$SU(3)$-flavor analysis of hadronic bottom baryon decays

By

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I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.

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List of Publications

Publications included in the thesis

1. **"Non-leptonic beauty baryon decays and CP-asymmetries based on $SU(3)$-Flavor analysis,"**

   Shibasis Roy, R. Sinha and N. G. Deshpande,

   Phys. Rev. D **101***, (2020) no.3, 036018.*

2. **"Beauty baryon non-leptonic decays into decuplet baryons and CP-asymmetries based on $SU(3)$-Flavor analysis,"**

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   Phys. Rev. D ***102*** *(2020) no.5, 053007*.

Other publications

1. **"Lepton mass effects and angular observables in $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$,"**

   Shibasis Roy, Ria Sain, Rahul Sinha

   Phys. Rev. D **96** *(2017) no.11, 116005.*

2. **"Radiative b-baryon decays to measure the photon and b-baryon polarization,"**

   L. M. Garcia Martin, B. Jashal, F. Martinez Vidal, A. Oyanguren,

   S. Roy, R. Sain and R. Sinha,

   Eur. Phys. J. C **79** *(2019) no.7, 634.*

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Presented Posters and Talks in School / Conferences

1. Talk title:

   Angular observables in $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$ decay,

   Workshop on High Energy Physics Phenomenology XV

   14-23 December, 2017

   Indian Institute of Science Education and Research (IISER), Bhopal

2. Poster title:

   Lepton mass effects and angular observables in $\Lambda_b \to \Lambda(\to p\pi)\ell^+\ell^-$,

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   Quy Nhon, Vietnam.

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   Amplitude relations in b-baryon decays based on $SU(3)$-flavor analysis

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Dedicated

to

my parents.

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Summary

The focus of this thesis is the application of $SU(3)$-flavor symmetry to decompose the decay amplitudes of two body hadronic charmless bottom baryon decays. The decay amplitudes are conveniently expressed in terms of $SU(3)$-invariant amplitudes and appropriate $SU(3)$ Clebsch-Gordon coefficients. To begin with, the most general effective Hamiltonian is considered that connects an initial anti-triplet bottom baryon to the final state octet or decuplet baryon and a pseudoscalar meson. The number of independent $SU(3)$-invariant amplitudes equals the total number of distinct decay modes in this case resulting in no relations between the decay modes. Subsequently, the dimension-6 effective Hamiltonian responsible for hadronic weak decays of b-quarks is assumed. With this choice, a significant reduction in the number of independent $SU(3)$-invariant amplitudes is observed which indicates that there are several amplitude relations between different decay modes. Of particular interest are the amplitude relations between two decay modes that can be translated to decay rate and CP-asymmetry relations. The effect of $SU(3)$-breaking on these relations is also explored. In absence of robust theoretical predictions, the decay rate and CP-asymmetry relations derived using $SU(3)$-flavor symmetry provides a qualitative understanding of the bottom baryon hadronic decays that can also be tested in experiments in near future.

Contents

Synopsis

The Standard Model (SM) of particle physics has had unprecedented success when it comes to describing the fundamental interactions of elementary particles. Over the years, the evidence in favor of the SM has only grown with more and more experimental breakthroughs culminating in the discovery of the Higgs boson, the last missing link of the puzzle. Despite of its overwhelming triumph, the SM fails to provide satisfactory explanations to a number of theoretical issues and empirical observations starting with the matter-antimatter asymmetry of the observable universe. In fact, the baryon-to-photon ratio, a measure of the matter-antimatter asymmetry, inferred from primordial nucleosynthesis and anisotropies in cosmic microwave background is many orders of magnitude higher than the SM predictions. One of the necessary conditions for such matter-antimatter asymmetry is sizable CP-violation at high energies. However, the only known source of CP-violation in the SM arising from the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is inadequate to explain the observed matter-antimatter asymmetry. A number of beyond Standard Model (BSM) theories have been postulated to address this issue by adding new degrees of freedom at a high energy scale. This may also resolve some of the other unanswered questions in the SM like the existence of dark matter, instabilities in the Higgs mass and Higgs vacuum expectation value and tiny neutrino masses to name a few. Besides looking for the imprints of new degrees of freedom actively in high energy collider experiments, one can also study its effects on low energy loop mediated processes in the SM.

Direct and indirect CP violation in hadronic B-meson decays have been measured extensively in *BABAR*, Belle and most recently at LHCb. In a number of cases, these measurements are in good agreements with the theoretical predictions. Naturally, one expects b-baryon CP-asymmetries [1–3] to be of similar magnitude as observed in B-meson decays since the underlying quark level transition are the same for the two cases. Moreover, a non-vanishing CP-asymmetry is a measure of direct CP violation as baryons and antibaryons do not undergo mixing due to baryon number conservation. So far, all CP-asymmetry measurements in bottom baryon decays [4–10] have been consistent with CP conservation hypothesis. The situation may change, however, when the statistical errors go down as LHCb gears up to analyze a significant number [11] of hadronic b-baryons decays in its subsequent runs [12].

In absence of robust theoretical predictions for bottom baryon hadronic decay, we can still use the approximate symmetries of the SM to relate different processes once some of them are observed in experiments. Since, any high-energy BSM theory must reproduce the correct low energy (energy scale $\sim m_b$) behavior and symmetries, these relations should remain unaffected. In particular, the $SU(3)$-flavor symmetry of the light quarks have successfully explained a number of experimental observations in the past. We focus on hadronic bottom baryon charmless decays using $SU(3)$ symmetry to a light octet baryon-octet meson [13] or decuplet baryon-octet meson [14] pair.

In this thesis we analyze all possible strangeness changing and strangeness preserving two body weak decays of bottom baryons. The general formalism allows us to $SU(3)$ decompose decay amplitudes in terms of a set of independent $SU(3)$-reduced amplitudes. As the number of possible decays are higher than number of independent $SU(3)$ parameters, we can derive all possible amplitude sum rules. We further derive CP-asymmetry relations from this amplitude sum rules that are expected to hold to a given accuracy in the

SM. The implications of $SU(3)$-breaking but isospin conserving effects are also pointed out in detail. We hope not only to test these relations but also to estimate the unknown $SU(3)$ parameters directly form experimental data as it becomes available in future.

$SU(3)$-analysis of two-body hadronic decays of b-baryons

The $SU(3)$ symmetry is an approximate symmetry [15] of the SM that considers the light quarks namely up (u), down (d) and strange (s) belonging to a triplet of $SU(3)$-flavor symmetry. Mesons and baryons which are bound states of a quark-antiquark pair and three quarks respectively can also be arranged such that they transform as particular representations under $SU(3)$-flavor. Despite of not being an exact symmetry, $SU(3)$-flavor can be used to derive amplitude relations [16–18] between different decay modes which hold remarkably well in practice. The $SU(3)$ decomposition of physical amplitudes describing a decay process involves writing it in terms of reduced matrix elements of explicit $SU(3)$ operators with appropriate coefficients. The procedure [18] is an application of Wigner-Eckart theorem for the $SU(3)$ group where the reduced matrix elements are all possible $SU(3)$ invariants with Clebsch-Gordon (CG) coefficients connecting the basis involving physical states to the group theoretic basis.

The case of interest, namely all possible strangeness preserving ($\Delta S = 0$) and strangeness changing ($\Delta S = -1$) transitions of an anti-triplet ($\bar{\mathbf{3}}$ of $SU(3)$) bottom baryon to charmless a) octet baryon and an octet meson [13] b) decuplet baryon and an octet meson final states are considered [14]. The allowed $SU(3)$ representations of the final state being an octet baryon and an octet meson or a decuplet baryon and an octet meson is given below;

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8}_1 \oplus \mathbf{8}_2 \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}, \tag{1}$$

$$\mathbf{10} \otimes \mathbf{8} = \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35}. \tag{2}$$

The most general Hamiltonian [19] $\mathcal{H}$ which connects the initial (i) and final (f) states via the matrix elements $\langle f|\mathcal{H}|i\rangle$, consists of exactly those representations [20] $\mathbf{R}$ appearing in $\mathbf{f}\otimes\bar{\mathbf{i}}$, where the labels i and f denote both physical states and $SU(3)$ representations. The expression of the amplitudes in terms of reduced $SU(3)$ amplitudes is given as,

$$\mathcal{A}(i \to f_b f_m) = (-1)^{I_3-\frac{Y}{2}-\frac{T}{3}} \sum_{\substack{\{f,R\} \\ Y^b+Y^m=Y^f,\, Y^f-Y^i=Y^H \\ I_3^b+I_3^m=I_3^f,\, I_3^f-I_3^i=I_3^H}} C^{I_3^b\, I_3^m\, I_3^f}_{I^b\, I^m\, I^f} \begin{pmatrix} \mathbf{f_b} & \mathbf{f_m} & \mathbf{f} \\ (Y^b,I^b) & (Y^m,I^m) & (Y^f,I^f) \end{pmatrix} \begin{pmatrix} \mathbf{f} & \bar{\mathbf{i}} & \mathbf{R} \\ (Y^f,I^f) & (-Y^i,I^i) & (Y^H,I^H) \end{pmatrix} C^{I_3^f\, -I_3^i\, I_3^H}_{I^f\, I^i\, I^H} \langle \mathbf{f} \parallel \mathbf{R_I} \parallel \mathbf{i}\rangle, \tag{3}$$

where, $C^{a,b,c}_{A,B,C}$ are the $SU(2)$ Clebsch-Gordon coefficients and

$$\begin{pmatrix} \mathbf{R_a} & \mathbf{R_b} & \mathbf{R_c} \\ (Y^a,I^a) & (Y^b,I^b) & (Y^c,I^c) \end{pmatrix}. \tag{4}$$

are the $SU(3)$ isoscalar coefficients [21,22] obtained by coupling the representations $\mathbf{R_a}\otimes\mathbf{R_b}\to\mathbf{R_c}$. T is the triality of the initial state conjugate $\bar{\mathbf{i}}$ and the phase factor appearing in front ensures that correct signs are assigned to the individual initial b-baryon anti-triplet. Given a form of effective Hamiltonian ($\mathcal{H}_{\text{eff}}$), it can be $SU(3)$ decomposed,

$$\mathcal{H}_{\text{eff}} = \sum_{\substack{\{Y,I,I_3\} \\ \mathbf{R}}} \mathcal{F}^{\{Y,I,I_3\}}_{\mathbf{R}} \mathbf{R_I}, \tag{5}$$

where $\mathcal{F}^{\{Y,I,I_3\}}_{\mathbf{R}}$ depends on the $SU(3)$ CG coefficients appearing in front of the $SU(3)$ representations ($\mathbf{R_I}$). Moreover $\mathcal{F}^{\{Y,I,I_3\}}_{\mathbf{R}}$ also contains additional factors entering Eq. (4.8) in form of Wilson coefficients and CKM elements. It is also important to note that by knowing the dynamical coefficients for different isospin values in a given $SU(3)$ representation, one can drop the isospin Casimir label (I) and express the Wigner-Eckart reduced matrix element $\langle \mathbf{f} \,||\, \mathbf{R} \,||\, \mathbf{i} \rangle$, in its usual form, independent of the isospin I label. By using completeness of $SU(3)$ CG coefficients up to a phase factor,

$$\langle \mathbf{f} \,||\, \mathbf{R_I} \,||\, \mathbf{i} \rangle = \underbrace{\mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \sqrt{\frac{\text{dim f}}{\text{dim R}}}}_{\text{dynamical Coeff. of } \mathcal{H}} \langle \mathbf{f} \,||\, \mathbf{R} \,||\, \mathbf{i} \rangle. \tag{6}$$

Apriori, the $SU(3)$ analysis of these decays can be performed without a particular set of dynamical assumptions while accounting for the effects of an arbitrarily broken $SU(3)$ flavor symmetry where the number of $SU(3)$-reduced amplitudes exactly match the number of decay modes. In practice, the lowest order effective Hamiltonian [19] for charmless b-baryon decays is assumed consisting of parts [20] that transform as $\mathbf{3}$, $\overline{\mathbf{6}}$, $\mathbf{15}$ under $SU(3)$-flavor and all the higher $SU(3)$ representations end up not contributing as long as exact $SU(3)$ symmetry is considered. For quick reference, the $SU(3)$ decomposition of the tree, gluonic and electroweak part of the effective Hamiltonian are given as,

$$\begin{aligned}\frac{\sqrt{2}\mathcal{H}_{\mathrm{T}}}{4G_F} = &\Bigg\{\lambda_u^s\left[\frac{(C_1+C_2)}{2}\left(-\mathbf{15}_1 - \frac{1}{\sqrt{2}}\mathbf{15}_0 - \frac{1}{\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right) + \frac{(C_1-C_2)}{2}\left(\overline{\mathbf{6}}_1 + \mathbf{3}_0^{(\bar{\mathbf{3}})}\right)\right] \\ &+ \lambda_u^d\left[\frac{(C_1+C_2)}{2}\left(-\frac{2}{\sqrt{3}}\mathbf{15}_{3/2} - \frac{1}{\sqrt{6}}\mathbf{15}_{1/2} - \frac{1}{\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right) + \frac{(C_1-C_2)}{2}\left(-\overline{\mathbf{6}}_{1/2} + \mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right)\right]\Bigg\},\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\sqrt{2}\mathcal{H}_{\mathrm{g}}}{4G_F} = &\Big\{-\lambda_t^s\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_0^{(\mathbf{6})} + (C_3-C_4)\mathbf{3}_0^{(\bar{\mathbf{3}})}\right] - \lambda_t^d\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_{1/2}^{(\mathbf{6})} + (C_3-C_4)\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right] \\ &- \lambda_t^s\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_0^{(\mathbf{6})} + (C_5-C_6)\mathbf{3}_0^{(\bar{\mathbf{3}})}\right] - \lambda_t^d\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_{1/2}^{(\mathbf{6})} + (C_5-C_6)\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right]\Big\},\end{aligned} \tag{8}$$

$$\frac{\sqrt{2}\mathcal{H}_{\rm EWP}}{4G_F} = \Bigg\{-\lambda_t^s\Bigg[\frac{(C_9+C_{10})}{2}\Bigg(-\frac{3}{2}\mathbf{15}_1 - \frac{3}{2\sqrt{2}}\mathbf{15}_0 + \frac{1}{2\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\Bigg) + \frac{(C_9-C_{10})}{2}\Bigg(\frac{3}{2}\overline{\mathbf{6}}_1 + \frac{1}{2}\mathbf{3}_0^{(\overline{\mathbf{3}})}\Bigg)\Bigg]$$
$$-\lambda_t^d\Bigg[\frac{(C_9+C_{10})}{2}\Bigg(-\sqrt{3}\,\mathbf{15}_{3/2} - \frac{1}{2}\sqrt{\frac{3}{2}}\mathbf{15}_{1/2} + \frac{1}{2\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\Bigg) + \frac{(C_9-C_{10})}{2}\Bigg(-\frac{3}{2}\overline{\mathbf{6}}_{1/2} + \frac{1}{2}\mathbf{3}_{1/2}^{(\overline{\mathbf{3}})}\Bigg)\Bigg]\Bigg\}. \quad (9)$$

This restricts the number of independent $SU(3)$-reduced amplitudes to ten for the octet baryon-octet meson

$$\langle 8_1 \,||\, \mathbf{3} \,||\, \overline{3}\rangle,\quad \langle 8_2 \,||\, \mathbf{3} \,||\, \overline{3}\rangle,\quad \langle 8_1 \,||\, \overline{\mathbf{6}} \,||\, \overline{3}\rangle,\quad \langle 8_2 \,||\, \overline{\mathbf{6}} \,||\, \overline{3}\rangle,\quad \langle 8_1 \,||\, \mathbf{15} \,||\, \overline{3}\rangle$$
$$\langle 8_2 \,||\, \mathbf{15} \,||\, \overline{3}\rangle,\quad \langle 10 \,||\, \mathbf{15} \,||\, \overline{3}\rangle,\quad \langle \overline{10} \,||\, \overline{\mathbf{6}} \,||\, \overline{3}\rangle,\quad \langle 27 \,||\, \mathbf{15} \,||\, \overline{3}\rangle,\quad \langle 1 \,||\, \mathbf{3} \,||\, \overline{3}\rangle \quad (10)$$

and five for decuplet baryon-octet meson

$$\langle 8 \,||\, \mathbf{3} \,||\, \overline{3}\rangle,\qquad \langle 8 \,||\, \overline{\mathbf{6}} \,||\, \overline{3}\rangle,\qquad \langle 8 \,||\, \mathbf{15} \,||\, \overline{3}\rangle$$
$$\langle 10 \,||\, \mathbf{15} \,||\, \overline{3}\rangle,\qquad \langle 27 \,||\, \mathbf{15} \,||\, \overline{3}\rangle \quad (11)$$

final states. The total number of decay modes for octet baryon-octet meson and decuplet baryon-octet meson final states are 44 and 40 respectively. We have also discussed $SU(3)$-singlet meson final states that need to be considered separately. In addition, we provide an alternative approach in terms of familiar quark flavor-flow diagrams introduced for meson decays specifically for the study of bottom baryons into a decuplet baryon and an octet meson. While the $SU(3)$-flavor decomposition of decay amplitudes and the diagrammatic approach are completely equivalent, the latter can sometimes highlight the dynamical picture of the processes as we found out in the last case. Regardless of the method, the number of distinct decay modes are higher than the number of independent $SU(3)$-parameters resulting in several amplitude relations between different decay modes. The relations for the b-baryon decaying to a) octet baryon-octet meson case [13];

$$\begin{aligned}
\mathcal{A}(\Lambda_b^0 \to \Sigma^- K^+) &= \mathcal{A}(\Xi_b^0 \to \Xi^- \pi^+), \quad \mathcal{A}(\Lambda_b^0 \to p^+ \pi^-) = \mathcal{A}(\Xi_b^0 \to \Sigma^+ K^-),\\
\mathcal{A}(\Xi_b^- \to n K^-) &= \mathcal{A}(\Xi_b^- \to \Xi^0 \pi^-), \quad \mathcal{A}(\Xi_b^- \to \Xi^- K^0) = \mathcal{A}(\Xi_b^- \to \Sigma^- \overline{K}^0),\\
\mathcal{A}(\Xi_b^0 \to \Xi^- K^+) &= \mathcal{A}(\Lambda_b^0 \to \Sigma^- \pi^+), \quad \mathcal{A}(\Xi_b^0 \to \Sigma^- \pi^+) = \mathcal{A}(\Lambda_b^0 \to \Xi^- K^+),\\
\mathcal{A}(\Xi_b^0 \to \Sigma^+ \pi^-) &= \mathcal{A}(\Lambda_b^0 \to p^+ K^-), \quad \mathcal{A}(\Xi_b^0 \to n \overline{K}^0) = \mathcal{A}(\Lambda_b^0 \to \Xi^0 K^0),\\
\mathcal{A}(\Xi_b^0 \to p^+ K^-) &= \mathcal{A}(\Lambda_b^0 \to \Sigma^+ \pi^-), \quad \mathcal{A}(\Xi_b^0 \to \Xi^0 K^0) = \mathcal{A}(\Lambda_b^0 \to n \overline{K}^0),
\end{aligned} \tag{12}$$

b) decuplet baryon-octet meson case [14];

$$\begin{aligned}
&\mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) = \mathcal{A}(\Xi_b^0 \to \Delta^+ K^-), \quad \mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0) = \mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K^0}),\\
&\mathcal{A}(\Lambda_b^0 \to \Delta^+ \pi^-) = \mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-), \quad \mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) = \mathcal{A}(\Xi_0^b \to \Sigma'^0 \overline{K^0}),\\
&\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) = \mathcal{A}(\Xi_b^0 \to \Sigma'^+ \pi^-) = -\mathcal{A}(\Lambda_b^0 \to \Delta^0 \overline{K^0}) = -\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0),\\
&\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) = -\frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-) = \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^- \to \Delta^0 K^-) = -\mathcal{A}(\Xi_b^- \to \Sigma'^0 \pi^-),\\
&\mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) = \mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+) = \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Omega^- K^+)\\
&= \frac{1}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Delta^+ \pi^-) = \mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+) = \mathcal{A}(\Xi_b^0 \to \Sigma'^- \pi^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^- K^+),\\
&\mathcal{A}(\Xi_b^- \to \Sigma'^- \overline{K^0}) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Xi'^- \eta_8) = -\frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Omega^- K^0)\\
&= \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K^0}) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Sigma'^- \eta_8) = -\mathcal{A}(\Xi_b^- \to \Xi'^- K^0) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^- \pi^0).
\end{aligned} \tag{13}$$

are obeyed by both the tree and penguin part of the decay amplitudes. However once we include $SU(3)$-breaking effects, all the above mentioned relations are broken except the one given below,

$$\mathcal{A}(\Lambda_b^0 \to \Delta^0 \overline{K^0}) = -\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) \tag{14}$$

Two interesting isospin triangle relations for bottom baryons decaying to a decuplet baryon-octet meson pair,

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^-\pi^+) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) = 0, \tag{15}$$

$$\sqrt{6}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^-\pi^+) + \sqrt{3}\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = 0, \tag{16}$$

continue to hold even after $SU(3)$-breaking effects are considered. An analogous triangle relation that survives $SU(3)$-breaking effects for bottom baryons decaying to an octet baryon-octet meson pair is,

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) + \mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) = 0. \tag{17}$$

The decays mentioned Eq (12) and Eq (5.32) receive contribution from multiple partial waves [23],

$$\mathcal{A} = V_{ub}V_{uq}^*\mathcal{A}^l_{\text{tree}} + V_{tb}V_{tq}^*\mathcal{A}^l_{\text{penguin}} \tag{18}$$

where $V_{ub}V_{uq}^*$, $V_{tb}V_{tq}^*$ are the CKM elements, $q = \{d, s\}$ and $l = 0, 1$ or $l = 1, 2$ for the octet baryon-octet meson or decuplet baryon-octet meson final state. For each partial wave a δ^a_{CP} relation can be derived based on the tree and penguin amplitude relations since,

$$\delta^l_{CP}(\mathcal{B}_b \to \mathcal{B}\mathcal{M}) = -4\mathbf{J} \times \text{Im}\Big[\mathcal{A}^{l*}_{\text{tree}}(\mathcal{B}_b \to \mathcal{B}\mathcal{M})\mathcal{A}^l_{\text{penguin}}(\mathcal{B}_b \to \mathcal{B}\mathcal{M})\Big], \tag{19}$$

J-being the Jarlskog invariant. There are additional phase space factors that are required to translate these δ^a_{CP} relations to A_{CP} relations which can be actually measured in experiments. In the U-spin limit [24], CP violation relations can be experimentally verified using the relation [24–27],

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{B}_j\mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \to \mathcal{B}_m\mathcal{M}_n)} \simeq -\frac{\tau_{\mathcal{B}_{bi}}}{\tau_{\mathcal{B}_{bl}}}\frac{\mathcal{BR}(\mathcal{B}_{bl} \to \mathcal{B}_m\mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{B}_j\mathcal{M}_k)}, \tag{20}$$

where i, j, k and l, m, n are indices corresponding to the various baryons belonging to the above mentioned δ_{CP} relations.

Conclusion

This synopsis contains a brief summary of our work on hadronic bottom baryon decays using $SU(3)$-flavor symmetry. Our approach facilitates an $SU(3)$ decomposition of the decays in terms of $SU(3)$-reduced amplitudes without any particular set of assumptions about the underlying dynamics. Several amplitude relations and CP asymmetry relations are derived for bottom baryons decaying to an octet or decuplet light baryon and an octet meson. So far, despite of the promising hints, the results [5, 6, 8] are consistent with CP-conservation hypothesis. Once CP-asymmetry is measured in some of these related decay modes, it can serve as a test of $SU(3)$-symmetry where $SU(3)$-breaking effects can be studied systematically. Moreover, we hope to estimate the unknown $SU(3)$ parameters directly from the data once sufficient number of these decays are measured in near future.

Plan of the thesis

- The first chapter will contain an overview of the symmetries of the Standard Model.
- The second chapter will introduce different representations of $SU(3)$ and their tensor products.
- In chapter three we use $SU(3)$-flavor symmetry to categorize decays of anti-triplet bottom baryons to an octet baryon and an octet meson.
- In chapter four we extend our analysis by studying decays of anti-triplet bottom baryons to a decuplet baryon and an octet meson using $SU(3)$-flavor symmetry.
- Chapter five will contain a discussion of the results obtained in the thesis and general outlook.

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Chapter 1

Introduction

The discovery of Higgs boson in 2012 has ushered in a new era of particle physics essentially completing the Standard Model picture of elementary particles. This journey of unearthing the fundamental interactions was far from straightforward and looking back, it is almost hard to believe how far we have come from the discovery of the first elementary particle, namely the electron in 1897 by J.J. Thompson. While experimental advances have brought a paradigm shift in the way we understand the universe today, the central role of symmetries as the underlying physical principle has largely remained unchanged. In fact, the Standard Model is a fascinating tale of exact symmetries and broken symmetries. It all began with Lorentz symmetry that ensured universality of physical laws in all inertial frames. Then electormagnetic interactions were cast in the language of *quantum field theory* as a manifestation of *abelian gauge symmetry*. The invariance principle under local transformations of the $U(1)$ group demanded a massless gauge field, the photon. The Fermi theory of weak interaction [28], proposed to explain nuclear β-decays, did not fit into a symmetry principle at first. The extremely short range interaction required a massive mediator which was in tension with the gauge symmetry principle. In addition, experimental observations indicated that weak interactions maximally violated parity (**P**) and charge-conjugation (**C**) symmetry which were presumed to be good symmetries of nature [29, 30]. The resolution to this conundrum was the remarkable proposal [31, 32] of a local $SU(2) \otimes U(1)$ symmetry *spontaneously broken* down to the familiar $U(1)$ electromagnetic gauge symmetry. In the process, the mediators of weak interaction become massive while the photon's mass remains identically zero. A major breakthrough on our understanding of strong interactions came when fermionic *quarks* were proposed [15] as constituents of baryons and mesons, collectively known as hadrons. Initially, three quark 'flavors' connected by a $SU(3)$-flavor symmetry, were introduced to explain the structure of hadrons. While constructing a completely antisymmetric wave function of a baryon made out of three identical quarks, it was apparent that there must be yet another symmetry under which the quarks are antisymmetric. This was a compelling argument [33] in favor of a $SU(3)$ symmetry which was later found to be gauge symmetry [34] that described *quantum chromodynamics*.

While the symmetries shed light on the structure of the SM, we still need to assign specific values to the parameters of the model [35] in order to make numerical predictions. Assuming massless neutrinos, the SM has 18 unknown parameters that can only be fixed from experimental inputs. The complex phase is one such parameter that is solely responsible for all CP-violating phenomena in the SM. Weak interactions not only violate **C** and **P** individually but also CP which makes CP non-conservation a feature of the SM [36,37]. However, unlike **C** and **P** violation which is due to the chiral nature of weak interactions, there is no dynamical reason for CP violation in SM. Moreover, the CP violation from the SM is inadequate to explain the matter-antimatter asymmetry of the observable universe. Since a potential enhancement in CP-asymmetry observations cannot be ruled out, we look for those effects in hadronic b-baryon decays. A non-vanishing CP-asymmetry measurement in b-baryon decays is the first step towards that goal. It is also a measure of direct CP violation as baryons and antibaryons do not undergo mixing due to baryon number conservation. So far, all CP-asymmetry measurements in bottom baryon decays [5, 6, 8–10] have been consistent with CP conservation hypothesis. The situation may change, however, when the statistical errors go down as LHCb gears up to analyze a significant number [11] of hadronic b-baryons decays in its subsequent runs [12, 38, 39].

The theoretical predictions for CP-violation in bottom baryon hadronic decay is also at an early stage. The perturbative QCD (pQCD) [40, 41] and QCD factorization (QCDF) [42] methods are used extensively to make quantitative predictions for decay rates and CP asymmetries in two body nonleptonic B-meson decays. Both of these approaches rely on the fact that the calculation of the hadronic matrix element of B meson decays into two light hadrons can be factorized into a part containing the QCD form factor and another new, unknown, non-local form factor. As it turns out, in a number of these non-leptonic two-body B-decays, the decay matrix element is dominated by the term proportional to the QCD form factor (accounting for the factorizable contribution) whereas the nonlocal form factor term encoding the non-factorizable effect introduces subleading perturbative corrections. Moreover, the QCD form factor, calculated in pQCD, and treated as an external input parameter in case of QCDF, are in good agreement with each other resulting in similar numerical predictions. In case of bottom baryon charmless decays, $\Lambda_b^0 \to p\pi$ and $\Lambda_b^0 \to pK$ transitions are studied in pQCD [2] and to a somewhat lesser extent in QCD factorization [43] approach. In the conventional pQCD scenario, the factoziable contribution are approximately two orders of magnitude smaller than the non-factorizable contribution [2]. In contrast, the branching fraction of $\Lambda_b^0 \to p\pi$, calculated by using light cone sum rule form factors [44], indicate a dominant factorizable contribution when compared to the experimental data allowing only moderate non-factorizable effect.

Alternatively, one can use a general framework based on $SU(3)$-flavor symmetry to analyze such weak decays of bottom baryons which are all decomposed in terms of a few $SU(3)$-invariant amplitudes. These invariant amplitudes can be mapped to quantities that are calculable employing pQCD or QCDF techniques. Once sufficient number of branching ratios and CP asymmetries are measured in experiments, these invariant amplitudes, treated as fit parameters, can be determined directly from the data.

At the same time, using $SU(3)$-flavor symmetry, we can acquire a qualitative understanding of hadronic b-baryon decays in terms of relations between branching fractions and CP-asymmetries of different decay modes. We will direct our efforts towards this goal throughout the thesis.

Chapter 2

The Standard Model and its symmetries

It is well known that *continuous symmetry operations* correspond to Lie groups. The Standard Model respects a set of local and global continuous symmetries which means the symmetry of the SM can be expressed in terms of Lie groups. In particular, the SM successfully describes three of the four fundamental interactions namely the strong, weak and electromagnetic interactions using the local unitary product group $SU(3)_{\text{color}} \otimes SU(2)_L \otimes U(1)_Y$. There are also approximate global symmetries like flavor symmetries which are not exact but play an important role in our understanding of the results observed in experiments. We will briefly recount the implications of all these symmetries in this chapter [45–47].

2.1 Strong interactions

Originally proposed to restore complete antisymmetry to baryon wavefunctions, the theory of strong interactions or *Quantum chromodynamics* (QCD) is based on the *color* property of quarks. QCD is described by a local, non-abelian $SU(3)_{\text{color}}$ symmetry where gluon is the mediator of strong interactions. The conserved quantity associated with this symmetry is *color*. A quark for any given flavor can be represented as a triplet (**3**) under

$SU(3)_{\text{color}}$ having three colors;

$$q(x) \equiv \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix}. \tag{2.1}$$

The quark colors transform in the fundamental representation (**3**) under $SU(3)_{\text{color}}$, that is;

$$q(x) \to q'(x) = Uq(x) = \exp\Big[-ig_s \frac{\lambda_a}{2}\theta_a(x)\Big]q(x), \tag{2.2}$$

where λ_a are the Gell-Mann matrices and the index a runs from 1 to 8. $\theta_a(x)$ are functions of space-time which implies that we need to introduce eight vector boson fields namely the gluons that transforms as an octate in the adjoint representation of $SU(3)_{\text{color}}$ to keep the free Dirac Lagrangian invariant. The meaning of these different representations of $SU(3)$ will be clearer when they are introduced in the next chapter. The QCD Lagrangian density involves writing all the terms that are $SU(3)_{\text{color}}$-invariant,

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \sum_f \bar{q}^f_i\left(i\not{D}^{ij} - m_f\delta^{ij}\right)q^f_j \tag{2.3}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor and q^f_i are all quark flavors f with a and i being the color index in the adjoint representation and fundamental representation of $SU(3)$ respectively. The different quantities introduced above are,

- $\not{D}^{ij} : \gamma_\alpha(\delta^{ij}\partial^\alpha - ig_s\mathcal{A}^\alpha_b T^{ij}_b)$, $T^{ij}_b = \lambda^{ij}_b/2$, g_s being the strong coupling, .

- $G^a_{\mu\nu}(x) : \partial_\mu\mathcal{A}^a_\nu(x) - \partial_\nu\mathcal{A}^a_\mu(x) + g_s f^{abc}\mathcal{A}^b_\mu(x)\mathcal{A}^c_\nu(x)$, $\mathcal{A}^a_\mu$ the gluon field, f^{abc} the structure constants in the adjoint representation of $SU(3)$.

To preserve the local $SU(3)$ invariance of the QCD Lagrangian described in Eq (2.3), the gluon fields $\mathcal{A}_a^\mu(x)$ must transform as,

$$\mathcal{A}_a^{'\mu} T_a \rightarrow U T_a U^{-1} \mathcal{A}_a^\mu + \frac{i}{g_s} (\partial^\mu U) U^{-1} \tag{2.4}$$

which in its infinitesimal form looks like,

$$\mathcal{A}_a^{'\mu} = \mathcal{A}_a^\mu + g_s f_{abc} \theta_b \mathcal{A}_c^\mu + \partial^\mu \theta_a. \tag{2.5}$$

It is also clear from the flavor diagonal mass term that strong force conserves the flavor of the quark, i.e. no change of quark flavors.

2.2 Electroweak interactions

The Fermi theory of weak interaction and electromagnetism was unified into electroweak interaction by Glashow, Weinberg and Salam using local $SU(2)_L \otimes U(1)_Y$ invariance of the SM Lagrangian. We begin by noting that, the fermions are chiral, which means the left chiral fermion $\psi_L \equiv \frac{(1-\gamma_5)}{2}\psi$ and right chiral fermions $\psi_R \equiv \frac{(1+\gamma_5)}{2}\psi$ transform as a doublet of $SU(2)_L$ and a singlet under $SU(2)_L$ respectively. The conserved quantum number is called *weak isospin* ($\mathcal{T}$) to distinguish it from the isospin symmetry respected by strong interactions. In addition, the left-chiral and right-chiral fermions transform differently under $U(1)_Y$ while conserving *weak hypercharge* (Y). Apart from the fermions, the transformation properties of the vector bosons and the scalar Higgs boson under local $SU(2)_L \otimes U(1)_Y$ is also provided in the table

The electroweak Lagrangian density consists of all those terms which are invariant under the $SU(2)_L \otimes U(1)_Y$ symmetry,

$$\mathcal{L}_{EW} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \tag{2.6}$$

Particle type	Generations	Color $SU(3)_c$	Weak Isospin $SU(2)_L$	Hypercharge $U(1)_Y$	Electric Charge $U(1)_Q$
Q_L^i	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	**3**	**2**	$\frac{1}{3}$	
			$\mathcal{T}_3(u_L, c_L, t_L) = \frac{1}{2}$		$\frac{2}{3}$
			$\mathcal{T}_3(d_L, s_L, b_L) = -\frac{1}{2}$		$-\frac{1}{3}$
L_L^i	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	**1**	**2**	-1	
			$\mathcal{T}_3(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}) = \frac{1}{2}$		0
			$\mathcal{T}_3(e_L, \mu_L, \tau_L) = -\frac{1}{2}$		-1
u_R^i	u_R, c_R, t_R	**3**	**1**	$\frac{4}{3}$	$\frac{2}{3}$
d_R^i	d_R, s_R, b_R	**3**	**1**	$-\frac{2}{3}$	$-\frac{1}{3}$
e_R^i	e_R, μ_R, τ_R	**1**	**1**	-2	-1
$W_\mu^\pm$		**1**	$\mathcal{T}_3(W^\pm) = \pm 1$	0	± 1
Z_μ^0, A_μ^0		**1**	$\mathcal{T}_3(Z^0, A^0) = 0$	0	0
G_μ		**8**	**1**	0	0
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$		**1**	**2**	1	
			$\mathcal{T}_3(\phi^+) = \frac{1}{2}$		
			$\mathcal{T}_3(\phi^0) = -\frac{1}{2}$		

Table 2.1: Representation of elementary particles under SM symmetry group

2.2.1 Kinetic term

$\mathcal{L}_{kin}$ contains all dimension-4 kinetic terms for vector bosons, fermions and fermion-vector beson interactions,

$$\mathcal{L}_{kin} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_n \left(Q^n_L \not{D} Q^n_L + L^n_L \not{D} L^n_L + u^n_R \not{D} u^n_R + d^n_R \not{D} d^n_R + e^n_R \not{D} e^n_R \right). \tag{2.7}$$

Here W^a_μ and B_μ are the massless vector bosons associated with the local $SU(2)_L$ and $U(1)_Y$ symmetries respectively. The index a in W^a_μ runs over the three generators of the $SU(2)$ group given by $\mathcal{T}^a = \frac{1}{2}\sigma^a$, $[\mathcal{T}^a, \mathcal{T}^b] = i\epsilon^{abc}\mathcal{T}^c$ σ^a being the 2×2 Pauli matrices.

The generator of hypercharge Y in this space acts like identity and therefore commutes with the three generators of $SU(2)_L$. The field strength tensors are

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.8}$$

and $\not{D} = \gamma^\beta(\partial_\beta + ig\mathcal{T}^a W^a_\beta + \frac{i}{2}g^{'} Y B_\mu)$. The gauge couplings for the groups $SU(2)_L$ and $U(1)_Y$ are g and $g^{'}$ respectively. The left-chiral quark and lepton $SU(2)_L$ doublets are denoted by Q^n_L and L^n_L respectively,

$$Q^n_L : \begin{pmatrix} u^n_L \\ d^n_L \end{pmatrix}, \qquad L^n_L : \begin{pmatrix} \nu^n_L \\ e^n_L \end{pmatrix} \tag{2.9}$$

with $n = 1, 2, 3$ being the generation index of the fermions. In contrast, the right-chiral fermions u^n_R, d^n_R and e^n_R are $SU(2)_L$ singlets, i.e $\mathcal{T}^a f^n_R = 0$. In order to make a direct correspondence with vector bosons having definite electric charge we switch to the physical basis by a transformation,

$$\begin{pmatrix} W^+_\mu \\ W^-_\mu \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} \qquad \begin{pmatrix} A^0_\mu \\ Z^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} \tag{2.10}$$

where the electric charge is given by $Q = \mathcal{T}_3 + Y/2$ and $\tan\theta_w = \frac{g^{'}}{g}$. The Weinberg angle θ_w is a parameter of the SM and its relation to the electric charge in fundamental units is $e = \frac{g}{\sin\theta_w}$. After this transformation, $Q(W^\pm) = \pm 1$ and $Q(Z^0) = Q(A^0) = 0$ that corresponds to the charged W-bosons and the two neutral bosons A^0 and Z^0. We immediately notice that any mass term of the form $F^\mu F_\mu$ for the vector bosons would explicitly break the $SU(2)_L \otimes U(1)_Y$ invariance of the kinetic term. Moreover, we are forbidden to write a fermion Dirac mass term of the form $m_f \overline{f}_L f_R$ as it also breaks $SU(2)_L \otimes U(1)_Y$ invarince of the kinetic term since the right-chiral and left-chiral fermions transform differently under $SU(2)_L \otimes U(1)_Y$. At this point, we have to discuss the Higgs mechanism which solves both of these problems and generates masses for vector bosons and fermions without breaking the $SU(2)_L \otimes U(1)_Y$ invariance at the Lagrangian level.

