \lambda_5 v^2$. The neutral particles of the inert doublet become degenerate if $\lambda_5 \to 0$. This also causes the masses of the SM neutrinos to vanish. It is also the λ_5 term that breaks lepton number by two units and thus will be used for leptogenesis ultimately leading to baryogenesis. Both SM neutrino masses and the ratio of the net baryon number density to the radiation density are very small quantities, therefore making the smallness of λ_5 technically natural in the 't Hooft sense [85].

3.3.1 Casas-Ibarra parametrization

The usual leptogenesis cases have N_i masses above $\mathcal{O}(10^9)$ GeV which is far above the reach of near future experiments. It is worthwhile to bring the scale lower to say 10 TeV masses for the lightest N_i as that will bring the model nearer to being tested at various experiments including collider based ones. Also, to start with an initial equilibrium abundance of the $\mathbb{Z}_2$ odd fermions, we will need to choose atleast some of the Yukawa couplings of the order of $10^{-3} - 10^{-4}$ for $\mathcal{O}(10\text{ TeV})$ N_i masses. One must make sure that the light neutrino masses obtained from a given choice of Yukawa couplings is consistent with the cosmological limit on the sum of neutrino masses, $\sum_i m_i \leq 0.11$ eV [2], as well as the neutrino oscillation data [86]. This can be ensured by taking the neutrino masses as input and writing the Yukawa matrix using the so-called Casas-Ibarra parametrization [87].

The neutrino mass matrix in Eq. (3.14) can be written in a form resembling the type-I seesaw formula:

$$M_\nu \;=\; Y\widetilde{M}^{-1}Y^T\,, \tag{3.16}$$

where $\widetilde{M}$ is a diagonal matrix composed of functions of corresponding elements of the diagonal mass matrix of the $\mathbb{Z}_2$ odd fermions as defined below

$$\widetilde{M}_i \;=\; \frac{2\pi^2}{\lambda_5}\zeta_i\frac{2M_i}{v^2}\,, \tag{3.17}$$

$$\text{and}\quad \zeta_i \;=\; \left(\frac{M_i^2}{8(m_{H^0}^2 - m_{A^0}^2)}\left[L_i(m_{H^0}^2) - L_i(m_{A^0}^2)\right]\right)^{-1}\,. \tag{3.18}$$

The light neutrino mass matrix (3.16) is diagonalized by the usual PMNS mixing matrix U, which is determined from the neutrino oscillation data (up to the Majorana phases) to give the diagonal mass matrix D_ν as:

$$D_\nu \;=\; U^\dagger M_\nu U^* \;=\; \mathrm{diag}(m_1, m_2, m_3)\,, \tag{3.19}$$

with m_1, m_2, m_3 being the masses of the three SM neutrinos. In the normal hierarchy, $m_1 \lesssim m_2 < m_3$ while in the inverted hierarchy, $m_2 \gtrsim m_1 > m_3$.

We can then write the Yukawa coupling matrix that satisfies the neutrino data in terms of the masses of the neutrinos and the $\mathbb{Z}_2$ odd fermions as

$$Y \;=\; U D_\nu^{1/2} O \widetilde{M}^{1/2}\,, \tag{3.20}$$

where O is an arbitrary complex orthogonal matrix. When written in this form, the choices for the Yukawa matrix elements will satisfy the neutrino data automatically.

TeV scale leptogenesis constrains only the lightest neutrino mass [88] without saying anything about the hierarchy of neutrino masses (normal or inverted). Therefore, without loss of generality, we work in the normal hierarchy.

3.4 Baryogenesis

Inflation ends any pre-existing asymmetry between the number densities of baryons and anti-baryons by diluting them exponentially. This necessitates the generation of asymmetry afresh to account for the observations of the Universe being almost completely made of matter with negligible anti-matter. In the present time, leptogenesis has emerged as one of the strongest contenders for generating the baryon asymmetry of the Universe because of its phenomenological implications and testability in various experiments including neutrino, dark matter and collider experiments. The choice of the scotogenic model to have a common framework for inflation, dark matter, neutrino masses and baryogenesis is particularly well-motivated. We have already shown that the inert doublet coupled non-minimally to gravity can play the part of the inflaton and later freeze-out near the EW phase transition to give the dark matter relic. The Yukawa interaction between the inert doublet and the $\mathbb{Z}_2$ odd fermions at the loop level can generate masses for the SM neutrinos, thereby connecting neutrino mass generation to the inflationary physics and the dark matter phenomenology. The scotogenic model also has plenty of scope for generating a net baryon asymmetry via leptogenesis using the out-of-equilibrium decay of the lightest $\mathbb{Z}_2$ odd fermion [88–93].

The usual leptogenesis also sometimes called the vanilla leptogenesis scenario with hierarchical N_i masses i.e. three distinct masses for all the three distinct N_i's [94] has an absolute lower bound on the mass of the lightest N_i (let's say N_1 without loss of generality) of $M_1 \gtrsim 10^9$ GeV [95, 96]. It is possible that this lower bound can be brought down to about 10^6 GeV if flavour and thermal effects are included [97]. A similar lower bound with values similar to the vanilla leptogenesis can be derived in the scotogenic model with only one or at most two $\mathbb{Z}_2$ odd fermions in the strong washout regime. However, with three such fermions, the bound can be lowered to around 10 TeV [88] without explicitly including flavour and thermal effects and without resorting to a resonant enhancement of the CP-asymmetry [98, 99].

We take the following hierarchical setup of benchmark values for the masses of the $\mathbb{Z}_2$ odd fermions:

$$M_1 = 10 \text{ TeV}, \tag{3.21}$$

$$M_2 = 50 \text{ TeV}, \tag{3.22}$$

$$M_3 = 100 \text{ TeV}, \tag{3.23}$$

to derive constraints on the model parameter space that involves allowed values of the CP violating coupling of the inert doublet λ_5 and the mass of the neutral scalar component of the inert doublet which is the dark matter candidate. Once sufficient amount of these fermions are produced during and after reheating, they become a part of the equilibrium. Their CP asymmetric decay produces a net $B-L$ number density, n_{B-L}. Sphaleron processes occurring in the Universe till the EW phase transition convert the $B-L$ number density to a net excess of baryon number density represented by the ratio of baryons to photons, η_B defined as

$$\eta_B = \frac{3}{4}\frac{g_*^0}{g_*} a_{\rm sph} n_{B-L}^f \simeq 9.2 \times 10^{-3}\, n_{B-L}^f\,, \tag{3.24}$$

where $a_{\rm sph} = \frac{8}{23}$ is the sphaleron conversion factor in the scotogenic model (obtained by taking into account two scalar doublets instead of just one in the vanilla leptogenesis) and $g_* = 110.75$ is the effective degrees of freedom in the relativistic plasma during the time of the final lepton asymmetry. g_* is obtained by taking into account two scalar doublets but not the $\mathbb{Z}_2$ odd fermions which have already fallen out of equilibrium (required for generation of lepton asymmetry). The other degree of freedom parameter, $g_*^0 = \frac{43}{11}$ counts the number of relativistic degrees of freedom at the recombination epoch.

For a hierarchical mass scenario where $M_1 \ll M_2, M_3$, the lepton asymmetry produced by the decays of N_2 and N_3 are negligible as they become suppressed due to the strong washout effects caused by N_1 or $N_{2,3}$ mediated interactions. Therefore for obtaining the final lepton asymmetry which is then converted to the baryon asymmetry, only N_1 decays are relevant.

3.4.1 Setting up the Boltzmann equations

Since the n_{B-L} is created by CP asymmetric decays of N_1, we need to numerically solve simultaneous Boltzmann equations for the N_1 number density, n_{N_1}, and the $B-L$ number density n_{B-L}. We use equilibrium initial condition for the number density of N_1 and a zero initial n_{B-L}. The typical Yukawa couplings for models with extra fermions which are involved in generating SM neutrino masses are so small that the models require new physics e.g. existence of another gauge boson Z'. However, for reasonably large Yukawa values, these fermions can enter the equilibrium during or shortly after the reheating phase. The interactions that aid in attaining equilibrium are $W/Z, H^{\pm}/H_0/A_0 \to N_i, \ell^{\pm}/\nu_i$. If at least some of the Yukawa couplings are of the order $10^{-3} - 10^{-4}$, the interaction rate can go above the Hubble rate early in the history of the Universe helping the $\mathbb{Z}_2$ fermions to attain equilibrium.

