

# PHENOMENOLOGY OF B MESON AND CP VIOLATION

*By*

CHANDRADEW SHARMA

The Institute of Mathematical Sciences, Chennai

*A thesis submitted to the  
Board of Studies in Physical Sciences*

*In partial fulfillment of requirements*

*For the Degree of*

DOCTOR OF PHILOSOPHY

*of*

HOMI BHABHA NATIONAL INSTITUTE



October, 2008



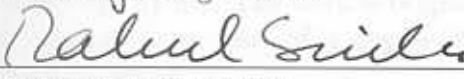
# Homi Bhabha National Institute

## Recommendations of the Viva Voce Board

### DECLARATION

As members of the Viva Voce Board, we recommend that the dissertation prepared by Chandradew Sharma entitled "Phenomenology of B meson and CP violation" may be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

  
\_\_\_\_\_  
Chairman - R. Jaganathan Date : 9/3/2009

  
\_\_\_\_\_  
Convenor - Rahul Sinha Date : 9-3-2009

  
\_\_\_\_\_  
Member 1 - Rahul Basu Date : 9/3/2009

  
\_\_\_\_\_  
Member 2 - M. V. N. Murthy Date : 9/3/2009

  
\_\_\_\_\_  
Member 3 - S. Uma Sankar Date : 9.3.2009

  
\_\_\_\_\_  
Dean - T. R. Govindarajan Date : 9.3.2009

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to HBNI.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it may be accepted as fulfilling the dissertation requirement.

  
\_\_\_\_\_  
Guide - Rahul Sinha Date : 9-3-2009

  
\_\_\_\_\_  
Co-Guide - Rahul Basu Date : 9/3/2009

## DECLARATION

I, hereby declare that the investigation presented in this thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part of a degree/diploma at this or any other Institution/University.

*Chandradew Sharma*  
Chandradew Sharma.

**In Fond Memory of My Grandmother**

# Acknowledgement

I joined the Institute of Mathematical Sciences in August 2002 and I thank all my teachers and colleagues who were involved in the year and a half long academic programme that followed.

It is difficult to overstate my gratitude to my Ph.D. supervisor, Prof. Rahul Sinha. With his enthusiasm, his inspiration, and his great efforts to explain things clearly and simply, he helped to make Physics fun for me. Throughout my research period, he provided encouragement, sound advice, good teaching, good company, and lots of good ideas. I would have been lost without him. I sincerely thank him.

I would also like to thank and acknowledge my thesis co-adviser, Prof. Rahul Basu, for having taught me, with patience, Quantum Field Theory and the art of numerical skills. I want to thank and acknowledge Prof. Nita Sinha for her help, encouragement and collaboration. I also want to thank all my collaborators: Prof. David London, Prof. C. S. Kim, Prof. Sechul Oh, Prof. P. N. Pandita, Maxime Imbeault and Yoon, Yeo Woong for their scintillating and lively discussions.

I wish to thank Prof. M. V. N. Murthy for suggesting me through the writing of the thesis. I also thank to Prof. Indumathi for having taught me particle physics. I would also like to thank Prof. Rajasekaran and Prof. H.S. Mani for many enjoyable interaction. I would be always indebted to Prof. R. Jagannathan and Prof. T.R.G. for his encouraging role in the process of submission of the thesis to HBNI.

I would like to address special thank to my mentor Mr. Qamar Takhyan for giving me the flavour of science, mathematics and the philosophy of life.

I must thank all my teachers: Late Mr. Dashrath Prasad, Late Mr. Chitranjan Majhi, Mr. Aswani Verma, Late Mr. Gulshan Vison, Mr. Manik Sen, Mr. Vinod Mahato, Mr. Sahebram Mahato ....

I thank all my fellow colleagues which are too many to mention always stood by my side asking over and over again. I would like to extend my special thanks to Rahul Muthu, Kinjal Banerjee, Ved Prakash Gupta, Alok Kumar and Sheerazuddin for their help in preparation of the thesis.

I am grateful to the library and the computer staffs for their help. I wish to acknowledge all the non-academic staffs for their work. I must thank the institute and its academic and administrative members for providing excellent academic atmosphere and facilities for research work.

I must thank my wife Manisha for putting up with my late hours, my spoiled weekends, my bad temper, but above all for putting up with me and surviving the ordeal.

Finally, I thank my grand father, uncles and parents for their encouragement and support which was necessary for all this to have happened.

# Abstract

Recent experimental results for the  $B \rightarrow K\pi$  modes, show deviations from Standard Model (SM) expectations based on estimates of hadronic parameters; the discrepancies are commonly referred to as the “ $B \rightarrow K\pi$  puzzle.” The discrepancies are now reduced, nevertheless it is important to understand the implication of NP on these modes. The  $B \rightarrow K\pi$  modes have certain inherent limitations. These discrepancies may lead to a signal of New Physics (NP), physics beyond the SM. We show that NP which affects B decays with any topological amplitude can be absorbed by redefinitions of the SM amplitudes. Hence, there are no clean signals of NP in such decays unless there is an accurate theoretical estimate of parameters or a justifiable approximation can be made.

The four  $B \rightarrow K\pi$  decay modes can be expressed in terms of 6 topological parameters;  $T, C, P, P_{EW}, P_{EW}^C, P_{uc}$  and  $\gamma$ . Experiment can yield at most 9 observables: four each of the branching ratios and direct CP asymmetries and one time-dependent CP asymmetry. Clearly, the 9 observables are insufficient to determine all the 12 theoretical parameters needed to describe these decay modes model independently.

We study  $B \rightarrow K^*\rho$  modes that are analogues of the much studied  $B \rightarrow K\pi$  modes with  $B$  decaying to two vector mesons instead of pseudoscalar mesons, using topological amplitudes in the quark diagram approach. We show how  $B \rightarrow K^*\rho$  modes can be used to obtain many more observables than those for  $B \rightarrow K\pi$  modes, even though the quark level subprocesses of both modes are exactly the same since there are three helicities for each of the modes of the amplitudes for  $B \rightarrow K^*\rho$ . Hence, the number of amplitudes for the  $B \rightarrow K^*\rho$  modes is three times that for the  $B \rightarrow K\pi$  modes, *i.e.* there are 36 theoretical parameters. The four  $B \rightarrow K^*\rho$  decay modes can experimentally yield at most 35 observables. Therefore, all the theoretical parameters can be determined in terms of the observables and  $\gamma$  without any model-dependent assumption. If we measure  $\gamma$  from somewhere else, then we have full information about topological amplitudes in terms of experimental observables. Hence we can test the SM and probe NP effects as well.

We demonstrate how  $B \rightarrow K^*\rho$  can also be used to verify if there exist any relations between theoretical parameters, such as the hierarchy relations between the topological amplitudes and possible relations between the strong phases. Conversely, if there exist reliable theoretical estimates of amplitudes and strong phases, the presence of New physics could be probed. We show that if the tree and color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins, it is not only possible



to verify the validity of such relations but also to have a clean measurement of New Physics parameters.

In conclusion, a few aspects of  $B$  physics were studied and we emphasize on two main points: “ $B \rightarrow K\pi$  puzzle” and “the signal of new physics”.

## List Of Publications

### Total Publications:

1. Radiative neutralino production in low energy supersymmetric models, Rahul Basu, P. N. Pandita, **Chandradew Sharma**, Phys. Rev. **D 77**, 115009, 2008, arXiv:0711.2121[hep-ph].
2. Generalized analysis on  $B \rightarrow K^* \rho$  within and beyond the Standard Model: Can it help understand the  $B \rightarrow K \pi$  puzzle? C. S. Kim, Sechul Oh, **Chandradew Sharma**, Rahul Sinha, Yeo Woong Yoon, Phys. Rev. **D76**:074019,2007, arXiv:0706.1150[hep-ph].
3. Patterns of new physics in B decays, Maxime Imbeault, David London, **Chandradew Sharma**, Nita Sinha, Rahul Sinha, Phys. Lett. **B653**: 254-258,2007, hep-ph/0608169.
4. Angular analysis of B decaying into  $J/\psi$  tensor,  $J/\psi$  vector and  $J/\psi$  scalar modes, **Chandradew Sharma**, Rahul Sinha, Phys. Rev. **D73**:014016,2006, hep-ph/0504178.

### This thesis is based on two publications :

1. Patterns of new physics in B decays, Maxime Imbeault, David London, Chandradew Sharma, Nita Sinha, Rahul Sinha, Phys. Lett. **B653**:254-258,2007, hep-ph/0608169.
2. Generalized analysis on  $B \rightarrow K^* \rho$  within and beyond the Standard Model: Can it help understand the  $B \rightarrow K \pi$  puzzle? C. S. Kim, Sechul Oh, Chandradew Sharma, Rahul Sinha, Yeo Woong Yoon, Phys. Rev. **D76**:074019,2007, arXiv:0706.1150[hep-ph].



# CONTENTS

<b>Abstract</b>	<b>ix</b>
<b>List of Publications</b>	<b>x</b>
<b>List of Figures</b>	<b>xv</b>
<b>List of Tables</b>	<b>xvii</b>
<b>1 Introduction to B Physics and CP Violation</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Discrete Symmetry . . . . .	3
1.2.1 Parity . . . . .	3
1.2.2 Charge-Conjugation . . . . .	5
1.2.3 Time-Reversal . . . . .	5
1.2.4 $CP$ . . . . .	6
1.2.5 $CPT$ . . . . .	7
1.3 Basics of the $CP$ Phenomenology in $B$ meson Decays . . . . .	8
1.3.1 Charged and neutral $B$ meson decays . . . . .	8
1.3.2 Neutral $B$ meson mixing . . . . .	9
1.3.3 CP violation in B system . . . . .	11
1.4 The Standard Model . . . . .	14
1.4.1 Wolfenstein parametrization . . . . .	22
1.4.2 Limitation of Standard Model . . . . .	24
1.4.3 Summary . . . . .	25
<b>2 Study of B meson in Standard Model</b>	<b>27</b>
2.1 Effective Hamiltonian . . . . .	28
2.2 Leptonic decays . . . . .	31
2.3 Semileptonic decays . . . . .	32
2.4 Radiative decays . . . . .	34

2.5	Nonleptonic decays . . . . .	34
2.6	Extraction of CKM angles . . . . .	36
2.6.1	Extraction of angle $\beta$ . . . . .	37
2.6.2	Extraction of angle $\alpha$ . . . . .	40
2.6.3	Extraction of angle $\gamma$ . . . . .	42
2.7	Evidence of Direct CP Asymmetry . . . . .	43
2.8	Hint of New Physics . . . . .	43
2.9	Summary . . . . .	44
<b>3</b>	<b>Study of New Physics</b> . . . . .	<b>45</b>
3.1	Introduction . . . . .	45
3.2	Reparametrization Invariant . . . . .	46
3.3	Pattern of New Physics in $B$ Decays . . . . .	48
3.4	Summary . . . . .	59
<b>4</b>	<b><math>B \rightarrow K\pi</math> Puzzle</b> . . . . .	<b>61</b>
4.1	Introduction . . . . .	61
4.2	Solution of $B \rightarrow K\pi$ Puzzle through $B \rightarrow K^*\rho$ . . . . .	63
4.2.1	Formalism for $B \rightarrow K^*\rho$ decays . . . . .	64
4.2.2	Extracting contributions of various topologies . . . . .	67
4.2.3	Testing the hierarchy of topological amplitudes and possible relations between their strong phases . . . . .	72
4.2.4	Isolating signals of New Physics in $B \rightarrow K^*\rho$ modes . . . . .	75
4.2.5	Summary . . . . .	77
<b>5</b>	<b>Summary</b> . . . . .	<b>79</b>
<b>A</b>	<b>Determination of <math>A_\lambda^f</math> and <math>\overline{A}_\lambda^f</math> with observables</b> . . . . .	<b>83</b>
A.1	Determination of the magnitude $A_\lambda^f$ and $\overline{A}_\lambda^f$ . . . . .	83
A.2	Determination of the phases of $A_\lambda^f$ and $\overline{A}_\lambda^f$ . . . . .	83
<b>B</b>	<b>All Possible Relations</b> . . . . .	<b>87</b>
B.1	Case I: $y_\lambda^{ij} \neq 1$ . . . . .	87
B.1.1	$\tilde{P}_\lambda > \tilde{T}_\lambda > \tilde{P}_\lambda^{EW} > \tilde{C}_\lambda > \tilde{P}_{C\lambda}^{EW} > \tilde{A}_\lambda$ . . . . .	87
B.1.2	$\delta_\lambda^C = \delta_\lambda^P$ . . . . .	87
B.1.3	$\delta_\lambda^{CEW} = \delta_\lambda^P$ . . . . .	87
B.1.4	$\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	87
B.1.5	$\delta_\lambda^{CEW} = \delta_\lambda^C$ . . . . .	88
B.1.6	$\delta_\lambda^A = \delta_\lambda^T$ . . . . .	88
B.1.7	$\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	88

B.1.8	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	88
B.1.9	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	88
B.1.10	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	88
B.1.11	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	88
B.1.12	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	89
B.1.13	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	89
B.1.14	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	89
B.1.15	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	89
B.1.16	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	89
B.1.17	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ . . . . .	90
B.1.18	$\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ . . . . .	90
B.1.19	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	90
B.1.20	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	90
B.1.21	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	91
B.1.22	$\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ and $\delta_\lambda^C = \delta_\lambda^P$ . . . . .	91
B.1.23	$\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ and $\delta_\lambda^{CEW} = \delta_\lambda^P$ . . . . .	91
B.1.24	$\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ and $\delta_\lambda^{CEW} = \delta_\lambda^C$ . . . . .	91
B.1.25	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ and $\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ . . . . .	92
B.1.26	Reparametrization of observables . . . . .	92
B.2	Case II: $y_\lambda^{ij} \approx 1$ . . . . .	93
B.2.1	$\tilde{P}_\lambda > \tilde{T}_\lambda > \tilde{P}_\lambda^{EW} > \tilde{C}_\lambda > \tilde{P}_{C\lambda}^{EW} > \tilde{A}_\lambda$ . . . . .	93
B.2.2	$\delta_\lambda^C = \delta_\lambda^P$ . . . . .	93
B.2.3	$\delta_\lambda^{CEW} = \delta_\lambda^P$ . . . . .	93
B.2.4	$\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	93
B.2.5	$\delta_\lambda^{CEW} = \delta_\lambda^C$ . . . . .	94
B.2.6	$\delta_\lambda^A = \delta_\lambda^T$ . . . . .	94
B.2.7	$\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	94
B.2.8	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	94
B.2.9	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	94
B.2.10	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	94
B.2.11	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	95
B.2.12	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	95
B.2.13	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	95
B.2.14	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	95
B.2.15	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	95
B.2.16	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^A = \delta_\lambda^{EW}$ . . . . .	96
B.2.17	$\delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$ . . . . .	96
B.2.18	$\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$ . . . . .	96
B.2.19	$\delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^{EW} = \delta_\lambda^T$ . . . . .	96
B.2.20	$\delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T$ . . . . .	97

B.2.21	$\delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^{EW}$	97
B.2.22	$\delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$	97
B.2.23	$\delta_\lambda^{CEW} = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$	98
B.2.24	$\delta_\lambda^{CEW} = \delta_\lambda^C$ and $\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$	98
B.2.25	$\delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P$ and $\delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$	98
B.2.26	Reparametrisation of observables for case II	99

## Bibliography

## LIST OF FIGURES

1.1	Unitarity triangle . . . . .	22
1.2	The unitarity triangle of the CKM matrix. . . . .	24
2.1	Tree diagrams ( $q_1, q_2 \in \{u, c\}$ ). . . . .	28
2.2	QCD penguin diagrams ( $q_1 = q_2 \in \{u, d, c, s\}$ ). . . . .	28
2.3	Electroweak penguin diagrams ( $q_1 = q_2 \in \{u, d, c, s\}$ ). . . . .	29
2.4	Box diagrams contributing to $B_q^0 - \bar{B}_q^0$ mixing ( $q \in \{d, s\}$ ). . . . .	35
2.5	The current situation in the $\bar{\rho} - \bar{\eta}$ plane. . . . .	36
2.6	Feynman diagrams contributing to $B_d^0 \rightarrow J/\psi K_S$ . The dashed lines in the penguin topology represent a colour-singlet exchange. . . . .	37
2.7	Complex triangle of (a) Eq. (2.56) and (b) Eq. (2.57). . . . .	42
3.1	Various topological equivalent diagrams . . . . .	56





## LIST OF TABLES

1.1	Parity Operator of Free Fields. . . . .	4
1.2	Charge-Conjugation Operator of Free Fields. . . . .	6
1.3	Time reversal operator of Free Fields. . . . .	6
1.4	The SM field content. $(Q_c, Q_L)_{Q_{Y/2}}$ lists color, weak isospin and hypercharge assignments of a given field, respectively, and $Q_c = 1$ , $Q_L = 1$ or $Q_{Y/2} = 0$ represent a singlet under the respective group transformations. The $T_3$ isospin operator is $+1/2$ ( $-1/2$ ) when acting on the upper (lower) component of an isospin doublet (and zero otherwise). . . . .	17
2.1	Exclusive branching ratios in different theoretical models. All numbers are to be multiplied by $10^{-6}$ . . . . .	34
2.2	Observed branching ratios in $10^{-6}$ . . . . .	34



## CHAPTER 1

# INTRODUCTION TO B PHYSICS AND CP VIOLATION

### 1.1 INTRODUCTION

One of the most difficult problems in physics is to explain why the universe is made up mostly of matter, rather than consisting of equal parts of matter and antimatter. According to the most successful theory of the origin and evolution of the cosmos (the Big Bang), matter and anti-matter should have produced in equal amounts. It was demonstrated by Sakharov that to create an imbalance between matter and antimatter from an initial condition of balance, certain conditions must be satisfied - there must exist baryon number violation, an asymmetry of physical laws under charge conjugation and also an asymmetry of physical laws under the product of charge conjugation and parity. Further there must exist thermal non-equilibrium.

The aim of the study of elementary particle physics is to understand the nature of fundamental particles and their interactions, and to get the answers to some of the most fundamental questions about Nature, especially what it is made up of and what holds it together. Our current knowledge on elementary particle physics is gathered in the Standard Model (SM) [1], a result of mammoth experimental and theoretical effort spanning more than fifty years. It is extremely successful in explaining almost all gathered experimental data, yet there are strong indications that it is not the final answer to all the questions on the nature of elementary particles and their interactions. The SM is regarded as theoretically incomplete in part due to the gauge hierarchy or naturalness problem and the origin of CP violation. The quest of theoretical and experimental activity is to probe for physics beyond the SM.

It was assumed until mid fifties that parity is a good symmetry. However, violation of the parity symmetry predicted by Lee and Yang was observed soon in the weak decays of Cobalt-60 by Wu *et al.* [4]. Most physicists still assumed that the combined charge-parity (CP) symmetry was not violated. However, Cronin and Fitch found the evidence for CP violation in the decay process of neutral kaons.

In 1977, a team of physicists, led by Leon M. Lederman, working on the historic experiment E288 ( $p+p \rightarrow X(X \rightarrow \mu^+\mu^-)+ \text{anything}$ ) in the proton beam line of the Fermilab fixed target areas discovered a new particle [2]. This is acknowledged to be the best way to detect any new particle decaying into  $\mu^+\mu^-$ . The new particle “Upsilon” – the bound state of a  $b$  quark and a  $\bar{b}$  antiquark has a mass of about 9.5 GeV. The glorious era of  $B$  meson physics started with this famous experiment.

Charged and neutral  $B$  mesons are composed of a quark-antiquark pair; one quark is always a ”bottom” quark and the another partner is a light quark, either of “up”, “down”, or “strange” type. The heavy  $b$  quark decays weakly and the  $B$  meson thus has a long life time of 1.5 ps. This makes it feasible to study CP violation in  $B$  decays. It was realised that interesting signals of CP violation can occur in  $B$  meson decays [5]. It was also shown that CP violating phases can be measured with hadronic uncertainty (to a very good approximation) using  $B$  meson decays

In SM, the masses and mixing of quarks have a common origin. They arise from the Yukawa interactions with the Higgs doublets. The Yukawa coupling matrices (not Hermitian) are diagonalised by bi-unitary transformation consisting of two different unitary matrices. The combination of these two different unitary matrices is known as Cabibo-Kobayashi-Maskawa (CKM) matrix. There are  $\frac{1}{2}(n-1)(n-2)$  independent complex phases for any  $n \times n$  unitary matrix. Thus in the three-generation SM, there is only one single complex phase, which is responsible for CP violation. This in essence is the Kobayashi Maskawa [6] hypothesis for introducing CP violation in the SM. Like the neutral  $K$  meson, where CP violation is observed, the neutral  $B^0$  meson also oscillates to its antiparticle the  $\bar{B}^0$  meson. Hence, experiments with  $B$  meson were planed to learn more about CP violation. The main goal of these experiments is to measure CP violating phases and test the consistency of the CKM hypothesis in the SM. In 2001, the BaBar Experiment at SLAC [7] and the Belle Experiment at the KEK [8] in Japan, observed CP violation in decays of the  $B$  mesons. Although the degree of CP violation currently observed in experiment is consistent with the SM, it is not enough to account for the baryon-antibaryon asymmetry determined from astronomical observations. Thus, it is expected that mechanisms other

than the SM CP violation must be responsible for the dominance of matter over antimatter in today’s universe. The Kobayashi-Maskawa hypothesis has been verified to be the dominant mechanism of CP violation, However, CP violation beyond the SM may still exist and show signals in B decays. Such signals of New Physics (NP), physics beyond the SM, have attracted both theoretical and experimental attraction.

A promising class of NP signals are Flavour Changing Neutral Current (FCNC) processes, which are mediated by amplitudes involving virtual loops that might include contributions from unobserved particles and interactions. These processes include  $b \rightarrow s$  penguin amplitudes, occurring in decays such as  $B \rightarrow K\pi$  etc, which have been observed and studied for signatures of CP violation. Although several hints of deviations from SM predictions have been seen in these rare modes, no definitive inconsistencies have been established. The data on the charged and neutral  $B \rightarrow K\pi$  decays which are sizeably affected by electroweak penguin contributions, have moved close towards the SM predictions, which are almost unchanged, thereby reducing the “ $B \rightarrow K\pi$  puzzle.”

Unfortunately, we still cannot draw definite conclusions about the presence of NP in the  $B \rightarrow K\pi$  system (and other  $b \rightarrow s$  penguin decays, such as  $B^0 \rightarrow \phi K_S$ ). The main motivation of this thesis is to resolve the “ $B \rightarrow K\pi$  puzzle.” We investigate when NP signals can be distinguished from hadronic uncertainties, if at all possible.

## 1.2 DISCRETE SYMMETRY

A discrete symmetry is a symmetry that describes non-continuous changes in a system, *e.g.* an equilateral triangle possesses discrete rotational symmetry, as only rotations by multiples of  $60^\circ$  will preserve the triangle’s original appearance. We will mainly discuss three types of discrete symmetry, namely: Parity, Charge Conjugation and Time Reversal.

### 1.2.1 PARITY

To distinguish between left and right, we use the concept of parity. The law of conservation of parity means complete symmetry between the left and right hands. Parity symmetry ( $P$ ), consists in the invariance of physics under a discrete transformation that changes the sign of the space coordinates  $x, y, z$ , *i.e.*  $P(x, y, z) = (-x, -y, -z)$  or in other words  $P\vec{r} = -\vec{r}$ . This corresponds to the inversion of the three coordinate axes through the origin, a transformation that changes the handedness of the system

of axes. A right-handed system becomes left-handed under a parity transformation. So the only relevant point as far as parity transformation is concerned is whether the process is invariant under mirror reflection. If it is, then we call the interaction responsible for that process invariant under parity  $Pf(x, y, z) \equiv f(-x, -y, -z) = f(x, y, z)$ , otherwise it is violated  $Pf(x, y, z) \equiv f(-x, -y, -z) \neq f(x, y, z)$ . Applying two parity transformations in succession is equivalent to no transformation at all, *i.e.*  $P^2(x, y, z) = P(-x, -y, -z) = (x, y, z)$ . There are some physical quantities that change sign under parity transformation - velocity, momentum, electric dipole moment, helicity etc, and some not - angular momentum, spin, magnetic dipole moment, etc. We can divide the physical quantities on the basis of parity transformation

All vectors and pseudo-scalars change their sign under parity transformation, and all scalars and pseudo-vectors do not change their sign under this transformation. Till date, parity violation is confirmed only in weak interaction, and other interactions preserve the parity symmetry. Since Hamiltonian of Classical Mechanics and Quantum Electrodynamics (QED) are invariant under the parity transformation, we can define the parity operator for fermion and boson fields in Quantum Mechanics. Since parity operator is unitary and QED Hamiltonian ( $H$ ) is invariant under parity transformation, *i.e.*  $[P, H] = 0$ ; this implies  $PHP^\dagger = H$  because  $P^{-1} = P^\dagger$ . We define the parity operator of the fields as is shown in Table 1.1.

Field	Parity Operator	Transformed Field under parity
Scalar	$P \phi(t, \vec{r}) P^\dagger$	$\exp(i\alpha_p) \phi((t, -\vec{r}))$
Pseudoscalar	$P \phi(t, \vec{r}) P^\dagger$	$-\exp(i\alpha_p) \phi((t, -\vec{r}))$
Vector	$P A_\mu(t, \vec{r}) P^\dagger$	$A^\mu(t, -\vec{r})$
Pseudovector	$P A_\mu(t, \vec{r}) P^\dagger$	$-A^\mu(t, -\vec{r})$
Spinor of 1st kind	$P \psi(t, \vec{r}) P^\dagger$	$\exp(i\beta_p) \gamma^0 \psi((t, -\vec{r}))$
Spinor of 2nd kind	$P \bar{\psi}(t, \vec{r}) P^\dagger$	$\exp(-i\beta_p) \bar{\psi}((t, -\vec{r})) \gamma^0$

Table 1.1: Parity Operator of Free Fields.

Until 1957 physicists believed this symmetry to hold for all physical processes. In 1956-1957 Wu, *et al.* found a clear violation of parity conservation in the beta decay of Cobalt-60. Therefore, parity is conserved in electromagnetism, strong interactions and gravity but not in weak interaction. The SM incorporates parity violation by expressing the weak interaction as a chiral gauge interaction. Only the left-handed components of particles and right-

handed components of antiparticles participate in weak interactions in the SM. This implies that parity is not a symmetry of our universe.

### 1.2.2 CHARGE-CONJUGATION

Like parity ( $P$ ) symmetry, charge-conjugation ( $C$ ) symmetry consists in the invariance of physics under a discrete transformation that flips only the sign of the charge of a particle, *i.e.* charge-conjugation symmetry means the symmetry of physical laws under a charge-conjugation transformation. This symmetry is related to the existence of an antiparticle for every particle, a prediction of relativistic Quantum Mechanics, that is confirmed by experiment through the discovery of the positron [9] and antiproton [10]. Charge symmetry assumes that antiparticles behave in exactly the same way as the corresponding particles. It is just convention that electron, proton, etc are considered particles while positron, antiproton, etc are considered antiparticles. Under charge conjugation transformation, a particle would be converted into an antiparticle and vice-versa.

Charge conjugation invariance is true for strong and electromagnetic interactions, but not for weak interactions. It is found that only left-handed neutrinos and only right-handed antineutrinos are involved in weak processes. Charge conjugation ( $C$ ) changes a left-handed neutrino into a left-handed antineutrino and right-handed neutrino to a right-handed antineutrino. But processes involving right handed-neutrinos or left-handed antineutrinos are not seen. So it is clear that charge conjugation symmetry does not apply to weak processes. This property is known as the “maximal violation” of C-symmetry in the weak interaction. Using this information, we can define the charge-conjugation operator for fermion and boson fields in Quantum Mechanics. Since charge-conjugation operator is unitary and QED Hamiltonian ( $H$ ) is invariant under charge-conjugation transformation, *i.e.*  $[C, H] = 0$ ; this implies  $CHC^\dagger = H$  because  $C^{-1} = C^\dagger$ . We define the charge-conjugation operator of the fields as is shown in Table 1.2.

Charge conjugation changes the sign of all quantum numbers (electrical charge, baryon number, lepton number, flavor charges, isospin z-component, magnetic moment) but does not change these – mass, linear momentum, spin, chirality.

### 1.2.3 TIME-REVERSAL

Time reversal transformation, usually denoted by ( $T$ ), consists of changing the sign of the time coordinate, *i.e.* under T transformation  $t \rightarrow -t$ . The



Field	Charge-Conjugation Operator	Transformed Field
Scalar of 1st kind	$C \phi(t, \vec{r}) C^\dagger$	$\exp(i\alpha_c) \phi^\dagger((t, \vec{r}))$
Scalar of 2nd kind	$C \phi^\dagger(t, \vec{r}) C^\dagger$	$\exp(-i\alpha_c) \phi((t, \vec{r}))$
Vector	$C A_\mu(t, \vec{r}) C^\dagger$	$-A_\mu(t, \vec{r})$
Pseudovector	$C B_\mu(t, \vec{r}) C^\dagger$	$B_\mu(t, \vec{r})$
Spinor of 1st kind	$C \psi(t, \vec{r}) C^\dagger$	$\exp(i\beta_c) \psi^c((t, \vec{r}))$
Spinor of 2nd kind	$C \bar{\psi}(t, \vec{r}) C^\dagger$	$\exp(-i\beta_c) \bar{\psi}^c((t, \vec{r}))$

Table 1.2: Charge-Conjugation Operator of Free Fields.

time reversal operator reverses momentum and spin and also flips the sign of the time component of a state. If we look at any particle collision, we can relate it to another particle collision with all momenta reversed in direction and all angular momenta likewise reversed.

In classical mechanics, a velocity reverses under the operation of T, but an acceleration does not; consequently the laws of mechanics are time reversal invariant. This invariance is also exact in strong and electromagnetic processes, but not in weak interactions [11]. The time reversal operator for fermion and boson fields can be defined in Quantum Mechanics if we assume that QED Hamiltonian is invariant under this transformation. Since time reversal operator is anti-unitary and QED Hamiltonian ( $H$ ) is invariant under time reversal transformation, *i.e.*  $[T, H] = 0$ ; this implies  $THT^{-1} = H$ . We define the time reversal operator of the fields as is shown in Table 1.3

Field	time reversal Operator	Transformed Field
Scalar of 1st kind	$T \phi(t, \vec{r}) T^{-1}$	$\exp(i\alpha_t) \phi((-t, \vec{r}))$
Scalar of 2nd kind	$T \phi^\dagger(t, \vec{r}) T^{-1}$	$\exp(-i\alpha_t) \phi^\dagger((-t, \vec{r}))$
Vector	$T A_\mu(t, \vec{r}) T^{-1}$	$A^\mu(-t, \vec{r})$
Pseudovector	$T B_\mu(t, \vec{r}) T^{-1}$	$B^\mu(-t, \vec{r})$
Spinor of 1st kind	$T \psi(t, \vec{r}) T^{-1}$	$\exp(i\beta_t) \gamma^1 \gamma^2 \psi((-t, \vec{r}))$
Spinor of 2nd kind	$T \bar{\psi}(t, \vec{r}) T^{-1}$	$\exp(-i\beta_t) \bar{\psi}((-t, \vec{r})) \gamma^2 \gamma^1$

Table 1.3: Time reversal operator of Free Fields.

#### 1.2.4 CP

$CP$  is the product of two symmetries: charge conjugation ( $C$ ) and parity ( $P$ ). The  $CP$  symmetry means that all physical laws would preserve their

form when a charge-inversion transformation and a parity-inversion transformation are done simultaneously. Explicitly, the  $C$  operation reverses all additive quantum numbers such as electric charge, hypercharge, strangeness, etc, while the  $P$  transformation “inverts” the coordinate system and the orientation of all objects in it:  $x \rightarrow -x, y \rightarrow -y, z \rightarrow -z$ . If the spin is aligned with the velocity, the particle is referred to as having “positive helicity.” If the spin is anti-parallel to the velocity direction, the particle has “negative helicity.” Under a  $P$  transformation, the velocity direction is reversed but the spin direction is not; thus a positive helicity particle  $\rightarrow$  negative helicity antiparticle and vice versa. So under a  $CP$  transformation, a negative helicity electron becomes a positive helicity positron.

In 1964, James W. Cronin and Val L. Fitch found that the long-lived neutral K meson does decay into two pi mesons. If  $CP$  was conserved, the short-lived variety of K meson would always decay into two  $\pi$  mesons, whereas the long-lived variety of K meson would always decay into three  $\pi$  mesons. Hence, Cronin and Fitch found an example of  $CP$  violation [12]. After that in 2001,  $CP$  violation is also confirmed in B mesons at the BaBar Experiment and the Belle Experiment [7, 8]. Therefore,  $CP$  violation is confirmed in weak interaction but no  $CP$  violation occurs in the Strong and electromagnetic interactions.

The SM Lagrangian is invariant under  $CP$  transformation if we do not include the Cabibo Kobayashi Masikawa ( $CKM$ ) matrix in the Lagrangian for third generation. Therefore, the only source of  $CP$  violation in SM is the complex phase of  $CKM$  matrix [13]. This tiny  $CP$  violation is unable to explain the matter-antimatter difference in the universe. It seems that the SM does not accurately predict this discrepancy. It is hoped that in the future new sources of  $CP$  violation will be found to resolve this discrepancies.

### 1.2.5 $CPT$

$CPT$  is the product of the three operations: charge conjugation, parity, and time reversal. In the late 1950s the violation of P-symmetry, C-symmetry and T-symmetry were revealed in weak decay process. For a short time, the CP-symmetry was believed to be preserved by all physical phenomena, but that was later found to be false, too. On the other hand, there is a theorem that derives the preservation of  $CPT$  symmetry for all of physical phenomena assuming the correctness of quantum laws and Lorentz invariance. Specifically, the  $CPT$  theorem states that any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have  $CPT$  symmetry. A consequence of this derivation is that a violation of  $CPT$  automatically

indicates a Lorentz violation. Till date, there is no signal of  $CPT$  violation. All particle physics theories are based on relativistic quantum field theory.  $CPT$  invariance is an exact property in the theory. If experiments discover that  $CPT$  invariance is not exact, it would require us to develop a new kind of theory.

### 1.3 BASICS OF THE $CP$ PHENOMENOLOGY IN $B$ MESON DECAYS

We present a general formalism for  $CP$  violation in the decay of a pseudoscalar meson  $B$  that might be a charged or a neutral [14]. While constructing a field theory we always require locality, Lorentz invariant and hermiticity of Lagrangian. That is sufficient to make any field theory invariant under  $CPT$  transformation. In many theories  $CP$  and  $T$  are separately invariant. All the field theories that had been studied upto that time had automatic  $CP$  conservation. After the experimental discovery of  $CP$  violation, people started to find the origin of  $CP$  violation and also wanted to find which theory is providing it.  $CP$  non conservation shows up a rate difference between two processes that are the  $CP$  conjugate to one-another. The phases of each partial amplitude may be changed at will and is meaningless, but the relative phase of two partial amplitudes is rephasing invariant and has observable effects. Only phases which are rephasing invariant have a physical meaning and lead to  $CP$  violation. There are mainly two kinds of phases that may arise in transition amplitudes:

- Weak or  $CP$ -odd phase: a weak phase is defined to be one which has opposite signs in the transition amplitude for a process and in the transition amplitude for its  $CP$ -conjugate process. Weak phases usually originate from complex couplings in the Lagrangian.
- Strong or  $CP$ -even phase: a strong phase has the same sign in the transition amplitudes for two  $CP$ -conjugate processes.

#### 1.3.1 CHARGED AND NEUTRAL $B$ MESON DECAYS

Till date,  $CP$  violation has been observed only in weak interaction and hence we concentrate only on weak Hamiltonian  $H$  in this sub-section. We define decay amplitudes of  $B$  and its  $CP$  conjugate  $\bar{B}$  to a multi-particle final state  $f$  and its  $CP$  conjugate  $\bar{f}$  as

$$A_f = \langle f|H|B\rangle, \quad \bar{A}_f = \langle f|H|\bar{B}\rangle \quad (1.1)$$

$$A_{\bar{f}} = \langle \bar{f} | H | B \rangle, \quad \bar{A}_{\bar{f}} = \langle \bar{f} | H | \bar{B} \rangle \quad (1.2)$$

The action of  $CP$  on these states introduces phases  $\xi_B$  and  $\xi_f$  that depend on their flavor content, according to

$$CP|B\rangle = e^{\xi_B}|\bar{B}\rangle, \quad CP|f\rangle = e^{\xi_f}|\bar{f}\rangle \quad (1.3)$$

$$CP|\bar{B}\rangle = e^{-\xi_B}|B\rangle, \quad CP|\bar{f}\rangle = e^{-\xi_f}|f\rangle \quad (1.4)$$

If  $CP$  is conserved by the dynamics,  $[CP, H] = 0$ , then  $A_f$  and  $\bar{A}_{\bar{f}}$  have the same magnitude and an arbitrary unphysical relative phase

$$\bar{A}_{\bar{f}} = e^{(\xi_f - \xi_B)} A_f \implies |\bar{A}_{\bar{f}}| = |A_f| \quad (1.5)$$

For final  $CP$  eigenstate (*i.e.*  $\bar{f} = f$ ),

$$|\bar{A}_f| = |A_f| \quad (1.6)$$

### 1.3.2 NEUTRAL $B$ MESON MIXING

A state that is initially a superposition of  $B^0$  and  $\bar{B}^0$  say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\bar{B}^0\rangle \quad (1.7)$$

will evolve in time acquiring components that describe all possible decay final states  $\{f_1, f_2, \dots\}$ , that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \dots \quad (1.8)$$

Since we are interested in computing only the values of  $a(t)$  and  $b(t)$  and the times  $t$  in which we are interested are much larger than the typical strong interaction scale, we can use a simplified formalism in which  $c_i(t)$  are neglected, *i.e.* the simplified time evolution is determined by a  $2 \times 2$  effective Hamiltonian  $H$  that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as  $H$ , can be written in terms of Hermitian matrices  $M$  and  $\Gamma$  as

$$H = M - \frac{i}{2}\Gamma. \quad (1.9)$$

$M$  and  $\Gamma$  are associated with  $(B^0, \bar{B}^0) \rightarrow (B^0, \bar{B}^0)$  transitions via off-shell (dispersive), and on-shell (absorptive) intermediate states, respectively.