2.2.2 Higgs interactions

In order to not explicitly break the $SU(2)_L \otimes U(1)_Y$ invariance of the electroweak Lagrangian we use the idea of *spontaneous symmetry breaking* (SSB). This implies that the ground state of the theory is not invariant under $SU(2)_L \otimes U(1)_Y$ while the actual Lagrangian is still invariant under the original local symmetry. After SSB, the ground state of the symmetry breaking field picks up a particular direction in the group space. Clearly, if the field responsible for SSB is anything other than a scalar then the vacuum expectation value of that field prefers a certain direction in spacetime, which would break Lorentz invariance. Therefore, we can only introduce a scalar field Φ which is a doublet under $SU(2)_L$

$$\Phi : \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = 1/\sqrt{2}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{2.11}$$

and charged under $U(1)_Y$ to construct the $\mathcal{L}_{Higgs}$ and $\mathcal{L}_{Yukawa}$ terms. The Lagrangin density for this scalar field containing Φ is given by,

$$\mathcal{L}_{Higgs} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - \mu^2(\Phi^\dagger\Phi) - \lambda(\Phi^\dagger\Phi)^2 \qquad (\lambda > 0) \tag{2.12}$$

For spontaneous breaking of $SU(2)_L \otimes U(1)_L$ we must have $\mu^2 < 0$ where we define the vacuum expectation value (VEV) v;

$$v^2 = \frac{-\mu^2}{\lambda} \tag{2.13}$$

is real. The potential of Φ is now given by, $V(\Phi) = \lambda(|\Phi|^2 - \frac{v^2}{2})$ where there is an infinite number of degenerate states with minimum energy satisfying $|\Phi| = \frac{v}{\sqrt{2}}$. By making a particular choice where

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0, \langle\phi_3\rangle = v/\sqrt{2} \tag{2.14}$$

we see that while $SU(2)_L \otimes U(1)_Y$ gets broken by the ground state of Φ, the $U(1)_Q$ generator remains unbroken;

$$Q\langle\Phi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0, \qquad T^a\langle\Phi\rangle \neq 0. \tag{2.15}$$

This unbroken subgroup is identified as $U(1)_{EM}$ with Q being the generator. A spontaneously broken continuous global symmetry gives rise to massless spin-0 *Goldstone bosons* (GB) that correspond to the broken generators. In $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ breaking, the three massless GBs get 'eaten up' by the longitudinal components of three previously massless vector bosons $W^\pm$ and Z^0. These vector bosons acquire masses as a result. This is commonly known as the *Higgs mechanism*. The choice made in (2.14) ensures that only physical degrees of freedom appear and around the vacuum Φ is expanded as,

$$\Phi(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}. \tag{2.16}$$

The vector bosons get their mass from the $(D_\mu\Phi^\dagger)(D^\mu\Phi)$ term leaving a photon massless,

$$m_W = \frac{1}{2}gv, \qquad m_Z = \frac{v}{2}\sqrt{g^2+g'^2}, \qquad m_{A^0} = 0 \tag{2.17}$$

A consequence of generating mass for vector bosons through SSB without breaking $SU(2)_L \otimes U(1)_Y$ explicitly also solves the problem of renormalizability of a theory having non-Abelian local symmetry with massive vector bosons.

2.2.3 Yukawa interaction

We have already alluded to the fact that mass terms of the form $m_f \overline{f}_L f_R$ are non-inavraint under $SU(2)_L \otimes U(1)_Y$ symmetry. With the help of the scalar field Φ charged under $SU(2)_L \otimes U(1)_Y$ introduced before, we can write down the terms

$$\mathcal{L}_Y = -\sum_{i,j} y_d^{ij} \overline{Q_L}^i \Phi d_R^j - \sum_{i,j} y_u^{ij} \overline{Q_L}^i \tilde{\Phi} u_R^j - \sum_{i,j} y_\ell^{ij} \overline{L_L}^i \Phi e_R^j + h.c. \tag{2.18}$$

that are actually $SU(2)_L \otimes U(1)_Y$ invariant. Once the electrically neutral component of this scalar field Φ picks up a vacuum expectation value after SSB, these terms generate fermion masses,

$$\mathcal{L}_Y = -\sum_{i,j} \frac{v y_d^{ij}}{\sqrt{2}} \overline{d_L}^i d_R^j - \sum_{i,j} \frac{v y_u^{ij}}{\sqrt{2}} \overline{u_L}^i u_R^j - \sum_{i,j} \frac{v y_\ell^{ij}}{\sqrt{2}} \overline{e_L}^i e_R^j + h.c. \tag{2.19}$$

According to the notation used in Eq (2.18) and Eq (2.19):

- Q_L^i, L_ℓ^i are quark and lepton $SU(2)_L$ doublets,
- u_R^i, d_R^i, e_R^i are up, down and charged lepton $SU(2)_L$ singlets.
- Φ is the scalar field, $\tilde{\Phi} = i\sigma_2 \Phi$
- i, j generation indices, $y_d^{ij}, y_u^{ij}, y_\ell^{ij}$ up, down and charged lepton Yukawa couplings.
- Mass matrix for up-type quarks $M_u^{ij} = \frac{v y_u^{ij}}{\sqrt{2}}$, down-type quarks $M_d^{ij} = \frac{v y_d^{ij}}{\sqrt{2}}$ and charged leptons $M_\ell^{ij} = \frac{v y_\ell^{ij}}{\sqrt{2}}$

Since the complex Yukawa coupling matrices for fermions (y_f^{ij}) need not be diagonal in the generation indices, we need to make the following unitary transformations to obtain diagonal fermion mass matrices,

$$u_R \to V_{u_R} u_R,\ d_R \to V_{d_R} d_R,\ u_L \to V_{u_L} u_L,\ d_L \to V_{d_L} d_L \tag{2.20}$$

that leaves the fermion kinetic term in Eq (2.7) invariant. The matrices V_{u_L} and V_{d_L} need not be equal and therefore W boson-fermion charged current interaction introduces the unitary matrix $V^{\dagger}_{u_L} V_{d_L}$ in the quark sector,

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\Big[\overline{u_{Lm}}(V^{\dagger}_{u_L})^{ml}\gamma^{\mu}W^{+}_{\mu}(V_{d_L})^{ln}d_{Ln} + \overline{d_{Lm}}(V^{\dagger}_{d_L})^{ml}\gamma^{\mu}W^{-}_{\mu}(V_{u_L})^{ln}u_{Ln}\Big]. \tag{2.21}$$

that generates interaction between up-quark family and down-quark family belonging to different generations. In contrast, the neutral-current interactions engender Z-boson-fermion or photon-fermion coupling that are diagonal in the generation space,

$$\mathcal{L}_{nc} = \frac{-g}{\cos\theta_w}\overline{f}\gamma^{\mu}\Big[\mathcal{T}^3 P_L - Q\sin^2\theta_w\Big]f Z^0_{\mu} - eQ\overline{f}\gamma^{\mu}f A^0_{\mu}. \tag{2.22}$$

The unitary matrix $V^{\dagger}_{u_L} V_{d_L}$ defined earlier is known as the Cabibbo Kobayashi Maskawa (CKM) matrix [48]. While the most general 3×3 unitary matrix consists of 9 parameters, a number of them can be absorbed by redefining the quark field leaving us with three angles and a phase. This phase is the *only* source of CP-violation in the quark sector of the SM. The CKM matrix V_{CKM} is often parameterized as follows,

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad c_{ij} : \cos\theta_{ij}, s_{ij} : \sin\theta_{ij} \tag{2.23}$$

where the three mixing angles are θ_{12}, θ_{23}, θ_{13} and δ is the CP-violating phase. To identify an invariant measure of CP violation in SM independent of basis choices, the *Jarlskog invariant* (**J**) is defined,

$$\mathrm{Im}[V_{ij}V_{mn}V^{*}_{in}V_{mj}] = \mathbf{J}\sum_{kl}\epsilon_{imk}\epsilon_{jnl} \tag{2.24}$$

where V_{ij} is the matrix elements of V_{CKM} with no sum over repeated indices. According to the parameterization of V_{CKM} in Eq (2.23);

$$\mathbf{J} = Im(V_{22}V_{13}V_{23}^{*}V_{12}^{*}) = c_{12}c_{23}c_{13}^{2}s_{12}s_{23}s_{13}\sin\delta. \tag{2.25}$$

which is non-vanishing not only because of V_{CKM} having a $\delta \neq 0$ but also for all mixing angle being non-zero ($\theta_{ij} \neq 0$).

2.3 Global symmetries and approximate symmetries

So far, we have outlined the local symmetries that are imposed on the SM in order to identify all possible allowed interactions between elementary particles. The global symmetries, in contrast, are not inputs in the SM and actually emerge from the fermion content and continuous symmetries of the SM. A number of those global symmetries arise purely from our unwillingness to write non-renormalizable interaction (operators with mass dimension higher than 4) at the Lagrangian level. For example, a term like

$$Q_L Q_L Q_L L_\ell \tag{2.26}$$

preserves the SM local symmetries but appears as a dimension-6 operator. By ignoring such type of terms which are automatically suppressed we discover the *accidental symmetries* of the SM. Some particular global symmetries are broken when one or multiple parameters are non zero. However, as long as those parameters are considered '*small*' compared to a relevant dimensional quantity, these symmetries can prove to be quite useful to make predictions. These are the *approximate symmetries* of the SM. We discuss important aspects of some of those symmetries below.

Custodial symmetry

We focus particularly on the $\mathcal{L}_{Higgs}$ part of the SM Lagrangian and we immediately notice that the scalar potential which is responsible for SSB has a bigger symmetry than $SU(2)\otimes U(1)$. We recall that before SSB, the potential described in Eq (2.12) which is a function of $\Phi^\dagger\Phi$,

$$\Phi^\dagger\Phi = \frac{1}{2}(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2), \qquad \phi_i : \text{real} \tag{2.27}$$

has an $SO(4)$ symmetry. Since $SO(4)$ symmetry is equivalent to a $SU(2)\otimes SU(2)$ symmetry, which is bigger than the $SU(2)\otimes U(1)$ local symmetry, there is a residual symmetry in this potential. After SSB, the ϕ_3 component picks up a VEV and the $SO(4)$ symmetry is broken down to an $SO(3)$-symmetry. From equivalence of group algebra $SO(3)$ can be identified as an $SU(2)$ and we are left with the symmetry breaking pattern,

$$SU(2)\times SU(2) \to SU(2) \tag{2.28}$$

before and after SSB of the scalar potential. This remnant symmetry is called the *custodial symmetry*. The ramification of this symmetry is three vector bosons with equal mass after SSB if the SM local symmetry was only $SU(2)_L$. This is the limiting case of the actual SM group $SU(2)_L\otimes U(1)_Y$ with the gauge coupling $g^{'}\to 0$ for the $U(1)_Y$ group. Due to the presence of the $U(1)_Y$ having $g^{'}\neq 0$ the custodial symmetry indicates a modified relation between the masses of the charged W-bosons and neutral Z^0-boson,

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_w} = 1 \tag{2.29}$$

at tree level. In absence of custodial $SU(2)$ symmetry breaking this continues to hold to all orders in perturbation theory. In reality, at loop level, Yukawa couplings and gauge-boson interactions would produce custodial $SU(2)$ violating effect, and thereby violate the mass relation quoted in Eq (2.29). However this breaking is small and deviations of the ρ-parameter from unity can be reliably estimated.

Flavor symmetries

The term *flavor* is used to describe copies or generations of fermions which are assigned identical quantum numbers under the SM gauge group. In absence of Yukawa interactions $y^{ij}_{u,d,e} = 0$, a larger $U(3)^5$ global symmetry [49–52] is manifest in the kinetic term of the fermions having three generations, given in Eq (2.7). This corresponds to independent unitary transformations in flavor space for five fermion fields,

$$\mathcal{G}^{\text{global}}_{SM}(y^{ij} = 0) = U(3)^5 = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{L_\ell} \otimes SU(3)_{e_R} \otimes U(1)^5 \tag{2.30}$$

Once Yukawa interactions are switched on, we are left with

$$\mathcal{G}^{\text{global}}_{SM}(y^{ij} \neq 0) = U(1)_B \otimes U(1)_L \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau \tag{2.31}$$

which are all *accidental symmetries* of the SM and therefore can be broken by non-renormalizable interactions in principle. These $U(1)$ symmetries correspond to total baryon number, total lepton number, e lepton flavor, μ lepton flavor and τ lepton flavor conservation respectively.

The $SU(3)$ symmetry of three light quarks given by up (u), down (d) and strange (s) gives rise to a flavor symmetry that is most relevant to this thesis. The symmetry is an artifact of low energy QCD where the masses of the light quarks can be taken to be equal which are small compared to Λ_{QCD}. This assertion works well for the u and d quarks but not for s quark which has a mass ($m_s \sim 100$MeV) that is non-negligible compared to $\Lambda_{\text{QCD}} \sim 300$MeV. Nevertheless, we consider this as an *approximate* symmetry. We notice that in absence of mass terms the kinetic terms for u, d, s-flavors,

$$\mathcal{L}_{kin} = iu_R \not{D} u_R + iu_L \not{D} u_L + id_R \not{D} d_R + id_L \not{D} d_L + is_R \not{D} s_R + is_L \not{D} s_L \tag{2.32}$$

have a $SU(3)_L \otimes SU(3)_R$ global symmetry. This symmetry is spontaneously broken down to $SU(3)_V = SU(3)_{L=R}$ subgroup by the ground state of QCD through non-zero expectation value of the qurak-antiquark condensates;

$$\langle u\bar{u}\rangle = \langle d\bar{d}\rangle = \langle s\bar{s}\rangle = V^3. \tag{2.33}$$

Moreover, the mass term for the light quarks already breaks the $SU(3)_L \otimes SU(3)_R$ symmetry explicitly. The result of $SU(3)_L \otimes SU(3)_R \to SU(3)_V$ breaking is 8 massive pseudo-Goldstone bosons representing the octet of pseduscalar mesons:

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta_8. \end{pmatrix} \tag{2.34}$$

Light baryons which are also bound states of three up, down or strange quarks is described by the $SU(3)_L \otimes SU(3)_R \to SU(3)_V$ breaking. Under $SU(3)_V$, the product of three quark $SU(3)$ triplets give a decuplet, two octets and a singlet of baryon. The well known proton and neutron sit in the octet representation of the baryon,

$$\mathcal{B} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p^+ \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n^0 \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix} \tag{2.35}$$

whereas the spin-3/2 delta baryons belongs to the baryon decuplet. This understanding of light mesons and baryons came long before we knew about the existence of quarks thanks to the remarkable observation of Gell-Mann's *eightfold way*. The $SU(3)_V$ from now on will be identified as the $SU(3)_{\text{flavor}}$-symmetry. The three $SU(2)$ subgroups of $SU(3)_{\text{flavor}}$ known as I-spin, U-spin, V-spin correspond to an interchange symmetry between u-quark and d-quark, d-quark and s-quark, u-quark and s-quark respectively. In the following chapters we will elucidate the use of this $SU(3)_{\text{flavor}}$-symmetry to study physical processes.

Chapter 3

The group $SU(3)$

$SU(3)$ is defined as the group of all unitary 3×3 matrices having a determinant equaling unity. If x_i is a complex 3-vector, the group action on that vector is the following:

$$U : x^i \to U^i_j x^j \tag{3.1}$$

where the space of all x^i form a basis for a representation of $SU(3)$. The group elements of $SU(3)$ are written in terms of 8 parameters θ_a,

$$U = \exp(-iT_a\theta_a) \tag{3.2}$$

where T^a's are the 8 traceless *generators* of the group with an implied sum over the index a. In case θ_as are real then T_as are all hermetian. A brief outline of some important representations of $SU(3)$ are provided below.

Fundamental representation

This 3-dimensional representation, which follows from the definition of the group, is called the fundamental representation of the group. The generators in this representation are given by 3×3 traceless Hermitian matrices which are also known as the **Gell-Mann** matrices,

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$
$$\lambda_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{3.3}$$

The generators in the fundamental representation are chosen in such a way that they satisfy the relations,

$$\mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \quad [T_a, T_b] = \frac{i}{2} f_{abc} T_c \tag{3.4}$$

that necessitates the identification of T_as as,

$$T_a = \frac{\lambda_a}{2}. \tag{3.5}$$

The λ^a also follow the relations,

$$\sum_{a=1}^{8} \lambda^a_{ij}\lambda^a_{kl} = 2\delta_{il}\delta_{kj} - \frac{2}{3}\delta_{ij}\delta_{kl}, \tag{3.6}$$
$$\sum_{a=1}^{8} \lambda^a_{ij}\lambda^a_{kl} = \frac{16}{9}\delta_{il}\delta_{kj} - \frac{1}{3}\sum_{a=1}^{8} \lambda^a_{il}\lambda^a_{kj} \tag{3.7}$$

The structure constants f_{abc}s are totally antisymmetric in its three indices. The non-zero entries are given by,

$$f_{123} = 1,\ f_{147} = f_{246} = f_{257} = f_{345} = -f_{156} = -f_{367} = \frac{1}{2},\ f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \tag{3.8}$$

The fundamental representation is denoted as the **3** of $SU(3)$.

Antifundamental representation

Another inequivalent 3-dimensional representation of $SU(3)$ group is possible. A general element of this representation is obtained by taking a complex conjugate of Eq (3.2);

$$U^* = \exp(iT_a^*\theta_a) \tag{3.9}$$

which implies the following relation between the generators of the fundamental and anti-fundamental representations,

$$(T_a)_{\text{anti-fundamental}} = -(T_a^*)_{\text{fundamental}}. \tag{3.10}$$

All the generators of the antifundamental representation cannot be related to the generators in the fundamental representation by a similarity transformation i.e.

$$-T_a^* \neq S T_a S^{-1} \tag{3.11}$$

for any constant unitary 3×3 matrix S, which is why this representation is not equivalent to the fundamental representation. The fundamental representation is denoted as the $\mathbf{\overline{3}}$ of $SU(3)$.

Adjoint representation

The adjoint representation of $SU(3)$ acts on the vector space of its Lie algebra and the dimension of such a representation is given by the number of generators which in this case is 8. The structure constants themselves generate this representation of the algebra once we define a matrix such that,

$$(T_a^{\text{adj}})_{bc} = -if_{abc}. \tag{3.12}$$

Since f_{abc}s are all real, the generators of the adjoint representation is purely imaginary.

We now turn our attention to representations [21, 53] induced on the space of mixed tensors with p upper indices and q lower indices as $D(p.q)$, or simply (p,q) in short. Here the distinction between the upper and lower indices are important as the there are two inequivalent 3-dimensional representations, namely the **3** and the $\overline{\mathbf{3}}$. Moreover, antisymmetric part of any tensor can be expressed in terms of a tensor of lower rank and the completely antisymmetric $\epsilon_{i_1 j_1 k_1}$ and $\epsilon^{i_2 j_2 k_2}$ tensors:

$$A^{[i_1,i_2]\dots}_{a_1 b_1 \dots} = \epsilon^{i_1 i_2 k} B^{\dots}_{k a_1 b_1 \dots}. \tag{3.13}$$

The mixed tensor can also be made traceless by using the $\delta^{i_1}_{j_1}$ invariant tensor and tensors of lower ranks. Therefore the mixed tensor (p,q) under consideration is

- purely symmetric in all p upper indices,
- purely symmetric in all q lower indices,
- traceless.

This family of tensors form inequivalent irreducible representations (IR) of $SU(3)$. To calculate the dimension of such a representation (p,q), the space of all completely symmetric tensors with p upper indices and q lower indices need to be decomposed into the space of all symmetric tensors while respecting the traceless property. An IR having only p upper indices has the dimension equal to the number of independent components,

$$\sum_{\sigma=0}^{p}(p-\sigma-1) = \frac{1}{2}(p+1)(p+2). \tag{3.14}$$

Similarly an IR having only q lower indices has the dimension,

$$\sum_{\sigma=0}^{q}(q-\sigma-1) = \frac{1}{2}(q+1)(q+2). \tag{3.15}$$

A mixed tensor totally symmetric in its p upper and q lower components has therefore $n_1 = \frac{1}{4}(p+1)(p+2)(q+1)(q+2)$ independent components. The trace of a mixed tensor totally symmetric in its p upper and totally symmetric in its q lower components has $n_2 = \frac{1}{4}p(p+1)q(q+1)$ independent components all identically zero due to the traceless property of (p,q). Therefore the number of independent components as well as the dimension of (p,q) is,

$$\dim(p,q) = (n_1 - n_2) = \frac{1}{2}(p+1)(q+1)(p+q+2). \tag{3.16}$$

Sometimes this dimension is used as a label for particular $SU(3)$ representation, although it may turn out to be ambiguous as several choices of (p,q) can give rise to same number. Since (p,q) and (q,p) have the same dimension, they are distinguished by labeling a representation by its dimension if $p > q$, and by its dimension with a bar if q is greater than p. Examples of some commonly used representations are given below;

$$(1,0) \Rightarrow \mathbf{3},\ (0,1) \Rightarrow \bar{\mathbf{3}},\ (1,1) \Rightarrow \mathbf{8},\ (3,0) \Rightarrow \mathbf{10},\ (2,1) \Rightarrow \mathbf{15},\ (4,0) \Rightarrow \mathbf{15}'. \tag{3.17}$$

It is important to identify the isospin and hypercharge subgroups inside a $SU(3)$ representation which are known to be good symmetries of the strong interaction. The subgroup $SU(2)\otimes U(1)$ does this job where the $SU(2)$ and $U(1)$ corresponds to the isospin and the hypercharge symmetry respectively. For example, consider the 3×3 matrices forming the fundamental, or **3** of $SU(3)$ acting on the space of states consisting of three-element column vectors. The subgroup $SU(2)$, identified with the isospin group can be assumed to consist of matrices of the form

$$U_2 = \exp(i\kappa^a \tau^a), \qquad \tau^a : \text{Pauli matrices} \tag{3.18}$$

mixing only the first two entries of the 3-dimensional column vector, leaving the third one unchanged. This means that under this subgroup, the **3** representation of $SU(3)$ breaks into a doublet, represented by the two upper elements of the column vector, and a singlet, which is the lowest element. The $U(1)$ quantum number of each entry is proportional to the corresponding element of the diagonal matrix λ_8. Inspecting the explicit form of λ_8 in Eq (3.3), we conclude that the $U(1)$ quantum number is equal for the two states in the $SU(2)$ doublet. It is obviously the case since $U(1)$ commutes with the $SU(2)$ and so the states in any $SU(2)$ representation should have the same $U(1)$ property. We can therefore write the **3** of $SU(3)$ as,

$$\mathbf{3} = (\frac{1}{2}, \frac{1}{3}) \oplus (0, -\frac{2}{3}) \tag{3.19}$$

The anti-fundamental $\overline{\mathbf{3}}$ of $SU(3)$ is expressed as a complex conjugate of Eq (3.19),

$$\overline{\mathbf{3}} = (\frac{1}{2}, -\frac{1}{3}) \oplus (0, \frac{2}{3}) \tag{3.20}$$

where only the $U(1)$ charges switch sign while the complex conjugation of the 2-dimensional representation of $SU(2)$ maps onto itself. Clearly, the $SU(3)$ invariant or identity representation should be a singlet under $SU(2)$ and having a $U(1)$ charge 0,

$$\mathbf{1} = (0, 0). \tag{3.21}$$

More generally, for an IR representation $(n, 0)$ and $(0, m)$ the isospin-hypercharge decomposition is given by,

$$
\begin{aligned}
(n,0) &= \Big(\frac{n}{2}\Big)_{\frac{n}{3}} \oplus \Big(\frac{n-1}{2}\Big)_{\frac{n}{3}-1} \cdots \oplus \Big(0\Big)_{-\frac{2n}{3}}, \\
(0,m) &= \Big(\frac{m}{2}\Big)_{-\frac{m}{3}} \oplus \Big(\frac{m-1}{2}\Big)_{-\frac{m}{3}+1} \cdots \oplus \Big(0\Big)_{\frac{2m}{3}}
\end{aligned} \tag{3.22}
$$

that is used to write down the isospin-hypercharge decomposition of the representation (n,m);

	$\left(\frac{n}{2}\right)_{\frac{n}{3}}$	$\left(\frac{n-1}{2}\right)_{\frac{n}{3}-1}$	$\cdots\cdots$	$\left(0\right)_{-\frac{2n}{3}}$
$\left(\frac{m}{2}\right)_{-\frac{m}{3}}$	$\left(\frac{n}{2}\otimes\frac{m}{2}\right)_{\frac{n-m}{3}}$	$\left(\frac{n-1}{2}\otimes\frac{m}{2}\right)_{\frac{n-m-1}{3}}$	$\cdots\cdots$	$\left(\frac{m}{2}\right)_{\frac{-2n-m}{3}}$
$\left(\frac{m-1}{2}\right)_{-\frac{m}{3}+1}$	$\left(\frac{n}{2}\otimes\frac{m-1}{2}\right)_{\frac{n-m+1}{3}}$			
$\vdots$	$\vdots$			
$\left(0\right)_{\frac{2m}{3}}$	$\left(\frac{n}{2}\right)_{\frac{2m+n}{3}}$			

3.1 Direct product of two $SU(3)$ representations

The direct product of two $SU(3)$ representations (n,m) and (n',m') is given by the general formula [53],

$$
(n,m)\otimes(n',m') = \sum_{i=0}^{\min(n,m')} \sum_{j=0}^{\min(n',m)} \oplus(n-i,n'-j;m-j,m'-i) \tag{3.23}
$$

where the sum indicates a direct sum over the objects given by $(n,n';m,m')$. The representation $(n,n';m,m')$ is defined as that representation which has for its basis the set of all tensors having $(n+n')$ upper indices and $(m+m')$ lower indices, that are completely symmetric among the first n upper indices, completely symmetric among the last n' upper indices, completely symmetric among the first m lower indices, completely symmetric among the last m' lower indices while being traceless. We need to convert this $(n,n';m,m')$ into direct sums of IR's. Operationally we want to decompose an arbitrary tensor from the basis of $(n,n^{'};m,m^{'})$ into a sum of linear combinations of traceless tensors symmetric under interchange of all of its upper and lower indices. Consider an arbitrary tensor

$$\mathcal{H}^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}} \tag{3.24}$$

which has first n upper indices $i_1\ldots i_n$, last $n^{'}$ upper indices $i_{n+1}\ldots i_{n+n'}$ and first m lower indices $j_1\ldots j_m$, last $m^{'}$ lower indices $j_{m+1}\ldots j_{m+m'}$ all symmetric under interchange. We want to reexpress $\mathcal{H}^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}}$ such that the new tensors are fully symmetric under interchange of any upper indices. Without any loss of generality we choose two indices i_1 and i_{n+1} that are not symmetrized and use the property that any arbitrary tensor can be written in terms of symmetric and antisymmetric tensor. Therefore,

$$\mathcal{H}^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}} = S^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}} + A^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}} \tag{3.25}$$

where the two indices i_1 and i_{n+1} that are anti-symmetric under interchange in $A^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots}$ can be expressed in terms of the fully anti-symmetric ϵ^{lmn} tensor as,

$$A^{i_1\ldots i_n i_{n+1}\ldots i_{n+n'}}_{j_1\ldots j_m j_{m+1}\ldots j_{m+m'}} = \epsilon^{k i_1 i_{n+1}} S^{i_2\ldots i_n i_{n+2}\ldots i_{n+n'}}_{k j_1\ldots j_m j_{m+1}\ldots j_{m+m'}}. \tag{3.26}$$

This $S^{i_2\ldots i_n i_{n+2}\ldots i_{n+n'}}_{k j_1\ldots j_m j_{m+1}\ldots j_{m+m'}}$ tensor is already completely symmetric in its lower indices as contraction of any pair of lower indices with the ϵ tensor identically vanishes. The upshot of this procedure is that by removing pairs of upper indices and by adding a lower index or by removing pairs of lower indices, and adding an upper index we can symmetrize the arbitrary tensor $\mathcal{H}$ until the resulting tensor becomes completely symmetric in its upper and lower indices. But we cannot remove a pair of upper indices and a pair of lower indices simultaneously, since removal of a pair of upper (lower) indices makes the tensor already completely symmetric in its lower (upper) indices. In summary, the object $(n,n^{'};m,m^{'})$

can be decomposed as a direct sum of $SU(3)$-IRs given below [53],

$$\begin{aligned}(n,n';m,m') = (n+n',m+m') &\oplus \sum_{a=1}^{\min(n,n')} (n+n'-2a, m+m'+a) \\ &\oplus \sum_{b=1}^{\min(n,n')} (n+n'+b, m+m'-2b). \qquad (3.27)\end{aligned}$$

As a concrete example we study the direct product of two IRs $(1,1)$ with $(1,1)$;

$$\begin{gathered}(1,1)\otimes(1,1) = (1,1;1,1)\oplus(0,1,1,0)\oplus(1,0,0,1)\oplus(0,0,0,0) \\ (1,1;1,1) = (2,2)\oplus(0,3)\oplus(3,0), \qquad (1,0;1,0) = (1,1) \\ (0,1;1,0) = (1,1), \qquad (0,0;0,0) = (0,0) \qquad (3.28)\end{gathered}$$

In terms of the dimension of a representation using Eq (3.16) we get,

$$\mathbf{8}\otimes\mathbf{8} = \mathbf{27}\oplus\overline{\mathbf{10}}\oplus\mathbf{10}\oplus\mathbf{8}_1\oplus\mathbf{8}_2\oplus\mathbf{1} \qquad (3.29)$$

Alternatively we can use Young tableaux [47] to find the tensor product of two irreducible representations of $SU(3)$. We will discuss Young tableaux specifically for $SU(3)$ that follow a set of rules given below,

- The Young tableaux corresponding to the fundamental representation (**3** of $SU(3)$) will consist of just one box, since this representation acts on states with one index (See Eq. (3.1)). That is,

$$x^i \equiv \square \qquad (3.30)$$

- For any other representation, there will be more boxes. If any two of the indices are symmetric, we put the corresponding boxes in the same row. If the indices are antisymmetric, the boxes are in the same column.

- A Young tableau is a diagram of left-justified rows of boxes where any row cannot be longer than the row on top of it.

- Any column cannot contain more than 3 boxes.

- Any column with exactly 3 boxes can be crossed out since it corresponds to the trivial representation (the singlet), Therefore any Young tableaux for $SU(3)$ can have at most two rows of boxes.

- In order to connect any Young tableaux with the $SU(3)$ irreducible representation (p,q) we note that q equals the number of boxes in the second row. p is given by the difference between the number of boxes in the first row and the second row.

- The complex conjugate of a given IR is represented by a tableaux obtained by switching every column of k boxes with a column of $(3-k)$ boxes.

The general recipe to find the tensor product of two $SU(3)$ IRs using Young tableaux is also outlined,

- Write the two tableaux which correspond to the direct product of IRs and label successive rows of the first and second tableau with indices A, B and a, b such that no two same indices appear in one column.

$$\begin{array}{|c|c|}\hline A & A \\ \hline B \\ \cline{1-1}\end{array} \otimes \begin{array}{|c|c|}\hline a & a \\ \hline b \\ \cline{1-1}\end{array}$$

- Attach the boxes from the second to the first tableau, one a time following the order a then b in all the possible way. The resulting diagrams should be valid Young tableaux with no two a or b in the same column. This also means no row should be longer than a row above it.

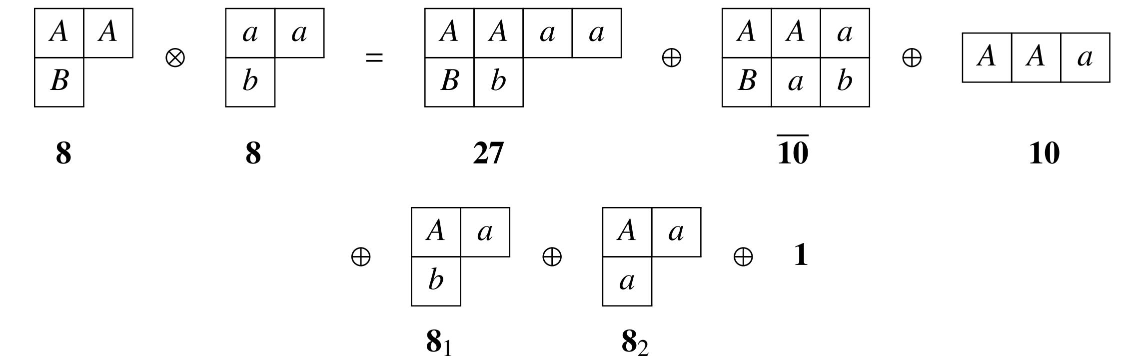
- If the resulting tableaux has a column of 3 boxes, that column is to be deleted and the rest as the tableaux is treated as a valid tableaux.

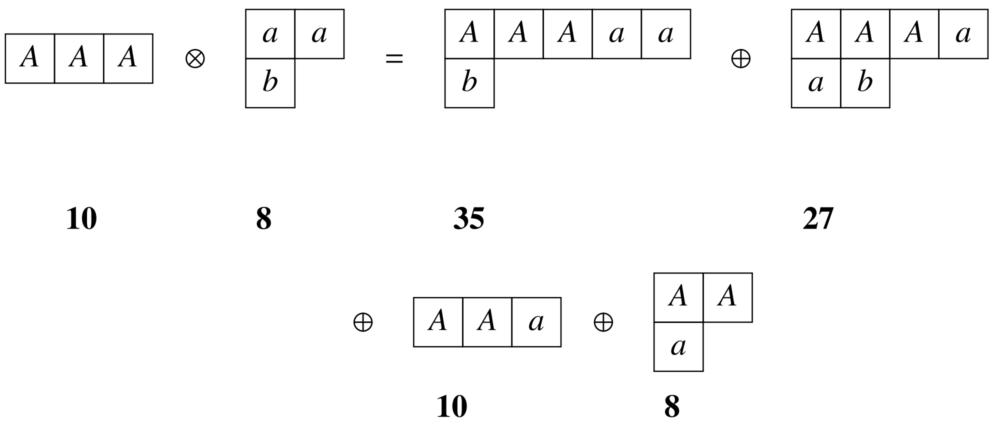
- Two tableaux with the same shape but labels distributed differently have to be kept. If two tableaux are identical only one has to be accounted for.

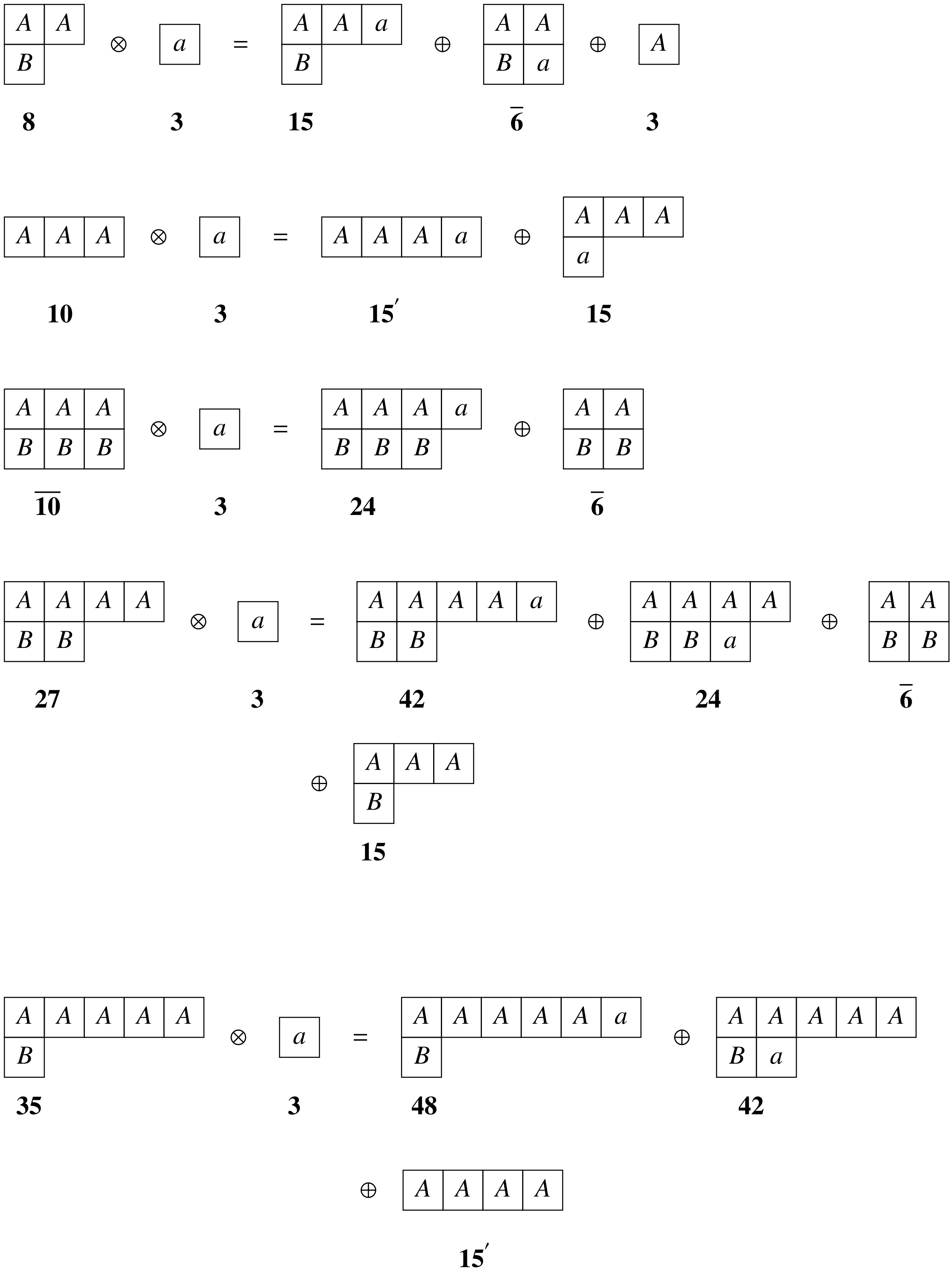
- While counting the labels from the first row from right to left, then the second row again from right to left, at any given box position the number of b cannot be greater than the number of a. For example a sequence (to be read from left to right) *aba* or *aab* is allowed but *baa* is not.

- In general, we will be able to form multiple tableaux by following the rules above. Each such solution will represent an $SU(3)$ IR in the tensor product.