The Fig. 3.3 shows that for a Yukawa value of 10^{-3}, equilibrium can be attained around $\mathcal{O}(10^{10}\,\text{GeV})$ while with a 10^{-4} Yukawa value, there is equilibrium around

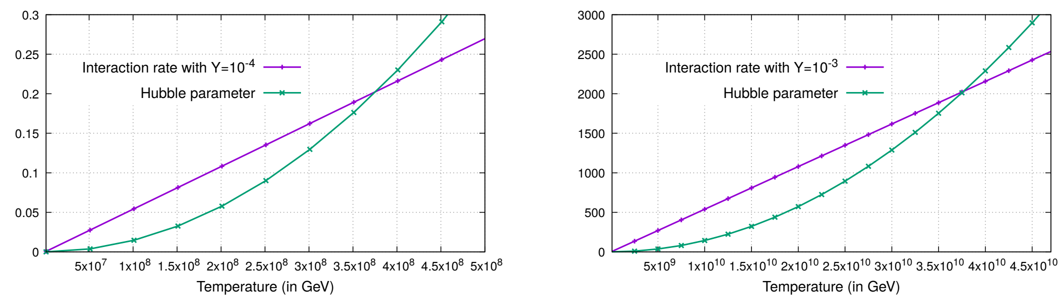

Figure 3.3: *The interaction rate is less than the Hubble rate in the early Universe but it overtakes the Hubble rate below $\mathcal{O}(10^8$ GeV$)$ for $Y = 10^{-4}$ (left) and $\mathcal{O}(10^{10}$ GeV$)$ for $Y = 10^{-3}$ (right).*

$\mathcal{O}(10^8\,\text{GeV})$. The N_i continue to remain in equilibrium till upto 10 TeV where they fall out of equilibrium and start generating the lepton asymmetry. The lepton asymmetry around the sphaleron temperature taken around 140 GeV where the EW phase transition starts is taken as the final asymmetry that gets converted to the baryon asymmetry.

Since $B-L$ calculation requires out of equilibrium interactions, we must compare the Hubble parameter and the decays rates for the interactions $N_1 \to \ell\Phi_2, \bar{\ell}\Phi_2^*$. We thus define the decay parameter

$$K_{N_1} = \frac{\Gamma_{N_1}}{H(z=1)}\,, \tag{3.25}$$

where Γ_{N_1} is the total decay rate of N_1 and $H(z=1)$ is the Hubble rate evaluated at $z = \frac{M_1}{T} = 1$ with T being the temperature in GeV of the Universe. K_{N_1} also determines the strong and weak washout regimes with strong washout occurring when $K_{N_1} > 4$ and weak washout for $K_{N_1} < 1$. Our choice of TeV scale mass for dark matter and 10-100 TeV scale masses for the $\mathbb{Z}_2$ odd fermions, succesful leptogenesis requires being in the weak washout regime [88]. With the Yukawa couplings given in Eq. (3.20), the decay rate of N_1 is

$$\Gamma_{N_1} \quad = \quad \frac{M_1}{8\pi}(Y^\dagger Y)_{11}\left[1-\left(\frac{m_{\rm DM}}{M_1}\right)^2\right]^2 \;\equiv\; \frac{M_1}{8\pi}(Y^\dagger Y)_{11}\,(1-\eta_1)^2\,. \quad (3.26)$$

The benchmark N_i mass values and the dark matter mass calculated from relic abundance calculation done in Sec 2.4 ensure that for Yukawa value satisfying the neutrino mass data as obtained in Eq. (3.20), we always remain in the weak washout regime.

We also need the CP asymmetry parameter, ε_1, which is the ratio of the difference between the decays to lepton and anti-lepton to the total decay to calculate the n_{B-L}. If we ignore all flavour effects by summing over all the flavours, the form the CP asymmetry parameter simplifies to

$$\varepsilon_1 \;=\; \frac{1}{8\pi(Y^\dagger Y)_{11}}\sum_{j\neq 1}{\rm Im}\left[(Y^\dagger Y)^2_{1j}\right]\left[f(r_{j1},\eta_1)-\frac{\sqrt{r_{j1}}}{r_{j1}-1}(1-\eta_1)^2\right]\,, \quad (3.27)$$

$$\text{where} \qquad f(r_{j1},\eta_1) \;=\; \sqrt{r_{j1}}\left[1+\frac{1-2\eta_1+r_{j1}}{(1-\eta_1)^2}\;\ln\left(\frac{r_{j1}-\eta_1^2}{1-2\eta_1+r_{j1}}\right)\right]\,, \quad (3.28)$$

$$\text{and} \qquad r_{j1} \;=\; \left(\frac{M_j}{M_1}\right)^2\,,\; \eta_1 \;\equiv\; \left(\frac{m_{\rm DM}}{M_1}\right)^2\,. \quad (3.29)$$

The simultaneous Boltzmann equations for n_{N_1} and n_{B-L} are given by

$$\frac{dn_{N_1}}{dz} = -D_1(n_{N_1} - n_{N_1}^{\rm eq})\,, \tag{3.30}$$

$$\frac{dn_{B-L}}{dz} = -\varepsilon_1 D_1(n_{N_1} - n_{N_1}^{\rm eq}) - W_1 n_{B-L}\,, \tag{3.31}$$

respectively [94], where $n_{N_1}^{\rm eq} = \frac{z^2}{2}K_2(z)$ is the equilibrium number density of N_1 (with $K_i(z)$ being the modified Bessel function of i-th kind). D_1 measures the total decay rate with respect to the Hubble rate and is given by

$$D_1 \equiv \frac{\Gamma_1}{Hz} = K_{N_1} z \frac{K_1(z)}{K_2(z)}\,, \tag{3.32}$$

The total washout rate $W_1 \equiv \frac{\Gamma_W}{Hz}$ measures the total washout rate. It has two contributions – one due to inverse decays $\ell\Phi_2, \bar{\ell}\Phi_2^* \to N_1$ denoted by W_{ID}, and, the other due to the $\Delta L = 2$ scatterings $\ell\Phi_2 \leftrightarrow \bar{\ell}\Phi_2^*,\ \ell\ell \leftrightarrow \Phi_2^*\Phi_2^*$ which violate lepton number by two units. They are given by

$$W_{\rm ID} = \frac{1}{4}K_{N_1} z^3 K_1(z)\,, \tag{3.33}$$

$$W_{\Delta L=2} \simeq \frac{18\sqrt{10}\,M_{\rm Pl}}{\pi^4 g_\ell \sqrt{g_*} z^2 v^4}\left(\frac{2\pi^2}{\lambda_5}\right)^2 M_1 \bar{m}_\zeta^2\,, \tag{3.34}$$

respectively [88], where $\eta_1 \ll 1$ is used to approximate $W_{\Delta L=2}$ to the value in Eq. (3.34). g_ℓ is the internal degrees of freedome of the SM leptons and $\bar{m}\zeta$ is the effective neutrino mass parameter given by

$$\bar{m}_{\zeta}^2 \simeq 4\zeta_1^2 m_1^2 + \zeta_2 m_2^2 + \zeta_3^2 m_3^2 \,, \tag{3.35}$$

where m_i are the neutrino mass eigenvalues and ζ_i are defined in Eq. (3.18).

Looking at the equations in the setup, we can note that the baryon asymmetry depends only on λ_5 and is independent of the other quartic couplings in the theory, justifying the name CP violating coupling given to it. The final value of $B-L$ number density, n_{B-L} obtained after solving the Boltzmann equations in Eqs. (3.30) and (3.31) is substituted in Eq. (3.24) to obtain the value of the baryon asymmetry of the Universe. We show the result for η_B calculated for a benchmark value of $m_{H^0} = 1.5$ TeV (also the mass of the dark matter in this case) as a function of λ_5 in Fig. 3.4

The horizontal line in Fig. 3.4 marks the Planck 2018 value of the observed value of $\eta_B^{\rm obs} = (6.04 \pm 0.08) \times 10^{-10}$ at 68% confidence level which translates to a baryon density of $\Omega_B h^2 = 0.0224 \pm 0.0001$ [2]. We find that $\lambda_5 \simeq 6.9 \times 10^{-5}$ for this benchmark point. The Yukawa matrix elements calculated from Eq. (3.20) using the data from SM neutrino mass square differences for normal hierarchy require that those matrix elements corresponding to the coupling of the lightest $\mathbb{Z}_2$ odd fermion, N_1, must be of the order $\mathcal{O}(10^{-8} - 10^{-9})$ while those involving $N_{2,3}$ are of order $\mathcal{O}(10^{-4} - 10^{-3})$. Such a distribution of the Yukawa matrix elements is needed because we are in the weak washout regime with $K_{N_1} < 1$, hence smaller $(Y^\dagger Y)_{11}$. Looking at Eq. (3.26), we notice that smaller $(Y^\dagger Y)_{11}$ means a smaller source term

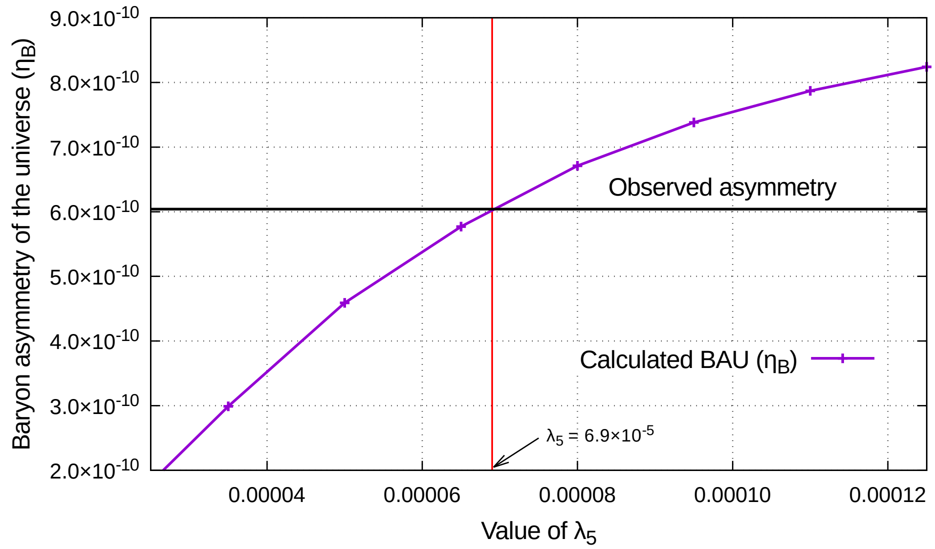

Figure 3.4: *The baryon-to-photon ratio as a function of* $|\lambda_5|$ *for benchmark DM mass of 1.5 TeV. The horizontal line gives the observed value, as inferred from Planck 2018 data [2].*

which requires a larger CP asymmetry and hence larger $(Y^\dagger Y)_{1j}$, $(j = 2, 3)$ terms to make up.