Diagonal elements of  $M$  and  $\Gamma$  are associated with the flavor-conserving transitions  $B^0 \rightarrow B^0$  and  $\bar{B}^0 \rightarrow \bar{B}^0$  while off-diagonal elements are associated with flavor-changing transitions  $B^0 \rightarrow \bar{B}^0$ .

Let the eigenvalues of  $M$  be  $m_H$  and  $m_L$ , and similarly the eigenvalues of  $\Gamma$  be  $\Gamma_H$  and  $\Gamma_L$ .

Under Wigner Weisskopf formalism [15], we can describe the time evolution of the state vector as

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle. \quad (1.10)$$

and the effective Schrödinger equation as

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle \quad (1.11)$$

Under the assumption of  $CPT$  invariance, the eigenvector of  $H$  may be written as

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad |B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle. \quad (1.12)$$

Where  $|B^0\rangle$  and  $|\bar{B}^0\rangle$  are flavour eigen states, and  $|B_H\rangle$  and  $|B_L\rangle$  weak eigen states (mass eigen states). On solving the Eq. (1.11) under  $CPT$  invariance, we can get

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \quad (1.13)$$

For convenience, we define some notations as follows

$$m \equiv \frac{m_H + m_L}{2}, \quad \Gamma \equiv \frac{\Gamma_H + \Gamma_L}{2}, \quad (1.14)$$

$$\Delta m \equiv m_H - m_L, \quad \Delta \Gamma \equiv \Gamma_H - \Gamma_L, \quad (1.15)$$

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}, \quad u \equiv -\frac{y}{x}. \quad (1.16)$$

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}, \quad \delta \equiv |p|^2 - |q|^2 \quad (1.17)$$

If we assume that Hamiltonian is  $CP$  invariant then

$$M_{12}^* = e^{2i\xi} M_{12} \quad \Gamma_{12}^* = e^{2i\xi} \Gamma_{12} \quad (1.18)$$

Consequently, we have

$$\frac{q}{p} = \pm e^{i\xi} \implies \left|\frac{q}{p}\right| = 1 \quad (1.19)$$

Therefore, we denote the ratio of mixing parameters in  $B$  system as:

$$\frac{q_B}{p_B} = \eta_f e^{2i\phi_M} \quad (1.20)$$

where,  $\eta_f$  may be positive or negative and  $\xi = 2\phi_M$ .

If the Hamiltonian is  $CP$  invariant ( $|p_B| = |q_B|$ ), then  $\langle B_L | B_H \rangle = |p_B|^2 - |q_B|^2 = 0$ , *i.e.*  $\delta_B \equiv |p_B|^2 - |q_B|^2 = 0$ . Therefore  $\delta_B$  measures  $CP$  violation in  $B$  meson due to mixing. With time being mass eigenstates  $B_H$  and  $B_L$  will evolve with evolution laws with well-defined masses and decay widths. Thus, we can write the time dependent mass eigenstates  $B_H(t)$  and  $B_L(t)$  under  $CPT$  invariant as:

$$|B_H(t)\rangle = e^{-i(M_H - \frac{i}{2}\Gamma_H)t} |B_H\rangle, \quad |B_L(t)\rangle = e^{-i(M_L - \frac{i}{2}\Gamma_L)t} |B_L\rangle \quad (1.21)$$

Using Eqs. (1.12) and (1.21), we can get the time dependent of flavour eigenstates as:

$$|B^0(t)\rangle = g_+(t) |B^0\rangle + \frac{q_B}{p_B} g_-(t) |\bar{B}^0\rangle, \quad (1.22)$$

$$|\bar{B}^0(t)\rangle = \frac{p_B}{q_B} g_-(t) |B^0\rangle + g_+(t) |\bar{B}^0\rangle \quad (1.23)$$

where

$$|g_{\pm}(t)|^2 = \frac{1}{4} \left[ e^{-\Gamma_L t} + e^{-\Gamma_H t} \pm 2 e^{-\Gamma t} \cos(\Delta m t) \right] \quad (1.24)$$

$$g_+(t)^* g_-(t) = \frac{1}{4} \left[ e^{-\Gamma_L t} - e^{-\Gamma_H t} + 2 i e^{-\Gamma t} \sin(\Delta m t) \right], \quad (1.25)$$

### 1.3.3 CP VIOLATION IN B SYSTEM

There are three types of  $CP$  violation:

- $CP$  violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its  $CP$ -conjugate process have different magnitudes, *i.e.*  $|\bar{A}_f| \neq |A_f|$ .
- $CP$  violation in mixing, which occurs when the two neutral mass eigenstate admixtures cannot be chosen to be  $CP$ -eigenstates, *i.e.*  $\left| \frac{q_B}{p_B} \right| \neq 1$ .
- $CP$  violation in the interference of decays with and without mixing, which occurs in decays into flavour-blind final states that are common to both states, *i.e.*  $\text{Im}\lambda_f \neq 0$ .

In B decays, it is generally assumed, based on both experimental and theoretical arguments, that there is no CP violation in the mixing: There are then two possible forms of CP violation: direct CP violation, and CP violation in the interference between the mixing and the decays. According to superweak theory, there is no CP violation in the decay amplitudes. We define CP asymmetries in B mesons on the basis that  $\Delta\Gamma = 0$  and  $\left|\frac{q_B}{p_B}\right| = 1$ . Using these approximation we get

$$\begin{aligned}\Gamma[B^0(t) \rightarrow f] &= \frac{|A_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda_f|^2 + (1 - |\lambda_f|^2) \cos \Delta mt + 2 \text{Im} \lambda_f \sin \Delta mt], \\ \Gamma[\bar{B}^0(t) \rightarrow f] &= \frac{|A_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda_f|^2 - (1 - |\lambda_f|^2) \cos \Delta mt - 2 \text{Im} \lambda_f \sin \Delta mt]\end{aligned}\tag{1.26}$$

For the time-integrated decay rates

$$\begin{aligned}\Gamma[B^0 \rightarrow f] &= \frac{|A_f|^2}{2\Gamma} \left[ 1 + |\lambda_f|^2 + \frac{1 - |\lambda_f|^2 + 2x \text{Im} \lambda_f}{1 + x^2} \right], \\ \Gamma[\bar{B}^0 \rightarrow f] &= \frac{|A_f|^2}{2\Gamma} \left[ 1 + |\lambda_f|^2 - \frac{1 - |\lambda_f|^2 + 2x \text{Im} \lambda_f}{1 + x^2} \right],\end{aligned}\tag{1.27}$$

Now we define the time dependent CP - violating asymmetry that  $B^0$  and  $\bar{B}^0$  decays into CP eigenstate using the decay rates Eqs. (1.26) and (1.27)

$$\begin{aligned}A_{CP}(t) &\equiv \frac{\Gamma[B^0(t) \rightarrow f] - \Gamma[\bar{B}^0(t) \rightarrow f]}{\Gamma[B^0(t) \rightarrow f] + \Gamma[\bar{B}^0(t) \rightarrow f]} \\ &= a^{dir} \cos \Delta mt + a^{int} \sin \Delta mt,\end{aligned}\tag{1.28}$$

where

$$a^{dir} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad a^{int} = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}\tag{1.29}$$

Similarly we can define time independent CP- violating asymmetry that  $B^0$  and  $\bar{B}^0$  decays into CP eigenstate using the decay rates Eqs. (1.26) and (1.27)

$$\begin{aligned}A_{CP} &\equiv \frac{\Gamma[B^0 \rightarrow f] - \Gamma[\bar{B}^0 \rightarrow f]}{\Gamma[B^0 \rightarrow f] + \Gamma[\bar{B}^0 \rightarrow f]} \\ &= \frac{a^{dir} + x a^{int}}{1 + x^2}\end{aligned}\tag{1.30}$$

$a^{dir}$  measures direct CP violation and  $a^{int}$  measures interference CP violation.

Let us consider the decay of a  $B$  meson (model independently) into some specific final state  $f$ . where,  $B$  stands for  $B^+$ ,  $B_d^0$  or  $B_s^0$ . We can then parametrize the decay amplitudes as

$$A_f = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}, \quad (1.31)$$

$$\bar{A}_{\bar{f}} = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}, \quad (1.32)$$

where  $\phi_1$  and  $\phi_2$  are two CP-odd weak phases<sup>1</sup>;  $A_1$  and  $A_2$  are the magnitudes of the corresponding terms; and  $\delta_1$  and  $\delta_2$  are the corresponding CP-even strong phases.

For the discussion about neutral  $B$  meson decays, we need Eqs. (1.31), (1.32), and also the mixing parameter Eq. (1.20).

Therefore,

$$\lambda_f = \frac{q_B \bar{A}_f}{p_B A_f} \quad (1.33)$$

$$= \eta_f e^{-2i\phi_{1f}} \frac{1 + r e^{i(\phi_{1f} - \phi_{2f})} e^{i\delta}}{1 + r e^{-i(\phi_{1f} - \phi_{2f})} e^{i\delta}}, \quad (1.34)$$

where  $\phi_{1f} \equiv \phi_1 - \phi_M$ ,  $\phi_{2f} \equiv \phi_2 - \phi_M$ ,  $\delta = \delta_2 - \delta_1$ , and  $r = A_2/A_1$ . Hence,  $\phi_{1f} - \phi_{2f} = \phi_1 - \phi_2$ . We have assumed that  $|q_B/p_B| = 1$ , meaning that the CP violation in  $B - \bar{B}$  mixing is negligible. It is known

that  $\lambda_f$  is measurable from the decay rates through

$$S_f \equiv \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} = -\eta_f \frac{\sin(2\phi_{1f}) + 2r \sin(\phi_{1f} + \phi_{2f}) \cos \delta + r^2 \sin(2\phi_{2f})}{1 + 2r \cos(\phi_{1f} - \phi_{2f}) \cos \delta + r^2}, \quad (1.35)$$

$$C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} = \frac{2r \sin(\phi_{1f} - \phi_{2f}) \sin \delta}{1 + 2r \cos(\phi_{1f} - \phi_{2f}) \cos \delta + r^2}, \quad (1.36)$$

since

$$\lambda_f = \frac{1}{1 + C_f} \left( \pm \sqrt{1 - C_f^2 - S_f^2} + i S_f \right) \quad (1.37)$$

For simplicity, we will assume in the following that  $S_f$  and  $C_f$  can be measured with absolute precision.

---

<sup>1</sup>Here  $\phi_i$  do not refer to the angles of Unitarity triangles (UT). Through the thesis we have used  $\alpha$ ,  $\beta$  and  $\gamma$  for the angles of the UT. For historical reasons, both the notations ( $\alpha$  ( $\phi_2$ ),  $\beta$  ( $\phi_1$ ) and  $\gamma$  ( $\phi_3$ )) are shown in Fig. 1.1



If  $C_f = 0$ , then the Eq. (1.36) implies that: i)  $r = 0$  (and there is only one amplitude/weak phase); or that ii)  $\phi_{1f} = \phi_{2f}$  (and there is only one weak phase); or that iii)  $\delta_1 = \delta_2$ . In the last case, we can always find a magnitude  $A_3$  and a weak phase  $\phi_3$  such that

$$A_f = (A_1 e^{i\phi_1} + A_2 e^{i\phi_2}) e^{i\delta_1} = A_3 e^{i\phi_3} e^{i\delta_1}, \quad (1.38)$$

$$\bar{A}_{\bar{f}} = \eta_f (A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2}) e^{i\delta_1} = \eta_f A_3 e^{-i\phi_3} e^{i\delta_1}. \quad (1.39)$$

These equalities are satisfied by

$$A_3^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2), \quad (1.40)$$

$$e^{-2i\phi_3} = \frac{A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2}}{A_1 e^{i\phi_1} + A_2 e^{i\phi_2}} = \frac{1 + r e^{i(\phi_1 - \phi_2)}}{1 + r e^{-i(\phi_1 - \phi_2)}} e^{-2i\phi_1}, \quad (1.41)$$

from which we can always determine  $\phi_3$ , because the numerator and the denominator on the RHS of Eq. (1.41) are complex conjugate.

In cases i) and ii)  $S_f = -\eta_f \sin(2\phi_{1f})$ ; in case iii)  $S_f = -\eta_f \sin(2\phi_{3f})$ , where  $\phi_{3f} = \phi_3 - \phi_M$ . Hence, if  $C_f = 0$  then we are sure that the amplitudes can be written in terms of only one weak phase, which is measured through  $S_f$ . Therefore, we can say as: “if the decay amplitude is determined by only one weak phase, then we can relate experiment with theory; if more than one weak phase is involved, then we cannot”. Hence, if the decays of  $B^0$  and  $\bar{B}^0$  into  $CP$  eigenstate  $f$  is dominated by a single weak phase, then the weak phase can be determined easily but in the presence of more than one weak phase, the determination of weak phases is very difficult.

#### 1.4 THE STANDARD MODEL

The Standard Model, based on local symmetry ( $SU(3) \otimes SU(2) \otimes U(1)$ ) and consistent with quantum field theory [16], of particle physics describes three of the four known fundamental interactions (strong, weak and electromagnetism) between the elementary particles that make up all matter. In the mid sixties, Abdus Salam, Sheldon Glashow and Steven Weinberg proposed the unified description of the electromagnetism and the weak interaction, known as the electroweak theory and later it was also extended for strong interaction (quantum chromodynamics QCD) by David Politzer, Frank Wilczek and David Gross. To date, almost all experimental tests of the three forces described by the SM have agreed with its predictions.

The electroweak interaction is described by a gauge theory [17] based on the  $SU(2)_L \otimes U(1)_Y$  group, which is spontaneously broken via the Higgs

mechanism. The matter fields “leptons and quarks” are organized in families, with the left handed fermions belonging to weak isodoublets while the right handed components transform as weak isosinglets. The vector bosons,  $W^\pm$ ,  $Z^0$  and  $\gamma$ , that mediate the interactions are introduced via minimal coupling to the matter fields.

The SM is now on solid ground; the works have been supplemented by the proof of renormalizability of the theory [18], the introduction of three lepton and quark generations, and the proof of tree-level cancellation of flavour-changing neutral currents (FCNC) [19]. There have been numerous pioneering experiments, like the discovery of the  $J/\psi$  meson [20]; the existence of neutral current mediated processes [21]; the discovery of  $W^\pm$  and  $Z$  at the CERN  $p\bar{p}$  collider [22]; the discoveries of the  $\tau$  lepton [23], and the bottom [24] and the top quarks [25]; and observation of CP violation in K and B meson systems [28]. We are yet to find the most important ingredient of the SM, the Higgs boson [29].

There are many excellent books that deal with the SM in great detail for particle physics [30], however we will just mention some important points that would be relevant for the phenomenology of B meson.

If a local gauge symmetry is unbroken, the corresponding gauge bosons remain massless. The strong interaction is an exact symmetry, so the gluons are massless and  $SU(3)_c$  (the subscript stands for colour) is unbroken. However, weak gauge bosons  $W^\pm$  and  $Z$  are massive. The mass generation is one of the most elegant features of the GWS model. An unbroken  $SU(2) \times U(1)$  symmetry at a high energy scale, which is nothing but the unification of electromagnetic and weak forces, gets broken at a few hundreds of GeV; out of the four gauge bosons of the unbroken theory, three become massive ( $W^\pm$  and  $Z$ ), and one remains massless, which is the photon, and corresponds to an unbroken  $U(1)_{em}$  gauge symmetry. This symmetry breaking, which occurs due to the fact that the vacuum (and not the Lagrangian) is asymmetric under the gauge group and is, hence, known as spontaneous symmetry breaking (SSB), requires the existence of a scalar field. This is a doublet under weak  $SU(2)$ , and after SSB, a neutral scalar remains as its remnant. This is known as the Higgs boson and is the only missing component of the SM till now.

Apart from the bosons, there are fermions, divided into three generations with two leptons and two quarks apiece. Remember that left and right chiral particle states are different as far as weak interaction goes (the first is a doublet, and hence feels charge-changing weak interaction, while the second is a singlet, and does not feel it); also, leptons are singlet under  $SU(3)_c$  while

quarks are triplets. Taking neutrinos as massless <sup>2</sup> the number of fermion fields in each generation adds up to 15 ( $e_L, e_R, \nu_e, (u, d)_{L,R} \times 3$ ).

The electroweak Lagrangian density has three parts:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{higgs} + \mathcal{L}_{fermion} \quad (1.42)$$

where

$$\mathcal{L}_{gauge} = -\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \quad (1.43)$$

with

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\varepsilon_{abc} W_\mu^b W_\nu^c \quad (1.44)$$

and

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (1.45)$$

Here,  $W_\mu^a$  is the triplet vector field ( $a=1,2,3$ ) associated with  $SU(2)_L$  and  $B_\mu$  is the singlet field associated with the hypercharge gauge group  $U(1)_Y$ .  $g$  denotes the  $SU(2)$  gauge coupling constant.

$\mathcal{L}_{fermion}$ , which includes the fermion kinetic terms as well as its interaction with gauge and Higgs fields, looks like

$$\mathcal{L}_{fermion} = \sum_j \bar{\psi}_j^L i\gamma^\mu D_\mu \psi_j^L + \sum_j \bar{\psi}_j^R i\gamma^\mu D'_\mu \psi_j^R + \mathcal{L}_Y, \quad (1.46)$$

where  $\mathcal{L}_Y$ , discussed later, gives the Yukawa interaction between the fermions and the scalar doublet.

The  $SU(2) \times U(1)$  covariant derivative is

$$D_\mu = \partial_\mu + ig\frac{\sigma_a}{2}W_\mu^a + ig'\frac{Y}{2}B_\mu \quad (1.47)$$

where  $g'$  is the  $U(1)_Y$  coupling constant, and the hypercharge assignment is given by  $Q = I_3 + Y/2$ , where  $I_3$  is the third component of the weak isospin. For the right-chiral fermions, we use  $D'$ , for which the  $SU(2)$  part is absent (and of course  $Y$  is different). Table 1.1 lists the fermionic quantum numbers. This assignment makes the model anomaly-free.

$\mathcal{L}_{higgs}$  is given by

$$\mathcal{L}_{higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad (1.48)$$

---

<sup>2</sup>They are not. This is the first unambiguous signal of NP beyond the SM. However,  $\nu_{RS}$  can easily be accommodated in the SM, but they are peculiar in the sense that they do not have any gauge interaction.

Sector	Spin	Field
SU(3) gauge bosons (gluons) SU(2) gauge bosons U(1) gauge boson	1	$G \equiv (8, 1)_0$ $W \equiv (1, 3)_0$ $B \equiv (1, 1)_0$
Chiral matter (Three families: $a = 1, 2, 3$ )	$\frac{1}{2}$	$Q_a \equiv \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L \equiv (3, 2)_{\frac{1}{6}}$  $u_{Ra} \equiv (3, 1)_{\frac{2}{3}}$ $d_{Ra} \equiv (3, 1)_{-\frac{1}{3}}$ $L_a \equiv \begin{pmatrix} \nu_a \\ e_a^- \end{pmatrix}_L \equiv (1, 2)_{-\frac{1}{2}}$ $e_{Ra} \equiv (1, 1)_{-1}$
Symmetry breaking (the Higgs boson)	0	$\Phi \equiv \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \equiv (1, 2)_{-\frac{1}{2}}$

Table 1.4: The SM field content.  $(Q_c, Q_L)_{Q_{Y/2}}$  lists color, weak isospin and hypercharge assignments of a given field, respectively, and  $Q_c = 1$ ,  $Q_L = 1$  or  $Q_{Y/2} = 0$  represent a singlet under the respective group transformations. The  $T_3$  isospin operator is  $+1/2$  ( $-1/2$ ) when acting on the upper (lower) component of an isospin doublet (and zero otherwise).

where  $D_\mu$  is given by Eq. (1.47).  $V(\Phi)$  is the potential term for the doublet field  $\Phi$ . This field can be written as

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi_0 + i\phi_3) \end{pmatrix} \quad (1.49)$$

and after Spontaneous Symmetry Breaking,  $\phi_0$  gets a vacuum expectation value (VEV)  $\langle \phi_0 \rangle \equiv v \approx 246$  GeV. The numerical value is obtained from the comparison of  $W$  mass and the Fermi coupling constant  $G_F$ .

All fermions remain massless before SSB, since a mass term of the form  $-m\bar{\psi}\psi$  is forbidden from gauge invariance:  $\psi_R \equiv P_R\psi$  is a singlet under weak SU(2) while  $\psi_L \equiv P_L\psi$  is a doublet (we use the standard chirality projection operators  $P_{R(L)} = (1 + (-)\gamma_5)/2$ ). However, there is a gauge invariant fermion-scalar Yukawa interaction of the form

$$\mathcal{L}_Y = -f_{ij}^e \bar{l}_{iL} \Phi e_{jR} - f_{ij}^d \bar{q}_{iL} \Phi d_{jR} + h.c \quad (1.50)$$

which, after SSB, generates the charged lepton and down-type quark masses of the form

$$m_e = f^e v / \sqrt{2}. \quad (1.51)$$

Here  $i, j = 1, 2, 3$  are the generation indices. There are two points to note. First, up-type quark masses are obtained from the VEV of the field  $\tilde{\Phi} \equiv i\sigma_2 \Phi^*$ . Second, the mass matrices are *not diagonal* and can even be complex. This means that the weak eigenstates are not physical states; one should perform field rotations to make the mass matrices diagonal. In turn, the weak interaction becomes non-diagonal and can occur between particles of different generations.

As a convention, we denote the fields in the weak or flavour basis by a prime, and the corresponding unprimed states denote the physical fields. A general complex matrix can be diagonalized by a biunitary transformation. Let us write the diagonal down-type mass matrix  $(M_d)_{diag} \equiv \text{diag}(m_d, m_s, m_b) = \mathcal{D}_L^\dagger M_d \mathcal{D}_R$  where  $(M_d)_{ij} = f_{ij}^d v / \sqrt{2}$ . If the Lagrangian remains invariant, the fields must transform as

$$d'_R = \mathcal{D}_R d_R, \quad d'_L = \mathcal{D}_L d_L. \quad (1.52)$$

One gets a similar transformation for the up-type quarks, involving the unitary matrices  $\mathcal{U}_R$  and  $\mathcal{U}_L$ .

The charged weak current is

$$\bar{u}'_i \gamma^\mu (1 - \gamma_5) d'_i = \bar{u}_i \mathcal{U}_L^\dagger \mathcal{D}_L \gamma^\mu (1 - \gamma_5) d_i. \quad (1.53)$$

It is only the combination  $\mathcal{U}_L^\dagger \mathcal{D}_L$  that can be probed in charged current weak interaction. There is no way to extract any information on the individual matrices, and the right-hand matrices are completely unknown, except for their unitarity property. The combination  $\mathcal{U}_L^\dagger \mathcal{D}_L$  is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix and is denoted by  $V$ , so that the weak interaction takes the form

$$V_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) d_j W_\mu^+ + h.c. \quad (1.54)$$

Remember that the hermitian conjugate involves  $V_{ij}^*$ , which may be different from  $V_{ij}$ .

The neutral weak current is flavour diagonal to start with. Now, the combination  $\bar{q}_{L(R)} \gamma^\mu q_{L(R)}$  remains invariant under field rotation, since the rotation matrices are unitary. Thus, *there is no flavour-changing neutral current (FCNC) in the SM at the tree-level*. This is called the GIM mechanism [19].

The CKM matrix as shown in Eq.(1.53) specifies the misalignment between the up-type quarks and the down-type quarks. Apparently, this has 9 elements, and all can be complex; so the number of independent elements seems to be 18. For a general  $N \times N$  CKM matrix (for  $N$  generations, *i.e.*,  $2N$  quarks) the number of independent elements is  $2N^2$ ; but there are  $N^2$  constraints coming from  $V^\dagger V = V V^\dagger = \mathbf{1}$ , and  $(2N - 1)$  phases can be absorbed by quark field redefinitions. Thus, one has  $2N^2 - N^2 - (2N - 1) = (N - 1)^2$  independent elements in the  $N$ -generation CKM matrix. For our case ( $N = 3$ ), this is 4.

Not all these elements are real. To find out the number of real elements, we redo the exercise for an orthogonal  $N \times N$  matrix. There are  $N^2$  elements to start with, and  $N(N + 1)/2$  constraints coming from  $OO^T = O^T O = \mathbf{1}$  ( $N$  constraints with right hand side equal to 1, and  $N(N - 1)/2$  with r.h.s. equal to 0). There is no question of field redefinition, since the elements are all real. So we have  $N(N - 1)/2$  independent elements. Thus, in a  $N$  generation CKM matrix,  $N(N - 1)/2$  elements are real, and  $(N - 1)^2 - N(N - 1)/2 = (N - 1)(N - 2)/2$  elements are complex phases.

For a 3-generation matrix, there are 3 real elements and 1 complex phase. We will now see how this complex phase generates  $CP$  violation. This is not possible for two generations. Under  $CP$ , the particles are transformed to antiparticles, and the  $(V - A)$  current retains its Lorentz structure. Consider, for example, the coupling of  $W$  to  $b$  and  $u$  quarks. The Lagrangian is given by

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \left[ V_{ub} \bar{u}_L \gamma^\mu b_L W_\mu^+ + V_{ub}^* \bar{b}_L \gamma^\mu u_L W_\mu^- \right],$$

$$CP \mathcal{L} (CP)^{-1} = -\frac{g}{\sqrt{2}} [V_{ub} \bar{b}_L \gamma^\mu u_L W_\mu^- + V_{ub}^* \bar{u}_L \gamma^\mu b_L W_\mu^+], \quad (1.55)$$

which is the same only if  $V_{ub} = V_{ub}^*$ . This shows why  $CP$  violation is related to complex phases in couplings. Yet this is only a necessary but not a sufficient condition for a theory to violate  $CP$ . A phase rotation of the quark fields in the  $CP$  transformed Lagrangian changes the phases of the couplings. If we can, in this way, rotate the phases in  $CP \mathcal{L} (CP)^{-1}$  back into those in  $\mathcal{L}$ , then  $CP$  is conserved. In our example the choice  $\phi^b - \phi^u = 2 \arg V_{ub}$  would transform  $CP \mathcal{L} (CP)^{-1}$  back into  $\mathcal{L}$ . We will later show that to obtain  $CP$  violation, one needs at least two competing amplitudes for a process, and it is the interference phenomenon that leads to  $CP$  violation.

There are many possible parametrizations of the CKM matrix. The standard parametrization, recommended by the Particle Data Group [28], is

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (1.56)$$

where  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$ ,  $i$  and  $j$  being generation labels ( $i, j = 1, 2, 3$ ).  $\delta$  is the phase necessary for  $CP$  violation.  $c_{ij}$  and  $s_{ij}$  can all be chosen to be positive and  $\delta$  may vary in the range  $0 \leq \delta \leq 2\pi$ . However, the measurements of  $CP$  violation in K decays force  $\delta$  to be in the range  $0 < \delta < \pi$ . From experiments, we know that  $s_{13}$  and  $s_{23}$  are small numbers:  $\mathcal{O}(10^{-3})$  and  $\mathcal{O}(10^{-2})$ , respectively. Consequently, to an excellent accuracy  $c_{13} = c_{23} = 1$  and the four independent parameters are given as

$$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta \quad (1.57)$$

The first three can be extracted from the tree level decays by the quark transitions of  $s \rightarrow u$ ,  $b \rightarrow u$  and  $b \rightarrow c$  respectively. The phase  $\delta$  can be extracted from  $CP$  violating transitions or loop processes sensitive to  $|V_{td}|$ . The latter fact is based on the observation that for  $0 \leq \delta \leq \pi$ , as required by the analysis of  $CP$  violation in the K system, there is a one-to-one correspondence between  $\delta$  and  $|V_{td}|$  given by

$$|V_{td}| = \sqrt{a^2 + b^2 - 2ab \cos \delta}, \quad a = |V_{cd}V_{cb}|, \quad b = |V_{ud}V_{ub}|. \quad (1.58)$$

The  $CP$  violating phase is always multiplied by the very small  $s_{13}$ . This is reflective of the fact that  $CP$  is an almost exact symmetry of nature; the

violation is small, irrespective of the actual value of  $\delta$ . However once the four parameters in the CKM matrix have been determined, it is often useful to make a change of basic parameters in order to see the structure of the result more transparently. This brings us to the Wolfenstein parametrization.

The unitarity of the CKM matrix leads to the following set of equations:

$$\begin{aligned}
V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* &= 0 \\
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \\
V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* &= 0 \\
V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* &= 0 \\
V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* &= 0 \\
V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* &= 0.
\end{aligned} \tag{1.59}$$

These equations originate from the orthogonality of different columns and rows. They are of particular interest since they can be represented as six ‘‘unitarity’’ triangles in the complex plane. They are invariant under any phase transformation of the quark fields; this is just equivalent to rotating the triangle in the complex plane.

Since the angles and the sides of these triangles remain unchanged and therefore are independent of the CKM phase convention, these quantities are physical observables. It can be shown that all six triangles have the same area which is related to the measure of CP violation  $J_{\text{CP}}$ , known as the Jarlskog parameter:

$$|J_{\text{CP}}| \equiv \text{Im}(V_{i\alpha}V_{i\beta}^*V_{j\beta}V_{j\alpha}^*) = 2A_{\Delta}, \tag{1.60}$$

where  $i, j$  are up-type quark indices,  $\alpha, \beta$  are down-type quark indices (no summation is implied), and  $A_{\Delta}$  denotes the area of the unitarity triangles.

Only two of these six triangles, given by the second and the fifth relationships, look like triangles; others are very squashed in nature. This can be easily shown by considering the order of each side in terms of the small parameter  $\lambda$ ; only for these two triangles all sides are of order  $\lambda^3$ . The second relationship, known as the *bd triangle*, can be probed in B decays.

Let us concentrate on the *bd* triangle:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{1.61}$$

The angles of the unitarity triangle are geometrically defined as

$$\alpha \equiv \arg(-V_{tb}^*V_{td}/V_{ub}^*V_{ud}), \tag{1.62}$$

$$\beta \equiv \arg(-V_{cb}^*V_{cd}/V_{tb}^*V_{td}), \tag{1.63}$$

$$\gamma \equiv \arg(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd}). \tag{1.64}$$



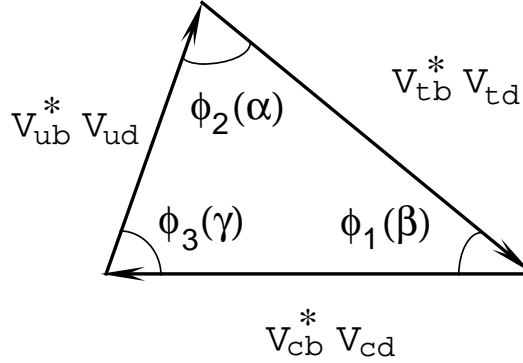


Figure 1.1: Unitarity triangle

These angles do not necessarily agree with the  $CP$  angles to be measured in experiments. Phenomenologically this triangle is very interesting as it involves simultaneously the elements  $V_{ub}$ ,  $V_{cb}$  and  $V_{td}$  which are under extensive discussion at present.

The angles  $\beta$  and  $\gamma$  of the unitarity triangle are related directly to the complex phases of the CKM-elements  $V_{td}$  and  $V_{ub}$ , respectively, through

$$V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}. \quad (1.65)$$

The angles ( $\alpha$ ,  $\beta$  and  $\gamma$ ) of CKM unitarity triangle are related by the relation

$$\alpha + \beta + \gamma = 180^\circ. \quad (1.66)$$

#### 1.4.1 WOLFENSTEIN PARAMETRIZATION

This is an approximate parametrization of the CKM matrix in which each element is expanded as a power series in the small parameter  $\lambda = |V_{us}| = 0.22$  [27, 28]. At the leading order,  $V$  can be written as

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \quad (1.67)$$

and the previous representation is replaced by

$$\lambda, \quad A, \quad \varrho, \quad \eta. \quad (1.68)$$

Here  $\lambda$  is small ( $\approx 0.22$ ), so it is sufficient to keep only the first few terms in this expansion. It is certainly more transparent than the standard parametrization. If one requires sufficient level of accuracy, the higher order terms in  $\lambda$  have to be included. There is no unanimous prescription for this, but one of the more commonly used ways is as follows [31]:

$$\begin{aligned}
V_{ud} &= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6), \\
V_{us} &= \lambda + \mathcal{O}(\lambda^7), \\
V_{ub} &= A\lambda^3(\varrho - i\eta), \\
V_{cd} &= -\lambda + \frac{1}{2}A^2\lambda^5 [1 - 2(\varrho + i\eta)] + \mathcal{O}(\lambda^7), \\
V_{cs} &= 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) + \mathcal{O}(\lambda^6), \\
V_{cb} &= A\lambda^2 + \mathcal{O}(\lambda^8), \\
V_{td} &= A\lambda^3 \left[ 1 - (\varrho + i\eta)(1 - \frac{1}{2}\lambda^2) \right] + \mathcal{O}(\lambda^7), \\
V_{ts} &= -A\lambda^2 + \frac{1}{2}A(1 - 2\varrho)\lambda^4 - i\eta A\lambda^4 + \mathcal{O}(\lambda^6), \\
V_{tb} &= 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6).
\end{aligned} \tag{1.69}$$

Note that by definition  $V_{ub}$  remains unchanged and the corrections to  $V_{us}$  and  $V_{cb}$  appear only at  $\mathcal{O}(\lambda^7)$  and  $\mathcal{O}(\lambda^8)$ , respectively. Consequently to an excellent accuracy we have

$$V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ub} = A\lambda^3(\varrho - i\eta), \quad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}), \tag{1.70}$$

with

$$\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \tag{1.71}$$

To include the next-to-leading  $\mathcal{O}(\lambda^5)$  terms, we note first that

$$V_{cd}V_{cb}^* = -A\lambda^3 + \mathcal{O}(\lambda^7). \tag{1.72}$$

Thus to an excellent accuracy  $V_{cd}V_{cb}^*$  is real with  $|V_{cd}V_{cb}^*| = A\lambda^3$ . Keeping  $\mathcal{O}(\lambda^5)$  corrections and rescaling all terms in (1.61) by  $A\lambda^3$  we find

$$\frac{1}{A\lambda^3}V_{ud}V_{ub}^* = \bar{\varrho} + i\bar{\eta}, \quad \frac{1}{A\lambda^3}V_{td}V_{tb}^* = 1 - (\bar{\varrho} + i\bar{\eta}) \tag{1.73}$$

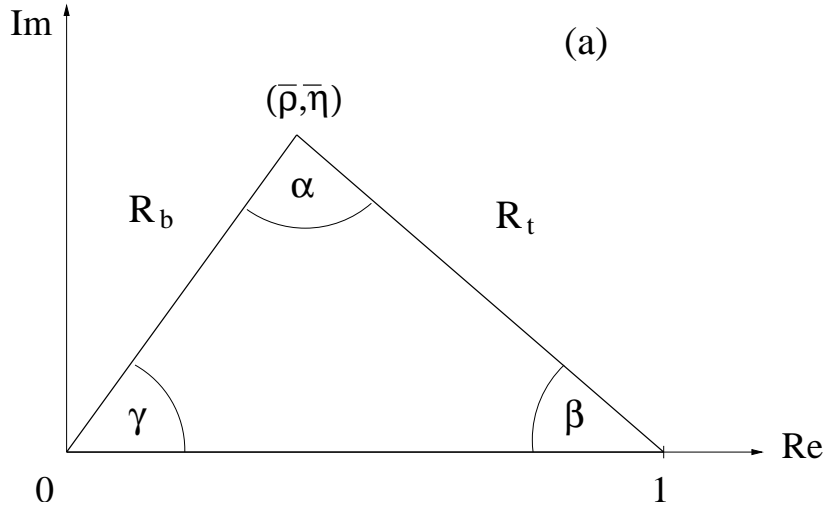


Figure 1.2: The unitarity triangle of the CKM matrix.

Thus we can represent Eq. (1.61) as the unitarity triangle in the complex  $(\bar{\rho}, \bar{\eta})$  plane. This is shown in fig. 1.2. The advantage of this generalization of the Wolfenstein parametrization over other generalizations found in the literature is the absence of relevant corrections to  $V_{us}$ ,  $V_{cb}$  and  $V_{ub}$  and an elegant change in  $V_{td}$  which allows a simple generalization of the so-called unitarity triangle beyond the leading order.

#### 1.4.2 LIMITATION OF STANDARD MODEL

The SM is not the end of the story, however successful it might be. It is at best an effective theory, valid upto some energy scale. There are enough reasons to believe that some NP will appear around that energy scale. The reasons are as follows.

The Higgs boson mass is not protected by any symmetry. There is no apparent reason why it should be of the order of a few hundreds of GeV, and not  $10^{19}$  GeV. So, unless there is some NP to protect the Higgs mass, this is a case of a very unnatural fine-tuning. There are many approaches to solve this problem: Technicolour type theories, where the Higgs is assumed to be a composite of two fermions; Supersymmetry, where new particles are proposed which cancel the large radiative corrections coming from the SM fields; Models with compactified extra dimensions, where the Planck scale is lowered to a few TeV by appealing to the fact that gravity is weak not because of a large Planck mass but due to a small intercept of higher

dimensional gravitational wave function with our physical world; and Little Higgs models, where the Higgs is constructed as a pseudo-Goldstone boson and hence has its mass protected.