Below are some of the results of tensor product of $SU(3)$ IR using the above mentioned rules that are useful for the next chapter;







This procedure of decomposing the direct product of two $SU(3)$ IR's into a direct sum of IRs is called an expansion in the Clebsch-Gordan series [54–57] for $SU(3)$. Here, $|\mathbf{r}_1, \alpha_1\rangle$, $|\mathbf{r}_2, \alpha_2\rangle$ and $|\mathbf{R}_n^{(m)}, A_n\rangle$ are the basis states for the representation $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{R}_n^{(m)}$ respectively. We can now define the Clebsch-Gordan coefficients [21,22] $G(\mathbf{R}_n^{(m)},A_n,\mathbf{r}_1,\alpha_1,\mathbf{r}_2,\alpha_2)$,

$$|\mathbf{r}_1,\alpha_1\rangle\otimes|\mathbf{r}_2,\alpha_2\rangle = \sum \underbrace{\langle \mathbf{R}_n^{(m)},A_n|\mathbf{r}_1,\alpha_1,\mathbf{r}_2,\alpha_2\rangle}_{G(\mathbf{R}_n^{(m)},A_n,\mathbf{r}_1,\alpha_1,\mathbf{r}_2,\alpha_2)} |\mathbf{R}_n^{(m)},A_n\rangle \tag{3.31}$$

that are nothing but the amplitudes for the projection of the product of two irreducible representations $\mathbf{r}_1$, $\mathbf{r}_2$ of $SU(3)$ onto the irreducible representations $\mathbf{R}_n$ found in the Clebsch-Gordan series in Eq (3.23),

$$|\mathbf{r}_1,\alpha_1\rangle\otimes|\mathbf{r}_2,\alpha_2\rangle = \sum_{\substack{i\\ \{m\}}} \oplus \mathbf{R}_n^{(m)} \tag{3.32}$$

Here, A_n, α_1, α_2 collectively denote the hypercharge (Y), isospin (I) and third component of isospin (I_z) for the representations $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{R}_n^{(m)}$ respectively. The index m in $\mathbf{R}_n^{(m)}$ denotes the degeneracy of a particular representation in the tensor product of representations. To uniquely define these CG coefficients, we have to define the relative phase of the basis vectors of the IR $\mathbf{R}_n^m$ with respect to the basis vectors in the product representation $\mathbf{r}_1\otimes\mathbf{r}_2$. In that regard,

- The internal phase convention, fixing the relative phases between states within a particular representation completely is achieved by adopting the Condon-Shortley phase convention [58] for the two subgroups of $SU(3)$, namely the isospin (I) and $V-$spin (in $SU(3)$-flavor the $V-$ spin operators interchange u and s quarks) operators. This implies that eigenvalues of the isospin- raising and lowering operators as well as the analogous raising and lowering operators of V-spin are real and positive. In this convention, all $SU(2)$ CG coefficients are real and they satisfy the property,

$$\langle I\,I_z|i_1\,i_{1z}\,i_2\,i_{2z}\rangle = (-1)^{I-i_1-i_2}\langle I\,I_z|i_2\,i_{2z}\,i_1\,i_{1z}\rangle. \tag{3.33}$$

- The overall phases of representations in the decomposition of the product of two irreducible representations is chosen in accordance with the convention of de Swart.

According to the convention, a state in $\mathbf{R}_n^{(m)}$ having the highest I_z that couples to the state in the first factor representation ($\mathbf{r}_1$) having the highest isospin as well as the the state in the second factor representation ($\mathbf{r}_2$) with the highest isospin should have a Clebsch-Gordon coefficient that is real and positive;

$$\langle \mathbf{R}, Y^h, I^h I_z^h | \mathbf{r}_1, y_1^h, \mathbf{r}_2, y_2^h, i_1^h i_{1z}^h i_2^h i_{2z}^h \rangle > 0. \tag{3.34}$$

These phase conventions ensure that the $SU(3)$ CG coefficients are all real [21, 22]. The $SU(3)$ Clebsch-Gordan coefficients can be factored into products of $SU(3)$ isoscalar factors and $SU(2)$ Clebsch-Gordan coefficients:

$$\langle \mathbf{R}, Y, I, I_z | \mathbf{r}, y, i, i_z, \mathbf{r}', y', i', i_z' \rangle = \begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y,i) & (y',i') & (Y,I) \end{pmatrix} \langle I, I_z | i, i_z, i', i_z' \rangle \tag{3.35}$$

where $\langle I, I_z | i, i_z, i', i_z' \rangle$ are the $SU(2)$ CG coefficients taking into account the Condon-Shortley convention and

$$F(\mathbf{R}, Y, I, \mathbf{r}, y, i, \mathbf{r}', y', i') = \begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y,i) & (y',i') & (Y,I) \end{pmatrix}. \tag{3.36}$$

$F(\mathbf{R}, Y, I, \mathbf{r}, y, i, \mathbf{r}', y', i')$ are the $SU(3)$ isoscalar factors. The order in which the $SU(3)$ representations are coupled is $\mathbf{r} \otimes \mathbf{r}' \to \mathbf{R}$. These isoscalar factors are all real in our chosen phase convention. Moreover, there are two symmetry relations involving the $SU(3)$ isoscalar factors;

A) If the order in which the representations are coupled is reversed (i.e. $\mathbf{r}' \otimes \mathbf{r} \to \mathbf{R}$) then the isoscalar factors pick up a phase factor;

$$\begin{pmatrix} \mathbf{r}' & \mathbf{r} & \mathbf{R} \\ (y',i') & (y,i) & (Y,I) \end{pmatrix} = (-1)^{I-i-i'} \xi(\mathbf{R}; \mathbf{r}, \mathbf{r}') \begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y,i) & (y',i') & (Y,I) \end{pmatrix}. \tag{3.37}$$

Here $\xi(\mathbf{R}; \mathbf{r}, \mathbf{r}')$ is the phase factor [22] that depends only on the identity element of $\mathbf{r}$, $\mathbf{r}'$

and **R** and the phase convention chosen above.

B) Conjugation operation on all three representations also give rise to a phase factor;

$$\begin{pmatrix} \bar{\mathbf{r}} & \overline{\mathbf{r}'} & \overline{\mathbf{R}} \\ (y,i) & (y',i') & (Y,I) \end{pmatrix} = (-1)^{I-i-i'}\zeta(\mathbf{R};\mathbf{r},\mathbf{r}')\begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (-y,i) & (-y',i') & (-Y,I) \end{pmatrix}. \tag{3.38}$$

Similar to the previous case, $\zeta(\mathbf{R};\mathbf{r},\mathbf{r}')$ is the phase factor [22] that depends only on the identity element of $\mathbf{r}$, $\mathbf{r}'$ and $\mathbf{R}$ and our chosen phase convention. As a corollary of Eqs. (3.37) and (3.38),

$$\xi(\overline{\mathbf{R}};\bar{\mathbf{r}},\overline{\mathbf{r}'}) = \xi(\mathbf{R};\mathbf{r},\mathbf{r}'), \tag{3.39}$$

$$\zeta(\mathbf{R};\mathbf{r}',\mathbf{r}) = \zeta(\mathbf{R};\mathbf{r},\mathbf{r}') \tag{3.40}$$

ζ and ξ can take values ± 1 in our phase convention. They are independent of the third component of isospin which is why we can choose the highest weight isospin state in the product representation that fixes ζ and ξ,

$$\begin{aligned} \zeta(\mathbf{R};\mathbf{r},\mathbf{r}') &= (-1)^{I_h-i_h-i_h'} \\ \xi(\mathbf{R};\mathbf{r},\mathbf{r}') &= (-1)^{I_h-i_h-i_h'}\frac{F(\mathbf{R},Y,I,\mathbf{r},y,i,\mathbf{r}',y',i')}{|F(\mathbf{R},Y,I,\mathbf{r},y,i,\mathbf{r}',y',i')|} \end{aligned} \tag{3.41}$$

Another important result [21, 59] involving the isoscalar factors is quoted below,

$$\begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y,i) & (y',i') & (Y,I) \end{pmatrix} = (-1)^{i+y/2}\upsilon(\mathbf{R};\mathbf{r},\mathbf{r}')\left(\frac{(2i'+1)\dim(\mathbf{R})}{(2I+1)\dim(\mathbf{r}')}\right)^{1/2}\begin{pmatrix} \mathbf{r} & \overline{\mathbf{R}} & \overline{\mathbf{r}'} \\ (y,i) & (-Y,I) & (-y',i') \end{pmatrix} \tag{3.42}$$

where $\upsilon(\mathbf{R};\mathbf{r},\mathbf{r}') = \pm 1$ and can be fixed in a similar method as ξ and ζ using the highest weight state in the product representation. The isoscalar factors relevant for this thesis are all listed in [21, 22]. The details of the derivation of those isoscalar factors are beyond the scope of this thesis and we henceforth will just use those values in the next chapter.

3.2 Wigner-Eckart theorem

The Wigner-Eckart theorem concerns the matrix element of an irreducible $SU(3)$ tensor operator between two basis states of $SU(3)$ irreducible representations. We have already noted that the irreducible representations act on basis states that are classified by the hypercharge (Y), isospin (I) and third component of isospin (I_z). Let us assume two basis states to be labeled by $|\mathbf{R}_1, Y_1, I_1, I_{1z}\rangle$ and $|\mathbf{R}_2, Y_2, I_2, I_{2z}\rangle$ respectively where $\mathbf{R}_1$ and $\mathbf{R}_2$ are two IRs of $SU(3)$. An irreducible tensor operator $T(\mathbf{R}, Y, I, I_z)$ acts on the basis state $|\mathbf{R}_1, Y_1, I_1, I_{1z}\rangle$ transforming as a direct product of two IRs $|\mathbf{R}, Y, I, I_z\rangle \otimes |\mathbf{R}_1, Y_1, I_1, I_{1z}\rangle$. The Wigner-Eckart theorem states [21],

$$\langle \mathbf{R}_2, Y_2, I_2, I_{2z}|T^{\mathbf{R}}_{Y,I,I_z}|\mathbf{R}_1, Y_1, I_1, I_{1z}\rangle = \langle \mathbf{R}_2||T^{\mathbf{R}}||\mathbf{R}_1\rangle \langle \mathbf{R}_2, Y_2, I_2, I_{2z}|\mathbf{R}, \mathbf{R}_1; Y, I, I_z, Y_1, I_1, I_{1z}\rangle \tag{3.43}$$

where $\langle \mathbf{R}_2||T^R||\mathbf{R}_1\rangle$ is a number, called the $SU(3)$-reduced matrix element that is independent of the hypercharge (Y), isospin (I) and third component of isospin (I_z) values of the basis states. The $SU(3)$ CG coefficient $\langle \mathbf{R}_2, Y_2, I_2, I_{2z}|\mathbf{R}, \mathbf{R}_1; Y, I, I_z, Y_1, I_1, I_{1z}\rangle$ is non-zero if and only if $\mathbf{R}_2$ appears in the tensor product $\mathbf{R} \otimes \mathbf{R}_1$. Since the left hand side of Eq. (3.43) transforms as a singlet of $SU(3)$ or in other words an $SU(3)$-invariant, it is the conjugate of $\mathbf{R}_2$ that forms a singlet with the $\mathbf{R} \otimes \mathbf{R}_1$. The $SU(3)$-reduced matrix elements are basis independent and are invariant under $SU(3)$ by themselves.

The initial and final states and the effective Hamiltonian in this thesis transform as irreducible representations of $SU(3)$-flavor. The Wigner-Eckart theorem implies the matrix elements for a transition from an initial state to final state mediated by the Hamiltonian depend only a few reduced matrix elements which we will enumerate in next chapter. These reduced matrix elements serve as a basis using which a group theory decomposition of all possible decay amplitudes of different processes can be achieved.

Chapter 4

$SU(3)$-flavor analysis of bottom baryon decaying to an octet baryon and an octet meson

The contents of this chapter is based on a joint work [13] with Rahul Sinha and N.G. Deshpande.

4.1 Prologue

The bottom quark is the only third generation quark that hadronizes and forms b-hadrons that further decays to lighter hadrons and leptons. The theoretical formalism developed to study heavy meson exclusive non-leptonic decays containing a $b(\overline{b})$-quark has proved to be remarkably successful when it comes to reliably predicting a large number of branching ratios and other decay parameters. Experimentally, as well, these predictions have been tested extensively and found to be in good agreement with most of the theory estimates. The combined effort of the theory and experimental collaborations have led to the observation and then measurement of direct and indirect CP-violation in B-meson in the last few decades. In the SM, we recall that the only source of CP-violation in electroweak quark flavor changing process is the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix is written as,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{4.1}$$

where every matrix element V_{ij} correspond to the magnitude and phase of the quark flavor changing process $i \to j$. B-meson non-leptonic decays typically involve underlying quark level processes such as $b \to u$ or $b \to c$ that are sensitive to the weak phases V_{ub} and V_{cb} respectively. A necessary condition for observable CP violating effect is two non-negligible amplitudes each having a different weak phase as well as strong phase contributing to the same process. For example, consider the amplitude for a process $i \to f$,

$$\mathcal{A}(i \to f) = e^{i\phi_1} e^{i\alpha_1} |a_1| + e^{i\phi_2} e^{i\alpha_2} |a_2| \tag{4.2}$$

The amplitude of the CP-conjugate process $\bar{i} \to \bar{f}$ is given by,

$$\mathcal{A}(\bar{i} \to \bar{f}) = e^{i\phi_1} e^{-i\alpha_1} |a_1| + e^{i\phi_2} e^{-i\alpha_2} |a_2| \tag{4.3}$$

where the two weak phases α_1, α_2 switch sign but the strong phases ϕ_1, ϕ_2 do not. The direct CP-asymmetry is given by,

$$a_{CP} = \frac{|\mathcal{A}(i \to f)|^2 - |\mathcal{A}(\bar{i} \to \bar{f})|^2}{|\mathcal{A}(i \to f)|^2 + |\mathcal{A}(\bar{i} \to \bar{f})|^2} = \frac{-2|a_1||a_2| \sin(\phi_1 - \phi_2) \sin(\alpha_1 - \alpha_2)}{|a_1|^2 + |a_2|^2 + 2\cos(\phi_1 - \phi_2) \sin(\alpha_1 - \alpha_2)}. \tag{4.4}$$

A non-vanishing CP-asymmetry demands that both the strong phase difference and weak phase difference are non-zero. Since the same $b \to u$ or $b \to c$ quark level transition and therefore weak phases mediate b-baryon decays we can hope to observe CP-violation in bottom baryon decays as well. Bottom baryons are three quark bound states with one of the quarks being the b-quark in contrast to a B-meson which is made of a quark-antiquark bound state. In addition, the b-baryons also carry a spin providing a unique opportunity to analyze its weak decay in the SM. A quantitative estimate of such non-leptonic weak decays involve using perturbative QCD (pQCD) as one of schemes of factorization [40]. While such factorizable terms, in most cases, have the dominant contribution on B-meson decay matrix elements, the same cannot be said for two-body hadronic weak decays of b-baryons. It was pointed out [2] that non-factorizable effects dominate over factorizable contribution in Λ_b^0 decays such as $\Lambda_b \to pK$ and $\Lambda_b \to p\pi$. In another work, the two light quarks inside b-baryon are approximated to form a scalar diquark. In this picture, the hadronic weak decay of b-baryon is effectively treated in QCD factorization [42] scheme, which is known to work well in B-meson decaying to two mesons. Predictions based on such calculations [43] are very important and more effort is needed in this regard.

Alternatively, we can decompose the decay amplitudes of a class of processes into a likely smaller set of invariant amplitudes classified according to their transformation properties under $SU(3)$-flavor symmetry. This approach is closely related to the diagrammatic approach where quark flavor-flow topologies serve as the invariant amplitudes and certain dynamical assumptions are invoked to include or neglect contributions from particular diagrams. It is quite straightforward to derive several model independent amplitude relations in these approaches that can be tested in experiments. Ultimately these invariant amplitudes are treated as parameters that can be determined from an overcomplete set of measurements.

A large b-baryon production fraction at LHCb encourages us to adopt the data-driven approach mentioned above. In particular, LHCb is expected to collect a substantial data set of two-body weak decays of beauty-baryons [11,38,39] into charmless baryons and pseudoscalar mesons. Previously, such progress in the theoretical understanding of beauty meson decays [16–18, 20, 27, 42, 60–94] came hand-in-hand through experimental advances at flavor factories Belle and Babar [95, 96] as well as in LHCb [38, 39, 97–100]. The general framework of $SU(3)$ analysis in beauty mesons as well as charm meson decays [101–112] into two pseudoscalars (PP), pseudoscalar-vector boson (PV), and two vector mesons (VV) successfully predicted several amplitude sum-rules and relationships between CP asymmetries for various decay modes. While attempts have been made, a comprehensive $SU(3)$-flavor analysis of hadronic beauty-baryons decaying into an octet or singlet of light baryons and a pseudoscalar meson is so far missing in the literature. In contrast to the methodology employed in [2, 3, 25, 26, 43, 113–130] for bottom and charmed hadron decays, our approach [18] facilitates an $SU(3)$ decomposition of the decays in terms of $SU(3)$-reduced amplitudes without any particular set of assumptions about the underlying dynamics.

The number of independent $SU(3)$-reduced amplitudes for any given initial and final state is exactly calculable and relations between decay amplitudes emerge naturally once the set of independent $SU(3)$-reduced amplitudes is smaller than the total number of possible decays. The counting of independent $SU(3)$ reduced amplitudes draws on the choice of the effective Hamiltonian, which in the most general case, indicate 44 independent reduced $SU(3)$ amplitudes equaling the number of all possible $\Delta S = -1$ and $\Delta S = 0$ processes. In practice, the dimension-6 effective Hamiltonian that mediates such hadronic decays of bottom baryons allow only ten independent reduced $SU(3)$ amplitudes. We therefore obtain amplitude relations between the decay modes and derive them explicitly. Moreover, our methodology naturally allows for a systematic study of the $SU(3)$-breaking effects at the level of decay amplitudes, order by order expanded in the $SU(3)$ breaking parameter. Since the number of independent $SU(3)$-reduced amplitudes increases, some of the amplitude relations derived earlier may no longer hold which we subsequently identify. Starting with the symmetries of the effective Hamiltonian, we relate or neglect reduced $SU(3)$ amplitudes to derive several sum rules relations between amplitudes while indicating more general relations that continue to hold when the $SU(3)$ symmetry is no longer exact. Some of these amplitude relations can be turned into CP-asymmetry relations that we also explore. This study is crucial for a detailed analysis of the CP asymmetry measurements in bottom baryons decays at the CDF and LHCb in recent times [4–9, 99, 100].

The approach to decompose the decay amplitudes in terms of reduced $SU(3)$ amplitudes is presented in Sec. 4.2. The results are summarized in Appendix. A.0.0.1-Appendix A.0.0.6. In Sec. 4.3 we perform the $SU(3)$ decomposition of unbroken effective hadronic weak decay Hamiltonian. The relations between the amplitudes for beauty baryon decays into octets of light baryons and pseudoscalar mesons are derived in Sec. 4.5. The effects of $SU(3)$ breaking on account of s-quark mass are considered in Sec. 4.5.1. The corresponding relations between CP asymmetries are derived in Sec. 4.6. We finally conclude in Sec. 4.7.

4.2 Application of $SU(3)$ to decay amplitudes

The $SU(3)$ decomposition of physical amplitudes describing a decay process involves writing it in terms of reduced matrix elements of explicit $SU(3)$ operators with appropriate coefficients. The procedure is a straightforward application of Wigner-Eckart theorem for the group $SU(3)$ where the reduced matrix elements are all possible $SU(3)$ invariants with Clebsch-Gordon (CG) coefficients connecting the basis involving physical hadronic $SU(3)$ states to the group theoretic basis.

The most general Hamiltonian $\mathcal{H}$ which connects [18] the initial and final states via the matrix elements $\langle f|\mathcal{H}|i\rangle$, consists of exactly those representations $\mathbf{R}$ appearing in $\mathbf{f}\otimes\bar{\mathbf{i}}$, where the labels i and f denote both physical states and $SU(3)$ representations. It is important to note that in addition to the usual $SU(3)$ CG coefficients that arise from coupling $\mathbf{f}\otimes\bar{\mathbf{i}}$, the most general effective Hamiltonian ($\mathcal{H}$) itself involves unknown coefficients appearing in front of every $SU(3)$ representation. *A priori*, these coefficients are all independent of each other which get determined once a particular form of effective Hamiltonian is assumed. The states of $SU(3)$ representations are uniquely distinguished when in addition to the I_3 and Y values, the isospin Casimir I^2 is also specified. We note that this is not in contradiction with the Wigner-Eckart theorem since the isospin label in the reduced $SU(3)$ amplitudes are there to merely indicate our ignorance about coefficients that may turn out to be unequal for different components of a given representation of the effective Hamiltonian. The full reduced $SU(3)$ amplitude is thus described by $\langle f|\mathbf{R}_I|i\rangle$. The expression of the amplitudes in terms of reduced $SU(3)$ amplitudes is concisely given as,

$$\mathcal{A}(i \to f_b f_m) = (-1)^{I_3-\frac{Y}{2}-\frac{T}{3}} \sum_{\substack{\{f,R\} \\ Y^b+Y^m=Y^f,\, Y^f-Y^i=Y^H \\ I_3^b+I_3^m=I_3^f,\, I_3^f-I_3^i=I_3^H}} C^{I_3^b\, I_3^m\, I_3^f}_{I^b\, I^m\, I^f} \begin{pmatrix} \mathbf{f_b} & \mathbf{f_m} & \mathbf{f} \\ (Y^b,I^b) & (Y^m,I^m) & (Y^f,I^f) \end{pmatrix} \begin{pmatrix} \mathbf{f} & \bar{\mathbf{i}} & \mathbf{R} \\ (Y^f,I^f) & (-Y^i,I^i) & (Y^H,I^H) \end{pmatrix} C^{I_3^f\, -I_3^i\, I_3^H}_{I^f\, I^i\, I^H} \langle \mathbf{f} \,||\, \mathbf{R_I} \,||\, \mathbf{i} \rangle, \tag{4.5}$$

where, $C^{a,b,c}_{A,B,C}$ are the $SU(2)$ Clebsch-Gordon coefficients and

$$\begin{pmatrix} \mathbf{R_a} & \mathbf{R_b} & \mathbf{R_c} \\ (Y^a,I^a) & (Y^b,I^b) & (Y^c,I^c) \end{pmatrix}. \tag{4.6}$$

are the $SU(3)$ isoscalar coefficients obtained by coupling the representations $\mathbf{R_a} \otimes \mathbf{R_b} \to \mathbf{R_c}$. The initial b-baryon ($\mathbf{i}$) belongs to a $\bar{\mathbf{3}}$ of $SU(3)$ and given as

$$\mathcal{B}_b = \begin{pmatrix} \Xi_b^- & -\Xi_b^0 & \Lambda_b^0 \end{pmatrix}. \tag{4.7}$$

The conjugate of the initial state is used in $SU(3)$-decomposition of decay amplitude, the factor $(-1)^{I_3-Y/2-T/3}$ is put that takes care of the correct phase factor appearing in front of $\overline{\Xi_b^-}$, $\overline{\Xi_b^0}$, $\overline{\Lambda_b^0}$. The triality (T) of an $SU(3)$ representation with m and n fundamental and anti-fundamental indices, i.e. an (m,n) of $SU(3)$ is given by $T(m,n) = (m-n)\,\mathbf{mod}\,3$. To summarize, $T = 1$ and I_3, Y of $\bar{\mathbf{i}}$ are assumed in Eq. (4.5). The symmetry properties of the $SU(3)$ isoscalar factor and its role in obtaining the $SU(3)$ CG coefficients [21, 22, 54, 55, 58, 131] is outlined in Chapter 3. The amplitude is written with specific attention to the order in which the representations are coupled, the final state representations are coupled via $\mathbf{f}_b \otimes \mathbf{f}_m \to \mathbf{f}$, where the product (f) is then coupled through the conjugate of the initial representation or equivalently $\mathbf{f} \otimes \bar{\mathbf{i}} \to \mathcal{H}$. This ensures that all possible $SU(3)$ representations are generated in case of the most general effective Hamiltonian.

Given a form of effective Hamiltonian ($\mathcal{H}_{\text{eff}}$), it can be $SU(3)$ decomposed,

$$\mathcal{H}_{\text{eff}} = \sum_{\substack{\{Y,I,I_3\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \mathbf{R}_{\mathbf{I}}, \tag{4.8}$$

where $\mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}}$ depends on the $SU(3)$ CG coefficients appearing in front of the $SU(3)$ representations $\mathbf{R}_{\mathbf{I}}$. Moreover $\mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}}$ also contains additional factors entering Eq. (4.8) in form of Wilson coefficients and CKM elements. We hereby note that by knowing the dynamical coefficients for different isospin values in a given $SU(3)$ representation, one can now drop the isospin Casimir label (I) and express the Wigner-Eckart reduced matrix element $\langle \mathbf{f} \parallel \mathbf{R} \parallel \mathbf{i} \rangle$, in its usual form, independent of the isospin I label. By using completeness of $SU(3)$ CG coefficients up to a phase factor,

$$\langle \mathbf{f} \parallel \mathbf{R}_{\mathbf{I}} \parallel \mathbf{i} \rangle = \underbrace{\mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \sqrt{\frac{\text{dim f}}{\text{dim R}}}}_{\text{dynamical Coeff. of } \mathcal{H}} \langle \mathbf{f} \parallel \mathbf{R} \parallel \mathbf{i} \rangle. \tag{4.9}$$

The origin of this formula lies in Eq (3.42). Alternatively, we can directly start with the given form of effective Hamiltonian in Eq. (4.8) and perform an $SU(3)$ decomposition of the decay amplitude;

$$\mathcal{A}(i \to f_b f_m) = \sum_{\substack{\{Y^H, I^H, I_3^H\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{Y,I,I_3\}} \sum_{\substack{\{f\} \\ Y^b+Y^m=Y^f,\, Y^f=Y^i+Y^H \\ I_3^b+I_3^m=I_3^f,\, I_3^f=I_3^i+I_3^H}} C^{I_3^b\, I_3^m\, I_3^f}_{I^b\, I^m\, I^f} \begin{pmatrix} \mathbf{f_b} & \mathbf{f_m} & \mathbf{f} \\ (Y^b, I^b) & (Y^m, I^m) & (Y^f, I^f) \end{pmatrix}$$
$$\begin{pmatrix} \mathbf{R} & \mathbf{i} & \mathbf{f} \\ (Y^H, I^H) & (Y^i, I^i) & (Y^f, I^f) \end{pmatrix} C^{I_3^H\, I_3^i\, I_3^f}_{I^H\, I^i\, I^f} \langle \mathbf{f} \,||\, \mathbf{R} \,||\, \mathbf{i} \rangle. \quad (4.10)$$

The case of our interest, namely, $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{B}(\mathbf{8})\,\mathcal{M}(\mathbf{8})$, where $\mathcal{B}_b$, the initial anti-triplet ($\overline{\mathbf{3}}$) beauty-baryon undergoes a charmless decay into an octet baryon ($\mathcal{B}$) and an octet pseudoscalar meson ($\mathcal{M}$), is described by a Hamiltonian with $\Delta Q = 0$ and $\Delta S = -1, 0$ (equivalently given in ΔI_3 and ΔY representation). The expressions for Q and S in terms of Y, I_3 and T is given,

$$Q = I_3 + \frac{Y}{2} + q_b,$$
$$S = Y - \frac{1}{3} T, \quad (4.11)$$

where q_b is the electric charge of the bottom quark. The possible decays can be divided into two sub classes, namely the $\Delta S = 0$ and $\Delta S = -1$ transitions. The allowed final state $SU(3)$ representations (**f**) are; **1**, $\mathbf{8}_1$, $\mathbf{8}_2$, **10**, $\overline{\mathbf{10}}$, **27**. It is also self evident that to form singlets of $SU(3)$ the tensor product of effective Hamiltonian and initial states should contain $SU(3)$ representations that transforms as **1**, **8**, **10**, $\overline{\mathbf{10}}$, **27**. There are 22 physical process possible for $\Delta S = -1$ and another 22 for $\Delta S = 0$. In Appendix A.0.0.1 and Appendix A.0.0.4 respectively each of these decay modes are decomposed in terms of the $SU(3)$ reduced amplitudes that add upto 44. Since the physical η and η' mesons are admixtures of octet η_8 and singlet η_1 mesons, a study of $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{B}(\mathbf{8})\,\mathcal{M}(\mathbf{1})$ is also necessary. Therefore one has to take into account 8 (4 each for $\Delta S = 0$ and $\Delta S = -1$) additional independent $SU(3)$ amplitudes which are also described in Appendix A.0.0.7

and Appendix A.0.0.8.

We emphasize that this way of counting accounts for a complete set of reduced amplitudes [18], regardless of the specific form of interaction Hamiltonian. In particular, this decomposition holds even if the $SU(3)$ symmetry is arbitrarily broken and there is no physical reason to organize particles in $SU(3)$ multiplets. At this point, every process is independent and to find relations among them requires assuming a specific form of the interaction Hamiltonian.

4.3 $SU(3)$ decomposition of unbroken effective Hamiltonian

The lowest order effective Hamiltonian [19, 132, 133] for charmless b-baryon decays consists $\Delta S = -1$ and $\Delta S = 0$ parts. Each part is composed from the operators $Q_1, \ldots, Q_{10}$. The complete Hamiltonian can be written as:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}}\Big[\lambda_u^{(s)}\Big(C_1(Q_1^{(u,s)} - Q_1^{(c,s)}) + C_2(Q_2^{(u,s)} - Q_2^{(c,s)})\Big) - \lambda_t^{(s)} \sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(s)} \sum_{i=3}^{10} C_i Q_i^{(s)}$$
$$+\lambda_u^{(d)}\Big(C_1(Q_1^{(u,d)} - Q_1^{(c,d)}) + C_2(Q_2^{(u,d)} - Q_2^{(c,d)})\Big) - \lambda_t^{(d)} \sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(d)} \sum_{i=3}^{10} C_i Q_i^{(d)}\Big], \tag{4.12}$$

where $V_{ub}V_{us}^* = \lambda_u^s$, $V_{ub}V_{ud}^* = \lambda_u^d$, $V_{tb}V_{ts}^* = \lambda_t^s$, $V_{tb}V_{td}^* = \lambda_t^d$ are the CKM elements and C_i s are the Wilson coefficients evaluated at an energy scale of the order of bottom quark mass.

Q_1 and Q_2 are the "Tree" operators:

$$\begin{aligned}
Q_1^{(u,s)} &= (\overline{u}^i_L \gamma^\mu b^j_L)(\overline{s}^j_L \gamma_\mu u^i_L), \quad Q_1^{(u,d)} = (\overline{u}^i_L \gamma^\mu b^j_L)(\overline{d}^j_L \gamma_\mu u^i_L), \\
Q_1^{(c,s)} &= (\overline{c}^i_L \gamma^\mu b^j_L)(\overline{s}^j_L \gamma_\mu c^i_L), \quad Q_1^{(c,d)} = (\overline{c}^i_L \gamma^\mu b^j_L)(\overline{d}^j_L \gamma_\mu c^i_L), \\
Q_2^{(u,s)} &= (\overline{u}^i_L \gamma^\mu b^i_L)(\overline{s}^j_L \gamma_\mu u^j_L), \quad Q_2^{(u,d)} = (\overline{u}^i_L \gamma^\mu b^i_L), (\overline{s}^j_L \gamma_\mu u^j_L) \\
Q_2^{(c,s)} &= (\overline{c}^i_L \gamma^\mu b^i_L)(\overline{s}^j_L \gamma_\mu c^j_L), \quad Q_2^{(c,d)} = (\overline{c}^i_L \gamma^\mu b^i_L)(\overline{d}^j_L \gamma_\mu c^j_L).
\end{aligned} \tag{4.13}$$

We have replaced the s-quark with a d-quark in second line of Eq (4.12) for the tree operators. Clearly the $\Delta S = -1$ and $\Delta S = 0$ processes are mediated by the first line and second line of Eq (4.12). $Q_3, \ldots, Q_6$ are the "Gluonic Penguin" operators:

$$\begin{aligned}
Q_3^{(s)} &= (\overline{s}^i_L \gamma^\mu b^i_L) \sum_{q=u,d,s} (\overline{q}^j_L \gamma_\mu q^j_L) \\
Q_4^{(s)} &= (\overline{s}^i_L \gamma^\mu b^j_L) \sum_{q=u,d,s} (\overline{q}^j_L \gamma_\mu q^i_L) \\
Q_5^{(s)} &= (\overline{s}^i_L \gamma^\mu b^i_L) \sum_{q=u,d,s} (\overline{q}^j_R \gamma_\mu q^j_R) \\
Q_6^{(s)} &= (\overline{s}^i_L \gamma^\mu b^j_L) \sum_{q=u,d,s} (\overline{q}^j_R \gamma_\mu q^i_R).
\end{aligned} \tag{4.14}$$

Out of the four "EWP" (i.e. "Electroweak Penguins") Operators: $Q_7, \ldots, Q_{10}$, Q_7 and Q_8:

$$\begin{aligned}
Q_7^{(s)} &= \frac{3}{2}(\overline{s}^i_L \gamma^\mu b^i_L) \sum_{q=u,d,s} e_q(\overline{q}^j_R \gamma_\mu q^j_R), \\
Q_8^{(s)} &= \frac{3}{2}(\overline{s}^i_L \gamma^\mu b^j_L) \sum_{q=u,d,s} e_q(\overline{q}^j_R \gamma_\mu q^i_R),
\end{aligned} \tag{4.15}$$

are typically ignored in hadronic decays because of the smallness of C_7 and C_8 with respect to the other Wilson Coefficients.

The remaining "EWP" operators are:

$$
\begin{aligned}
Q_9^{(s)} &= \frac{3}{2}(\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^j), \\
Q_{10}^{(s)} &= \frac{3}{2}(\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^i).
\end{aligned}
\tag{4.16}
$$

Once again we have replaced the s-quark with a d-quark in the second line of Eq (4.12) for all the penguin operators. We note that the tree operators $Q_1^{(c)}$ and $Q_2^{(c)}$ can give rise to "charming penguin" like contributions [68–72] which basically mean penguin diagrams with charm (c) quark going in the loop. Similar to B-meson charmless decays, the insertion of $Q_1^{(c)}$ and $Q_2^{(c)}$ tree operators in penguin diagrams cannot be neglected in comparison to tree diagrams where $Q_1^{(u)}$ and $Q_2^{(u)}$ contributes. Obviously the usual QCD penguin operators $Q_3 - Q_6$ as well as the electroweak penguin operators contribute to the penguin diagrams having an internal top quark. We have already used the unitarity of the CKM matrix and the relations

$$
\lambda_u^{(s)} + \lambda_c^{(s)} + \lambda_t^{(s)} = 0, \quad \lambda_u^{(d)} + \lambda_c^{(d)} + \lambda_t^{(d)} = 0 \tag{4.17}
$$

to express the first line of Eq (4.12) with $V_{ub}V_{us}^*$ and $V_{tb}V_{ts}^*$ appearing in front of tree and penguin operators respectively. Similarly in the second line of Eq (4.12) the CKM elements appearing in front of the tree and penguin operators are $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$. We will now focus on the $SU(3)$ properties of each of these dimension-6 operators.

$\mathcal{H}_{\text{eff}}$ is a linear combinations of four quark operators of the form $(\bar{q}_1 b)(\overline{q_2} q_3)$. These operators transform as $\mathbf{3} \otimes \mathbf{3} \otimes \overline{\mathbf{3}}$ under $SU(3)$-flavor and can be decomposed into sums of irreducible operators corresponding to irreducible $SU(3)$ representations: $\mathbf{15}, \overline{\mathbf{6}}, \mathbf{3}^{(\mathbf{6})}, \mathbf{3}^{(\overline{\mathbf{3}})}$ where the superscript index: '6' ('$\overline{3}$') indicates the origin of $\mathbf{3}$ out of the two possible representations arising from the tensor product of q_1 and q_2. Through out this thesis, the chosen convention for the $SU(3)$ triplet representation of quarks (q_i) and its conjugate denoting the anti-quarks ($\overline{q_i}$) consist of the flavor states;

$$q_i = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \overline{q_i} = \begin{pmatrix} \overline{d} \\ -\overline{u} \\ \overline{s} \end{pmatrix} \tag{4.18}$$

According to the sign convention chosen in Eq. (4.18), the meson wavefunctions are given as,

$$\begin{gathered} K^+ = u\overline{s}, \qquad K^- = -s\overline{u}, \qquad K^0 = d\overline{s}, \qquad \overline{K}^0 = s\overline{d} \\ \pi^+ = u\overline{d}, \qquad \pi^- = -d\overline{u}, \qquad \pi^0 = \frac{1}{\sqrt{2}}(d\overline{d} - u\overline{u}) \\ \eta_8 = -\frac{1}{2\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s}) \quad \eta_1 = -\frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}) \end{gathered}$$

The physical mesons $\eta, \eta^{'}$ are related to the η_8 and η_1 through the $SO(2)$ rotation,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} \tag{4.19}$$

where the definition of θ is consistent with the overall notation for the meson wavefunctions as well as agreeing with the phenomenologically determined value of θ. In the following table the four quark operators, which appear in $\mathcal{H}_{\text{eff}}$, are decomposed using $SU(3)$ Clebsch-Gordan tables. It is worthwhile to note that in the Hamiltonian operators appear as $\overline{q}_1\,\overline{q_2}\,q_3$ whereas in Table 4.1 they are expressed conveniently as $q_1\,q_2\overline{q}_3$. With the help of Table 4.1, the effective Hamiltonian can be expressed in terms of operators having definite $SU(3)$ transformation properties. The tree part consisting of Q_1 and Q_2 is $SU(3)$ decomposed below [77],

	$15_{I=1}$	$15_{I=0}$	$\overline{6}_{I=1}$	$3^{(6)}_{I=0}$	$3^{(\overline{3})}_{I=0}$	$15_{I=3/2}$	$15_{I=1/2}$	$\overline{6}_{I=1/2}$	$3^{(6)}_{I=1/2}$	$3^{(\overline{3})}_{I=1/2}$
$us\overline{u}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$					
$su\overline{u}$	$-1/2$	$-1/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$1/2$					
$sd\overline{d}$	$1/2$	$-1/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$1/2$					
$ds\overline{d}$	$1/2$	$-1/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$-1/2$					
$ss\overline{s}$		$1/\sqrt{2}$		$-1/\sqrt{2}$						
$ud\overline{u}$						$-1/\sqrt{3}$	$-1/\sqrt{24}$	$1/2$	$-1/\sqrt{8}$	$-1/2$
$du\overline{u}$						$-1/\sqrt{3}$	$-1/\sqrt{24}$	$-1/2$	$-1/\sqrt{8}$	$1/2$
$dd\overline{d}$						$1/\sqrt{3}$	$-1/\sqrt{6}$		$-1/\sqrt{2}$	
$ds\overline{s}$							$\sqrt{3}/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$1/2$
$sd\overline{s}$							$\sqrt{3}/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$

Table 4.1: Operator Decomposition

$$\frac{\sqrt{2}\mathcal{H}_{\mathrm{T}}}{4G_F} = \Bigg\{\lambda_u^s\left[\frac{(C_1+C_2)}{2}\left(-\mathbf{15}_1 - \frac{1}{\sqrt{2}}\mathbf{15}_0 - \frac{1}{\sqrt{2}}\mathbf{3}_0^{(6)}\right) + \frac{(C_1-C_2)}{2}\left(\overline{\mathbf{6}}_1 + \mathbf{3}_0^{(\overline{3})}\right)\right] + \lambda_u^d\left[\frac{(C_1+C_2)}{2}\left(-\frac{2}{\sqrt{3}}\mathbf{15}_{3/2} - \frac{1}{\sqrt{6}}\mathbf{15}_{1/2} - \frac{1}{\sqrt{2}}\mathbf{3}_{1/2}^{(6)}\right) + \frac{(C_1-C_2)}{2}\left(-\overline{\mathbf{6}}_{1/2} + \mathbf{3}_{1/2}^{(\overline{3})}\right)\right]\Bigg\}, \tag{4.20}$$

Since the c-quark is a singlet under $SU(3)$ flavor, the Q_1^c and Q_2^c transforms as a single $\mathbf{3}$ under $SU(3)$. We cannot distinguish between the three $\mathbf{3}$ contributions as they all transform identically under $SU(3)$. In case of QCD penguin operators, the quark pair is produced from a gluon which is $SU(3)$-flavor singlet. Naturally, under $SU(3)$-flavor, $Q_3 \ldots Q_6$ should transform as a $\mathbf{3}$, a fact that is reflected in the explicit decomposition of those operators,

$$\frac{\sqrt{2}\mathcal{H}_{\mathrm{g}}}{4G_F} = \Bigg\{-\lambda_t^s\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_0^{(6)} + (C_3-C_4)\mathbf{3}_0^{(\overline{3})}\right] - \lambda_t^d\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_{1/2}^{(6)} + (C_3-C_4)\mathbf{3}_{1/2}^{(\overline{3})}\right] - \lambda_t^s\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_0^{(6)} + (C_5-C_6)\mathbf{3}_0^{(\overline{3})}\right] - \lambda_t^d\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_{1/2}^{(6)} + (C_5-C_6)\mathbf{3}_{1/2}^{(\overline{3})}\right]\Bigg\}, \tag{4.21}$$

Finally we have the electroweak penguin operators where the quark pair is produced from a photon or an off-shell Z-boson. Here the electric charge of the quarks has to be taken into account. The $SU(3)$-flavor decomposition of the electroweak penguin operators are given below:

$$\frac{\sqrt{2}\mathcal{H}_{\text{EWP}}}{4G_F} = \left\{-\lambda_t^s\left[\frac{(C_9+C_{10})}{2}\left(-\frac{3}{2}\mathbf{15}_1 - \frac{3}{2\sqrt{2}}\mathbf{15}_0 + \frac{1}{2\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right) + \frac{(C_9-C_{10})}{2}\left(\frac{3}{2}\overline{\mathbf{6}}_1 + \frac{1}{2}\mathbf{3}_0^{(\bar{\mathbf{3}})}\right)\right]\right.$$
$$\left.-\lambda_t^d\left[\frac{(C_9+C_{10})}{2}\left(-\sqrt{3}\,\mathbf{15}_{3/2} - \frac{1}{2}\sqrt{\frac{3}{2}}\mathbf{15}_{1/2} + \frac{1}{2\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right) + \frac{(C_9-C_{10})}{2}\left(-\frac{3}{2}\overline{\mathbf{6}}_{1/2} + \frac{1}{2}\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right)\right]\right\}. \tag{4.22}$$

In absence of $SU(3)$ breaking due to s-quark mass, it is clear from Table 4.1 that higher $SU(3)$ representations like **24**, **42** and $\mathbf{15}'$ are absent in the unbroken Hamiltonian.