Another requirement for such low scale leptogenesis is that the lightest SM neutrino must be of the order $\mathcal{O}(10^{-11} - 10^{-12}$ eV. The masses of the other neutrinos are set as input from the neutrino mass square differences data.

Fig. 3.5 shows the role of varying dark matter masses in baryogenesis with respect to the values of the various couplings of the inert doublet namely $\lambda_{3,4,5}$. Assuming $\lambda_4 = \lambda_5$, we can change λ_3 to change the value of $\lambda_s = \lambda_3 + \lambda_4 + \lambda_5$ which goes into calculating the dark matter relic abundance and see its effect of baryogenesis. This effectively links baryogenesis and neutrino masses to dark matter phenomenology and by extension to inflation in the scotogenic model.

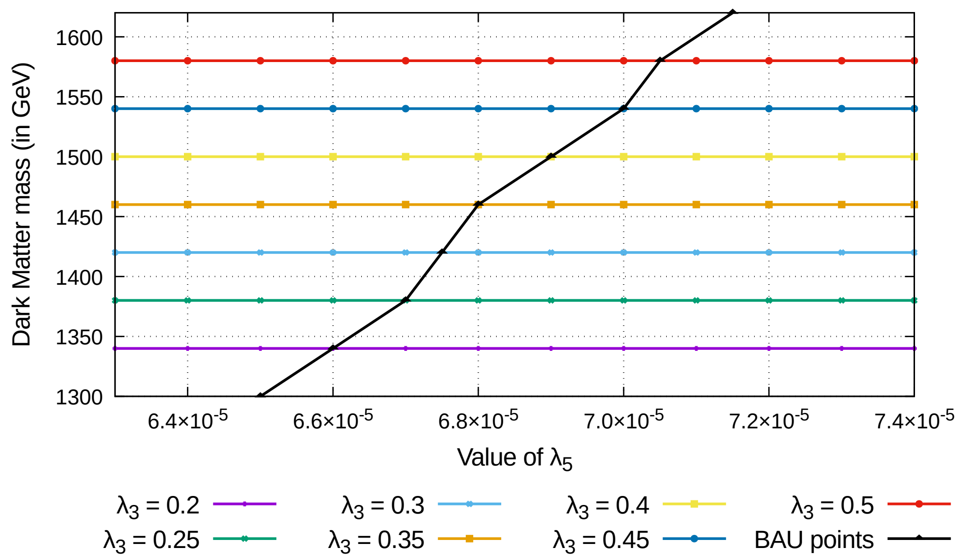

Figure 3.5: *DM mass as a function of* λ_3 *and* $|\lambda_5|$ *satisfying the relic density constraint. The black line across the graph shows the points that satisfy the baryogenesis constraint*

The baryogenesis constraints give a preferred range of $1.3-1.6$ TeV for dark matter mass. The upper bound comes from renormalization group running constraints of λ_s while the lower bounds can be explained from the allowed dark matter direct detection parameter ranges. The values of $\lambda_{1,2,3,4,5}$, the various Yukawa couplings and the gauge couplings must remain below perturbative bounds upto Planck scales for the model to explain low scale phenomena like dark matter, neutrino masses and baryogenesis as well as high scale phenomena like inflation and reheating.

3.5 Conclusion

In this way, we show that inflation, dark matter, baryogenesis and neutrino masses can be linked to each other in a simple and minimal model like the scotogenic model with the inert doublet particles playing the part of the inflaton because of their non-minimal coupling to gravity and later freezing out to become the dark matter relic candidate while producing neutrino masses and asymmetry in the lepton sector finally leading the baryon asymmetry of the Universe when they interact with the $\mathbb{Z}_2$ fermions. The complete model gives a lower bound on $\lambda_2 \gtrsim \frac{1}{60}$ because of constraints from reheating. The dark matter relic density calculation together with renormalization group considerations constrains the other quartic couplings between the inert doublet and the Higgs doublet, specially the λ_3 considering $\lambda_4, \lambda_5 \ll \lambda_3$. The Yukawa matrix elements are set by the neutrino mass data and λ_5 gets fixed by baryogenesis observations. Although it requires the lightest SM neutrino mass to be of $\mathcal{O}(10^{-11} - 10^{-12}$ eV), the low scale of leptogenesis in the model brings it closer to testability in various experiments. The dark matter direct detection cross sections in this model are also within range of near future experiments like XENONnT, DARWIN, LZ and PandaX-30T. To conclude, this model provides a quite successful, simple and very economical means of *linking inflation with dark matter, baryogenesis and neutrino masses.*

4 Exploring inflation by fermion condensates

This chapter uses parts of the preprint "Inflation and reheating with a fermionic field" [100].

4.1 Motivation

Using the scotogenic model with the inert doublet coupled non-minimally to gravity, we attained the goal of combining inflation, dark matter, baryogenesis and neutrino masses in a common framework. Although the model is very simple, quite economical and rich in phenomenology, it is not absolutely minimal as it involves two sets of new particles – first the inert scalar doublet and second, three extra SM singlet fermions – both of which are odd under an extra $\mathbb{Z}_2$ symmetry. The SM particles remain even under this extra symmetry. A truly minimal model would have only one type of extra particles. The inert doublet coupled non-minimally to gravity does explain inflation and dark matter by itself but it does not have a CP violating interaction term to generate baryon asymmetry. The SM singlet fermions which could be the right handed neutrinos can explain neutrino masses and baryogenesis via leptogenesis with CP violating Feynman diagrams at the loop level. They can also under certain conditions of stability be a dark matter candidate. But right handed neutrinos, being fermions, cannot explain inflation which requires fields that can have large occupation numbers and scalars form the simplest such fields.

Over the years, many people have considered using a fermion condensate as the inflaton. A spinor field interacting with a scalar non-linearly was studied in [101–105] to explain early universe anisotropy in the context of Bianchi type 1 cosmologies having an anisotropic space-time metric with different scale factors for different spatial directions. Another effect of such cases was an exponentially expanding quasi de-Sitter space-time which is similar to an inflating Universe scenario.

4.2 Noether symmetry

People have also studied the possibility of using the scalar or the pseudo-scalar bilinear of the spinor field as the inflaton in a homogeneous space-time [106–110]. In these works, a spinor bilinear coupled to gravity non-minimally is shown to create an accelerated expansion phase similar to inflation. [109, 110] also use a Noether symmetry argument to show when such a bilinear can give an accelerated expanding solution and when it will behave as a matter field for the following action

$$S = \int \sqrt{-g}\, d^4x \left(F(\Psi)R + \frac{i}{2}\left(\bar{\psi}\tilde{\gamma}^{\mu} D_{\mu}\psi - (D_{\mu}\bar{\psi}\tilde{\gamma}^{\mu}\psi\right) - V(\Psi) \right) , \tag{4.1}$$

where $\Psi = \bar{\psi}\psi$ or $i\bar{\psi}\gamma^5\psi$, $\tilde{\gamma}^\mu = e^\mu_\nu \gamma^\nu$ are the generalized gamma matrices with e^μ_ν being the *vierbeins* or the tetrad fields. The spinor covariant derivative D_μ is given by

$$D_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi; \;\; D_\mu \bar{\psi} = \partial_\mu \psi + \bar{\psi}\Omega_\mu \,, \tag{4.2}$$

where the spin connection Ω_μ is defined in terms of the generalized gamma matrices and the affine connection $\Gamma^\mu_{\nu\delta}$ as

$$\Omega_\mu = -\frac{1}{4} g_{\rho\sigma} \left[\Gamma^\rho_{\mu\delta} - e^\rho_b (\partial_\mu e^b_\delta) \right] \tilde{\gamma}^\delta \tilde{\gamma}^\sigma \,. \tag{4.3}$$

The equations of motion for the spinor and its adjoint are:

$$\dot{\psi} + \frac{3}{2} H\psi + i\gamma^0 \psi V' - i6(\dot{H} + 2H^2)\gamma^0 \psi F' \;\; = \;\; 0 \,, \tag{4.4}$$

$$\dot{\bar{\psi}} + \frac{3}{2} H\bar{\psi} - i\bar{\psi}\gamma^0 V' + i6(\dot{H} + 2H^2)\bar{\psi}\gamma^0 F' \;\; = \;\; 0 \,, \tag{4.5}$$

where the prime denotes a derivative with respect to the bilinear Ψ. Eqs. (4.6) and (4.5) can be combined to yield the equation of motion for the bilinear:

$$\dot{\Psi} + 3H\Psi = 0 \,, \tag{4.6}$$

such that

$$\Psi = \frac{\Psi_0}{a^3} \text{ where the constant } \Psi_0 \text{ is the initial value of the bilinear} \,. \tag{4.7}$$

We note the results of the Noether symmetry arguments here:

- *Case 1:* $F' = 0$

  In this case, there is no coupling of the spinor field to gravity. The action can be broken into distinct gravitational and spinor parts. The potential can be obtained to be [109, 110]:

$$V = m\Psi \,. \tag{4.8}$$

  The spinor energy density and the pressure density are given as follows:

$$\rho_s = \frac{\lambda\Psi_0}{a^3} \,, \tag{4.9}$$

$$p_s = 0 \,, \tag{4.10}$$

  which is the typical cosmological solution for a matter dominated Universe. If ψ is the only field present in the Universe, there is no accelerated expansion and thus no inflation in this case.