The three gauge couplings do not unify in the SM. So, the SM cannot lead to a unified theory of strong and electroweak interactions. This is an aesthetic objection, but Supersymmetry provides a nice way to gauge coupling unification and hence a Grand Unified Theory.

The number of free parameters is too large in the SM for any fundamental theory: the nine Yukawa couplings (not counting the neutrinos), four CKM parameters, three gauge couplings, Higgs mass, VEV of the scalar field, and the  $\theta$ -parameter related with the strong CP problem. It is hoped that a more fundamental theory will relate some of them.

Experimental observation of neutrino mass and oscillations cannot be accounted for in the SM. One has to introduce the neutrino masses by hand.

The GIM cancellation holds at the tree-level of the SM; it is violated in loop-mediated decays (*e.g.*,  $b \rightarrow s\gamma$ ), and can also be violated at the tree-level for extensions of the SM.

As is well-known, the couplings depend on the energy scale at which they are measured; in common parlance, they ‘run’. In the processes considered, the triple and quartic gluon couplings enter only through the running of the QCD coupling constant and in higher order QCD corrections to weak decays. The quartic electroweak couplings do not enter our discussion at the level of approximations considered.

The photonic and gluonic vertices are vectorlike ( $V$ ), the  $W^\pm$  vertices are purely  $V - A$ , whereas the  $Z$  vertices involve both  $V - A$  and  $V + A$  structures. The Higgs coupling can always be neglected unless it couples to the top quark, and in some exceptional cases, to the bottom quark. We expect that any theory which tries to answer these puzzles should leave some low-energy signatures.

### 1.4.3 SUMMARY

The phenomenology of the  $B$  system and CP violation is very active and exciting field of research. We started in Chapter 1 with a short introduction of  $B$  physics and CP violation. We discussed mainly three types of discrete symmetry (Parity, Charge-Conjugation and Time reversal) briefly. We discussed CP and CPT symmetry, as well. These symmetries are very important tools to study CP violation. Basics of the CP Phenomenology in  $B$  meson decays (charged and neutral  $B$  meson decays, and neutral  $B$  meson mixing) were discussed, too. We discussed various types of CP violation in

*B* system. We also presented a very brief discussion of the SM. The source of CP violation in SM is only through CKM matrix elements. The important parametrizations (Standard Parametrization, Wolfenstein Parametrization) were also described briefly. We also discussed the limitation of the SM at the end this chapter.

## CHAPTER 2

# STUDY OF B MESON IN STANDARD MODEL

We point out some salient features of the SM dynamics that affect B physics. The dominant decay channel of the  $b$  quark is  $b \rightarrow c$ . This leads to the semileptonic and nonleptonic decays involving  $J/\psi$  or D mesons in the final state. They have comparatively large branching ratios; so, even though they are useful to determine the shape of the unitary triangle (UT), a better place to look for indirect signals of NP is the rare B decay channels. They are of following types:

- *Leptonic*—They involve the channels  $B^+ \rightarrow \ell^+ \nu$  and  $B^0(B_s) \rightarrow \ell^+ \ell^-$ . The first one is essentially a replica of the  $\pi^+$  decay. The second one proceeds through electromagnetic penguins and  $W$ -mediated boxes.  $B_s \rightarrow \mu^+ \mu^-$  is expected to be a good candidate where potential NP (*e.g.*, supersymmetry) could show up.
- *Semileptonic*—The decays  $B \rightarrow X_{s,d} \ell^+ \nu$  and  $B \rightarrow X_{s,d} \ell^+ \ell^-$  fall in this category. They are mediated by the same operators; however, for the exclusive channels the hadronic matrix element brings some extra theoretical uncertainty in the predictions.
- *Radiative*—They include the inclusive channel  $B \rightarrow X_{s,d} \gamma$  and its exclusive counterparts, and are controlled by the magnetic penguin operators.
- *Nonleptonic*—They include all nonleptonic decays of  $b$  not involving a charm quark. Notable examples are  $B \rightarrow \phi K_S$ ,  $B \rightarrow \pi\pi$ ,  $B \rightarrow \eta' K$ , etc

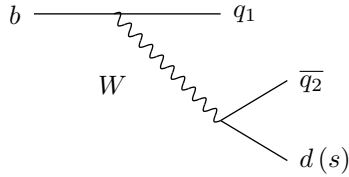


Figure 2.1: Tree diagrams ( $q_1, q_2 \in \{u, c\}$ ).

## 2.1 EFFECTIVE HAMILTONIAN

We use low-energy effective Hamiltonians [32], which are calculated by making use of the “operator product expansion”, yielding transition amplitudes of the following structure:

$$\langle f|H_{\text{eff}}|i\rangle = \frac{G_{\text{F}}}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k C_k(\mu) \langle f|Q_k(\mu)|i\rangle. \quad (2.1)$$

Here  $G_{\text{F}}$  denotes Fermi’s constant,  $\lambda_{\text{CKM}}$  is a CKM factor, and  $\mu$  denotes a renormalization scale. The technique of the operator product expansion allows us to separate the short-distance contributions to this transition amplitude from the long-distance ones, which are described by perturbative quantities  $C_k(\mu)$  (“Wilson coefficient functions”) and non-perturbative quantities  $\langle f|Q_k(\mu)|i\rangle$  (“hadronic matrix elements”), respectively. The  $Q_k$  are local operators, which are generated through the electroweak interactions and the interplay with QCD, and govern “effectively” the decay in question. The Wilson coefficients are – simply speaking – the scale-dependent couplings of the vertices described by the  $Q_k$ .

Let us consider the quark-level process  $b \rightarrow c\bar{u}s$ , which originates from a tree diagram of the kind shown in Fig. 2.1, as a simple illustration. If we

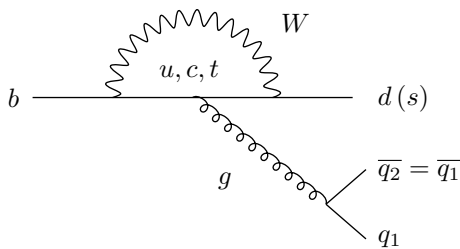


Figure 2.2: QCD penguin diagrams ( $q_1 = q_2 \in \{u, d, c, s\}$ ).

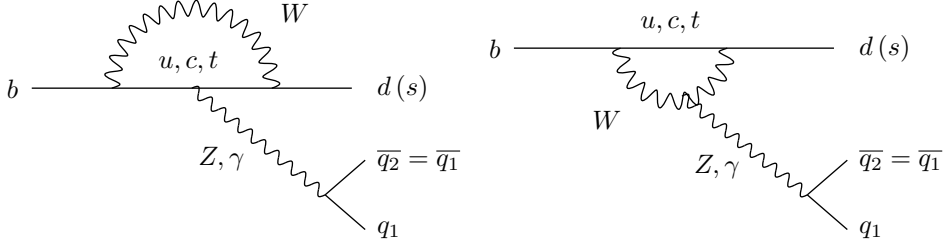


Figure 2.3: Electroweak penguin diagrams ( $q_1 = q_2 \in \{u, d, c, s\}$ ).

“integrate out” the  $W$  boson having four-momentum  $k$ , i.e. use the relation

$$\frac{g_{\nu\mu}}{k^2 - M_W^2} \xrightarrow{k^2 \ll M_W^2} -\frac{g_{\nu\mu}}{M_W^2} \equiv -\left(\frac{8G_F}{\sqrt{2}g_2^2}\right) g_{\nu\mu}, \quad (2.2)$$

we arrive at the following low-energy effective Hamiltonian:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb} O_2, \quad (2.3)$$

with the “current–current” operator

$$O_2 \equiv [\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) u_\alpha] [\bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\beta] \quad (2.4)$$

and the Wilson coefficient  $C_2 = 1$ ;  $\alpha$  and  $\beta$  are the  $SU(3)_C$  indices of QCD. Taking now QCD effects, i.e. the exchange of gluons, into account and performing a proper “matching” between the full and the effective theories, a second current–current operator,

$$O_1 \equiv [\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta] [\bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha], \quad (2.5)$$

is generated, involving a Wilson coefficient  $C_1(\mu)$ . Due to the impact of QCD, also the Wilson coefficient of  $O_2$  acquires now a renormalization-scale dependence and deviates from one. The results for the  $C_k(\mu)$  contain terms of  $\log(\mu/M_W)$ , which become large for  $\mu = O(m_b)$ , the typical scale governing the hadronic matrix elements of the four-quark operators  $O_k$ . In order to deal with these large logarithms, “renormalization-group-improved” perturbation theory offers the appropriate tool [33]. The fact that  $\langle f | H_{\text{eff}} | i \rangle$  in (2.1) *cannot* depend on the renormalization scale  $\mu$  implies a renormalization group equation, which has a solution of the following form:

$$\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W). \quad (2.6)$$



Here the “evolution matrix”  $\hat{U}(\mu, M_W)$  connects the initial values  $\vec{C}(M_W)$  encoding the whole *short-distance* physics at high-energy scales with the coefficients at scales at the level of a few GeV. Following these lines,

$$\alpha_s^n \left[ \log \left( \frac{\mu}{M_W} \right) \right]^n \text{ (LO), } \quad \alpha_s^n \left[ \log \left( \frac{\mu}{M_W} \right) \right]^{n-1} \text{ (NLO), } \quad \dots \quad (2.7)$$

can be systematically summed up, where “LO” and “NLO” stand for the leading and next-to-leading order approximations, respectively. If we apply the unitarity of the CKM matrix, we find that the corresponding CKM factors are related through

$$V_{ur}^* V_{ub} + V_{cr}^* V_{cb} + V_{tr}^* V_{tb} = 0, \quad (2.8)$$

Finally, using (2.8) to eliminate  $V_{tr}^* V_{tb}$ , we obtain an effective Hamiltonian of the following structure:

- *Current-Current:*

$$O_1 = (\bar{c}b)_{8,V-A} (\bar{s}c)_{8,V-A}, \quad O_2 = (\bar{c}b)_{1,V-A} (\bar{s}c)_{1,V-A}. \quad (2.9)$$

Only a typical combination  $\bar{s}c$  is shown; there may be other combinations.

- *QCD Penguins:*

$$\begin{aligned} O_{3(4)} &= (\bar{s}b)_{1(8),V-A} \sum_q (\bar{q}q)_{1(8),V-A}, \\ O_{5(6)} &= (\bar{s}b)_{1(8),V-A} \sum_q (\bar{q}q)_{1(8),V+A}. \end{aligned} \quad (2.10)$$

The sum runs over all the lighter flavours ( $u, d, s, c$ ).

- *Electroweak Penguins:*

$$\begin{aligned} O_{7(8)} &= \frac{3}{2} (\bar{s}b)_{1(8),V-A} \sum_q e_q (\bar{q}q)_{1(8),V+A}, \\ O_{9(10)} &= \frac{3}{2} (\bar{s}b)_{1(8),V-A} \sum_q e_q (\bar{q}q)_{1(8),V-A}. \end{aligned} \quad (2.11)$$

- *Magnetic Penguins:*

$$O_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s} \sigma^{\mu\nu} (1+\gamma_5) b F_{\mu\nu}, \quad O_{8G} = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a \quad (2.12)$$

Here  $\alpha$  and  $\beta$  are colour indices and  $T^a$  are the SU(3) generators.

- $\Delta B = 2$  Operators:

$$O(\Delta B = 2) = (\bar{d}b)_{1,V-A}(\bar{d}b)_{1,V-A} \quad (2.13)$$

This is only relevant for the SM calculation of  $B^0 - \bar{B}^0$  box.

- *Semileptonic Operators:*

$$O_{9V} = (\bar{d}b)_{1,V-A}(\bar{e}e)_V \quad O_{10A} = (\bar{d}b)_{1,V-A}(\bar{e}e)_A \quad (2.14)$$

This also contributes to leptonic decays. Again, this basis is for the SM only.

The subscripts 1 and 8 denote whether the currents are in singlet-singlet or octet-octet combination of colour SU(3), and  $V - (+)A$  stands for the Lorentz structure of  $1 - (+)\gamma_5$ .

## 2.2 LEPTONIC DECAYS

In the SM the  $B^+ \rightarrow \ell^+ \nu$  decay is of particular interest, due to its sensitivity to both the meson decay constant and the CKM matrix elements  $V_{ub}$ . In the matrix elements, only pseudoscalar and axial vector operators contribute. They are given by the PCAC [34] relations:

$$\begin{aligned} \langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^+(p) \rangle &= i f_B p_B^\mu, \\ \langle 0 | \bar{u} \gamma_5 b | B^+(p) \rangle &= -i f_B \frac{m_B^2}{m_b + m_u}. \end{aligned} \quad (2.15)$$

Assuming that the neutrino is massless we get the helicity suppressed branching fraction

$$Br(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} |V_{ub}|^2 f_b \tau_B m_B m_\ell^2 \left[ 1 - \frac{m_\ell^2}{m_B^2} \right]^2. \quad (2.16)$$

For different lepton flavours the theoretical predictions [35] are

$$Br(e^+ \nu) = 6.9 \times 10^{-12}, \quad Br(\mu^+ \nu) = 2.9 \times 10^{-7}, \quad Br(\tau^+ \nu) = 6.6 \times 10^{-5}. \quad (2.17)$$

The upper bounds on the branching fractions are  $9.8 \times 10^{-6}$  and  $1.7 \times 10^{-6}$  respectively for  $e$  and  $\mu$  modes. But branching ratio for  $\tau$  mode has been reported as  $1.4 \pm 0.4 \times 10^{-4}$  [28].

The decay  $B_q \rightarrow \ell^+ \ell^-$ , where  $q = d$  or  $s$  and  $\ell = e, \mu$  or  $\tau$ , proceeds through loop diagrams and is of fourth order in the weak coupling. In the

SM, the dominant contributions to this decay come from the  $W$  box and  $Z$  penguin diagrams. Because the contributions with a top quark in the loop are dominant, at low energies of order  $m_b$  the decay can be described by a local  $\bar{b}q\bar{\ell}\ell$  coupling via the effective Hamiltonian (2.14), and the branching fraction is given by

$$Br(B^0 \rightarrow \ell^+ \ell^-) = \frac{G_F^2}{8\pi} f_B^2 \tau_B m_B^3 \sqrt{\left(1 - \frac{4m_\ell^2}{m_B^2}\right)} \left[ |C_P^{\ell\bar{\ell}} - \frac{2m_\ell}{m_B} C_A^{\ell\bar{\ell}}|^2 + \left(1 - \frac{4m_\ell^2}{m_B^2}\right) |C_P^{\ell'\bar{\ell}'}|^2 \right] \quad (2.18)$$

In the SM  $C_P^{\ell\bar{\ell}}$  and  $C_P^{\ell'\bar{\ell}'}$  arise from penguin diagram with physical and unphysical neutral scalar exchange, and are suppressed by a factor  $(m_b/m_W)^2$ . The decay rate is controlled by the coefficient

$$\left[ C_A^{\ell\bar{\ell}} \right]_{SM} = \frac{\alpha V_{tb} V_{td}^*}{\sqrt{8\pi} \sin^2 \theta_w} Y(x_t) \quad (2.19)$$

where  $x_t \equiv \frac{m_t^2}{m_W^2}$ ,  $\sin^2 \theta_w$  is the weak mixing angle, and the function  $Y(x_t)$  is given by  $Y(x_t) \approx 1.03 Y_0(x)$ , where

$$Y_0(x) = \frac{x}{8} \left[ \frac{x-4}{x-1} + \frac{3x}{(x-1)^2} \log(x) \right] \quad (2.20)$$

For different lepton flavour the SM branching fractions are [31]

$$Br(e^+ e^-) = 2.6 \times 10^{-15}, \quad Br(\mu^+ \mu^-) = 1.1 \times 10^{-10}, \quad Br(\tau^+ \tau^-) = 3.1 \times 10^{-8}. \quad (2.21)$$

These numbers show that purely leptonic decays are too rare to be observed unless they are significantly enhanced by NP. The upper bounds on the branching fractions are [28]:  $11.3 \times 10^{-8}$  ( $e^+ e^-$ ),  $1.5 \times 10^{-8}$  ( $\mu^+ \mu^-$ ), and  $4.1 \times 10^{-3}$  ( $\tau^+ \tau^-$ ).

### 2.3 SEMILEPTONIC DECAYS

The semileptonic inclusive decay  $B \rightarrow X_{s,d} \ell^+ \ell^-$ , originating from the parton level process  $b \rightarrow s(d) \ell^+ \ell^-$ , can be calculated using the effective Hamiltonian formalism as:

$$A(B \rightarrow X_s \ell^+ \ell^-) = \frac{\sqrt{2} G_F \alpha}{\pi} V_{tb} V_{ts}^* [C_9^{eff} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \ell + C_{10} \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell - 2C_7^{eff} m_b \bar{s}_L i \sigma^{\mu\nu} \frac{q_\nu}{q^2} b_R \bar{\ell} \gamma_\mu \ell], \quad (2.22)$$

where  $q^2$  is the momentum transferred to the lepton pair. The WCs  $C_7$  and  $C_9$  contain, apart from the RG evolutions of  $C_7$  and  $C_9$  at the weak scale, mixing effects with operators  $O_{1-6}$  (for  $C_9$ ) and  $O_2$  and  $O_8$  (for  $C_7$ ); hence the superscript. There is also a sizable long-distance coming from  $B \rightarrow K^{(*)}\psi$  and  $\psi \rightarrow \ell^+\ell^-$ , where  $\psi$  is a generic vector  $c\bar{c}$  state.

For the exclusive decays, one needs to compute the following matrix elements, given by

$$\langle K(p')|\bar{s}\gamma^\mu b|B(p)\rangle = (p+p')^\mu F_1(q^2) + \frac{m_B^2 - m_K^2}{q^2} (F_0(q^2) - F_1(q^2)) \quad (2.23)$$

$$\langle K(p')|\bar{s}i\sigma^{\mu\nu}q_\nu b|B(p)\rangle = \left[ (p+p')^\mu q^2 - (m_B^2 - m_K^2)q^\mu \right] \frac{F_T(q^2)}{m_B + m_K}. \quad (2.24)$$

$$\begin{aligned} \langle K^*(p', \epsilon)|\bar{s}\gamma^\mu(1 - \gamma_5)b|B(p)\rangle &= \epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^*p_\alpha p'_\beta \frac{2V(q^2)}{m_B + m_{K^*}} \\ &- i\epsilon^{*\mu}(m_B + m_{K^*})A_1(q^2) + i(\epsilon^*.q)(p+p')^\mu \frac{A_2(q^2)}{m_B + m_{K^*}} \\ &+ i(\epsilon^*.q)\frac{2m_{K^*}}{q^2} (A_3(q^2) - A_0(q^2))q^\mu. \end{aligned} \quad (2.25)$$

$$\begin{aligned} \langle K^*(p', \epsilon)|\bar{s}\sigma^{\mu\nu}q_\nu \frac{(1 + \gamma_5)}{2} b|B(p)\rangle &= i\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^*p_\alpha p'_\beta 2T_1(q^2) \\ &+ \left[ \epsilon^{*\mu}(m_B^2 - m_{K^*}^2) - (\epsilon^*.q)(p+p')^\mu \right] T_2(q^2) \\ &+ (\epsilon^*.q) \left[ q^\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p+p')^\mu \right] T_3(q^2). \end{aligned} \quad (2.26)$$

Here,  $q = p - p'$  is the momentum transferred to the lepton pair,  $F_1(0) = F_0(0)$ ,  $A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2)$ ,  $A_3(0) = A_1(0)$ , and  $T_1(0) = iT_2(0)$ . The value of  $T_1(0)$  is fixed from  $B \rightarrow K^*\gamma$  decay rate. These form factors have been estimated by different groups using different techniques; table 2.1 summarises the SM predictions [36, 37, 38, 39, 40, 41]. The branching ratios ( in  $10^{-6}$ ) of these observed decay modes [28] are given in this Table 2.2:

Here  $\ell^+\ell^-$  indicates an average over all leptonic channels. There are upper limits on lepton-flavour violating modes. This places tight constraints on the parameter spaces of NP. Decay asymmetries and final state lepton polarizations are also sensitive observables on the structure of NP.

Technique	$Br(B \rightarrow K\ell^+\ell^-)$	$Br(B \rightarrow K^*e^+e^-)$	$Br(B \rightarrow K^*\mu^+\mu^-)$
Lattice QCD		1.15	1.15
QCD sum rules	0.3	1	1
Light cone sum	$0.25 \pm 0.07$		0.95
Quark model1	$0.62 \pm 0.13$	$2.1 \pm 0.7$	$1.5 \pm 0.6$
Quark model2	0.5		1.4
Quark model3	$0.59 \pm 0.22$	$3.4 \pm 1.3$	$2.2 \pm 0.9$

Table 2.1: Exclusive branching ratios in different theoretical models. All numbers are to be multiplied by  $10^{-6}$ .

Decay modes	Br. (in $10^{-6}$ )	Decay modes	Br. (in $10^{-6}$ )
$B \rightarrow Ke^+e^-$	$0.42 \pm 0.06$	$B \rightarrow K^*e^+e^-$	$1.24 \pm 0.19$
$B \rightarrow K\mu^+\mu^-$	$0.47 \pm 0.06$	$B \rightarrow K^*\mu^+\mu^-$	$1.08 \pm 0.15$
$B \rightarrow K\ell^+\ell^-$	$0.43 \pm 0.04$	$B \rightarrow K^*\ell^+\ell^-$	$1.00 \pm 0.11$

Table 2.2: Observed branching ratios in  $10^{-6}$ .

## 2.4 RADIATIVE DECAYS

The inclusive decay  $B \rightarrow X_s\gamma$ , and its parton-level counterpart  $b \rightarrow s\gamma$ , proceeds through the magnetic penguin operator  $O_{7\gamma}$ . The complete NLO calculation is available and the theoretical prediction of  $Br(B \rightarrow X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4}$  agrees pretty well with the experimental result of  $(3.56 \pm 0.25) \times 10^{-4}$  [28].  $B \rightarrow X_d\gamma$  is CKM suppressed; the theoretical estimate is

$$0.017 < \frac{Br(B \rightarrow X_d\gamma)}{Br(B \rightarrow X_s\gamma)} < 0.074 \quad (2.27)$$

where the uncertainty is largely due to the poorly determined CKM elements. Determination of this ratio should give a clean measurement of  $|V_{td}/V_{ts}|$ .

## 2.5 NONLEPTONIC DECAYS

For the exploration of CP violation, non-leptonic  $B$  decays play the key role. The final states of such transitions consist only of quarks, and they are mediated by  $b \rightarrow q_1 \bar{q}_2 d (s)$  quark-level processes, with  $q_1, q_2 \in \{u, d, c, s\}$ . There are basically two types of topologies contributing to such decays: “tree” and “penguin” topologies. The latter consist of gluonic (QCD) and electroweak (EW) penguins. The corresponding leading-order Feynman diagrams are

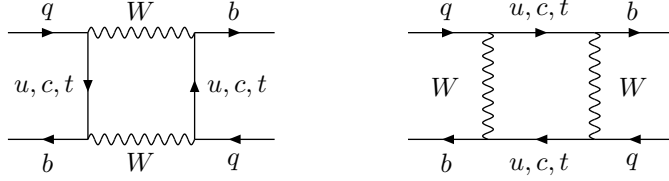


Figure 2.4: Box diagrams contributing to  $B_q^0-\overline{B}_q^0$  mixing ( $q \in \{d, s\}$ ).

shown in Figs. (2.1), (2.2) and (2.3). We can categorize the non-leptonic  $b \rightarrow q_1 \bar{q}_2 d (s)$  decays, depending on the flavour content of their final states as follows:

- $q_1 \neq q_2 \in \{u, c\}$ : *only* tree diagrams contribute.
- $q_1 = q_2 \in \{u, c\}$ : tree *and* penguin diagrams contribute.
- $q_1 = q_2 \in \{d, s\}$ : *only* penguin diagrams contribute.

We can use low-energy effective Hamiltonian for the description of weak  $B$ -meson decays, as well as  $B_q^0-\overline{B}_q^0$  mixing. At a renormalization scale  $\mu = O(m_b)$ , the Wilson coefficients of the current-current operators are  $C_1(\mu) = O(10^{-1})$  and  $C_2(\mu) = O(1)$ , whereas those of the penguin operators are as large as  $O(10^{-2})$ . Although short-distance part is tamed, the long-distance piece still suffers large theoretical uncertainties. Non-leptonic decay “ $\overline{B} \rightarrow \overline{f}$ ” is given by the hadronic matrix elements  $\langle \overline{f} | Q_k(\mu) | \overline{B} \rangle$  of the four-quark operators. It is very difficult to solve this hadronic matrix element; An easy way to solve it by the product of the matrix elements of two quark currents under favourable “factorization scale”  $\mu = \mu_F$ . This procedure can be justified in the large- $N_C$  approximation,[42, 43] however, it is in general not on solid ground. Interesting theoretical progress could be made through the development of the QCD factorization (QCDF) [44] and perturbative QCD (PQCD) [45] approaches, the soft collinear effective theory (SCET), [46] and QCD light-cone sum-rule methods. [47] An important target of these methods is given by  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays. Since the data indicate large non-factorizable corrections, [48, 49, 50] the long-distance contributions to these decays remain a theoretical challenge. The best way to deal with all such problems is through model independent way. We will discuss in a bit detail about the model independent way to explain the data what it is confronting with the theoretical model’s predictions.

Within the SM,  $B^0-\overline{B}^0$  mixing arises from the box diagrams shown in Fig. 2.4 and it is dominated by only single weak phase. In this special case,

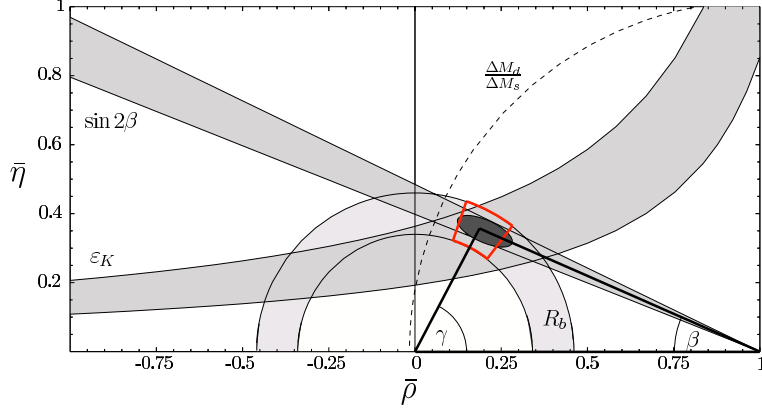


Figure 2.5: The current situation in the  $\bar{\rho}$ - $\bar{\eta}$  plane.

there is no direct CP violation ( $a^{dir} = 0$ , *i.e.*  $|\lambda_f| = 1$  or  $|A_f| = |\bar{A}_f|$ ) and no indirect CP violation (*i.e.*  $\frac{q_B}{p_B} = \eta_f e^{2i\phi_M}$ ) but there is CP violation only due to interference between decay and mixing. Therefore, we can write the SM amplitude for this process as

$$A_f = |A_f| e^{i\delta} e^{i\phi_w} \quad (2.28)$$

where  $\delta$  is strong phase, invariant under CP transformation, and  $\phi_w$  is a weak phase coming from CKM matrix and it changes under CP transformation. The CP-conjugate amplitude for this process is

$$\bar{A}_f = |A_f| e^{i\delta} e^{-i\phi_w} \quad (2.29)$$

$$\lambda_f = \eta_f e^{i(2\phi_M - 2\phi_w)} \implies \text{Im}\lambda_f = \eta_f \sin 2(\phi_M - \phi_w) \quad (2.30)$$

we define  $\phi = \phi_M - \phi_w$ .

The corresponding time-dependent CP asymmetry now takes the following simple form:

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow \bar{f})}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow \bar{f})} \Big|_{\Delta\Gamma=0} = \eta_f \sin 2\phi \sin(\Delta mt), \quad (2.31)$$

and allows an elegant determination of  $\sin 2\phi$ .

## 2.6 EXTRACTION OF CKM ANGLES

The extraction of CKM angles are essential to study of CP violation. Theory is used to convert experimental data into contours in the  $\bar{\rho}$ - $\bar{\eta}$  plane,

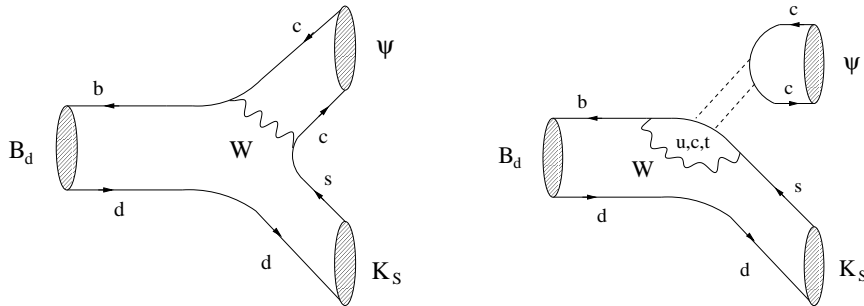


Figure 2.6: Feynman diagrams contributing to  $B_d^0 \rightarrow J/\psi K_S$ . The dashed lines in the penguin topology represent a colour-singlet exchange.

where semileptonic  $b \rightarrow u\bar{\nu}_\ell, c\bar{\nu}_\ell$  decays and  $B_{d,s}^0 - \bar{B}_{d,s}^0$  mixing allow us to determine the UT sides  $R_b \equiv \left| \frac{V_{ud} V_{ub}}{V_{cd} V_{cb}} \right|$  and  $R_t \equiv \left| \frac{V_{td} V_{tb}}{V_{cd} V_{cb}} \right|$ , respectively, *i.e.* to fix two circles in the  $\bar{\rho}-\bar{\eta}$  plane. On the other hand, the indirect CP violation in the neutral kaon system described by  $\varepsilon_K$  can be transformed into a hyperbola. It is possible to convert measurements of CP-violating effects in  $B$ -meson decays into direct information on the UT angles. The most prominent example is the determination of  $\sin 2\beta$  through  $B_d \rightarrow J/\psi K_S$ .

In Fig. (2.5), the shaded dark ellipse is the result of a CKM fit [52], the straight lines represent the measurement of  $\sin 2\beta$  and the quadrangle corresponds to a determination of  $\alpha$  from  $B_d \rightarrow \pi^+ \pi^-$  decays, etc, [28, 26, 52].

The data is mostly consistent with the SM. Additionally, the recent data for  $B \rightarrow \pi\rho, \rho\rho$  as well as  $B_d \rightarrow D^{(*)\pm} \pi^\mp$  and  $B \rightarrow DK$  decays give constraints for the UT that are also in accordance with the KM mechanism. But there is still hope to encounter deviations from the SM despite this consistent picture. The  $B$ -meson system provides a variety of processes for the exploration of CP violation [26]. Rare  $B$  and  $K$  meson decays, originating from loop effects in the SM, provide complementary insights into flavour physics.

### 2.6.1 EXTRACTION OF ANGLE $\beta$

The decay mode “ $B_d^0 \rightarrow J/\psi K_S$ ” (at quark level transition  $\bar{b} \rightarrow \bar{c}c\bar{s}$ ) is very important to extract one of the CKM angle “ $\beta$ ”. Both diagrams (tree and penguin like topologies, Fig.(2.6) are needed. We can write the correspond-



ing amplitude as[54]

$$A(B_d^0 \rightarrow J/\psi K_S) = \lambda_c^{(s)} (A_t^{c'} + A_{\text{pen}}^{c'}) + \lambda_u^{(s)} A_{\text{pen}}^{u'} + \lambda_t^{(s)} A_{\text{pen}}^{t'}, \quad (2.32)$$

where  $A_t^{c'}$  denotes the “tree” processes and the  $A_{\text{pen}}^{q'}$  describe the contributions from penguin topologies with internal  $q$  quarks ( $q \in \{u, c, t\}$ ). These penguin amplitudes take into account both QCD and EW penguin contributions. We define the CKM factors for the transition  $\bar{b} \rightarrow \bar{s}$  as:

$$\lambda_q^{(s)} \equiv V_{qb}^* V_{qs} \quad (2.33)$$

Now we use the unitarity of the CKM matrix to eliminate  $\lambda_t^{(s)}$  through  $\lambda_t^{(s)} = -\lambda_u^{(s)} - \lambda_c^{(s)}$ , and the Wolfenstein parametrization, it can be written as:

$$A(B_d^0 \rightarrow J/\psi K_S) \propto [1 + \lambda^2 a e^{i\theta} e^{i\gamma}], \quad (2.34)$$

where the hadronic parameter  $a e^{i\theta}$  measures, the ratio of penguin- to tree-diagram-like contributions to  $B_d^0 \rightarrow J/\psi K_S$ . Since this parameter enters in a doubly Cabibbo-suppressed way, we can neglect the term to good approximation and using the definition of  $a^{dir}$  and  $a^{mix}$

$$a^{dir}(B_d^0 \rightarrow J/\psi K_S) = 0, \quad a^{mix}(B_d \rightarrow J/\psi K_S) = -\sin 2\beta. \quad (2.35)$$

where  $\phi = 2\beta$  in the SM.

The present status of  $\sin 2\beta$  for the decay mode  $B_d \rightarrow J/\psi K_S$  is given as follows:

$$\sin 2\beta = \begin{cases} 0.660 \pm 0.036 \pm 0.012 & \text{(BaBar [55])} \\ 0.643 \pm 0.038 & \text{(Belle [56])}, \end{cases} \quad (2.36)$$

yielding the world average for all charmonium modes [57]

$$\sin 2\beta = 0.672 \pm 0.024, \quad (2.37)$$

which agrees well with the results of the “standard analysis” of the unitarity triangle, implying  $0.6 \leq \sin 2\beta \leq 0.9$ .

We are expecting that the experimental accuracy of the measurement of  $\sin 2\beta$  will be increased by one order of magnitude in near future. Therefore, it is possible to obtain deeper insights (due to high accuracy and precision) into the theoretical uncertainties affecting Eq. (2.35) due to penguin contributions.

Although the agreement between (2.37) and the results of the CKM fits is striking, there is still some room for NP that may hide in  $a^{mix}(B_d \rightarrow J/\psi K_S)$ . The key quantity is actually  $\beta$ , which is fixed through  $\sin 2\beta = 0.65 \pm 0.025$  up to a twofold ambiguity,

$$\beta = (21.1 \pm 0.9)^\circ \vee (68.9 \pm 1.0)^\circ. \quad (2.38)$$

Here the former solution would be in perfect agreement with the range implied by the CKM fits,  $40^\circ \leq 2\beta \leq 60^\circ$ , whereas the latter would correspond to NP. The two solutions can be distinguished through a measurement of the sign of  $\cos 2\beta$ : in the case of  $\cos 2\beta = +0.74 > 0$ , we would conclude  $\beta = 21^\circ$ , whereas  $\cos 2\beta = -0.74 < 0$  would point towards  $\beta = 69^\circ$ , *i.e.* NP. There are several ways to resolve the two-fold ambiguity in the extraction of  $2\beta$  [58, 59, 60, 61] but they are rather challenging practically. The BaBar and Belle collaborations have performed measurements of  $\sin 2\beta$  and  $\cos 2\beta$  in time-dependent transversity analyses of the pseudoscalar to vector-vector decay  $B^0 \rightarrow J/\psi K^*$ , where  $\cos 2\beta$  enters as a factor in the interference between CP-even and CP-odd amplitudes.

An important testing ground for the SM description of CP violation is also provided by the decay mode  $B \rightarrow \phi K$ . The decay modes are governed by QCD penguin processes [63], and EW penguins [64, 65]. Therefore,  $B \rightarrow \phi K$  modes represent a sensitive probe for NP, too. In the SM, we have the following relations [66, 67, 68, 69]:

$$a^{dir}(B_d \rightarrow \phi K_S) = 0 + \mathcal{O}(\lambda^2) \quad (2.39)$$

$$a^{mix}(B_d \rightarrow \phi K_S) = a^{mix}(B_d \rightarrow J/\psi K_S) + \mathcal{O}(\lambda^2). \quad (2.40)$$

All the modes  $B_d \rightarrow \phi K_S$ ,  $B^\pm \rightarrow \phi K^\pm$  can be used to get the whole picture [69] but NP still cannot be distinguished from the SM [53, 69].

Till date, the summary of experimental report is as follows:

$$a^{dir}(B_d \rightarrow \phi K_S) = -0.01 \pm 0.12 \quad (2.41)$$

$$a^{mix}(B_d \rightarrow \phi K_S) = 0.39 \pm 0.17 \quad (2.42)$$

Unfortunately, the experimental uncertainties are still very large. Because of  $a^{mix}(B_d \rightarrow J/\psi K_S) = -0.65 \pm 0.025$  (see (2.35) and (2.37)), there were already speculations about new-physics effects in  $B_d \rightarrow \phi K_S$  [72]. In this context, it is interesting to note that there are more data available [28]:

$$a^{dir}(B_d \rightarrow \eta' K_S) = -0.04 \pm 0.20 \quad (2.43)$$

$$a^{mix}(B_d \rightarrow \eta' K_S) = 0.43 \pm 0.17 \quad (2.44)$$

$$a^{dir}(B_d \rightarrow K^+ K^- K_S) = 0.07 \pm 0.36 \pm 0.08 \quad (2.45)$$

$$a^{mix}(B_d \rightarrow K^+ K^- K_S) = -0.74 \pm 0.11. \quad (2.46)$$

The corresponding modes are governed by the same quark-level transitions as  $B_d \rightarrow \phi K_S$ . Consequently, there is not clear cut signals of NP in  $B_d \rightarrow \phi K_S$  [62]. It is expected that the experimental situation will be improved significantly in the future.