4.4 Allowed $SU(3)$-reduced matrix elements

The $SU(3)$-reduced elements are $SU(3)$-invariants that can be constructed out of the given initial state, final state and effective Hamiltonian. We recall that before our choice of a particular form of the effective Hamiltonian we had all total 44 $SU(3)$-reduced elements equaling the total number of all $\Delta S = -1$ and $\Delta S = 0$ process, indicated in Appendix A.0.0.1 and Appendix A.0.0.4. We had to treat those $SU(3)$-reduced elements as independent amplitudes since we didn't know beforehand the dynamical coefficients that could connect one to the other. Now, from the tree and electroweak part of the Hamiltonian we can project out the coefficients corresponding to the **15** part of the Hamiltonian and write down the following relations between reduced matrix elements regardless of the initial and final states,

$$\frac{\langle \mathbf{f} \,\|\, \mathbf{15_0} \,\|\, \mathbf{i}\rangle}{\langle \mathbf{f} \,\|\, \mathbf{15_1} \,\|\, \mathbf{i}\rangle} = \frac{1}{\sqrt{2}}, \quad \frac{\langle \mathbf{f} \,\|\, \mathbf{15_{1/2}} \,\|\, \mathbf{i}\rangle}{\langle \mathbf{f} \,\|\, \mathbf{15_{3/2}} \,\|\, \mathbf{i}\rangle} = \frac{1}{2\sqrt{2}} \tag{4.23}$$

$$\frac{\lambda_t^d\langle \mathbf{f} \,\|\, \mathbf{15_0} \,\|\, \mathbf{i}\rangle_{\text{T}}}{\lambda_t^s\langle \mathbf{f} \,\|\, \mathbf{15_{1/2}} \,\|\, \mathbf{i}\rangle_{\text{T}}} = \sqrt{3}, \quad \frac{\lambda_t^d\langle \mathbf{f} \,\|\, \mathbf{15_0} \,\|\, \mathbf{i}\rangle_{\text{EWP}}}{\lambda_t^s\langle \mathbf{f} \,\|\, \mathbf{15_{1/2}} \,\|\, \mathbf{i}\rangle_{\text{EWP}}} = \sqrt{3} \tag{4.24}$$

In case of several different operator structures contributing to the Hamiltonian as is the case in Eq. (4.12), the relations between reduced matrix elements are expressed in the following way,

$$\frac{\langle \mathbf{f} \,||\, \mathbf{R_I} \,||\, \mathbf{i} \rangle}{\langle \mathbf{f} \,||\, \mathbf{R_{I'}} \,||\, \mathbf{i} \rangle} = \frac{\sum_l C_l C_l}{\sum_m C_m C'_m}, \tag{4.25}$$

where, the $C_i^{(')}$ are the coefficients of the different components of the Hamiltonian and C_j's are the CG coefficients and the sums extend over all the corresponding contributions to the Hamiltonian. In addition, the absence of some of the $SU(3)$ representations in the Hamiltonian is a consequence of the vanishing dynamical coefficients corresponding to the reduced matrix elements $\langle \mathbf{f} \,||\, \mathbf{42} \,||\, \mathbf{i} \rangle$, $\langle \mathbf{f} \,||\, \mathbf{24} \,||\, \mathbf{i} \rangle$ and $\langle \mathbf{f} \,||\, \mathbf{15}' \,||\, \mathbf{i} \rangle$, regardless of the I value and initial and final states. We shall see how these representations may contribute when we discuss the effects of $SU(3)$-breaking later. Finally, we are left with ten $SU(3)$-reduced matrix elements,

$$\begin{aligned} &\langle 8_1 \,||\, \mathbf{3} \,||\, \overline{3} \rangle, \quad \langle 8_2 \,||\, \mathbf{3} \,||\, \overline{3} \rangle, \quad \langle 8_1 \,||\, \overline{\mathbf{6}} \,||\, \overline{3} \rangle, \quad \langle 8_2 \,||\, \overline{\mathbf{6}} \,||\, \overline{3} \rangle, \quad \langle 8_1 \,||\, \mathbf{15} \,||\, \overline{3} \rangle \\ &\langle 8_2 \,||\, \mathbf{15} \,||\, \overline{3} \rangle, \quad \langle 10 \,||\, \mathbf{15} \,||\, \overline{3} \rangle, \quad \langle \overline{10} \,||\, \overline{\mathbf{6}} \,||\, \overline{3} \rangle, \quad \langle 27 \,||\, \mathbf{15} \,||\, \overline{3} \rangle, \quad \langle 1 \,||\, \mathbf{3} \,||\, \overline{3} \rangle \end{aligned} \tag{4.26}$$

which are all independent of each other.

4.5 Amplitude relations

The $\Delta S = -1$ and $\Delta S = 0$ decay amplitudes and the reduced $SU(3)$ elements are expressed as column matrices $\mathcal{A}$ and $\mathcal{R}$ respectively and related by the matrix equation,

$$\mathcal{A}_{\text{tree}} = \mathbb{T}\mathcal{R}, \tag{4.27}$$

$$\mathcal{A}_{\text{penguin}} = \mathbb{P}\mathcal{R} \tag{4.28}$$

where $\mathbb{T}$ and $\mathbb{P}$ are the coefficient matrices. $\mathbb{T}$ and $\mathbb{P}$ contain those coefficients that relate the tree and penguin part of the effective Hamiltonian in Eq. (4.20)-(4.22) to the physical initial and final states. The total decay amplitude can always be written as a sum of the tree and penguin part,

$$\mathcal{A} = \mathcal{A}_{\text{tree}} + \mathcal{A}_{\text{penguin}}. \tag{4.29}$$

The rank of matrix $\mathbb{T}$ is lower than the total number of decay modes suggesting that not all of the reduced $SU(3)$ matrix elements are independent. The number of actually independent reduced $SU(3)$ matrix elements are equal to the rank of matrix $\mathbb{T}$. Since the choice of $SU(3)$-reduced amplitudes remain the same while considering gluonic and EW penguin operators, the coefficient matrix $\mathbb{P}$, also has the same rank as matrix $\mathbb{T}$. It is clear from the previous section that the rank of $\mathbb{T}$ or $\mathbb{P}$ matrix is ten, which is the number of independent $SU(3)$-reduced amplitudes. The number of amplitude relations can now be estimated unambiguously which is the difference between the total number decay modes and rank of $\mathbb{T}(\mathbb{P})$. It is advantageous to factor out the CKM elements $\lambda_{u,t}^{s,d}$ that was previously inside the $SU(3)$-reduced amplitudes. The explicit form of $\mathbb{T}$ and $\mathbb{P}$ for $\Delta S = -1$ and $\Delta S = 0$ processes without the overall CKM factors are provided in Appendix A.0.0.2, Appendix A.0.0.5 and Appendix A.0.0.3, Appendix A.0.0.6 respectively. Moreover, a spin-1/2 state decaying to another spin-1/2 state and a spin-0 state can go through two possible relative angular momentum states. Thus we have to distinguish between $l = 0$ and $l = 1$ relative angular momentum final states while decomposing the decay amplitude in terms of tree and penguin reduced amplitudes,

$$\begin{aligned} \mathcal{A}^{\mathsf{S}} &= \lambda_u^q \mathcal{A}_{\text{tree}}^{\mathsf{S}} + \lambda_t^q \mathcal{A}_{\text{penguin}}^{\mathsf{S}}, \\ \mathcal{A}^{\mathsf{P}} &= \lambda_u^q \mathcal{A}_{\text{tree}}^{\mathsf{P}} + \lambda_t^q \mathcal{A}_{\text{penguin}}^{\mathsf{P}}, \end{aligned} \tag{4.30}$$

where $q = s, d$ denote the $\Delta S = -1, 0$ process, S and P denote the S-wave ($l = 0$) and P-wave ($l = 1$) amplitudes of the decay. The coefficient matrices are now redefined as $\mathcal{T}$ and $\mathcal{P}$ with entries that are nothing but products of Wilson Coefficients (C_i) and Clebsch-Gordon coefficients. Of course, the number of independent rows remain unchanged and the matrix equations take the form,

$$\mathcal{A}^i_{\text{tree}} = \mathcal{T}\mathcal{R} \qquad \mathcal{A}^i_{\text{penguin}} = \mathcal{P}\mathcal{R} \tag{4.31}$$

for i = S-wave, i = P-wave part. In case of gluonic penguins, $\overline{\mathbf{6}}$ and $\mathbf{15}$ of $SU(3)$ are absent which result in a smaller set of independent reduced $SU(3)$ matrix elements. This implies additional amplitude relations between decay modes, some of which are violated once the electroweak penguins are taken into account in the unbroken Hamiltonian. We include electroweak penguins that have parts transforming as $\mathbf{3}$, $\overline{\mathbf{6}}$ and $\mathbf{15}$ of $SU(3)$ and retain all the reduced $SU(3)$ matrix elements. As a result, the amplitude relations derived hold for the gluonic penguin part as well as the electroweak penguin part of the unbroken Hamiltonian. We begin with identifying the identical rows of the $\mathcal{T}$ matrix which readily gives the simplest amplitude relations for the tree part,

$$\mathcal{A}_{\text{tree}}(\Lambda_b^0 \to \Sigma^- K^+) = \mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Xi^- \pi^+), \tag{4.32}$$

$$\mathcal{A}_{\text{tree}}(\Lambda_b^0 \to p^+ \pi^-) = \mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Sigma^+ K^-), \tag{4.33}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^- \to n K^-) = \mathcal{A}_{\text{tree}}(\Xi_b^- \to \Xi^0 \pi^-), \tag{4.34}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^- \to \Xi^- K^0) = \mathcal{A}_{\text{tree}}(\Xi_b^- \to \Sigma^- \overline{K}^0), \tag{4.35}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Xi^- K^+) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to \Sigma^- \pi^+), \tag{4.36}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Sigma^- \pi^+) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to \Xi^- K^+), \tag{4.37}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Sigma^+ \pi^-) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to p^+ K^-), \tag{4.38}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to n \overline{K}^0) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to \Xi^0 K^0), \tag{4.39}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to p^+ K^-) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to \Sigma^+ \pi^-), \tag{4.40}$$

$$\mathcal{A}_{\text{tree}}(\Xi_b^0 \to \Xi^0 K^0) = \mathcal{A}_{\text{tree}}(\Lambda_b^0 \to n \overline{K}^0), \tag{4.41}$$

and the same set of relations derived from the matrix $\mathcal{P}$ corresponding to the penguin part of the effective Hamiltonian. There are several triangle relations connecting the $\Delta S = -1$

decays modes;

$$\begin{aligned}
\mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) + \mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) + 2\mathcal{A}(\Lambda_b^0 \to \Sigma^0\pi^0) &= 0, \\
\mathcal{A}(\Xi_b^- \to \Xi^-\pi^0) - \sqrt{3}\mathcal{A}(\Xi_b^- \to \Xi^-\eta_8) + \sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma^-\overline{K^0}) &= 0, \\
\mathcal{A}(\Xi_b^- \to \Sigma^0 K^-) - \sqrt{3}\mathcal{A}(\Xi_b^- \to \Lambda^0 K^-) + \sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi^0\pi^-) &= 0, \\
\mathcal{A}(\Xi_b^0 \to \Xi^-\pi^+) - \mathcal{A}(\Lambda_b^0 \to \Xi^- K^+) + \mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) &= 0, \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+ K^-) - \mathcal{A}(\Lambda_b^0 \to p^+ K^-) + \mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) &= 0,
\end{aligned} \tag{4.42}$$

as well as the $\Delta S = 0$ decay modes;

$$\begin{aligned}
\mathcal{A}(\Xi_b^- \to \Sigma^0\pi^-) - \sqrt{3}\mathcal{A}(\Xi_b^- \to \Lambda^0\pi^-) - \sqrt{2}\mathcal{A}(\Xi_b^- \to nK^-) &= 0, \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\pi^0) - \sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi^- K^0) - \sqrt{3}\mathcal{A}(\Xi_b^- \to \Sigma^-\eta_8) &= 0, \\
\mathcal{A}(\Xi_b^0 \to \Sigma^-\pi^+) - \mathcal{A}(\Xi_b^0 \to \Xi^- K^+) - \mathcal{A}(\Lambda_b^0 \to \Sigma^- K^+) &= 0, \\
\mathcal{A}(\Xi_b^0 \to p^+ K^-) - \mathcal{A}(\Xi_b^0 \to \Sigma^+\pi^-) + \mathcal{A}(\Lambda_b^0 \to p^+\pi^-) &= 0.
\end{aligned} \tag{4.43}$$

The simplest amplitude relations for the case of $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \otimes \mathbf{1}_{\mathcal{M}}$ involving the $SU(3)$ singlet η_1 are indicated,

$$\begin{aligned}
\mathcal{A}(\Xi_b^0 \to \Xi^0\eta_1) &= \mathcal{A}(\Lambda_b^0 \to n\eta_1), \\
\mathcal{A}(\Xi_b^- \to \Xi^-\eta_1) &= \mathcal{A}(\Xi_b^- \to \Sigma^-\eta_1),
\end{aligned} \tag{4.44}$$

along with triangle relation for $\Delta S = -1$ processes

$$\mathcal{A}(\Lambda_b^0 \to \Lambda\eta_1) - \tfrac{1}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Sigma^0\eta_1) - \tfrac{\sqrt{2}}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Xi^0\eta_1) = 0$$

and for $\Delta S = 0$ processes,

$$\mathcal{A}(\Lambda_b^0 \to n\eta_1) + \frac{\sqrt{3}}{\sqrt{2}}\mathcal{A}(\Xi_b^0 \to \Lambda^0\eta_1) - \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^0 \to \Sigma^0\eta_1) = 0.$$

While there is no ground state $SU(3)$ singlet Λ baryon, there can be $l = 1$ excited state spin-3/2 Λ_s^{0*}-baryon, for which one can derive amplitude relations equivalent to the case of $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \otimes \mathbf{8}_{\mathcal{M}}$;

$$\mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\overline{K^0}) = \mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}K^0)$$
$$\mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\eta_8) = \mathcal{A}(\Xi_b^- \to \Lambda_s^{0*}K^-) \tag{4.45}$$

triangle $\Delta S = -1$ relations:

$$\mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}\pi_0) - \frac{1}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}\eta_8) + \frac{\sqrt{2}}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\overline{K^0}) = 0, \tag{4.46}$$

triangle $\Delta S = 0$ relations:

$$-\frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\eta_8) + \mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\pi_0) - \frac{\sqrt{2}}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}K^0) = 0. \tag{4.47}$$

The S-wave and P-wave parts of the decay amplitudes behave identically under $SU(3)$ decomposition and the relations hold individually for the tree and penguin part of the all the above mentioned amplitude relations. Finally, we consider the trivial case of $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \otimes \mathbf{1}_{\mathcal{M}}$ where the final state baryon and meson are both $SU(3)$ singlets. The only relevant decay, $\Lambda_b^0 \to \Lambda_s^{*0}\eta_1$, satisfying the $SU(3)$ quantum numbers involve a single reduced $SU(3)$ amplitude matching with the counting of the number of possible independent $SU(3)$-reduced amplitudes. This concludes our discussion of all possible $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \otimes \mathbf{8}_{\mathcal{M}}$, $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \otimes \mathbf{1}_{\mathcal{M}}$, $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \otimes \mathbf{8}_{\mathcal{M}}$, $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \otimes \mathbf{1}_{\mathcal{M}}$ decays of b-baryons. More $SU(3)$ relations are obtained by starting from the $\mathbb{T}(\mathbb{P})$ matrix and expressing the dependent rows as a linear combination of the independent ones. We do not list those relations here as they are not particularly illuminating.

4.5.1 $SU(3)$ breaking effect

While isospin symmetry holds to a good approximation, the $SU(3)$ symmetry of the light quarks is broken by the mass of the s quark (m_s). To incorporate such $SU(3)$ violating effects on decay amplitudes, we parametrize the breaking of flavor $SU(3)$ by the following interaction [101, 103, 110, 134–136],

$$\delta\mathcal{H} = \epsilon\,\overline{q}\,\lambda_8\,q \tag{4.48}$$

where λ_8 is the Gell-Mann matrix that contributes to the $SU(3)$-breaking and the breaking parameter ϵ depends on m_s. The $SU(3)$ structure of the unbroken Hamiltonian is modified by this term and to the first order in strange quark mass, the broken Hamiltonian is made of the following $SU(3)$ representations [103],

$$\begin{aligned}(\mathbf{3}\oplus\overline{\mathbf{6}}\oplus\mathbf{15})\otimes(\mathbf{1}+\epsilon\mathbf{8}) = (\mathbf{3}\oplus\overline{\mathbf{6}}\oplus\mathbf{15}) + \epsilon(&\mathbf{3}_i\oplus\overline{\mathbf{6}}_i\oplus\mathbf{15}_1\oplus\mathbf{15}_2\oplus\mathbf{15}_3^1 \\ &\oplus\mathbf{15}_3^2\oplus\mathbf{15}'\oplus\mathbf{24}_2\oplus\mathbf{24}_3\oplus\mathbf{42}),\end{aligned} \tag{4.49}$$

where the subscript $i = 1,2,3$ indicates the origin of that representation from $\mathbf{3}$,$\overline{\mathbf{6}}$, $\mathbf{15}$ respectively. The set of reduced $SU(3)$ amplitudes thus gets enlarged and there are less number of relations as a result. The isospin relation,

$$\mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) + \mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) + 2\mathcal{A}(\Lambda_b^0 \to \Sigma^0\pi^0) = 0 \tag{4.50}$$

continues to hold even after including the $SU(3)$ breaking effect to the linear order. There are other amplitude relations that can be derived on more general grounds. For instance, the isospin symmetry of the unbroken Hamiltonian forbids a $\Delta I = 2$ and $\Delta I = 5/2$ transition. As a consequence, the $SU(3)$-reduced matrix elements $\langle \mathbf{f} \,||\, \mathbf{R}_{I=2} \,||\, \mathbf{i}\rangle$ and $\langle \mathbf{f} \,||\, \mathbf{R}_{I=5/2} \,||\, \mathbf{i}\rangle$ must have a vanishing contribution to the decay amplitude for arbitrary initial and final states. Such $SU(3)$ breaking but isospin conserving relations are given below,

$$\frac{\mathcal{A}(\Xi_b^0 \to \Sigma^0 \bar{K}^0)}{3} + \frac{\mathcal{A}(\Xi_b^0 \to \Sigma^+ K^-)}{3\sqrt{2}} + \frac{\mathcal{A}(\Xi_b^0 \to \Xi^0 \pi^0)}{3} + \frac{\mathcal{A}(\Xi_b^0 \to \Xi^- \pi^+)}{3\sqrt{2}} + \frac{\mathcal{A}(\Xi_b^- \to \Sigma^0 K^-)}{3} + \frac{\mathcal{A}(\Xi_b^- \to \Sigma^- \bar{K}^0)}{3\sqrt{2}} + \frac{\mathcal{A}(\Xi_b^- \to \Xi^0 \pi^-)}{3\sqrt{2}} + \frac{\mathcal{A}(\Xi_b^- \to \Xi^- \pi^0)}{3} + \frac{\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma^0 \pi^0)}{3} + \frac{\mathcal{A}(\Lambda_b^0 \to \Sigma^- \pi^+)}{3\sqrt{2}} + \frac{\mathcal{A}(\Lambda_b^0 \to \Sigma^+ \pi^-)}{3\sqrt{2}} = 0, \quad (4.51)$$

$$\frac{\mathcal{A}(\Xi_b^0 \to \Sigma^0 \bar{K}^0)}{\sqrt{6}} + \frac{\mathcal{A}(\Xi_b^0 \to \Sigma^+ K^-)}{2\sqrt{3}} - \frac{\mathcal{A}(\Xi_b^0 \to \Xi^0 \pi^0)}{\sqrt{6}} - \frac{\mathcal{A}(\Xi_b^0 \to \Xi^- \pi^+)}{2\sqrt{3}} + \frac{\mathcal{A}(\Xi_b^- \to \Sigma^0 K^-)}{\sqrt{6}} + \frac{\mathcal{A}(\Xi_b^- \to \Sigma^- \bar{K}^0)}{2\sqrt{3}} - \frac{\mathcal{A}(\Xi_b^- \to \Xi^0 \pi^-)}{2\sqrt{3}} - \frac{\mathcal{A}(\Xi_b^- \to \Xi^- \pi^0)}{\sqrt{6}} = 0. \quad (4.52)$$

The same set of relations hold for the penguin parts as well.

4.6 CP relations

The total decay rate for a two body decay of a spin-1/2 anti-triplet b-baryon ($\mathcal{B}_b$) to a spin 0 pseudo-scalar ($\mathcal{M}$) meson and a spin 1/2 baryon ($\mathcal{B}$) has the following form [23, 25, 54, 137–139]

$$\Gamma(\mathcal{B}_b \to \mathcal{B}\mathcal{M}) = \frac{|\mathbf{p}_{\mathcal{B}}|}{8\pi m_{\mathcal{B}_b}^2}\left[|S|^2 + |P|^2\right] \quad (4.53)$$

where $|\mathbf{p}_{\mathcal{B}}|$ is the momentum of the final state baryon given by,

$$|\mathbf{p}_{\mathcal{B}}| = \frac{1}{2m_{\mathcal{B}_b}}\sqrt{m_{\mathcal{B}_b}^4 + m_{\mathcal{B}}^4 + m_{\mathcal{M}}^4 - 2m_{\mathcal{B}_b}^2 m_{\mathcal{B}}^2 - 2m_{\mathcal{B}_b}^2 m_{\mathcal{M}}^2 - 2m_{\mathcal{M}}^2 m_{\mathcal{B}}^2}. \quad (4.54)$$

The S-wave and P-wave contributions [23, 54, 116] in Eq. (4.53) are factored into kinematic factors and decay amplitudes $\mathcal{A}^{\mathsf{S}}$ and $\mathcal{A}^{\mathsf{P}}$,

$$\begin{aligned} S &= \sqrt{2m_{\mathcal{B}_b}(E_{\mathcal{B}}+m_{\mathcal{B}})}\mathcal{A}^{\mathsf{S}} \\ P &= \sqrt{2m_{\mathcal{B}_b}(E_{\mathcal{B}}-m_{\mathcal{B}})}\mathcal{A}^{\mathsf{P}} \end{aligned} \tag{4.55}$$

where $\mathcal{A}^{\mathsf{S}}$ and $\mathcal{A}^{\mathsf{P}}$ are expressed in terms of $SU(3)$-reduced amplitudes defined in Eq. (4.30). The decay rate,

$$\Gamma = \frac{|\mathbf{p}_{\mathcal{B}}|}{4\pi}\frac{(E_{\mathcal{B}}+m_{\mathcal{B}})}{m_{\mathcal{B}_b}}\Big[|\mathcal{A}^{\mathsf{S}}|^2+\Big(\frac{|\mathbf{p}_{\mathcal{B}}|}{E_{\mathcal{B}}+m_{\mathcal{B}}}\Big)^2|\mathcal{A}^{\mathsf{P}}|^2\Big] \tag{4.56}$$

and A_{CP} are defined subsequently as [135],

$$\begin{aligned} A_{CP} =& \frac{\Gamma(\mathcal{B}_b\to\mathcal{B}\mathcal{M})-\Gamma(\overline{\mathcal{B}}_b\to\overline{\mathcal{B}}\overline{\mathcal{M}})}{\Gamma(\mathcal{B}_b\to\mathcal{B}\mathcal{M})+\Gamma(\overline{\mathcal{B}}_b\to\overline{\mathcal{B}}\overline{\mathcal{M}})} \\ =& \frac{\Delta_{CP}(\mathcal{B}_b\to\mathcal{B}\mathcal{M})}{2\tilde{\Gamma}(\mathcal{B}_b\to\mathcal{B}\mathcal{M})}, \end{aligned} \tag{4.57}$$

where, $\tilde{\Gamma}(\mathcal{B}_b\to\mathcal{B}\mathcal{M}) = \frac{1}{2}(\Gamma(\mathcal{B}_b\to\mathcal{B}\mathcal{M})+\Gamma(\overline{\mathcal{B}}_b\to\overline{\mathcal{B}}\overline{\mathcal{M}}))$. We note that the amplitude relations quoted in Eqs. (4.32)-(4.41) are actually U-spin relations that are no longer valid when $SU(3)$-breaking effects are considered. Nevertheless, we express CP relation among those modes [24] relying on the identity $\mathrm{Im}(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -\mathrm{Im}(V_{ub}V_{us}^*V_{tb}^*V_{ts}) = \mathbf{J}$, where $\mathbf{J}$ is the well known Jarlskog invariant. Notice that Eqs. (4.53) and (4.57) imply that A_{CP} is the sum of CP violation in S and P waves. We define a quantity $\delta^a_{CP} = |\mathcal{A}^a|^2-|\bar{\mathcal{A}}^a|^2$, for the partial wave a, where $a = \{\mathsf{S}, \mathsf{P}\}$ and $\mathcal{A}^a$ are defined in Eq. (4.55) with phase-space factors removed from the respective partial waves. By definition,

$$\delta^a_{CP}(\mathcal{B}_b\to\mathcal{B}\mathcal{M}) = -4\mathbf{J}\times\mathrm{Im}\Big[\mathcal{A}^{a*}_{\mathrm{T}}(\mathcal{B}_b\to\mathcal{B}\mathcal{M})\mathcal{A}^a_{\mathrm{P}}(\mathcal{B}_b\to\mathcal{B}\mathcal{M})\Big]. \tag{4.58}$$

Based on amplitude relations for the tree and penguin parts obtained in Eqs. (4.32)–(4.41) the following ten δ^a_{CP} relations are obtained,

$$\begin{aligned}
\delta^a_{CP}(\Lambda^0_b \to \Sigma^- K^+) &= -\,\delta^a_{CP}(\Xi^0_b \to \Xi^- \pi^+),\\
\delta^a_{CP}(\Lambda^0_b \to p^+ \pi^-) &= -\,\delta^a_{CP}(\Xi^0_b \to \Sigma^+ K^-),\\
\delta^a_{CP}(\Xi^-_b \to n K^-) &= -\,\delta^a_{CP}(\Xi^-_b \to \Xi^0 \pi^-),\\
\delta^a_{CP}(\Xi^-_b \to \Xi^- K^0) &= -\,\delta^a_{CP}(\Xi^-_b \to \Sigma^- \overline{K}^0),\\
\delta^a_{CP}(\Xi^0_b \to \Xi^- K^+) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Sigma^- \pi^+),\\
\delta^a_{CP}(\Xi^0_b \to \Sigma^- \pi^+) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Xi^- K^+),\\
\delta^a_{CP}(\Xi^0_b \to \Sigma^+ \pi^-) &= -\,\delta^a_{CP}(\Lambda^0_b \to p^+ K^-),\\
\delta^a_{CP}(\Xi^0_b \to n \overline{K}^0) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Xi^0 K^0),\\
\delta^a_{CP}(\Xi^0_b \to p^+ K^-) &= -\,\delta^a_{CP}(\Lambda^0_b \to \Sigma^+ \pi^-),\\
\delta^a_{CP}(\Xi^0_b \to \Xi^0 K^0) &= -\,\delta^a_{CP}(\Lambda^0_b \to n \overline{K}^0),
\end{aligned} \tag{4.59, 4.60}$$

for both $a = \mathsf{S}$ and $a = \mathsf{P}$. Finally we obtain A_{CP} relations using,

$$A_{CP}(\mathcal{B}_b \to \mathcal{B}\mathcal{M}) = \frac{\tau_{\mathcal{B}_b}}{\mathcal{BR}(\mathcal{B}_b \to \mathcal{B}\mathcal{M})} \Delta_{CP}(\mathcal{B}_b \to \mathcal{B}\mathcal{M}), \tag{4.61}$$

where, $\tau_{\mathcal{B}_b}$ is the lifetime of the beauty-baryon. The relation between Δ_{CP} and δ_{CP} is,

$$\Delta_{CP} = \frac{|\mathbf{p}_{\mathcal{B}}|}{4\pi} \frac{(E_{\mathcal{B}} + m_{\mathcal{B}})}{m_{\mathcal{B}_b}} \Big[\delta^{\mathsf{S}}_{CP} + \Big(\frac{|\mathbf{p}_{\mathcal{B}}|}{E_{\mathcal{B}} + m_{\mathcal{B}}}\Big)^2 \delta^{\mathsf{P}}_{CP}\Big] \tag{4.62}$$

Since, Δ_{CP} depends on the masses of the initial and final baryons as well as the final state meson [23, 115], some approximation is needed to obtain A_{CP} relations between various modes. In the U-spin limit [24], by ignoring $\mathbf{p}_{\mathcal{B}}$ and $m_{\mathcal{B}}$ differences, CP violation relations can be experimentally verified between the modes connected by $SU(3)$-flavor symmetry using the relation,

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{B}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \to \mathcal{B}_m \mathcal{M}_n)} \simeq -\frac{\tau_{\mathcal{B}_{bi}}}{\tau_{\mathcal{B}_{bl}}} \frac{\mathcal{BR}(\mathcal{B}_{bl} \to \mathcal{B}_m \mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{B}_j \mathcal{M}_k)}, \tag{4.63}$$

where i, j, k and l, m, n are indices corresponding to the various hadrons belonging to the above mentioned δ_{CP} relations. There is a further simplification in case $i = l$, resulting in

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{B}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bi} \to \mathcal{B}_m \mathcal{M}_n)} \simeq -\frac{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{B}_m \mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{B}_j \mathcal{M}_k)}, \tag{4.64}$$

where the uncertainties due to lifetime measurement also cancel out [25]. Alternatively, if the longitudinal polarization of the daughter baryon can be measured from an angular distribution study of the final states, one can estimate the relative strength of the P-wave contribution [23,116] in the total decay width. The longitudinal polarization of the daughter baryon is given by,

$$\alpha = \frac{2\text{Re}(\mathcal{A}^{\mathsf{S}*}\mathcal{A}^{\mathsf{P}})|\mathbf{p}_{\mathcal{B}}|/E_{\mathcal{B}} + m_{\mathcal{B}}}{|\mathcal{A}^{\mathsf{S}}|^2 + |\mathcal{A}^{\mathsf{P}}|^2(|\mathbf{p}_{\mathcal{B}}|/E_{\mathcal{B}} + m_{\mathcal{B}})^2} \tag{4.65}$$

The P-wave contribution can now be systematically taken into account resulting in a more reliable prediction for A_{CP} relations. These relations serve as an important test of flavor $SU(3)$ symmetry in beauty-baryon non-leptonic decays and one can compare these findings with the analogous decays of bottom mesons to have a better understanding of the $SU(3)$ flavor symmetry breaking pattern.

4.7 Summary

We consider a general framework for hadronic beauty-baryon decays into octet or singlet of light baryon and a pseudoscalar meson, based on $SU(3)$ decomposition of the decay amplitudes. We show that in the most general case, the 44 distinct decay modes require 44 independent reduced $SU(3)$ amplitudes to describe all possible $\Delta S = -1$ and $\Delta S = 0$

processes. In practice, the dimension-6 effective Hamiltonian that mediates such non-leptonic decays of bottom baryons allows only 10 independent reduced $SU(3)$ amplitudes. As a consequence there must exist relations between the decay amplitudes. We explicitly derive several sum rules relations between decay amplitudes as well as relations between CP asymmetries. Moreover, we systematically study the $SU(3)$-breaking effects in the decay amplitudes at leading order in the $SU(3)$ breaking parameter. We further identify amplitude relations that survives even when the $SU(3)$ flavor symmetry is no longer exact.

Chapter 5

$SU(3)$-flavor analysis of bottom baryon decaying into a decuplet baryon and an octet meson

The contents of this chapter is based on a joint work [14] with Rahul Sinha and N.G. Deshpande.

5.1 Prologue

In the last chapter we set up the formalism to study charmless decays of beauty baryons into an octet baryon and an octet pseudoscalar meson using $SU(3)$-flavor symmetry. Since decuplet baryon are known to exist, it is natural to ask whether we can observe a decuplet baryon and a pseudo-scalar or vector meson in b-baryon decays. Experimental evidences support this claim and in fact LHCb is well on its course to observe CP violation in regions of the phase space that contain quasi-two body or three body decays of b-baryons featuring a decuplet baryon in the final or intermediate state. There are two important attributes of beauty-baryon decaying into a decuplet baryon and an octet pseudoscalar meson that

Decay mode	Invariant-mass requirements (in Mev/c^2)
$\Lambda_b^0 \to p^+\pi^-\pi^+\pi^-$	
$\Lambda_b^0 \to \Delta^{++}(1232)\pi^-\pi^-$	$1078 < m(p^+\pi^-) < 1432$
$\Lambda_b^0 \to N^0(1520)\rho^0(770)$	$1078 < m(p^+\pi^-) < 1800$ and $m(\pi^+\pi^-) < 1100$
$\Lambda_b^0 \to p^+a_1^-(1260)$	$419 < m(\pi^+\pi^-\pi^+) < 1500$
$\Lambda_b^0 \to p^+K^-\pi^+\pi^-$	
$\Lambda_b^0 \to \Delta^{++}(1232)K^-\pi^-$	$1078 < m(p^+\pi^-) < 1432$
$\Lambda_b^0 \to N^0(1520)K^{*0}(892)$	$1078 < m(p^+\pi^-) < 1800$ and $750 < m(\pi^+K^-) < 1100$
$\Lambda_b^0 \to \Lambda^0(1520)\rho^0(770)$	$1460 < m(p^+K^-) < 1580$ and $m(\pi^+\pi^-) < 1100$
$\Lambda_b^0 \to p^+\,K_1^-(1410)$	$1200 < m(K^-\pi^+\pi^-) < 1600$
$\Lambda_b^0 \to p^+K^-K^+K^-$	
$\Lambda_b^0 \to \Lambda^0(1520)\phi(1020)$	$1460 < m(p^+K^-) < 1600$ and $1005 < m(K^+K^-) < 1040$
$\Lambda_b^0 \to (pK^-)_{\text{high-mass}}\,\phi(1020)$	$m(p^+K^-) > 1600$ and $1005 < m(K^+K^-) < 1040$

Table 5.1: Observed resonances in multibody decays of bottom baryons [9]

is worth pointing out. Firstly, all factorizable amplitudes vanish and non-factorizable contributions play the dominant role in these decays. Secondly, only five $SU(3)$-flavor parameters are required to describe all possible decays that can be determined from the data as it becomes available. This motivates us to extend our analysis to anti-triplet ($\overline{\mathbf{3}}$) beauty-baryon ($\mathcal{B}_b$) decaying into a decuplet baryon ($\mathcal{D}$) and an octet pseudoscalar meson ($\mathcal{M}$) i.e. $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$ in this chapter. In addition, we also provide an alternative approach in terms of quark diagrams and compare with the $SU(3)$ decomposition in the limit of exact $SU(3)$-flavor symmetry. Some of these quark diagrams do not contribute in exact $SU(3)$-flavor symmetry limit, as we will see in detail later. This has implications on the number of free parameters that are required to describe all possible b-baryon decays to a decuplet baryon and an octet meson. By pursuing both the $SU(3)$-reduction of decay amplitudes and the diagrammatic prescription we are in a position to crosscheck our assumptions directly in terms of experimentally measurable quantities.