- *Case 2:* $F' \neq 0$

  This case signifies a non-minimal coupling between the spinor and gravity and has two sets of solutions

$$\text{First, } F = \xi\Psi^{1/3} + C_1, \quad V = m\Psi \,, \tag{4.11}$$

$$\text{and second, } F = \xi\Psi + C_2, \quad V = m\Psi \,, \tag{4.12}$$

  where $C_{1,2}$ are constants. The solution set in Eq. (4.12) is interesting for an inflationary cosmology as it is for this set that we get a spinor pressure density that is negative of the spinor energy density [109, 110]:

$$\rho_s = -\frac{\lambda\Psi_0}{2a^3}, \tag{4.13}$$
$$p_s = \frac{\lambda\Psi_0}{2a^3} = -\rho_s. \tag{4.14}$$

This is the typical equation of state for an fluid which can cause an accelerated expansion of the Universe if it dominates over all other fields. It may be possible in this case for the spinor bilinear to be the inflaton field.

4.3 Reheating

When $F = \frac{1}{2}(M_{Pl}^2 - \xi\Psi)$ ($F' \neq 0$ case), the Hubble parameter can be written as:

$$H^2 = \frac{\lambda\Psi}{3M_{Pl}^2 + 6\xi\Psi} \tag{4.15}$$

Eq. (4.6) tells us the behaviour of Ψ as time elapses. If it is the inflaton and we assume 60 e-foldings expansion during inflation, the values of Ψ at the start and end of inflation can be so arranged that $\xi\Psi \gg M_{Pl}^2$ at the beginning of inflation and $\xi\Psi \ll M_{Pl}^2$ at the end of inflation. Using $N = Ht_0 = 60$ (where $t = 0$ is taken as the start of inflation, t_0 is the end of inflation), we get

$$t_0 = 60\sqrt{\frac{6\xi}{m}}. \tag{4.16}$$

If there are Yukawa interactions between the spinor ψ and an SM scalar H and an SM fermion ℓ, $Y\bar{\psi}H\ell + h.c.$, these interactions will become comparable to the bare potential term of $m\Psi = m\bar{\psi}\psi$ when Ψ is small such that decays of ψ into other particles can start. Otherwise, a warm inflation scenario can be considered where the inflaton decays during inflation itself. The bare potential term can be interpreted as the mass of ψ. The decay rate of ψ is then given by

$$\Gamma = \frac{mY^*Y}{8\pi}\,. \tag{4.17}$$

Inflaton will decay to SM particles to reheat the Universe if $\Gamma \gtrsim H$. There is a short matter dominated phase after inflation if $\xi\Psi$ has fallen much below M_{Pl}^2. In this phase $H = \frac{2}{3t}$. To the first approximation, reheating starts at time t_0 where $H = \frac{2}{3t_0}$. Therefore the decay condition is [100]

$$\Gamma \gtrsim \frac{2}{3t_0}\,, \tag{4.18}$$

$$\xi^{1/2} \gtrsim \frac{4\pi}{45\sqrt{6m}Y^*Y}\,. \tag{4.19}$$

This is shown in Fig. 4.1 for a range of the mass and the Yukawa values. It can be seen that for most of the parameter space, ξ takes on very large values of the order $\gtrsim 10^8$. This might create further problems like loss of unitarity at scales much below the Planck scale. Values of ξ similar to those in Higgs inflation or s-inflation or the inert doublet inflation (of the $\mathcal{O}(10^4 - 10^5)$) can be possible only for spinor masses above $10^5 - 10^6$ GeV and with Yukawa values $Y \lesssim 10^{-3}$. Of course this analysis is a first approximation and quite simple. It is possible that there are other dynamics that can change these results at such high scales.

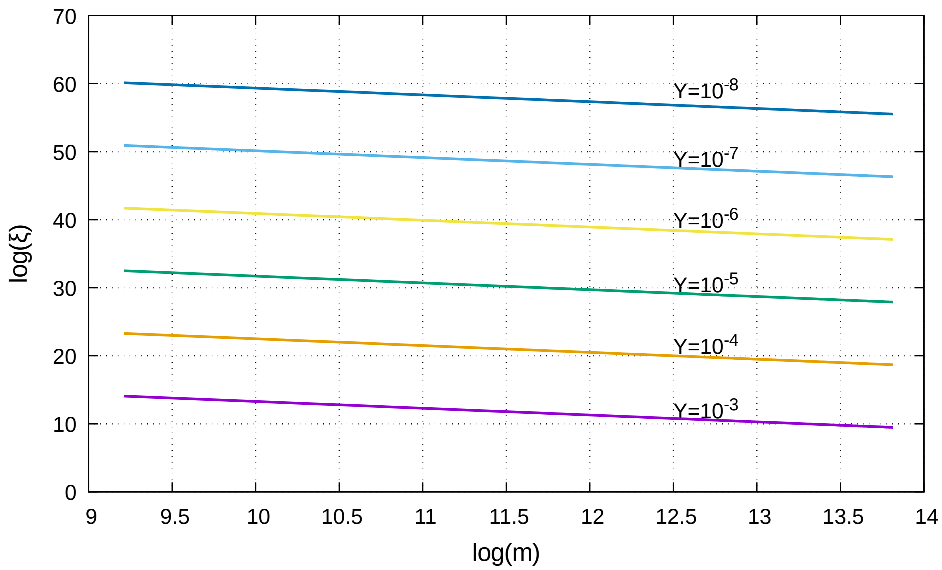

Figure 4.1: *Variation of ξ as a function of m and Y. The unit of m is GeV. Both the axes are log-scaled to the base e.*

We also calculate the reheating temperature [100]

$$\rho_\gamma = \frac{\rho_0 e^{\Gamma t_0} t_0^{3\omega}}{\Gamma^\omega t^{4\omega}} \Gamma(\omega+1, \Gamma t_0, \Gamma t)\,, \tag{4.20}$$

$$T_r = \left(\frac{30\rho_\gamma}{\pi^2 g_*}\right)^{1/4}\,. \tag{4.21}$$

where $\Gamma(a,b,c)$ is the generalized incomplete Gamma function, g_* is the number of relativistic degrees of freedom which is 109 for the SM. ω is a parameter which is $2/3$ for a matter dominated Universe and $1/2$ for a radiation dominated Universe. The transition from $\omega = 2/3$ to $\omega = 1/2$ occurs somewhere in the reheating phase.

Fig. 4.2 shows the reheating temperature as the function of the mass of the spinor and its Yukawa coupling strength with the value of the spinor bilinear at the end of inflation taken to be $\xi\Psi_e = 10^{-3}M_{Pl}^2$

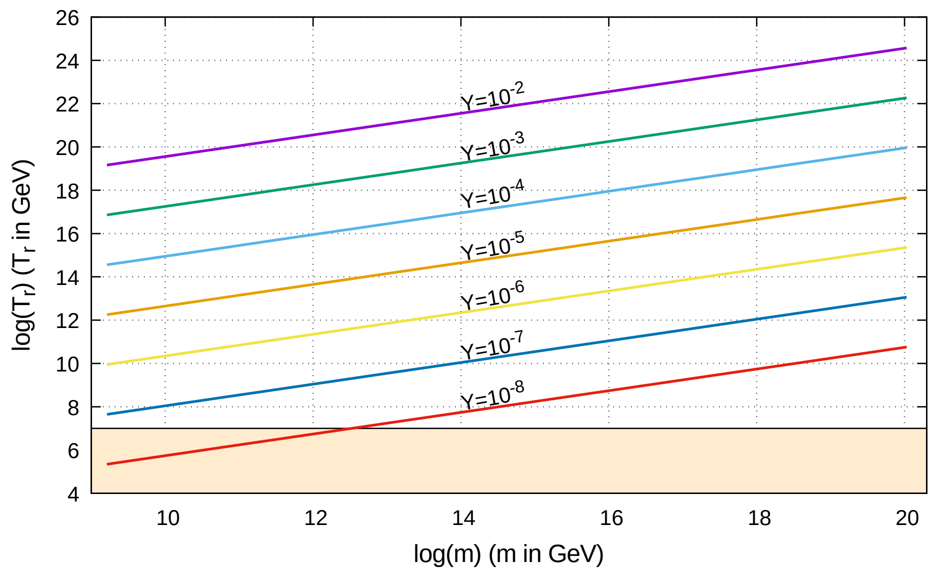

Figure 4.2: *Variation of the reheating temperature T_r with mass of the spinor m and its Yukawa coupling Y. The lower shaded region is the region below 1 TeV. If the reheating temperature is sufficiently above the EW scale, we are left with plenty of time for other things to happen like dark matter freeze-out and generation of baryon asymmetry before the EW symmetry breaking occurs.*