### 2.6.2 EXTRACTION OF ANGLE $\alpha$

The CKM angle  $\gamma$  can be determined by the mode  $B_d^0 \rightarrow \pi^+ \pi^-$  ( $\bar{b} \rightarrow \bar{u} u \bar{d}$  quark-level transitions) in the  $B$  factories. The corresponding decay amplitude can then be written as [73]:

$$A(B_d^0 \rightarrow \pi^+ \pi^-) = \lambda_u^{(d)} (A_t^u + A_{\text{pen}}^u) + \lambda_c^{(d)} A_{\text{pen}}^c + \lambda_t^{(d)} A_{\text{pen}}^t. \quad (2.47)$$

On using the unitarity of the CKM matrix and Wolfenstein parametrization, we obtain

$$A(B_d^0 \rightarrow \pi^+ \pi^-) \propto [e^{i\gamma} - d e^{i\theta}], \quad (2.48)$$

where

$$d e^{i\theta} \equiv \frac{1}{R_b} \left( \frac{A_{\text{pen}}^c - A_{\text{pen}}^t}{A_{\text{CC}}^u + A_{\text{pen}}^u - A_{\text{pen}}^t} \right) \quad (2.49)$$

measures the ratio of penguin to tree contributions in  $B_d \rightarrow \pi^+ \pi^-$ . In contrast to the  $B_d^0 \rightarrow J/\psi K_S$  amplitude (2.34), this parameter does not enter in (2.48) in a doubly Cabibbo-suppressed way, thereby leading to the well-known ‘‘penguin problem’’ in  $B_d \rightarrow \pi^+ \pi^-$ . If we had negligible penguin contributions, i.e.  $d = 0$ , the corresponding CP-violating observables were given as follows:

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-) = 0, \quad \mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-) = \sin(2\beta + 2\gamma) = -\sin 2\alpha, \quad (2.50)$$

The phases  $2\beta = \phi$  and  $\gamma$  enter directly in the  $B_d \rightarrow \pi^+ \pi^-$  observables. Consequently, we can fix  $\phi$  through  $B_d \rightarrow J/\psi K_S$  and we can use  $B_d \rightarrow \pi^+ \pi^-$  to probe  $\alpha$ . Measurements of CP asymmetries in the modes  $B_d \rightarrow \pi^+ \pi^-$  are already available [28]:

$$a^{dir}(B_d \rightarrow \pi^+ \pi^-) = -0.38 \pm 0.17, \quad (2.51)$$

$$a^{mix}(B_d \rightarrow \pi^+ \pi^-) = 0.61 \pm 0.08 \quad (2.52)$$

where the errors in brackets are the ones increased by the PDG scaling-factor procedure [76]. For direct CP violation at this level, it is required large penguin contributions with large CP-conserving strong phases. A significant impact of penguins on  $B_d \rightarrow \pi^+\pi^-$  is also indicated by data on  $B \rightarrow K\pi, \pi\pi$  decays, as well as by theoretical considerations [77, 78]. Consequently, it is already evident that the penguin contributions to  $B_d \rightarrow \pi^+\pi^-$  cannot be neglected.

The penguin problem can be tackled in many ways to extract the weak phases from the CP-violating  $B_d \rightarrow \pi^+\pi^-$  observables; but the isospin analysis of the  $B \rightarrow \pi\pi$  system [79] is the best theoretical way to extract the  $\alpha$ . This can be achieved by using isospin to relate the amplitudes of  $B_d^0 \rightarrow \pi^+\pi^-$ ,  $B_d^0 \rightarrow \pi^0\pi^0$ , and  $B_u^+ \rightarrow \pi^+\pi^0$ . The  $B$  has only  $I = \frac{1}{2}$  state and  $\pi\pi$  final states can have only  $I = 0$  or  $I = 2$ . The tree diagram can lead to either  $I = 0$  or  $I = 2$  final states but gluonic penguin can lead to only  $I = 0$  state; i.e. the  $\Delta I = \frac{3}{2}$  operator occurs purely as a tree diagram, but  $\Delta I = \frac{1}{2}$  operator has both tree and penguin diagrams. Therefore,  $B_u^+ \rightarrow \pi^+\pi^0$  arises only from tree contribution. The amplitudes for  $B_d^0 \rightarrow \pi^+\pi^-$ ,  $B_d^0 \rightarrow \pi^0\pi^0$  and  $B_u^+ \rightarrow \pi^+\pi^0$  can be written in terms of the  $I = 0$  and  $I = 2$  pieces as [80]:

$$1/\sqrt{2}\mathcal{A}^{+-} = A_2 - A_0, \quad (2.53)$$

$$\mathcal{A}^{00} = 2A_2 + A_0, \quad (2.54)$$

$$\mathcal{A}^{+0} = 3A_2, \quad (2.55)$$

where  $A_0$  and  $A_2$  for  $I = 0$  and  $I = 2$ , respectively. Using the above relations, it is easy to get the triangle relations

$$\frac{1}{\sqrt{2}}\mathcal{A}^{+-} + \mathcal{A}^{00} = \mathcal{A}^{+0}, \quad (2.56)$$

$$\frac{1}{\sqrt{2}}\overline{\mathcal{A}}^{+-} + \overline{\mathcal{A}}^{00} = \overline{\mathcal{A}}^{+0}. \quad (2.57)$$

Sides of the above triangle are obtained from the magnitude of decay amplitudes. Consequently  $|A_2|$  are obtained directly from  $|\mathcal{A}^{+0}|$ .  $|A_0|$  and  $\cos\theta$  ( $\theta$  is the angle between  $A_2$  and  $A_0$ ) are also obtained from the triangle. Under the assumption that direct CP asymmetry is zero and using the Eqs. (1.17), (1.19), (1.26) and (1.29), it is possible to obtain  $\sin 2\alpha$  from the two modes:  $B_d^0 \rightarrow \pi^+\pi^-$  and  $B_d^0 \rightarrow \pi^0\pi^0$ . The angle  $\alpha$  can also be measured from  $B \rightarrow \rho\pi$  mode [83] and from  $B \rightarrow \rho\rho$  mode [84]. There are also various other ways to obtain  $\alpha$  [73, 85, 86, 87, 88, 89].

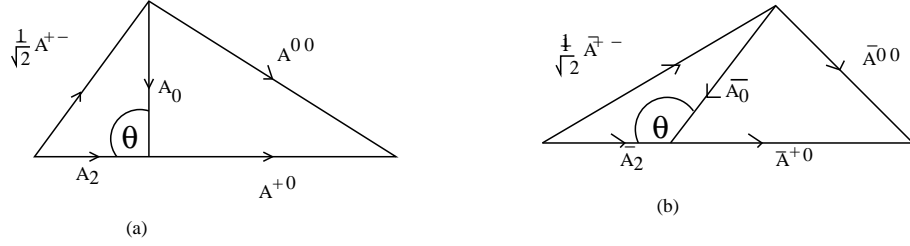


Figure 2.7: Complex triangle of (a) Eq. (2.56) and (b) Eq. (2.57).

### 2.6.3 EXTRACTION OF ANGLE $\gamma$

The determination of the angle  $\gamma$  of the unitarity triangle of the CKM matrix is regarded as a challenge for  $B$ -physics experiments. In the SM,  $\gamma$  is the relative phase between  $b \rightarrow \bar{c}us$  and  $b \rightarrow \bar{c}us$  transition; *i.e.*  $\bar{u}c(u\bar{c})$  hadronize into single  $D^0(\bar{D}^0)$  meson, which is subsequently seen as a CP eigenstate and lead to a common final state. Therefore, these two channels interfere and giving CP violating effects.

The Gronau-London-Wyler (GLW) method extracts the angle  $\gamma$  from the measurements of the branching ratios of the six processes,  $B^+ \rightarrow K^+D^0$ ,  $B^+ \rightarrow K^+\bar{D}^0$ ,  $B^+ \rightarrow K^+(f_{CP})_D$ , and their CP-conjugate processes; where  $(f_{CP})_D$  is a CP eigenstate (*e.g.*  $f_{CP} = \pi^+\pi^-$ , etc.). The two interfering amplitudes have a CP violating phase difference  $\gamma$ . But the problem in this method is that CP violating asymmetries tend to be small since  $B^+ \rightarrow K^+D^0$  is color suppressed, whereas  $B^+ \rightarrow K^+\bar{D}^0$  is color allowed. The former amplitude is an order of magnitude smaller than the latter. To overcome this, Atwood-Dunietz-Soni (ADS) suggested that analysis should be performed with non-CP eigenstates; especially appealing are modes  $f$  such that  $D^0 \rightarrow f$  is doubly Cabibbo suppressed while  $\bar{D}^0 \rightarrow f$  is Cabibbo allowed (*e.g.*  $f = K^+\pi^-$ , etc.). As a result, the interfering amplitudes become comparable. By measuring the branching ratios of the decay of the D mesons, the CKM angle  $\gamma$  can be extracted.

$$\gamma = \begin{cases} (92 \pm 41 \pm 11 \pm 12)^\circ & (\text{BaBar [55]}) \\ (53 \pm 16 \pm 3 \pm 9)^\circ & (\text{Belle [56]}), \end{cases} \quad (2.58)$$

The precision on the angle  $\gamma$  from  $B \rightarrow DK$  has to be improved to at least 5% to match the precision of its indirect estimate from a global fit to CKM parameters. At present, it has large error but it's precision will be improved in LHCb. LHCb measurement of the angle  $\gamma$  and the indirect determination of  $\gamma$  will provide a stringent test of the SM.

## 2.7 EVIDENCE OF DIRECT CP ASYMMETRY

CP violation arises via the interference of at least two diagrams with comparable amplitudes but different CP conserving and violating phases. In golden mode ( $B \rightarrow J/\psi K_S$ ), mixing induced CP violation has been established. In B meson system, sizable direct CP violation is also expected in the SM. The first experimental evidence for direct CP violation in B mesons was shown by Belle for the decay mode  $B^0 \rightarrow \pi^+\pi^-$  [93]. This result shows large interference between penguin and tree diagrams [94]. Belle [95] and Babar [96] have also reported direct CP violation in  $B^0 \rightarrow K^+\pi^-$ .

## 2.8 HINT OF NEW PHYSICS

Constraining the angles  $\alpha$ ,  $\beta$  and  $\gamma$  by CP asymmetries is complementary to these CP conserving measurements. The asymmetry measurements involve the discrete ambiguities in the angles, which ought to be resolved [97]. In near future, we will have better understanding not only of the CKM parameters but also of inconsistencies, if any beyond the SM level. It would be important to identify the source of the inconsistencies in a model-independent way and then we can hunt for new theory to explain the existing data.

Physics beyond the SM can modify CKM phenomenology and predictions for CP asymmetries by introducing additional contributions in three types of amplitudes:

- $B^0 - \bar{B}^0$  and  $B_s^0 - \bar{B}_s^0$  mixing amplitudes.
- Penguin decay amplitudes (both gluonic and electroweak).
- Tree decay amplitudes (both color allowed and color suppressed).

Large variety of models can show the possibility of 1st type. The mixing amplitudes can be identified either by measuring asymmetries which lie outside the allowed range, or by comparison with mixing-unrelated constraints. New contributions in decay amplitudes [98] are usually small and can be detected by comparing asymmetries in different processes. Extremely small asymmetries under KM hypothesis are particularly sensitive to new amplitudes.

We list a few examples of signals for NP.

- Rate enhancement beyond the SM predictions for electroweak penguin decays,  $B \rightarrow X_{d,s}\ell^+\ell^-$ ,  $B^0/B_s \rightarrow \ell^+\ell^-$ .

- Lepton asymmetry  $A_{sl} \geq \mathcal{O}(10^{-2})$ .
- Different asymmetries in  $B^0(t) \rightarrow \psi K_S, \phi K_S, \eta' K_S$ .
- Sizable asymmetries in  $b \rightarrow s\gamma$  or  $B_s \rightarrow \psi\phi$ .
- Contradictory constraints on  $\gamma$  from  $B \rightarrow K\pi, B \rightarrow DK, B_s \rightarrow D_s K$ .

## 2.9 SUMMARY

In this chapter, we have reviewed some important features of  $B$  meson decays in the frame-work of SM. We discussed various important decay channels (Leptonic, Semileptonic, Radiative and Nonleptonic) with the help of effective Hamiltonian. Babar and Belle group have established CP violation in the  $B$  system. We discussed briefly the extraction of the CKM angles  $\alpha, \beta$  and  $\gamma$ . The main goal is to overconstrain the unitarity triangle as much as possible so that we can perform a stringent test of the KM mechanism of CP violation. For this reason many important benchmark modes ( $B \rightarrow \pi\pi, B \rightarrow \phi K$  and  $B \rightarrow K\pi$  decays, etc) are being explored. We also explored the possible hint of NP beyond the SM. In the near future, we can understand better the physics regarding CP violation.

## CHAPTER 3

# STUDY OF NEW PHYSICS

### 3.1 INTRODUCTION

CP violations in  $B$  decays are a sensitive probe of NP in the quark sector, because they may differ from the SM predictions if the sources of CP violation are beyond the CKM phase of the SM. This can occur in the following ways:

- If the unitarity of the three-generation CKM matrix does not hold.
- If there are convincing contributions to  $B - \bar{B}$  mixing beyond the box diagram with intermediate top quarks.
- The constraints on the CKM parameters may change if there are significant new contributions to  $B - \bar{B}$  mixing and to  $\epsilon_K$ .
- $\Gamma_{12} \ll M_{12}$ . To violate this relation, one needs a new dominant contribution to tree decays of  $B$  mesons, which is highly unlikely.
- The relevant decay processes are dominated by SM diagrams. Consequently, it is highly unlikely that NP (primarily takes place at a high energy scale) would compete with weak tree decays. Therefore, there is a high chance to get the significant contributions from NP for penguin dominated decays.

Within the SM, CP asymmetries in both  $B$  decays and  $B - \bar{B}$  mixing are determined by combinations of CKM elements. The asymmetries then measure the relative phase between these combinations. Unitarity of the CKM matrix relates these phases to angles of the unitarity triangles. In models with NP, unitarity of the three-generation charged-current mixing matrix



may be lost and consequently the relation between the CKM phases and angles of the unitarity triangle violated [99].

Thus, when studying CP asymmetries in models of NP, we look for violation of the unitarity constraints and, even more importantly, for contributions to  $B - \bar{B}$  mixing that are different in phase and not much smaller in magnitude than the SM contribution. But it is better to study about CP asymmetries model independently. If we take the value of branching ratios, CP asymmetries from experiment and we want to extract the NP effect beyond the SM, then we can say that it is a clean signal of NP. But sometimes it is not possible to extract NP effect even if there is a contribution from NP. We discuss the impact of reparametrization invariance, physical results are invariant under this reparametrization, to search for the pattern of NP in  $B$  decays.

### 3.2 REPARAMETRIZATION INVARIANT

The source of CP violation in the SM of electroweak interactions is one single irremovable phase in the CKM matrix. We can exploit  $B$  physics experiments to uncover NP effects [?, 100, 101]. The extraction of CKM matrix elements is complicated due to hadronic uncertainty. In a few cases, it is possible to remove such hadronic matrix elements and we can relate experimental observables with parameters in the original Lagrangian of the electroweak theory. To study the NP, we can write the matrix elements in two parts one coming from the SM and another from beyond the SM. We know that any complex number can be written as linear combination of two known complex number; e.g. let  $2 + i$  and  $5 + 3i$  are known (given) complex number and we can write  $9 + 5i = 2(2 + i) + 1(5 + 3i)$ , here  $2 + i$  and  $5 + 3i$  are considered as basis vectors. Therefore, any decay amplitude can be described by *any two weak phases* considered as basis phases chosen completely at random (as long as they do not differ by a multiple of  $180^\circ$ ); we can change the basis utilized to describe the weak phases from  $\{\phi_1, \phi_2\}$  into  $\{\phi'_1, \phi'_2\}$  but physical results cannot be changed under such a reparametrization, we refer to this property as “reparametrization invariance” [102] of the decay amplitudes.

Let us consider the decay of a  $B$  meson (model independently) into some specific final state  $f$ . where,  $B$  stands for  $B^+$ ,  $B_d^0$  or  $B_s^0$ . We can then parametrize the decay amplitudes as

$$A_f = \tilde{A}_1 e^{i\phi_1} e^{i\delta_1} + \tilde{A}_2 e^{i\phi_2} e^{i\delta_2}, \quad (3.1)$$

$$\bar{A}_{\bar{f}} = \tilde{A}_1 e^{-i\phi_1} e^{i\delta_1} + \tilde{A}_2 e^{-i\phi_2} e^{i\delta_2}, \quad (3.2)$$

where  $\phi_1$  and  $\phi_2$  are two CP-odd weak phases;  $A_1$  and  $A_2$  are the magnitudes of the corresponding terms; and  $\delta_1$  and  $\delta_2$  are the corresponding CP-even strong phases. It is easy to show that a third weak phase  $\phi_3$  in  $A_f$  and  $\bar{A}_{\bar{f}}$  can be described in terms of  $\phi_1$  and  $\phi_2$  as:

$$\begin{aligned} e^{i\phi_3} &= ce^{i\phi_1} + de^{i\phi_2}, \\ e^{-i\phi_3} &= ce^{-i\phi_1} + de^{-i\phi_2}, \end{aligned} \quad (3.3)$$

where  $c$  and  $d$  are such that

$$\begin{aligned} c &= \frac{\sin(\phi_3 - \phi_2)}{\sin(\phi_1 - \phi_2)}, \\ d &= \frac{\sin(\phi_3 - \phi_1)}{\sin(\phi_2 - \phi_1)}, \end{aligned} \quad (3.4)$$

which are valid if  $\phi_1 - \phi_2 \neq n\pi$ , with  $n$  integer, meaning that the same cannot be done with only one weak phase.

We can use this result to write any amplitude, with an arbitrary number  $N$  of distinct weak phases, in terms of only two.

$$\begin{aligned} A_f &= \tilde{A}_1 e^{i\phi_1} e^{i\tilde{\delta}_1} + \tilde{A}_2 e^{i\phi_2} e^{i\tilde{\delta}_2} + \sum_{k=3}^N \tilde{A}_k e^{i\phi_k} e^{i\tilde{\delta}_k} \\ &= A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}, \end{aligned} \quad (3.5)$$

where

$$\begin{aligned} A_1 e^{i\delta_1} &= \tilde{A}_1 e^{i\tilde{\delta}_1} + \sum_{k=3}^N c_k \tilde{A}_k e^{i\tilde{\delta}_k}, \\ A_2 e^{i\delta_2} &= \tilde{A}_2 e^{i\tilde{\delta}_2} + \sum_{k=3}^N d_k \tilde{A}_k e^{i\tilde{\delta}_k}, \end{aligned} \quad (3.6)$$

and

$$\begin{aligned} c_k &= \frac{\sin(\phi_k - \phi_2)}{\sin(\phi_1 - \phi_2)}, \\ d_k &= \frac{\sin(\phi_k - \phi_1)}{\sin(\phi_2 - \phi_1)}. \end{aligned} \quad (3.7)$$

We can also describe the decay amplitudes with different sets of weak phases  $\{\phi'_1, \phi'_2\}$  as our basis.

Using Eqs. (3.3) and (3.4), then it is easy to show that

$$\begin{aligned} A_f &= \tilde{A}_1 e^{i\phi_1} e^{i\delta_1} + \tilde{A}_2 e^{i\phi_2} e^{i\delta_2}, \\ &= A'_1 e^{i\phi'_1} e^{i\delta'_1} + A'_2 e^{i\phi'_2} e^{i\delta'_2}, \end{aligned} \quad (3.8)$$

as long as

$$\begin{aligned} A'_1 e^{i\delta'_1} &= \tilde{A}_1 e^{i\delta_1} \frac{\sin(\phi_1 - \phi'_2)}{\sin(\phi'_1 - \phi'_2)} + \tilde{A}_2 e^{i\delta_2} \frac{\sin(\phi_2 - \phi'_2)}{\sin(\phi'_1 - \phi'_2)}, \\ A'_2 e^{i\delta'_2} &= \tilde{A}_1 e^{i\delta_1} \frac{\sin(\phi_1 - \phi'_1)}{\sin(\phi'_2 - \phi'_1)} + \tilde{A}_2 e^{i\delta_2} \frac{\sin(\phi_2 - \phi'_1)}{\sin(\phi'_2 - \phi'_1)}. \end{aligned} \quad (3.9)$$

Using Eq. (3.9), we can relate the parameters needed to describe the decay amplitudes with two different choices for the pair of weak phases used as a basis. Therefore, weak phases may be chosen completely at will. Of course, physical results cannot change under such a reparametrization; we refer to this property as “reparametrization invariance”.

### 3.3 PATTERN OF NEW PHYSICS IN $B$ DECAYS

Now we discuss in details about the possible pattern of new physics in  $B$  Decays and their impact. We use the diagrammatic approach to write the amplitudes for  $B$  decays. Using this approach, the amplitudes are written in terms of diagrams: the color-favored and color-suppressed tree amplitudes  $T$  and  $C$ , the gluonic penguin amplitude  $P$ , the color-favored and color-suppressed electroweak penguin amplitudes  $P_{EW}$  and  $P_{EW}^C$ , the annihilation and exchange amplitudes  $A$  and  $E$ , the penguin-annihilation amplitude  $PA$ , the penguin exchange amplitude  $EP$ , and color-suppressed electroweak penguin exchange amplitude  $EP_{EW}^C$  [103].

In the SM, the  $\bar{b} \rightarrow \bar{q}$  transitions ( $q = d, s$ ) involve the three CKM structures  $V_{ub}^* V_{uq}$ ,  $V_{cb}^* V_{cq}$ , and  $V_{tb}^* V_{tq}$ . A decay amplitude may be written as

$$A(\bar{b} \rightarrow \bar{q}) = V_{ub}^* V_{uq} A_u + V_{cb}^* V_{cq} A_c + V_{tb}^* V_{tq} A_t, \quad (3.10)$$

where the  $A_i$  ( $i = u, c, t$ ) involve the relevant hadronic matrix elements with the corresponding CP-even strong phases. Using the unitarity of the CKM matrix,

$$V_{ub}^* V_{uq} + V_{cb}^* V_{cq} + V_{tb}^* V_{tq} = 0, \quad (3.11)$$

From Eq. (3.11), we can eliminate  $V_{cb}^* V_{cq}$  as:

$$V_{cb}^* V_{cq} = -V_{ub}^* V_{uq} - V_{tb}^* V_{tq}, \quad (3.12)$$

Using Eq. (3.12), we can rewrite the Eq. (3.10) as:

$$\begin{aligned}
A(\bar{b} \rightarrow \bar{q}) &= V_{ub}^* V_{uq} A_u - (V_{ub}^* V_{uq} + V_{tb}^* V_{tq}) A_c + V_{tb}^* V_{tq} A_t, \\
&= V_{ub}^* V_{uq} (A_u - A_c) + V_{tb}^* V_{tq} (A_t - A_c), \\
&= V_{ub}^* V_{uq} A_{uc} + V_{tb}^* V_{tq} A_{tc}.
\end{aligned} \tag{3.13}$$

where, we define  $A_{ij} \equiv (A_i - A_j)$ .

Hence a decay amplitude can be written in terms of only two weak phases with three possibilities as:

$$A(\bar{b} \rightarrow \bar{q}) = V_{ub}^* V_{uq} A_{uc} + V_{tb}^* V_{tq} A_{tc}. \tag{3.14}$$

$$A(\bar{b} \rightarrow \bar{q}) = V_{ub}^* V_{uq} A_{ut} + V_{cb}^* V_{cq} A_{ct}. \tag{3.15}$$

$$A(\bar{b} \rightarrow \bar{q}) = V_{cb}^* V_{cq} A_{cu} + V_{tb}^* V_{tq} A_{tu}. \tag{3.16}$$

the relations among the above parametrization have been discussed in detail by Gronau and Rosner in [104]. In the SM, it is very difficult to calculate exactly the amplitudes  $A_i$  (hadronic matrix elements). Hence, in the context of the SM, Eqs. (3.14) through (3.16) provide us with three natural choices for the pair of weak phases  $\{\phi_1, \phi_2\}$  chosen as the basis for Eq. (3.5).

Using Wolfenstein Parametrization and Eq. (3.14), we can write a decay amplitude for  $\bar{b} \rightarrow \bar{s}$  transition

$$A(\bar{b} \rightarrow \bar{s}) = \mathcal{A}'_{uc} e^{i\gamma} - \mathcal{A}'_{tc}. \tag{3.17}$$

where, we define  $\mathcal{A}'_{uc} e^{i\gamma} \equiv V_{ub}^* V_{us} A_{uc}$  and  $-\mathcal{A}'_{tc} \equiv V_{tb}^* V_{ts} A_{tc}$

Similarly a decay amplitude for  $\bar{b} \rightarrow \bar{d}$  transition can be written as:

$$A(\bar{b} \rightarrow \bar{d}) = \mathcal{A}''_{uc} e^{i\gamma} + \mathcal{A}''_{tc} e^{-i\beta}. \tag{3.18}$$

where,  $\mathcal{A}''_{uc} e^{i\gamma} \equiv V_{ub}^* V_{ud} A_{uc}$  and  $-\mathcal{A}''_{tc} e^{-i\beta} \equiv V_{tb}^* V_{td} A_{tc}$ .

In this thesis, it is assumed that isospin is a good symmetry, even for any new physics beyond the SM. We use isospin symmetries to relate different  $B$  decay amplitudes and thus reduce the number of independent diagrammatic amplitudes. Some of the decays where this approach is particularly useful include  $B \rightarrow \pi\pi$ ,  $B \rightarrow K\pi$ ,  $B \rightarrow \rho\rho$ , etc.

We now consider the contribution of NP to these decays. There are a variety of ways of parametrizing this NP, but in this thesis we propose a diagrammatic approach which allows each diagram to be individually modified:  $De^{i\phi_{SM}} \rightarrow De^{i\phi_{SM}} + NPe^{i\phi_{NP}}$ . The addition of NP effects in terms of diagrams is equivalent to the inclusion of NP operators in terms of quarks [105]. Now, NP matrix elements can in general include several contributions, and may not have a well-defined weak phase. However, individual matrix elements do have well-defined phases. Therefore, the addition of these individual NP matrix elements to the diagrams is considered one by one.

Though our discussions are general, as an example  $B \rightarrow K\pi$  decay modes are focused on because present data are at odds with the predictions of the SM [106]. This discrepancy is known as “ $K\pi$ -puzzle,” .

The quark subprocess for the decay channel  $B \rightarrow K\pi$  is  $\bar{b} \rightarrow \bar{u}u\bar{s}$ . Using diagrammatic approach, we can write the amplitudes for  $B \rightarrow K\pi$  decay channel in the SM. The decay channel  $B^+ \rightarrow K^0\pi^+$  consists basically five diagrams: the annihilation amplitude ( $A$ ), the gluonic penguin amplitude ( $P$ ), color-suppressed electroweak penguin amplitude ( $P_{EW}^C$ ), penguin exchange amplitude ( $EP$ ) and color suppressed electroweak penguin exchange amplitude ( $EP_{EW}^C$ ).

In the SM the  $B^+ \rightarrow K^0\pi^+$  amplitudes can be written as:

$$\begin{aligned}
A(B^+ \rightarrow K^0\pi^+) &= AV_{ub}^*V_{us} + \sum_{i=u,c,t} (P_i - \frac{1}{3}P_{EWi}^C + EP_i + \frac{2}{3}EP_{EWi}^C) V_{ib}^*V_{is} \\
&= (A + P_{uc} - \frac{1}{3}P_{EWuc}^C + EP_{uc} + \frac{2}{3}EP_{EWuc}^C) V_{ub}^*V_{us} \\
&+ (P_{tc} - \frac{1}{3}P_{EWtc}^C + EP_{tc} + \frac{2}{3}EP_{EWtc}^C) V_{tb}^*V_{ts} \\
&= (A' + P'_{uc} - \frac{1}{3}P'_{EWuc}{}^C + EP'_{uc} + \frac{2}{3}EP'_{EWuc}{}^C) e^{i\gamma} \\
&- (P'_{tc} - \frac{1}{3}P'_{EWtc}{}^C + EP'_{tc} + \frac{2}{3}EP'_{EWtc}{}^C) \\
A(B^+ \rightarrow K^0\pi^+) &= -\mathcal{P}' + \mathcal{A}' e^{i\gamma} \tag{3.19}
\end{aligned}$$

where, in order to simplify the expressions, we have introduced

$$\mathcal{P}' \equiv P'_{tc} - \frac{1}{3}P'_{EWtc}{}^C + EP'_{tc} + \frac{2}{3}EP'_{EWtc}{}^C,$$

and

$$\mathcal{A}' \equiv A' + P'_{uc} - \frac{1}{3}P'_{EWuc}{}^C + EP'_{uc} + \frac{2}{3}EP'_{EWuc}{}^C.$$

Similarly the decay channel  $B^+ \rightarrow K^+\pi^0$  consists eight diagrams: the color-favored tree amplitude ( $T$ ), the color-suppressed tree amplitude ( $C$ ), the annihilation amplitude ( $A$ ), the gluonic penguin amplitude ( $P$ ), color-favored electroweak penguin amplitude ( $P_{EW}$ ) and color-suppressed electroweak penguin amplitude ( $P_{EW}^C$ ). In the SM the  $B^+ \rightarrow K^+\pi^0$  amplitudes can be written as:

$$\begin{aligned}
-\sqrt{2}A(B^+ \rightarrow K^+\pi^0) &= (T + C + A) V_{ub}^* V_{us} \\
&+ \sum_{i=u,c,t} (P_i + P_{EWi} + \frac{2}{3}P_{EWi}^C + EP_i + \frac{2}{3}EP_{EWi}^C) V_{ib}^* V_{is} \\
&= (T + C + A) V_{ub}^* V_{us} \\
&+ (P_{uc} + P_{EWuc} + \frac{2}{3}P_{EWuc}^C + EP_{uc} + \frac{2}{3}EP_{EWuc}^C) V_{ub}^* V_{us} \\
&+ (P_{tc} + P_{EWtc} + \frac{2}{3}P_{EWtc}^C + EP_{tc} + \frac{2}{3}EP_{EWtc}^C) V_{tb}^* V_{ts} \\
&= ((T + P_{uc} + \frac{2}{3}P_{EWuc}^C + EP_{uc} - \frac{1}{3}EP_{EWuc}^C) \\
&+ (C - P_{uc} + P_{EWuc} + \frac{1}{3}P_{EWuc}^C - EP_{EWuc}^C + \frac{1}{3}EP_{EWuc}^C) \\
&+ (A + P_{uc} - \frac{1}{3}P_{EWuc}^C + EP_{uc} + \frac{2}{3}EP_{EWuc}^C)) V_{ub}^* V_{us} \\
&+ ((P_{tc} - \frac{1}{3}P_{EWtc}^C + EP_{tc} + \frac{2}{3}EP_{EWtc}^C) \\
&+ (P_{EWtc} + EP_{EWtc}^C) + (P_{EWtc}^C - EP_{EWtc}^C)) V_{tb}^* V_{ts} \\
&= ((T' + P'_{uc} + \frac{2}{3}P'_{EWuc}^C + EP'_{uc} - \frac{1}{3}EP'_{EWuc}^C) \\
&+ (C' - P'_{uc} + P'_{EWuc} + \frac{1}{3}P'_{EWuc}^C - EP'_{EWuc}^C + \frac{1}{3}EP'_{EWuc}^C) \\
&+ (A' + P'_{uc} - \frac{1}{3}P'_{EWuc}^C + EP'_{uc} + \frac{2}{3}EP'_{EWuc}^C)) e^{i\gamma} \\
&- ((P'_{tc} - \frac{1}{3}P'_{EWtc}^C + EP'_{tc} + \frac{2}{3}EP'_{EWtc}^C) \\
&+ (P'_{EWtc} + EP'_{EWtc}^C) + (P'_{EWtc}^C - EP'_{EWtc}^C)) \\
\sqrt{2}A(B^+ \rightarrow K^+\pi^0) &= (\mathcal{P}' + \mathcal{P}'_{EW} + \mathcal{P}'_{EW}^C) - (\mathcal{T}' + \mathcal{C}' + \mathcal{A}') e^{i\gamma}. \tag{3.20}
\end{aligned}$$

where, we have introduced

$$\begin{aligned}
\mathcal{P}' &\equiv P'_{tc} - \frac{1}{3}P'_{EWtc}^C + EP'_{tc} + \frac{2}{3}EP'_{EWtc}^C, \\
\mathcal{A}' &\equiv A' + P'_{uc} - \frac{1}{3}P'_{EWuc}^C + EP'_{uc} + \frac{2}{3}EP'_{EWuc}^C,
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}' &\equiv T' + P'_{uc} + \frac{2}{3}P'_{EWuc} + EP'_{uc} - \frac{1}{3}EP'_{EWuc}, \\
\mathcal{C}' &\equiv C' - P'_{uc} + P'_{EWuc} + \frac{1}{3}P'_{EWuc} - EP'_{EWuc} + \frac{1}{3}EP'_{EWuc}, \\
\mathcal{P}'_{EW} &\equiv P'_{EWtc} + EP'_{EWtc},
\end{aligned}$$

and

$$\mathcal{P}'_{EW} \equiv P'_{EWtc} - EP'_{EWtc}.$$

Similarly we can write the decay amplitude of  $B^0 \rightarrow K^+\pi^-$

$$\begin{aligned}
-A(B^0 \rightarrow K^+\pi^-) &= TV_{ub}^*V_{us} \\
&+ \sum_{i=u,c,t} (P_i + \frac{2}{3}P_{EWi}^C + EP_i - \frac{1}{3}EP_{EWi}^C) V_{ib}^*V_{is} \\
&= \mathcal{T}' e^{i\gamma} - (\mathcal{P}' + \mathcal{P}'_{EW}) \\
A(B^0 \rightarrow K^+\pi^-) &= (\mathcal{P}' + \mathcal{P}'_{EW}) - \mathcal{T}' e^{i\gamma}. \tag{3.21}
\end{aligned}$$

The decay amplitude for  $B^0 \rightarrow K^0\pi^0$  can be written as

$$\begin{aligned}
\sqrt{2}A(B^0 \rightarrow K^0\pi^0) &= -CV_{ub}^*V_{us} \\
&+ \sum_{i=u,c,t} (P_i - P_{EWi} - \frac{1}{3}P_{EWi}^C + EP_i - \frac{1}{3}EP_{EWi}^C) V_{ib}^*V_{is} \\
&= -\mathcal{C}' e^{i\gamma} - (\mathcal{P}' - \mathcal{P}'_{EW}) \\
\sqrt{2}A(B^0 \rightarrow K^0\pi^0) &= -(\mathcal{P}' - \mathcal{P}'_{EW}) - \mathcal{C}' e^{i\gamma}. \tag{3.22}
\end{aligned}$$

Hence, in the SM the four  $B \rightarrow K\pi$  amplitudes are given by

$$\begin{aligned}
A(B^+ \rightarrow K^0\pi^+) &= -\mathcal{P}' + \mathcal{A}' e^{i\gamma}, \\
\sqrt{2}A(B^+ \rightarrow K^+\pi^0) &= (\mathcal{P}' + \mathcal{P}'_{EW} + \mathcal{P}'_{EW}^C) \\
&\quad - (\mathcal{T}' + \mathcal{C}' + \mathcal{A}') e^{i\gamma}, \tag{3.23} \\
A(B^0 \rightarrow K^+\pi^-) &= (\mathcal{P}' + \mathcal{P}'_{EW}^C) - \mathcal{T}' e^{i\gamma}, \\
\sqrt{2}A(B^0 \rightarrow K^0\pi^0) &= -(\mathcal{P}' - \mathcal{P}'_{EW}) - \mathcal{C}' e^{i\gamma}.
\end{aligned}$$

Now we add new physics contribution into  $B \rightarrow K\pi$  decays; *e.g.* let's consider first a NP contribution to the electroweak penguin:  $\mathcal{P}'_{EW} \rightarrow \mathcal{P}'_{EW} + N' e^{i\phi}$ . It has been shown that any complex number can be written in terms of two other pieces with arbitrary phases [reparametrization invariance (RI)] [107]. Since the  $B \rightarrow K\pi$  amplitudes involve the phases 0 and  $\gamma$ , we rewrite  $N' e^{i\phi}$  as  $N'_1 + N'_2 e^{i\gamma}$ . After the addition of this new physics, the amplitudes become

$$\begin{aligned}
A(B^+ \rightarrow K^0 \pi^+) &= -\mathcal{P}' + \mathcal{A}' e^{i\gamma}, \\
\sqrt{2}A(B^+ \rightarrow K^+ \pi^0) &= (\mathcal{P}' + \mathcal{P}'_{EW} + N'_1 + \mathcal{P}'_{EW}{}^C) \\
&\quad - (\mathcal{T}' + \mathcal{C}' - N'_2 + \mathcal{A}') e^{i\gamma}, \\
A(B^0 \rightarrow K^+ \pi^-) &= (\mathcal{P}' + \mathcal{P}'_{EW}{}^C) - \mathcal{T}' e^{i\gamma}, \\
\sqrt{2}A(B^0 \rightarrow K^0 \pi^0) &= -(\mathcal{P}' - \mathcal{P}'_{EW} - N'_1) - (\mathcal{C}' - N'_2) e^{i\gamma}.
\end{aligned} \tag{3.24}$$

With the new physics contributing to  $\mathcal{P}'_{EW}$ , only the amplitudes  $A(B^+ \rightarrow K^+ \pi^0)$  and  $A(B^0 \rightarrow K^0 \pi^0)$  are affected. Now  $N'_1$  and  $N'_2$  can be removed by making the following redefinitions:  $\mathcal{C}' \rightarrow \hat{\mathcal{C}}' \equiv \mathcal{C}' - N'_2$  and  $\mathcal{P}'_{EW} \rightarrow \hat{\mathcal{P}}'_{EW} \equiv \mathcal{P}'_{EW} + N'_1$ .

$$\begin{aligned}
A(B^+ \rightarrow K^0 \pi^+) &= -\mathcal{P}' + \mathcal{A}' e^{i\gamma}, \\
\sqrt{2}A(B^+ \rightarrow K^+ \pi^0) &= (\mathcal{P}' + \hat{\mathcal{P}}'_{EW} + \mathcal{P}'_{EW}{}^C) \\
&\quad - (\mathcal{T}' + \hat{\mathcal{C}}' + \mathcal{A}') e^{i\gamma}, \\
A(B^0 \rightarrow K^+ \pi^-) &= (\mathcal{P}' + \mathcal{P}'_{EW}{}^C) - \mathcal{T}' e^{i\gamma}, \\
\sqrt{2}A(B^0 \rightarrow K^0 \pi^0) &= -(\mathcal{P}' - \hat{\mathcal{P}}'_{EW}) - \hat{\mathcal{C}}' e^{i\gamma}.
\end{aligned} \tag{3.25}$$

Consequently, the above amplitudes (with NP contribution) now reduce to the same form as in the SM [Eq. (3.23)]. We can see the effects of this as follows: First, it is clear that the new physics cannot be detected directly through measurements of  $B \rightarrow K\pi$  decays. Second, we can see that there is indirect effect related to the size of  $\mathcal{P}'_{EW}$  and  $\mathcal{C}'$  ( $\mathcal{P}'_{EW}$  and  $\mathcal{C}'$  are modified). Similarly if we included the contribution of new physics to  $\mathcal{C}' e^{i\gamma}$  instead of  $\mathcal{P}'_{EW}$ , the amplitudes in the presence of this new physics would again have the same form as in the SM with the redefinitions  $\mathcal{C}' \rightarrow \hat{\mathcal{C}}' \equiv \mathcal{C}' + N'_2$  and  $\mathcal{P}'_{EW} \rightarrow \hat{\mathcal{P}}'_{EW} \equiv \mathcal{P}'_{EW} - N'_1$ .