5.2 Formalism

The formalism to study charmless decay of an anti-triplet ($\overline{\mathbf{3}}$) beauty-baryon into a decuplet baryon ($\mathcal{D}$) and an octet pseudoscalar meson ($\mathcal{M}$), i.e. $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$ is exactly the same as described in the last chapter. The possible decays can be divided into two sub classes, namely the $\Delta S = 0$ and $\Delta S = -1$ transitions. The allowed final state $SU(3)$ representations ($\mathbf{f}$) are; $\mathbf{8}$, $\mathbf{10}$, $\mathbf{27}$, $\mathbf{35}$. There are twenty physical process possible for $\Delta S = -1$ and another twenty for $\Delta S = 0$. In Appendix B.0.0.1 and Appendix B.0.0.2, each of these decay modes are decomposed in terms of the $SU(3)$ reduced amplitudes. The count of total number of $SU(3)$ reduced amplitudes add upto forty. Since the physical η and $\eta^{'}$ mesons are admixtures of octet η_8 and singlet η_1 mesons, a study of $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{1})$ is also necessary. Therefore one has to take into account four (two each for $\Delta S = -1$ and $\Delta S = 0$) additional independent $SU(3)$ amplitudes. Since no assumption about the particular form of effective Hamilton has been made yet, these forty four reduced amplitudes are all independent of each other and no amplitude relation exist between the decay modes.

Now we assume same dimension-6 Hamiltonian with $\Delta Q = 0$ and $\Delta S = -1, 0$ parts as given in Eq (4.12). For quick reference, the $SU(3)$ decomposition of the tree, gluonic and electroweak part of the effective Hamiltonian are rewritten,

$$\frac{\sqrt{2}\mathcal{H}_{\rm T}}{4G_F} = \Bigg\{\lambda_u^s\left[\frac{(C_1+C_2)}{2}\left(-\mathbf{15}_1 - \frac{1}{\sqrt{2}}\mathbf{15}_0 - \frac{1}{\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right) + \frac{(C_1-C_2)}{2}\left(\overline{\mathbf{6}}_1 + \mathbf{3}_0^{(\bar{\mathbf{3}})}\right)\right] + \lambda_u^d\left[\frac{(C_1+C_2)}{2}\left(-\frac{2}{\sqrt{3}}\mathbf{15}_{3/2} - \frac{1}{\sqrt{6}}\mathbf{15}_{1/2} - \frac{1}{\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right) + \frac{(C_1-C_2)}{2}\left(-\overline{\mathbf{6}}_{1/2} + \mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right)\right]\Bigg\}, \tag{5.1}$$

$$\frac{\sqrt{2}\mathcal{H}_{\rm g}}{4G_F} = \Big\{-\lambda_t^s\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_0^{(\mathbf{6})} + (C_3-C_4)\mathbf{3}_0^{(\bar{\mathbf{3}})}\right] - \lambda_t^d\left[-\sqrt{2}(C_3+C_4)\mathbf{3}_{1/2}^{(\mathbf{6})} + (C_3-C_4)\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right] - \lambda_t^s\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_0^{(\mathbf{6})} + (C_5-C_6)\mathbf{3}_0^{(\bar{\mathbf{3}})}\right] - \lambda_t^d\left[-\sqrt{2}(C_5+C_6)\mathbf{3}_{1/2}^{(\mathbf{6})} + (C_5-C_6)\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right]\Big\}, \tag{5.2}$$

$$\frac{\sqrt{2}\mathcal{H}_{\rm EWP}}{4G_F} = \Bigg\{-\lambda_t^s\left[\frac{(C_9+C_{10})}{2}\left(-\frac{3}{2}\mathbf{15}_1 - \frac{3}{2\sqrt{2}}\mathbf{15}_0 + \frac{1}{2\sqrt{2}}\mathbf{3}_0^{(\mathbf{6})}\right) + \frac{(C_9-C_{10})}{2}\left(\frac{3}{2}\overline{\mathbf{6}}_1 + \frac{1}{2}\mathbf{3}_0^{(\bar{\mathbf{3}})}\right)\right] -\lambda_t^d\left[\frac{(C_9+C_{10})}{2}\left(-\sqrt{3}\,\mathbf{15}_{3/2} - \frac{1}{2}\sqrt{\frac{3}{2}}\mathbf{15}_{1/2} + \frac{1}{2\sqrt{2}}\mathbf{3}_{1/2}^{(\mathbf{6})}\right) + \frac{(C_9-C_{10})}{2}\left(-\frac{3}{2}\overline{\mathbf{6}}_{1/2} + \frac{1}{2}\mathbf{3}_{1/2}^{(\bar{\mathbf{3}})}\right)\right]\Bigg\}. \tag{5.3}$$

With this particular choice of effective Hamiltonian and allowed final state $SU(3)$ representations, there are five independent $SU(3)$-reduced matrix elements:

$$\langle 8 \,\|\, \mathbf{3} \,\|\, \bar{3}\rangle, \qquad \langle 8 \,\|\, \overline{\mathbf{6}} \,\|\, \bar{3}\rangle, \qquad \langle 8 \,\|\, \mathbf{15} \,\|\, \bar{3}\rangle$$
$$\langle 10 \,\|\, \mathbf{15} \,\|\, \bar{3}\rangle, \qquad \langle 27 \,\|\, \mathbf{15} \,\|\, \bar{3}\rangle \tag{5.4}$$

and the $SU(3)$-reduced amplitudes arising from $\mathcal{H}_{\rm T}$ are conveniently expressed;

$$\begin{aligned}
c_8 &= -(C_1+C_2)\langle 8 \,\|\, \mathbf{3}^{(6)} \,\|\, \overline{3}\rangle + \sqrt{2}(C_1-C_2)\langle 8 \,\|\, \mathbf{3}^{(\overline{3})} \,\|\, \overline{3}\rangle,\\
b_8 &= (C_1-C_2)\langle 8 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3}\rangle,\\
a_8 &= (C_1+C_2)\langle 8 \,\|\, \mathbf{15} \,\|\, \overline{3}\rangle,\\
a_{10} &= (C_1+C_2)\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3}\rangle,\\
a_{27} &= (C_1+C_2)\langle 27 \,\|\, \mathbf{15} \,\|\, \overline{3}\rangle.
\end{aligned} \tag{5.5}$$

The decay amplitudes for all possible $\Delta S = -1$ and $\Delta S = 0$ processes are expressed using Eq (5.5) and are given in Table 5.2 and Table 5.3 respectively.

An alternate description of decay amplitudes is obtained in terms of topological quark diagrams. The symmetry properties of the final state decuplet baryons allow five possible diagrams starting from a flavor anti-triplet b-baryon whose light quarks are in a flavor anti-symmetric state. The flavor flow in five topologies [27] given in Figure 5.1 are described below;

$$\begin{aligned}
E_1 &: B_{[ij]}D^{\{kmj\}}M^l_k H^i_{lm}\\
E_2 &: B_{[ij]}D^{\{kmj\}}M^l_k H^i_{ml}\\
P_u &: B_{[ij]}D^{\{kjl\}}M^i_k H^m_{ml}\\
T &: B_{[ij]}D^{\{lmj\}}M^i_k H^k_{ml}\\
E_3 &: B_{[ij]}D^{\{klm\}}M^j_m H^i_{kl}
\end{aligned}$$

where $B_{[ij]}$, $D^{\{klm\}}$, M^x_y are flavor wavefunctions of the initial anti-triplet ($\overline{\mathbf{3}}$) b-baryon, the final state decuplet baryon (**10**) and the octet meson (**8**) respectively. Those flavor wavefunctions are given in Eq (B.1) and Eq (B.2). The H^i_{ab} mediates the quark transition $b \to q_i \overline{q_a} q_b$ and for $\Delta S = -1$, $\Delta S = 0$ processes, the only non-zero contribution come from the elements $H^1_{13} = 1$ and $H^1_{12} = 1$ respectively. Those five independent topologies consist of three W-exchanges (E_1, E_1, E_3), one tree (T) and a penguin-like (P_q) (q being the flavor

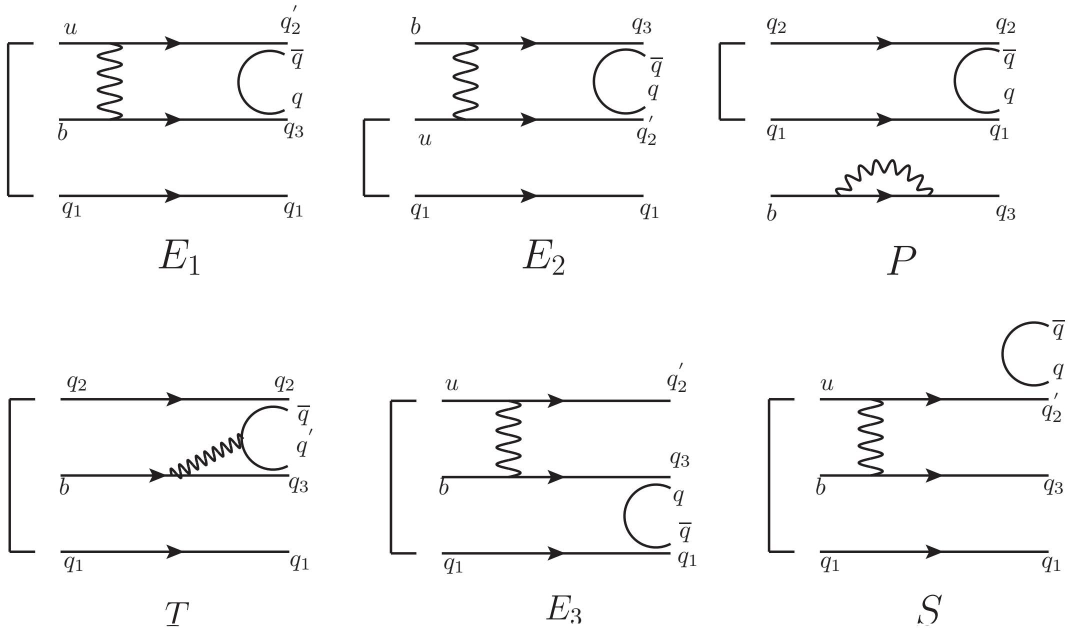


Figure 5.1: Topological diagrams contributing to tree amplitude for anti-triplet b-baryon hadronic decays. The first five diagrams contribute to $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$ processes. The last diagram contributes exclusively to $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{1})$ processes.

of the quark going in the loop) amplitude where the marked quarks are anti-symmetrized in the initial state baryon. The sixth diagram, denoted as S contributes exclusively to the decay modes containing the singlet η_1 meson. The mapping between the topological amplitudes and the $SU(3)$-reduced amplitudes is given below,

$$
\begin{aligned}
c_8 &= -\frac{1}{8}\Big(-\sqrt{10}E_1 + 3\sqrt{10}E_2 - 2\sqrt{10}E_3 + 8\sqrt{10}P_u + 2\sqrt{10}T\Big), \\
b_8 &= \frac{1}{4}\Big(\sqrt{15}E_2 - \sqrt{15}E_1\Big), \\
a_8 &= \frac{1}{24}\Big(5\sqrt{6}E_1 + 5\sqrt{6}E_2 - 10\sqrt{6}E_3 + 2\sqrt{6}T\Big), \\
a_{10} &= \frac{1}{3}\Big(2\sqrt{3}E_1 + 2\sqrt{3}E_2 + 2\sqrt{3}E_3 - \sqrt{3}T\Big), \\
a_{27} &= \sqrt{\frac{2}{3}}T.
\end{aligned}
\tag{5.6}
$$

The topologies E_3 and T are sometimes ignored in accordance with Korner-Pati-Woo theorem [140–151] based on "$\Delta I = 1/2$" rule which suggests that the quark pair produced in weak interaction ending up in the same baryon must be anti-symmetric in flavor space. However, the accuracy of such a statement is dependent on the modeling of the baryon-baryon transition and exact $SU(3)$ flavor symmetry. The details of the arguments made to ignore E_3 and T are reproduced below.

5.2.1 Lessons from Korner-Pati-Woo theorem

Considering only the tree operators in the effective Hamiltonian,

$$\mathcal{H}^{\text{eff}} = C_1 Q_1 + C_2 Q_2 \tag{5.7}$$

that can be recast as [140–144],

$$\mathcal{H}^{\text{eff}} = C^- Q^- + C^+ Q^+, \tag{5.8}$$

where $C^- = C_1 - C_2$, $C^+ = C_1 + C_2$ and

$$Q^- = \frac{1}{2}(Q_1 - Q_2) = \frac{1}{2}\Big((\overline{u}^i_L \gamma^\mu b^j_L)(\overline{d}^j_L \gamma_\mu u^i_L) - (\overline{u}^i_L \gamma^\mu b^i_L)(\overline{d}^j_L \gamma_\mu u^j_L)\Big) \tag{5.9}$$

$$Q^+ = \frac{1}{2}(Q_1 + Q_2) = \frac{1}{2}\Big((\overline{u}^i_L \gamma^\mu b^j_L)(\overline{d}^j_L \gamma_\mu u^i_L) + (\overline{u}^i_L \gamma^\mu b^i_L)(\overline{d}^j_L \gamma_\mu u^j_L)\Big). \tag{5.10}$$

Rewriting Q_1 and Q_2 we get,

$$Q_1 = \frac{1}{4}\Big[(\overline{u}^i)_\alpha [\gamma^\mu (1-\gamma_5)]_{\alpha\beta} (b^l)_\beta\Big]\Big[(\overline{d}^j)_\sigma [\gamma_\mu (1-\gamma_5)]_{\sigma\lambda} (u^k)_\lambda\Big]\delta^{lj}\delta^{ik}, \tag{5.11}$$

$$Q_2 = \frac{1}{4}\Big[(\overline{u}^i)_\alpha [\gamma^\mu (1-\gamma_5)]_{\alpha\beta} (b^l)_\beta\Big]\Big[(\overline{d}^j)_\sigma [\gamma_\mu (1-\gamma_5)]_{\sigma\lambda} (u^k)_\lambda\Big]\delta^{il}\delta^{jk} \tag{5.12}$$

We use the identity,

$$\frac{1}{2}\epsilon^{aij}\epsilon^{alk} = (\delta^{il}\delta^{jk} - \delta^{ik}\delta^{jl})$$

to write Q^- as,

$$Q^- = -\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4}\epsilon^{aij}\epsilon^{alk}\Big[(\overline{u}^i)_\alpha[\gamma^\mu(1-\gamma_5)]_{\alpha\beta}(b^l)_\beta\Big]\Big[(\overline{d}^j)_\sigma[\gamma_\mu(1-\gamma_5)]_{\sigma\lambda}(u^k)_\lambda\Big] \tag{5.13}$$

Using Firez transformation [152] one can recast Eq. (5.13) as [142],

$$Q^- = -\frac{1}{4}\frac{1}{2}\underbrace{\epsilon^{alk}\Big[(\overline{b}^l)^{\mathbf{C}}(1-\gamma_5)(u^k)\Big]}_{(bu)_{\overline{3}}}\underbrace{\epsilon^{aij}\Big[(\overline{u}^i)(1+\gamma_5)(d^j)^{\mathbf{C}}\Big]}_{(ud)^\dagger_{\overline{3}}} \tag{5.14}$$

where $q^{\mathbf{C}} = \mathbf{C}q^*$, $\mathbf{C}$ being the charge-conjugation operator. According to diquark mechanism [142, 143], the diquark $\epsilon^{aij}\Big[(\overline{u}^i)(1+\gamma_5)(d^j)^{\mathbf{C}}\Big]$ has a total spin 0 and transforms as a $\overline{\mathbf{3}}$ and $\overline{\mathbf{3}}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$ respectively. This diquark ending up completely in the final state baryon cannot produce[1] a decuplet baryon as any quark pair inside the decuplet baryon transforms as $\mathbf{6}$ and $\overline{\mathbf{3}}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$ with a total spin equaling 1. On the other hand, the operator Q^+ which is a product of $SU(3)_{\text{color}}$ sextet currents cannot produce a color singlet baryon. In diagrams T and E_3, the quark pair antisymmetric in color and flavor [140, 141] originating from weak interaction ends up in the decuplet baryon which is forbidden by the above mentioned argument [145–151]. Since the quark pair $(ud)_{\overline{3}}$ is in isospin 0 state[2], the total quark transition obeys $\Delta I = 1/2$ rule. In contrast, in diagrams E_1 and E_2, the diquark argument is not applicable as only one of the quarks from weak interaction form the final state baryon while the other ends up in the meson.

As a consequence, the number of independent diagrams reduces to three. If we demand an equivalent description of all possible decays in terms of $SU(3)$-reduced amplitudes, some of the reduced amplitudes can no longer be independent. The relation between the

[1]Assuming the baryon made of the diquark and a third quark transforming as a $\overline{\mathbf{3}}$ and $\mathbf{3}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$ the final state transforms as $SU(3)_{\text{color}}$: $\overline{\mathbf{3}}\otimes\mathbf{3} = \mathbf{1}\oplus\mathbf{8}$, $SU(3)_F$: $\overline{\mathbf{3}}\otimes\mathbf{3} = \mathbf{1}\oplus\mathbf{8}$. Since baryons are color singlet, a $\mathbf{6}$ of $SU(3)_{\text{color}}$ diquark can never form a baryon.

[2]Only allowed isospin values in a $\overline{\mathbf{3}}$ of $SU(3)_F$ is 0 or 1/2.

remaining diagrams and the $SU(3)$-reduced amplitudes is given below,

$$E_1 = \frac{\sqrt{3}a_{10}}{4} - \frac{2b_8}{\sqrt{15}}, \tag{5.15}$$

$$E_2 = \frac{\sqrt{3}a_{10}}{4} + \frac{2b_8}{\sqrt{15}}, \tag{5.16}$$

$$P_u = -\frac{\sqrt{3}a_{10}}{16} + \frac{c_8}{\sqrt{10}} - \frac{b_8}{\sqrt{15}} \tag{5.17}$$

where c_8, b_8, a_8, a_{10}, a_{27} are defined in Eq. (5.5).

Equivalently, the following relations hold for the $SU(3)$-reduced matrix elements,

$$a_{27} = 0 \qquad (C_1 \neq 0, C_2 \neq 0) \tag{5.18}$$

$$a_8 = \frac{5}{8\sqrt{2}}a_{10}. \tag{5.19}$$

We derive several interesting tests (Eqs. (5.33)–(5.34)) to verify if the contribution from the topological amplitudes E_3 and T are indeed suppressed. We, however have chosen to include the E_3 and T diagrams throughout the rest of this thesis.

The $SU(3)$-decomposition of decays and the diagrammatic approach are equivalent and imply that individual topological amplitudes cannot be expressed in terms of a single $SU(3)$-reduced amplitude and vice-versa in these two basis. An exception to this observation is the T diagram which is expressible entirely in terms of a single $SU(3)$-reduced amplitude a_{27} as is evident from Eq. (5.6). In context of the $\mathbf{10} \otimes \mathbf{1}$ transition, involving the singlet meson, it is worth mentioning that the only $SU(3)$-reduced amplitude, given by $\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle$ needs to be counted independent of the five $SU(3)$-reduced amplitude contributing to the $\mathbf{10} \otimes \mathbf{8}$ processes. The flavor-flow diagram S is expressed in terms of that single $SU(3)$-reduced amplitude. It is also important to note that the topology, P_u, originate purely from the tree operators i.e. O_1 and O_2 as emphasized in [68–72], even though it is denoted by P_u and referred to as a penguin topology. Moreover the W-exchange topologies being a four-quark tree-like structure also contribute to the tree amplitudes.

The transition induced by QCD penguin operators, given in Eq (4.21) is only a **3** under $SU(3)$ and corresponding $SU(3)$-reduced amplitude is identified with the 'penguin' amplitude P_t,

$$P_t = -\sqrt{2}(C_3 + C_4 + C_5 + C_6)\langle 8 \parallel \mathbf{3}^{(\mathbf{6})} \parallel \overline{3}\rangle + (C_3 - C_4 + C_5 - C_6)\langle 8 \parallel \mathbf{3}_0^{(\overline{\mathbf{3}})} \parallel \overline{3}\rangle. \tag{5.20}$$

5.2.2 Relating EWP and Tree amplitudes

We begin this section by noting that the **15** and $\overline{\mathbf{6}}$ part of the Hamiltonian described in Eqs. (4.20) and (4.22) relate the contributions to the decay from the tree and EWP operators [20, 63, 74] respectively, so as to effectively obey the following relations,

$$\begin{aligned}
\mathcal{H}^{\text{EWP}}_{\mathbf{15}}(\Delta S) &= -\frac{3}{2}\frac{\lambda_t^{s(d)}(C_9 + C_{10})}{\lambda_u^{s(d)}(C_1 + C_2)}\mathcal{H}^{\text{T}}_{\mathbf{15}}(\Delta S) \\
\mathcal{H}^{\text{EWP}}_{\overline{\mathbf{6}}}(\Delta S) &= -\frac{3}{2}\frac{\lambda_t^{s(d)}(C_9 - C_{10})}{\lambda_u^{s(d)}(C_1 - C_2)}\mathcal{H}^{\text{T}}_{\overline{\mathbf{6}}}(\Delta S)
\end{aligned} \tag{5.21}$$

These relations are valid independently for both $\Delta S = \{0, -1\}$ decays and remain unaffected by the QCD penguin operators since O_3 and O_4 transforms entirely as a **3** under $SU(3)$. Apriori, the $\mathbf{3}^{(6)}$ and $\mathbf{3}^{(\overline{3})}$ operators do not follow a simple relation for arbitrary values of C_1, C_2, C_9 and C_{10}. Since the set of $SU(3)$-reduced elements remains the same, in analogy to Eq. (5.5) one can define,

$$\begin{aligned}
c_8^{EW} &= (C_9 + C_{10})\langle 8 \parallel \mathbf{3}^{(6)} \parallel \overline{3}\rangle + \sqrt{2}(C_9 - C_{10})\langle 8 \parallel \mathbf{3}^{(\overline{3})} \parallel \overline{3}\rangle \\
b_8^{EW} &= -\frac{3}{2}(C_9 - C_{10})\langle 8 \parallel \overline{\mathbf{6}} \parallel \overline{3}\rangle \\
a_8^{EW} &= -\frac{3}{2}(C_9 + C_{10})\langle 8 \parallel \mathbf{15} \parallel \overline{3}\rangle \\
a_{10}^{EW} &= -\frac{3}{2}(C_9 + C_{10})\langle 10 \parallel \mathbf{15} \parallel \overline{3}\rangle \\
a_{27}^{EW} &= -\frac{3}{2}(C_9 + C_{10})\langle 27 \parallel \mathbf{15} \parallel \overline{3}\rangle.
\end{aligned} \tag{5.22}$$

The EWP reduced amplitudes $b_8^{EW}, a_8^{EW}, a_{10}^{EW}$ and a_{27}^{EW} are expressed in terms of b_8, a_8, a_{10} and a_{27} defined in Eq. (5.5);

$$b_8^{EW} = -\frac{3}{2}\frac{C_9 - C_{10}}{C_1 - C_2} b_8, \quad a_8^{EW} = -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2} a_8,$$
$$a_{10}^{EW} = -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2} a_{10}, \quad a_{27}^{EW} = -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2} a_{27} \tag{5.23}$$

Using numerical values of the Wilson coefficients to leading logarithmic order we obtain,

$$\frac{C_9 + C_{10}}{C_1 + C_2} = -1.139\,\alpha, \qquad \frac{C_9 - C_{10}}{C_1 - C_2} = -1.107\,\alpha. \tag{5.24}$$

To a good approximation [20] these two ratios of Wilson coefficients can be taken to be a common value [19, 133] given by κ;

$$\kappa = \frac{C_9 + C_{10}}{C_1 + C_2} = \frac{C_9 - C_{10}}{C_1 - C_2} \simeq -1.12\,\alpha. \tag{5.25}$$

With this additional assumption, Eq. (5.23) implies that the $b_8^{EW}, a_8^{EW}, a_{10}^{EW}$ and a_{27}^{EW} are proportional to the tree reduced amplitudes b_8, a_8, a_{10} and a_{27}

$$b_8^{EW} = -\frac{3}{2}\kappa b_8, \qquad a_8^{EW} = -\frac{3}{2}\kappa a_8,$$
$$a_{10}^{EW} = -\frac{3}{2}\kappa a_{10}, \qquad a_{27}^{EW} = -\frac{3}{2}\kappa a_{27}. \tag{5.26}$$

The equivalence of $SU(3)$-reduced amplitudes to topological diagrams, allows one to interpret P_i^{EW} as electroweak quark diagrams with one insertion of the electroweak penguin operator;

$$P_{E_i}^{EW} = -\frac{3}{2}\kappa E_i \tag{5.27}$$
$$P_T^{EW} = -\frac{3}{2}\kappa T \tag{5.28}$$

where, $i = \{1, 2, 3\}$.

$\Delta S = -1$	$SU(3)-$reduced amplitude	Topological diagrams
$\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-)$	$\frac{1}{60}\left(-5\sqrt{2}a_{10} - 3a_{27} - 8a_8 + 4\sqrt{10}b_8\right)$	$-\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Lambda_b^0 \to \Delta^0 \bar{K}^0)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} + 3a_{27} + 8a_8 - 4\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-)$	$\frac{1}{60}\left(-5\sqrt{2}a_{10} + 15a_{27} + 10a_8 + 2\sqrt{15}c_8 - 2\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(-E_3 + P_u + T)$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0)$	$\frac{1}{30}\left(-3a_{27} - 3a_8 - \sqrt{15}c_8\right)$	$-\frac{1}{2\sqrt{6}}(E_2 - E_3 + 2P_u + T)$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \eta_8)$	$-\frac{1}{10\sqrt{3}}\left(3a_{27} - 2a_8 + \sqrt{10}b_8\right)$	$\frac{1}{6\sqrt{2}}(2E_1 - E_2 - E_3 - T)$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} + 15a_{27} - 10a_8 - 2\sqrt{15}c_8 + 2\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(E_3 - P_u)$
$\mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-)$	$\frac{1}{30}\left(-5\sqrt{2}a_{10} + 6a_{27} + a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(-E_1 - E_3 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \bar{K}^0)$	$\frac{1}{60}\left(10a_{10} + 9\sqrt{2}a_{27} - \sqrt{2}a_8 - \sqrt{30}c_8 - 2\sqrt{5}b_8\right)$	$\frac{1}{2\sqrt{3}}(E_1 + E_3 - P_u)$
$\mathcal{A}(\Xi_b^0 \to \Xi'^0 \pi^0)$	$\frac{1}{120}\left(-10a_{10} - 27\sqrt{2}a_{27} - 2\left(\sqrt{2}a_8 + \sqrt{30}c_8 + 2\sqrt{5}b_8\right)\right)$	$-\frac{1}{2\sqrt{3}}(E_2 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \to \Xi'^0 \eta_8)$	$\frac{1}{120}\left(10\sqrt{3}a_{10} + 3\sqrt{6}a_{27} - 2\left(\sqrt{6}a_8 + 3\sqrt{10}c_8 + 2\sqrt{15}b_8\right)\right)$	$\frac{1}{6}(2E_1 - E_2 + 2E_3 - 3P_u - T)$
$\mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \to \Omega^- K^+)$	$\frac{1}{60}\left(5\sqrt{6}a_{10} - 3\sqrt{3}a_{27} + 2\sqrt{3}a_8 + 6\sqrt{5}c_8 + 2\sqrt{30}b_8\right)$	$\frac{1}{\sqrt{2}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-)$	$\frac{1}{60}\left(12\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8\right)$	$\frac{1}{2\sqrt{3}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \to \Sigma'^- \bar{K}^0)$	$\frac{1}{30}\left(3a_{27} + 3a_8 - \sqrt{15}c_8 + \sqrt{10}b_8\right)$	$-\frac{1}{\sqrt{6}}P_u$
$\mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-)$	$\frac{1}{30}\left(-12a_{27} + 3a_8 - \sqrt{15}c_8 + \sqrt{10}b_8\right)$	$-\frac{1}{\sqrt{6}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0)$	$\frac{1}{60}\left(-3\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8\right)$	$\frac{1}{2\sqrt{3}}P_u$
$\mathcal{A}(\Xi_b^- \to \Xi'^- \eta_8)$	$\frac{1}{60}\left(3\sqrt{6}a_{27} + 3\sqrt{6}a_8 - 3\sqrt{10}c_8 + 2\sqrt{15}b_8\right)$	$-\frac{1}{2}P_u$
$\mathcal{A}(\Xi_b^- \to \Omega^- K^0)$	$\frac{1}{30}\left(-3\sqrt{3}a_{27} - 3\sqrt{3}a_8 + 3\sqrt{5}c_8 - \sqrt{30}b_8\right)$	$\frac{1}{\sqrt{2}}P_u$

Table 5.2: $\mathcal{B}_b(\bar{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$ decay $\Delta S = -1$ transitions

As mentioned earlier, there is no simple relation between the **3** part of the EWP Hamiltonian to the tree part. There are, however, decays where the **3** part of the Hamiltonian cannot contribute to the formation of final states which require a pure $\Delta I = 1$ or $\Delta I = 3/2$ transition. For example, the decay $\Lambda_b^0 \to \Delta^+ K^-$ receives contribution from $\overline{\mathbf{6}}$ and **15** part of the effective Hamiltonian and in this particular case the ratio of EWP and tree contributions is given entirely by the simple ratio $-3/2\kappa(\lambda_t^s/\lambda_u^s)$. The $SU(3)$-reduced amplitudes for the penguin operators are provided in Appendix. B.0.0.5 and Appendix. B.0.0.6.

$\Delta S = 0$	$SU(3)$ – reduced amplitude	Topological Diagram
$\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-)$	$\frac{1}{30}\left(-5\sqrt{2}a_{10} + 6a_{27} + a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(-E_1 - E_3 + P_u + T)$
$\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0)$	$\frac{1}{120}\left(10a_{10} - 9\sqrt{2}a_{27} - 4\left(\sqrt{2}a_8 + \sqrt{30}c_8 + 2\sqrt{5}b_8\right)\right)$	$\frac{1}{2\sqrt{3}}(E_1 - E_2 + E_3 - 2P_u - T)$
$\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_8)$	$\frac{1}{24}\left(-2\sqrt{3}a_{10} - 3\sqrt{6}a_{27}\right)$	$-\frac{1}{6}(E_1 + E_2 + E_3 + T)$
$\mathcal{A}(\Lambda_b^0 \to \Delta^-\pi^+)$	$\frac{1}{60}\left(5\sqrt{6}a_{10} - 3\sqrt{3}a_{27} + 2\sqrt{3}a_8 + 6\sqrt{5}c_8 + 2\sqrt{30}b_8\right)$	$\frac{1}{\sqrt{2}}(E_2 + P_u)$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0)$	$\frac{1}{60}\left(10a_{10} + 9\sqrt{2}a_{27} - \sqrt{2}a_8 - \sqrt{30}c_8 - 2\sqrt{5}b_8\right)$	$\frac{1}{2\sqrt{3}}(E_1 + E_3 - P_u)$
$\mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \to \Delta^+ K^-)$	$\frac{1}{60}\left(-5\sqrt{2}a_{10} + 15a_{27} + 10a_8 + 2\sqrt{15}c_8 - 2\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(-E_3 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K}^0)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} + 15a_{27} - 10a_8 - 2\sqrt{15}c_8 + 2\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(E_3 - P_u)$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-)$	$\frac{1}{60}\left(-5\sqrt{2}a_{10} - 3a_{27} - 8a_8 + 4\sqrt{10}b_8\right)$	$-\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^0\pi^0)$	$\frac{1}{60}\left(-12a_{27} + 3a_8 - \sqrt{15}c_8 - 3\sqrt{10}b_8\right)$	$\frac{1}{2\sqrt{6}}(E_1 - E_2 - P_u - T)$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^0\eta_8)$	$\frac{1}{60}\left(-5\sqrt{3}a_8 - 3\sqrt{5}c_8 + \sqrt{30}b_8\right)$	$-\frac{1}{6\sqrt{2}}(E_1 + E_2 - 2E_3 + 3P_u + T)$
$\mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} + 3a_{27} + 8a_8 - 4\sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Xi_b^0 \to \Xi'^- K^+)$	$\frac{1}{60}\left(5\sqrt{2}a_{10} - 3a_{27} + 2\left(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8\right)\right)$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^- \to \Delta'^0 K^-)$	$\frac{1}{30}\left(12a_{27} - 3a_8 + \sqrt{15}c_8 - \sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K}^0)$	$\frac{1}{30}\left(3\sqrt{3}a_{27} + 3\sqrt{3}a_8 - 3\sqrt{5}c_8 + \sqrt{30}b_8\right)$	$-\frac{1}{\sqrt{2}}P_u$
$\mathcal{A}(\Xi_b^- \to \Sigma'^0\pi^-)$	$\frac{1}{60}\left(-12\sqrt{2}a_{27} + 3\sqrt{2}a_8 - \sqrt{30}c_8 + 2\sqrt{5}b_8\right)$	$-\frac{1}{2\sqrt{3}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \to \Sigma'^-\pi^0)$	$\frac{1}{60}\left(-3\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8\right)$	$\frac{1}{2\sqrt{3}}P_u$
$\mathcal{A}(\Xi_b^- \to \Sigma'^-\eta_8)$	$\frac{1}{60}\left(3\sqrt{6}a_{27} + 3\sqrt{6}a_8 - 3\sqrt{10}c_8 + 2\sqrt{15}b_8\right)$	$-\frac{1}{2}P_u$
$\mathcal{A}(\Xi_b^- \to \Xi'^- K^0)$	$\frac{1}{30}\left(-3a_{27} - 3a_8 + \sqrt{15}c_8 - \sqrt{10}b_8\right)$	$\frac{1}{\sqrt{6}}P_u$

Table 5.3: $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$ decay $\Delta S = 0$ transitions

5.3 Amplitude relations

The complete decay amplitude is given in terms of tree and penguin $SU(3)$-reduced amplitudes and the CKM elements,

$$\mathcal{A} = \lambda_u^q \mathcal{A}_{\text{tree}} + \lambda_t^q \mathcal{A}_{\text{penguin}}, \tag{5.29}$$

where $q = s, d$ denote the $\Delta S = -1, 0$ process. Since the $SU(3)$ operators appear in EW and tree part of the Hamiltonian in a particular combination, the same amplitude relations between $\Delta S = -1$ and $\Delta S = 0$ processes are satisfied by the EWP part and the tree part. Moreover, the decay products of a spin-1/2 particle decaying to a spin-3/2 and a spin-0 particle can be in relative angular momentum $l = 1$ or $l = 2$ state. The decay amplitude is further decomposed into P-wave and D-wave parts;

$$\mathcal{A}^{\rm P} = \lambda_u^q \mathcal{A}^{\rm P}_{\rm tree} + \lambda_t^q \mathcal{A}^{\rm P}_{\rm penguin}, \tag{5.30}$$

$$\mathcal{A}^{\rm D} = \lambda_u^q \mathcal{A}^{\rm D}_{\rm tree} + \lambda_t^q \mathcal{A}^{\rm D}_{\rm penguin}. \tag{5.31}$$

Finally, the following decay amplitude relations are obtained for both the P-wave and D-wave parts;

$$\begin{aligned}
&\mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) = \mathcal{A}(\Xi_b^0 \to \Delta^+ K^-),\\
&\mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0) = \mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K^0}),\\
&\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = \mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-),\\
&\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) = \mathcal{A}(\Xi_0^b \to \Sigma'^0 \overline{K^0}),
\end{aligned} \tag{5.32}$$

$$\begin{aligned}
&\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) = \mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-) = -\mathcal{A}(\Lambda_b^0 \to \Delta^0 \overline{K^0}) = -\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0),\\
&\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) = -\frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^- \to \Xi'^0\pi^-) = \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^- \to \Delta^0 K^-) = -\mathcal{A}(\Xi_b^- \to \Sigma'^0\pi^-),\\
&\mathcal{A}(\Lambda_b^0 \to \Sigma'^-\pi^+) = \mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^-\pi^+) = \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Omega^- K^+)\\
&= \frac{1}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = \mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+) = \mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^- K^+),\\
&\mathcal{A}(\Xi_b^- \to \Sigma'^- \overline{K^0}) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi'^-\pi^0) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Xi'^-\eta_8) = -\frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Omega^- K^0)\\
&= \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K^0}) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Sigma'^-\eta_8) = -\mathcal{A}(\Xi_b^- \to \Xi'^- K^0) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^-\pi^0).
\end{aligned}$$

The additional amplitude relations that emerge after applying the arguments from Korner-Pati-Woo theorem are highlighted in the boxes,

$$\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = \mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-) \boxed{= -\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) = -\sqrt{2}\mathcal{A}(\Xi_0^b \to \Sigma'^0 \overline{K^0}),} \tag{5.33}$$

$$\mathcal{A}(\Lambda_b^0 \to \Sigma'^-\pi^+) = \mathcal{A}(\Lambda_b^0 \to \Xi'^-K^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^-\pi^+) = \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Omega^-K^+)$$
$$= \frac{1}{\sqrt{3}}\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = \mathcal{A}(\Lambda_b^0 \to \Sigma'^-K^+) = \mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+) = \mathcal{A}(\Xi_b^0 \to \Xi'^-K^+)$$
$$\boxed{= -\sqrt{2}\mathcal{A}(\Xi_b^0 \to \Xi'^0\pi^0)} \tag{5.34}$$

$$\mathcal{A}(\Xi_b^- \to \Sigma'^-\overline{K^0}) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi'^-\pi^0) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Xi'^-\eta_8) = -\frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Omega^-K^0)$$
$$= \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \to \Delta'^-\overline{K^0}) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \to \Sigma'^-\eta_8) = -\mathcal{A}(\Xi_b^- \to \Xi'^-K^0) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^-\pi^0)$$
$$\boxed{= -\mathcal{A}(\Xi_b^0 \to \Delta^+K^-) = \mathcal{A}(\Xi_b^0 \to \Delta^0\overline{K^0}) = -\mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) = \mathcal{A}(\Lambda_b^0 \to \Xi'^0K^0)}$$
$$\boxed{= -\mathcal{A}(\Xi_b^- \to \Delta^0K^-) = \sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^0\pi^-) = -\sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^0K^-) = \mathcal{A}(\Xi_b^- \to \Xi'^0\pi^-)}. \tag{5.35}$$

In case of $\overline{\mathbf{3}} \to \mathbf{10} \otimes \mathbf{1}$ processes, it is clear from Eq (B.3) and Eq (B.4) that the only amplitude relation between the two modes is,

$$\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_1) = \sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\eta_1) \tag{5.36}$$

Coming back to the $\overline{\mathbf{3}} \to \mathbf{10} \otimes \mathbf{8}$ processes we write down the $\Delta S = -1$ triangle relations (without using Korner-Pati-Woo theorem),

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^-\pi^+) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) = 0, \tag{5.37}$$
$$\mathcal{A}(\Lambda_b^0 \to \Delta^+K^-) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) - \mathcal{A}(\Xi_b^0 \to \Sigma'^+K^-) = 0, \tag{5.38}$$
$$\mathcal{A}(\Xi_b^0 \to \Sigma'^0\overline{K^0}) - \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_b^0 \to \Delta^+K^-) - \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_b^0 \to \Xi'^0K^0) = 0, \tag{5.39}$$
$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) - \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_b^0 \to \Xi'^0K^0) - \mathcal{A}(\Xi_b^0 \to \Xi'^0\pi^0) = 0, \tag{5.40}$$
$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\eta_8) + \sqrt{\frac{3}{2}}\mathcal{A}(\Lambda_b^0 \to \Xi'^0K^0) - \mathcal{A}(\Xi_b^0 \to \Xi'^0\eta_8) = 0, \tag{5.41}$$
$$\frac{2}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Sigma'^0\overline{K^0}) - \mathcal{A}(\Xi_b^0 \to \Xi'^0\eta_8) + \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Xi'^0\pi^0) = 0, \tag{5.42}$$