4.4 Power spectrum

It is possible to create an exponentially expanding Universe out of spinors. However, any realistic inflationary model using spinors can only be constructed if the various observables like the power spectrum and the spectral index can be calculated and matched with experiments. Such a calculation was done in [111] where a spinor with a potential that is a function of the scalar bilinear (an example is the conventional mass term of the fermion $m\bar{\psi}\psi$) was used to study inflation. The fermion was not coupled to gravity in their model. They consider *classical*, homogeneous spinor fields in the Heisenberg picture which is interpreted as the expectation value of the spinor in an appropriate state $|s\rangle$,

$$\psi_{cl} = \langle s|\hat{\psi}|s\rangle, \tag{4.22}$$

where the spinor operator field $\hat{\psi}$ obeys the Dirac equation $i\gamma^\mu\partial_\mu\hat{\psi} - m\hat{\psi} = 0$. The expectation value of the Dirac equation in the state $|s\rangle$ gives

$$i\gamma^\mu\partial_\mu\psi_{cl} - m\psi_{cl} = 0. \tag{4.23}$$

Therefore the classical spinor ψ_{cl} obeys the Dirac equation. To calculate the perturbations, they write $\psi = \psi_{cl} + \delta\psi$ where $\delta\psi$ is the fluctuation in the spinor field over the the classical spinor ψ_{cl}. Then the power spectrum P_s is given by the relation

$$\langle\beta(t,\vec{x})\beta(t,\vec{x}+\vec{r})\rangle = \int \frac{dk}{k}\frac{\sin(kr)}{kr}P_s\,, \tag{4.24}$$

where the variable β is defined as

$$\beta = \frac{\delta\bar{\psi}\psi + \bar{\psi}\delta\psi}{\bar{\psi}\psi}. \tag{4.25}$$

They found that at large scales, the power spectrum is approximated by

$$P_s \sim -\frac{k^3}{2\pi^3\Theta}|\Gamma(\nu)|^2\sin\left(\frac{\pi m}{H}\right)\,, \tag{4.26}$$

where H is the Hubble parameter, $\Gamma(\nu)$ is the Gamma function and Θ is a time-independent constant given by $\Theta = a^3\bar{\psi}\psi$. This form of the power spectrum means that the spectral index is $n = 4$ which is not consistent observations from Planck 2018 [1] finding $n = 0.9649 \pm 0.0042$. They conclude therefore that inflation driven by a spinor cannot produce a scale-invariant power spectrum. Eq. (4.26) also shows that for a massless spinor, there is no perturbation.

This calculation is done in a scenario with classical spinors which are not coupled to gravity. It can be interesting to see if a spinor coupled to gravity non-minimally can have a scale invariant power spectrum or not. A non-minimal gravity coupling can modify the power spectrum by modifying the effective potential with the curvature coming into picture. Calculation of the power spectrum in curvaton models can be found in [112–115].

4.5 Right handed neutrino condensates

The natural questions after using spinor bilinears that are actually fermion condensates for explaining inflation are the scale of condensation and why such a condensation will take place in the first place. Bardeen-Hill-Lindner (BHL) in [116] (see also [117] and the associated erratum) used top quark condensates as a model for a composite Higgs field [118]. It has also been shown how new interactions at some high energy scale Λ can trigger formation of low energy condensates mimicking a scalar in Nambu–Jona-Lasinio mechanism [119–121].

In a manner similar to forming top quark condensates in the BHL model with a mechanism similar to the Nambu–Jona-Lasinio mechanism, the sterile right handed neutrino condensates can form [122] at low energies due to some new interactions at high energy scale Λ which can be used to drive inflation. An effective interaction term with four fermion coupling for the right handed neutrino N is introduced to describe physics below the cut-off scale (Λ, which is taken as the Planck scale):

$$G(\bar{N}^c N)(\bar{N} N^c)\,, \tag{4.27}$$

where G is the dimension-full coupling constant and the c in the superscript denotes charge conjugation. An auxiliary scalar field ϕ devoid of a kinetic term is introduced to account for the condensation effects

$$-\, m_0^2 \phi^\dagger \phi + g_0(\bar{N}^c N \phi + h.c.)\,, \tag{4.28}$$

with g_0 is given by the relation $G = g_0^2/m_0^2$. ϕ can be integrated out to reproduce the four Fermi interaction term. To study the low energy condensates, the high frequency modes of the right handed neutrino are integrated out. The full effective Lagrangian with the induced kinetic term and the potential terms for the auxiliary field ϕ are calculated in [122]. Since ϕ is a complex field, it can be written as $\phi = \phi_0 e^{i\theta}$ (the scalar ϕ has a continuous $U(1)$ symmetry). θ is either a Nambu-Goldstone boson or a pseudo Nambu-Goldstone boson depending on whether the $U(1)$ symmetry remains intact or is explicitly broken in the model. In a *natural inflation* scenario with pseudo Nambu-Goldstone bosons [123], the phase θ becomes the inflaton with the Hubble rate given by

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\left[\frac{\dot{\theta}^2}{2} + V(\theta)\right] , \tag{4.29}$$

where $V(\theta)$ is the effective potential for θ.

This model was further built upon to combine baryogenesis in the same framework as inflation in [124].

4.6 Conclusion

All these works show that although challenging, it is possible to study inflation using fermions. Typically fermions are avoided because they cannot form large occupation number states due to Fermi-Dirac statistics. Therefore they cannot have a large classical value to drive inflation. This can be worked around by using fermion bilinears which mimic a scalar particle – effectively meaning that a condensate is driving inflation. However, such condensate inflation models have their own problems which include calculations of the spectral index which has been observed by various experiments to a high accuracy. One also needs to address the inherent mechanism that forces condensation to occur during the early Universe era. If all of this is done successfully, a model using right handed neutrino condensate can provide the most minimal case study for combining inflation, baryogenesis, dark matter and neutrino masses.

5 Conclusions and future prospects

In this thesis, we have explored links between inflation, dark matter, baryogenesis and neutrino masses. All of these are highly interesting and have inspired many people to actively invest into researching them. In this work, instead of focussing on any one, we sought to combine all the four into one common framework. This was done in two steps

1. **Step 1**: Merge inflaton and dark matter into one field or field multiplet. We used the $\mathbb{Z}_2$ odd inert doublet coupled non-minimally to gravity for this. The conformally transformed field became the inflaton while the neutral scalar became the dark matter relic later.

2. **Step 2**: Add neutrino masses and baryogenesis (via leptogenesis) to the mix. This was done by adding three $\mathbb{Z}_2$ odd, SM-singlet fermions. Their CP asymmetric decay produced lepton asymmetry, subsequently transformed into the baryon asymmetry by the sphaleron processes. These also generate neutrino

masses radiatively.

In **Chapter 2**, we showed the first step in detail. We computed the conformal transformation of the metric and the fields to end up with a Starobinsky class potential sitting in the sweet spot of all observations. The slow-roll parameters were obtained and they in turn allowed us to calculate the tensor-to-scalar ratio of perturbations and the scalar spectral index which matched with observations quite well. The power spectrum calculation gave us a relation between the non-minimal coupling and the quartic self-coupling of the inert doublet. Reheating dynamics put a lower bound on the quartic self-coupling of the inert doublet. At the end of the reheating era, the inert doublet interactions with Higgs doublet and the gauge bosons put it into equilibrium from which its components freeze-out and the neutral scalar gives the dark matter relic.

The second step was shown in **Chapter 3**. We added three $\mathbb{Z}_2$ odd SM-singlet fermions to the model. We found that this addition does not disturb either the inflation and reheating set-up or the dark matter abundance calculations owing to the smallness of the Yukawa couplings. The neutrinos of the standard model got masses radiatively by the presence of the inert doublet particles and the $\mathbb{Z}_2$ odd fermions in the loop. The lightest $\mathbb{Z}_2$ fermion undergoes a CP asymmetric decay due to the interference of the tree level decay diagram with the one loop vertex correction and self-energy diagrams, thus producing a lepton asymmetry. This process is called leptogenesis. Owing to the $B-L$ conservation and $B+L$ violation of the standard model, the sphaleron processes in the Universe before electroweak symmetry breaking convert the lepton asymmetry to the baryon asymmetry of the Universe.

We had set out to construct a minimal model for combining inflation with dark matter, baryogenesis and neutrino masses. The scotogenic model is very simple, economical and very minimal. It does the job very well. However, it contains two different sets of particles. The most minimal set-up would have only one type of beyond the standard model particles. Therefore, in **Chapter 4** we looked towards fermions to see if they could drive inflation. The right handed neutrinos can easily generate neutrino masses, baryon asymmetry of the Universe and by some stabilizing feature can also become the dark matter candidate. If they could drive inflation, it would be the ideal minimal combination of all the four phenomena. We saw that it is possible to have an exponentially expanding solution along with a subsequent reheating phase. However, this can be a realistic only if the spectral index and other relevant inflationary observables can match with experiments.