If we add the new physics to other diagrams, the effect is similar. In all cases, we can write the new amplitudes in the same form as that of the SM. Hence we cannot see a clean signal of NP in the  $B \rightarrow K\pi$  modes. There are two cases. (i) The new physics is added to  $\mathcal{T}'$ :  $\mathcal{T}' e^{i\gamma} \rightarrow \mathcal{T}' e^{i\gamma} + N' e^{i\phi}$ .  $A(B^+ \rightarrow K^+ \pi^0)$  and  $A(B^0 \rightarrow K^+ \pi^-)$  are affected. One makes the redefinitions  $\mathcal{T}' \rightarrow \hat{\mathcal{T}}' \equiv \mathcal{T}' + N'_2$  and  $\mathcal{P}'_{EW}{}^C \rightarrow \hat{\mathcal{P}}'_{EW}{}^C \equiv \mathcal{P}'_{EW}{}^C - N'_1$ . The situation is the same, apart from a change of relative sign in the redefinitions, if the new physics is added to  $\mathcal{P}'_{EW}{}^C$ . (ii) The new physics is added to  $\mathcal{P}'$ :  $\mathcal{P}' \rightarrow \mathcal{P}' + N' e^{i\phi}$ . Now all four amplitudes are affected. One makes the redefinitions  $\mathcal{P}' \rightarrow \hat{\mathcal{P}}' \equiv \mathcal{P}' + N'_1$  and  $\mathcal{A}' \rightarrow \hat{\mathcal{A}}' \equiv \mathcal{A}' - N'_2$ ,  $\mathcal{C}' \rightarrow \hat{\mathcal{C}}' \equiv \mathcal{C}' + N'_2$ ,  $\mathcal{T}' \rightarrow \hat{\mathcal{T}}' \equiv \mathcal{T}' - N'_2$ . Once again the situation is the same. if the new physics is added to  $\mathcal{A}'$ , the situation is still the same.



We, therefore conclude that, regardless of how one adds the new physics, it is always possible to write the  $B \rightarrow K\pi$  amplitudes as in the SM. This implies that there is no clean signal of new physics in  $B \rightarrow K\pi$  decays. The reason for this is reparametrization invariance [107] and the fact that the  $B \rightarrow K\pi$  amplitudes involve two phases.

We can see similar results in  $B \rightarrow \pi\pi$  decays ( $\bar{b} \rightarrow \bar{d}$ ). Using the redefinitions:

$$\begin{aligned}\mathcal{P}'' &\equiv P''_{tc} - \frac{1}{3}P''_{EWtc} + PA''_{tc} + EP''_{tc} - \frac{1}{3}EP''_{EWtc}, \\ \mathcal{P}''_{uc} &\equiv P''_{uc} + E'' - \frac{1}{3}P''_{EWuc} + PA''_{uc} + EP''_{uc} - \frac{1}{3}EP''_{EWuc}, \\ \mathcal{T}'' &\equiv T'' + P''_{EWuc}, \\ \mathcal{C}'' &\equiv C'' + P''_{EWuc}, \\ \mathcal{P}''_{EW} &\equiv P''_{EWtc}\end{aligned}$$

and

$$\mathcal{P}''_{EW}{}^C \equiv P''_{EWtc}{}^C,$$

the complete SM amplitudes for these decays are

$$\begin{aligned}-\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= (\mathcal{T}'' + \mathcal{C}'') e^{i\gamma} + (\mathcal{P}''_{EW} + \mathcal{P}''_{EW}{}^C) e^{-i\beta}, \\ -A(B^0 \rightarrow \pi^+\pi^-) &= (\mathcal{T}'' + \mathcal{P}''_{uc}) e^{i\gamma} + (\mathcal{P}''_{EW} + \mathcal{P}'') e^{-i\beta}, \\ -\sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) &= (\mathcal{C}'' - \mathcal{P}''_{uc}) e^{i\gamma} + (\mathcal{P}''_{EW} - \mathcal{P}'') e^{-i\beta}.\end{aligned}\quad (3.26)$$

It is easy to see that if new physics enters any of the diagrams above, we can divide it into two pieces as  $N e^{i\phi} = N_1 e^{i\gamma} + N_2 e^{-i\beta}$ , and absorbed through redefinitions of the SM diagrams. Hence no clean NP signal is possible in the  $B \rightarrow \pi\pi$  either. Once again, just as in the  $B \rightarrow K\pi$  case, we find that new physics introduced in  $\mathcal{P}''_{EW}$  or  $\mathcal{C}''$  modifies  $\mathcal{P}''_{EW}$  and  $\mathcal{C}''$  amplitudes simultaneously. The case for the pairs  $(\mathcal{T}'', \mathcal{P}''_{EW})$  and  $(\mathcal{P}'', \mathcal{P}''_{uc})$  are similar.

In fact, there are significant differences between  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays. The  $\mathcal{P}''_{EW}$  and  $\mathcal{P}''_{EW}{}^C$  contributions in  $B \rightarrow \pi\pi$  decays are expected to be tiny and are hence justifiably ignored within the SM. If there is new physics, the amplitudes cannot be recast in the SM form if  $\mathcal{P}''_{EW}$  and  $\mathcal{P}''_{EW}{}^C$  are neglected. Hence, direct CP violation in  $B^+ \rightarrow \pi^+\pi^0$  is a clean signal of new physics. Thus, it is only if  $\mathcal{P}''_{EW}$  and  $\mathcal{P}''_{EW}{}^C$  are kept that one can conclude that no clean NP signal is possible.

Similarly we can easily analysed other  $B$  decays if we look at the quark-level topologies.  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  decays are  $\bar{b} \rightarrow \bar{s}u\bar{u}$  and  $\bar{b} \rightarrow \bar{d}u\bar{u}$ ,

respectively. The analysis of all decays of the form  $\bar{b} \rightarrow \bar{d}u\bar{u}$ ,  $\bar{b} \rightarrow \bar{s}u\bar{u}$ ,  $\bar{b} \rightarrow \bar{d}c\bar{c}$ , or  $\bar{b} \rightarrow \bar{s}c\bar{c}$  is thus identical to those above. If we keep all SM diagrams, any NP effects can be absorbed by redefining the SM diagrams, so that there are no clean signals of NP in such decays. Decays of the form  $\bar{b} \rightarrow \bar{d}d\bar{d}$ ,  $\bar{b} \rightarrow \bar{s}d\bar{d}$ ,  $\bar{b} \rightarrow \bar{d}s\bar{s}$ , or  $\bar{b} \rightarrow \bar{s}s\bar{s}$  are slightly different because  $T$  and  $C$  cannot enter (i.e. they are “pure penguin”). For example, consider  $B^0 \rightarrow K^0\bar{K}^0$ . The amplitude for this decay is

$$A(B^0 \rightarrow K^0\bar{K}^0) = \mathcal{P}_{uc}e^{i\gamma} + \mathcal{P}e^{-i\beta}, \quad (3.27)$$

where  $\mathcal{P}$  and  $\mathcal{P}_{uc}$  are as defined above except there is no  $E$  contribution here. It is easy to see that NP added to any of the diagrams can once again be absorbed by redefinitions of the amplitudes. Thus, there can be no clean signal of NP in  $B^0 \rightarrow K^0\bar{K}^0$  decays.

Hence, we can say that there are *no clean signals of NP in B decays which receive penguin contributions*. This result was earlier stated in Ref. [108] in weak form (only by modifying one of the weak phases by the contribution of NP). But our conclusion is more robust as it is obtained when an arbitrary NP amplitude is introduced. If we make some approximation in the SM amplitudes, then only we can get a signal of NP in modes with penguin contributions. The approximation must be such a way that NP altered amplitudes cannot be recast in the SM form. *The signal of NP so obtained will be reliable, if the approximation is justifiable.*

The clean signals of NP are possible only in the pure tree-level decays  $\bar{b} \rightarrow \bar{u}c\bar{d}$ ,  $\bar{b} \rightarrow \bar{c}u\bar{d}$ ,  $\bar{b} \rightarrow \bar{u}c\bar{s}$ , and  $\bar{b} \rightarrow \bar{c}u\bar{s}$ . Through the measurements of direct CP violation, NP can be detected because these decays have only one weak phase. However, any NP effects are purely tree-level and are therefore much suppressed [109]. In modes with penguin contributions, our observation that NP -modified amplitudes retain the SM form has two additional consequences which we discuss in detail.

$K\pi$  puzzle can be solved if we assume that a large electroweak-penguin amplitude is resulting from NP. Several fits to the  $B \rightarrow K\pi$  data [106] is indicating that the electroweak-penguin amplitude is indeed large, but it is difficult to accommodate the data without demanding a larger-than-expected  $\mathcal{C}'$  amplitude. A priori no one would expect a tree-level amplitude to be considerably affected by NP. However, as Indeed it is illustrated above that the simultaneous effect of NP on  $\mathcal{C}'$  and  $\mathcal{P}'_{EW}$  is not an accident. In fact, a second conclusion that emerges from the above discussion is that *NP always affects diagrammatic amplitudes in pairs*. Therefore, we find that the following pairs of amplitudes are simultaneously affected by NP: i)  $\mathcal{P}'_{EW}$  and

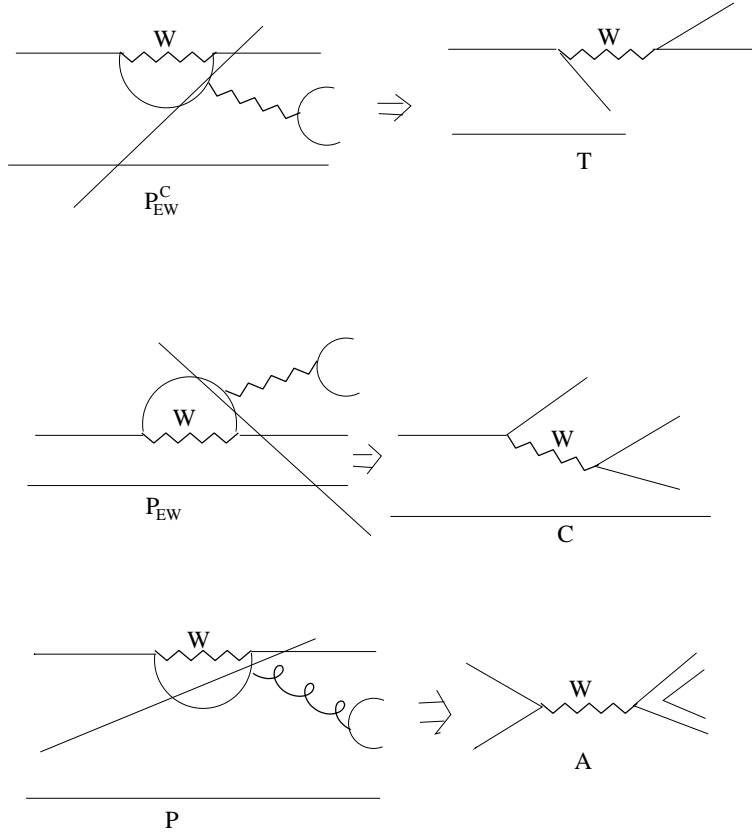


Figure 3.1: Various topological equivalent diagrams

$\mathcal{C}'$  ( $\mathcal{P}'_{EW}$  and  $\mathcal{C}'$ ), ii)  $\mathcal{P}'_{EW}$  and  $\mathcal{T}'$  ( $\mathcal{P}'_{EW}$  and  $\mathcal{T}'$ ), iii)  $\mathcal{P}'$  and  $\mathcal{A}'$  ( $\mathcal{P}'$  and  $\mathcal{A}'$ ).

These pairs of diagrams are topologically equivalent in the sense that if they contribute to any decay mode, they always appear together, shown in fig 3.1. Hence, NP must contribute to specific pairs of amplitudes simultaneously. The fact that fits to the  $B \rightarrow K\pi$  data require a large  $\mathcal{C}'$  may well indicate a contribution of NP to  $\mathcal{P}'_{EW}$ .

Now, the  $B \rightarrow K\pi$  modes have been used to measure the weak phase  $\gamma$ . It is important to examine the effect of NP on this weak-phase measurement. It is shown that all the isospin-related  $B \rightarrow K\pi$  amplitudes have the same form as in the SM even in the presence of NP. Therefore, the measured weak phase with or without NP should remain the same. However,  $\gamma$  cannot be extracted without resorting to some approximations because the number of theoretical parameters is more than the number of independent observables.

when no approximations are made, NP does not alter the SM form of the amplitudes. However, if some of the diagrammatic amplitudes are neglected, then the NP -modified amplitudes may not have the SM form. Whenever we make the approximations such that the amplitudes with the addition of NP retain the SM form, then the *weak phase measured remains unaltered from its SM value, even in the presence of NP*. Therefore, it explains nicely why fits to the  $B \rightarrow K\pi$  data [106] often yield  $\gamma$  consistent with the CKM fits [52]. In case the approximations made are such that the amplitudes in the presence of NP cannot be recast in the SM form, the addition of NP will result in adding to the number of theoretical parameters, rendering the weak-phase measurement impossible without further assumptions. If  $\gamma$  is nevertheless measured and is found to differ from the SM value, the deviation may be either due to NP or due to the invalid approximations used. In this sense a discrepancy in the measured value of  $\gamma$  is not necessarily an unambiguous signal of NP.

We see that there are more theoretical parameters than observables in  $B \rightarrow K\pi$  modes. At best there can be 9 independent observables ( 4 branching ratios and 4 direct CP asymmetries and 1 time-dependent CP asymmetry) for  $B_d^0 \rightarrow K^0\pi^0$ . But there are six diagrammatic amplitudes and two weak phases  $\gamma$  and  $\beta$ , resulting in a total of 13 parameters in this decay mode [Eq. (3.23)]. We can take the measurement of  $\sin 2\beta$  in  $B^0(t) \rightarrow J/\psi K_S$ :  $\sin 2\beta = 0.65 \pm 0.025$  [26] and leaving 12 theoretical unknowns. It is therefore clear that one needs to make some approximation in order to analyze these modes.

In Ref. [103], the relative sizes of the amplitudes were estimated to be roughly

$$\begin{aligned} 1 : |P'_{tc}|, \quad \mathcal{O}(\bar{\lambda}) : |T'|, |P'_{EW}|, \\ \mathcal{O}(\bar{\lambda}^2) : |C'|, |P'_{EW}{}^C|, \quad \mathcal{O}(\bar{\lambda}^3) : |A'|, \end{aligned} \quad (3.28)$$

where  $\bar{\lambda} \sim 0.2$ . We can use these SM estimates as a guide to neglect diagrammatic amplitudes and reduce the number of parameters.

If only the “large” diagrams ( $P'_{tc}$ ,  $T'$ ,  $P'_{EW}$ ) are retained, the  $B \rightarrow K\pi$  amplitudes can be written

$$\begin{aligned} A(B^+ \rightarrow K^0\pi^+) &\simeq -P'_{tc}, \\ \sqrt{2}A(B^+ \rightarrow K^+\pi^0) &\simeq P'_{tc} + P'_{EW} - T'e^{i\gamma}, \\ A(B^0 \rightarrow K^+\pi^-) &\simeq P'_{tc} - T'e^{i\gamma}, \\ \sqrt{2}A(B^0 \rightarrow K^0\pi^0) &\simeq -P'_{tc} + P'_{EW}. \end{aligned} \quad (3.29)$$

The signals of NP are possible in the above approximate form since some of the amplitudes have a well-defined phase and RI is lost. For example, the direct CP asymmetry in  $B^+ \rightarrow K^0\pi^+$  and  $B^0 \rightarrow K^0\pi^0$  vanishes. Similarly, the mixing-induced CP asymmetry in  $B^0(t) \rightarrow K^0\pi^0$  should be equal to that in  $B^0(t) \rightarrow J/\psi K_S$ . If we get any deviation from these expectations may imply presence of NP. But we can conclude this only on the validity of the approximations made in the above parametrization.

Another NP signal can be found as follows. In the SM, the following sum rule holds approximately [111]:

$$\Gamma(B^0 \rightarrow K^+\pi^-) + \Gamma(B^+ \rightarrow K^0\pi^+) \approx 2\Gamma(B^+ \rightarrow K^+\pi^0) + 2\Gamma(B^0 \rightarrow K^0\pi^0) . \quad (3.30)$$

The error in this sum rule is quadratic in the subdominant terms ( $T'$ ,  $P'_{EW}$ ). If it is found not to hold, again NP will be deduced.

In a fit to the  $B \rightarrow K\pi$  data, smaller diagrams can be retained. First, we observe that not all the diagrams are independent. In the SM, to a good approximation, the diagrams  $P'_{EW}$  and  $P'^C_{EW}$  are related to  $T'$  and  $C'$  using flavor SU(3) [112]:

$$\begin{aligned} \frac{P'_{EW}}{T'} &\simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] R , \\ \frac{P'^C_{EW}}{C'} &\simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] R . \end{aligned} \quad (3.31)$$

Here, the  $c_i$  are Wilson coefficients [32] and

$$R \equiv \left| \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} \right| = \frac{1}{\lambda^2} \frac{\sin(\beta + \gamma)}{\sin \beta} . \quad (3.32)$$

The  $B \rightarrow K\pi$  amplitudes thus depend on eleven theoretical parameters: the magnitudes and relative strong phases of  $P'_{tc}$ ,  $T'$ ,  $C'$ ,  $P'_{uc}$  and  $A'$ , and the weak phases  $\beta$  and  $\gamma$ . The phase  $\beta$  can be taken from the measurement of  $\sin 2\beta$  in  $B^0(t) \rightarrow J/\psi K_S$  and leaving ten theoretical unknowns. However, there are only nine  $B \rightarrow K\pi$  measurements: four CP-averaged branching ratios, four direct CP asymmetries, and one mixing-induced CP asymmetry (in  $B^0(t) \rightarrow K^0\pi^0$ ). In order to perform a fit, it is therefore necessary to make an approximation to reduce the number of theoretical parameters, i.e. certain diagrams must be neglected.

There are also other approximate parametrization for the  $B \rightarrow K\pi$  amplitudes considered in literature [113] is

$$A(B^+ \rightarrow K^0\pi^+) \simeq -P'_{tc} ,$$

$$\begin{aligned}
\sqrt{2}A(B^+ \rightarrow K^+\pi^0) &\simeq P'_{tc} + P'_{EW} - (T' + C')e^{i\gamma}, \\
A(B^0 \rightarrow K^+\pi^-) &\simeq P'_{tc} - T'e^{i\gamma}, \\
\sqrt{2}A(B^0 \rightarrow K^0\pi^0) &\simeq -P'_{tc} + P'_{EW} - C'e^{i\gamma}.
\end{aligned} \tag{3.33}$$

$C'$  is retained even though it is subdominant, as it is claimed that the fit is extremely poor without retaining  $C'$ . Leaving aside the merits of such an assumption, we see that in Eq. (3.33) we have 9 variables and 9 observables. Hence, all the variables can be solved in terms of observables. It is straightforward to see that if new physics contributes to  $P'_{EW}$  or  $C'$  it can be reabsorbed using RI. This is because  $P'_{EW}$  and  $C'$  appear simultaneously in the amplitudes. Since the amplitudes have the same form with or without new physics the value of  $\gamma$  measured under this approximation *would not differ from the SM value even in the presence of NP*. If we get disagreement in the value of  $\gamma$ , it must indicate a failure of the assumptions made. If NP contributes to other topologies it cannot be reabsorbed using RI and the value of  $\gamma$  cannot be measured unless further approximations are made.

Given that the pairing of  $P'_{EW}$  and  $C'$  is the reason for fits to the  $B \rightarrow K\pi$  data finding a value for  $\gamma$  consistent with the SM CKM fits, it is puzzling why fits to the  $B \rightarrow K\pi$  data using Eq. (3.29) (e.g., Ref. [114]) also result in a value for  $\gamma$  in agreement with the SM. The explanation lies in the large error in the data, due to which the two parametrizations in Eqs. (3.29) and (3.33) cannot be distinguished. This argument is vindicated by the fits performed in Ref. [114] when NP is included. If NP is added to  $P'_{EW}$  in Eq. (3.29), it can be recast in the form of Eq. (3.33) with  $C'$  being replaced entirely by a NP contribution and  $P'_{EW}$  being redefined. Only new physics of this kind (scenario (i) of Ref. [114]) results in the best fits and  $\gamma$  is remarkably consistent with the SM. The point is that the agreement of  $\gamma$  obtained with the SM value does not a priori rule out new physics, but rather strengthens the arguments in favor of NP contributing to  $P'_{EW}$ .

### 3.4 SUMMARY

We point out that a generic  $B \rightarrow f$  decay amplitude may be written as a sum of two terms, corresponding to a pair of weak phases  $\{\phi_1, \phi_2\}$  chosen completely at will (as long as they do not differ by a multiple of  $180^\circ$ ). Clearly, physical observables may not depend on this choice; we designate this property by “reparametrization invariance”.

We explore some of the unusual features of reparametrization invariance. we have made three main points. First, there are no clean signals of NP in

any  $B$  decay which receives penguin contributions. In order to obtain a signal of NP, it is necessary to either have an accurate theoretical estimate of parameters or to make a justifiable approximation. Second, we have noted that in all decays with penguin contributions, NP always affects diagrammatic amplitudes in pairs. The diagrams  $\mathcal{P}'_{EW}$  and  $C'$ ,  $\mathcal{P}'^C_{EW}$  and  $\mathcal{T}'$ , and  $\mathcal{P}'$  and  $\mathcal{A}'$  are simultaneously affected by NP. Fits to  $B \rightarrow K\pi$  data suggest larger-than-expected  $P^C_{EW}$  and  $C'$  contributions. In view of our observation, the requirement of large  $C'$  may be a sign of NP. Finally, we have shown that if NP contributes in such a way that the amplitudes retain the SM form using RI, the weak phase obtained will not be altered due to the presence of NP. This provides a natural explanation of the result that several fits to  $B \rightarrow K\pi$  data with varying approximations yield  $\gamma$  in accord with the SM. The observation of a large  $C'$  and  $\gamma$  consistent with the SM in  $B \rightarrow K\pi$  decays provide substantial circumstantial evidence in favor of NP [148].

## CHAPTER 4

# $B \rightarrow K\pi$ PUZZLE

### 4.1 INTRODUCTION

Excellent progress of the  $B$  factory experiments throws light on the study of rare  $B$  decays, which are crucial for testing the SM and detecting any hints beyond the SM.  $B$  factories has been producing large numbers of  $B$  mesons, providing accurate measurements of branching ratios and direct CP asymmetry for many modes; the  $B \rightarrow PP$ ,  $B \rightarrow PV$  and  $B \rightarrow VV$  modes, where  $P$  and  $V$  denote a pseudoscalar meson and a vector meson.  $B \rightarrow K\pi$  decays are of great importance not only to investigate NP beyond the SM but also to examine the hadronic parameters and CKM matrix elements within the SM. The special property of penguin dominance for the  $B \rightarrow K\pi$  modes provides us a golden opportunity to study NP in the  $b \rightarrow s$  penguin processes. The  $B \rightarrow K\pi$  modes has been studied extensively for quite a long time. For example, the angle  $\gamma$  of the CKM unitary triangle has been determined in model independent ways using  $B \rightarrow K\pi$  over the past ten years even before the new generation of the experimental data came out [116, 117, 118]. Many elaborate theoretical predictions based on QCDF [119], PQCD [120, 121] and SCET [122] have been also produced for physical observables in the  $B \rightarrow K\pi$  modes within the SM.

NP beyond the SM in  $B \rightarrow K\pi$  decays has been inspired by experimental results as follows. First, the ratios

$$R_c \equiv 2 \frac{\text{BR}(B^+ \rightarrow K^+\pi^0)}{\text{BR}(B^+ \rightarrow K^0\pi^+)} \quad \text{and} \quad R_n \equiv \frac{1}{2} \frac{\text{BR}(B^0 \rightarrow K^+\pi^-)}{\text{BR}(B^0 \rightarrow K^0\pi^0)} \quad (4.1)$$

are expected to satisfy  $R_c \approx R_n$  within the SM [118]. Before ICHEP-2008, those experimental values had shown a significant discrepancy, but as time passes they were getting closer to each other [123]. Current data updated by



Sep 2008 in HFAG [26] show  $R_c = 1.11 \pm 0.05$  and  $R_n = 0.99 \pm 0.05$ , which are consistent with the SM expectation. On the other hand, the CP asymmetry measurements still show a disagreement with the SM prediction. The SM naively expects  $\mathcal{A}_{CP}(B^0 \rightarrow K^+\pi^-) \approx \mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0)$  for the direct CP asymmetry and  $(\sin 2\beta)_{K_S\pi^0} \approx (\sin 2\beta)_{c\bar{c}s} = 0.675$  for the mixing-induced CP asymmetry. But the current experimental data show

$$\mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0) - \mathcal{A}_{CP}(B^0 \rightarrow K^+\pi^-) = 0.13 \pm 0.02, \quad (4.2)$$

$$(\sin 2\beta)_{K_S\pi^0} - (\sin 2\beta)_{c\bar{c}s} = -0.1 \pm 0.15. \quad (4.3)$$

The recent PQCD result for the difference of the above direct CP asymmetries is  $0.08 \pm 0.09$ , which is actually consistent with the data. However, the PQCD prediction  $\mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0)_{PQCD} = -0.01^{+0.03}_{-0.05}$  still has some difference from the current experimental data  $\mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0)_{EXP} = 0.027 \pm 0.029$ . Moreover, the difference of the mixing-induced CP asymmetry from the PQCD prediction is  $0.065 \pm 0.04$ , which shows still some deviation from the data.

On the one hand, the electroweak penguin (EWP) processes in the  $B \rightarrow K\pi$  decays has drawn lots of attention for studying NP for a long time, especially based on various specific NP scenarios such as SUSY models [125], flavor-changing  $Z'$  models [126], four generation models [127], and so on, and on the other hand, numerous model-independent attempts have been also made in search of NP within the quark diagram approach. Many authors made known that the anomalous behaviors of the experimental data could be accommodated with the enhancement of the EWP amplitude [128, 129] as well as an additional weak phase in the electroweak sector [130, 131, 132]. A few authors have also found that the color-suppressed tree amplitude would be the main source of NP in the  $B \rightarrow K\pi$  modes [132, 133]. However, any NP contribution can be absorbed into the SM amplitudes always in pair, due to re-parametrization invariance (RI) of decay amplitudes [102, 148]; for example, both the color-suppressed tree and the EWP amplitude. Hence, the large enhancement of the color-suppressed tree amplitude can be understood by a NP contribution to the EWP amplitude with a non-zero NP weak phase. The study of two-body hadronic  $B$  decays provides good opportunities to test the SM and more to probe possible NP effects beyond the SM. Large numbers of  $B$  mesons have been produced at the  $B$  factories enabling accurate measurements of branching ratios and direct CP asymmetry for many modes. Polarization measurements for several such modes have been reported, as well.

Recent experimental results [26, 135, 136] for the  $B \rightarrow K\pi$  mode, show deviations from SM expectations; the discrepancy, commonly being referred

to as the “ $B \rightarrow K\pi$  puzzle.” The dominant quark level subprocesses for  $B \rightarrow K\pi$  decays are  $b \rightarrow s\bar{q}q$  ( $q = u, d$ ) penguin processes which are potentially sensitive to NP effects. Many efforts have been made to resolve the puzzle [129, 155]. A model-independent study shows that the experimental data strongly indicate large enhancements of both the electroweak (EW) penguin and the color-suppressed tree contributions [155]. The  $B \rightarrow K\pi$  modes have certain inherent limitations. The four  $B \rightarrow K\pi$  decay modes can experimentally yield at most 9 observables: four each of the branching ratios and direct CP asymmetries and one time-dependent CP asymmetry. Clearly, the 9 observables are insufficient to determine all the 12 theoretical parameters [155] needed to describe these decay modes. One hence needs to make some assumptions. Traditionally, assumptions have often been made on sizes of the topological amplitudes as well as on the strong phases of the different topologies.

#### 4.2 SOLUTION OF $B \rightarrow K\pi$ PUZZLE THROUGH $B \rightarrow K^*\rho$

The  $B \rightarrow VV$  modes, where  $V$  denotes a vector meson, have the advantage that they provide many more observables, compared with those being measured in  $B \rightarrow PP$  (e.g.,  $B \rightarrow K\pi$ ) or  $B \rightarrow VP$  (e.g.,  $B \rightarrow K^*\pi$ ) modes, where  $P$  denotes a pseudoscalar meson, due to spins of the final state vector mesons. Since the first observation of  $B \rightarrow K^*\phi$  by CLEO Collaboration, several  $B$  decays to two charmless vector mesons, such as  $B \rightarrow K^*\rho$  and  $B \rightarrow \rho\rho$ , have been reported by BABAR and BELLE Collaboration [26, 135, 138, 139, 140, 141].

The  $B \rightarrow K^*\rho$  modes ( $B \rightarrow VV$ ) are the analogues of  $B \rightarrow K\pi$  modes ( $B \rightarrow PP$ ) at the quark level processes because the quark level processes of both modes are exactly the same. Therefore, it is assumed that if there appear any NP effects through  $B \rightarrow K\pi$ , then similar new physics effects will appear through  $B \rightarrow K^*\rho$  as well. However, the study of  $B \rightarrow VV$  modes requires performing an angular analysis in order to obtain the helicity amplitudes. While angular analysis is often regarded as an additional complication needed due to the presence of both CP-even and CP-odd components that dilute the time dependent CP asymmetry, it can provide an impressive gain in terms of the large number of observables. In fact, preliminary polarization measurement for  $B^+ \rightarrow K^{*+}\rho^0$ ,  $B^+ \rightarrow K^{*0}\rho^+$  and  $B^0 \rightarrow K^{*0}\rho^0$  have already been done [26, 135, 138, 141].

We have studied  $B \rightarrow K^*\rho$  decays in a model-independent approach. We have shown that the  $B \rightarrow K^*\rho$  modes result in a total of 35 independent observables in comparison to the  $B \rightarrow K\pi$  modes that yield 9 observables.

However, we need 36 independent parameters for theoretical description of  $B \rightarrow K^* \rho$ , which is still one short of the number of possible observables. If we assume that one of the parameter  $\gamma$  is measured somewhere else, then we can determine all the remaining parameters in terms of observables and therefore we can resolve the “ $B \rightarrow K\pi$  puzzle.” First, we will determine all the theoretical parameters (except  $\gamma$ ) describing the decay amplitudes of  $B \rightarrow K^* \rho$  in a model-independent way in terms of experimental observables. We will then compare these determined theoretical parameters with the corresponding model estimates. It is very important for improving model calculations, like QCD factorization [142], perturbative QCD [143], and so on. Secondly, certain tests of the SM are discussed that may reveal NP effects if they appear in  $B \rightarrow K^* \rho$  decays. Any indication of NP effects in  $B \rightarrow K^* \rho$  will provide valuable hints on possible NP effects in  $B \rightarrow K\pi$  decays because both decay modes are same at the quark level. We write the decay amplitudes of  $B \rightarrow K^* \rho$  in terms of linear combinations of the topological amplitudes in the quark diagram approach [103]. Then we focus on how to extract all the theoretical parameters, including the magnitudes of the topological amplitudes and their strong phases, in terms of experimental observables. Finally, it is possible to determine all the parameters in analytic forms. We also propose tests of conventional hierarchy relations between the topological amplitudes and of possible relations between the relevant strong phases within the SM. A breakdown of these relations may indicate possible NP contributions appearing in  $B \rightarrow K^* \rho$  decays, as well as in the analogous mode  $B \rightarrow K\pi$ . One could hence verify if NP is the source of the “ $B \rightarrow K\pi$  puzzle.”

#### 4.2.1 FORMALISM FOR $B \rightarrow K^* \rho$ DECAYS

On the basis of quark diagram approach, the decay amplitudes for  $B \rightarrow K^* \rho$  modes can be written in terms of the topological amplitudes similar to Eq. (3.23) as follows:

$$A_\lambda^{0+} \equiv A_\lambda(B^+ \rightarrow K^{*0} \rho^+) = -\mathcal{P}'_\lambda + \mathcal{A}'_\lambda e^{i\gamma}, \quad (4.4)$$

$$\begin{aligned} \sqrt{2}A_\lambda^{+0} \equiv \sqrt{2}A_\lambda(B^+ \rightarrow K^{*+} \rho^0) &= (\mathcal{P}'_\lambda + \mathcal{P}'_{EW,\lambda} + \mathcal{P}'_{EW,\lambda}{}^C) \\ &\quad - (\mathcal{T}'_\lambda + \mathcal{C}'_\lambda + \mathcal{A}'_\lambda) e^{i\gamma}, \end{aligned} \quad (4.5)$$

$$A_\lambda^{+-} \equiv A_\lambda(B^0 \rightarrow K^{*+} \rho^-) = (\mathcal{P}'_\lambda + \mathcal{P}'_{EW,\lambda}{}^C) - \mathcal{T}'_\lambda, \quad (4.6)$$

$$\sqrt{2}A_\lambda^{00} \equiv \sqrt{2}A_\lambda(B^0 \rightarrow K^{*0}\rho^0) = -(\mathcal{P}'_\lambda - \mathcal{P}'_{EW,\lambda}) - \mathcal{C}'_\lambda e^{i\gamma} . \quad (4.7)$$

where the subscript  $\lambda = \{0, \parallel, \perp\}$  denotes the helicity of the amplitudes. We follow and generalize the notation used in Ref. [155].