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) + \mathcal{A}(\Xi_b^0 \to \Xi'^-\pi^+) = 0, \tag{5.43}$$

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) + \mathcal{A}(\Lambda_b^0 \to \Xi'^-K^+) = 0, \tag{5.44}$$

$$\mathcal{A}(\Lambda_b^0 \to \Delta^0\overline{K^0}) - \mathcal{A}(\Lambda_b^0 \to \Sigma'^+\pi^-) + \mathcal{A}(\Xi_b^0 \to \Sigma'^+K^-) = 0, \tag{5.45}$$

$$\mathcal{A}(\Lambda_b^0 \to \Delta^+K^-) - \mathcal{A}(\Lambda_b^0 \to \Xi'^0K^0) + \sqrt{2}\mathcal{A}(\Xi_b^0 \to \Sigma'^0\overline{K^0}) = 0, \tag{5.46}$$

$$\mathcal{A}(\Lambda_b^0 \to \Sigma'^0\pi^0) + \frac{1}{2}\mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) + \frac{1}{2\sqrt{3}}\mathcal{A}(\Xi_b^0 \to \Omega^-K^+) = 0. \tag{5.47}$$

The $\Delta S = 0$ triangle relations are also obtained,

$$\sqrt{6}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^-\pi^+) + \sqrt{3}\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = 0, \tag{5.48}$$

$$\sqrt{3}\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_8) - \mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + 2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0K^0) = 0, \tag{5.49}$$

$$\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) - \mathcal{A}(\Xi_b^0 \to \Delta^+K^-) + \mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-) = 0, \tag{5.50}$$

$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0K^0) - \mathcal{A}(\Xi_0^b \to \Delta^0\overline{K^0}) + \mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-) = 0, \tag{5.51}$$

$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) - \mathcal{A}(\Xi_b^0 \to \Delta^0\overline{K^0}) - 2\mathcal{A}(\Xi_b^0 \to \Sigma'^0\pi^0) = 0, \tag{5.52}$$

$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_8) + \sqrt{3}\mathcal{A}(\Xi_b^0 \to \Delta^0\overline{K^0}) - 2\mathcal{A}(\Xi_b^0 \to \Sigma'^0\eta_8) = 0, \tag{5.53}$$

$$\mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = 0, \tag{5.54}$$

$$\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) - \mathcal{A}(\Xi_b^0 \to \Delta^+K^-) + \mathcal{A}(\Xi_b^0 \to \Xi'^0K^0) = 0, \tag{5.55}$$

$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Sigma'^0K^0) - \mathcal{A}(\Xi_b^0 \to \Delta^0\overline{K^0}) - \mathcal{A}(\Xi_b^0 \to \Xi'^0K^0) = 0, \tag{5.56}$$

$$\mathcal{A}(\Xi_b^0 \to \Xi'^-K^+) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) = 0, \tag{5.57}$$

$$\sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^-K^+) = 0. \tag{5.58}$$

5.4 $SU(3)$ breaking effects

The modified Hamiltonian after including linear $SU(3)$ breaking effects is exactly the same as given in Eq (4.49). $SU(3)$ breaking effects will induce higher $SU(3)$ representations and some of the amplitude relations will cease to hold as there are now thirteen independent $SU(3)$-reduced matrix elements. The isospin relation,

$$\mathcal{A}(\Lambda_b^0 \to \Delta^0 \overline{K^0}) = -\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) \tag{5.59}$$

and isospin triangle relations, for $\Delta S = -1$,

$$2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) = 0, \tag{5.60}$$

and for $\Delta S = 0$,

$$\sqrt{6}\mathcal{A}(\Lambda_b^0 \to \Delta^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^- \pi^+) + \sqrt{3}\mathcal{A}(\Lambda_b^0 \to \Delta^+ \pi^-) = 0, \tag{5.61}$$

continue to hold. In addition, arbitrary $SU(3)$-breaking but isospin conserving effects still forbid $\Delta I = 2$ and $\Delta I = \frac{5}{2}$ transitions which results in general amplitude sum rules,

$$\begin{aligned}
&\mathcal{A}(\Lambda_b^0 \to \Delta^0 \overline{K}^0) + \mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) + 2\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) + \\
&\quad \mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) - \sqrt{2}\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \overline{K}^0) - \mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-) \\
&\quad - \sqrt{2}\mathcal{A}(\Xi_b^0 \to \Xi'^0 \pi^0) - \mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+) + \sqrt{2}\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) \\
&\quad + \mathcal{A}(\Xi_b^- \to \Sigma'^- \overline{K}^0) + \mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-) + \sqrt{2}\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0) = 0
\end{aligned} \tag{5.62}$$

$$\begin{aligned}
&2\sqrt{3}\mathcal{A}(\Lambda_b^0 \to \Delta^0 \pi^0) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \to \Delta^- \pi^+) + \sqrt{6}\mathcal{A}(\Lambda_b^0 \to \Delta^+ \pi^-) - \sqrt{6}\mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K}^0) \\
&\quad - \sqrt{6}\mathcal{A}(\Xi_b^0 \to \Delta^+ K^-) - 2\sqrt{6}\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \pi^0) - \sqrt{6}\mathcal{A}(\Xi_b^0 \to \Sigma'^- \pi^+) \\
&\quad - \sqrt{6}\mathcal{A}(\Xi_b^0 \to \Sigma'^+ \pi^-) + \sqrt{6}\mathcal{A}(\Xi_b^- \to \Delta^0 K^-) + \sqrt{2}\mathcal{A}(\Xi_b^- \to \Delta^- \overline{K}^0) \\
&\quad + 2\sqrt{3}\mathcal{A}(\Xi_b^- \to \Sigma'^0 \pi^-) + \mathcal{A}(\Xi_b^- \to \Sigma'^- \pi^0) = 0.
\end{aligned} \tag{5.63}$$

5.5 CP relations

The general decay amplitude for a spin-1/2 b-baryon ($\mathcal{B}_b$) to a spin 0 pseudo-scalar ($\mathcal{M}$) and a spin-3/2 ($\mathcal{D}$) is given by,

$$\mathfrak{M} = -iq_\mu \overline{u}^\mu_{\mathcal{D}}(a + b\gamma_5)u_{\mathcal{B}_b}, \tag{5.64}$$

where $\overline{u}^\mu_{\mathcal{D}}$ is the Rarita-Schwinger spinor for the spin-3/2 decuplet baryon, q_μ is the momentum of the pseudo-scalar meson and $u_{\mathcal{B}_b}$ is the spinor for the initial spin-1/2 b-baryon. The two coefficients a and b have dimension GeV^{-1} and contain the CKM elements as well as the same flavor structure as $\mathcal{A}_{\text{tree}}$ and $\mathcal{A}_{\text{penguin}}$. The total decay rate for an unpolarized b-baryon described by the decay amplitude of the form in Eq.(5.64) is given by,

$$\Gamma(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) = \frac{|\mathbf{p}_{\mathcal{D}}|}{8\pi m^2_{\mathcal{B}_b}}\Big(\frac{2}{3}\frac{m^2_{\mathcal{B}_b}}{m^2_{\mathcal{D}}}2m_{\mathcal{B}_b}(E_{\mathcal{D}} + m_{\mathcal{D}})|\mathbf{p}_{\mathcal{D}}|^2\Big) \Big(|a|^2 + \frac{|\mathbf{p}_{\mathcal{D}}|^2}{(E_{\mathcal{D}} + m_{\mathcal{D}})^2}|b|^2\Big) \tag{5.65}$$

where $|\mathbf{p}_{\mathcal{D}}|$ is the magnitude of the 3-momentum of the decuplet baryon in the rest frame of the initial b-baryon. The decay products can be in any one of the two possible relative angular momentum states, $l = 1$ and $l = 2$ identified as P-wave and D-wave respectively. The kinematic factors for P-wave and D-wave are given by,

$$P = \sqrt{\frac{2}{3}}\frac{|\mathbf{p}_{\mathcal{D}}|}{m_{\mathcal{D}}}\sqrt{2m^3_{\mathcal{B}_b}(E_{\mathcal{D}} + m_{\mathcal{D}})}\mathcal{A}^{\text{P}}, \tag{5.66}$$

$$D = \sqrt{\frac{2}{3}}\frac{|\mathbf{p}_{\mathcal{D}}|}{m_{\mathcal{D}}}\sqrt{2m^3_{\mathcal{B}_b}(E_{\mathcal{D}} - m_{\mathcal{D}})}\mathcal{A}^{\text{D}} \tag{5.67}$$

where $\mathcal{A}^{\mathsf{P}}$ and $\mathcal{A}^{\mathsf{D}}$ are defined in Eq. (5.30) and Eq. (5.31). In terms of the P-wave and D-wave contributions introduced, the total decay rate Γ is;

$$\Gamma(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) = \frac{|\mathbf{p}_{\mathcal{D}}|}{8\pi m_{\mathcal{B}_b}^2}(|P|^2 + |D|^2) \tag{5.68}$$

$$= \frac{m_{\mathcal{B}_b}(E_{\mathcal{D}} + m_{\mathcal{D}})|\mathbf{p}_{\mathcal{D}}|^3}{6\pi m_{\mathcal{D}}^2} \times \left(|\mathcal{A}^{\mathsf{P}}|^2 + \frac{|\mathbf{p}_{\mathcal{D}}|^2}{(E_{\mathcal{D}} + m_{\mathcal{D}})^2}|\mathcal{A}^{\mathsf{D}}|^2\right). \tag{5.69}$$

As before, A_{CP} is defined as,

$$A_{CP} = \frac{\Gamma(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) - \Gamma(\overline{\mathcal{B}_b} \to \overline{\mathcal{D}}\overline{\mathcal{M}})}{\Gamma(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) + \Gamma(\overline{\mathcal{B}_b} \to \overline{\mathcal{D}}\overline{\mathcal{M}})} = \frac{\Delta_{CP}(\mathcal{B}_b \to \mathcal{D}\mathcal{M})}{2\tilde{\Gamma}(\mathcal{B}_b \to \mathcal{D}\mathcal{M})}, \tag{5.70}$$

where,

$$\tilde{\Gamma}(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) = \frac{1}{2}(\Gamma(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) + \Gamma(\overline{\mathcal{B}_b} \to \overline{\mathcal{D}}\overline{\mathcal{M}})).$$

A_{CP} is the sum of CP violating contribution from the δ_{CP}^{P} and δ_{CP}^{D} with appropriate phase-space factor multiplied:

$$A_{CP}(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) = \frac{\tau_{\mathcal{B}_b}}{\mathcal{BR}(\mathcal{B}_b \to \mathcal{D}\mathcal{M})} \frac{m_{\mathcal{B}_b}(E_D + m_D)|\mathbf{p}_{\mathcal{D}}|^3}{6\pi m_{\mathcal{D}}^2} \left(\delta_{CP}^{\mathsf{P}} + \frac{|\mathbf{p}_{\mathcal{D}}|^2}{(E_D + m_D)^2}\delta_{CP}^{\mathsf{D}}\right) \tag{5.71}$$

where $\tau_{\mathcal{B}_b}$ is the lifetime of the beauty-baryon.

By definition,

$$\delta_{CP}^{l}(\mathcal{B}_b \to \mathcal{D}\mathcal{M}) = -4\mathbf{J} \times \mathrm{Im}\left[\mathcal{A}_{\text{tree}}^{l*}(\mathcal{B}_b \to \mathcal{D}\mathcal{M})\mathcal{A}_{\text{penguin}}^{l}(\mathcal{B}_b \to \mathcal{D}\mathcal{M})\right], \tag{5.72}$$

$\mathbf{J}$ being the Jarlskog invariant. Based on amplitude relations for the tree and penguin parts obtained in Eq. (5.32) the following δ^a_{CP} relations [23, 137, 138] are obtained,

$$\delta^l(\Lambda_b^0 \to \Sigma'^+\pi^-) = -\delta^l(\Xi_b^0 \to \Delta^+ K^-),$$
$$\delta^l(\Lambda_b^0 \to \Xi'^0 K^0) = -\delta^l(\Xi_b^0 \to \Delta^0 \overline{K^0}),$$
$$\delta^l(\Xi_b^0 \to \Sigma'^+ K^-) = -\delta^l(\Lambda_b^0 \to \Delta^+\pi^-),$$
$$\delta^l(\Xi_0^b \to \Sigma'^0 \overline{K^0}) = -\delta^l(\Lambda_b^0 \to \Sigma'^0 K^0),$$
$$\delta^l(\Xi_b^- \to \Sigma'^0 K^-) = \frac{1}{2}\delta^l(\Xi_b^- \to \Xi'^0\pi^-) = -\frac{1}{2}\delta^l(\Xi_b^- \to \Delta^0 K^-) = -\delta^l(\Xi_b^- \to \Sigma'^0\pi^-),$$
$$\delta^l(\Lambda_b^0 \to \Sigma'^-\pi^+) = \delta^l(\Lambda_b^0 \to \Xi'^- K^+) = \delta^l(\Xi_b^0 \to \Xi'^-\pi^+) = \frac{1}{3}\delta^l(\Xi_b^0 \to \Omega^- K^+)$$
$$= -\frac{1}{3}\delta^l(\Lambda_b^0 \to \Delta^+\pi^-) = -\delta^l(\Lambda_b^0 \to \Sigma'^- K^+) = -\delta^l(\Xi_b^0 \to \Sigma'^-\pi^+) = -\delta^l(\Xi_b^0 \to \Xi'^- K^+),$$
$$\delta^l(\Xi_b^- \to \Sigma'^- \overline{K^0}) = 2\delta^l(\Xi_b^- \to \Xi'^-\pi^0) = \frac{2}{3}\delta^l(\Xi_b^- \to \Xi'^-\eta_8) = \frac{1}{3}\delta^l(\Xi_b^- \to \Omega^- K^0)$$
$$= -\frac{1}{3}\delta^l(\Xi_b^- \to \Delta^- \overline{K^0}) = -\frac{2}{3}\delta^l(\Xi_b^- \to \Sigma'^-\eta_8) = -\delta^l(\Xi_b^- \to \Xi'^- K^0) = -2\delta^l(\Xi_b^- \to \Sigma'^-\pi^0).$$

for both $l = \mathsf{P}$ and $l = \mathsf{D}$. Since, A_{CP} depends on the masses of the initial and final baryons as well as the final state meson [23, 115], some approximation is needed to obtain A_{CP} relations between various modes. In the U-spin limit [24], by ignoring $\mathbf{p}_{\mathcal{D}}$ and $m_{\mathcal{D}}$ differences between such modes, CP violation relations can be experimentally verified using the relation [24, 27, 130],

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{D}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \to \mathcal{D}_m \mathcal{M}_n)} \simeq -\frac{\tau_{\mathcal{B}_{bi}}}{\tau_{\mathcal{B}_{bl}}} \frac{\mathcal{BR}(\mathcal{B}_{bl} \to \mathcal{D}_m \mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{D}_j \mathcal{M}_k)}, \tag{5.73}$$

where i, j, k and l, m, n are indices corresponding to the various baryons belonging to the above mentioned δ_{CP} relations. There is a further simplification in case $i = l$, resulting in

$$\frac{A_{CP}(\mathcal{B}_{bi} \to \mathcal{D}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bi} \to \mathcal{D}_m \mathcal{M}_n)} \simeq -\frac{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{D}_m \mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \to \mathcal{D}_j \mathcal{M}_k)}, \tag{5.74}$$

where the uncertainties due to lifetime measurement also cancel out [25]. The decay asymmetry parameter (α) can be measured from an angular distribution study of the final states provided that the subsequent decay of the decuplet baryon is parity violating. The relative strength of the D-wave contribution [23, 116] is extracted from α;

$$\alpha = \frac{2\mathrm{Re}(\mathcal{A}^{\mathsf{P}*}\mathcal{A}^{\mathsf{D}})|\mathbf{p}_{\mathcal{D}}|/E_{\mathcal{D}} + m_{\mathcal{D}}}{|\mathcal{A}^{\mathsf{P}}|^2 + |\mathcal{A}^{\mathsf{D}}|^2(|\mathbf{p}_{\mathcal{D}}|/E_{\mathcal{D}} + m_{\mathcal{D}})^2}. \tag{5.75}$$

By systematically taking into account both partial wave contributions, a reliable prediction for A_{CP} relations is possible.

5.6 Summary

We have explored hadronic anti-triplet ($\overline{\mathbf{3}}$) beauty-baryon into a decuplet baryon ($\mathcal{D}$) and an octet pseudoscalar meson ($\mathcal{M}$), i.e. $\mathcal{B}_b(\overline{\mathbf{3}}) \to \mathcal{D}(\mathbf{10})\,\mathcal{M}(\mathbf{8})$, based on $SU(3)$ decomposition of the decay amplitudes in a general framework. This extends our previous analysis [13] of the anti-triplet beauty baryon decays into the octet or singlet of a light baryon and a pseudoscalar meson and completes the application of the method to decays involving any non-charmed baryon. We have shown that in the most general case, the forty distinct decay modes require forty independent reduced $SU(3)$ amplitudes to describe all possible $\Delta S = -1$ and $\Delta S = 0$ processes. The dimension-6 effective Hamiltonian and allowed final state $SU(3)$ representations constrain the number of independent $SU(3)$-reduced matrix elements to five. An alternative approach in terms of quark diagrams is also provided and compared with the $SU(3)$ decomposition in the limit of exact $SU(3)$-flavor symmetry. We explicitly demonstrate a one to one correspondence between the quark-diagrams and $SU(3)$-reduced matrix elements. Both the approaches indicate that there exist several amplitude relations between different decay modes. We explicitly derive those sum rules relations between decay amplitudes as well as relations between CP asymmetries. We further probe the $SU(3)$-breaking effects in the decay amplitudes at leading order in the $SU(3)$-breaking parameter and identify those amplitude relations that survive even when the $SU(3)$ flavor symmetry is arbitrarily broken.

Chapter 6

Conclusions and outlook

The Standard Model is undoubtedly the most significant step towards understanding the interactions between fundamental particles. Over the years, we have realized that symmetries are at the heart of the SM which is highlighted by a large class of experimental observations till date. In fact, this proof of principle have enabled us to construct beyond Standard Model theories with additional symmetries in order to explain some of the unanswered issues of SM like matter-antimatter asymmetry, the existence of dark matter, instabilities in the Higgs mass and Higgs vacuum expectation value and tiny neutrino masses. While the direct signature of such BSM physics are being actively pursued in high energy experiments, we must not overlook its imprints on precision measurements of various low energy decays of hadrons. In that regard, the interest in non-leptonic weak decays of bottom baryons has been steadily increasing due to the possibility of observing them in large numbers at LHCb in near future. Despite of its similarities with B-meson hadronic decays, the theoretical understanding of analogous b-baryon decays are rather limited and as a result theoretical predictions suffer from a large amount of uncertainty. In this thesis, we have provided a general framework based on $SU(3)$-flavor symmetry to analyze such non-leptonic charmless weak decays of bottom baryons.

First, we focus on all possible strangeness changing and strangeness preserving decays of $SU(3)$-flavor anti-triplet ($\overline{\mathbf{3}}$) b-baryons to an octet baryon and a pseudoscalar meson. We systematically build up the formalism where we decompose individual decay processes in terms of a few independent $SU(3)$-reduced amplitudes. Starting with an arbitrary Hamiltonian we first demonstrate that the number of independent $SU(3)$-reduced amplitudes is exactly equal to the number of distinct decay process. From that point, we choose the unbroken dimension-6 effective Hamiltonian and narrow down the number of independent $SU(3)$ parameters to 10. This opens up the possibility to derive a number of amplitude relations between decay modes. We restrict ourselves to amplitude relations involving two decay modes and three decay modes. Since the same relations hold for the CP conjugate processes we then attempt to convert those amplitude relations to relations between CP-asymmetry observables. We then consider $SU(3)$-breaking effects to the linear order in the breaking parameter and find that the number of independent $SU(3)$-reduced amplitudes increases. As a consequence, most of the amplitude relations derived previously cease to hold and the remaining unbroken ones are identified by us. Our framework also allows us to identify a number of general amplitude relations between decay modes that hold even when $SU(3)$-flavor symmetry breaking is not restricted to linear order.

Next, we extend our analysis of $SU(3)$-flavor anti-triplet ($\overline{\mathbf{3}}$) b-baryon charmless decays to a decuplet baryon and a pseudoscalar meson final state. We apply the same formalism developed in the previous case to find only 5 independent $SU(3)$-reduced amplitudes describing all possible decays with our choice of the dimension-6 effective Hamiltonian. We also provide an alternate description of the decays in terms of five topological flavor-flow diagrams and show its equivalence with the $SU(3)$-decomposition of amplitude method. The diagrammatic approach highlight the fact that a couple of flavor-flow toplogies have a vanishing contribution in the $SU(3)$-flavor limit. This further reduces the number of independent amplitudes to 3. In order to test this assertion, we have come up with number of amplitude relations that will only be satisfied if the number of independent parameters are indeed three. On the other hand, the amplitude relations inferred directly from the five independent $SU(3)$-reduced amplitudes are also listed. The linear $SU(3)$-breaking effects incorporated in the effective Hamiltonian once again introduces more parameters. We still find a number of amplitude relation that remain unaffected. The CP-asymmetry relations are derived in the end.

In both of these works we have focused on deriving relations between decay modes that can be verified in LHCb in its future runs. In absence of robust theoretical estimates, this approach would provide at least a qualitative understanding of b-baryon decays. In light of the recent data on $B \to K\pi$ decays that are related by isospin, it is particularly important to test analogous isospin relations that are unaffected by $SU(3)$-breaking corrections. However, there are several other interesting directions one can take. To begin with, we can directly fit the independent $SU(3)$ parameters to the actual data once we measure a number of branching ratios and CP-asymmetries of different decay modes. This approach is particularly relevant for b-baryons decaying to a decuplet baryon-pseudoscalar meson pair where non-factorizable contributions dominate the decay but requires only a few $SU(3)$ parameters to fit the data. Interestingly, all amplitude relations and CP-asymmetry relations for b-baryon decaying to a ground state baryon and vector meson can be obtained by just replacing the pseudoscalar meson with a vector meson in the previously derived relations in this thesis. In addition to two body decays, we can consider three-body decays of b-baryons which are also being observed at LHCb. If CP-violation is conclusively measured in a particular three body decay, it provides a strong motivation to identify all those decay modes that are related by $SU(3)$-flavor. This can be achieved quite straightforwardly using the methodology provided in this thesis.

The formalism developed in this thesis can be applied to other weak decays of heavy hadrons. Perhaps the most important application of this technique is in hadronic weak decays of charm mesons and baryons where the massive data set from Belle II, BESIII and LHCb can be used effectively to estimate the $SU(3)$ parameters and test a number of amplitude relations in near future.

Appendix A

A.0.0.1 $SU(3)$-decomposition of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Sigma^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to p^+K^-) \\
\mathcal{A}(\Lambda_b^0 \to n\bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^-K^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^0K^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0K^-) \\
\mathcal{A}(\Xi_b^- \to \Xi^-\pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0\pi^0)
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{3}{2\sqrt{10}} & \frac{3}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & 0 & 0 & 0 & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{2} & \frac{1}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\
-\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & 0 & 0 & 0 & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{2} & \frac{1}{3} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\
-\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{5}} & -\frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{6\sqrt{10}} & \frac{1}{6\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{\sqrt{\frac{3}{5}}}{2} & \frac{2}{3} & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & 0 & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\
-\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{4\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{4} & 0 & 0 & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 & 0 \\
\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & 0 & -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 & 0 \\
\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{4\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{4} & 0 & 0 & -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\
\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & \frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{15}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & 0 & -\frac{1}{3\sqrt{10}} & \frac{1}{6\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{\frac{3}{5}}}{4} & \frac{1}{6} & -\frac{1}{3\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{4} & -\frac{1}{4\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\
-\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & 0 & -\frac{1}{3\sqrt{10}} & \frac{1}{6\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{\frac{3}{5}}}{4} & \frac{1}{6} & -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} \\
\frac{1}{4} & -\frac{1}{4} & \frac{1}{2\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{\frac{3}{5}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & -\frac{\sqrt{\frac{3}{2}}}{4} & \frac{1}{4\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\
-\frac{1}{4} & \frac{1}{4} & \frac{1}{2\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{\frac{3}{5}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{\sqrt{\frac{3}{2}}}{4} & -\frac{1}{4\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\
-\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & \frac{1}{\sqrt{30}} & -\sqrt{\frac{2}{15}} & -\frac{\sqrt{\frac{3}{5}}}{4} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & \frac{1}{6\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{3}{4\sqrt{10}} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{3\sqrt{2}} & -\frac{1}{3} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{6}} \\
\frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & -\frac{1}{\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{\sqrt{\frac{3}{5}}}{4} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & \frac{1}{6\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{3}{4\sqrt{10}} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{3\sqrt{2}} & -\frac{1}{3} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & \frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{15}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & 0 & \frac{1}{3\sqrt{10}} & -\frac{1}{6\sqrt{5}} & \frac{3}{4\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{4} & -\frac{1}{6} & \frac{1}{3\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{1}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} \\
\frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & -\frac{1}{\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{\sqrt{\frac{3}{5}}}{4} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & -\frac{1}{6\sqrt{5}} & \frac{1}{6\sqrt{10}} & -\frac{3}{4\sqrt{10}} & \frac{\sqrt{\frac{3}{10}}}{4} & -\frac{1}{3\sqrt{2}} & \frac{1}{3} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{6}} \\
-\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & 0 & \frac{1}{3\sqrt{10}} & -\frac{1}{6\sqrt{5}} & \frac{3}{4\sqrt{5}} & -\frac{\sqrt{\frac{3}{5}}}{4} & -\frac{1}{6} & \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{4} & -\frac{1}{4\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\
-\frac{1}{4} & \frac{1}{4} & -\frac{1}{2\sqrt{30}} & -\sqrt{\frac{2}{15}} & \frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{\frac{3}{5}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & -\frac{\sqrt{\frac{3}{2}}}{4} & \frac{1}{4\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\
\frac{1}{4} & -\frac{1}{4} & -\frac{1}{2\sqrt{30}} & -\sqrt{\frac{2}{15}} & \frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{\frac{3}{5}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{\sqrt{\frac{3}{10}}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{\sqrt{\frac{3}{2}}}{4} & -\frac{1}{4\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\
-\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & \frac{1}{\sqrt{30}} & -\sqrt{\frac{2}{15}} & -\frac{\sqrt{\frac{3}{5}}}{4} & \frac{\sqrt{\frac{3}{5}}}{4} & 0 & -\frac{1}{6\sqrt{5}} & \frac{1}{6\sqrt{10}} & -\frac{3}{4\sqrt{10}} & \frac{\sqrt{\frac{3}{10}}}{4} & -\frac{1}{3\sqrt{2}} & \frac{1}{3} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}}
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \rangle \\
\langle 10 \,\|\, \mathbf{15'_1} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{24_1} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{42_1} \,\|\, \bar{3} \rangle \\
\langle \overline{10} \,\|\, \mathbf{24_1} \,\|\, \bar{3} \rangle \\
\langle \overline{10} \,\|\, \mathbf{\bar{6}_1} \,\|\, \bar{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{\bar{6}_1} \,\|\, \bar{3} \rangle \\
\langle 1 \,\|\, \mathbf{3_0} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{42_0} \,\|\, \bar{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{3_0} \,\|\, \bar{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{24_2} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{42_2} \,\|\, \bar{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{\bar{6}_1} \,\|\, \bar{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{3_0} \,\|\, \bar{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \rangle \\
\langle 10 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \rangle \\
\langle \overline{10} \,\|\, \mathbf{24_2} \,\|\, \bar{3} \rangle
\end{pmatrix}
$$

A.0.0.2 $SU(3)$-decomposition of tree part of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Sigma^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^-\pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to p^+K^-) \\
\mathcal{A}(\Lambda_b^0 \to n\bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^-K^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^0K^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0K^-) \\
\mathcal{A}(\Xi_b^- \to \Xi^-\pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0\pi^0)
\end{pmatrix}
=
\begin{pmatrix}
-\frac{C^+_{1,2}}{4\sqrt{3}} & -\frac{\sqrt{3}C^+_{1,2}}{10} & \frac{C^-_{1,2}}{4\sqrt{3}} & -\frac{C^+_{1,2}}{5\sqrt{3}} & \frac{C^-_{1,2}}{\sqrt{30}} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{9C^+_{1,2}}{40} & 0 & \frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{2\sqrt{15}} & 0 & 0 & 0 \\
-\frac{C^+_{1,2}}{12} & \frac{C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & \frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{2\sqrt{15}} & -\frac{C^+_{1,2}}{3\sqrt{5}} & \frac{C^-_{1,2}}{3\sqrt{2}} & 0 \\
\frac{C^+_{1,2}}{12} & \frac{C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & \frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{2\sqrt{15}} & \frac{C^+_{1,2}}{3\sqrt{5}} & -\frac{C^-_{1,2}}{3\sqrt{2}} & 0 \\
\frac{C^+_{1,2}}{4\sqrt{3}} & -\frac{\sqrt{3}C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{4\sqrt{3}} & -\frac{C^+_{1,2}}{5\sqrt{3}} & \frac{C^-_{1,2}}{\sqrt{30}} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{C^+_{1,2}}{40} & 0 & -\frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{2\sqrt{15}} & 0 & 0 & 0 \\
\frac{C^+_{1,2}}{12} & -\frac{7C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & \frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{4\sqrt{15}} & -\frac{\sqrt{5}C^+_{1,2}}{12} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
\frac{C^+_{1,2}}{12} & -\frac{C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & \frac{3C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{15}} & \frac{C^+_{1,2}}{12\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{12} & -\frac{7C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & \frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{4\sqrt{15}} & \frac{\sqrt{5}C^+_{1,2}}{12} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{12} & -\frac{C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & \frac{3C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{15}} & -\frac{C^+_{1,2}}{12\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
0 & \frac{C^+_{1,2}}{10} & \frac{C^-_{1,2}}{6} & \frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & -\frac{C^+_{1,2}}{4\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
0 & \frac{C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{6} & \frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & \frac{C^+_{1,2}}{4\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
0 & \frac{1}{5}\sqrt{\frac{3}{2}}C^+_{1,2} & 0 & \frac{1}{20}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{2C^-_{1,2}-\sqrt{2}C^+_{1,2}}{8\sqrt{5}} & \frac{1}{4}\sqrt{\frac{3}{10}}C^+_{1,2} & \frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(2C^-_{1,2}-\sqrt{2}C^+_{1,2}\right) \\
0 & \frac{1}{5}\sqrt{\frac{3}{2}}C^+_{1,2} & 0 & \frac{1}{20}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{2C^-_{1,2}-\sqrt{2}C^+_{1,2}}{8\sqrt{5}} & -\frac{1}{4}\sqrt{\frac{3}{10}}C^+_{1,2} & -\frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(\sqrt{2}C^+_{1,2}-2C^-_{1,2}\right) \\
0 & \frac{\sqrt{2}C^+_{1,2}}{5} & \frac{C^-_{1,2}}{3\sqrt{2}} & -\frac{3C^+_{1,2}}{20\sqrt{2}} & -\frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{40}\left(\sqrt{30}C^+_{1,2}-2\sqrt{15}C^-_{1,2}\right) & \frac{C^+_{1,2}}{4\sqrt{10}} & \frac{C^-_{1,2}}{12} & \frac{1}{24}\left(2\sqrt{3}C^-_{1,2}-\sqrt{6}C^+_{1,2}\right) \\
0 & \frac{\sqrt{2}C^+_{1,2}}{5} & -\frac{C^-_{1,2}}{3\sqrt{2}} & -\frac{3C^+_{1,2}}{20\sqrt{2}} & -\frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{40}\left(\sqrt{30}C^+_{1,2}-2\sqrt{15}C^-_{1,2}\right) & -\frac{C^+_{1,2}}{4\sqrt{10}} & -\frac{C^-_{1,2}}{12} & \frac{1}{24}\left(\sqrt{6}C^+_{1,2}-2\sqrt{3}C^-_{1,2}\right) \\
\frac{C^+_{1,2}}{6} & \frac{C^+_{1,2}}{5} & \frac{C^-_{1,2}}{6} & \frac{C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(C^+_{1,2}-\sqrt{2}C^-_{1,2}\right) & -\frac{C^+_{1,2}}{12\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{3C^+_{1,2}}{10\sqrt{2}} & -\frac{C^-_{1,2}}{3\sqrt{2}} & -\frac{C^+_{1,2}}{20\sqrt{2}} & -\frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{8}\sqrt{\frac{3}{5}}\left(2C^-_{1,2}-\sqrt{2}C^+_{1,2}\right) & -\frac{C^+_{1,2}}{12\sqrt{10}} & -\frac{C^-_{1,2}}{12} & \frac{1}{24}\left(2\sqrt{3}C^-_{1,2}-\sqrt{6}C^+_{1,2}\right) \\
-\frac{C^+_{1,2}}{6} & \frac{C^+_{1,2}}{5} & -\frac{C^-_{1,2}}{6} & \frac{C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(C^+_{1,2}-\sqrt{2}C^-_{1,2}\right) & \frac{C^+_{1,2}}{12\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{2\sqrt{6}} & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & 0 & -\frac{C^+_{1,2}}{20\sqrt{6}} & -\frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{2C^-_{1,2}-\sqrt{2}C^+_{1,2}}{8\sqrt{5}} & -\frac{C^+_{1,2}}{4\sqrt{30}} & -\frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(2C^-_{1,2}-\sqrt{2}C^+_{1,2}\right) \\
\frac{C^+_{1,2}}{2\sqrt{6}} & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & 0 & -\frac{C^+_{1,2}}{20\sqrt{6}} & -\frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{2C^-_{1,2}-\sqrt{2}C^+_{1,2}}{8\sqrt{5}} & \frac{C^+_{1,2}}{4\sqrt{30}} & \frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(\sqrt{2}C^+_{1,2}-2C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{3C^+_{1,2}}{10\sqrt{2}} & \frac{C^-_{1,2}}{3\sqrt{2}} & -\frac{C^+_{1,2}}{20\sqrt{2}} & -\frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{8}\sqrt{\frac{3}{5}}\left(2C^-_{1,2}-\sqrt{2}C^+_{1,2}\right) & \frac{C^+_{1,2}}{12\sqrt{10}} & \frac{C^-_{1,2}}{12} & \frac{1}{24}\left(\sqrt{6}C^+_{1,2}-2\sqrt{3}C^-_{1,2}\right)
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle \overline{10} \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle
\end{pmatrix}
$$

A.0.0.3 $SU(3)$-decomposition of penguin part of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\,\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Sigma^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^+ \pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^- \pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to p^+ K^-) \\
\mathcal{A}(\Lambda_b^0 \to n \bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^- K^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi^0 K^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^0 \pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^- \bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi^- \eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Xi^- \pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0 K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^- \pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0 \bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0 \eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0 \bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi^0 \pi^0)
\end{pmatrix}
=
\begin{pmatrix}
\frac{\sqrt{3}C^{+}_{9,10}}{8} & \frac{3\sqrt{3}C^{+}_{9,10}}{20} & -\frac{\sqrt{3}C^{-}_{9,10}}{8} & \frac{\sqrt{3}C^{+}_{9,10}}{10} & -\frac{1}{2}\sqrt{\frac{3}{10}}C^{-}_{9,10} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{27C^{+}_{9,10}}{80} & 0 & -\frac{3C^{+}_{9,10}}{20} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & 0 & 0 & 0 \\
\frac{C^{+}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{20} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{2\sqrt{5}} & -\frac{C^{-}_{9,10}}{2\sqrt{2}} & 0 \\
-\frac{C^{+}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{20} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{2\sqrt{5}} & \frac{C^{-}_{9,10}}{2\sqrt{2}} & 0 \\
-\frac{\sqrt{3}C^{+}_{9,10}}{8} & \frac{3\sqrt{3}C^{+}_{9,10}}{20} & \frac{\sqrt{3}C^{-}_{9,10}}{8} & \frac{\sqrt{3}C^{+}_{9,10}}{10} & -\frac{1}{2}\sqrt{\frac{3}{10}}C^{-}_{9,10} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{3C^{+}_{9,10}}{80} & 0 & \frac{3C^{+}_{9,10}}{20} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & 0 & 0 & 0 \\
-\frac{C^{+}_{9,10}}{8} & \frac{21C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & \frac{\sqrt{5}C^{+}_{9,10}}{8} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
-\frac{C^{+}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & -\frac{9C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & -\frac{C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
\frac{C^{+}_{9,10}}{8} & \frac{21C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & -\frac{\sqrt{5}C^{+}_{9,10}}{8} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
\frac{C^{+}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & -\frac{9C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & \frac{C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & -\frac{3C^{+}_{9,10}}{20} & -\frac{C^{-}_{9,10}}{4} & -\frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{3C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & -\frac{3C^{+}_{9,10}}{20} & \frac{C^{-}_{9,10}}{4} & -\frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{3C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & -\frac{3}{10}\sqrt{\frac{3}{2}}C^{+}_{9,10} & 0 & -\frac{3}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & -\frac{3}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & -\frac{\sqrt{3}C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
0 & -\frac{3}{10}\sqrt{\frac{3}{2}}C^{+}_{9,10} & 0 & -\frac{3}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & \frac{3}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & \frac{\sqrt{3}C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
0 & -\frac{3C^{+}_{9,10}}{5\sqrt{2}} & -\frac{C^{-}_{9,10}}{2\sqrt{2}} & \frac{9C^{+}_{9,10}}{40\sqrt{2}} & \frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{3C^{+}_{9,10}}{8\sqrt{10}} & -\frac{C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}} \\
0 & -\frac{3C^{+}_{9,10}}{5\sqrt{2}} & \frac{C^{-}_{9,10}}{2\sqrt{2}} & \frac{9C^{+}_{9,10}}{40\sqrt{2}} & \frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{3C^{+}_{9,10}}{8\sqrt{10}} & \frac{C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}} \\
-\frac{C^{+}_{9,10}}{4} & -\frac{3C^{+}_{9,10}}{10} & -\frac{C^{-}_{9,10}}{4} & -\frac{3C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & -\sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
-\frac{C^{+}_{9,10}}{4\sqrt{2}} & -\frac{9C^{+}_{9,10}}{20\sqrt{2}} & \frac{C^{-}_{9,10}}{2\sqrt{2}} & \frac{3C^{+}_{9,10}}{40\sqrt{2}} & \frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & \frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{8\sqrt{10}} & \frac{C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}} \\
\frac{C^{+}_{9,10}}{4} & -\frac{3C^{+}_{9,10}}{10} & \frac{C^{-}_{9,10}}{4} & -\frac{3C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & -\sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
\frac{1}{4}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & 0 & \frac{1}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & \frac{1}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & \frac{\sqrt{3}C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
-\frac{1}{4}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & 0 & \frac{1}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & -\frac{1}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & -\frac{\sqrt{3}C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
\frac{C^{+}_{9,10}}{4\sqrt{2}} & -\frac{9C^{+}_{9,10}}{20\sqrt{2}} & -\frac{C^{-}_{9,10}}{2\sqrt{2}} & \frac{3C^{+}_{9,10}}{40\sqrt{2}} & \frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & \frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{8\sqrt{10}} & -\frac{C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle \overline{10} \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle
\end{pmatrix}
$$