This work provides a lot of scope for future work, specially in the Fermi condensate inflation scenario. One can try to find justifications for formation of the condensate and look for high energy interactions that can create effective low energy condensates. Once that is done satisfactorily, one needs to calculate the spectral index and other parameters of inflation. On the other hand, for scalar inflation cases like the inert doublet inflation which we studied, future work can include stochastic techniques for inflation and dark matter. One can also try to look for situations where the inflaton decays during inflation itself which is a warm inflation scenario and potentially overcomes the swampland criteria. A decaying inflaton might be able to generate baryon asymmetry during or just after inflation itself as well.

Bibliography

[1] Y. Akrami *et al.* (Planck), "Planck 2018 results. X. Constraints on inflation," (2018), arXiv:1807.06211 [astro-ph.CO] .

[2] N. Aghanim *et al.* (Planck), "Planck 2018 results. VI. Cosmological parameters," (2018), arXiv:1807.06209 [astro-ph.CO] .

[3] H. B. Richer *et al.*, "The Lower main sequence and mass function of the globular cluster Messier 4," Astrophys. J. **574**, L151–L154 (2002), arXiv:astro-ph/0205086 [astro-ph] .

[4] B. M. S. Hansen *et al.*, "The white dwarf cooling sequence of the globular cluster messier 4," Astrophys. J. **574**, L155–L158 (2002), arXiv:astro-ph/0205087 [astro-ph] .

[5] E. Komatsu *et al.* (WMAP), "Seven-Year Wilkinson Microwave Anisotropy

Probe (WMAP) Observations: Cosmological Interpretation," Astrophys. J. Suppl. **192**, 18 (2011), arXiv:1001.4538 [astro-ph.CO] .

[6] M. Tanabashi *et al.* (Particle Data Group), "Review of Particle Physics," Phys. Rev. **D98**, 030001 (2018).

[7] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. **D23**, 347–356 (1981), [Adv. Ser. Astrophys. Cosmol.3,139(1987)].

[8] A. D. Linde, "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," *Second Seminar on Quantum Gravity Moscow, USSR, October 13-15, 1981*, Phys. Lett. **108B**, 389–393 (1982).

[9] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, "Reheating in Inflationary Cosmology: Theory and Applications," Ann. Rev. Nucl. Part. Sci. **60**, 27–51 (2010), arXiv:1001.2600 [hep-th] .

[10] A. D. Sakharov, "Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe," Pisma Zh. Eksp. Teor. Fiz. **5**, 32–35 (1967), [Usp. Fiz. Nauk161,no.5,61(1991)].

[11] M. Trodden, "Electroweak baryogenesis," Rev. Mod. Phys. **71**, 1463–1500 (1999), arXiv:hep-ph/9803479 [hep-ph] .

[12] G. Kauffmann, S. D. M. White, and B. Guiderdoni, "The Formation and

Evolution of Galaxies Within Merging Dark Matter Haloes," Mon. Not. Roy. Astron. Soc. **264**, 201 (1993).

[13] M. Mateo, "Dwarf galaxies of the Local Group," Ann. Rev. Astron. Astrophys. **36**, 435–506 (1998), arXiv:astro-ph/9810070 [astro-ph] .

[14] A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, "Where are the missing Galactic satellites?" Astrophys. J. **522**, 82–92 (1999), arXiv:astro-ph/9901240 [astro-ph] .

[15] B. Moore, S. Ghigna, F. Governato, G. Lake, T. R. Quinn, J. Stadel, and P. Tozzi, "Dark matter substructure within galactic halos," Astrophys. J. **524**, L19–L22 (1999), arXiv:astro-ph/9907411 [astro-ph] .

[16] J. S. Bullock, "Notes on the Missing Satellites Problem," (2010), arXiv:1009.4505 [astro-ph.CO] .

[17] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, "Too big to fail? The puzzling darkness of massive Milky Way subhaloes," Mon. Not. Roy. Astron. Soc. **415**, L40 (2011), arXiv:1103.0007 [astro-ph.CO] .

[18] A. R. Liddle and L. A. Ureña López, "Inflation, dark matter and dark energy in the string landscape," Phys. Rev. Lett. **97**, 161301 (2006), arXiv:astro-ph/0605205 [astro-ph] .

[19] F. Kahlhoefer and J. McDonald, "WIMP Dark Matter and Unitarity-Conserving Inflation via a Gauge Singlet Scalar," JCAP **1511**, 015 (2015), arXiv:1507.03600 [astro-ph.CO] .

[20] R. N. Lerner and J. McDonald, "Gauge singlet scalar as inflaton and thermal relic dark matter," Phys. Rev. **D80**, 123507 (2009), arXiv:0909.0520 [hep-ph] .

[21] A. Aravind, M. Xiao, and J.-H. Yu, "Higgs Portal to Inflation and Fermionic Dark Matter," Phys. Rev. **D93**, 123513 (2016), [Erratum: Phys. Rev.D96,no.6,069901(2017)], arXiv:1512.09126 [hep-ph] .

[22] T. Tenkanen, "Feebly Interacting Dark Matter Particle as the Inflaton," JHEP **09**, 049 (2016), arXiv:1607.01379 [hep-ph] .

[23] J.-O. Gong, H. M. Lee, and S. K. Kang, "Inflation and dark matter in two Higgs doublet models," JHEP **04**, 128 (2012), arXiv:1202.0288 [hep-ph] .

[24] N. Okada and Q. Shafi, "WIMP Dark Matter Inflation with Observable Gravity Waves," Phys. Rev. **D84**, 043533 (2011), arXiv:1007.1672 [hep-ph] .

[25] M. Fukugita and T. Yanagida, "Baryogenesis Without Grand Unification," Phys. Lett. **B174**, 45–47 (1986).

[26] R. Allahverdi, B. Dutta, and A. Mazumdar, "Unifying inflation and dark matter with neutrino masses," Phys. Rev. Lett. **99**, 261301 (2007), arXiv:0708.3983 [hep-ph] .

[27] K. Kohri, A. Mazumdar, and N. Sahu, "Inflation, baryogenesis and gravitino dark matter at ultra low reheat temperatures," Phys. Rev. **D80**, 103504 (2009), arXiv:0905.1625 [hep-ph] .

[28] M. Shaposhnikov and I. Tkachev, "The nuMSM, inflation, and dark matter," Phys. Lett. **B639**, 414–417 (2006), arXiv:hep-ph/0604236 [hep-ph] .

[29] S. M. Boucenna, S. Morisi, Q. Shafi, and J. W. F. Valle, "Inflation and majoron dark matter in the seesaw mechanism," Phys. Rev. **D90**, 055023 (2014), arXiv:1404.3198 [hep-ph] .

[30] A. Salvio, "A Simple Motivated Completion of the Standard Model below the Planck Scale: Axions and Right-Handed Neutrinos," Phys. Lett. **B743**, 428–434 (2015), arXiv:1501.03781 [hep-ph] .

[31] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, "Unifying inflation with the axion, dark matter, baryogenesis and the seesaw mechanism," Phys. Rev. Lett. **118**, 071802 (2017), arXiv:1608.05414 [hep-ph] .

[32] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, "Standard Model—axion—seesaw—Higgs portal inflation. Five problems of particle physics and cosmology solved in one stroke," JCAP **1708**, 001 (2017), arXiv:1610.01639 [hep-ph] .

[33] S. Choubey and A. Kumar, "Inflation and Dark Matter in the Inert Doublet Model," JHEP **11**, 080 (2017), arXiv:1707.06587 [hep-ph] .

[34] N. G. Deshpande and E. Ma, "Pattern of Symmetry Breaking with Two Higgs Doublets," Phys. Rev. **D18**, 2574 (1978).

[35] R. Barbieri and A. Strumia, "The 'LEP paradox'," in *4th Rencontres du Viet-*

nam: Physics at Extreme Energies (Particle Physics and Astrophysics) Hanoi, Vietnam, July 19-25, 2000 (2000) arXiv:hep-ph/0007265 [hep-ph] .

[36] R. Barbieri, L. J. Hall, and V. S. Rychkov, “Improved naturalness with a heavy Higgs: An Alternative road to LHC physics,” Phys. Rev. **D74**, 015007 (2006), arXiv:hep-ph/0603188 [hep-ph] .

[37] J. A. Casas, J. R. Espinosa, and I. Hidalgo, “Expectations for LHC from naturalness: modified versus SM Higgs sector,” Nucl. Phys. **B777**, 226–252 (2007), arXiv:hep-ph/0607279 [hep-ph] .

[38] L. Lopez Honorez, E. Nezri, J. F. Oliver, and M. H. G. Tytgat, “The Inert Doublet Model: An Archetype for Dark Matter,” JCAP **0702**, 028 (2007), arXiv:hep-ph/0612275 [hep-ph] .