Now we again redefine the topological amplitudes as:

$$\mathcal{A}'_\lambda \equiv \tilde{A}_\lambda e^{i\delta_\lambda^A}, \quad (4.8)$$

$$\mathcal{P}'_\lambda \equiv \tilde{P}_\lambda e^{i\delta_\lambda^P}, \quad (4.9)$$

$$\mathcal{T}'_\lambda \equiv \tilde{T}_\lambda e^{i\delta_\lambda^T}, \quad (4.10)$$

$$\mathcal{C}'_\lambda \equiv \tilde{C}_\lambda e^{i\delta_\lambda^C}, \quad (4.11)$$

$$\mathcal{P}'_{EW,\lambda} \equiv \tilde{P}_\lambda^{EW} e^{i\delta_\lambda^{EW}}, \quad (4.12)$$

$$\mathcal{P}'_{EW,\lambda}^C \equiv \tilde{P}_{C,\lambda}^{EW} e^{i\delta_\lambda^{CEW}}. \quad (4.13)$$

Now we re-express Eqs. (4.4)–(4.7) as

$$A_\lambda^{0+} = e^{i\gamma} \tilde{A}_\lambda e^{i\delta_\lambda^A} - \tilde{P}_\lambda e^{i\delta_\lambda^P}, \quad (4.14)$$

$$A_\lambda^{+0} = -\frac{1}{\sqrt{2}} \left[ e^{i\gamma} (\tilde{T}_\lambda e^{i\delta_\lambda^T} + \tilde{C}_\lambda e^{i\delta_\lambda^C} + \tilde{A}_\lambda e^{i\delta_\lambda^A}) - (\tilde{P}_\lambda e^{i\delta_\lambda^P} + \tilde{P}_\lambda^{EW} e^{i\delta_\lambda^{EW}} + \tilde{P}_{C,\lambda}^{EW} e^{i\delta_\lambda^{CEW}}) \right], \quad (4.15)$$

$$A_\lambda^{+-} = -\left[ e^{i\gamma} \tilde{T}_\lambda e^{i\delta_\lambda^T} - (\tilde{P}_\lambda e^{i\delta_\lambda^P} + \tilde{P}_{C,\lambda}^{EW} e^{i\delta_\lambda^{CEW}}) \right], \quad (4.16)$$

$$A_\lambda^{00} = -\frac{1}{\sqrt{2}} \left[ e^{i\gamma} \tilde{C}_\lambda e^{i\delta_\lambda^C} + (\tilde{P}_\lambda e^{i\delta_\lambda^P} - \tilde{P}_\lambda^{EW} e^{i\delta_\lambda^{EW}}) \right], \quad (4.17)$$

where the  $\gamma$  and  $\delta_\lambda$ 's are the weak phase and the relevant strong phases, respectively. And the amplitudes for these 4 decay modes and their conjugate modes can be related by isospin symmetry[80] as:

$$\frac{1}{\sqrt{2}} (A_\lambda^{0+} - A_\lambda^{+-}) = A_\lambda^{00} - A_\lambda^{+0}, \quad (4.18)$$

$$\frac{1}{\sqrt{2}} (\bar{A}_\lambda^{0+} - \bar{A}_\lambda^{+-}) = \bar{A}_\lambda^{00} - \bar{A}_\lambda^{+0}. \quad (4.19)$$

There are three helicities for each of the modes of the amplitudes for  $B \rightarrow K^*\rho$ . We can express all these amplitudes and their conjugates as

$$\begin{aligned} Amp(B \rightarrow K^*\rho) &= A_0 g_0 + A_\parallel g_\parallel + i A_\perp g_\perp, \\ Amp(\bar{B} \rightarrow K^*\rho) &= \bar{A}_0 g_0 + \bar{A}_\parallel g_\parallel - i \bar{A}_\perp g_\perp, \end{aligned} \quad (4.20)$$

where the  $g_\lambda$  are the coefficients of the helicity amplitudes written in the linear polarization basis. The  $g_\lambda$  depend only on the angles describing the

kinematics [144]. We can denote the helicity amplitudes (and their conjugate amplitudes) for the four  $K^*\rho$  modes as  $A_\lambda^{0+}, A_\lambda^{+-}, A_\lambda^{+0}, A_\lambda^{00}$ , (and  $\bar{A}_\lambda^{0+}, \bar{A}_\lambda^{+-}, \bar{A}_\lambda^{+0}, \bar{A}_\lambda^{00}$ ). Hence, the number of amplitudes for the  $B \rightarrow K^*\rho$  modes is three times that for the  $B \rightarrow K\pi$  modes. In principle, it is possible to measure many more observables in the  $B \rightarrow K^*\rho$  case compared to the  $B \rightarrow K\pi$  case. In the  $B \rightarrow K^*\rho$  modes, without including the interference terms between helicities, we have three times the number of observables (*i.e.*, 27 observables) in comparison to the  $K\pi$  modes (*i.e.*, 9 observables). In addition to these observables, there are many more of observables result from the interference terms between the helicities. We can examine in detail the number of observables available in  $B \rightarrow K^*\rho$ .

The time dependent decay for  $B \rightarrow f$ , where  $f$  is one of the  $K^*\rho$  final state, may be expressed as

$$\Gamma(\bar{B}(t) \rightarrow f) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left( \Lambda_{\lambda\sigma}^f \pm \Sigma_{\lambda\sigma}^f \cos(\Delta Mt) \mp \rho_{\lambda\sigma}^f \sin(\Delta Mt) \right) g_\lambda g_\sigma \quad (4.21)$$

where

$$\begin{aligned} B_\lambda^f &\equiv \Lambda_{\lambda\lambda}^f = \frac{1}{2}(|A_\lambda^f|^2 + |\bar{A}_\lambda^f|^2), & \Sigma_{\lambda\lambda}^f &= \frac{1}{2}(|A_\lambda^f|^2 - |\bar{A}_\lambda^f|^2), \\ \Lambda_{\perp i}^f &= -\text{Im}(A_\perp^f A_i^{f*} - \bar{A}_\perp^f \bar{A}_i^{f*}), & \Lambda_{\parallel 0}^f &= \text{Re}(A_\parallel^f A_0^{f*} + \bar{A}_\parallel^f \bar{A}_0^{f*}), \\ \Sigma_{\perp i}^f &= -\text{Im}(A_\perp^f A_i^{f*} + \bar{A}_\perp^f \bar{A}_i^{f*}), & \Sigma_{\parallel 0}^f &= \text{Re}(A_\parallel^f A_0^{f*} - \bar{A}_\parallel^f \bar{A}_0^{f*}), \\ \rho_{\perp i}^f &= \text{Re}\left(e^{-i\phi_M^q} [A_\perp^{f*} \bar{A}_i^f + A_i^{f*} \bar{A}_\perp^f]\right), & \rho_{\perp\perp}^f &= \text{Im}\left(e^{-i\phi_M^q} A_\perp^{f*} \bar{A}_\perp^f\right), \\ \rho_{\parallel 0}^f &= -\text{Im}\left(e^{-i\phi_M^q} [A_\parallel^{f*} \bar{A}_0^f + A_0^{f*} \bar{A}_\parallel^f]\right), & \rho_{ii}^f &= -\text{Im}\left(e^{-i\phi_M^q} A_i^{f*} \bar{A}_i^f\right), \\ & & & (\lambda, \sigma = \{0, \parallel, \perp\}, i = \{0, \parallel\}). \end{aligned} \quad (4.22)$$

We can measure all these 18 observables only for the CP eigenstate of  $K^{*0}\rho^0$ . The other 3 modes are not CP eigenstates so that time dependent asymmetry cannot be measured. But we can measure  $\Lambda_{\lambda\sigma}^f$  and  $\Sigma_{\lambda\sigma}^f$  for each of these modes, resulting in a total of 12 observables for each of the 3 modes:  $B^0 \rightarrow K^{*+}\rho^-, B^+ \rightarrow K^{*0}\rho^+$  and  $B^+ \rightarrow K^{*+}\rho^0$ . Hence, we can get a total of 54 observables. However, due to the isospin relations in Eqs. (4.18) and (4.19), the number of independent amplitudes is 18. *i.e.* at best we can get total of 35 independent informations related to 18 magnitudes of the amplitudes and their 17 relative phases. Thus, only 35 of the above 54 observables are independent.

We can describe the modes  $B \rightarrow K^* \rho$  using isospin analogous to the  $B \rightarrow K \pi$  modes. Since there are three helicity states, the amplitudes corresponding to the different topologies carry a helicity index and may be denoted by  $\tilde{T}_\lambda, \tilde{C}_\lambda, \tilde{A}_\lambda, \tilde{P}_\lambda, \tilde{P}_\lambda^{EW}$ , and  $\tilde{P}_{C,\lambda}^{EW}$ . Therefore, there are 18 amplitudes each with its own strong phase denoted by  $\delta_\lambda^T, \delta_\lambda^C, \delta_\lambda^A, \delta_\lambda^P, \delta_\lambda^{EW}$  and  $\delta_\lambda^{CEW}$ , respectively. Since we can measure only relative strong phases, so the number of strong phases become 17. Thus, we require 36 parameters: 18 (real) amplitudes, 17 strong phases and  $\gamma$ . Despite the large number of observables in the  $K^* \rho$  case, we still have one more parameter than the observables.

In the next section, we discuss how to determine all the theoretical parameters, such as the magnitudes and strong phases of the topological amplitudes, in term of the observables.

#### 4.2.2 EXTRACTING CONTRIBUTIONS OF VARIOUS TOPOLOGIES

We can describe the  $B \rightarrow K^* \rho$  modes by a total of 36 parameters. However, it is not possible to obtain more than 35 independent informations from the measurements. Thus, it is only possible to solve for the parameters with respect to one unknown parameter namely  $\gamma$ . It is well known that the weak phase  $\gamma$  can be measured through certain  $B$  decay processes, such as  $B \rightarrow D^{(*)} K^{(*)}$  [134, 135]. In this section, we present analytic solutions to all the parameters with respect to  $\gamma$ . To simplify expression we introduce some new notation. We define

$$y_\lambda^f = \sqrt{1 - \left( \frac{\Sigma_{\lambda\lambda}^f}{\Lambda_{\lambda\lambda}^f} \right)^2}, \quad (4.23)$$

$$\alpha_\lambda^{ij} = \arg(A_\lambda^{ij}), \quad \bar{\alpha}_\lambda^{ij} = \arg(\bar{A}_\lambda^{ij}), \quad (4.24)$$

where  $(ij) = (0+), (+0), (+-), (00)$  and  $\lambda = \{0, \parallel, \perp\}$ .

We first find the phases  $\alpha_\lambda^{ij}$  and  $\bar{\alpha}_\lambda^{ij}$  in terms of observables. Then, using the  $\alpha_\lambda^{ij}$  and  $\bar{\alpha}_\lambda^{ij}$ , we determine the amplitudes  $\tilde{T}_\lambda, \tilde{C}_\lambda, \tilde{A}_\lambda, \tilde{P}_\lambda, \tilde{P}_\lambda^{EW}$ ,  $\tilde{P}_{C,\lambda}^{EW}$  as well as the strong phases  $\delta_\lambda^T, \delta_\lambda^C, \delta_\lambda^A, \delta_\lambda^P, \delta_\lambda^{EW}$ , and  $\delta_\lambda^{CEW}$  given in Eqs. (4.14)–(4.17).

The relative sizes among these topological amplitudes are roughly estimated [145] as

$$\begin{aligned} 1 & : |V_{tb}^* V_{ts} P_{tc}|, \\ \mathcal{O}(\bar{\lambda}) & : |V_{ub}^* V_{us} T|, |V_{tb}^* V_{ts} P^{EW}|, \end{aligned}$$

$$\begin{aligned}
\mathcal{O}(\bar{\lambda}^2) &: |V_{ub}^* V_{us} C|, |V_{tb}^* V_{ts} P_C^{EW}|, \\
\mathcal{O}(\bar{\lambda}^3) &: |V_{ub}^* V_{us} A|, |V_{ub}^* V_{us} P_{uc}|,
\end{aligned} \tag{4.25}$$

where  $\bar{\lambda} \sim 0.2$ . We can roughly estimate the relative size of  $|V_{ub}^* V_{us} P_{uc}|$  as

$$\left| \frac{V_{ub}^* V_{us} P_{uc}}{V_{tb}^* V_{ts} P_{tc}} \right| \sim \bar{\lambda}^2 \left| \frac{P_{uc}}{P_{tc}} \right|. \tag{4.26}$$

Here  $|P_u|$  and  $|P_c|$  are smaller than  $|P_t|$  [146], and  $0.2 < |P_{uc}/P_{tc}| < 0.4$  within the perturbative calculation [147]. Hence, it can be assumed that  $|(V_{ub}^* V_{us} P_{uc})/(V_{tb}^* V_{ts} P_{tc})| \sim \mathcal{O}(\bar{\lambda}^3)$  for our analysis.

Since the estimation of the annihilation contribution ( $\tilde{A}/\tilde{P} \sim \mathcal{O}(\bar{\lambda}^3)$  where  $\bar{\lambda} \sim 0.2$ ) is very small, so it can be neglected [149]. After neglecting the annihilation terms We can write the decay amplitudes of the  $K^{*0}\rho^+$  modes as

$$\tilde{P}_\lambda = |A_\lambda^{0+}|, \tag{4.27}$$

$$\delta_\lambda^P = \alpha_\lambda^{0+} - \pi, \tag{4.28}$$

$$\bar{\alpha}_\lambda^{0+} = \alpha_\lambda^{0+}. \tag{4.29}$$

These relations imply that the direct CP asymmetry of the  $K^{*0}\rho^+$  mode vanishes:  $\Sigma_{\lambda\lambda}^{0+} = 0$  or  $y_\lambda^{0+} = 1$ . We can set  $\alpha_0^{0+} = \pi$  or  $\delta_0^P = 0$  without loss of generality. Then the phases  $\alpha_\parallel^{0+}$  and  $\alpha_\perp^{0+}$  can be obtained from the relative phases  $(\alpha_\parallel^{0+} - \alpha_0^{0+})$  and  $(\alpha_\perp^{0+} - \alpha_0^{0+})$  that are determined from the angular analysis through the measurement of  $\Lambda_{\perp 0}^{0+}$ ,  $\Sigma_{\perp 0}^{0+}$ ,  $\Lambda_{\parallel 0}^{0+}$ ,  $\Sigma_{\parallel 0}^{0+}$ . Subsequently all the  $\delta_\lambda^P$  and  $\bar{\alpha}_\lambda^{0+}$  for  $\lambda = \{0, \parallel, \perp\}$  are determined from Eqs. (4.28) and (4.29), up to a discrete ambiguity. We can remove this ambiguity by using theoretical estimates [139]. Now we re-parameterize for each helicity state of every relevant phase, such as  $\delta_\lambda^T$ ,  $\delta_\lambda^C$ ,  $\alpha_\lambda^{+-}$ , etc., as the relative phase to  $\delta_\lambda^P$ . For example, the strong phase  $\delta_\parallel^T$  should be understood as  $(\delta_\parallel^T - \delta_\parallel^P)$ .

Now, using the isospin analysis, we determine  $\alpha_\lambda^{ij}$ ,  $\bar{\alpha}_\lambda^{ij}$  in terms of the observables. The isospin relations between the decay amplitudes for  $B \rightarrow K^*\rho$  and their conjugate modes are the same as those given in Eqs. (4.18) and (4.19). Eq. (4.18) can be rewritten as

$$\frac{1}{\sqrt{2}} \left( |A_\lambda^{0+}| e^{i\alpha_\lambda^{0+}} - |A_\lambda^{+-}| e^{i\alpha_\lambda^{+-}} \right) = |A_\lambda^{00}| e^{i\alpha_\lambda^{00}} - |A_\lambda^{+0}| e^{i\alpha_\lambda^{+0}}, \tag{4.30}$$

where  $A_\lambda^{ij} \equiv |A_\lambda^{ij}| e^{i\alpha_\lambda^{ij}}$  and  $\alpha_0^{0+} = \pi$  ( $\lambda = \{0, \parallel, \perp\}$ ). In Eq. (4.30) we see that it is possible to measure all the magnitudes  $|A_\lambda^{ij}|$  of the decay

amplitudes and the relative phases  $(\alpha_{\parallel}^{ij} - \alpha_0^{ij})$  and  $(\alpha_{\perp}^{ij} - \alpha_0^{ij})$ . Therefore, the three isospin relations given in Eq. (4.30) are described by only three independent parameters,  $\alpha_0^{+-}$ ,  $\alpha_0^{00}$  and  $\alpha_0^{+0}$ , for the three helicity states  $\lambda = \{0, \parallel, \perp\}$ . Since we have 3 independent complex equations with these 3 real parameters for  $\lambda = \{0, \parallel, \perp\}$ , we can solve all these equations to determine the parameters  $\alpha_0^{0+}$ ,  $\alpha_0^{+-}$  and  $\alpha_0^{+0}$ . As a result, all the 12 magnitudes  $|A_{\lambda}^{ij}|$  and the 12 phases  $\alpha_{\lambda}^{ij}$  are completely determined. Details of the solutions of the phases and magnitudes of  $A_{\lambda}^{ij}$  (and  $\bar{A}_{\lambda}^{ij}$ ) are given in Appendix A.

For the CP conjugate decay modes, one can use the same method as the above by starting with the isospin relations:

$$\frac{1}{\sqrt{2}} \left( \bar{A}_{\lambda}^{0+} e^{i\bar{\alpha}_{\lambda}^{0+}} - \bar{A}_{\lambda}^{+-} e^{i\bar{\alpha}_{\lambda}^{+-}} \right) = \bar{A}_{\lambda}^{00} e^{i\bar{\alpha}_{\lambda}^{00}} - \bar{A}_{\lambda}^{+0} e^{i\bar{\alpha}_{\lambda}^{+0}}, \quad (4.31)$$

where  $\bar{A}_{\lambda}^{ij} \equiv A_{\lambda}^{ij} e^{i\bar{\alpha}_{\lambda}^{ij}}$  and  $\bar{\alpha}_0^{0+} = \alpha_0^{0+} = \pi$  ( $\lambda = \{0, \parallel, \perp\}$ ). Thus, in Eq. (4.31) for  $\lambda = \{0, \parallel, \perp\}$  there are only three independent parameters  $\bar{\alpha}_0^{+-}$ ,  $\bar{\alpha}_0^{00}$  and  $\bar{\alpha}_0^{+0}$  and we can determine these by solving the three independent complex equations. Consequently, we can determine all the 12  $\bar{A}_{\lambda}^{ij}$  and the 12  $\bar{\alpha}_{\lambda}^{ij}$ .

Now we define the following useful parameters :

$$X_{\lambda} e^{i\delta_{\lambda}^X} = |A_{\lambda}^{+-}| e^{i\alpha_{\lambda}^{+-}} - \tilde{P}_{\lambda}, \quad \bar{X}_{\lambda} e^{i\bar{\delta}_{\lambda}^X} = |\bar{A}_{\lambda}^{+-}| e^{i\bar{\alpha}_{\lambda}^{+-}} - \tilde{P}_{\lambda}, \quad (4.32)$$

$$Y_{\lambda} e^{i\delta_{\lambda}^Y} = \sqrt{2} |A_{\lambda}^{00}| e^{i\alpha_{\lambda}^{00}} + \tilde{P}_{\lambda}, \quad \bar{Y}_{\lambda} e^{i\bar{\delta}_{\lambda}^Y} = \sqrt{2} |\bar{A}_{\lambda}^{00}| e^{i\bar{\alpha}_{\lambda}^{00}} + \tilde{P}_{\lambda} \quad (4.33)$$

Since everything on the right-hand side of these equations has been found, we can determine for each helicity state all the 8 parameters  $X_{\lambda}$ ,  $\bar{X}_{\lambda}$ ,  $Y_{\lambda}$ ,  $\bar{Y}_{\lambda}$ ,  $\delta_{\lambda}^X$ ,  $\bar{\delta}_{\lambda}^X$ ,  $\delta_{\lambda}^Y$ ,  $\bar{\delta}_{\lambda}^Y$  on the left-hand side in terms of the known parameters by directly solving the 4 complex equations. Then we re-express Eqs. (4.16) and (4.17) as

$$X_{\lambda} e^{i\delta_{\lambda}^X} = -e^{i\gamma} \tilde{T}_{\lambda} e^{i\delta_{\lambda}^T} + \tilde{P}_{C,\lambda}^{EW} e^{i\delta_{\lambda}^{CEW}}, \quad (4.34)$$

$$Y_{\lambda} e^{i\delta_{\lambda}^Y} = -e^{i\gamma} \tilde{C}_{\lambda} e^{i\delta_{\lambda}^C} + \tilde{P}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}}. \quad (4.35)$$

These two complex equations together with their CP conjugate mode equations (*i.e.*, 8 real equations) include 8 real parameters (the magnitudes and strong phases of 4 topological amplitudes) for each  $\lambda$  that can be determined in terms of the observables as: From Eqn. (4.34) and its CP conjugate, we have

$$X_{\lambda} e^{i\delta_{\lambda}^X} = -e^{i\gamma} \tilde{T}_{\lambda} e^{i\delta_{\lambda}^T} + \tilde{P}_{C,\lambda}^{EW} e^{i\delta_{\lambda}^{CEW}}, \quad (4.36)$$



$$\bar{X}_\lambda e^{i\bar{\delta}_\lambda^X} = -e^{-i\gamma} \tilde{T}_\lambda e^{i\delta_\lambda^T} + \tilde{P}_{C,\lambda}^{EW} e^{i\delta_\lambda^{CEW}}. \quad (4.37)$$

From Eqn. (4.32),

$$\begin{aligned} X_\lambda e^{i\delta_\lambda^X} - \bar{X}_\lambda e^{i\bar{\delta}_\lambda^X} &= (|A_\lambda^{+-}| e^{i\alpha_\lambda^{+-}} - \tilde{P}_\lambda) - (|\bar{A}_\lambda^{+-}| e^{i\bar{\alpha}_\lambda^{+-}} - \tilde{P}_\lambda) \\ &= |A_\lambda^{+-}| e^{i\alpha_\lambda^{+-}} - |\bar{A}_\lambda^{+-}| e^{i\bar{\alpha}_\lambda^{+-}} \\ &= |A_\lambda^{+-}| (\cos \alpha_\lambda^{+-} + i \sin \alpha_\lambda^{+-}) \\ &\quad - |\bar{A}_\lambda^{+-}| (\cos \bar{\alpha}_\lambda^{+-} + i \sin \bar{\alpha}_\lambda^{+-}) \\ &= (|A_\lambda^{+-}| \cos \alpha_\lambda^{+-} - |\bar{A}_\lambda^{+-}| \cos \bar{\alpha}_\lambda^{+-}) \\ &\quad + i(|A_\lambda^{+-}| \sin \alpha_\lambda^{+-} - |\bar{A}_\lambda^{+-}| \sin \bar{\alpha}_\lambda^{+-}) \end{aligned} \quad (4.38)$$

From Eqns. (4.36) and (4.37), we have

$$\begin{aligned} X_\lambda e^{i\delta_\lambda^X} - \bar{X}_\lambda e^{i\bar{\delta}_\lambda^X} &= -(e^{i\gamma} - e^{-i\gamma}) \tilde{T}_\lambda e^{i\delta_\lambda^T} \\ &= -2i \sin \gamma (\cos \delta_\lambda^T + i \sin \delta_\lambda^T) T_\lambda \\ &= 2 \sin \gamma \sin \delta_\lambda^T T_\lambda - 2i \sin \gamma \cos \delta_\lambda^T \tilde{T}_\lambda \end{aligned} \quad (4.39)$$

On comparing Eqns. (4.38) and (4.39), we have

$$2 \sin \gamma \sin \delta_\lambda^T \tilde{T}_\lambda = |A_\lambda^{+-}| \cos \alpha_\lambda^{+-} - |\bar{A}_\lambda^{+-}| \cos \bar{\alpha}_\lambda^{+-} \quad (4.40)$$

$$-2 \sin \gamma \cos \delta_\lambda^T \tilde{T}_\lambda = |A_\lambda^{+-}| \sin \alpha_\lambda^{+-} - |\bar{A}_\lambda^{+-}| \sin \bar{\alpha}_\lambda^{+-} \quad (4.41)$$

Now squaring and adding Eqns. (4.40) and (4.41), we get

$$\begin{aligned} 4 \sin^2 \gamma \tilde{T}_\lambda^2 &= |A_\lambda^{+-}|^2 + |\bar{A}_\lambda^{+-}|^2 - 2|A_\lambda^{+-}| |\bar{A}_\lambda^{+-}| \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-}) \\ &= 2B_\lambda^{+-} - \sqrt{4|A_\lambda^{+-}|^2 |\bar{A}_\lambda^{+-}|^2 \cos(\alpha_\lambda^{+-} - \bar{\alpha}_\lambda^{+-})} \end{aligned} \quad (4.42)$$

Since

$$\begin{aligned} \sqrt{4|A_\lambda^{+-}|^2 |\bar{A}_\lambda^{+-}|^2} &= \sqrt{(|A_\lambda^{+-}|^2 + |\bar{A}_\lambda^{+-}|^2)^2 - (|A_\lambda^{+-}|^2 - |\bar{A}_\lambda^{+-}|^2)^2} \\ &= \sqrt{4(B_\lambda^{+-})^2 - 4(\Sigma_\lambda^{+-})^2} \\ &= 2B_\lambda^{+-} \sqrt{1 - \left(\frac{\Sigma_\lambda^{+-}}{B_\lambda^{+-}}\right)^2} \\ &= 2B_\lambda^{+-} y_\lambda^{+-} \end{aligned} \quad (4.43)$$

Therefore,

$$\begin{aligned}
4 \sin^2 \gamma \tilde{T}_\lambda^2 &= 2B_\lambda^{+-} - 2B_\lambda^{+-} y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-}) \\
&= 2B_\lambda^{+-} [1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-})] \\
\tilde{T}_\lambda &= \sqrt{\frac{B_\lambda^{+-}}{2 \sin^2 \gamma} [1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-})]} \quad (4.44)
\end{aligned}$$

From Eqns. (4.40) and (4.41), we get the value of  $\tan \delta_\lambda^T$ .

$$\tan \delta_\lambda^T = -\frac{|\bar{A}_\lambda^{+-}| \cos \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \cos \alpha_\lambda^{+-}}{|\bar{A}_\lambda^{+-}| \sin \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \sin \alpha_\lambda^{+-}}, \quad (4.45)$$

Similarly we can solve for other parameters in terms of observables as:

$$\tilde{T}_\lambda = \sqrt{\frac{B_\lambda^{+-}}{2 \sin^2 \gamma} [1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-})]}, \quad (4.46)$$

$$\tilde{C}_\lambda = \sqrt{\frac{B_\lambda^{00}}{\sin^2 \gamma} [1 - y_\lambda^{00} \cos(\bar{\alpha}_\lambda^{00} - \alpha_\lambda^{00})]}, \quad (4.47)$$

$$\tilde{P}_{C,\lambda}^{EW} = \sqrt{\frac{1}{4 \sin^2 \gamma} [X_\lambda^2 + \bar{X}_\lambda^2 - 2X_\lambda \bar{X}_\lambda \cos(\bar{\delta}_\lambda^X - \delta_\lambda^X + 2\gamma)]}, \quad (4.48)$$

$$\tilde{P}_\lambda^{EW} = \sqrt{\frac{1}{4 \sin^2 \gamma} [Y_\lambda^2 + \bar{Y}_\lambda^2 - 2Y_\lambda \bar{Y}_\lambda \cos(\bar{\delta}_\lambda^Y - \delta_\lambda^Y + 2\gamma)]}. \quad (4.49)$$

And the strong phases are

$$\tan \delta_\lambda^T = -\frac{|\bar{A}_\lambda^{+-}| \cos \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \cos \alpha_\lambda^{+-}}{|\bar{A}_\lambda^{+-}| \sin \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \sin \alpha_\lambda^{+-}}, \quad (4.50)$$

$$\tan \delta_\lambda^C = -\frac{|\bar{A}_\lambda^{00}| \cos \bar{\alpha}_\lambda^{00} - |A_\lambda^{00}| \cos \alpha_\lambda^{00}}{|\bar{A}_\lambda^{00}| \sin \bar{\alpha}_\lambda^{00} - |A_\lambda^{00}| \sin \alpha_\lambda^{00}}, \quad (4.51)$$

$$\tan \delta_\lambda^{CEW} = -\frac{|\bar{A}_\lambda^{+-}| \cos(\bar{\alpha}_\lambda^{+-} + \gamma) - |A_\lambda^{+-}| \cos(\alpha_\lambda^{+-} - \gamma)}{|\bar{A}_\lambda^{+-}| \sin(\bar{\alpha}_\lambda^{+-} + \gamma) - |A_\lambda^{+-}| \sin(\alpha_\lambda^{+-} - \gamma) - 2|A_\lambda^{0+}| \sin \gamma} \quad (4.52)$$

$$\tan \delta_\lambda^{EW} = -\frac{|\bar{A}_\lambda^{00}| \cos(\bar{\alpha}_\lambda^{00} + \gamma) - |A_\lambda^{00}| \cos(\alpha_\lambda^{00} - \gamma)}{|\bar{A}_\lambda^{00}| \sin(\bar{\alpha}_\lambda^{00} + \gamma) - |A_\lambda^{00}| \sin(\alpha_\lambda^{00} - \gamma) + \sqrt{2} |A_\lambda^{0+}| \sin \gamma} \quad (4.53)$$

It is shown that all the hadronic parameters can be cast in terms of the observables and only one unknown parameter  $\gamma$ . If we assume that  $\gamma$  is measured from somewhere else, then we can achieve a model-independent understanding of which hadronic parameter is dominating in these modes.

In near future experiments, we can get the necessary information to extract each hadronic parameter. For example, we can determine the parameters—the color-suppressed tree ( $\tilde{C}_\lambda$ ) and the EW penguin ( $\tilde{P}_\lambda^{EW}$ ) amplitudes by using the relevant observables expected to be measured in the near future and the formulas given in Eqs. (4.47) and (4.49). Then, we can compare the determined parameters with theoretical predictions, one can then further investigate possible NP effects(if any) appearing in  $B \rightarrow K^* \rho$  decay processes [150]. In the next section it is discussed in details how we can determine these parameters in terms of the observables to verify the hierarchy relations between the topological amplitudes that are conventionally assumed to be true in the SM. Then it is also discussed various ways to test the validity of assumptions equating the strong phases of a certain set of topological amplitudes.

#### 4.2.3 TESTING THE HIERARCHY OF TOPOLOGICAL AMPLITUDES AND POSSIBLE RELATIONS BETWEEN THEIR STRONG PHASES

All the topological amplitudes and strong phases can be estimated in terms of the observables and  $\gamma$ . One can conclude that if there exist any obtained relations between the theoretical parameters they must also result in relations among the observables. We first derive certain relations between the observables that can test the conventional hierarchy between the topological amplitudes within the SM. It may be expected that

$$\frac{\tilde{T}_\lambda}{\tilde{P}_\lambda} \approx \frac{\tilde{P}_\lambda^{EW}}{\tilde{P}_\lambda} \approx \bar{\lambda}, \quad \frac{\tilde{C}_\lambda}{\tilde{P}_\lambda} \approx \frac{\tilde{P}_{C,\lambda}^{EW}}{\tilde{P}_\lambda} \approx \bar{\lambda}^2 \quad (4.54)$$

in analogy to the expectations [103] for the modes  $B \rightarrow K\pi$  because the two modes are topologically equivalent.

The topological amplitudes  $\tilde{P}_\lambda$ ,  $\tilde{A}_\lambda$ ,  $\tilde{T}_\lambda$ ,  $\tilde{C}_\lambda$ ,  $\tilde{P}_\lambda^{EW}$  and  $\tilde{P}_{C,\lambda}^{EW}$  have been expressed in terms of the observables and  $\gamma$  in the previous section. It is therefore easy to see that there must exist a relation between the observables and  $\gamma$  that must hold as a consequence of the hierarchy between the

topological amplitudes. The relations (4.54) indicate the hierarchy relation  $\tilde{P}_\lambda > \tilde{T}_\lambda \approx \tilde{P}_\lambda^{EW} > \tilde{C}_\lambda \approx \tilde{P}_{C,\lambda}^{EW}$  which must hold within the SM.

A simple approach would be to test the hierarchy  $\tilde{P}_\lambda > \tilde{T}_\lambda > \tilde{C}_\lambda$ , which would imply the following relation:

$$\begin{aligned} 2 \sin^2 \gamma B_\lambda^{0+} &> B_\lambda^{+-} \left[ 1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-}) \right] > \\ 2B_\lambda^{00} \left[ 1 - y_\lambda^{00} \cos(\bar{\alpha}_\lambda^{00} - \alpha_\lambda^{00}) \right]. \end{aligned} \quad (4.55)$$

To test the hierarchy  $\tilde{P}_\lambda^{EW} > \tilde{P}_{C,\lambda}^{EW}$ , one can test the following relation:

$$\begin{aligned} Y_\lambda^2 + \bar{Y}_\lambda^2 - 2Y_\lambda \bar{Y}_\lambda \cos(\bar{\delta}_\lambda^Y - \delta_\lambda^Y + 2\gamma) &> \\ X_\lambda^2 + \bar{X}_\lambda^2 - 2X_\lambda \bar{X}_\lambda \cos(\bar{\delta}_\lambda^X - \delta_\lambda^X + 2\gamma). \end{aligned} \quad (4.56)$$

Besides the above relation, simple tests verifying the hierarchy of  $\tilde{P}_\lambda^{EW}$  and  $\tilde{P}_{C,\lambda}^{EW}$  can be derived. Assuming that  $\tilde{P}_{C,\lambda}^{EW} = \bar{\lambda}^2 \tilde{P}_\lambda$  in Eq. (4.16), it can be shown that

$$\frac{B_\lambda^{+-} \left[ 1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-} + 2\gamma) \right]}{2 \sin^2 \gamma B_\lambda^{0+}} = 1 + \mathcal{O}(\bar{\lambda}^2) \quad (4.57)$$

Similarly assuming that  $P_\lambda^{EW} = \bar{\lambda} P_\lambda$  in Eq. (4.17), it can be found that

$$\frac{B_\lambda^{00} \left[ 1 - y_\lambda^{00} \cos(\bar{\alpha}_\lambda^{00} - \alpha_\lambda^{00} + 2\gamma) \right]}{\sin^2 \gamma B_\lambda^{0+}} = 1 + \mathcal{O}(\bar{\lambda}) \quad (4.58)$$

The relations  $\tilde{T}_\lambda \approx \tilde{P}_\lambda^{EW}$  and  $\tilde{C}_\lambda \approx \tilde{P}_{C,\lambda}^{EW}$  would imply that

$$2B_\lambda^{+-} \left[ 1 - y_\lambda^{+-} \cos(\bar{\alpha}_\lambda^{+-} - \alpha_\lambda^{+-}) \right] \approx Y_\lambda^2 + \bar{Y}_\lambda^2 - 2Y_\lambda \bar{Y}_\lambda \cos(\bar{\delta}_\lambda^Y - \delta_\lambda^Y + 2\gamma), \quad (4.59)$$

$$4B_\lambda^{00} \left[ 1 - y_\lambda^{00} \cos(\bar{\alpha}_\lambda^{00} - \alpha_\lambda^{00}) \right] \approx X_\lambda^2 + \bar{X}_\lambda^2 - 2X_\lambda \bar{X}_\lambda \cos(\bar{\delta}_\lambda^X - \delta_\lambda^X + 2\gamma) \quad (4.60)$$

Testing the hierarchy  $\tilde{T}_\lambda > \tilde{P}_{C,\lambda}^{EW}$  and  $\tilde{P}_\lambda^{EW} > \tilde{C}_\lambda$  is rather simple. We note that  $\tilde{T}_\lambda$  and  $\tilde{C}_\lambda$  can be rewritten as

$$\tilde{T}_\lambda = \sqrt{\frac{1}{4 \sin^2 \gamma} \left[ X_\lambda^2 + \bar{X}_\lambda^2 - 2X_\lambda \bar{X}_\lambda \cos(\bar{\delta}_\lambda^X - \delta_\lambda^X) \right]}, \quad (4.61)$$

$$\tilde{C}_\lambda = \sqrt{\frac{1}{4 \sin^2 \gamma} \left[ Y_\lambda^2 + \bar{Y}_\lambda^2 - 2Y_\lambda \bar{Y}_\lambda \cos(\bar{\delta}_\lambda^Y - \delta_\lambda^Y) \right]}. \quad (4.62)$$

Comparing these equations with Eqs. (4.48) and (4.49), we find that

$$\tilde{T}_\lambda > \tilde{P}_{C,\lambda}^{EW} \implies \sin(\bar{\delta}^X - \delta^X + \gamma) < 0, \quad (4.63)$$

$$\tilde{P}_\lambda^{EW} > \tilde{C}_\lambda \implies \sin(\bar{\delta}^Y - \delta^Y + \gamma) > 0. \quad (4.64)$$

The hierarchy assumption between the magnitudes of topological amplitudes can be verified by using the relations (4.55)–(4.64). These relations will provide a litmus test to verify the above hierarchy assumption. The same hierarchy relation is expected to hold in  $B \rightarrow K\pi$  modes because the decay modes  $B \rightarrow K\pi$  and  $B \rightarrow K^*\rho$  are equivalent at quark level. We can also derive several relations in terms of the observables and  $\gamma$  by assuming relations between the strong phases of the topological amplitudes. We can use these relations to verify the assumptions often made between these strong phases, such as  $\delta_\lambda^C \approx \delta_\lambda^{CEW} \approx \delta_\lambda^P$  and  $\delta_\lambda^{EW} \approx \delta_\lambda^T$ , expecting to hold within the SM. We will discuss only a few such relations that test some common assumptions being made on the strong phases; however more general relations between observables under various assumptions on strong phases without neglecting the annihilation contribution are also available in Appendix B.

We can start with the interesting relation  $\delta_\lambda^C = \delta_\lambda^P$ . In our convention ( $\delta_\lambda^P \equiv 0$ ), this implies  $\delta_\lambda^C = 0$ . So from Eq. (4.51) we get the relation

$$|\bar{A}_\lambda^{00}| \cos \bar{\alpha}_\lambda^{00} = |A_\lambda^{00}| \cos \alpha_\lambda^{00}. \quad (4.65)$$

If  $\delta_\lambda^C = 0$ , then the real part of the amplitude  $A_\lambda^{00}$  is the same as that of its CP conjugate amplitude  $\bar{A}_\lambda^{00}$ , which is just the restatement of the relation (4.65). Since the topological amplitude  $\tilde{C}_\lambda$  is estimated to be very small ( $\tilde{C}_\lambda = \bar{\lambda}^2 \tilde{P}_\lambda$ ) in the SM, it is also expected from Eq. (4.17) that in the SM the direct CP asymmetry in the  $K^{*0}\rho^0$  mode almost vanishes. Similarly the assumption  $\delta_\lambda^{CEW} = \delta_\lambda^P$  leads to the relation

$$|\bar{A}_\lambda^{+-}| \cos(\bar{\alpha}_\lambda^{+-} + \gamma) = |A_\lambda^{+-}| \cos(\alpha_\lambda^{+-} - \gamma), \quad (4.66)$$

obtained from Eq. (4.52). Finally, from the assumption  $\delta_\lambda^T = \delta_\lambda^{EW}$ , we get the relation

$$\frac{|\bar{A}_\lambda^{+-}| \cos \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \cos \alpha_\lambda^{+-}}{|\bar{A}_\lambda^{+-}| \sin \bar{\alpha}_\lambda^{+-} - |A_\lambda^{+-}| \sin \alpha_\lambda^{+-}} = \frac{|\bar{A}_\lambda^{00}| \sin \bar{\alpha}_\lambda^{00} + |A_\lambda^{00}| \sin \alpha_\lambda^{00}}{2|A_\lambda^{00}| \cos \alpha_\lambda^{00} + \sqrt{2}|A_\lambda^{0+}|} [1 + \mathcal{O}(\bar{\lambda}^2)], \quad (4.67)$$

where we have used  $\tilde{C}_\lambda = \bar{\lambda}^2 \tilde{P}_\lambda$ .