A.0.0.4 $SU(3)$-decomposition of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to n\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^- K^+) \\
\mathcal{A}(\Lambda_b^0 \to p^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to n\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0 K^0) \\
\mathcal{A}(\Xi_b^- \to nK^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0\pi^-) \\
\mathcal{A}(\Xi_b^- \Xi^- K^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\pi^0) \\
\mathcal{A}(\Xi_b^0 \Xi^- K^+) \\
\mathcal{A}(\Xi_b^0 \Sigma^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+\pi^-) \\
\mathcal{A}(\Xi_b^0 \to n\overline{K^0}) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\pi^0) \\
\mathcal{A}(\Xi_b^0 \to p^+K^-) \\
\mathcal{A}(\Xi_b^0 \Xi^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\pi^0)
\end{pmatrix}
=
\begin{pmatrix}
\frac{1}{\sqrt5} & \frac{1}{2\sqrt5} & \frac{1}{\sqrt5} & \frac{\sqrt{\frac35}}{2} & \frac{1}{\sqrt{10}} & -\frac{\sqrt{\frac{3}{10}}}{4} & -\frac{\sqrt{\frac{3}{10}}}{4} & -\frac{1}{4\sqrt5} & \frac{\sqrt{\frac32}}{4} & \frac{\sqrt{\frac32}}{4} & \frac14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{\frac{2}{15}}}{3} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{\frac{2}{15}}}{3} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{15}} & \frac{3}{4\sqrt5} & \frac{3}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac14 & -\frac14 & -\frac{1}{2\sqrt6} & \frac{1}{2\sqrt2} & \frac{1}{2\sqrt6} & \frac{\sqrt{\frac53}}{6} & \frac{\sqrt{\frac23}}{3} & \frac{\sqrt{\frac53}}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{\frac{2}{15}}}{3} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{\frac{2}{15}}}{3} & -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{15}} & \frac{3}{4\sqrt5} & \frac{3}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & \frac14 & \frac14 & \frac{1}{2\sqrt6} & -\frac{1}{2\sqrt2} & -\frac{1}{2\sqrt6} & \frac{\sqrt{\frac53}}{6} & \frac{\sqrt{\frac23}}{3} & \frac{\sqrt{\frac53}}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{3\sqrt{15}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{15}} & \frac{1}{2\sqrt5} & \frac{1}{\sqrt{30}} & -\frac{3}{4\sqrt{10}} & -\frac{3}{4\sqrt{10}} & -\frac{\sqrt{\frac35}}{4} & -\frac{1}{4\sqrt2} & -\frac{1}{4\sqrt2} & -\frac{1}{4\sqrt3} & -\frac12 & -\frac{1}{2\sqrt3} & \frac{\sqrt{\frac56}}{3} & \frac{2}{3\sqrt3} & \frac{\sqrt{\frac56}}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{\sqrt5} & \frac{1}{2\sqrt5} & \frac{1}{\sqrt5} & -\frac{\sqrt{\frac35}}{2} & -\frac{1}{\sqrt{10}} & -\frac{\sqrt{\frac{3}{10}}}{4} & -\frac{\sqrt{\frac{3}{10}}}{4} & -\frac{1}{4\sqrt5} & -\frac{\sqrt{\frac32}}{4} & -\frac{\sqrt{\frac32}}{4} & -\frac14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{3\sqrt{15}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{15}} & -\frac{1}{2\sqrt5} & -\frac{1}{\sqrt{30}} & -\frac{3}{4\sqrt{10}} & -\frac{3}{4\sqrt{10}} & -\frac{\sqrt{\frac35}}{4} & \frac{1}{4\sqrt2} & \frac{1}{4\sqrt2} & \frac{1}{4\sqrt3} & \frac12 & \frac{1}{2\sqrt3} & \frac{\sqrt{\frac56}}{3} & \frac{2}{3\sqrt3} & \frac{\sqrt{\frac56}}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac25}}{3} & \frac{1}{\sqrt{15}} & -\frac{3}{4\sqrt5} & \frac{1}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & \frac14 & -\frac{1}{12} & -\frac{1}{2\sqrt6} & -\frac{1}{6\sqrt2} & \frac{1}{2\sqrt6} & \frac{1}{6\sqrt{15}} & \frac{1}{3\sqrt6} & -\frac{\sqrt{\frac53}}{6} & \frac13 & -\frac{1}{3\sqrt2} & \frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt2} & 0 & 0 \\
\frac{2}{3\sqrt5} & -\frac{2}{3\sqrt5} & -\frac{1}{3\sqrt5} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{10}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt5} & 0 & 0 & 0 & \frac{1}{4\sqrt3} & -\frac14 & \frac{1}{6\sqrt{10}} & \frac16 & -\frac{\sqrt{\frac52}}{6} & -\frac{1}{\sqrt6} & -\frac{1}{2\sqrt3} & -\frac{1}{\sqrt{15}} & 0 & 0 & 0 \\
\frac{2}{3\sqrt5} & -\frac{2}{3\sqrt5} & -\frac{1}{3\sqrt5} & \frac{1}{\sqrt{15}} & -\frac{1}{\sqrt{10}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt5} & 0 & 0 & 0 & -\frac{1}{4\sqrt3} & \frac14 & \frac{1}{6\sqrt{10}} & \frac16 & -\frac{\sqrt{\frac52}}{6} & \frac{1}{\sqrt6} & \frac{1}{2\sqrt3} & -\frac{1}{\sqrt{15}} & 0 & 0 & 0 \\
\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{2}{15}}}{3} & \frac{\sqrt{\frac25}}{3} & -\frac{1}{\sqrt{15}} & -\frac{3}{4\sqrt5} & \frac{1}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac14 & \frac{1}{12} & \frac{1}{2\sqrt6} & \frac{1}{6\sqrt2} & -\frac{1}{2\sqrt6} & \frac{1}{6\sqrt{15}} & \frac{1}{3\sqrt6} & -\frac{\sqrt{\frac53}}{6} & -\frac13 & \frac{1}{3\sqrt2} & \frac{1}{\sqrt{10}} & \frac{1}{3\sqrt2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{3\sqrt5} & -\frac{1}{\sqrt{30}} & 0 & 0 & 0 & \frac{1}{2\sqrt2} & -\frac{1}{6\sqrt2} & -\frac{1}{2\sqrt3} & \frac{1}{12} & -\frac{1}{4\sqrt3} & \frac{\sqrt{\frac56}}{2} & -\frac{1}{2\sqrt3} & -\frac{1}{2\sqrt{30}} & -\frac{1}{3\sqrt2} & \frac16 & 0 & -\frac13 & -\frac{1}{\sqrt5} & 0 \\
0 & 0 & 0 & -\frac{1}{3\sqrt5} & \frac{1}{\sqrt{30}} & 0 & 0 & 0 & -\frac{1}{2\sqrt2} & \frac{1}{6\sqrt2} & \frac{1}{2\sqrt3} & -\frac{1}{12} & \frac{1}{4\sqrt3} & \frac{\sqrt{\frac56}}{2} & -\frac{1}{2\sqrt3} & -\frac{1}{2\sqrt{30}} & \frac{1}{3\sqrt2} & -\frac16 & 0 & \frac13 & -\frac{1}{\sqrt5} & 0 \\
\frac{1}{6\sqrt{30}} & -\frac{\sqrt{\frac56}}{3} & \frac{\sqrt{\frac{2}{15}}}{3} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt5} & \frac{1}{2\sqrt5} & 0 & 0 & -\frac13 & \frac{1}{\sqrt6} & -\frac{1}{6\sqrt2} & \frac{1}{2\sqrt6} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt6} & \frac{\sqrt{\frac53}}{6} & -\frac16 & -\frac{1}{3\sqrt2} & -\frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt2} & 0 & \frac{1}{2\sqrt2} \\
\frac{1}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt5} & -\frac{3}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac14 & \frac{1}{12} & \frac{1}{2\sqrt6} & \frac{1}{6\sqrt2} & -\frac{1}{2\sqrt6} & \frac{\sqrt{\frac53}}{6} & -\frac{1}{3\sqrt6} & -\frac{1}{6\sqrt{15}} & \frac16 & \frac{1}{3\sqrt2} & 0 & -\frac{\sqrt2}{3} & \frac{1}{\sqrt{10}} & \frac{1}{2\sqrt2} \\
\frac{1}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt5} & -\frac{3}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & \frac14 & -\frac{1}{12} & -\frac{1}{2\sqrt6} & -\frac{1}{6\sqrt2} & \frac{1}{2\sqrt6} & \frac{\sqrt{\frac53}}{6} & -\frac{1}{3\sqrt6} & -\frac{1}{6\sqrt{15}} & -\frac16 & -\frac{1}{3\sqrt2} & 0 & \frac{\sqrt2}{3} & \frac{1}{\sqrt{10}} & \frac{1}{2\sqrt2} \\
\frac{7}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{4\sqrt5} & -\frac{1}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & \frac14 & -\frac{5}{12} & \frac{1}{2\sqrt6} & \frac{1}{6\sqrt2} & -\frac{1}{2\sqrt6} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt6} & \frac{\sqrt{\frac53}}{6} & \frac16 & \frac{1}{3\sqrt2} & -\frac{1}{\sqrt{10}} & \frac{1}{3\sqrt2} & 0 & -\frac{1}{2\sqrt2} \\
\frac{\sqrt{\frac25}}{3} & -\frac{\sqrt{\frac25}}{3} & -\frac{1}{3\sqrt{10}} & -\frac{1}{\sqrt{30}} & \frac{1}{2\sqrt5} & \frac{\sqrt{\frac35}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & 0 & -\frac{1}{2\sqrt6} & \frac{1}{2\sqrt2} & -\frac{1}{6\sqrt5} & -\frac{1}{3\sqrt2} & \frac{\sqrt5}{6} & -\frac{1}{2\sqrt3} & \frac{1}{\sqrt6} & \sqrt{\frac{2}{15}} & 0 & 0 & 0 \\
-\frac{1}{6\sqrt{30}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{30}} & 0 & 0 & -\frac{1}{4\sqrt5} & \frac{3}{4\sqrt5} & -\frac{\sqrt{\frac{3}{10}}}{2} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{\frac53}}{3} & -\frac{\sqrt{\frac23}}{3} & -\frac{1}{3\sqrt{15}} & 0 & 0 & 0 & 0 & \sqrt{\frac25} & -\frac{1}{2\sqrt2} \\
\frac{1}{6\sqrt{30}} & -\frac{\sqrt{\frac56}}{3} & \frac{\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt5} & \frac{1}{2\sqrt5} & 0 & 0 & \frac13 & -\frac{1}{\sqrt6} & \frac{1}{6\sqrt2} & -\frac{1}{2\sqrt6} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt6} & \frac{\sqrt{\frac53}}{6} & \frac16 & \frac{1}{3\sqrt2} & -\frac{1}{\sqrt{10}} & \frac{1}{3\sqrt2} & 0 & \frac{1}{2\sqrt2} \\
\frac{7}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{2\sqrt{\frac{2}{15}}}{3} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{4\sqrt5} & -\frac{1}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac14 & \frac{5}{12} & -\frac{1}{2\sqrt6} & -\frac{1}{6\sqrt2} & \frac{1}{2\sqrt6} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt6} & \frac{\sqrt{\frac53}}{6} & -\frac16 & -\frac{1}{3\sqrt2} & -\frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt2} & 0 & -\frac{1}{2\sqrt2} \\
-\frac{\sqrt{\frac{3}{10}}}{2} & -\sqrt{\frac{3}{10}} & \sqrt{\frac{3}{10}} & 0 & 0 & \frac{1}{4\sqrt5} & -\frac{3}{4\sqrt5} & \frac{\sqrt{\frac{3}{10}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt2} \\
\frac{\sqrt{\frac25}}{3} & -\frac{\sqrt{\frac25}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{\sqrt{30}} & -\frac{1}{2\sqrt5} & \frac{\sqrt{\frac35}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & 0 & \frac{1}{2\sqrt6} & -\frac{1}{2\sqrt2} & -\frac{1}{6\sqrt5} & -\frac{1}{3\sqrt2} & \frac{\sqrt5}{6} & \frac{1}{2\sqrt3} & -\frac{1}{\sqrt6} & \sqrt{\frac{2}{15}} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\langle 27 \parallel \mathbf{15_{1/2}} \parallel \overline{3} \rangle \\
\langle 27 \parallel \mathbf{24_{1/2}} \parallel \overline{3} \rangle \\
\langle 27 \parallel \mathbf{42_{1/2}} \parallel \overline{3} \rangle \\
\langle \overline{10} \parallel \mathbf{24_{1/2}} \parallel \overline{3} \rangle \\
\langle \overline{10} \parallel \mathbf{\overline{6}_{1/2}} \parallel\parallel \overline{3} \rangle \\
\langle 8_1 \parallel \mathbf{3_{1/2}} \parallel \overline{3} \rangle \\
\langle 8_1 \parallel \mathbf{15_{1/2}} \parallel \overline{3} \rangle \\
\langle 8_1 \parallel \mathbf{\overline{6}_{1/2}} \parallel \overline{3} \\
\langle 8_2 \parallel \mathbf{3_{1/2}} \parallel \overline{3} \\
\langle 8_2 \parallel \mathbf{15_{1/2}} \parallel \overline{3} \\
\langle 8_2 \parallel \mathbf{\overline{6}_{1/2}} \parallel \overline{3} \\
\langle 10 \parallel \mathbf{15_{3/2}} \parallel \overline{3} \\
\langle 10 \parallel \mathbf{15'_{3/2}} \parallel \overline{3} \\
\langle 27 \parallel \mathbf{15_{3/2}} \parallel \overline{3} \\
\langle 27 \parallel \mathbf{24_{3/2}} \parallel \overline{3} \\
\langle 27 \parallel \mathbf{42_{3/2}} \parallel \overline{3} \\
\langle 10 \parallel \mathbf{15_{1/2}} \parallel \overline{3} \\
\langle 10 \parallel \mathbf{24_{3/2}} \parallel \overline{3} \\
\langle 8_1 \parallel \mathbf{15_{3/2}} \parallel \overline{3} \\
\langle 8_2 \parallel \mathbf{15_{3/2}} \parallel \overline{3} \\
\langle 27 \parallel \mathbf{42_{5/2}} \parallel \overline{3} \\
\langle 1 \parallel \mathbf{3_{1/2}} \parallel \overline{3}
\end{pmatrix}
$$

A.0.0.5 $SU(3)$-decomposition of tree part of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to n\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^- K^+) \\
\mathcal{A}(\Lambda_b^0 \to p^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to n\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0 K^0) \\
\mathcal{A}(\Xi_b^- \to nK^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0\pi^-) \\
\mathcal{A}(\Xi_b^-\Xi^- K^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\pi^0) \\
\mathcal{A}(\Xi_b^0\Xi^- K^+) \\
\mathcal{A}(\Xi_b^0\Sigma^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+\pi^-) \\
\mathcal{A}(\Xi_b^0 \to n\overline{K^0}) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\pi^0) \\
\mathcal{A}(\Xi_b^0 \to p^+K^-) \\
\mathcal{A}(\Xi_b^0\Xi^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\pi^0)
\end{pmatrix}
=
\begin{pmatrix}
0 & -\frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & -\frac{C^-_{1,2}}{2\sqrt{6}} & \frac{C^+_{1,2}}{20\sqrt{6}} & \frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{1}{40}\left(\sqrt{10}C^+_{1,2}-2\sqrt{5}C^-_{1,2}\right) & -\frac{C^+_{1,2}}{4\sqrt{30}} & -\frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(2C^-_{1,2}-\sqrt{2}C^+_{1,2}\right) \\
-\frac{C^+_{1,2}}{6} & -\frac{C^+_{1,2}}{5} & -\frac{C^-_{1,2}}{6} & -\frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & \frac{C^+_{1,2}}{12\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
\frac{C^+_{1,2}}{6} & -\frac{C^+_{1,2}}{5} & \frac{C^-_{1,2}}{6} & -\frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & -\frac{C^+_{1,2}}{12\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
\frac{C^+_{1,2}}{3\sqrt{2}} & -\frac{3C^+_{1,2}}{10\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{20\sqrt{2}} & \frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{40}\left(\sqrt{30}C^+_{1,2}-2\sqrt{15}C^-_{1,2}\right) & \frac{C^+_{1,2}}{12\sqrt{10}} & \frac{C^-_{1,2}}{12} & \frac{1}{24}\left(\sqrt{6}C^+_{1,2}-2\sqrt{3}C^-_{1,2}\right) \\
0 & -\frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{2\sqrt{6}} & \frac{C^+_{1,2}}{20\sqrt{6}} & \frac{C^-_{1,2}}{4\sqrt{15}} & 0 & \frac{1}{40}\left(\sqrt{10}C^+_{1,2}-2\sqrt{5}C^-_{1,2}\right) & \frac{C^+_{1,2}}{4\sqrt{30}} & \frac{C^-_{1,2}}{4\sqrt{3}} & \frac{1}{8}\left(\sqrt{2}C^+_{1,2}-2C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{3\sqrt{2}} & -\frac{3C^+_{1,2}}{10\sqrt{2}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{20\sqrt{2}} & \frac{C^-_{1,2}}{4\sqrt{5}} & 0 & \frac{1}{40}\left(\sqrt{30}C^+_{1,2}-2\sqrt{15}C^-_{1,2}\right) & -\frac{C^+_{1,2}}{12\sqrt{10}} & -\frac{C^-_{1,2}}{12} & \frac{1}{24}\left(2\sqrt{3}C^-_{1,2}-\sqrt{6}C^+_{1,2}\right) \\
0 & \frac{C^+_{1,2}}{10} & \frac{C^-_{1,2}}{6} & \frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & -\frac{C^+_{1,2}}{4\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
0 & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{2\sqrt{6}} & -\frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & -\frac{C^-_{1,2}}{2\sqrt{15}} & 0 & \frac{1}{20}\left(\sqrt{10}C^+_{1,2}-2\sqrt{5}C^-_{1,2}\right) & 0 & 0 & 0 \\
0 & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & -\frac{C^-_{1,2}}{2\sqrt{6}} & -\frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & -\frac{C^-_{1,2}}{2\sqrt{15}} & 0 & \frac{1}{20}\left(\sqrt{10}C^+_{1,2}-2\sqrt{5}C^-_{1,2}\right) & 0 & 0 & 0 \\
0 & \frac{C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{6} & \frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & 0 & \frac{1}{4}\sqrt{\frac{3}{5}}\left(\sqrt{2}C^-_{1,2}-C^+_{1,2}\right) & \frac{C^+_{1,2}}{4\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
0 & \frac{C^+_{1,2}}{2\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{C^+_{1,2}}{2\sqrt{10}} & -\frac{C^-_{1,2}}{6} & \frac{1}{12}\left(\sqrt{6}C^+_{1,2}-2\sqrt{3}C^-_{1,2}\right) \\
0 & \frac{C^+_{1,2}}{2\sqrt{2}} & \frac{C^-_{1,2}}{6\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{C^+_{1,2}}{2\sqrt{10}} & \frac{C^-_{1,2}}{6} & \frac{1}{12}\left(2\sqrt{3}C^-_{1,2}-\sqrt{6}C^+_{1,2}\right) \\
-\frac{C^+_{1,2}}{12} & -\frac{C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & -\frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{2\sqrt{15}} & -\frac{C^+_{1,2}}{3\sqrt{5}} & \frac{C^-_{1,2}}{3\sqrt{2}} & 0 \\
\frac{C^+_{1,2}}{12} & \frac{7C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & -\frac{C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{15}} & -\frac{\sqrt{5}C^+_{1,2}}{12} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
-\frac{C^+_{1,2}}{12} & \frac{7C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & -\frac{C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{15}} & \frac{\sqrt{5}C^+_{1,2}}{12} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
\frac{C^+_{1,2}}{12} & \frac{C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & -\frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{4\sqrt{15}} & \frac{C^+_{1,2}}{12\sqrt{5}} & \frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) \\
-\frac{C^+_{1,2}}{4\sqrt{3}} & 0 & \frac{C^-_{1,2}}{4\sqrt{3}} & \frac{C^+_{1,2}}{4\sqrt{3}} & -\frac{C^-_{1,2}}{2\sqrt{30}} & 0 & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{5}} & 0 & 0 & 0 \\
0 & \frac{13C^+_{1,2}}{40} & 0 & \frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{4\sqrt{15}} & 0 & 0 & 0 \\
\frac{C^+_{1,2}}{12} & -\frac{C^+_{1,2}}{40} & \frac{C^-_{1,2}}{12} & -\frac{C^+_{1,2}}{10} & 0 & \frac{1}{24}\left(\sqrt{3}C^+_{1,2}-\sqrt{6}C^-_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{2\sqrt{15}} & \frac{C^+_{1,2}}{3\sqrt{5}} & -\frac{C^-_{1,2}}{3\sqrt{2}} & 0 \\
-\frac{C^+_{1,2}}{12} & \frac{C^+_{1,2}}{40} & -\frac{C^-_{1,2}}{12} & -\frac{3C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{\sqrt{2}C^-_{1,2}-C^+_{1,2}}{4\sqrt{15}} & -\frac{C^+_{1,2}}{12\sqrt{5}} & -\frac{C^-_{1,2}}{6\sqrt{2}} & \frac{1}{12}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) \\
0 & -\frac{3C^+_{1,2}}{40} & 0 & -\frac{C^+_{1,2}}{20} & \frac{C^-_{1,2}}{2\sqrt{10}} & \frac{1}{24}\left(\sqrt{6}C^-_{1,2}-\sqrt{3}C^+_{1,2}\right) & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{15}} & 0 & 0 & 0 \\
\frac{C^+_{1,2}}{4\sqrt{3}} & 0 & -\frac{C^-_{1,2}}{4\sqrt{3}} & \frac{C^+_{1,2}}{4\sqrt{3}} & -\frac{C^-_{1,2}}{2\sqrt{30}} & 0 & \frac{C^+_{1,2}-\sqrt{2}C^-_{1,2}}{4\sqrt{5}} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle \overline{10} \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle
\end{pmatrix}
$$

A.0.0.6 $SU(3)$-decomposition of penguin part of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to n\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^- K^+) \\
\mathcal{A}(\Lambda_b^0 \to p^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to n\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Lambda^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma^0 K^0) \\
\mathcal{A}(\Xi_b^- \to nK^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Lambda^0\pi^-) \\
\mathcal{A}(\Xi_b^- \Xi^- K^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma^-\pi^0) \\
\mathcal{A}(\Xi_b^0 \Xi^- K^+) \\
\mathcal{A}(\Xi_b^0 \Sigma^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^+\pi^-) \\
\mathcal{A}(\Xi_b^0 \to n\overline{K^0}) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma^0\pi^0) \\
\mathcal{A}(\Xi_b^0 \to p^+K^-) \\
\mathcal{A}(\Xi_b^0 \Xi^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Lambda^0\pi^0)
\end{pmatrix}
=
\begin{pmatrix}
0 & \frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{4}\sqrt{\frac{3}{2}}C^{-}_{9,10} & -\frac{1}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & \frac{1}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & \frac{\sqrt{3}C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
\frac{C^{+}_{9,10}}{4} & \frac{3C^{+}_{9,10}}{10} & \frac{C^{-}_{9,10}}{4} & \frac{3C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
-\frac{C^{+}_{9,10}}{4} & \frac{3C^{+}_{9,10}}{10} & -\frac{C^{-}_{9,10}}{4} & \frac{3C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
-\frac{C^{+}_{9,10}}{2\sqrt{2}} & \frac{9C^{+}_{9,10}}{20\sqrt{2}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{3C^{+}_{9,10}}{40\sqrt{2}} & -\frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{8\sqrt{10}} & -\frac{C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}} \\
0 & \frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{4}\sqrt{\frac{3}{2}}C^{-}_{9,10} & -\frac{1}{40}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{8}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{5}} & -\frac{1}{8}\sqrt{\frac{3}{10}}C^{+}_{9,10} & -\frac{\sqrt{3}C^{-}_{9,10}}{8} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2} \\
\frac{C^{+}_{9,10}}{2\sqrt{2}} & \frac{9C^{+}_{9,10}}{20\sqrt{2}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{3C^{+}_{9,10}}{40\sqrt{2}} & -\frac{3C^{-}_{9,10}}{8\sqrt{5}} & 0 & -\frac{1}{2}\sqrt{\frac{3}{5}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{8\sqrt{10}} & \frac{C^{-}_{9,10}}{8} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{3}} \\
0 & -\frac{3C^{+}_{9,10}}{20} & -\frac{C^{-}_{9,10}}{4} & -\frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{3C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & -\frac{1}{4}\sqrt{\frac{3}{2}}C^{-}_{9,10} & \frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{4}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{5}} & 0 & 0 & 0 \\
0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{4}\sqrt{\frac{3}{2}}C^{-}_{9,10} & \frac{3}{20}\sqrt{\frac{3}{2}}C^{+}_{9,10} & \frac{1}{4}\sqrt{\frac{3}{5}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{5}} & 0 & 0 & 0 \\
0 & -\frac{3C^{+}_{9,10}}{20} & \frac{C^{-}_{9,10}}{4} & -\frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & 0 & \sqrt{\frac{3}{10}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{3C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & -\frac{3C^{+}_{9,10}}{4\sqrt{2}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{3C^{+}_{9,10}}{4\sqrt{10}} & \frac{C^{-}_{9,10}}{4} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{3}} \\
0 & -\frac{3C^{+}_{9,10}}{4\sqrt{2}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{3C^{+}_{9,10}}{4\sqrt{10}} & -\frac{C^{-}_{9,10}}{4} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{3}} \\
\frac{C^{+}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{20} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & \frac{C^{+}_{9,10}}{2\sqrt{5}} & -\frac{C^{-}_{9,10}}{2\sqrt{2}} & 0 \\
-\frac{C^{+}_{9,10}}{8} & -\frac{21C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & \frac{\sqrt{5}C^{+}_{9,10}}{8} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
\frac{C^{+}_{9,10}}{8} & -\frac{21C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & -\frac{\sqrt{5}C^{+}_{9,10}}{8} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
-\frac{C^{+}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & \frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & -\frac{C^{+}_{9,10}}{8\sqrt{5}} & -\frac{C^{-}_{9,10}}{4\sqrt{2}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
\frac{\sqrt{3}C^{+}_{9,10}}{8} & 0 & -\frac{\sqrt{3}C^{-}_{9,10}}{8} & -\frac{\sqrt{3}C^{+}_{9,10}}{8} & \frac{1}{4}\sqrt{\frac{3}{10}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{10}} & 0 & 0 & 0 \\
0 & -\frac{39C^{+}_{9,10}}{80} & 0 & -\frac{3C^{+}_{9,10}}{40} & \frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & 0 & 0 & 0 \\
-\frac{C^{+}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{80} & -\frac{C^{-}_{9,10}}{8} & \frac{3C^{+}_{9,10}}{20} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \sqrt{\frac{2}{15}}-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10} & -\frac{C^{+}_{9,10}}{2\sqrt{5}} & \frac{C^{-}_{9,10}}{2\sqrt{2}} & 0 \\
\frac{C^{+}_{9,10}}{8} & -\frac{3C^{+}_{9,10}}{80} & \frac{C^{-}_{9,10}}{8} & \frac{9C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & \frac{C^{+}_{9,10}}{8\sqrt{5}} & \frac{C^{-}_{9,10}}{4\sqrt{2}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{6}} \\
0 & \frac{9C^{+}_{9,10}}{80} & 0 & \frac{3C^{+}_{9,10}}{40} & -\frac{3C^{-}_{9,10}}{4\sqrt{10}} & \frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{2\sqrt{6}} & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{30}} & 0 & 0 & 0 \\
-\frac{\sqrt{3}C^{+}_{9,10}}{8} & 0 & \frac{\sqrt{3}C^{-}_{9,10}}{8} & -\frac{\sqrt{3}C^{+}_{9,10}}{8} & \frac{1}{4}\sqrt{\frac{3}{10}}C^{-}_{9,10} & 0 & -\frac{-\frac{1}{4\sqrt{2}}C^{+}_{9,10}-\frac{1}{4}C^{-}_{9,10}}{\sqrt{10}} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle \overline{10} \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_1 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{15} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \overline{\mathbf{6}} \,\|\, \overline{3} \rangle \\
\langle 8_2 \,\|\, \mathbf{3} \,\|\, \overline{3} \rangle
\end{pmatrix}
$$

A.0.0.7 $SU(3)$-decomposition of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \mathbf{1}_{\mathcal{M}}$ for most general effective Hamiltonian

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \to n\eta_1) \\ \mathcal{A}(\Xi_b^0 \to \Lambda^0\eta_1) \\ \mathcal{A}(\Xi_b^0 \to \Sigma^0\eta_1) \\ \mathcal{A}(\Xi_b^- \to \Sigma^-\eta_1) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{\frac{3}{2}}}{2} & \frac{1}{2} & \frac{\sqrt{\frac{3}{2}}}{2} & 0 \\ -\frac{1}{4} & -\frac{\sqrt{\frac{3}{2}}}{2} & \frac{3}{4} & 0 \\ \frac{\sqrt{3}}{4} & -\frac{1}{2\sqrt{2}} & -\frac{1}{4\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{\sqrt{\frac{3}{2}}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \,\|\, \mathbf{3}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \overline{\mathbf{6}}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{3/2} \,\|\, \overline{3} \rangle \end{pmatrix}$$

A.0.0.8 $SU(3)$-decomposition of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{8}_{\mathcal{B}} \mathbf{1}_{\mathcal{M}}$ for most general effective Hamiltonian

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \to \Lambda^0\eta_1) \\ \mathcal{A}(\Lambda_b^0 \to \Sigma^0\eta_1) \\ \mathcal{A}(\Xi_b^- \to \Xi^-\eta_1) \\ \mathcal{A}(\Xi_b^0 \to \Xi^0\eta_1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{\frac{3}{2}}}{2} & \frac{1}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ \frac{\sqrt{\frac{3}{2}}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \,\|\, \mathbf{3}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \overline{\mathbf{6}}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{1} \,\|\, \overline{3} \rangle \end{pmatrix}$$

A.0.0.9 $SU(3)$-decomposition of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$\begin{pmatrix} \mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\eta_8) \\ \mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\pi_0) \\ \mathcal{A}(\Xi_b^- \to \Lambda_s^{0*}\pi^-) \\ \mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}K^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & -\frac{\sqrt{\frac{3}{2}}}{2} & \frac{3}{4} & 0 \\ \frac{\sqrt{3}}{4} & -\frac{1}{2\sqrt{2}} & -\frac{1}{4\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{\sqrt{\frac{3}{2}}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{\sqrt{\frac{3}{2}}}{2} & \frac{1}{2} & \frac{\sqrt{\frac{3}{2}}}{2} & 0 \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \,\|\, \mathbf{3}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \overline{\mathbf{6}}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{1/2} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{3/2} \,\|\, \overline{3} \rangle \end{pmatrix}$$

A.0.0.10 $SU(3)$-decomposition of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{1}_{\mathcal{B}} \mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}\pi^0) \\ \mathcal{A}(\Lambda_b^0 \to \Lambda_s^{0*}\eta_8) \\ \mathcal{A}(\Xi_b^- \to \Lambda_s^{0*}K^-) \\ \mathcal{A}(\Xi_b^0 \to \Lambda_s^{0*}\overline{K^0}) \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{\frac{3}{2}}}{2} & \frac{1}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ \frac{\sqrt{\frac{3}{2}}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \,\|\, \mathbf{3}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \overline{\mathbf{6}}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{0} \,\|\, \overline{3} \rangle \\ \langle \mathbf{8} \,\|\, \mathbf{15}_{1} \,\|\, \overline{3} \rangle \end{pmatrix}$$

Appendix B

According to the phase convention chosen for quark (q_i) and anti-quark ($\overline{q_i}$) flavor states in Eq. (4.18), the initial b-baryon anti triplet $B_{[ij]}$ consists of the following states;

$$\Lambda_b^0 = \frac{1}{\sqrt{2}}(ud - du)b, \quad \Xi_b^0 = \frac{1}{\sqrt{2}}(us - su)b, \quad \Xi_b^- = \frac{1}{\sqrt{2}}(ds - sd)b. \tag{B.1}$$

The decuplet baryons have a completely symmetric flavor wave-function:

$$\begin{gathered}\Delta^{++} = uuu, \quad \Delta^- = ddd, \quad \Delta^+ = \frac{1}{\sqrt{3}}(uud + udu + duu), \quad \Delta^0 = \frac{1}{\sqrt{3}}(udd + ddu + dud),\\ \Sigma'^+ = \frac{1}{\sqrt{3}}(uus + usu + suu), \quad \Sigma'^0 = \frac{1}{\sqrt{6}}(uds + usd + dus + dsu + sud + sdu),\\ \Sigma'^- = \frac{1}{\sqrt{3}}(dds + dsd + sdd), \quad \Xi'^0 = \frac{1}{\sqrt{6}}(uss + sus + ssu),\\ \Xi'^- = \frac{1}{\sqrt{3}}(dss + ssd + sds), \quad \Omega^- = sss.\end{gathered} \tag{B.2}$$

The $SU(3)$-decomposition of $\overline{\mathbf{3}} \to \mathbf{10} \otimes \mathbf{1}$ $\Delta S = 0$ processes:

$$\mathcal{A}(\Lambda_b^0 \to \Delta^0 \eta_1) = -\frac{C_{1,2}^+}{\sqrt{6}}(\lambda_u^d - \frac{3}{2}\kappa\lambda_t^d)\langle 10 \parallel \mathbf{15} \parallel \bar{3}\rangle, \qquad \mathcal{A}(\Xi_b^- \to \Sigma'^- \eta_1) = 0, \tag{B.3}$$

$\Delta S = -1$ processes:

$$\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \eta_1) = -\frac{C_{1,2}^+}{2\sqrt{3}}(\lambda_u^s - \frac{3}{2}\kappa\lambda_t^s)\langle 10 \parallel \mathbf{15} \parallel \bar{3}\rangle, \qquad \mathcal{A}(\Xi_b^- \to \Xi'^- \eta_1) = 0 \tag{B.4}$$