[39] M. Gustafsson, E. Lundstrom, L. Bergstrom, and J. Edsjo, “Significant Gamma Lines from Inert Higgs Dark Matter,” Phys. Rev. Lett. **99**, 041301 (2007), arXiv:astro-ph/0703512 [ASTRO-PH] .

[40] E. Lundstrom, M. Gustafsson, and J. Edsjo, “The Inert Doublet Model and LEP II Limits,” Phys. Rev. **D79**, 035013 (2009), arXiv:0810.3924 [hep-ph] .

[41] T. Hambye, F. S. Ling, L. Lopez Honorez, and J. Rocher, “Scalar Multiplet Dark Matter,” JHEP **07**, 090 (2009), [Erratum: JHEP05,066(2010)], arXiv:0903.4010 [hep-ph] .

[42] E. M. Dolle and S. Su, “The Inert Dark Matter,” Phys. Rev. **D80**, 055012 (2009), arXiv:0906.1609 [hep-ph] .

[43] L. Lopez Honorez and C. E. Yaguna, "The inert doublet model of dark matter revisited," JHEP **09**, 046 (2010), arXiv:1003.3125 [hep-ph] .

[44] L. Lopez Honorez and C. E. Yaguna, "A new viable region of the inert doublet model," JCAP **1101**, 002 (2011), arXiv:1011.1411 [hep-ph] .

[45] N. Birrell and P. Davies, *Quantum fields in curved space* (Cambridge University Press, 1984).

[46] D. I. Kaiser, "Conformal Transformations with Multiple Scalar Fields," Phys. Rev. **D81**, 084044 (2010), arXiv:1003.1159 [gr-qc] .

[47] A. A. Starobinsky, "Spectrum of relict gravitational radiation and the early state of the universe," JETP Lett. **30**, 682–685 (1979), [Pisma Zh. Eksp. Teor. Fiz.30,719(1979)].

[48] X. Calmet and I. Kuntz, "Higgs Starobinsky Inflation," Eur. Phys. J. **C76**, 289 (2016), arXiv:1605.02236 [hep-th] .

[49] B. Garbrecht and T. Prokopec, "Baryogenesis from the amplification of vacuum fluctuations during inflation," Phys. Rev. **D78**, 123501 (2008), arXiv:0706.2594 [astro-ph] .

[50] A. D. Dolgov and D. P. Kirilova, "ON PARTICLE CREATION BY A TIME DEPENDENT SCALAR FIELD," Sov. J. Nucl. Phys. **51**, 172–177 (1990), [Yad. Fiz.51,273(1990)].

[51] J. H. Traschen and R. H. Brandenberger, "Particle Production During Out-of-equilibrium Phase Transitions," Phys. Rev. **D42**, 2491–2504 (1990).

[52] L. Kofman, A. D. Linde, and A. A. Starobinsky, "Reheating after inflation," Phys. Rev. Lett. **73**, 3195–3198 (1994), arXiv:hep-th/9405187 [hep-th] .

[53] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, "Universe reheating after inflation," Phys. Rev. **D51**, 5438–5455 (1995), arXiv:hep-ph/9407247 [hep-ph] .

[54] F. Bezrukov, D. Gorbunov, and M. Shaposhnikov, "On initial conditions for the Hot Big Bang," JCAP **0906**, 029 (2009), arXiv:0812.3622 [hep-ph] .

[55] N. Chakrabarty, D. K. Ghosh, B. Mukhopadhyaya, and I. Saha, "Dark matter, neutrino masses and high scale validity of an inert Higgs doublet model," Phys. Rev. **D92**, 015002 (2015), arXiv:1501.03700 [hep-ph] .

[56] J. Garcia-Bellido, D. G. Figueroa, and J. Rubio, "Preheating in the Standard Model with the Higgs-Inflaton coupled to gravity," Phys. Rev. **D79**, 063531 (2009), arXiv:0812.4624 [hep-ph] .

[57] J. Repond and J. Rubio, "Combined Preheating on the lattice with applications to Higgs inflation," JCAP **1607**, 043 (2016), arXiv:1604.08238 [astro-ph.CO] .

[58] E. Ma, "Verifiable radiative seesaw mechanism of neutrino mass and dark matter," Phys. Rev. **D73**, 077301 (2006), arXiv:hep-ph/0601225 [hep-ph] .

[59] A. Dasgupta and D. Borah, "Scalar Dark Matter with Type II Seesaw," Nucl. Phys. **B889**, 637–649 (2014), arXiv:1404.5261 [hep-ph] .

[60] M. Cirelli, N. Fornengo, and A. Strumia, "Minimal dark matter," Nucl. Phys. **B753**, 178–194 (2006), arXiv:hep-ph/0512090 [hep-ph] .

[61] M. Gustafsson, S. Rydbeck, L. Lopez-Honorez, and E. Lundstrom, "Status of the Inert Doublet Model and the Role of multileptons at the LHC," Phys. Rev. **D86**, 075019 (2012), arXiv:1206.6316 [hep-ph] .

[62] A. Goudelis, B. Herrmann, and O. Stal, "Dark matter in the Inert Doublet Model after the discovery of a Higgs-like boson at the LHC," JHEP **09**, 106 (2013), arXiv:1303.3010 [hep-ph] .

[63] A. Arhrib, Y.-L. S. Tsai, Q. Yuan, and T.-C. Yuan, "An Updated Analysis of Inert Higgs Doublet Model in light of the Recent Results from LUX, PLANCK, AMS-02 and LHC," JCAP **1406**, 030 (2014), arXiv:1310.0358 [hep-ph] .

[64] M. A. Díaz, B. Koch, and S. Urrutia-Quiroga, "Constraints to Dark Matter from Inert Higgs Doublet Model," Adv. High Energy Phys. **2016**, 8278375 (2016), arXiv:1511.04429 [hep-ph] .

[65] P. Gondolo and G. Gelmini, "Cosmic abundances of stable particles: Improved analysis," Nucl. Phys. **B360**, 145–179 (1991).

[66] E. W. Kolb and M. S. Turner, "The Early Universe," Front. Phys. **69**, 1–547 (1990).

[67] G. Jungman, M. Kamionkowski, and K. Griest, "Supersymmetric dark matter," Phys. Rept. **267**, 195–373 (1996), arXiv:hep-ph/9506380 [hep-ph] .

[68] E. Aprile *et al.* (XENON), "Physics reach of the XENON1T dark matter experiment," JCAP **1604**, 027 (2016), arXiv:1512.07501 [physics.ins-det] .

[69] D. S. Akerib *et al.* (LZ), "LUX-ZEPLIN (LZ) Conceptual Design Report," (2015), arXiv:1509.02910 [physics.ins-det] .

[70] J. Aalbers *et al.* (DARWIN), "DARWIN: towards the ultimate dark matter detector," JCAP **1611**, 017 (2016), arXiv:1606.07001 [astro-ph.IM] .

[71] J. Liu, X. Chen, and X. Ji, "Current status of direct dark matter detection experiments," Nature Phys. **13**, 212–216 (2017), arXiv:1709.00688 [astro-ph.CO] .

[72] D. Borah, P. S. B. Dev, and A. Kumar, "TeV scale leptogenesis, inflaton dark matter and neutrino mass in a scotogenic model," Phys. Rev. **D99**, 055012 (2019), arXiv:1810.03645 [hep-ph] .

[73] S. Weinberg, "Cosmological Production of Baryons," Phys. Rev. Lett. **42**, 850–853 (1979).

[74] E. W. Kolb and S. Wolfram, "Baryon Number Generation in the Early Universe," Nucl. Phys. **B172**, 224 (1980), [Erratum: Nucl. Phys.B195,542(1982)].

[75] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, "On the Anomalous

Electroweak Baryon Number Nonconservation in the Early Universe," Phys. Lett. **155B**, 36 (1985).

[76] C. S. Fong, E. Nardi, and A. Riotto, "Leptogenesis in the Universe," Adv. High Energy Phys. **2012**, 158303 (2012), arXiv:1301.3062 [hep-ph] .

[77] P. Minkowski, "$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?" Phys. Lett. **B67**, 421–428 (1977).

[78] R. N. Mohapatra and G. Senjanovic, "Neutrino Mass and Spontaneous Parity Violation," Phys. Rev. Lett. **44**, 912 (1980).

[79] T. Yanagida, "HORIZONTAL SYMMETRY AND MASSES OF NEUTRINOS," *Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe: Tsukuba, Japan, February 13-14, 1979*, Conf. Proc. **C7902131**, 95–99 (1979).

[80] M. Gell-Mann, P. Ramond, and R. Slansky, "Complex Spinors and Unified Theories," *Supergravity Workshop Stony Brook, New York, September 27-28, 1979*, Conf. Proc. **C790927**, 315–321 (1979), arXiv:1306.4669 [hep-th] .

[81] S. L. Glashow, "The Future of Elementary Particle Physics," *Cargese Summer Institute: Quarks and Leptons Cargese, France, July 9-29, 1979*, NATO Sci. Ser. B **61**, 687 (1980).

[82] J. Schechter and J. W. F. Valle, "Neutrino Masses in SU(2) x U(1) Theories," Phys. Rev. **D22**, 2227 (1980).

[83] A. Das, T. Nomura, H. Okada, and S. Roy, "Generation of a radiative neutrino mass in the linear seesaw framework, charged lepton flavor violation, and dark matter," Phys. Rev. **D96**, 075001 (2017), arXiv:1704.02078 [hep-ph] .