The validity of the several relations derived above, or the degree to which they fail to hold, will shed light on the possible origins of the “ $B \rightarrow K\pi$  puzzle,” and therefore help in uncovering possible new physics contributions.

#### 4.2.4 ISOLATING SIGNALS OF NEW PHYSICS IN $B \rightarrow K^*\rho$ MODES

In the previous section we have derived some possible relations between topological amplitudes and strong phases. If we assume that these relations hold within the SM, then a violation of these relations would indicate about NP beyond SM. The expressions in Eqs. (4.14)–(4.17) for the decay amplitudes  $B \rightarrow K^*\rho$  modes describe not only the SM contributions but also *any* possible NP effects that contribute to the amplitude. For example, Let’s consider the contribution of NP with an amplitude  $N_\lambda e^{i\delta_\lambda} e^{i\phi_{NP}}$ . This amplitude can be re-expressed using reparametrization invariance [102] as a sum of two contributions with one term having no weak phase and the other term having a weak phase  $\gamma$ , *i.e.*  $N_\lambda e^{i\delta_\lambda} e^{i\phi_{NP}} \equiv N_1^\lambda e^{i\delta_\lambda} + N_2^\lambda e^{i\delta_\lambda} e^{i\gamma}$ , where  $N_1^\lambda$  and  $N_2^\lambda$  are determined purely in terms of  $\phi_{NP}$  and  $\gamma$ . As an explicit example let us consider NP contributing via the EW penguin to amplitudes in Eqs. (4.15) and (4.17). Using reparametrization invariance it can easily be absorbed by redefining the amplitudes  $\tilde{P}_\lambda^{EW}$  and  $\tilde{C}_\lambda$ , so that the amplitudes in Eqs. (4.15) and (4.17) retain the same form. In general NP contributing to any of the topological amplitudes can be easily absorbed so that the amplitudes in Eqs. (4.14)–(4.17) retain the same form. However, if there exist relations between the amplitudes or strong phases, the  $B \rightarrow K^*\rho$  amplitudes would differ from the SM form in the presence of NP. Since the number of independent SM parameters is reduced. Therefore we, not only be able to see signals of NP but also solve for NP parameters. In this section we consider two cases to explore this possibility. We first discuss the consequence of relations between  $\tilde{P}_\lambda^{EW} e^{i\delta_\lambda^{EW}}$  and  $\tilde{T}_\lambda e^{i\delta_\lambda^T}$ , and  $\tilde{P}_{C,\lambda}^{EW} e^{i\delta_\lambda^{CEW}}$  and  $\tilde{C}_\lambda e^{i\delta_\lambda^C}$  that are expected to hold in the SM. We next consider the case where the strong phases are related in the SM. Let us consider a NP contribution in the EW penguins, to amplitudes in Eqs. (4.15) and (4.17). It is assumed that NP contributes with an amplitude  $N_\lambda e^{i\delta_\lambda^N} e^{i\phi_{NP}} \equiv N_1^\lambda e^{i\delta_\lambda^N} + N_2^\lambda e^{i\delta_\lambda^N} e^{i\gamma}$ . We have shown that using angular-analysis we can solve for all the parameters in Eqs. (4.14)–(4.17). In particular we can measure  $\tilde{P}_\lambda^{EW}$ ,  $\tilde{P}_{C,\lambda}^{EW}$ ,  $\tilde{T}_\lambda$ ,  $\tilde{C}_\lambda$ ,  $\delta_\lambda^{EW}$ ,  $\delta_\lambda^{CEW}$ ,  $\delta_\lambda^T$ ,  $\delta_\lambda^C$  in terms of  $\gamma$ . Note that these measured values include any NP contributions that may be present. In fact, if NP contributes via the EW penguins, the SM amplitudes (defined by calligraphic characters)– $\tilde{P}_\lambda^{EW}$  and  $\tilde{C}_\lambda$  are the only ones modified by NP, but they cannot them-

selves be measured. The other amplitudes are unmodified by NP and hence we need not distinguish the SM amplitudes from the amplitudes defined in Eqs. (4.14)–(4.17).

Using flavor SU(3), we can relate the  $\Delta I = 3/2$  parts of the tree and electroweak penguin Hamiltonians(SM part) as [112]

$$\mathcal{H}_{\Delta I=3/2}^{EW} = -\frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \left| \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} \right| \mathcal{H}_{\Delta I=3/2}^{tree}. \quad (4.68)$$

Hence,  $\tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}}$  and  $\tilde{\mathcal{P}}_{C,\lambda}^{EW} e^{i\delta_{\lambda}^{CEW}}$  are related to  $\tilde{T}_{\lambda} e^{i\delta_{\lambda}^T}$  and  $\tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C}$  :

$$\begin{aligned} \tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}} &\simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \left| \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} \right| \tilde{T}_{\lambda} e^{i\delta_{\lambda}^T} \equiv \zeta \tilde{T}_{\lambda} e^{i\delta_{\lambda}^T}, \\ \tilde{\mathcal{P}}_{C,\lambda}^{EW} e^{i\delta_{\lambda}^{CEW}} &\simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \left| \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} \right| \tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C} \equiv \zeta \tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C}, \end{aligned} \quad (4.69)$$

where the  $c_i$  are Wilson coefficients [32]. Using the SM relations in Eq. (4.69), we can see that

$$\begin{aligned} \zeta \tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C} &= \zeta \tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C} + \zeta N_2^{\lambda} e^{i\delta_{\lambda}^N}, \\ &= \tilde{\mathcal{P}}_{C,\lambda}^{EW} e^{i\delta_{\lambda}^{CEW}} + \zeta N_2^{\lambda} e^{i\delta_{\lambda}^N}, \end{aligned} \quad (4.70)$$

$$\begin{aligned} \tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}} &= \tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}} + N_1^{\lambda} e^{i\delta_{\lambda}^N}, \\ &= \zeta \tilde{T}_{\lambda} e^{i\delta_{\lambda}^T} + N_1^{\lambda} e^{i\delta_{\lambda}^N}. \end{aligned} \quad (4.71)$$

Eqs. (4.70) and (4.71) form four relations in terms of only three unknowns  $N_1^{\lambda}$ ,  $N_2^{\lambda}$  and  $\delta_{\lambda}^N$  for each  $\lambda$ . Hence we can solve  $N_1^{\lambda}$ ,  $N_2^{\lambda}$  and  $\delta_{\lambda}^N$ . the weak phase of NP  $\phi_{NP}$  can also be obtained from the relation

$$\frac{N_1^{\lambda}}{N_2^{\lambda}} = \frac{\sin(\gamma - \phi_{NP})}{\sin \phi_{NP}}. \quad (4.72)$$

We have enough observables even to solve for  $\zeta$ . Therefore, we not only measure NP but also test the SU(3) assumption under the above assumptions made.

We now assume that in the SM the strong phases are related such that  $\delta_{\lambda}^C = \delta_{\lambda}^{CEW} = \delta_{\lambda}^P$  and  $\delta_{\lambda}^{EW} = \delta_{\lambda}^T$ . Then we can see that

$$\tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^{EW}} = \tilde{\mathcal{P}}_{\lambda}^{EW} e^{i\delta_{\lambda}^T} + N_1^{\lambda} e^{i\delta_{\lambda}^N} \quad (4.73)$$

$$\tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^C} = \tilde{\mathcal{C}}_{\lambda} e^{i\delta_{\lambda}^P} + N_2^{\lambda} e^{i\delta_{\lambda}^N} \quad (4.74)$$

Now we can see that there are four equations, but now in terms of five unknowns  $\tilde{\mathcal{P}}_\lambda^{EW}$ ,  $\tilde{\mathcal{C}}_\lambda$ ,  $N_1^\lambda$ ,  $N_2^\lambda$  and  $\delta_\lambda^N$ ; *i.e.* it is not possible to solve all five unknowns. Hence if the strong phases are related in the SM, the failure of the relations  $\delta_\lambda^C = \delta_\lambda^{CEW} = \delta_\lambda^P$  and  $\delta_\lambda^{EW} = \delta_\lambda^T$  can be tested. Hence, it is not possible to get clean signal of NP under the assumptions on strong phases; the failure of the relations between strong phases could be due to NP or simply due to hadronic effects within the SM.

We have emphasized that it is generally impossible to have a clean signal of NP due to reparameterization invariance [148]. It is not always possible to independently test the hadronic assumption and at the same time cleanly measure the NP parameters. However, we do demonstrate that if the tree and color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins by the well known relations of Eq. (4.69), *it is possible not only to verify the validity of these relations but also to have a clean measurement of NP parameters.* Therefore, an angular analysis in  $B \rightarrow K^*\rho$  is essential not only to establish cleanly the validity of the relations in Eq. (4.69) but also at the same time to cleanly probe for new physics.

#### 4.2.5 SUMMARY

We have performed a detailed study of the  $B \rightarrow K^*\rho$  decays using a model-independent approach. It was shown that  $B \rightarrow K^*\rho$  modes have a distinct advantage due the large number of independent observables that can be measured. In comparison to the  $B \rightarrow K\pi$  modes that yield only 9 independent observables, the  $B \rightarrow K^*\rho$  modes result in as many as 35 independent observables. Since  $B \rightarrow K\pi$  and  $B \rightarrow K^*\rho$  have the same quark level subprocess, the study of  $B \rightarrow K^*\rho$  may well shed light on the well known “ $B \rightarrow K\pi$  puzzle.” The relevant decay amplitudes were decomposed into linear combinations of the topological amplitudes with their respective strong phases assuming isospin. We have pointed out that the amplitude written this way are the most general ones and included contributions not only from the SM but also any NP that might exist. We obtained explicit model-independent expressions for all the topological amplitudes and their strong phases in terms of observables and the weak phase  $\gamma$ . With  $\gamma$  measured using other modes, our results are the first in literature to estimate the topological amplitudes and strong phases purely in terms of observables, for the  $B \rightarrow K\pi$  analogous modes. We further suggested clean tests to verify if there existed any hierarchy relations among topological amplitudes analogous to the ones conventionally assumed to be existed for  $B \rightarrow K\pi$  in the SM. In addition, we presented tests that would verify any equality be-



tween the strong phases of the topological amplitudes. A model independent understanding of the relative sizes of the topological amplitudes and relations between their strong phases could provide valuable insights into NP searches. Generally it is not in general possible to independently test the hadronic assumption and at the same time cleanly measure the NP parameters, we show one example where it is possible to do both. We demonstrated that if the tree and color-suppressed trees were related to the electroweak penguins and color-suppressed electroweak penguins. Therefore it is shown that it is not only possible to verify the validity of such relations but also to cleanly measure NP parameters [151].

## CHAPTER 5

# SUMMARY

In this thesis, a few aspects of  $B$  physics were studied. We now summarise the main features. In Chapter 1, we begin with a short introduction of  $B$  physics and CP violation. We briefly discussed the discrete symmetries C (Charge-Conjugation), P (Parity) and T (Time reversal). We extended the discussion to CP and CPT symmetries as well. Basics of the CP phenomenology in  $B$  meson decays were briefly reviewed. We discussed various types of CP violation in  $B$  system. We also presented a very brief discussion of the SM. The source of CP violation in the Standard Model (SM) is only through CKM matrix elements. The two important parametrizations of CKM matrix, Standard Parametrization and Wolfenstein Parametrization, were described briefly. We discussed the limitation of the SM and reasons to believe in the existence of physics beyond SM. Finally we explored how  $B$  mesons can be used to seek evidence of New Physics (NP).

In chapter 2, we reviewed some important features of  $B$  meson decays in the framework of the SM. We discussed various important decay channels with the help of effective Hamiltonians. Extraction of weak phases  $\alpha$ ,  $\beta$  and  $\gamma$  of the unitary triangle were also discussed briefly. The main goal is to overconstrain the unitarity triangle as much as possible so that we can perform a stringent test of the KM mechanism of CP violation.

In third chapter, we examined the pattern of NP if it enters in any particular quark-level diagram. We have shown how NP contribution to amplitudes can be absorbed into SM topological amplitudes. Consequently we have shown the difficulties in distinguishing the NP signals from the SM. We point out that a generic  $B \rightarrow f$  decay amplitude may be written as a sum of two terms, corresponding to a pair of weak phases  $\{\phi_1, \phi_2\}$  chosen completely at will (as long as they do not differ by a multiple of  $180^\circ$ ). Clearly, physical observables may not depend on this choice; we designate

this property by “reparametrization invariance”.

We have explored some of the unusual features of reparametrization invariance. We have made three main points. First, there is no clean signal of NP in any  $B$  decay which receives penguin contributions. In order to obtain a signal of NP, it is necessary to either have an accurate theoretical estimate of parameters or to make a justifiable approximation. Second, we have noted that in all decays with penguin contributions, NP always affects diagrammatic amplitudes in pairs; *e.g.* in  $B \rightarrow K\pi$  decay, the diagrams  $P'_{EW}$  and  $C'$ ,  $P'_{EW}$  and  $T'$ , and  $P'$  and  $P'_{uc}$  are simultaneously affected by NP. Fits to  $B \rightarrow K\pi$  data suggest larger-than-expected  $P'_{EW}$  and  $C'$  contributions. In view of our observation, the requirement of large  $C'$  may be a sign of NP. Finally, we have shown that if NP contributes in such a way that the amplitudes retain the SM form using reparametrization invariance, the weak phase obtained will not be altered due to the presence of NP. This provides a natural explanation of the result that has several fits to  $B \rightarrow K\pi$  data with varying approximations yielding  $\gamma$  in accord with the SM. The observation of a large  $C'$  and  $\gamma$  consistent with the SM in  $B \rightarrow K\pi$  decays provide ample circumstantial evidence in favor of NP.

In chapter four, we have studied the “ $B \rightarrow K\pi$  Puzzle” in details. We have also studied how it is possible to solve the “ $B \rightarrow K\pi$  Puzzle” using  $B \rightarrow K^*\rho$  mode. We have performed a detailed study of the  $B \rightarrow K^*\rho$  decays using a model-independent approach. It was shown that  $B \rightarrow K^*\rho$  modes had a distinct advantage due the large number of independent observables that could be measured. In comparison to the  $B \rightarrow K\pi$  modes that yield only 9 independent observables, the  $B \rightarrow K^*\rho$  modes result in as many as 35 independent observables. Since  $B \rightarrow K\pi$  and  $B \rightarrow K^*\rho$  have the same quark level subprocess, the study of  $B \rightarrow K^*\rho$  may well shed light on the well known “ $B \rightarrow K\pi$  puzzle.” The relevant decay amplitudes were decomposed into linear combinations of the topological amplitudes with their respective strong phases assuming isospin. We have pointed out that the amplitude written this way are the most general ones and included contributions not only from the SM but also any NP that might exist.

We have obtained explicit model-independent expressions for all the topological amplitudes and their strong phases in terms of observables and the weak phase  $\gamma$ . With  $\gamma$  measured using other modes, our results are the first in literature to estimate the topological amplitudes and strong phases purely in terms of observables, for any mode which receives  $B \rightarrow K\pi$  like topological contributions. We further suggested clean tests to verify if there existed any hierarchical relations among topological amplitudes analogous to the ones conventionally assumed to exist for  $B \rightarrow K\pi$  in the SM. In addition,

we have presented tests that would verify any equality between the strong phases of the topological amplitudes. A model independent understanding of the relative sizes of the topological amplitudes and relations between their strong phases could provide valuable insights into NP investigations. Generally, it is not possible to independently test the hadronic assumption and at the same time cleanly measure the NP parameters; however we have shown one example where it is possible to do both. We have demonstrated that if the tree and color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins, it is not only possible to verify the validity of such relations but also to cleanly measure NP parameters.



## APPENDIX A

# DETERMINATION OF $A_\lambda^f$ AND $\overline{A}_\lambda^f$ WITH OBSERVABLES

### A.1 DETERMINATION OF THE MAGNITUDE $A_\lambda^f$ AND $\overline{A}_\lambda^f$

The branching ratios (BRs) and direct CP asymmetries of the decay modes  $B \rightarrow K^* \rho$  are measured experimentally [134, 135]. Using the measured values of BRs and direct CP asymmetry of each helicity for the decay modes,  $|A_\lambda^f|$  (magnitude of  $A_\lambda^f$ ) can be determined straightforwardly. The direct CP asymmetry is defined as  $a_\lambda^f \equiv \frac{\Sigma_{\lambda\lambda}^f}{B_\lambda^f}$ , where  $\Sigma_{\lambda\lambda}^f$  and  $B_\lambda^f$  are defined in Eq. (4.22), and  $f$  is one of the final states of  $K^* \rho$ . Therefore,  $|A_\lambda^f|$  and  $|\overline{A}_\lambda^f|$  can be written as

$$|A_\lambda^f|^2 = B_\lambda^f + \Sigma_{\lambda\lambda}^f \quad \text{and} \quad |\overline{A}_\lambda^f|^2 = B_\lambda^f - \Sigma_{\lambda\lambda}^f. \quad (\text{A.1})$$

### A.2 DETERMINATION OF THE PHASES OF $A_\lambda^f$ AND $\overline{A}_\lambda^f$

Let us first try to find out the phases  $\alpha_\lambda^{ij}$  of  $A_\lambda^f$ . Since the relative phases  $(\alpha_\parallel^{ij} - \alpha_0^{ij})$  and  $(\alpha_\perp^{ij} - \alpha_0^{ij})$  can be measured in experiment, one needs to determine only  $\alpha_0^{ij}$ . We express the three equations of Eq. (4.30) explicitly with three unknown parameters  $\alpha_0^{+-}$ ,  $\alpha_0^{00}$ , and  $\alpha_0^{+0}$ :

$$\begin{aligned} \frac{1}{\sqrt{2}} \left[ |A_0^{0+}| e^{i\pi} - |A_0^{+-}| e^{i\alpha_0^{+-}} \right] &= |A_0^{00}| e^{i\alpha_0^{00}} - |A_0^{+0}| e^{i\alpha_0^{+0}}, \\ \frac{1}{\sqrt{2}} \left[ |A_\parallel^{0+}| e^{i\pi} - |A_\parallel^{+-}| e^{i(\alpha_0^{+-} + \tilde{\phi}_\parallel^{+-})} \right] &= |A_\parallel^{00}| e^{i(\alpha_0^{00} + \tilde{\phi}_\parallel^{00})} - |A_\parallel^{+0}| e^{i(\alpha_0^{+0} + \tilde{\phi}_\parallel^{+0})}, \end{aligned}$$

$$\frac{1}{\sqrt{2}} \left[ |A_{\perp}^{0+}| e^{i\pi} - |A_{\perp}^{+-}| e^{i(\alpha_0^{+-} + \tilde{\phi}_{\perp}^{+-})} \right] = |A_{\perp}^{00}| e^{i(\alpha_0^{00} + \tilde{\phi}_{\perp}^{00})} - |A_{\perp}^{+0}| e^{i(\alpha_0^{+0} + \tilde{\phi}_{\perp}^{+0})}, \quad (\text{A.2})$$

where  $\tilde{\phi}_{\parallel}^{ij}$  and  $\tilde{\phi}_{\perp}^{ij}$  are defined in terms of the observables  $\phi_{\parallel}^{ij}$  ( $\equiv \alpha_{\parallel}^{ij} - \alpha_0^{ij}$ ) and  $\phi_{\perp}^{ij}$  ( $\equiv \alpha_{\perp}^{ij} - \alpha_0^{ij}$ ) such that

$$\tilde{\phi}_{\parallel}^{ij} = \phi_{\parallel}^{ij} - \phi_{\parallel}^{0+} \quad \text{and} \quad \tilde{\phi}_{\perp}^{ij} = \phi_{\perp}^{ij} - \phi_{\perp}^{0+}. \quad (\text{A.3})$$

Here we remind that in our convention each phase  $\alpha_{\parallel(\perp)}^{ij}$  has been defined as the relative phase to  $\delta_{\parallel(\perp)}^P = \alpha_{\parallel(\perp)}^{0+} - \alpha_0^{0+} \equiv \phi_{\parallel(\perp)}^{0+}$ . Then we can re-write Eq. (A.2) as the matrix equation

$$\mathbf{S}\mathbf{X} = \mathbf{A}, \quad (\text{A.4})$$

where the matrix  $\mathbf{S}$  and the column vectors  $\mathbf{X}$  and  $\mathbf{A}$  are given by

$$S_{11} = \frac{1}{\sqrt{2}} |A_0^{+-}|, \quad S_{12} = 0, \quad S_{13} = |A_0^{00}|, \\ S_{14} = 0, \quad S_{15} = -|A_0^{+0}|, \quad S_{16} = 0, \quad (\text{A.5})$$

$$S_{21} = 0, \quad S_{22} = \frac{1}{\sqrt{2}} |A_0^{+-}|, \quad S_{23} = 0, \\ S_{24} = |A_0^{00}|, \quad S_{25} = 0, \quad S_{26} = -|A_0^{+0}|, \quad (\text{A.6})$$

$$S_{31} = \frac{1}{\sqrt{2}} |A_{\parallel}^{+-}| \cos \tilde{\phi}_{\parallel}^{+-}, \quad S_{32} = -\frac{1}{\sqrt{2}} |A_{\parallel}^{+-}| \sin \tilde{\phi}_{\parallel}^{+-}, \quad S_{33} = |A_{\parallel}^{00}| \cos \tilde{\phi}_{\parallel}^{00}, \\ S_{34} = -|A_{\parallel}^{00}| \sin \tilde{\phi}_{\parallel}^{00}, \quad S_{35} = -|A_{\parallel}^{+0}| \cos \tilde{\phi}_{\parallel}^{+0}, \quad S_{36} = |A_{\parallel}^{+0}| \sin \tilde{\phi}_{\parallel}^{+0}, \quad (\text{A.7})$$

$$S_{41} = \frac{1}{\sqrt{2}} |A_{\parallel}^{+-}| \sin \tilde{\phi}_{\parallel}^{+-}, \quad S_{42} = \frac{1}{\sqrt{2}} |A_{\parallel}^{+-}| \cos \tilde{\phi}_{\parallel}^{+-}, \quad S_{43} = |A_{\parallel}^{00}| \sin \tilde{\phi}_{\parallel}^{00}, \\ S_{44} = |A_{\parallel}^{00}| \cos \tilde{\phi}_{\parallel}^{00}, \quad S_{45} = -|A_{\parallel}^{+0}| \sin \tilde{\phi}_{\parallel}^{+0}, \quad S_{46} = -|A_{\parallel}^{+0}| \cos \tilde{\phi}_{\parallel}^{+0}, \quad (\text{A.8})$$

$$S_{51} = \frac{1}{\sqrt{2}} |A_{\perp}^{+-}| \cos \tilde{\phi}_{\perp}^{+-}, \quad S_{52} = -\frac{1}{\sqrt{2}} |A_{\perp}^{+-}| \sin \tilde{\phi}_{\perp}^{+-}, \quad S_{53} = |A_{\perp}^{00}| \cos \tilde{\phi}_{\perp}^{00}, \\ S_{54} = -|A_{\perp}^{00}| \sin \tilde{\phi}_{\perp}^{00}, \quad S_{55} = -|A_{\perp}^{+0}| \cos \tilde{\phi}_{\perp}^{+0}, \quad S_{56} = |A_{\perp}^{+0}| \sin \tilde{\phi}_{\perp}^{+0}, \quad (\text{A.9})$$

$$S_{61} = \frac{1}{\sqrt{2}} |A_{\perp}^{+-}| \sin \tilde{\phi}_{\perp}^{+-}, \quad S_{62} = \frac{1}{\sqrt{2}} |A_{\perp}^{+-}| \cos \tilde{\phi}_{\perp}^{+-}, \quad S_{63} = |A_{\perp}^{00}| \sin \tilde{\phi}_{\perp}^{00}, \\ S_{64} = |A_{\perp}^{00}| \cos \tilde{\phi}_{\perp}^{00}, \quad S_{65} = -|A_{\perp}^{+0}| \sin \tilde{\phi}_{\perp}^{+0}, \quad S_{66} = -|A_{\perp}^{+0}| \cos \tilde{\phi}_{\perp}^{+0} \quad (\text{A.10})$$

$$\mathbf{X} = \left( \cos \alpha_0^{+-} \quad \sin \alpha_0^{+-} \quad \cos \alpha_0^{00} \quad \sin \alpha_0^{00} \quad \cos \alpha_0^{+0} \quad \sin \alpha_0^{+0} \right)^T \quad (\text{A.11})$$

$$\mathbf{A} = \left( -\frac{1}{\sqrt{2}} A_0^{0+} \quad 0 \quad -\frac{1}{\sqrt{2}} A_{\parallel}^{0+} \quad 0 \quad -\frac{1}{\sqrt{2}} A_{\perp}^{0+} \quad 0 \right)^T \quad (\text{A.12})$$

where  $\mathbf{X}$  is the column vector to be determined. One can easily solve this matrix equation by calculating the inverse matrix of  $\mathbf{S}$ . The solution  $\mathbf{X}$  is given by

$$\mathbf{X} = \mathbf{S}^{-1} \mathbf{A} . \quad (\text{A.13})$$

We note that in Eq. (A.13) both cosine and sine of each phase  $\alpha_0^{+0}, \alpha_0^{00}, \alpha_0^{0+}$  can be determined, which results in removing discrete ambiguities associated with trigonometrical functions of the solution.

By using exactly the same method as above, one can also find the phases  $\bar{\alpha}_\lambda^{ij}$  of the CP conjugate amplitudes  $\bar{A}_\lambda^f$ .





## APPENDIX B

# ALL POSSIBLE RELATIONS

B.1 CASE I:  $y_\lambda^{ij} \neq 1$

B.1.1  $\tilde{P}_\lambda > \tilde{T}_\lambda > \tilde{P}_\lambda^{EW} > \tilde{C}_\lambda > \tilde{P}_{C\lambda}^{EW} > \tilde{A}_\lambda$

$$\begin{aligned}
& B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] > B_\lambda^{+-}[1 - y_\lambda^{+-} \cos \theta_\lambda^{+-}] > \\
& (2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)] + B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]) \\
& - 2\sqrt{2B_\lambda^{00}B_\lambda^{0+}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \cos \chi_1} > \\
& 2B_\lambda^{00}[1 - y_\lambda^{00} \cos \theta_\lambda^{00}] > \\
& (B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)] + B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]) \\
& - 2\sqrt{B_\lambda^{+-}B_\lambda^{0+}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \cos \chi_2} > \\
& B_\lambda^{0+}[1 - y_\lambda^{0+} \cos \theta_\lambda^{0+}] \tag{B.1}
\end{aligned}$$

B.1.2  $\delta_\lambda^C = \delta_\lambda^P$

$$2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 = \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \tag{B.2}$$

B.1.3  $\delta_\lambda^{CEW} = \delta_\lambda^P$

$$\sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \sin^2 \frac{\chi_2}{2}} = 0 \tag{B.3}$$

B.1.4  $\delta_\lambda^{EW} = \delta_\lambda^T$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)] \sin \chi_3} = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \sin \chi_4} \tag{B.4}$$

$$\text{B.1.5} \quad \delta_\lambda^{CEW} = \delta_\lambda^C$$

$$\sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 = \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \quad (\text{B.5})$$

$$\text{B.1.6} \quad \delta_\lambda^A = \delta_\lambda^T$$

$$B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 = \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \quad (\text{B.6})$$

$$\text{B.1.7} \quad \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \quad (\text{B.7})$$

$$\text{B.1.8} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 = \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \quad (\text{B.8})$$

$$\text{B.1.9} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 = \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1$$

$$B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 = \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \quad (\text{B.9})$$

$$\text{B.1.10} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 = \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \quad (\text{B.10})$$

$$\text{B.1.11} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$\sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} = 0$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \quad (\text{B.11})$$

$$\text{B.1.12} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$\sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} = 0$$

$$B_\lambda^{0+} [-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 = \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \quad (\text{B.12})$$

$$\text{B.1.13} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} = 0$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \quad (\text{B.13})$$

$$\text{B.1.14} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$\sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 = \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \quad (\text{B.14})$$

$$\text{B.1.15} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$\sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 = \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6$$

$$B_\lambda^{0+} [-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 = \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \quad (\text{B.15})$$

$$\text{B.1.16} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 = \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6$$

$$\sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 = \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \quad (\text{B.16})$$

$$\text{B.1.17} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \end{aligned} \quad (\text{B.17})$$

$$\text{B.1.18} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$\begin{aligned} B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.18})$$

$$\text{B.1.19} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW} \quad \text{AND} \quad \delta_\lambda^{EW} = \delta_\lambda^T$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \end{aligned} \quad (\text{B.19})$$

$$\text{B.1.20} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW} \quad \text{AND} \quad \delta_\lambda^A = \delta_\lambda^T$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \\ B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \end{aligned} \quad (\text{B.20})$$

$$\text{B.1.21} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW} \quad \text{AND} \quad \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.21})$$

$$\text{B.1.22} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW} \quad \text{AND} \quad \delta_\lambda^C = \delta_\lambda^P$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.22})$$

$$\text{B.1.23} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW} \quad \text{AND} \quad \delta_\lambda^{CEW} = \delta_\lambda^P$$

$$\begin{aligned} \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.23})$$

$$\text{B.1.24} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW} \quad \text{AND} \quad \delta_\lambda^{CEW} = \delta_\lambda^C$$

$$\begin{aligned} \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \\ B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.24})$$

$$\text{B.1.25} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW} \quad \text{AND} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$\begin{aligned} 2B_\lambda^{00}[1 + \cos^2 \gamma - 2y_\lambda^{00} \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi_1 &= \Sigma_{\lambda\lambda}^{00} \sin 2\gamma \cos \chi_1 \\ \sqrt{[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)][1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin^2 \frac{\chi_2}{2} &= 0 \\ \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_5 &= \sqrt{B_\lambda^{+-}[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]} \sin \chi_6 \\ B_\lambda^{0+}[-\cos \gamma + y_\lambda^{0+} \cos(\theta_\lambda^{0+} + \gamma)] \sin \chi_4 &= \Sigma_{\lambda\lambda}^{0+} \sin \gamma \cos \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_3 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_4 \\ \sqrt{2B_\lambda^{00}[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]} \sin \chi_7 &= \sqrt{B_\lambda^{0+}[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \sin \chi_8 \end{aligned} \quad (\text{B.25})$$

### B.1.26 REPARAMETRIZATION OF OBSERVABLES

where  $\chi_1 = \{\pm(\xi_1 + \xi_2) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+}), \pm(\xi_1 - \xi_2) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+})\}$ ,  
 $\chi_2 = \{\pm(\pi - \xi_1 - \xi_3) + (\alpha_\lambda^{+-} - \alpha_\lambda^{0+}), \pm(\pi - \xi_1 + \xi_3) + (\alpha_\lambda^{+-} - \alpha_\lambda^{0+})\}$ ,  
 $\chi_3 = \{\pm(\pi - \xi_2 + \xi_4) + (\alpha_\lambda^{00} - \alpha_\lambda^{+-} + \gamma), \pm(\pi - \xi_2 - \xi_4) + (\alpha_\lambda^{00} - \alpha_\lambda^{+-} + \gamma)\}$ ,  
 $\chi_4 = \{\pm(\pi - \xi_1 + \xi_4) + (\alpha_\lambda^{0+} - \alpha_\lambda^{+-} + \gamma), \pm(\pi - \xi_1 - \xi_4) + (\alpha_\lambda^{0+} - \alpha_\lambda^{+-} + \gamma)\}$ ,  
 $\chi_5 = \{\pm(\pi - \xi_1 + \xi_5) + (\alpha_\lambda^{0+} - \alpha_\lambda^{00} + \gamma), \pm(\pi - \xi_1 - \xi_5) + (\alpha_\lambda^{0+} - \alpha_\lambda^{00} + \gamma)\}$ ,  
 $\chi_6 = \{\pm(\xi_3 + \xi_5) + (\alpha_\lambda^{+-} - \alpha_\lambda^{00} + \gamma), \pm(\xi_3 - \xi_5) + (\alpha_\lambda^{+-} - \alpha_\lambda^{00} + \gamma)\}$ ,  
 $\chi_7 = \{\pm(2\pi - \xi_2 - \xi_6) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+} + \gamma), \pm(\xi_2 - \xi_6) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+} + \gamma)\}$ ,  
 $\chi_8 = \{\pm(2\pi - \xi_1 - \xi_6) + \gamma, \pm(\xi_1 - \xi_6) + \gamma\}$ . and

$$\xi_1 = \arccos \left( \frac{\sin \gamma + y_\lambda^{0+} \sin(\theta_\lambda^{0+} + \gamma)}{\sqrt{2[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]}} \right) \quad (\text{B.26})$$

$$\xi_2 = \arccos \left( \frac{\sin \gamma + y_\lambda^{00} \sin(\theta_\lambda^{00} + \gamma)}{\sqrt{2[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]}} \right) \quad (\text{B.27})$$

$$\xi_3 = \arccos \left( \frac{\sin \gamma + y_\lambda^{+-} \sin(\theta_\lambda^{+-} + \gamma)}{\sqrt{2[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]}} \right) \quad (\text{B.28})$$

$$\xi_4 = \arccos \left( \frac{\sin \gamma - y_\lambda^{+-} \sin(\theta_\lambda^{+-} + \gamma)}{\sqrt{2[1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)]}} \right) \quad (\text{B.29})$$

$$\xi_5 = \arccos \left( \frac{\sin \gamma - y_\lambda^{00} \sin(\theta_\lambda^{00} + \gamma)}{\sqrt{2[1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)]}} \right) \quad (\text{B.30})$$

$$\xi_6 = \arccos \left( \frac{\sin \gamma - y_\lambda^{0+} \sin(\theta_\lambda^{0+} + \gamma)}{\sqrt{2[1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]}} \right) \quad (\text{B.31})$$

$$\begin{aligned}
(\tilde{P}_\lambda^{EW})^2 &= \frac{1}{2\sin^2\gamma} \left( 2B_\lambda^{00} [1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)] + B_\lambda^{0+} [1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \right. \\
&\quad \left. - 2\sqrt{2B_\lambda^{00} B_\lambda^{0+} [1 - y_\lambda^{00} \cos(\theta_\lambda^{00} + 2\gamma)] [1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \cos\chi_1 \right). \tag{B.32}
\end{aligned}$$

$$\begin{aligned}
(\tilde{P}_{C\lambda}^{EW})^2 &= \frac{1}{2\sin^2\gamma} \left( B_\lambda^{+-} [1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)] + B_\lambda^{0+} [1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)] \right. \\
&\quad \left. - 2\sqrt{B_\lambda^{+-} B_\lambda^{0+} [1 - y_\lambda^{+-} \cos(\theta_\lambda^{+-} + 2\gamma)] [1 - y_\lambda^{0+} \cos(\theta_\lambda^{0+} + 2\gamma)]} \cos\chi_2 \right). \tag{B.33}
\end{aligned}$$

## B.2 CASE II: $y_\lambda^{ij} \approx 1$

$y_\lambda^{ij} \approx 1, \Sigma_{\lambda\lambda}^{ij} \approx 0, B_\lambda^{ij} = |\mathcal{A}_\lambda^{ij}|^2 [1 - y_\lambda^{ij} \cos(\theta_\lambda^{ij} + 2\gamma)] = 2\sin^2(\frac{\theta_\lambda^{ij}}{2} + \gamma),$   
 $\xi_1 = \frac{\theta_\lambda^{0+}}{2}$  and procedures are same

$$B.2.1 \quad \tilde{P}_\lambda > \tilde{T}_\lambda > \tilde{P}_\lambda^{EW} > \tilde{C}_\lambda > \tilde{P}_{C\lambda}^{EW} > \tilde{A}_\lambda$$

$$\begin{aligned}
&B_\lambda^{0+} \sin^2(\frac{\theta_\lambda^{0+}}{2} + \gamma) > B_\lambda^{+-} \sin^2(\frac{\theta_\lambda^{+-}}{2}) > [2B_\lambda^{00} \sin^2(\frac{\theta_\lambda^{00}}{2} + \gamma) \\
&+ B_\lambda^{0+} \sin^2(\frac{\theta_\lambda^{0+}}{2} + \gamma) - 2\sqrt{2B_\lambda^{00} B_\lambda^{0+}} \sin(\frac{\theta_\lambda^{00}}{2} + \gamma) \sin(\frac{\theta_\lambda^{0+}}{2} + \gamma) \cos\chi_1] > \\
&2B_\lambda^{00} \sin^2(\frac{\theta_\lambda^{00}}{2}) > [B_\lambda^{+-} \sin^2(\frac{\theta_\lambda^{+-}}{2} + \gamma) + B_\lambda^{0+} \sin^2(\frac{\theta_\lambda^{0+}}{2} + \gamma) \\
&- 2\sqrt{B_\lambda^{+-} B_\lambda^{0+}} \sin(\frac{\theta_\lambda^{+-}}{2} + \gamma) \sin(\frac{\theta_\lambda^{0+}}{2} + \gamma) \cos\chi_2] > B_\lambda^{0+} \sin^2(\frac{\theta_\lambda^{0+}}{2}) \tag{B.34}
\end{aligned}$$

$$\begin{aligned}
B.2.2 \quad \delta_\lambda^C &= \delta_\lambda^P \\
&[1 + \cos^2\gamma - 2\cos\gamma \cos(\theta_\lambda^{00} + \gamma)] \sin\chi_1' = 0 \tag{B.35}
\end{aligned}$$

$$\begin{aligned}
B.2.3 \quad \delta_\lambda^{CEW} &= \delta_\lambda^P \\
&\sin(\frac{\theta_\lambda^{+-}}{2} + \gamma) \sin(\frac{\theta_\lambda^{0+}}{2} + \gamma) \sin^2\frac{\chi_2'}{2} = 0 \tag{B.36}
\end{aligned}$$

$$\begin{aligned}
B.2.4 \quad \delta_\lambda^{EW} &= \delta_\lambda^T \\
&\sqrt{2B_\lambda^{00}} \sin(\frac{\theta_\lambda^{00}}{2} + \gamma) \sin\chi_3' = \sqrt{B_\lambda^{0+}} \sin(\frac{\theta_\lambda^{0+}}{2} + \gamma) \sin\chi_4' \tag{B.37}
\end{aligned}$$



$$\begin{aligned}
\text{B.2.5} \quad \delta_\lambda^{CEW} &= \delta_\lambda^C \\
\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 &= \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6 \quad (\text{B.38})
\end{aligned}$$

$$\text{B.2.6} \quad \delta_\lambda^A = \delta_\lambda^T$$

From Eq.(34) and (17)