B.0.0.1 $SU(3)$-decomposition of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{D}}\,\mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0 \bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Omega^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- \bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \eta_8) \\
\mathcal{A}(\Xi_b^- \to \Omega^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
\sqrt{\tfrac{2}{15}} & \sqrt{\tfrac{2}{15}} & \tfrac{1}{2\sqrt{3}} & \tfrac{1}{2\sqrt{3}} & \tfrac{1}{6\sqrt{5}} & \tfrac{1}{6\sqrt{5}} & \tfrac{1}{2} & \tfrac{1}{6\sqrt{2}} & \tfrac{1}{2\sqrt{2}} & \tfrac{1}{2\sqrt{30}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & \tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\tfrac{2}{15}} & -\sqrt{\tfrac{2}{15}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{6\sqrt{5}} & -\tfrac{1}{6\sqrt{5}} & \tfrac{1}{2} & -\tfrac{1}{6\sqrt{2}} & \tfrac{1}{2\sqrt{2}} & -\tfrac{1}{2\sqrt{30}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\tfrac{1}{\sqrt{30}} & -\tfrac{1}{\sqrt{30}} & \tfrac{1}{2\sqrt{3}} & \tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{1}{6} & -\tfrac{1}{2\sqrt{2}} & -\tfrac{1}{6\sqrt{2}} & \tfrac{1}{2\sqrt{30}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & \tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{21}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{\sqrt{\frac{3}{5}}}{2} & -\tfrac{\sqrt{\frac{2}{5}}}{3} & -\tfrac{2}{3\sqrt{5}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{3} & 0 & -\tfrac{1}{3\sqrt{2}} & 0 & \sqrt{\tfrac{3}{10}} & 0 & \tfrac{1}{\sqrt{5}} & 0 & \tfrac{1}{2\sqrt{5}} & \tfrac{\sqrt{\frac{3}{5}}}{2} & \tfrac{\sqrt{\frac{2}{5}}}{3} & \tfrac{2}{3\sqrt{5}} & 0 & 0 \\
-\tfrac{1}{\sqrt{10}} & -\tfrac{1}{\sqrt{10}} & 0 & 0 & \tfrac{1}{\sqrt{15}} & \tfrac{1}{\sqrt{15}} & 0 & \tfrac{1}{\sqrt{6}} & 0 & \tfrac{1}{\sqrt{10}} & 0 & \tfrac{3}{\sqrt{35}} & 0 & \tfrac{1}{\sqrt{7}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{6} & \tfrac{1}{2\sqrt{2}} & -\tfrac{1}{6\sqrt{2}} & -\tfrac{1}{2\sqrt{30}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{21}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{\sqrt{\frac{3}{5}}}{2} & -\tfrac{\sqrt{\frac{2}{5}}}{3} & -\tfrac{2}{3\sqrt{5}} & 0 & 0 \\
\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{3\sqrt{5}} & -\tfrac{1}{3\sqrt{5}} & 0 & -\tfrac{1}{3\sqrt{2}} & 0 & \tfrac{1}{\sqrt{30}} & 0 & \sqrt{\tfrac{3}{35}} & 0 & \tfrac{1}{\sqrt{21}} & \tfrac{1}{2\sqrt{5}} & \tfrac{\sqrt{\frac{3}{5}}}{2} & -\tfrac{1}{\sqrt{10}} & -\tfrac{1}{\sqrt{5}} & 0 & 0 \\
\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{3\sqrt{5}} & -\tfrac{1}{3\sqrt{5}} & 0 & -\tfrac{1}{3\sqrt{2}} & 0 & \tfrac{1}{\sqrt{30}} & 0 & \sqrt{\tfrac{3}{35}} & 0 & \tfrac{1}{\sqrt{21}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{\sqrt{\frac{3}{5}}}{2} & \tfrac{1}{\sqrt{10}} & \tfrac{1}{\sqrt{5}} & 0 & 0 \\
-\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & \tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{6} & 0 & -\tfrac{1}{3\sqrt{2}} & \sqrt{\tfrac{2}{15}} & \tfrac{1}{\sqrt{30}} & -\tfrac{\sqrt{\frac{3}{35}}}{2} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{21}} & -\tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{15}} & -\tfrac{1}{3\sqrt{10}} & \tfrac{1}{6\sqrt{5}} & \tfrac{1}{\sqrt{6}} & \tfrac{1}{2\sqrt{3}} \\
\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{6}} & \tfrac{1}{2\sqrt{6}} & -\tfrac{7}{6\sqrt{10}} & \tfrac{1}{3\sqrt{10}} & \tfrac{1}{3\sqrt{2}} & \tfrac{1}{6} & -\tfrac{1}{3} & \tfrac{1}{2\sqrt{15}} & \tfrac{1}{\sqrt{15}} & \sqrt{\tfrac{3}{70}} & -\tfrac{1}{\sqrt{10}} & -\tfrac{5}{2\sqrt{42}} & \tfrac{1}{2\sqrt{10}} & -\tfrac{1}{2\sqrt{30}} & \tfrac{1}{6\sqrt{5}} & -\tfrac{1}{6\sqrt{10}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{6}} \\
\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{15}} & \tfrac{1}{4\sqrt{6}} & -\tfrac{1}{4\sqrt{6}} & \tfrac{11}{12\sqrt{10}} & -\tfrac{2\sqrt{\frac{2}{5}}}{3} & -\tfrac{1}{3\sqrt{2}} & \tfrac{1}{12} & \tfrac{1}{3} & \tfrac{\sqrt{\frac{5}{3}}}{4} & \tfrac{1}{\sqrt{15}} & 0 & -\tfrac{1}{\sqrt{10}} & -\tfrac{\sqrt{\frac{7}{6}}}{4} & \tfrac{1}{2\sqrt{10}} & -\tfrac{1}{2\sqrt{30}} & \tfrac{7}{12\sqrt{5}} & -\tfrac{7}{12\sqrt{10}} & \tfrac{1}{4\sqrt{3}} & \tfrac{1}{4\sqrt{6}} \\
\tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{1}{4\sqrt{2}} & \tfrac{1}{4\sqrt{2}} & \tfrac{1}{4\sqrt{30}} & \tfrac{1}{\sqrt{30}} & 0 & -\tfrac{1}{4\sqrt{3}} & 0 & \tfrac{3}{4\sqrt{5}} & 0 & -\tfrac{3}{\sqrt{70}} & 0 & \tfrac{3}{4\sqrt{14}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{1}{2\sqrt{10}} & -\tfrac{\sqrt{\frac{3}{5}}}{4} & \tfrac{\sqrt{\frac{3}{10}}}{4} & -\tfrac{1}{4} & \tfrac{3}{4\sqrt{2}} \\
-\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & -\tfrac{1}{4\sqrt{3}} & \tfrac{1}{4\sqrt{3}} & \tfrac{3}{4\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{6} & -\tfrac{1}{2\sqrt{2}} & \tfrac{1}{3\sqrt{2}} & \tfrac{1}{2\sqrt{30}} & \tfrac{1}{\sqrt{30}} & \tfrac{\sqrt{\frac{3}{35}}}{2} & -\tfrac{1}{2\sqrt{5}} & -\tfrac{5}{4\sqrt{21}} & -\tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{15}} & -\tfrac{7}{6\sqrt{10}} & \tfrac{7}{12\sqrt{5}} & -\tfrac{1}{2\sqrt{6}} & -\tfrac{1}{4\sqrt{3}} \\
-\tfrac{1}{\sqrt{10}} & \tfrac{1}{\sqrt{10}} & -\tfrac{1}{4} & \tfrac{1}{4} & -\tfrac{1}{4\sqrt{15}} & -\tfrac{1}{\sqrt{15}} & 0 & \tfrac{1}{2\sqrt{6}} & 0 & \tfrac{1}{2\sqrt{10}} & 0 & -\tfrac{1}{\sqrt{35}} & 0 & \tfrac{1}{4\sqrt{7}} & -\tfrac{\sqrt{\frac{3}{5}}}{2} & \tfrac{1}{2\sqrt{5}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{\sqrt{\frac{3}{5}}}{4} & -\tfrac{1}{2\sqrt{2}} & \tfrac{1}{4} \\
\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{6}} & \tfrac{1}{2\sqrt{6}} & -\tfrac{7}{6\sqrt{10}} & \tfrac{1}{3\sqrt{10}} & -\tfrac{1}{3\sqrt{2}} & \tfrac{1}{6} & \tfrac{1}{3} & \tfrac{1}{2\sqrt{15}} & -\tfrac{1}{\sqrt{15}} & \sqrt{\tfrac{3}{70}} & \tfrac{1}{\sqrt{10}} & -\tfrac{5}{2\sqrt{42}} & -\tfrac{1}{2\sqrt{10}} & \tfrac{1}{2\sqrt{30}} & -\tfrac{1}{6\sqrt{5}} & \tfrac{1}{6\sqrt{10}} & \tfrac{1}{2\sqrt{3}} & \tfrac{1}{2\sqrt{6}} \\
-\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & \tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{3}} & -\tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{6} & 0 & \tfrac{1}{3\sqrt{2}} & \sqrt{\tfrac{2}{15}} & -\tfrac{1}{\sqrt{30}} & -\tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{21}} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{15}} & \tfrac{1}{3\sqrt{10}} & -\tfrac{1}{6\sqrt{5}} & -\tfrac{1}{\sqrt{6}} & -\tfrac{1}{2\sqrt{3}} \\
-\tfrac{1}{\sqrt{30}} & \tfrac{1}{\sqrt{30}} & -\tfrac{1}{4\sqrt{3}} & \tfrac{1}{4\sqrt{3}} & \tfrac{3}{4\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{6} & -\tfrac{1}{2\sqrt{2}} & -\tfrac{1}{3\sqrt{2}} & \tfrac{1}{2\sqrt{30}} & -\tfrac{1}{\sqrt{30}} & \tfrac{\sqrt{\frac{3}{35}}}{2} & \tfrac{1}{2\sqrt{5}} & -\tfrac{5}{4\sqrt{21}} & \tfrac{1}{2\sqrt{5}} & -\tfrac{1}{2\sqrt{15}} & \tfrac{7}{6\sqrt{10}} & -\tfrac{7}{12\sqrt{5}} & \tfrac{1}{2\sqrt{6}} & \tfrac{1}{4\sqrt{3}} \\
\tfrac{1}{2\sqrt{15}} & -\tfrac{1}{2\sqrt{15}} & \tfrac{1}{4\sqrt{6}} & -\tfrac{1}{4\sqrt{6}} & \tfrac{11}{12\sqrt{10}} & -\tfrac{2\sqrt{\frac{2}{5}}}{3} & \tfrac{1}{3\sqrt{2}} & \tfrac{1}{12} & -\tfrac{1}{3} & \tfrac{\sqrt{\frac{5}{3}}}{4} & -\tfrac{1}{\sqrt{15}} & 0 & \tfrac{1}{\sqrt{10}} & -\tfrac{\sqrt{\frac{7}{6}}}{4} & -\tfrac{1}{2\sqrt{10}} & \tfrac{1}{2\sqrt{30}} & -\tfrac{7}{12\sqrt{5}} & \tfrac{7}{12\sqrt{10}} & -\tfrac{1}{4\sqrt{3}} & -\tfrac{1}{4\sqrt{6}} \\
-\tfrac{1}{2\sqrt{5}} & \tfrac{1}{2\sqrt{5}} & \tfrac{1}{4\sqrt{2}} & -\tfrac{1}{4\sqrt{2}} & -\tfrac{1}{4\sqrt{30}} & -\tfrac{1}{\sqrt{30}} & 0 & \tfrac{1}{4\sqrt{3}} & 0 & -\tfrac{3}{4\sqrt{5}} & 0 & \tfrac{3}{\sqrt{70}} & 0 & -\tfrac{3}{4\sqrt{14}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{1}{2\sqrt{10}} & -\tfrac{\sqrt{\frac{3}{5}}}{4} & \tfrac{\sqrt{\frac{3}{10}}}{4} & -\tfrac{1}{4} & \tfrac{3}{4\sqrt{2}} \\
\tfrac{1}{\sqrt{10}} & -\tfrac{1}{\sqrt{10}} & \tfrac{1}{4} & -\tfrac{1}{4} & \tfrac{1}{4\sqrt{15}} & \tfrac{1}{\sqrt{15}} & 0 & -\tfrac{1}{2\sqrt{6}} & 0 & -\tfrac{1}{2\sqrt{10}} & 0 & \tfrac{1}{\sqrt{35}} & 0 & -\tfrac{1}{4\sqrt{7}} & -\tfrac{\sqrt{\frac{3}{5}}}{2} & \tfrac{1}{2\sqrt{5}} & \tfrac{\sqrt{\frac{3}{10}}}{2} & -\tfrac{\sqrt{\frac{3}{5}}}{4} & -\tfrac{1}{2\sqrt{2}} & \tfrac{1}{4}
\end{pmatrix}
\begin{pmatrix}
\langle 8 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \\
\langle 8 \,\|\, \mathbf{6_1} \,\|\, \bar{3} \\
\langle 10 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \\
\langle 10 \,\|\, \mathbf{15'_1} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{15_1} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{24_1} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{24_2} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{42_1} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{42_2} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{42_1} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{42_2} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{48_1} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{48_2} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{15'_1} \,\|\, \bar{3} \\
\langle 8 \,\|\, \mathbf{3_0} \,\|\, \bar{3} \\
\langle 8 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \\
\langle 27 \,\|\, \mathbf{42_0} \,\|\, \bar{3} \\
\langle 10 \,\|\, \mathbf{15_0} \,\|\, \bar{3} \\
\langle 35 \,\|\, \mathbf{42_0} \,\|\, \bar{3}
\end{pmatrix}
$$

B.0.0.2 $SU(3)$-decomposition of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{D}}\,\mathbf{8}_{\mathcal{M}}$ for most general effective Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^-\pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0\pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Delta'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^-\pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
-\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{30}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{5}{3}}}{6} & -\frac{1}{3\sqrt{30}} & -\frac{\sqrt{\frac{2}{3}}}{3} & -\frac{\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{5}{3}}}{6} & \frac{1}{10} & \frac{\sqrt{6}}{5} & \frac{\sqrt{\frac{2}{7}}}{5} & \frac{\sqrt{\frac{3}{2}}}{5} & \frac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 \\
\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{15}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & \frac{2}{3\sqrt{15}} & \frac{\sqrt{\frac{5}{6}}}{12} & \frac{1}{3\sqrt{15}} & \frac{1}{6\sqrt{3}} & \frac{2}{3\sqrt{15}} & \frac{\sqrt{\frac{5}{6}}}{12} & -\frac{1}{20\sqrt{2}} & \frac{2\sqrt{3}}{5} & -\frac{1}{10\sqrt{7}} & \frac{\sqrt{3}}{5} & -\frac{1}{4\sqrt{42}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{\frac{3}{2}}}{4} & \frac{1}{4\sqrt{2}} & 0 & \frac{\sqrt{\frac{5}{2}}}{4} & 0 & \frac{1}{2} & 0 & \frac{\sqrt{\frac{5}{2}}}{4} & \frac{\sqrt{\frac{3}{2}}}{4} & 0 & \frac{\sqrt{\frac{3}{7}}}{2} & 0 & \frac{5}{4\sqrt{14}} & 0 & 0 & 0 & 0 \\
-\frac{\sqrt{\frac{3}{5}}}{2} & -\frac{\sqrt{\frac{3}{5}}}{2} & -\frac{1}{\sqrt{10}} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & -\frac{\sqrt{\frac{2}{5}}}{3} & \frac{\sqrt{5}}{12} & -\frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{2}} & -\frac{\sqrt{\frac{2}{5}}}{3} & \frac{\sqrt{5}}{12} & -\frac{\sqrt{3}}{20} & \frac{\sqrt{2}}{5} & -\frac{\sqrt{\frac{3}{14}}}{5} & \frac{1}{5\sqrt{2}} & -\frac{1}{4\sqrt{7}} & 0 & 0 & 0 & 0 \\
\frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{4}{3\sqrt{15}} & -\frac{\sqrt{\frac{5}{6}}}{6} & -\frac{2}{3\sqrt{15}} & -\frac{1}{3\sqrt{3}} & -\frac{4}{3\sqrt{15}} & -\frac{\sqrt{\frac{5}{6}}}{6} & \frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{7}} & 0 & \frac{5}{2\sqrt{42}} & 0 & 0 & 0 & 0 \\
-\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{30}} & -\frac{1}{4} & -\frac{1}{4\sqrt{3}} & \frac{4\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{5}{3}}}{12} & \frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{6}} & \frac{4\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{5}{3}}}{12} & \frac{1}{4} & 0 & \frac{1}{\sqrt{14}} & 0 & \frac{5}{4\sqrt{21}} & 0 & 0 & 0 & 0 \\
-\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{7}{6\sqrt{15}} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{\frac{2}{3}}}{3} & \frac{1}{6\sqrt{30}} & -\frac{1}{6\sqrt{15}} & \frac{7}{30} & \frac{\sqrt{\frac{3}{2}}}{10} & -\frac{\sqrt{\frac{2}{7}}}{5} & -\frac{\sqrt{\frac{3}{2}}}{5} & -\frac{1}{2\sqrt{21}} & -\frac{2\sqrt{\frac{2}{5}}}{3} & \frac{1}{3\sqrt{2}} & -\frac{3}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\
\frac{1}{2\sqrt{5}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{30}} & -\frac{4}{3\sqrt{15}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{6}} & -\frac{1}{6\sqrt{30}} & \frac{2}{3\sqrt{15}} & \frac{1}{15} & \frac{\sqrt{\frac{3}{2}}}{10} & \frac{3}{5\sqrt{14}} & -\frac{\sqrt{\frac{3}{2}}}{5} & -\frac{1}{\sqrt{21}} & \frac{2\sqrt{\frac{2}{5}}}{3} & -\frac{1}{3\sqrt{2}} & -\frac{3}{2\sqrt{10}} & -\frac{1}{6\sqrt{2}} \\
0 & \frac{2}{3\sqrt{5}} & -\sqrt{\frac{2}{15}} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{30}} & \frac{1}{6\sqrt{15}} & -\frac{7}{3\sqrt{30}} & -\frac{\sqrt{\frac{2}{3}}}{3} & \frac{\sqrt{\frac{5}{6}}}{6} & \frac{7}{6\sqrt{15}} & \frac{7}{30} & \frac{\sqrt{\frac{3}{2}}}{10} & -\frac{\sqrt{\frac{2}{7}}}{5} & -\frac{\sqrt{\frac{3}{2}}}{5} & -\frac{1}{2\sqrt{21}} & \frac{\sqrt{\frac{2}{5}}}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\
\frac{1}{4\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{\frac{3}{10}}}{2} & 0 & 0 & \frac{\sqrt{\frac{2}{15}}}{3} & \frac{\sqrt{\frac{5}{3}}}{6} & \frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{6}} & -\frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{1}{6\sqrt{15}} & \frac{3}{10} & \frac{\sqrt{\frac{3}{2}}}{5} & \frac{1}{5\sqrt{14}} & -\frac{\sqrt{6}}{5} & -\frac{\sqrt{\frac{3}{7}}}{2} & 0 & 0 & \frac{1}{\sqrt{10}} & 0 \\
\frac{\sqrt{\frac{3}{5}}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & -\frac{\sqrt{\frac{2}{5}}}{3} & \frac{1}{6\sqrt{5}} & \frac{\sqrt{\frac{2}{5}}}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{10}} & -\frac{\sqrt{5}}{6} & \frac{1}{2\sqrt{3}} & 0 & -\sqrt{\frac{3}{14}} & 0 & \frac{1}{2\sqrt{7}} & \sqrt{\frac{2}{15}} & 0 & 0 & \frac{1}{\sqrt{6}} \\
-\frac{1}{2\sqrt{5}} & \frac{\sqrt{5}}{6} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & -\frac{\sqrt{\frac{5}{6}}}{3} & \frac{2}{3\sqrt{15}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{6}} & \frac{11}{6\sqrt{30}} & -\frac{4}{3\sqrt{15}} & \frac{1}{15} & \frac{\sqrt{\frac{3}{2}}}{10} & \frac{3}{5\sqrt{14}} & -\frac{\sqrt{\frac{3}{2}}}{5} & -\frac{1}{\sqrt{21}} & -\frac{\sqrt{\frac{2}{5}}}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{10}} & -\frac{1}{6\sqrt{2}} \\
0 & -\frac{2}{3\sqrt{5}} & \sqrt{\frac{2}{15}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{1}{6\sqrt{15}} & -\frac{4\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{\frac{5}{6}}}{3} & \frac{\sqrt{\frac{5}{3}}}{6} & \frac{1}{6} & 0 & -\frac{1}{\sqrt{14}} & 0 & \frac{1}{2\sqrt{21}} & -\frac{\sqrt{\frac{2}{5}}}{3} & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{3\sqrt{2}} \\
-\frac{1}{2\sqrt{5}} & \frac{\sqrt{5}}{6} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & \frac{\sqrt{\frac{5}{6}}}{3} & -\frac{1}{6\sqrt{15}} & \frac{2\sqrt{\frac{2}{15}}}{3} & -\frac{1}{3\sqrt{6}} & -\frac{7}{3\sqrt{30}} & \frac{\sqrt{\frac{5}{3}}}{6} & \frac{1}{6} & 0 & -\frac{1}{\sqrt{14}} & 0 & \frac{1}{2\sqrt{21}} & -\frac{\sqrt{\frac{2}{5}}}{3} & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{3\sqrt{2}} \\
-\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & -\frac{1}{12} & \frac{1}{4\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{23}{12\sqrt{15}} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{\frac{2}{3}}}{3} & \frac{1}{6\sqrt{30}} & \frac{7}{12\sqrt{15}} & \frac{11}{60} & -\frac{\sqrt{\frac{3}{2}}}{10} & \frac{\sqrt{\frac{2}{7}}}{5} & \frac{\sqrt{\frac{3}{2}}}{5} & -\frac{5}{4\sqrt{21}} & \frac{\sqrt{\frac{2}{5}}}{3} & \frac{1}{3\sqrt{2}} & \frac{3}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\
\frac{\sqrt{\frac{3}{5}}}{2} & -\frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{10}} & \frac{1}{4\sqrt{3}} & -\frac{1}{4} & \frac{1}{3\sqrt{10}} & -\frac{7}{12\sqrt{5}} & -\frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{12\sqrt{5}} & \frac{7}{20\sqrt{3}} & -\frac{1}{10\sqrt{2}} & -\frac{\sqrt{\frac{3}{14}}}{5} & \frac{1}{5\sqrt{2}} & -\frac{1}{4\sqrt{7}} & -\sqrt{\frac{2}{15}} & -\frac{1}{\sqrt{6}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{6}} \\
\frac{1}{2\sqrt{10}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{\sqrt{15}} & 0 & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{\frac{3}{10}}}{2} & \frac{11}{30\sqrt{2}} & -\frac{\sqrt{3}}{10} & \frac{2}{5\sqrt{7}} & \frac{\sqrt{3}}{5} & -\frac{5}{2\sqrt{42}} & -\frac{1}{3\sqrt{5}} & \frac{1}{3} & -\frac{1}{2\sqrt{5}} & \frac{1}{6} \\
-\frac{1}{2\sqrt{10}} & \frac{1}{6\sqrt{10}} & \frac{1}{2\sqrt{15}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{30}} & \frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{30}} & \frac{4\sqrt{2}}{15} & -\frac{\sqrt{3}}{10} & -\frac{1}{10\sqrt{7}} & \frac{\sqrt{3}}{5} & -\sqrt{\frac{2}{21}} & \frac{1}{3\sqrt{5}} & -\frac{1}{3} & -\frac{1}{2\sqrt{5}} & -\frac{1}{6} \\
\frac{\sqrt{\frac{3}{10}}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt{5}} & 0 & 0 & -\frac{2}{3\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{2}{3\sqrt{5}} & -\frac{1}{6} & \frac{1}{3\sqrt{5}} & \frac{\sqrt{\frac{5}{2}}}{6} & -\frac{1}{2\sqrt{6}} & 0 & \frac{\sqrt{\frac{3}{7}}}{2} & 0 & -\frac{1}{2\sqrt{14}} & -\frac{1}{\sqrt{15}} & 0 & 0 & \frac{1}{\sqrt{3}} \\
-\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & \frac{2\sqrt{\frac{2}{15}}}{3} & \frac{1}{6\sqrt{15}} & -\frac{2\sqrt{\frac{2}{15}}}{3} & \frac{1}{3\sqrt{6}} & -\frac{\sqrt{\frac{2}{15}}}{3} & -\frac{\sqrt{\frac{5}{3}}}{6} & -\frac{1}{6} & 0 & \frac{1}{\sqrt{14}} & 0 & -\frac{1}{2\sqrt{21}} & \frac{\sqrt{\frac{2}{5}}}{3} & -\frac{\sqrt{2}}{3} & 0 & \frac{\sqrt{2}}{3}
\end{pmatrix}
\begin{pmatrix}
\langle 8 \,\|\, \mathbf{3}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 8 \,\|\, \mathbf{15}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 8 \,\|\, \overline{\mathbf{6}}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 10 \,\|\, \mathbf{15}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 10 \,\|\, \mathbf{15}'_{3/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{15}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{15}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{24}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{24}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{42}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{42}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{42}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{42}_{5/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{48}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{48}_{5/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{15}'_{3/2} \,\|\, \bar{3}\rangle \\
\langle 8 \,\|\, \mathbf{15}_{3/2} \,\|\, \bar{3}\rangle \\
\langle 10 \,\|\, \mathbf{15}_{1/2} \,\|\, \bar{3}\rangle \\
\langle 27 \,\|\, \mathbf{42}_{5/2} \,\|\, \bar{3}\rangle \\
\langle 35 \,\|\, \mathbf{42}_{1/2} \,\|\, \bar{3}\rangle
\end{pmatrix}
$$

B.0.0.3 $SU(3)$-decomposition of tree part of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{B}}\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+\pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0\pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0\eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^-\pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+\pi^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0\pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0\eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^-\pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Delta'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0\pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^-\pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^-\eta_8) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
-\frac{C^+_{1,2}}{3\sqrt{2}} & \frac{C^+_{1,2}}{5} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{12} & -\frac{3C^+_{1,2}}{20\sqrt{2}} & -\frac{C^+_{1,2}}{15\sqrt{2}} & -\frac{C^-_{1,2}}{3\sqrt{5}} & \frac{C^-_{1,2}}{\sqrt{15}} - \frac{C^+_{1,2}}{\sqrt{30}} \\
-\frac{C^+_{1,2}}{4\sqrt{3}} & -\frac{1}{4}\sqrt{\frac{3}{2}}C^+_{1,2} & 0 & 0 & 0 \\
\frac{C^+_{1,2}}{2\sqrt{6}} & -\frac{\sqrt{3}C^+_{1,2}}{20} & \frac{C^+_{1,2}}{10\sqrt{3}} & \frac{C^-_{1,2}}{\sqrt{30}} & \frac{C^+_{1,2}}{2\sqrt{5}} - \frac{C^-_{1,2}}{\sqrt{10}} \\
\frac{C^+_{1,2}}{6} & \frac{3C^+_{1,2}}{10\sqrt{2}} & -\frac{C^+_{1,2}}{30\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^-_{1,2}}{2\sqrt{15}} - \frac{C^+_{1,2}}{2\sqrt{30}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
-\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{4} & \frac{C^+_{1,2}}{6} & -\frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{4} & -\frac{C^+_{1,2}}{6} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^-_{1,2}}{\sqrt{30}} - \frac{C^+_{1,2}}{2\sqrt{15}} \\
-\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & -\frac{2C^+_{1,2}}{15} & \frac{1}{3}\sqrt{\frac{2}{5}}C^-_{1,2} & 0 \\
0 & -\frac{C^+_{1,2}}{5} & \frac{C^+_{1,2}}{20} & -\frac{C^-_{1,2}}{2\sqrt{10}} & \frac{C^-_{1,2}}{2\sqrt{30}} - \frac{C^+_{1,2}}{4\sqrt{15}} \\
0 & 0 & -\frac{C^+_{1,2}}{4\sqrt{3}} & \frac{C^-_{1,2}}{2\sqrt{30}} & \frac{C^-_{1,2}}{2\sqrt{10}} - \frac{C^+_{1,2}}{4\sqrt{5}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{20} & \frac{2C^+_{1,2}}{15} & -\frac{1}{3}\sqrt{\frac{2}{5}}C^-_{1,2} & 0 \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
0 & \frac{2C^+_{1,2}}{5} & -\frac{C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
0 & \frac{\sqrt{3}C^+_{1,2}}{10} & \frac{\sqrt{3}C^+_{1,2}}{10} & \frac{C^-_{1,2}}{\sqrt{30}} & \frac{C^-_{1,2}}{\sqrt{10}} - \frac{C^+_{1,2}}{2\sqrt{5}} \\
0 & -\frac{\sqrt{2}C^+_{1,2}}{5} & \frac{C^+_{1,2}}{10\sqrt{2}} & \frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^-_{1,2}}{2\sqrt{15}} - \frac{C^+_{1,2}}{2\sqrt{30}} \\
0 & -\frac{C^+_{1,2}}{10\sqrt{2}} & -\frac{C^+_{1,2}}{10\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^+_{1,2}}{2\sqrt{30}} - \frac{C^-_{1,2}}{2\sqrt{15}} \\
0 & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{2\sqrt{15}} & \frac{C^-_{1,2}}{2\sqrt{5}} - \frac{C^+_{1,2}}{2\sqrt{10}} \\
0 & -\frac{C^+_{1,2}}{10} & -\frac{C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}}
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \overline{\mathbf{6}} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \mathbf{3} \,\|\, \bar{3} \rangle
\end{pmatrix}
$$

B.0.0.4 $SU(3)$-decomposition of tree part of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{D}}\,\mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0 \bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Omega^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- \bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \eta_8) \\
\mathcal{A}(\Xi_b^- \to \Omega^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
-\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & -\frac{2C^+_{1,2}}{15} & \frac{1}{3}\sqrt{\frac{2}{5}}C^-_{1,2} & 0 \\
\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{20} & \frac{2C^+_{1,2}}{15} & -\frac{1}{3}\sqrt{\frac{2}{5}}C^-_{1,2} & 0 \\
-\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{4} & \frac{C^+_{1,2}}{6} & -\frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
0 & -\frac{C^+_{1,2}}{10} & -\frac{C^+_{1,2}}{10} & 0 & \frac{C^-_{1,2}}{\sqrt{30}} - \frac{C^+_{1,2}}{2\sqrt{15}} \\
0 & -\frac{\sqrt{3}C^+_{1,2}}{10} & \frac{C^+_{1,2}}{5\sqrt{3}} & -\frac{C^-_{1,2}}{\sqrt{30}} & 0 \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & \frac{C^+_{1,2}}{4} & -\frac{C^+_{1,2}}{6} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^-_{1,2}}{\sqrt{30}} - \frac{C^+_{1,2}}{2\sqrt{15}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
-\frac{C^+_{1,2}}{3\sqrt{2}} & \frac{C^+_{1,2}}{5} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{6} & \frac{3C^+_{1,2}}{10\sqrt{2}} & -\frac{C^+_{1,2}}{30\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^-_{1,2}}{2\sqrt{15}} - \frac{C^+_{1,2}}{2\sqrt{30}} \\
-\frac{C^+_{1,2}}{12} & -\frac{9C^+_{1,2}}{20\sqrt{2}} & -\frac{C^+_{1,2}}{30\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^-_{1,2}}{2\sqrt{15}} - \frac{C^+_{1,2}}{2\sqrt{30}} \\
\frac{C^+_{1,2}}{4\sqrt{3}} & \frac{1}{20}\sqrt{\frac{3}{2}}C^+_{1,2} & -\frac{C^+_{1,2}}{10\sqrt{6}} & -\frac{C^-_{1,2}}{2\sqrt{15}} & \frac{C^-_{1,2}}{2\sqrt{5}} - \frac{C^+_{1,2}}{2\sqrt{10}} \\
\frac{C^+_{1,2}}{6\sqrt{2}} & -\frac{C^+_{1,2}}{20} & \frac{C^+_{1,2}}{30} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^+_{1,2}}{2\sqrt{15}} - \frac{C^-_{1,2}}{\sqrt{30}} \\
\frac{C^+_{1,2}}{2\sqrt{6}} & -\frac{\sqrt{3}C^+_{1,2}}{20} & \frac{C^+_{1,2}}{10\sqrt{3}} & \frac{C^-_{1,2}}{\sqrt{30}} & \frac{C^+_{1,2}}{2\sqrt{5}} - \frac{C^-_{1,2}}{\sqrt{10}} \\
0 & \frac{\sqrt{2}C^+_{1,2}}{5} & -\frac{C^+_{1,2}}{10\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^+_{1,2}}{2\sqrt{30}} - \frac{C^-_{1,2}}{2\sqrt{15}} \\
0 & \frac{C^+_{1,2}}{10} & \frac{C^+_{1,2}}{10} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^-_{1,2}}{\sqrt{30}} - \frac{C^+_{1,2}}{2\sqrt{15}} \\
0 & -\frac{2C^+_{1,2}}{5} & \frac{C^+_{1,2}}{10} & \frac{C^-_{1,2}}{3\sqrt{10}} & \frac{C^-_{1,2}}{\sqrt{30}} - \frac{C^+_{1,2}}{2\sqrt{15}} \\
0 & -\frac{C^+_{1,2}}{10\sqrt{2}} & -\frac{C^+_{1,2}}{10\sqrt{2}} & -\frac{C^-_{1,2}}{6\sqrt{5}} & \frac{C^+_{1,2}}{2\sqrt{30}} - \frac{C^-_{1,2}}{2\sqrt{15}} \\
0 & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{1}{10}\sqrt{\frac{3}{2}}C^+_{1,2} & \frac{C^-_{1,2}}{2\sqrt{15}} & \frac{C^-_{1,2}}{2\sqrt{5}} - \frac{C^+_{1,2}}{2\sqrt{10}} \\
0 & -\frac{\sqrt{3}C^+_{1,2}}{10} & -\frac{\sqrt{3}C^+_{1,2}}{10} & -\frac{C^-_{1,2}}{\sqrt{30}} & \frac{C^+_{1,2}}{2\sqrt{5}} - \frac{C^-_{1,2}}{\sqrt{10}}
\end{pmatrix}
\begin{pmatrix}
\langle 10 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 27 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \mathbf{15} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \overline{\mathbf{6}} \,\|\, \bar{3} \rangle \\
\langle 8 \,\|\, \mathbf{3} \,\|\, \bar{3} \rangle
\end{pmatrix}
$$

B.0.0.5 $SU(3)$-decomposition of penguin part of $\Delta S = 0$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{D}} \mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+ \pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^- \pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Delta^0 \overline{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+ \pi^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^- \pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Delta'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Delta'^- \overline{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0 \pi^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- \pi^0) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- \eta_8) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
\frac{C^+_{9,10}}{2\sqrt{2}} & -\frac{3C^+_{9,10}}{10} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{8} & \frac{9C^+_{9,10}}{40\sqrt{2}} & \frac{C^+_{9,10}}{10\sqrt{2}} & \frac{C^-_{9,10}}{2\sqrt{5}} & \frac{2D}{\sqrt{15}} \\
\frac{\sqrt{3}C^+_{9,10}}{8} & \frac{3}{8}\sqrt{\frac{3}{2}}C^+_{9,10} & 0 & 0 & 0 \\
-\frac{1}{4}\sqrt{\frac{3}{2}}C^+_{9,10} & \frac{3\sqrt{3}C^+_{9,10}}{40} & -\frac{\sqrt{3}C^+_{9,10}}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C^-_{9,10} & -\sqrt{\frac{2}{5}}D \\
-\frac{C^+_{9,10}}{4} & -\frac{9C^+_{9,10}}{20\sqrt{2}} & \frac{C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{8} & -\frac{C^+_{9,10}}{4} & \frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{8} & \frac{C^+_{9,10}}{4} & -\frac{C^-_{9,10}}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\
\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & \frac{C^+_{9,10}}{5} & -\frac{C^-_{9,10}}{\sqrt{10}} & 0 \\
0 & \frac{3C^+_{9,10}}{10} & -\frac{3C^+_{9,10}}{40} & \frac{3C^-_{9,10}}{4\sqrt{10}} & \frac{D}{\sqrt{30}} \\
0 & 0 & \frac{\sqrt{3}C^+_{9,10}}{8} & -\frac{1}{4}\sqrt{\frac{3}{10}}C^-_{9,10} & \frac{D}{\sqrt{10}} \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{5} & \frac{C^-_{9,10}}{\sqrt{10}} & 0 \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
0 & -\frac{3C^+_{9,10}}{5} & \frac{3C^+_{9,10}}{20} & \frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
0 & -\frac{3\sqrt{3}C^+_{9,10}}{20} & -\frac{3\sqrt{3}C^+_{9,10}}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C^-_{9,10} & \sqrt{\frac{2}{5}}D \\
0 & \frac{3C^+_{9,10}}{5\sqrt{2}} & -\frac{3C^+_{9,10}}{20\sqrt{2}} & -\frac{C^-_{9,10}}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\
0 & \frac{3C^+_{9,10}}{20\sqrt{2}} & \frac{3C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\
0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C^+_{9,10} & -\frac{3}{20}\sqrt{\frac{3}{2}}C^+_{9,10} & -\frac{1}{4}\sqrt{\frac{3}{5}}C^-_{9,10} & \frac{D}{\sqrt{5}} \\
0 & \frac{3C^+_{9,10}}{20} & \frac{3C^+_{9,10}}{20} & \frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D
\end{pmatrix}
\begin{pmatrix}
\langle 10 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 27 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 8 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 8 \| \overline{\mathbf{6}} \| \bar{3} \rangle \\
\langle 8 \| \mathbf{3} \| \bar{3} \rangle
\end{pmatrix}
$$

B.0.0.6 $SU(3)$-decomposition of penguin part of $\Delta S = -1$ decay amplitudes for $\overline{\mathbf{3}}_{\mathcal{B}_b} \to \mathbf{10}_{\mathcal{B}} \mathbf{8}_{\mathcal{M}}$ for dim-6 unbroken Hamiltonian

$$
\begin{pmatrix}
\mathcal{A}(\Lambda_b^0 \to \Delta^+ K^-) \\
\mathcal{A}(\Lambda_b^0 \to \Delta^0 \bar{K}^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^+ \pi^-) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \pi^0) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^0 \eta_8) \\
\mathcal{A}(\Lambda_b^0 \to \Sigma'^- \pi^+) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^0 K^0) \\
\mathcal{A}(\Lambda_b^0 \to \Xi'^- K^+) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^+ K^-) \\
\mathcal{A}(\Xi_b^0 \to \Sigma'^0 \bar{K}^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \pi^0) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^0 \eta_8) \\
\mathcal{A}(\Xi_b^0 \to \Xi'^- \pi^+) \\
\mathcal{A}(\Xi_b^0 \to \Omega^- K^+) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^0 K^-) \\
\mathcal{A}(\Xi_b^- \to \Sigma'^- \bar{K}^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^0 \pi^-) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \pi^0) \\
\mathcal{A}(\Xi_b^- \to \Xi'^- \eta_8) \\
\mathcal{A}(\Xi_b^- \to \Omega^- K^0)
\end{pmatrix}
=
\begin{pmatrix}
\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & \frac{C^+_{9,10}}{5} & -\frac{C^-_{9,10}}{\sqrt{10}} & 0 \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{5} & \frac{C^-_{9,10}}{\sqrt{10}} & 0 \\
\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{8} & -\frac{C^+_{9,10}}{4} & \frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
0 & \frac{3C^+_{9,10}}{20} & \frac{3C^+_{9,10}}{20} & 0 & \sqrt{\frac{2}{15}}D \\
0 & \frac{3\sqrt{3}C^+_{9,10}}{20} & -\frac{\sqrt{3}C^+_{9,10}}{10} & \frac{1}{2}\sqrt{\frac{3}{10}}C^-_{9,10} & 0 \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & -\frac{3C^+_{9,10}}{8} & \frac{C^+_{9,10}}{4} & -\frac{C^-_{9,10}}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
\frac{C^+_{9,10}}{2\sqrt{2}} & -\frac{3C^+_{9,10}}{10} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{C^+_{9,10}}{4} & -\frac{9C^+_{9,10}}{20\sqrt{2}} & \frac{C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\
\frac{C^+_{9,10}}{8} & \frac{27C^+_{9,10}}{40\sqrt{2}} & \frac{C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\
-\frac{\sqrt{3}C^+_{9,10}}{8} & -\frac{3}{40}\sqrt{\frac{3}{2}}C^+_{9,10} & \frac{1}{20}\sqrt{\frac{3}{2}}C^+_{9,10} & \frac{1}{4}\sqrt{\frac{3}{5}}C^-_{9,10} & \frac{D}{\sqrt{5}} \\
-\frac{C^+_{9,10}}{4\sqrt{2}} & \frac{3C^+_{9,10}}{40} & -\frac{C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\
-\frac{1}{4}\sqrt{\frac{3}{2}}C^+_{9,10} & \frac{3\sqrt{3}C^+_{9,10}}{40} & -\frac{\sqrt{3}C^+_{9,10}}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C^-_{9,10} & -\sqrt{\frac{2}{5}}D \\
0 & -\frac{3C^+_{9,10}}{5\sqrt{2}} & \frac{3C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\
0 & -\frac{3C^+_{9,10}}{20} & -\frac{3C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\
0 & \frac{3C^+_{9,10}}{5} & -\frac{3C^+_{9,10}}{20} & -\frac{C^-_{9,10}}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\
0 & \frac{3C^+_{9,10}}{20\sqrt{2}} & \frac{3C^+_{9,10}}{20\sqrt{2}} & \frac{C^-_{9,10}}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\
0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C^+_{9,10} & -\frac{3}{20}\sqrt{\frac{3}{2}}C^+_{9,10} & -\frac{1}{4}\sqrt{\frac{3}{5}}C^-_{9,10} & \frac{D}{\sqrt{5}} \\
0 & \frac{3\sqrt{3}C^+_{9,10}}{20} & \frac{3\sqrt{3}C^+_{9,10}}{20} & \frac{1}{2}\sqrt{\frac{3}{10}}C^-_{9,10} & -\sqrt{\frac{2}{5}}D
\end{pmatrix}
\begin{pmatrix}
\langle 10 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 27 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 8 \| \mathbf{15} \| \bar{3} \rangle \\
\langle 8 \| \overline{\mathbf{6}} \| \bar{3} \rangle \\
\langle 8 \| \mathbf{3} \| \bar{3} \rangle
\end{pmatrix}
$$

Here we have used the following shorthand notation to express the tree ($\mathcal{T}$) and penguin ($\mathcal{P}$) matrices in a convenient form:

$$(C_{10} \pm C_9) = C^{\pm}_{9,10}, \qquad (C_1 \pm C_2) = C^{\pm}_{1,2},$$

$$(C_3 \pm C_4) = C^{\pm}_{3,4}, \qquad (C_5 \pm C_6) = C^{\pm}_{5,6},$$

and D is given by the particular combination of Wilson coefficients:

$$D = -\frac{1}{4\sqrt{2}}\left(C^{+}_{9,10} + \sqrt{2}C^{-}_{9,10}\right) + \left\{\sqrt{2}C^{+}_{3,4} - C^{-}_{3,4} + \sqrt{2}C^{+}_{5,6} - C^{-}_{5,6}\right\}. \tag{B.5}$$

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