[84] A. Merle and M. Platscher, "Running of radiative neutrino masses: the scotogenic model — revisited," JHEP **11**, 148 (2015), arXiv:1507.06314 [hep-ph] .

[85] G. 't Hooft, "Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking," *Recent Developments in Gauge Theories. Proceedings, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979*, NATO Sci. Ser. B **59**, 135–157 (1980).

[86] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, "Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity," JHEP **01**, 087 (2017), arXiv:1611.01514 [hep-ph] .

[87] J. A. Casas and A. Ibarra, "Oscillating neutrinos and $\mu \to e, \gamma$," Nucl. Phys. **B618**, 171–204 (2001), arXiv:hep-ph/0103065 [hep-ph] .

[88] T. Hugle, M. Platscher, and K. Schmitz, "Low-Scale Leptogenesis in the Scotogenic Neutrino Mass Model," Phys. Rev. **D98**, 023020 (2018), arXiv:1804.09660 [hep-ph] .

[89] E. Ma, "Common origin of neutrino mass, dark matter, and baryogenesis," Mod. Phys. Lett. **A21**, 1777–1782 (2006), arXiv:hep-ph/0605180 [hep-ph] .

[90] S. Kashiwase and D. Suematsu, "Baryon number asymmetry and dark matter in the neutrino mass model with an inert doublet," Phys. Rev. **D86**, 053001 (2012), arXiv:1207.2594 [hep-ph] .

[91] S. Kashiwase and D. Suematsu, "Leptogenesis and dark matter detection in a TeV scale neutrino mass model with inverted mass hierarchy," Eur. Phys. J. **C73**, 2484 (2013), arXiv:1301.2087 [hep-ph] .

[92] J. Racker, "Mass bounds for baryogenesis from particle decays and the inert doublet model," JCAP **1403**, 025 (2014), arXiv:1308.1840 [hep-ph] .

[93] J. D. Clarke, R. Foot, and R. R. Volkas, "Natural leptogenesis and neutrino masses with two Higgs doublets," Phys. Rev. **D92**, 033006 (2015), arXiv:1505.05744 [hep-ph] .

[94] W. Buchmuller, P. Di Bari, and M. Plumacher, "Leptogenesis for pedestrians," Annals Phys. **315**, 305–351 (2005), arXiv:hep-ph/0401240 [hep-ph] .

[95] S. Davidson and A. Ibarra, "A Lower bound on the right-handed neutrino mass from leptogenesis," Phys. Lett. **B535**, 25–32 (2002), arXiv:hep-ph/0202239 [hep-ph] .

[96] W. Buchmuller, P. Di Bari, and M. Plumacher, "Cosmic microwave background, matter - antimatter asymmetry and neutrino masses," Nucl. Phys. **B643**, 367–390 (2002), [Erratum: Nucl. Phys.B793,362(2008)], arXiv:hep-ph/0205349 [hep-ph] .

[97] K. Moffat, S. Pascoli, S. T. Petcov, H. Schulz, and J. Turner, "Three-flavored nonresonant leptogenesis at intermediate scales," Phys. Rev. **D98**, 015036 (2018), arXiv:1804.05066 [hep-ph] .

[98] A. Pilaftsis and T. E. J. Underwood, "Resonant leptogenesis," Nucl. Phys. **B692**, 303–345 (2004), arXiv:hep-ph/0309342 [hep-ph] .

[99] P. S. B. Dev, M. Garny, J. Klaric, P. Millington, and D. Teresi, "Resonant enhancement in leptogenesis," Int. J. Mod. Phys. **A33**, 1842003 (2018), arXiv:1711.02863 [hep-ph] .

[100] A. Kumar, "Inflation and Reheating with a Fermionic Field," (2018), arXiv:1811.12237 [gr-qc] .

[101] B. Saha and G. N. Shikin, "Interacting spinor and scalar fields in Bianchi type I universe filled with perfect fluid: Exact selfconsistent solutions," Gen. Rel. Grav. **29**, 1099–1113 (1997), arXiv:gr-qc/9609056 [gr-qc] .

[102] B. Saha, "Dirac spinor in Bianchi type I universe with time dependent gravitational and cosmological constants," Mod. Phys. Lett. **A16**, 1287–1296 (2001), arXiv:gr-qc/0009002 [gr-qc] .

[103] B. Saha, "Sinor field in Bianchi type I universe: Regular solutions," Phys. Rev. **D64**, 123501 (2001), arXiv:gr-qc/0107013 [gr-qc] .

[104] B. Saha and T. Boyadjiev, "Bianchi type I cosmolgy with scalar and spinor fields," Phys. Rev. **D69**, 124010 (2004), arXiv:gr-qc/0311045 [gr-qc] .

[105] B. Saha, "Nonlinear spinor field in Bianchi type-I cosmology: Inflation, isotropization, and late time acceleration," Phys. Rev. **D74**, 124030 (2006).

[106] M. O. Ribas, F. P. Devecchi, and G. M. Kremer, "Cosmological model with non-minimally coupled fermionic field," EPL **81**, 19001 (2008), arXiv:0710.5155 [gr-qc] .

[107] M. O. Ribas, F. P. Devecchi, and G. M. Kremer, "Fermions as sources of accelerated regimes in cosmology," Phys. Rev. **D72**, 123502 (2005), arXiv:gr-qc/0511099 [gr-qc] .

[108] L. P. Chimento, F. P. Devecchi, M. Forte, G. M. Kremer, M. O. Ribas, and L. L. Samojeden, "Fermionic cosmologies," *Proceedings, 5th International Workshop: Space-Time-Matter - current issues in quantum mechanics and beyond (DICE2010): Castello Pasquini, Castiglioncello, Italy, September 13-17, 2010*, J. Phys. Conf. Ser. **306**, 012052 (2011).

[109] R. C. de Souza and G. M. Kremer, "Noether symmetry for non-minimally coupled fermion fields," Class. Quant. Grav. **25**, 225006 (2008), arXiv:0807.1965 [gr-qc] .

[110] G. Grams, R. C. de Souza, and G. M. Kremer, "Fermion field as inflaton, dark energy and dark matter," Class. Quant. Grav. **31**, 185008 (2014), arXiv:1407.5481 [gr-qc] .

[111] C. Armendariz-Picon and P. B. Greene, "Spinors, inflation, and nonsingu-

lar cyclic cosmologies," Gen. Rel. Grav. **35**, 1637–1658 (2003), arXiv:hep-th/0301129 [hep-th] .

[112] S. Mollerach, "Isocurvature Baryon Perturbations and Inflation," Phys. Rev. **D42**, 313–325 (1990).

[113] A. D. Linde and V. F. Mukhanov, "Nongaussian isocurvature perturbations from inflation," Phys. Rev. **D56**, R535–R539 (1997), arXiv:astro-ph/9610219 [astro-ph] .

[114] T. Moroi and T. Takahashi, "Effects of cosmological moduli fields on cosmic microwave background," Phys. Lett. **B522**, 215–221 (2001), [Erratum: Phys. Lett.B539,303(2002)], arXiv:hep-ph/0110096 [hep-ph] .

[115] D. H. Lyth and D. Wands, "Generating the curvature perturbation without an inflaton," Phys. Lett. **B524**, 5–14 (2002), arXiv:hep-ph/0110002 [hep-ph] .

[116] W. A. Bardeen, C. T. Hill, and M. Lindner, "Minimal Dynamical Symmetry Breaking of the Standard Model," Phys. Rev. **D41**, 1647 (1990).

[117] H. Terazawa, "t quark mass predicted from a sum rule for lepton and quark masses," Phys. Rev. **D22**, 2921–2921 (1980), [Erratum: Phys. Rev.D41,3541(1990)].

[118] H. Terazawa, K. Akama, and Y. Chikashige, "Unified Model of the Nambu-Jona-Lasinio Type for All Elementary Particle Forces," Phys. Rev. **D15**, 480 (1977).

[119] Y. Nambu and G. Jona-Lasinio, "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. 1." Phys. Rev. **122**, 345–358 (1961), [,127(1961)].

[120] Y. Nambu and G. Jona-Lasinio, "DYNAMICAL MODEL OF ELEMENTARY PARTICLES BASED ON AN ANALOGY WITH SUPERCONDUCTIVITY. II," Phys. Rev. **124**, 246–254 (1961), [,141(1961)].

[121] Y. Nambu, "BCS mechanism, quasi supersymmetry, and fermion masses," in *Broken symmetry: Selected papers of Y. Nambu* (1988) pp. 1–10, [,406(1988)].

[122] G. Barenboim, "Inflation might be caused by the right: Handed neutrino," JHEP **03**, 102 (2009), arXiv:0811.2998 [hep-ph] .

[123] K. Freese, J. A. Frieman, and A. V. Olinto, "Natural inflation with pseudo - Nambu-Goldstone bosons," Phys. Rev. Lett. **65**, 3233–3236 (1990).

[124] G. Barenboim and J. Rasero, "Baryogenesis from a right-handed neutrino condensate," JHEP **03**, 097 (2011), arXiv:1009.3024 [hep-ph] .