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0 \quad (\text{B.39})$$

$$\text{B.2.7} \quad \delta_\lambda^A = \delta_\lambda^{EW}$$

and from Eqs. (40), (41)

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8 \quad (\text{B.40})$$

$$\text{B.2.8} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.41})$$

$$\text{B.2.9} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0 \quad (\text{B.42})$$

$$\text{B.2.10} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8 \quad (\text{B.43})$$

$$\text{B.2.11} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.44})$$

$$\text{B.2.12} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0 \quad (\text{B.45})$$

$$\text{B.2.13} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8 \quad (\text{B.46})$$

$$\text{B.2.14} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.47})$$

$$\text{B.2.15} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0 \quad (\text{B.48})$$

$$\text{B.2.16} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$\begin{aligned} \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 &= \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6 \\ \sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 &= \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8 \end{aligned} \quad (\text{B.49})$$

$$\text{B.2.17} \quad \delta_\lambda^C = \delta_\lambda^P = \delta_\lambda^{CEW}$$

$$\begin{aligned} [1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 &= 0 \\ \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} &= 0 \end{aligned} \quad (\text{B.50})$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6$$

$$\text{B.2.18} \quad \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.51})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

$$\text{B.2.19} \quad \delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^{EW} = \delta_\lambda^T$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6 \quad (\text{B.52})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4$$

$$\text{B.2.20} \quad \delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6 \quad (\text{B.53})$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0$$

$$\text{B.2.21} \quad \delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^{EW}$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6 \quad (\text{B.54})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

$$\text{B.2.22} \quad \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.55})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

$$\text{B.2.23} \quad \delta_\lambda^{CEW} = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.56})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

$$\text{B.2.24} \quad \delta_\lambda^{CEW} = \delta_\lambda^C \text{ AND } \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4 \quad (\text{B.57})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

$$\text{B.2.25} \quad \delta_\lambda^{CEW} = \delta_\lambda^C = \delta_\lambda^P \text{ AND } \delta_\lambda^A = \delta_\lambda^T = \delta_\lambda^{EW}$$

$$[1 + \cos^2 \gamma - 2 \cos \gamma \cos(\theta_\lambda^{00} + \gamma)] \sin \chi'_1 = 0$$

$$\sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin^2 \frac{\chi'_2}{2} = 0$$

$$\sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_5 = \sqrt{B_\lambda^{+-}} \sin\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin \chi'_6$$

$$\sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin \chi'_4 = 0 \quad (\text{B.58})$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_3 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_4$$

$$\sqrt{2B_\lambda^{00}} \sin\left(\frac{\theta_\lambda^{00}}{2} + \gamma\right) \sin \chi'_7 = \sqrt{B_\lambda^{0+}} \sin\left(\frac{\theta_\lambda^{0+}}{2} + \gamma\right) \sin \chi'_8$$

### B.2.26 REPARAMETRISATION OF OBSERVABLES FOR CASE II

where  $\chi'_1 = \{\pm(\frac{\theta_\lambda^{0+} + \theta_\lambda^{00}}{2}) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+}), \pm(\frac{\theta_\lambda^{0+} - \theta_\lambda^{00}}{2}) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+})\}$ ,  
 $\chi'_2 = \{\pm(\pi - (\frac{\theta_\lambda^{0+} + \theta_\lambda^{+-}}{2})) + (\alpha_\lambda^{+-} - \alpha_\lambda^{0+}), \pm(\pi - (\frac{\theta_\lambda^{0+} - \theta_\lambda^{+-}}{2})) + (\alpha_\lambda^{+-} - \alpha_\lambda^{0+})\}$ ,

$$\chi'_3 = \{\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{+-}}{2}\right)\right) - \frac{\theta_\lambda^{00}}{2}] + (\alpha_\lambda^{00} - \alpha_\lambda^{+-} + \gamma),$$

$$\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{+-}}{2}\right)\right) + \frac{\theta_\lambda^{00}}{2}] + (\alpha_\lambda^{00} - \alpha_\lambda^{+-} + \gamma),$$

$$\chi'_4 = \{\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{+-}}{2}\right)\right) - \frac{\theta_\lambda^{0+}}{2}] + (\alpha_\lambda^{0+} - \alpha_\lambda^{+-} + \gamma),$$

$$\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{+-}}{2}\right)\right) + \frac{\theta_\lambda^{0+}}{2}] + (\alpha_\lambda^{0+} - \alpha_\lambda^{+-} + \gamma)\},$$

$$\chi'_5 = \{\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{00}}{2}\right)\right) - \frac{\theta_\lambda^{0+}}{2}] + (\alpha_\lambda^{0+} - \alpha_\lambda^{00} + \gamma),$$

$$\pm[\arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{00}}{2}\right)\right) + \frac{\theta_\lambda^{0+}}{2}] + (\alpha_\lambda^{0+} - \alpha_\lambda^{00} + \gamma),$$

$$\chi'_6 = \{\pm(\pi - (\frac{\theta_\lambda^{+-}}{2} + \arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{00}}{2}\right)\right)) + (\alpha_\lambda^{+-} - \alpha_\lambda^{00} + \gamma),$$

$$\pm(\pi - (\frac{\theta_\lambda^{+-}}{2} - \arccos\left(\cot\left(\frac{\theta_\lambda^{+-}}{2} + \gamma\right) \sin\left(\frac{\theta_\lambda^{00}}{2}\right)\right)) + (\alpha_\lambda^{+-} - \alpha_\lambda^{00} + \gamma)\},$$

$$\chi'_7 = \{\pm(2\pi - \frac{\theta_\lambda^{0+}}{2} - \arccos\left(\cot\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right)\right)) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+} + \gamma),$$

$$\pm(\frac{\theta_\lambda^{0+}}{2} - \arccos\left(\cot\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right)\right)) + (\alpha_\lambda^{00} - \alpha_\lambda^{0+} + \gamma)\},$$

$$\chi'_8 = \{\pm(2\pi - \frac{\theta_\lambda^{00}}{2} - \arccos\left(\cot\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right)\right)) + \gamma, \pm(\frac{\theta_\lambda^{00}}{2} - \arccos\left(\cot\left(\frac{\theta_\lambda^{0+}}{2}\right) \sin\left(\frac{\theta_\lambda^{0+}}{2}\right)\right)) + \gamma\}.$$



## BIBLIOGRAPHY

- [1] S. L. Glashow, Nucl. Phys. **22**, 579, 1961; S. Weinberg, Phys. Rev. Lett. **19**, 1264, 1967; A. Salam, Elementary Particle Theory: Relativistic Groups and Analyticity, in *Proc. of the 8th Nobel Symposium*, Aspenäs garden 1968, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968); D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. David Politzer, Phys. Rev. Lett. **30**, 1346 (1973).
- [2] L. M. Lederman *et al.*, Phys. Rev. Lett. **36**, 1236 (1976); C. W. Darden *et al.*, Phys. Lett. B **76B**, 246 (1978).
- [3] N. Lockyer *et al.*, Phys. Rev. Lett. **51**, 1316 (1983).
- [4] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson, Phys. Rev. **105**, 1413 (1957).
- [5] M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
- [6] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [7] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **87**, 091801 (2001) [arXiv:hep-ex/0107013];
- [8] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **87**, 091802 (2001) [arXiv:hep-ex/0107061].
- [9] Carl D. Anderson, Physical Review **43** 491 (1933).
- [10] Owen Chamberlain *et al.*, Phys. Rev. **100**, 947 (1955).
- [11] H. A. Kramers, *Proc. Acad. Sci. Ameterdam* **33** 959 (1930); F. J. Dyson, J. Math. Phys. **3** 140 (1962); J. J. Sakurai, *Invariance Principles and Elementary Particles*, Princeton University Press, New Jersey (1964); R. G. Sachs, *The Physics of Time Reversal*, University of Chicago Press, Chicago (1987); T. D. Lee, *Particle Physics and Introduction to Field Theory*, Harwood Academic Publishers, New York (1988).



- [12] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1964).
- [13] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652, 1973; The mixing of quarks was first discussed in N. Cabibbo, *Phys. Rev. Lett.* **10**, 531, (1963).
- [14] G. C. Branco, L. Silva and J. A. Silva *CP Violation*, Oxford University Press (1999); I. I .BIGI and A. I. Sanda, *CP Violation*, Cambridge University Press; C. Jarlskog, *CP Violation*, World Scientific; J. F. Donoghue *CP Violation and The Limits of The Standard Model*, World Scientific; H. Y. Cheng and W. S. Hou, *B Physics and CP Violation*, World Scientific; Patrick J. O' Donnell, [arXiv:hep-ph/9211297]; Yosef Nir, [arXiv:hep-ph/9507290]; Michael Gronau,[arXiv:hep-ph/9611255]; Michael Feindt,[arXiv:hep-ph/9802380]; Alexander L. Kagan, [arXiv:hep-ph/9803368]; Michael Gronau, [arXiv:hep-ph/9908343]; R. Fleischer, [arXiv:hep-ph/0011323]; Mattias Neubert, [arXiv:hep-ph/0001334]; Helen Quinn, [arXiv:hep-ph/0111177]; Robert Fleischer, [arXiv:hep-ph/0210323].
- [15] V. F. Weisskopf and E. P. Wigner, *Z. Phys.* **63** 54 (1930), *Z. Phys.* **65** 18 (1930).
- [16] I. J. R. Aitchison, *Relativistic Quantum Mechanics*, Macmillan, London (1972); J. D. Bjorken and S. D. Drell,*Relativistic Quantum Mechanics*, McGraw-Hill, New York; R. P. Feynman, *The Feynman Lectures on Physics*, Addison Wesley, Reading Mass. (1963); C.Itzykson and J. B. Zuber, *Quantum Field Theory*, McGraw-Hill, New York (1980); T. D. Lee, *Particle Physics and Intoduction to Field theory*, Harwood Academic Publishers, Chur (1980); P. Ramond, *Field Theory, A Modern Primer*, Benjamin-Cummings, Reading Mass. (1981); J. J. Sakurai, *Advanced Quantum Mechanics*, Addison Wesley, Reading, Mass; M. D. Scanron, *Advanced Quantum Theory*, Springer-Verlag, New York; L. I. Schiff, *Quantum Mechanics*, McGraw-Hill, New York; Michael E. Peskin and Daniel V. Schroeder *An Introduction to quantum Field Theory*, Addison-Wesley Publishing Company (1995); Mandal, F. and Shaw, G. , *Quantum Field Theory*, Wiley, New York (1984); Ryder, Lewis H.,*Quantum Field Theory*, Cambridge University Press, Cambridge (1985); George, Serman, *Introduction to Quantum Field Theory*, Cambridge University Press, Cambridge (1993); John, Collins, *Renormalization*, Cambridge University Press, Cambridge (1984); Berestet-

- skii, V. B. , E. M. Lifshitz and L. P. Pitaevskii, *Quantum Electrodynamics*, Pergamon, Oxford (1982).
- [17] Bailin, D. and Love, A, *Introduction to Gauge Theory*, Adam Hilger, Bristol (1986); Aitchison, Ian J.R. and Anthony J. G., *Gauge Theories in Particle Physics*, Adam Hilger Bristol (1989); Quigg, Cris, *Gauge Theories of the strong, Weak, and Electromagnetic Interactions*, Benjamin/Cummings, Menlo Park, California, (1983); Taylor, J. C. , *Gauge Theories of Weak Interactions*, Cambridge University Press, Cambridge (1976); J. C. Taylor, *Gauge Theories of Weak Interaction*, Cambridge University Press, Cambridge, England
- [18] G. 't Hooft, Nucl. Phys. B **33**, 173, 1971; Nucl. Phys. B **35**, 167, (1971).
- [19] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285, (1970).
- [20] J. J. Aubert *et al.*, Phys. Rev. Lett. **33**, 1404, (1974); J.-E. Augustin *et al.*, Phys. Rev. Lett. **33**, 1406, (1974).
- [21] C. Y. Prescott *et al.*, Phys. Lett. B **77**, 347, (1978).
- [22] G. Arnison *et al.*, Phys. Lett. B **122**, 103, (1983); Phys. Lett. B **126**, 398, (1983); Phys. Lett. B **129**, 273, (1983); M. Banner *et al.*, Phys. Lett. B **122**, 476, (1983); P. Bagnaia *et al.*, Phys. Lett. B **129**, 130, (1983).
- [23] M. L. Perl *et al.*, Phys. Rev. Lett. **35**, 1489, (1975).
- [24] S. W. Herb *et al.*, Phys. Rev. Lett. **39**, 252, (1977); L. M. Lederman, in *Proc. of the 19th Int. Conf. on High Energy Physics*, Tokyo, ed. G. Takeda (1978).
- [25] F. Abe *et al.*, Phys. Rev. D **50**, 2966, (1994).
- [26] The *Heavy Flavor Averaging Group*:  
<http://www.slac.stanford.edu/xorg/hfag/index.html>.
- [27] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
- [28] C. Amsler *et al.* (Particle Data Group), Physics Letters **B667**, 1 (2008)  
<http://pdg.lbl.gov/2008>
- [29] P. W. Higgs, Phys. Rev. Lett. **12**, 132, (1964); Phys. Rev. Lett. **13**, 509, (1964).

- [30] F. Halzen and A.D. Martin, *Quarks and Leptons*, John Wiley; J.F. Donoghue, E. Golowich, and B.R. Holstein, *Dynamics of the Standard Model*, Cambridge U. Press.
- [31] A. J. Buras and R. Fleischer, *Adv. Ser. Direct. High Energy Phys.* **15**, 65 (1998); [arXiv:hep-ph/9704376]; A. J. Buras, arXiv:hep-ph/9806471; Yuval Grossman, Zoltan Ligeti, Enrico Nardi, *Phys. Rev. D* **55**, 2768 (1997).
- [32] Bauer, M., Stech, B., and Wirbel, M. *Zeitschrift für Physik*, **C29** 637 (1985), *Zeitschrift für Physik*, **C34** 103 (1987); Buras, A. J. and Fleischer, R. *Physics Letters* **B341**, 379 (1995); Buchalla, G., Buras, A. J., and Lautenbacher, M., *Review of Modern Physics* **68**, 1125 (1996); A. J. Buras, [hep-ph/9806471]; Buras, A. J. and Fleischer, *Heavy flavours II*, World Scientific, Singapore (1998); R. Fleischer, arXiv:hep-ph/0405091.
- [33] Robert Fleischer, arxiv:hep-ph/0505018.
- [34] Nambu, Y. *Phys. Rev. Lett.* **4**, 380, (1960); Gell-Mann, M. and Levy, M., *Nuovo Cimento* **16**, 705 (1960); Chou, K.-C., *Soviet Phys. JETP* **12**, 492 (1961); Adler, S. L., *Phys. Rev.* **139**, B1638, (1965).
- [35] G. Burdman, J. T. Goldman and D. Wyler, *Phys. Rev. D* **51**, 111 (1995) [arXiv:hep-ph/9405425].
- [36] C. Allton *et al.* (APE Collaboration), *Phys. Lett. B* **345**, 513, (1995); *Phys. Lett. B* **365**, 275, (1996); K. Bowler *et al.* (UKQCD Collaboration), *Nucl. Phys. B* **447**, 425, (1995); L. DelDebbio *et al.* (UKQCD Collaboration), *Phys. Lett. B* **416**, 392, (1998).
- [37] P. Colangelo *et al.*, *Phys. Rev. D* **53**, 3672, (1996), *Phys. Lett. B* **395**, 338, (1997).
- [38] T. M. Aliev *et al.*, *Phys. Lett. B* **400**, 194, (1997).
- [39] D. Melikhov, N. Nikitin and S. Simula, *Phys. Lett. B* **410**, 290, (1997).
- [40] W. Jaus and D. Wyler, *Phys. Rev. D* **41**, 3405, (1990); C. Q. Geng and C. P. Kao, *Phys. Rev. D* **54**, 5636, (1996).
- [41] A. Ali, T. Mannel, and T. Morozumi, *Phys. Lett. B* **273**, 505, (1991).
- [42] A.J. Buras and J.-M. Gérard, *Nucl. Phys.* **B264**, 371 (1986); A.J. Buras, J.-M. Gérard and R. Rückl, *Nucl. Phys.* **B268**, 16 (1986).

- [43] J.D. Bjorken, *Nucl. Phys. (Proc. Suppl.)* **B11**, 325 (1989); M. Dugan and B. Grinstein, *Phys. Lett.* **B255**, 583 (1991); H.D. Politzer and M.B. Wise, *Phys. Lett.* **B257**, 399 (1991).
- [44] M. Beneke, G. Buchalla, M. Neubert and C. Sachrajda, *Phys. Rev. Lett.* **83**, 1914 (1999); *Nucl. Phys.* **B591**, 313 (2000); *Nucl. Phys.* **B606**, 245 (2001).
- [45] H.-n. Li and H.L. Yu, *Phys. Rev.* **D53**, 2480 (1996); Y.Y. Keum, H.-n. Li and A.I. Sanda, *Phys. Lett.* **B504**, 6 (2001); Y.Y. Keum and H.-n. Li, *Phys. Rev.* **D63**, 074006 (2001).
- [46] C.W. Bauer, D. Pirjol and I.W. Stewart, *Phys. Rev. Lett.* **87**, 201806 (2001); C.W. Bauer, B. Grinstein, D. Pirjol and I.W. Stewart, *Phys. Rev.* **D67**, 014010 (2003).
- [47] A. Khodjamirian, *Nucl. Phys.* **B605**, 558 (2001); A. Khodjamirian, T. Mannel and B. Melic, *Phys. Lett.* **B571**, 75 (2003).
- [48] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, hep-ph/0410407.
- [49] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Phys. Rev. Lett.* **92**, 101804 (2004); *Nucl. Phys.* **B697**, 133 (2004).
- [50] C.W. Chiang, M. Gronau, J.L. Rosner and D.A. Suprun, *Phys. Rev.* **D70**, 034020 (2004).
- [51] A.J. Buras, F. Schwab and S. Uhlig, hep-ph/0405132.
- [52] *CKMfitter* Group: <http://ckmfitter.in2p3.fr/>;  
*UTfit* Collaboration: <http://utfit.roma1.infn.it/>.
- [53] R. Fleischer, *Phys. Rep.* **370**, 537 (2002).
- [54] R. Fleischer, *Eur. Phys. J.* **C10**, 299 (1999).
- [55] ∴ B. Aubert [The BABAR Collaboration], arXiv:0808.1903 [hep-ex].
- [56] K. F. Chen *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **98**, 031802 (2007) [arXiv:hep-ex/0608039].
- [57] The *Heavy Flavor Averaging Group*:  
<http://www.slac.stanford.edu/xorg/hfag/triangle/summer2008/index.shtml>
- [58] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. D* **71**, 032005 (2005) [arXiv:hep-ex/0411016].

- [59] R. Itoh *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **95**, 091601 (2005) [arXiv:hep-ex/0504030].
- [60] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. D* **74**, 091101 (2006) [arXiv:hep-ex/0608016].
- [61] J. Dalseno *et al.* [Belle Collaboration], *Phys. Rev. D* **76**, 072004 (2007) [arXiv:0706.2045 [hep-ex]].
- [62] Y. Nir, hep-ph/0109090.
- [63] D. London and R.D. Peccei, *Phys. Lett.* **B223**, 257 (1989); N.G. Deshpande and J. Trampetic, *Phys. Rev.* **D41**, 895 and 2926 (1990); J.-M. Gérard and W.-S. Hou, *Phys. Rev.* **D43**, 2909; *Phys. Lett.* **B253**, 478 (1991).
- [64] R. Fleischer, *Z. Phys.* **C62**, 81 (1994).
- [65] N.G. Deshpande and X.-G. He, *Phys. Lett.* **B336**, 471 (1994).
- [66] R. Fleischer, *Int. J. Mod. Phys.* **A12**, 2459 (1997).
- [67] Y. Grossman and M.P. Worah, *Phys. Lett.* **B395**, 241 (1997).
- [68] D. London and A. Soni, *Phys. Lett.* **B407**, 61 (1997).
- [69] R. Fleischer and T. Mannel, *Phys. Lett. B* **511** 240 (2001)
- [70] B. Aubert *et al.* (BaBar Collaboration), BABAR-CONF-02/016 [hep-ex/0207070]
- [71] K. Abe *et al.* (Belle Collaboration), BELLE-CONF-0201 [hep-ex/0207098]
- [72] G. Hiller, SLAC-PUB-9326 [hep-ph/0207356];  
M. Artuso *et al.*, arXiv:0801.1833 [hep-ph]. A. Datta, UDEM-GPP-TH-02-103 [hep-ph/0208016];  
M. Ciuchini and L. Silvestrini, hep-ph/0208087;  
M. Raidal, CERN-TH/2002-182 [hep-ph/0208091];  
B. Dutta, C.S. Kim and S. Oh, hep-ph/0208226;  
Jong-Phil Lee and Kang Young Lee, KIAS-P02054 [hep-ph/0209290]
- [73] R. Fleischer, *Phys. Lett. B* **459** 306 (1999)
- [74] B. Aubert *et al.* (BaBar Collaboration), BABAR-PUB-02-009 [hep-ex/0207055]

- [75] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **89** 071801 (2002)
- [76] K. Hagiwara *et al.* (Particle Data Group), Phys. Rev. D **66** 010001 (2002)
- [77] A.I. Sanda and K. Ukai, Prog. Theor. Phys. **107** 421 (2002)
- [78] Y.-Y. Keum, DPNU-02-30 [hep-ph/0209208]
- [79] M. Gronau and D. London, Phys. Rev. Lett. **65** 3381 (1990)
- [80] H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D **44**, 1454 (1991).
- [81] Y. Grossman and H.R. Quinn, Phys. Rev. D **58** 017504 (1998);  
M. Gronau, D. London, N. Sinha and R. Sinha, Phys. Lett. B **514** 315 (2001)
- [82] J. Charles, Phys. Rev. D **59** 054007 (1999)
- [83] H. R. Quinn and J. P. Silva, Phys. Rev. D **62**, 054002 (2000) [arXiv:hep-ph/0001290].
- [84] A. F. Falk, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D **69**, 011502 (2004) [arXiv:hep-ph/0310242].
- [85] R. Fleischer and T. Mannel, Phys. Lett. B **397** 269 (1997)
- [86] D. London, N. Sinha and R. Sinha, Phys. Rev. D **63** 054015 (2001)
- [87] C.-D. Lü and Z.-j. Xiao, BIHEP-TH-2002-22 [hep-ph/0205134]
- [88] M. Gronau and J.L. Rosner, Phys. Rev. D **65** 093012 (2002)
- [89] M. Gronau and J.L. Rosner, Phys. Rev. D **66** 053003 (2002)
- [90] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81**, 5076 (1998) [arXiv:hep-ph/9809311].
- [91] M. Gronau, J. L. Rosner and D. London, Phys. Rev. Lett. **73**, 21 (1994) [arXiv:hep-ph/9404282].
- [92] M. Neubert, JHEP **9902**, 014 (1999) [arXiv:hep-ph/9812396].
- [93] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **93**, 021601 (2004) [arXiv:hep-ex/0401029].

- [94] C. K. Chua, W. S. Hou and K. C. Yang, *Mod. Phys. Lett. A* **18**, 1763 (2003) [arXiv:hep-ph/0210002]; S. Barshay, L. M. Sehgal and J. van Leusen, *Phys. Lett. B* **591**, 97 (2004) [arXiv:hep-ph/0403049].
- [95] Y. Chao *et al.* [BELLE Collaboration], *Phys. Rev. D* **71**, 031502 (2005) [arXiv:hep-ex/0407025].
- [96] B. Aubert *et al.* [BaBar Collaboration], *Phys. Rev. Lett.* **93**, 131801 (2004) [arXiv:hep-ex/0407057].
- [97] . Grossman and H. R. Quinn, *Phys. Rev. D* , **56** 7529 (1997); L. Wolfenstein, *Phys. Rev. D* , **57** 6857 (1998).
- [98] Y. Grossman and M. P. Worah, *Phys. Letter B* **395** 241 (1997); D. London and A. Soni, *Phys. Letter B* **407** 61 (1997).
- [99] Y. Nir, arXiv:hep-ph/9309325.
- [100] The CKMfitter Group, J. Charles *et al.*, hep-ph/0406184.
- [101] J. P. Silva, hep-ph/0410351.
- [102] F. J. Botella and J. P. Silva, *Phys. Rev. D* **71**, 094008 (2005) [arXiv:hep-ph/0503136].
- [103] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, *Phys. Rev. D* **50**, 4529 (1994) [arXiv:hep-ph/9404283]; *Phys. Rev. D* **52**, 6374 (1995) [arXiv:hep-ph/9504327].
- [104] M. Gronau and J. L. Rosner, *Phys. Rev. D* **66**, 053003 (2002).
- [105] A. Datta and D. London, *Phys. Lett. B* **595**, 453 (2004); A. Datta, M. Imbeault, D. London, V. Page, N. Sinha and R. Sinha, *Phys. Rev. D* **71**, 096002 (2005). In these references NP strong phases were argued to be negligible. No such assumption is made in the present work.
- [106] For example, see S. Baek, *JHEP* **0607**, 025 (2006), and references therein.
- [107] F. J. Botella and J. P. Silva, *Phys. Rev. D* **71**, 094008 (2005); S. Baek, F. J. Botella, D. London and J. P. Silva, *Phys. Rev. D* **72**, 036004 (2005).
- [108] D. London, N. Sinha and R. Sinha, *Phys. Rev. D* **60**, 074020 (1999).

- [109] C. Dib, D. London and Y. Nir, *Int. J. Mod. Phys. A* **6**, 1253 (1991).
- [110] BELLE Collaboration, K. Abe *et al.*, hep-ex/0408111; BABAR Collaboration, B. Aubert *et al.*, hep-ex/0408127.
- [111] H. J. Lipkin, *Phys. Lett. B* **445**, 403 (1999); M. Gronau and J. L. Rosner, *Phys. Rev. D* **59**, 113002 (1999).
- [112] M. Neubert and J. L. Rosner, *Phys. Lett. B* **441**, 403 (1998), *Phys. Rev. Lett.* **81**, 5076 (1998); M. Gronau, D. Pirjol and T. M. Yan, *Phys. Rev. D* **60**, 034021 (1999) [Erratum-*ibid.* *D* **69**, 119901 (2004)]; M. Imbeault, A. L. Lemerle, V. Page and D. London, *Phys. Rev. Lett.* **92**, 081801 (2004).
- [113] C. S. Kim, S. Oh and C. Yu, *Phys. Rev. D* **72**, 074005 (2005).
- [114] S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, *Phys. Rev. D* **71**, 057502 (2005).
- [115] M. Imbeault, D. London, C. Sharma, N. Sinha and R. Sinha, *Phys. Lett. B* **653**, 254 (2007) [arXiv:hep-ph/0608169].
- [116] Y. Nir and H. R. Quinn, *Phys. Rev. Lett.* **67**, 541 (1991); R. Fleischer, *Phys. Lett. B* **365**, 399 (1996) [arXiv:hep-ph/9509204]; A. J. Buras and R. Fleischer, *Eur. Phys. J. C* **16**, 97 (2000) [arXiv:hep-ph/0003323]; M. Imbeault, A. L. Lemerle, V. Page and D. London, *Phys. Rev. Lett.* **92**, 081801 (2004) [arXiv:hep-ph/0309061].
- [117] R. Fleischer and T. Mannel, *Phys. Rev. D* **57**, 2752 (1998) [arXiv:hep-ph/9704423].
- [118] A. J. Buras and R. Fleischer, *Eur. Phys. J. C* **11**, 93 (1999) [arXiv:hep-ph/9810260].
- [119] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *Nucl. Phys. B* **606**, 245 (2001) [arXiv:hep-ph/0104110]; M. Beneke and M. Neubert, *Nucl. Phys. B* **675**, 333 (2003) [arXiv:hep-ph/0308039]; X. q. Li and Y. d. Yang, *Phys. Rev. D* **72**, 074007 (2005) [arXiv:hep-ph/0508079].
- [120] H. n. Li and H. L. Yu, *Phys. Rev. D* **53**, 2480 (1996) [arXiv:hep-ph/9411308]; Y. Y. Keum, H. n. Li and A. I. Sanda, *Phys. Lett. B* **504**, 6 (2001) [arXiv:hep-ph/0004004]; *Phys. Rev. D* **63**, 054008 (2001) [arXiv:hep-ph/0004173].



- [121] H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D **72**, 114005 (2005) [arXiv:hep-ph/0508041].
- [122] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004) [arXiv:hep-ph/0401188]; C. W. Bauer, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **74**, 034010 (2006) [arXiv:hep-ph/0510241]; A. R. Williamson and J. Zupan, Phys. Rev. D **74**, 014003 (2006) [Erratum-ibid. D **74**, 03901 (2006)] [arXiv:hep-ph/0601214].
- [123] R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **51**, 55 (2007) [arXiv:hep-ph/0702275].
- [124] E. Barberio *et al.*, , arXiv:hep-ex/0603003.
- [125] S. Khalil, Phys. Rev. D **72**, 035007 (2005) [arXiv:hep-ph/0505151]; R. Arnowitt, B. Dutta, B. Hu and S. Oh, Phys. Lett. B **633**, 748 (2006) [arXiv:hep-ph/0509233]; Phys. Lett. B **641**, 305 (2006) [arXiv:hep-ph/0606130].
- [126] V. Barger, C. W. Chiang, P. Langacker and H. S. Lee, Phys. Lett. B **598**, 218 (2004) [arXiv:hep-ph/0406126].
- [127] W. S. Hou, M. Nagashima and A. Soddu, Phys. Rev. Lett. **95**, 141601 (2005) [arXiv:hep-ph/0503072]; W. S. Hou, M. Nagashima, G. Raz and A. Soddu, JHEP **0609**, 012 (2006) [arXiv:hep-ph/0603097].
- [128] T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003) [arXiv:hep-ph/0306147]; A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **32**, 45 (2003) [arXiv:hep-ph/0309012]; Y. L. Wu and Y. F. Zhou, Phys. Rev. D **72**, 034037 (2005) [arXiv:hep-ph/0503077]; C. S. Kim, Y. J. Kwon, J. Lee and T. Yoshikawa, Prog. Theor. Phys. **116**, 143 (2006) [arXiv:hep-ph/0509015].
- [129] S. Mishima and T. Yoshikawa, Phys. Rev. D **70**, 094024 (2004) [arXiv:hep-ph/0408090]; Y. L. Wu and Y. F. Zhou, Phys. Rev. D **71**, 021701 (2005) [arXiv:hep-ph/0409221]; A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Acta Phys. Polon. B **36**, 2015 (2005) [arXiv:hep-ph/0410407]; X. G. He and B. H. J. McKellar, arXiv:hep-ph/0410098; S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D **71**, 057502 (2005) [arXiv:hep-ph/0412086]; T. Carruthers and B. H. J. McKellar, arXiv:hep-ph/0412202; S. Nandi and A. Kundu, arXiv:hep-ph/0407061; T. Morozumi, Z. H. Xiong and T. Yoshikawa, arXiv:hep-ph/0408297.

- [130] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. **92**, 101804 (2004) [arXiv:hep-ph/0312259]; Acta Phys. Polon. B **36**, 2015 (2005) [arXiv:hep-ph/0410407]; Eur. Phys. J. C **45**, 701 (2006) [arXiv:hep-ph/0512032]; S. Baek, JHEP **0607**, 025 (2006) [arXiv:hep-ph/0605094].
- [131] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Nucl. Phys. B **697**, 133 (2004) [arXiv:hep-ph/0402112].
- [132] S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D **71**, 057502 (2005) [arXiv:hep-ph/0412086]; S. Baek and D. London, arXiv:hep-ph/0701181.
- [133] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D **70**, 034020 (2004) [arXiv:hep-ph/0404073]; Y. Y. Charng and H. n. Li, Phys. Rev. D **71**, 014036 (2005) [arXiv:hep-ph/0410005]; X. G. He and B. H. J. McKellar, arXiv:hep-ph/0410098; C. S. Kim, S. Oh and C. Yu, Phys. Rev. D **72**, 074005 (2005) [arXiv:hep-ph/0505060].
- [134] Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag/> (2008).
- [135] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
- [136] K. Abe *et al.*, arXiv:hep-ex/0507045; K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0609006; K. Abe., arXiv:hep-ex/0609015; K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0608049; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607096; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607106; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0608003; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97**, 171805 (2006) [arXiv:hep-ex/0608036].
- [137] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D **72**, 074005 (2005) [arXiv:hep-ph/0505060].
- [138] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **91**, 171802 (2003) [arXiv:hep-ex/0307026].
- [139] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93**, 231804 (2004) [arXiv:hep-ex/0408017].

- [140] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607097; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607098; B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0607092; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **74**, 051102 (2006) [arXiv:hep-ex/0605017]; J. Zhang *et al.* [BELLE Collaboration], Phys. Rev. Lett. **91**, 221801 (2003) [arXiv:hep-ex/0306007]; K. F. Chen *et al.* [BELLE Collaboration], Phys. Rev. Lett. **94**, 221804 (2005) [arXiv:hep-ex/0503013]; A. Somov *et al.*, Phys. Rev. Lett. **96**, 171801 (2006) [arXiv:hep-ex/0601024].
- [141] K. Abe *et al.* [BELLE-Collaboration], Phys. Rev. Lett. **95**, 141801 (2005) [arXiv:hep-ex/0408102]; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97**, 201801 (2006) [arXiv:hep-ex/0607057].
- [142] M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003) [arXiv:hep-ph/0308039].
- [143] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D **63**, 054008 (2001) [arXiv:hep-ph/0004173].
- [144] N. Sinha and R. Sinha, Phys. Rev. Lett. **80**, 3706 (1998) [arXiv:hep-ph/9712502].
- [145] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994) [arXiv:hep-ph/9404283].
- [146] S. Baek, F. J. Botella, D. London and J. P. Silva, Phys. Rev. D **72**, 114007 (2005) [arXiv:hep-ph/0509322].
- [147] A. J. Buras and R. Fleischer, Phys. Lett. B **341**, 379 (1995) [arXiv:hep-ph/9409244].
- [148] M. Imbeault, D. London, C. Sharma, N. Sinha and R. Sinha [arXiv:hep-ph/0608169].
- [149] In this case, the number of parameters is still the same as that of informations. As 6 parameters ( $\tilde{A}_\lambda$ ,  $\delta_\lambda^A$ ) are neglected, the number of informations are also reduced by 6 (*i.e.*,  $\bar{A}_\lambda^{0+}$ ,  $\bar{\alpha}_\lambda^{0+}$ ) due to the vanishing direct CP asymmetry of the  $K^{*0}\rho^+$  mode.
- [150] D. London, N. Sinha and R. Sinha, Phys. Rev. D **69**, 114013 (2004) [arXiv:hep-ph/0402214].

- [151] C. S. Kim, S. Oh, C. Sharma, R. Sinha and Y. W. Yoon, Phys. Rev. D **76**, 074019 (2007) [arXiv:0706.1150 [hep-ph]].
- [152] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981).
- [153] M. Gronau, J. L. Rosner and D. London, Phys. Rev. Lett. **73**, 21 (1994) [arXiv:hep-ph/9404282]; O. F. Hernandez, D. London, M. Gronau and J. L. Rosner, Phys. Lett. B **333**, 500 (1994) [arXiv:hep-ph/9404281]; M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **52**, 6374 (1995) [arXiv:hep-ph/9504327]; C. S. Kim, D. London and T. Yoshikawa, Phys. Rev. D **57**, 4010 (1998) [arXiv:hep-ph/9708356].
- [154] T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003) [arXiv:hep-ph/0306147].
- [155] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D **72**, 074005 (2005) [arXiv:hep-ph/0505060].
- [156] Jose F Nieves, Palash B. Pal, American Journal Physics **72** (8) August 2004.
- [157] C.C.Nishi, American Journal Physics **73** (12) December 2005.
- [158] In the decay mode  $B^+ \rightarrow \pi K^+$ , there are no d quark in final state in the  $T$ ,  $C$  and  $P_C^{EW}$  topologies but in  $P$  and  $P^{EW}$  topologies, both u and d quarks are possible in final state. For the decay mode  $B^+ \rightarrow \pi^+ K^0$ , no  $u$  quark will be in final state i.e.  $\bar{s}, \bar{d}$  and  $d$  be in final state. In the decay mode  $B^0 \rightarrow \pi^- K^+$ , there is no d quark in final state. Only u quark is allowed in final state in  $C$  topology of the decay mode  $B^0 \rightarrow \pi^0 K^0$